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Modelling cascade dynamics of passenger flow congestion in urban rail transit network induced by train delay

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Abstract A research into congestion propagation mechanism of urban rail transit passenger flow in train delay scenario is of great significance to the formulation of coordinated limitation measures of passenger flow among the operation lines in urban rail transit networks. This paper presents a methodology for modelling cascade dynamics of passenger flow congestion. In this method, firstly, a computing method of network passenger flow states based on load entropy is proposed to describe the overall load conditions of passenger flow in the network. Secondly, a rail transit network is developed and the computation formulas of load and capacity of nodes have been established combining with a comprehensive consideration of nodes' topological passenger flow attributes. Thirdly, Trip Betweenness Centrality of nodes is proposed and calculated. Considering the function of rail transit line capacity and load distribution strategy, the passenger flow congestion in train delay scenario is established to calculate cascade dynamics of congestion and the state transition of urban rail transit network. A real case study on Beijing Metro Line 10 is used to demonstrate the proposed methodology. The results show that the load values of nodes and the distances from the initial failure nodes can be determined for the spreading scope of passenger flow congestion in disrupted metro line and adjacent metro lines.

1. Introduction

Urban rail transit (URT) has become people's first choice of commuting and travel and developed into a networked operation mode in largest cities around the world due to its convenience, efficiency and safety.

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With the operation mileages of URT networks constructed, the complexity of correlation among stations and rail lines has been increasing, and disruptive incidents such as train delays, passenger congestion problems have become as significant issues to be solved to avoid accidents and improve passenger service level. URT network congestion of passenger flow is an important issue concerned by the public, the government and the operation management department, especially under the oversaturated conditions. Congestion control and relief strategies are effective means to ensure the normal operation of URT. Here, cascading congestion refers to a functional failure of a series of stations due to passenger flow congestion caused by train delay at one station. Therefore, it is essential to conduct an in-depth analysis of congestion propagation mechanism of the network and reduce the impact in train delay scenario, which will provide the effective measures of passenger control in urban rail transit operation.

There are many studies on urban rail transit networks and passenger flow congestions, for example, study on network performance, network cascading failure analysis, the load states of stations, the distribution characteristics of passenger flows, metro disruptions. However, researches on congestion evolution modeling are relatively scarce due to dynamics and randomness.

A few studies have been carried out with the topographical mapping modelling in network performance analysis, which passenger betweenness centrality (PBC) is introduced to measure network performance [1]. Huang et al. [2] used entropy-TOPSIS method to evaluate urban rail transit system operation performance. Feng et al. (2017) analyzed weighted complex network of the Beijing subway system considering both train and passenger flow. Xu et al. [3] utilized trip data obtained from the Beijing Subway System to characterize individual passenger movement patterns.

Referring network cascading failure analysis, most studies typically focused on the complex network topology analysis of metro systems without network flows. As the basis for research on cascade failure, some scholars have proposed the concept of Node Load and then use degree or Betweenness or shortest paths [4–8] to define node load, or using the actual flow through nodes as Node Load in a direct manner [9] to describe cascading failure in the network. A new mitigation strategy was introduced to prevent or mitigate the cascading propagation on complex networks more efficiently [10]. The load redistribution strategies [11,12] of load mainly involve those in the nearest neighborhood, overall the network, and the remaining capacity according to nodes.

With respect to the description of the flow load state in the nodes and cascading failure in the network, the density of stations and the load index of sections regarding the passenger flow was presented. Some scholars proposed the definition of node load to characterize the significance of nodes in the network [13–15] as well as the flow load of nodes. The analysis of passenger flow distribution of urban rail transit is mainly aimed at the network characteristics [16–18]. Jianhua Zhang et al. have studied the topological characteristics and proposed two novel parameters called the functionality loss and connectivity for measuring the transport functionality and the connectivity of subway lines [16], and then calculated the average degrees of nodes, the average shortest path lengths and the average betweenness of nodes and edges of urban rail transit networks [17]. Sybil Derrible et al. [18]. have developed a specific method to analyze metro systems as networks, explained the concepts of scale-free and small-world networks and adapt them to metro systems. The imbalance and spatial distribution regarding the passenger flow are studied in Ref. [19].

As for passenger flow behavior, an evacuation strategy of metro station is investigated and proposed, a calculation formula is proposed for the metro station evacuation time [20]. Referring to unbalanced urban traffic network, a coordinated multimodal dynamic freight load balancing system was proposed to balance freight loads [21]. And after that a multimodal freight routing system based on optimization was evaluated with hard vehicle availability and capacity constraints [22].

Researches related to disruptions gradually increase. As discussed by Sun et al. [23], a novel approach was proposed to identify such disruptions and evaluate their influence on travel times and delays. Then abnormal passenger flow was divided into three characteristic types and analyzed in this paper. Cadarso et al. [24] studied the disruption management problem and explicitly dealt with the effects of the disruption on the passenger demand in rapid transit rail networks. In the paper [25], the resilience of a metro network is associated to the network performance loss triangle over the relevant timeline from the occurrence of a random or intentional disruption to full recovery.

As stated above, the previous researches on network performance analysis, network cascading failure analysis, the load state of station analysis, the distribution characteristics of passenger flows analysis, and metro disruption analysis have some problems in the analysis process, which can be summarized below.

a) Some scholars have presented the node load concept to describe the importance of node considering its topological attributes such as degree and Betweenness Centrality. But how to quantify and analyze the network global state based on both nodes' topological and passenger flow attributes, from the perspective of network decision maker?

b) Some researchers have established cascading propagation models of passenger flow such as Susceptible Infected Recovered (SIR)model, Cellular Automata (CA)model. While there are few studies on the evolution mechanism of passenger flow congestion with the employment of point congestion (the overloading state of passenger flow at stations) as key elements in the model. Only few studies on the evolution mechanism of passenger flow congestion considered point congestion (the overloading state of passenger flow at stations) in their models, which is insufficient.

The load definition of node and the passenger flow congestion model are presented in this paper, which the research can contribute:

a) For describing the congestion in stations, the load definition of node is proposed considering not only its topological attributes, but also its flow attributes and neighbors' topological attributes. The load entropy is developed to describe the whole network, which can be used to represent the overall characteristics of passenger flow in the network.

b) For revealing the congestion mechanism of passenger flow in URT network in train delay operation scenario, we have presented the passenger flow congestion model, considering both the distance between the functional abnormal node and normal node, and the function of throughput of normal nodes under normal conditions. And then the transitions of network passenger flow states are depicted.

The rest of this paper is organized as follows: Section 2 describes the calculation of network passenger flow state based on load entropy and the cascade dynamics of passenger flow congestion of URT network in train delay scenario. Section 3 depicts the simulation results and explains the characteristics of cascading propagation. In the first part of this section, the source of the data and the background of train delay event are introduced. In the second part, comparative analysis of the actual situation and the simulation results of network passenger flow state are established. Conclusions are provided based on the two cases of state transitions of cascading failures in networks in Section 4.

2. Methodology

2.1. Urban rail transit topological network

Urban Rail Transit Network (URTN) consists of metro lines which are composed of stations and sections. Considering the directions of trains, the topological network, named Urban Rail Transit Topological Network (URTTN) as shown in Fig. 1. In Fig. 1, blue, orange and purple indicate three operating lines, with different arrow types indicating the direction of train running, black spot indicating transfer station.

Station i is defined as node v_i. Stations i and j are connected by an edge that is represented by $e_{ij} = \{v_i \rightarrow v_j\}$. URTTN can be presented as

$$URTTN = \{V, E, T_v, F_v, P_e\} \tag{1}$$

$$\begin{cases} V = \{v_1, v_2, \cdots, v_n\} \\ E = \{e_{ij} | i, j \in n\} \\ T_v = \{k_i, BC_i, TBC_i, \cdots | i, j \in n\} \\ F_v = \{I_i(t), E_i(t), T_i(t), \cdots | i \in n\} \\ P_e = \{w_{e_{ij}}, \delta_{ij}, \cdots | i, j \in n\} \end{cases} \tag{2}$$

where, V, E, T_v, F_v, and P_e are the set of nodes, edges, topology attributes, flow attributes and path attributes, respectively. k_i is the i-th node degree. $w_{e_{ij}}$ is the weight of edge e_{ij}, BC_i is the Betweenness Centrality of v_i based on shortest path with $w_{e_{ij}} = 1$. TBC_i is the Trip Betweenness Centrality of v_i based on Most Selective Path (MSP). $I_i(t)$, $E_i(t)$ and $T_i(t)$, represent the ingress passenger flow volume of the station, the egress passenger flow volume of station and the transferring passenger flow volume, respectively. δ_{ij} is the Most Selective Path (MSP).

Definition 1 (*The Most Selective Path (MSP)*). In the actual operation of the metro lines, a passenger does not simply choose the shortest path as his travel route. A passenger usually weighs both the transfer times and the length of path. The Most Selective Path (MSP) refers to the path with the smallest $w_{e_{ij}}$ from v_i to v_j in the metro network, with the mathematical formula as follows,

$$\delta_{ij} = \min w_{e_{ij}} \tag{3}$$

$$w_{e_{ij}} = \kappa_1 r_{ij} + \kappa_2 N_{tr_{ij}} \tag{4}$$

where, δ_{ij} is the Most Selective Path (MSP). $N_{tr_{ij}}$ is the transfer times from v_i to v_j. r_{ij} is the distance from v_i to v_j and it is defined as the minimum number of nodes between v_i and v_j with the expression of $r_{ij} = |i - j|$. κ_1 and κ_2 are the weight coefficients that satisfy $\kappa_1 + \kappa_2 = 1$.

Definition 2 (*The Trip Betweenness Centrality (TBC) based on MSP*). The betweenness centrality of nodes reflects the degree of topological significance and takes the Most Selective Path (MSP) into consideration. A BC index, namely Trip Betweenness Centrality (TBC), is proposed to represent the relationship between MSP and TBC. Compared to BC, TBC can be used more effective to measure the attraction capacity of a node i.e., station in passenger flow. In other words, the higher TBC of a node is, the more attractive for passengers at this station is. TBC_i of the i-th node can be calculated by

$$TBC_i = \frac{\sum \int_{j,k \in G, j \neq k} d_{jk}(v_i)}{\sum \int_{j,k \in G, j \neq k} d_{jk}} \forall j, k \epsilon URTTN j \neq k \tag{5}$$

where d_{jk} is the total number of MSP between nodes v_j and v_k, $d_{jk}(v_i)$ is the total number of MSP passed through node v_i.

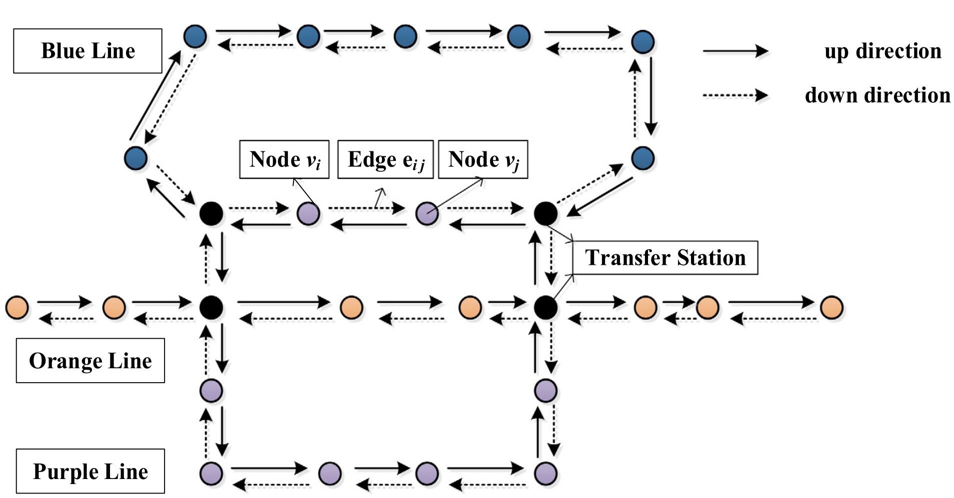


Fig. 1 An example of URTTN.

2.2. Calculation of network passenger flow states based on load entropy

2.2.1. The loads of nodes

The importance of a node is determined not only by topology location in the network and availability of appropriate alternatives, but also from the load of passengers. It is necessary to define node load from the perspective of the actual flow and topology properties to describe the importance of node in the network. In the meanwhile, the load, as an important parameter in calculating the characteristics of the entire network flow, offers a basic index to describe the cascading failure mechanism of the overload of network flow from the perspective of dynamic behavior, which also needs to be defined.

(1) The node load $L_i(t_0)$ and capacity $C_i(t_0)$ of URTTN

The node load $L_i(t_0)$ describes the load state of a passenger flow, which is related to the topological parameter of a node, the topological parameter of neighbor nodes and the passenger flow, which can be defined as

$$L_i(t_0) = \varphi(TBC_i \cdot fl_i(t_0))^{\beta_0} \left(TBC_i \cdot TBC_{i_1} \cdot \sum_{i_1 \in \Gamma_i} fl_{i_1}(t_0)\right)^{\beta_1} \left(TBC_{i_1} \cdot TBC_{i_2} \cdot \sum_{i_2 \in \Gamma_{i_1}} fl_{i_2}(t_0)\right)^{\beta_2} \tag{6}$$

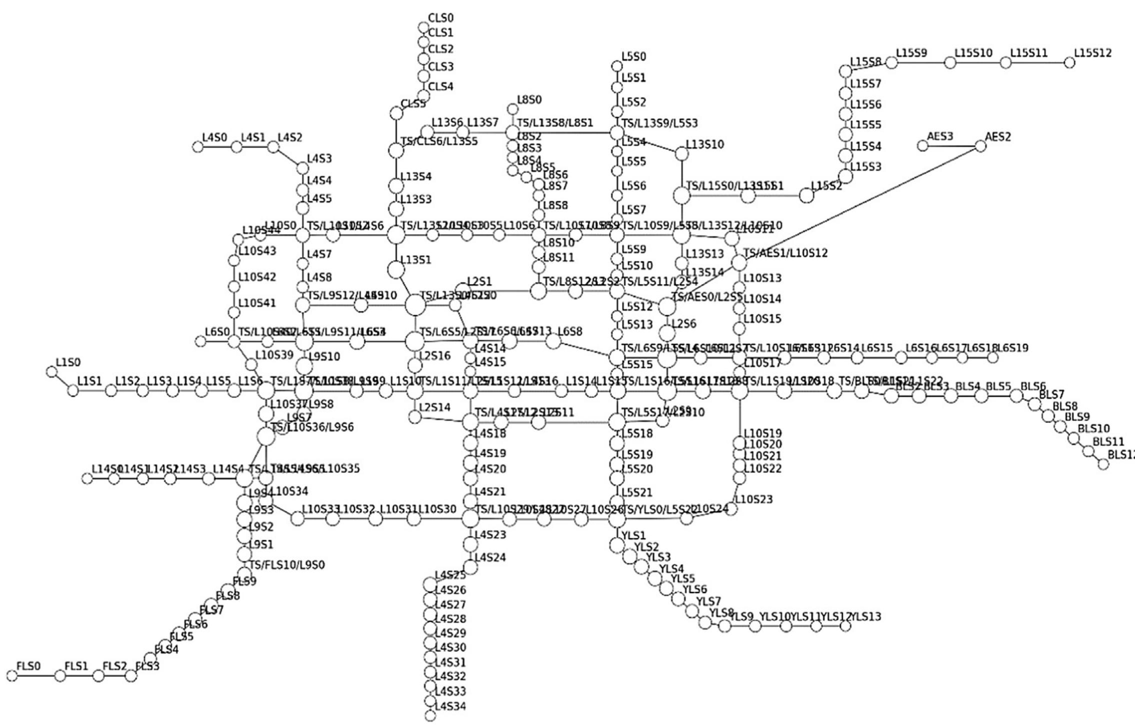


Fig. 2 The topological diagram of operation network regarding Beijing rail transit.

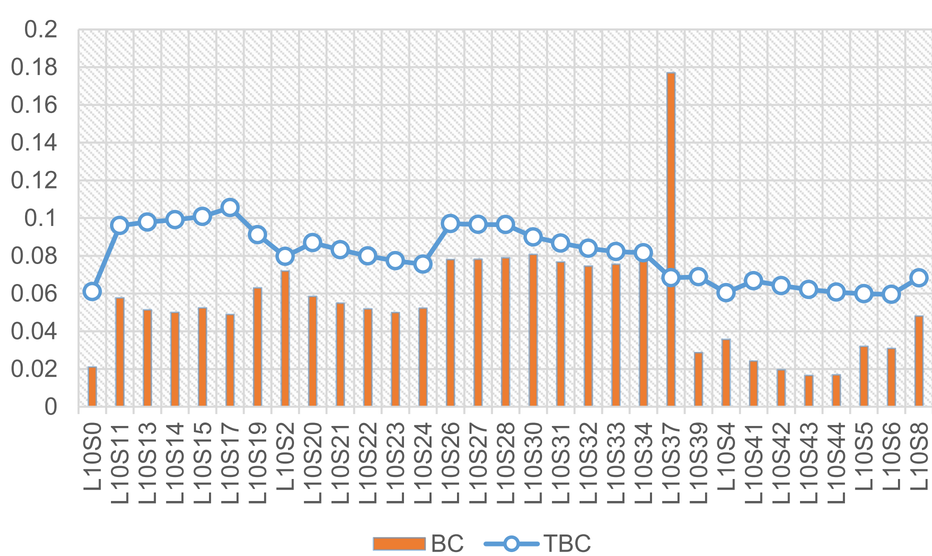


Fig. 3 The TBC_i and BC_i of Beijing Metro Line 10.

$$fl_i(t_0) = \zeta \cdot k_i \cdot \frac{I_i(t) + E_i(t) + T_i(t)}{\sum I_i(t) + E_i(t) + T_i(t)} \quad (7)$$

where $fl_i(t_0)$ is the parameter of passenger flow at time t_0 and $I_i(t), E_i(t),$ and $T_i(t)$ represent the ingress passenger flow volume of the station, the egress passenger flow volume of a station, i.e., a node and the transferring passenger flow volume, respectively. k_i is node degree. Γ_i is the neighbor set of v_i. $\beta_0, \beta_1, \beta_2$ are the influencing and controlling parameters of nodes themselves, their neighbors and the next neighbors, with φ, ζ as the adjustment parameter of load.

The capacity of a node is affected by cost constraints, determining the load capacity of these nodes is often "according to effective volume," therefore, the node capacity and the initial load $L_i(t_0)$ can be directly proportionally calculated

$$C_i(t_0) = (1 + \tau_a)L_i(t_0) \quad (8)$$

where τ_a is the "tolerance parameter," the redundant capacity for node i, and at the same time, it reflects the additional cost of node i, the additional burden on capacity and protection for node i. τ_a means the maximum allowance ratio for a station load in URTN.

(2) The description of cascading congestion propagation of passenger flow

The passenger flow congestion propagation in URTTN can be modelled based on the relationship between node load $L_i(t_0)$ and capacity $C_i(t_0)$ of a node.

Table 1 The top 10 stations of Line 10 after sorting by TBC_i and. BC_i

Station Number	TBC_i	Station Number	BC_i
L10S17	0.10553	L10S37	0.177016
L10S15	0.10081	L10S30	0.080753
L10S14	0.09912	L10S34	0.079279
L10S13	0.09787	L10S28	0.078948
L10S26	0.09704	L10S27	0.078156
L10S27	0.09663	L10S26	0.078045
L10S28	0.09655	L10S31	0.076737
L10S11	0.09604	L10S33	0.07554
L10S19	0.09118	L10S32	0.07449
L10S30	0.08999	L10S2	0.071967

The functional abnormal node in URTTN is related to the crowding degree of passengers in station. From a perspective of crowding in station, a node has three status, which cascading congestion scenario is defined as:

$$\begin{cases} L_i(t) \le C_i, normalnodev_i \\ C_i < L_i(t) < \xi C_i, partialfunctionalfailureofnodev_i \\ L_i(t) > C_i, thoroughfunctionalfailureofnodev_i \end{cases} \quad (9)$$

a) When $L_i(t) \le C_i$, this scenario means there is no functional failure at node v_i. Passengers on the platform can move freely; all passengers can board the trains.
b) When $C_i < L_i(t) < \xi C_i$, this scenario means there is a partial functional failure of node v_i, the platform is crowed, passengers can get off the train but not everyone can get on the train.
c) When $L_i(t) > C_i$, this scenario means there is a thorough functional failure of node v_i, the platform of this station is overloaded, the trains pass through the station without stopping, passenger flow congestion occurs and spreads in the network. Here, ξ is a system function resilience parameter, which reflects self-healing performance of transport function and is determined by actual maximum capacity of station platform, usually let $\xi = 1.3$ according to the actual operation experience.

2.2.2. The computation of network macro flow state

The state of passenger flow was calculated by some scholars [26–29]. Here we have presented the concept of load entropy to define the state of passenger flow in the Beijing Metro network. As an important concept to research a complicated system, entropy [30,31] can reflect the relationship between macro state variables and micro state ones in a complicated system as well as the evolution of the system with the employment of the changes of entropy. The decreasing of entropy means the system evolves from a disorder way to an order one, otherwise the opposite will follow.

Assume $L_i(t)$ is in the interval $[0, x*s)$ during the failure propagation. x successive intervals are defined as $[0,s), [s, 2*s), \cdots, [(x-1)*s, x*s)$ and then the computation is defined as follows:

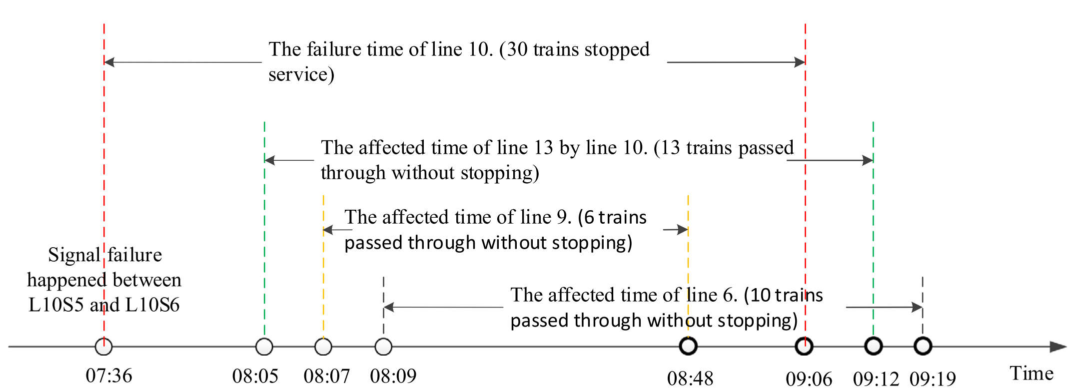


Fig. 4 The train delay event at Line 10.

$$H_n(t) = -\sum_{k=1} \int^{x} \frac{n_k(t)}{N} \log \frac{n_k(t)}{N} \tag{10}$$

where, $H_n(t)$ is the load entropy of the URTTN, $n_k(t)$ is the numbers of nodes, whose load value $L_i(t)$ falls within the k th interval $[(k-1)*s, k*s)$, and N is the sum of network nodes.

2.3. The cascade dynamics of passenger flow congestion model of URT network in a train delay scenario

According to the Eq. (9), the train delay scenario can be divided into two kinds of incidents, i.e., partial congestion and thorough out congestion of trains and platforms in the network. With the changing parameters of initial load regarding accidental nodes, the cascading congestion model of weighted URT network regarding passenger flow can be established by a subsequent analysis of the dynamic propagation processes of network overload after the failures of nodes.

For making the evolution model of load accord with the actual situation in the network, we consider the effect of time delays here. In case of the delay of the train, the overload in the failure node will lead to the time delay of congestion and the redistributed overload entirely in the network. This effect is expected to occur in the changes of the line capacity. In case of a node overload, the overload of each node in the network will not always be at a state of high load. As time goes by, the congestion will change in a dynamic way, with a self-restoration.

At the initial time of t_0, the initial load of the failure node v_i is $L_i(t_0)$. Assume that the load of the failure node is distributed to normal nodes according to redistribution rules, so there is an updating of the load of normal nodes in the network, with ΔL_i as load increment, i.e., $L_i(t_0) \rightarrow L_j(t_1) = L_j(t_0) + \Delta L_i$.

The load increment ΔL_i distributed to normal node v_j is related to the distance from nodes v_i and v_j, topology attributes and the flow of node v_j under normal historical conditions. ΔL_i can be obtained by:

$$\Delta L_i = \sum_{i=1} \int^{3} \rho_i g_i \tag{11}$$

$$g_1 = \gamma \left[\left(\frac{r_{ij}}{L_{en}} - \frac{1}{2} \right)^2 - v \right] \tag{12}$$

$$g_2 = -\varsigma TBC_j^{2} \tag{13}$$

$$g_3 = \vartheta R_{\ln} \left(\frac{I_j^{\prime}(t)}{Ih_j^{\prime}(t)} \right)^2 \tag{14}$$

where $g_1, g_2,$ andg_3 are the functions of load distributions of nodes, including the distance, topology attributes and passenger flow of normal nodes, respectively. $\rho_1, \rho_2,$ andρ_3 are respectively influencing and controlling parameters of $g_1, g_2,$ andg_3 under historical conditions. γ, v, ς, and ϑ are the parameters of model adjustment, respectively. r_{ij} represents the distance from nodes from v_i and v_j , which is defined the minimum hops of the two nodes, with the expression of $r_{ij} = |i - j|$. L_{en} is the max diameter of URTTN.

Different metro lines have different transportation capacity. In the delayed train operation scenario, the Transport Capacity will decrease correspondingly. Here let $R_{\ln}$ be the function of the actual transportation capacity with respect to the metro lines. $I_j^{\prime}(t)$ and $Ih_j^{\prime}(t)$ represent the discrete ingress passenger flow volume of the station per minute and per hour, respectively, under normal historical operating conditions. Given the difficulty in acquiring the data of passenger flow with the time interval of train departure, we use polynomial interpolation method [32–35] to discretize the passenger flow data of $Ih_j^{\prime}(t)$ to acquire $I_j^{\prime}(t)$ minutely and describe the evolution process of passenger flow state.

3. Case study

3.1. The topological network and the data

(1) Development of Beijing Metro topological network

Fig. 2 shows topological network based on Beijing Metro network in 2013. In this figure different sizes of circles show the different values of TBC_i regarding all nodes, with the construction of the topological diagram.

The analysis of Beijing Metro Line 10 (from L10S0 to L10S44) is used to demonstrate the proposed methodology.

Fig. 3 shows the calculation results of node topological significance by using Eq (5). Take station L10S37 as an example, its BC_i value is 0.177 only considering the factor of topology. But passengers do not simply choose the shortest path as their travel route. Once they consider both the transfer times and the length in the path, the betweenness centrality drops to 0.068. Station L10S37 (Name: Lianhuaqiao) is located in a small circle consisting of Line 1 (from L1S0 to L1S18), Line 10 and Line 9 (from L9S0 to L9S12). This leads to more transfers in travel routes and lower TBC_i value.

The median of TBC_i obtained by statistical analysis is 0.08158, and the median of BC_i obtained by statistical analysis is 0.05224. According to Table 1, the topological significance of nodes varies with TBC_i and BC_i changes. Considering both topology and transfer times factors, the topological significance of some stations with TBC_i index has increased com-

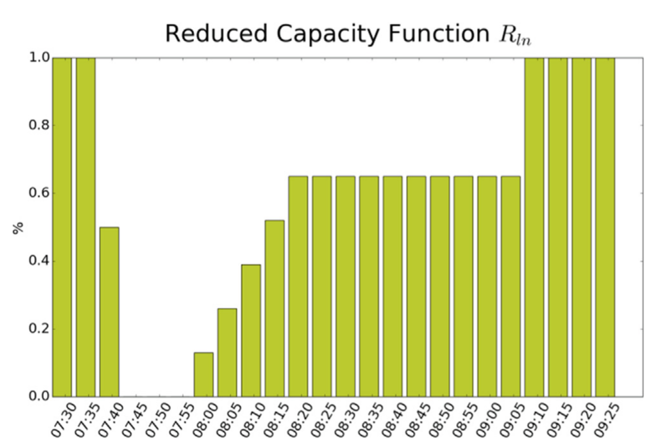


Fig. 5 The line capacity function of line 10 in a train delayed operation.

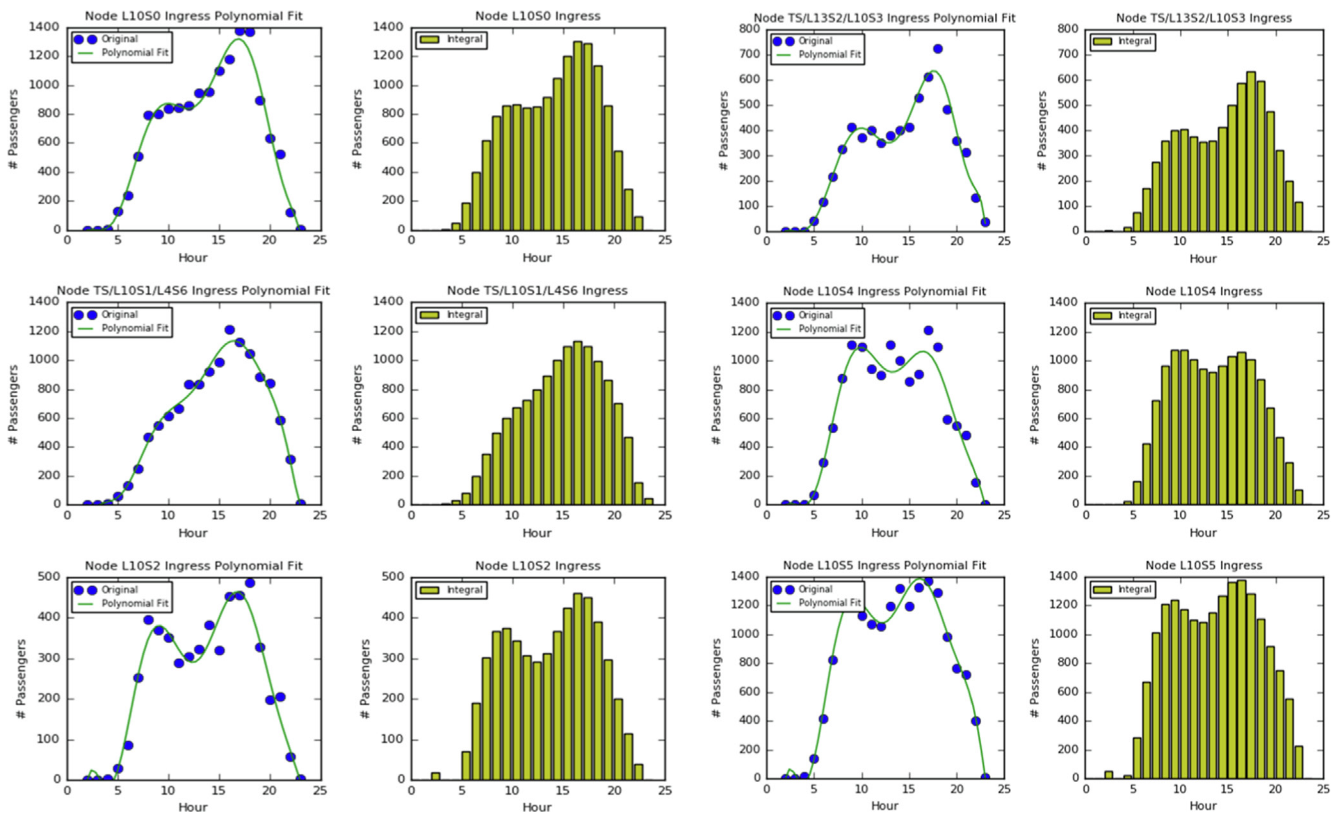


Fig. 6 The discretized ingress passenger flow of six stations in Line 10.

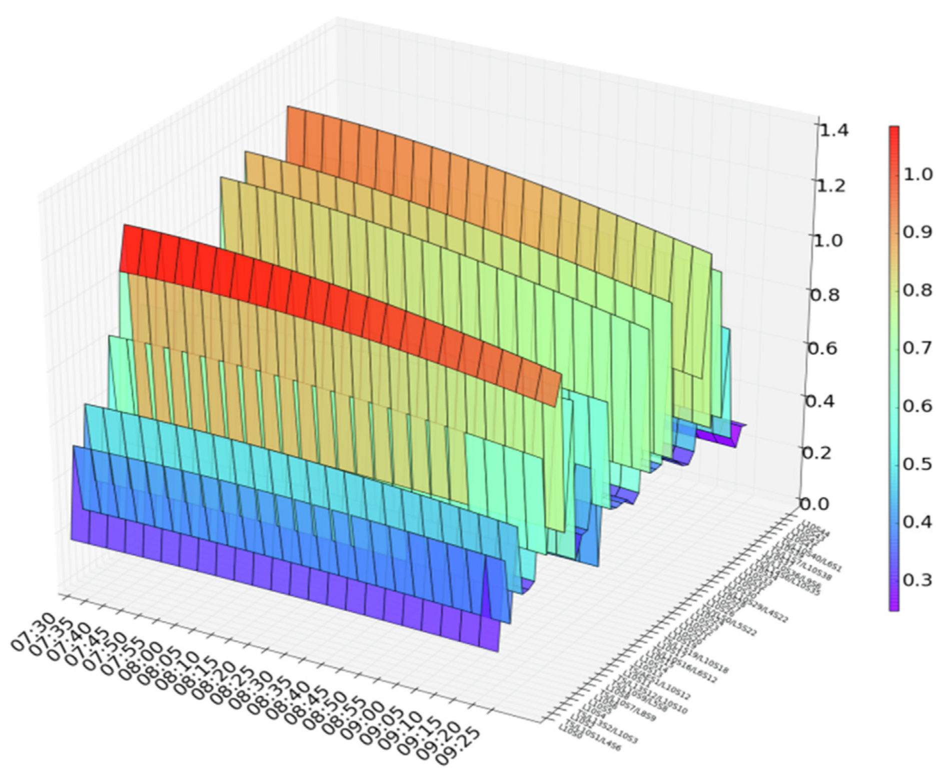


Fig. 7 The node load of line 10 at different time in a train delayed operation.

pared with BC_i index, such as L10S17, L10S15, L10S14, L10S13. The calculated topological significance of stations with TBC_i such as L10S26, L10S27, L10S28 are almost unchanged compared with BC_i index. Stations such as L10S37, L10S30, L10S34, L10S31, L10S33 and L10S32 are not in the top 10 stations list of TBC_i.

(2) The source of the passenger flow data and the background of train delay event

The data of passenger flow within the interval of 07:30–09:30 regarding the whole network of Beijing Metro on a given date of October 2013 are collected.

The truth of the train delay event is shown in as Fig. 4 to conduct a simulation of the state evolution process in the network in a delayed train operation scenario. This event arose from the turnout fault in a downward direction at Line 10.

According to Fig. 4, cascading failures is triggered by Signal failure happened in Section L10S5 to L10S6, and the begin-

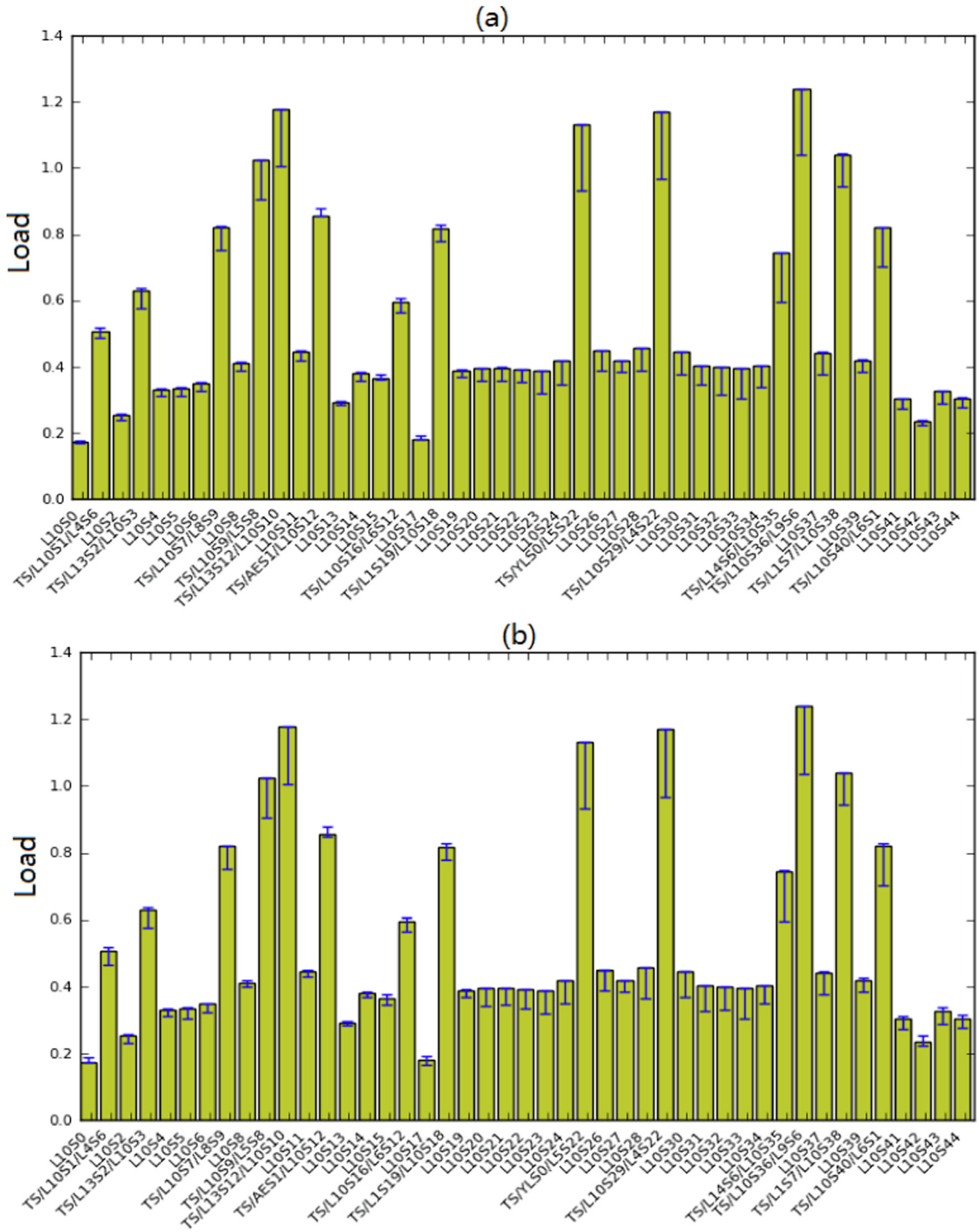


Fig. 8 The comparison between the actual situation of passenger flow **(a)** and the simulation results from state evolution model **(b)**.

ning and termination time intervals of 07:30–09:25 is set regarding the model simulation, with the data of passenger flow at 07:30 at the Station of L10S5 as the simulated initial load.

3.2. A comparative analysis of the actual situation and the simulation results of network passenger flow state

(1) Determination of the line capacity function R_{ln}

To determine the parameter R_{ln} in the simulation model, that is, the actual transport capacity function of the line, this paper gives the line capacity function of line 10 under the train delayed operation in October 2013 according to Fig. 4, as shown in Fig. 5.

In Fig. 5, The line capacity function normally equals one, when Signal failure happened, the driver received a dispatch instruction of lower speed, the line capacity drops. Then fault repair occurred and Line stopped service for a period, the line capacity reduces to zero. Finally, Line operation gradually recovered and the line capacity gets back to normal standard.

(2) The discretization of hourly passenger flow data $Ih_j^{'}(t)$

For calculating the node load and describing the evolution process of passenger flow state, the hourly passenger flow data $Ih_j^{'}(t)$ is discretized into the data of $I_j^{'}(t)$ on a minutely basis according to polynomial interpolation, see Fig. 6.

(3) The load of nodes in the network

The load of nodes in the network is calculated in accordance with the formula (6) in Part 2.2. Due to the limitation of the length of an article, we have chosen the load of nodes at Line 10 for an analysis, the outcome of which is shown as Fig. 7.

As can be seen from Fig. 7 that the load is composed of that of topology and flow. A bigger betweenness and passenger flow volume at a transference station will lead to a larger load value than that at ordinary stations. The transference stations like TS/L10S29/L4S22 (Name:Jiaomenxi), TS/L10S9/L55 (Name: South Huixinxijie), TS/L13S12/L10S10 (Name: Shaoyaoju), TS/AES1/L10S12 (Name: Sanyuanqiao), TS/YLS0/L5S22 (Name: Songjiazhuang), TS/L10S36/L9S6 (Name: Liuliqiao), and the load value of node TS/L1S7/L10S38 (Name: Gongzhufen) is comparatively bigger within the time interval of 09:00–09:25.

With the load value (the initial load) at the time of 07:30 as the basis (making a reference to the column in Fig. 8) and in accordance with the evolution model of passenger flow in the network in case of a delayed train operation, we have conducted a simulation of passenger flow state within the interval of 07:30–9:25, with the acquisition of the maximum value and the minimum value of the load at different stations at Line 10 in case of a delayed train operation (making a reference to the short vertical line in the interior of the column in Fig. 8). The simulation outcome of the model revealed in Fig. 8 roughly corresponds to the actual load trend of passenger flow.

Load changes have been revealed regarding the load increment of ΔL_i distributed from other nodes in the simulation model and the distributed functions g_1, g_2, g_3 of node load (see formulas (9)–(11)), as Fig. 9 shown. The three distributed functions of node load, as the decisive parameters of load increment, will have a direct effect on the load value at the next time.

With the computation of the load of each node in the network and in combination with the computation method of network state based on load entropy, we have acquired the

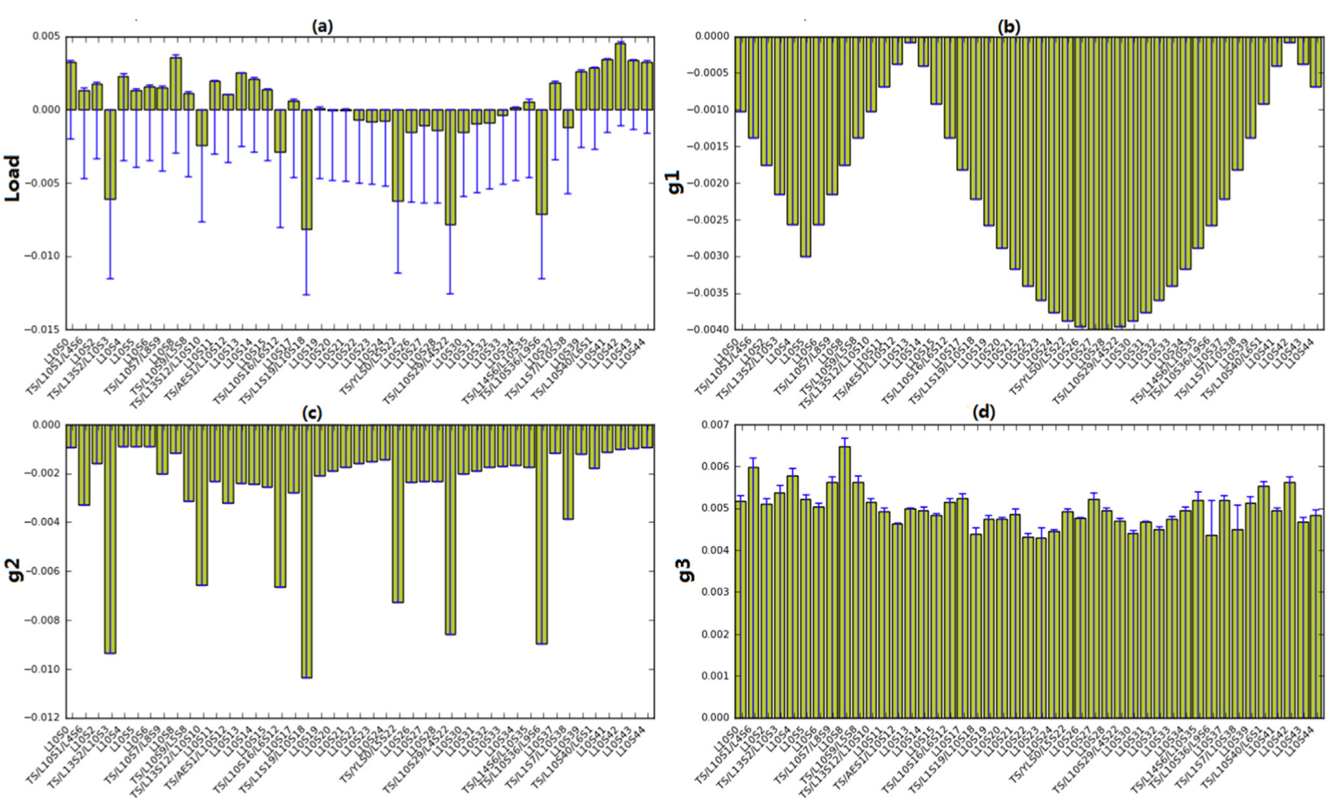
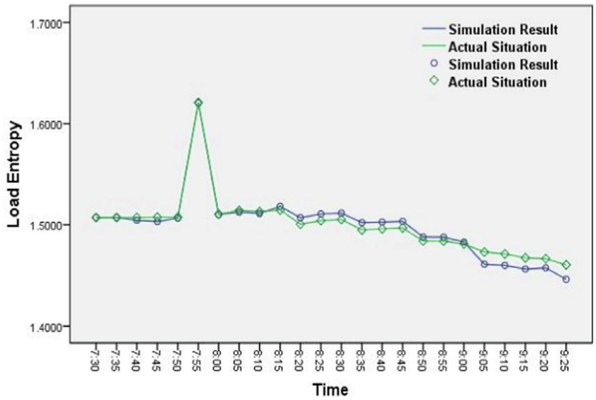


Fig. 9 The load increment (a) and the load distribution functions g_1, g_2, g_3 respectively (b), (c) and (d).



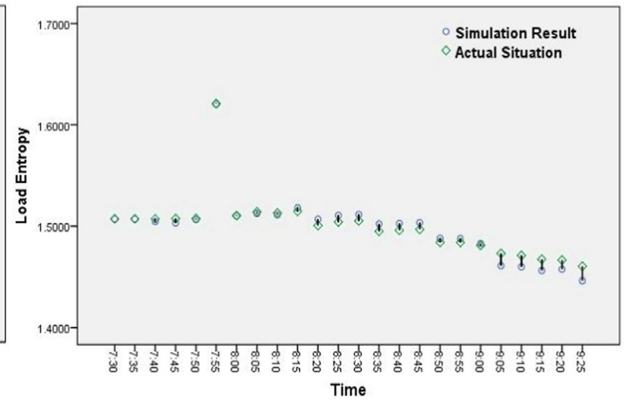


Fig. 10 The actual situation and the simulation result regarding passenger flow state of the network in case of a train delayed operation.

diagrammatic sketch regarding the state of passenger flow at different time which is shown in Fig. 10.

The changes in the value of load entropy have reflected the evolution process in the system of passenger flow of the network. We have discovered from a comparison that the acquired simulation result based on the proposed model is basically consistent with the actual data of passenger flow.

There are two typical cases of state transition in the chart. Firstly, within the time interval of 07:50 → 07:55, the value of $H_n(t)$ increases from 1.508 to 1.621, with the evolution of the passenger flow system from an order state to a disorder one, there is a serious imbalance of the load of the whole network and the subsequently continual congestion at the stations. Many lines are involved in this congestion. Secondly, within the time interval of 07:55 → 08:00, the value of $H_n(t)$ decrease from 1.621 to 1.511, there is a continual congestion at the stations of the network. The congestion is at one single line, i.e., Line 10, with no effect on the passenger flow at other lines. The reason is that the failure immediately aroused the attentions of government departments, and within a certain time of response, the government have taken necessary measures to require the trains to pass these stations at a low speed to resume the normal operation.

Within the time interval of 09:00 → 09:25, there is a relative balance of the load of the whole network, which means the state level of network passenger flow is in an acceptable range, and passenger flow in only several stations reach the saturation state.

4. Conclusions

In this paper, the conceptualized expression method of passenger flow state is innovatively presented based on the proposed definition of load. The node load concept describes the load state of passenger flow, which are related to the topological parameter of node, the topological parameter of neighbors and the passenger flow. A bigger betweenness and passenger flow volume at a transference station will lead to a larger load value than that at ordinary stations, such as transfer stations TS/L10S29/L4S22, TS/L10S9/L55 within the time interval 09:00–09:25.

The Most Selective Path is innovatively proposed which refers to the path with the smallest weight. considering the MSP, the Trip Betweenness Centrality (TBC), is proposed. Compared to BC, TBC is more effective to measure station's capacity of passenger flow attraction. Stations with higher TBC serve more passenger trips. When the subjective views factor of passengers is introduced, the topological significance of some stations with TBC_i index has increased compared with BC_i, such as L10S17, L10S15, L10S14, L10S13.

As the second contribution of this paper, the cascade dynamics of passenger flow congestion model of urban rail transit network in a train delay scenario, is established with a comprehensive consideration of TBC, the function of transportation capacity of the lines, the factor of congestion time delay and parameters of ingress volume at the stations. Taking Beijing Metro Line 10 as an example and in combination with the actual data of passenger flow, the outcome of simulation has established. What can be acquired from the cascading propagation model basically corresponds to the actual data of passenger flow.

There are two typical cases of state transition within the time interval 7:50–08:00, which means that serious imbalance of the passenger flow load in the entire network occur. The congestion of passenger flow arising from turnout failure at section L10S5 to L10S6 influences many other lines. At 08:55, there is a relative balance of load of the passenger flow in the whole network within the acceptable range of passengers. Next work, we will go on our research into the propagation rules of passenger flow congestion and the optimization of the function of line capacity in the model.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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