Spontaneous pattern formation with Salerno equations: ring-cavity feedback, static instabilities, and mean-field theory

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TALK ABSTRACT

In physics, the discrete nonlinear Schrödinger (dNLS) equation plays a key role in modelling wave propagation in periodic optical systems [Christodoulides and Joseph, Opt. Lett., vol. 13(9), 794–796 (1988)]. Architectures typically involve light confined to a set of waveguide channels with nearest-neighbour coupling and whose dielectric response has a local cubic nonlinearity. While the widely-used dNLS model is non-integrable, it possesses an integrable counterpart—the Ablowitz-Ladik (AL) equation [J. Math. Phys. vol. 17(6), 1011–1018 (1976)]—which is often of greater interest in applied mathematics research. The price paid for integrability is a nonlinear response that remains cubic but becomes nonlocal in a way that defies straightforward physical interpretations. In this presentation, our interest lies with the Salerno equation [Phys. Rev. A, vol. 46(11), 6856–6859 (1992)], which facilitates a simple linear interpolation between the dNLS and AL regimes.

Here, we consider the Salerno equation in the context of spontaneous pattern formation involving a discrete waveguide array and a ring-cavity arrangement. Feedback from the cavity—which comprises external periodic pumping, coupling-mirror losses, and mistuning relative to the pump wave—is accommodated via a single 'lumped' boundary condition applied on the input plane. The stationary plane-wave solutions of the cavity are detailed, and a linearized perturbation theory deployed to predict their robustness against small-amplitude periodic modulations. In this way, the most-unstable spatial frequency (hence the dominant length-scale of any emergent static patterns) can be identified from the threshold instability spectrum. The dNLS and AL spectra appear as special cases, and the long-wavelength asymptotics of all three models are consistent with the continuum nonlinear Schrödinger equation.

Extensive simulations of discrete cavities with a single transverse dimension have been carried out, with initialization corresponding to a plane-wave stationary state perturbed by low-level coloured noise. Those numerical calculations demonstrate the emergence of static cosine-type patterns, in good agreement with theory. We have also extended our considerations to capture a second transverse dimension in the Salerno equation. Simulations have yielded static hexagon patterns that appear to be stable across time.

We conclude with a foray into mean-field theory, which is typically used to model the space-time dynamics of a longitudinally-averaged cavity field. The resulting Salerno equation is of the discrete Ginzburg-Landau class, where cavity effects appear as additional forcing terms rather than through repeated application of a formal boundary condition. Results from pattern formation in both one and two transverse dimensions will be detailed.

Keywords: Discrete equations, instabilities, nonlinear waves, pattern formation.