**Exercise 10**

**Theory**

**Sampling continuous signals**

Sampling: process for converting a continuous signal into a discrete signal. The discrete form, , of a continuous signal, , with frequencies which are not higher than Hz, can be determined if its ordinates are taken at a sampling frequency () which is greater than i.e. the following mathematical relationship should hold: .

The continuous form of a sinusoidal signal with amplitude 2, frequency 5 Hz and phase rad would be expressed as: and its discrete version would be expressed as:

where n is a vector consisting of linearly spaced samples with spacing equal to: .

Thus, the signal starting from point 0 and finishing at point 1with can be written in Matlab as:

fs1 = 100;

t1 = 0:1/fs1:1;

f1=5;

x1 = 2\*sin(2\*pi\*f1\*t1);

stem(t1,x1)

title('Discrete-time signal');

**Fourier Transform, Discrete Fourier Transform, Fast Fourier Transform**

Fourier Transform: The Fourier Transform (FT) is a mathematical function which decomposes a time-domain signal into the frequencies from which this signal consists of. The FT of a signal , is a complex-valued function of frequency, . The absolute value of provides the magnitude of the frequency / frequencies which are present in and the argument of provides the phase content of . Please note that for this exercise, we will focus only on the magnitude content of the signal(s).

The FT of a signal, , is defined as: where

Discrete Fourier Transform: The Discrete Fourier Transform (DFT) is used when the signal is in discrete form in other words, when the signal is known at certain instants which are separated by sample times. The DFT is defined as:

Fast Fourier Transform: The Fast Fourier Transform (FFT) is an algorithmic implementation of the DFT.

**Display of a signal in the Time & Frequency domains and identification of its frequency content**

**Introduction:**

In Sections A and B the Steps are provided in order to display a Signal in the Time and in the Frequency domains.

A. A step-by-step guide in order to plot a signal in the time domain

1. *audioread* function uploads the signal and provides the sampling rate (number of samples per second).

2. Find the sampling period i.e. the inverse of sampling rate, . Sampling period, , is the time which elapses between two samples of the signal.

3. Find the total time length (tmax) of the signal by multiplying the sampling period by the total number of samples of the signal minus 1; consider that time should start from point 0.

4. Create the time axis which starts from 0 finishes at tmax in steps of .

5. Having developed the time axis, you can now plot the signal in the time domain.

B. A step-by-step guide in order to plot a signal in the frequency domain

1. *audioread* function uploads the signal and provides the sampling rate, fs.

2. Create the frequency axis which starts from and goes up to . The sampling frequency of a signal divided by 2, , corresponds to the maximum frequency value which can be represented unambiguously (Nyquist frequency) in the frequency domain. All the frequencies which are greater than will be aliased to a frequency from 0 to ; aliasing occurs when the frequency / frequencies of a signal cannot be distinguished due to insufficient sampling.

Note: or i.e. should be greater compared to the highest frequency of the signal () which is represented in the frequency domain. All the frequencies of this signal which are greater than will be aliased to a frequency from 0 to .

* In order to plot the signal in the frequency domain we develop the frequency axis ranging from to with a step of where *N* is the length of the time domain signal or in other words we develop a vector ranging from to and then we divide it into *N* equidistant spaces. Consequently, the x-axis (frequency axis) of the frequency domain signal consists of the same number of samples as the x-axis (time axis) of the time domain signal.

2. Evaluate the DFT of the signal using the FFT algorithm.

Given that the signal is synthetic, its amplitude can be retained in the frequency domain by multiplying the output of the FFT by the factor: .

Note 1: Due to the symmetry of the Fast Fourier Transform (FFT) algorithm, the frequencies appearing from 0 to will also appear in the frequency range from to 0 (mirror image).

Note 2: Utilizing the fftshift function, the zero-frequency component of the signal is shifted to the centre of the frequency spectrum.

3. Plot the magnitude of the signal by evaluating the absolute value of the signal’s shifted FFT (output of Step 2). In your plot use the frequency axis developed in Step 1.

Exercise

Plot, using Matlab, the signals: , and

in the time domain and identify their frequencies using the DFT.

Specifically:

1. Develop the following six figures each consisting of three subplots:

(i) Figure 1; Plot signals (subplot 1), (subplot 2) and (subplot 3) in their

continuous-time form. Each signal should start from point 0 and finish at point 1.

(ii) Figure 2; Plot signals (subplot 1), (subplot 2) and (subplot 3) in their

discrete-time form. Each signal should start from point 0 and finish at point 1.

(iii) Figure 3; Plot, in the frequency domain, the magnitude of the DFT of the signals: (subplot 1), (subplot 2) and (subplot 3) for . Identify the frequencies of these three signals.

(iv) Figure 4; Plot, in the frequency domain, the magnitude of the DFT of the signals: (subplot 1), (subplot 2) and (subplot 3) for . Identify the frequencies of these three signals.

(v) Figure 5; Plot, in the frequency domain, the magnitude of the DFT of the signals: (subplot 1), (subplot 2) and (subplot 3) for . Identify the frequencies of these three signals.

(vi) Figure 6; Plot, in the frequency domain, the magnitude of the DFT of the signals: (subplot 1), (subplot 2) and (subplot 3) for . Identify the frequencies of these three signals.

Each subplot should contain: Title, x-axis label and y-axis label.

2. Using Theory and Figures 3, 4, 5 and 6, comment on the importance of the sampling frequency in the detection of a signal’s frequency content.