Dynamics in a magnetic pendulum model: dipole-dipole interactions and chaos on a sphere

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POSTER ABSTRACT

In its most familiar incarnation, the magnetic pendulum comprises a magnetic bob that is free to swing in three-dimensional (3D) space. Below, a horizontal base-plane contains three similar magnets; each is arranged in the opposite-poles-attract configuration relative to the bob, and placed on the vertices of an equilateral triangle.

Peitgen, Jürgens, and Saupe proposed in the early 1990s a phenomenological equation of motion for a magnetic pendulum with two degrees of freedom ["Chaos and Fractals: New Frontiers in Science," Springer, 1992]. Their approach was to adopt a plan view of the pendulum, using Cartesian coordinates and a 2D damped simple harmonic oscillator (SHO) to represent a projection of the bob's position as it darts erratically to-and-fro above the base-plane magnets. Linear damping was accommodated by including a drag force proportional to negative velocity. Moreover, the magnetic interactions were assumed to originate from point sources which led naturally to an inverse-square force law. This toy model is appealing in its simplicity, though analysis by Motter *et al*. [Phys. Rev. Lett., vol. 111(19), art. no. 194101 (2013)] uncovered surprisingly rich behaviours. However, the damped-SHO paradigm and its ad hoc inverse-square rule mimicking magnetism does not provide an entirely satisfactory picture of the real system.

Here, a more physical model is proposed for the magnetic pendulum. At the outset, the problem is formulated by deploying Lagrangian dynamics. The bob, in reality, moves through space along the surface of a sphere whose centre coincides with the suspension point and whose constant radius provides a holonomic constraint. Hence, the natural coordinates to use turn out to be the azimuthal and polar angles. The restoring force of gravity is straightforward to include, and linear damping is introduced through a velocity-dependent potential. Crucially, magnetic dipole-dipole interactions—which have an intrinsic directional character—replace the assumption of inverse-square attraction. In these ways, the classic model changes non-trivially from the damped SHO-type to the damped spherical pendulum-type.

This presentation will look at the derivation and scaling of the governing equations, which are somewhat complicated in structure and computationally expensive to solve. There appear several additional quantities that must be specified in order to fully characterize the pendulum, increasing dramatically the size of the parameter space. The phenomenon of sensitive dependence on initial conditions will be demonstrated. Preliminary computations of the basins of attraction will also be shown, providing some qualitative evidence of fractality in the structure of their boundaries.

Keywords: Chaos, fractals, magnetism, nonlinear dynamics.