Multi-commodity Optimization of Peer-to-peer Energy Trading Resources in Smart Grid

Olamide Jogunola, Bamidele Adebisi, Kelvin Anoh, Augustine Ikpehai, Mohammad Hammoudeh, and Georgina Harris

Abstract—**Utility maximization is a major priority of prosumers participating in peer-to-peer energy trading and sharing (P2P-ETS). However, as more distributed energy resources integrate into the distribution network, the impact of the communication link becomes significant. We present a multi-commodity formulation that allows the dual-optimization of energy and communication resources in P2P-ETS. On one hand, the proposed algorithm minimizes the cost of energy generation and communication delay. On the other hand, it also maximizes the global utility of prosumers with fair resource allocation. We evaluate the algorithm in a variety of realistic conditions including a time-varying communication network with signal delay signal loss. The results show that the convergence is achieved in a fewer number of time steps than the previously proposed algorithms. It is further observed that the entities with a higher willingness to trade the energy acquire more satisfactions than others.**

Index Terms—**Distributed algorithm, social welfare, peer-to-peer energy trading and sharing, multi-commodity networks, economic dispatch, packet loss, peer-to-peer energy trading, distributed dual-gradient (DDG).**

I. Introduction

SINCE the past decade, the advances in technology have been driving the rise of distributed energy resources (DERs) at the community level [1]-[4]. These DERs create a chain of independent energy producers and consumers that coexist with capacities and demands of different energy generations [1]. The existence of these prosumers could result in power grid instability and unreliability if their energy supply and demand requirements are not properly coordinated. A common approach of energy coordination and control is utilizing distributed control algorithm to eliminate the single point of failure in centralized control systems [5], [6].

Distributed algorithms have been proposed in the literature for energy coordination [7] and in peer-to-peer (P2P) energy trading [5], [8]. In these algorithms, each prosumer keeps a local approximate value of its energy profile and communicates this estimated value directly to its connected neighbor. The energy profiles of all the prosumers converge to an optimal value over a communication network [9]. In [8] and [10], a perfect communication link is assumed between these prosumers, hence the typical communication issues are ignored in real networks.

In practical P2P networks with digital capabilities, end-to-end transaction may be affected by several communication-related factors including topology, jitters, latency, reliability and attenuation due to weather, physical environment or contingencies on the link and the number of P2P prosumers in the network. Other factors include link capacity and message size [11], [12]. This has been exemplified using a distributed consensus algorithm [13] that fails to converge in the presence of prolonged communication delays. Some recent research works consider the impact of the imperfect communication links on optimal dispatch of the energy among DERs [9], [14]-[17]. The performance metrics include communication delay [14], [15], time-varying topologies [16], time-varying directed network with delays [17], and unreliable communication links subject to packet drops [9]. While one or two network constraints of imperfect communication links are considered in these studies, a typical economic dispatch problem (EDP) should encompass more to incorporate the diverse generation mix of DERs in the power grid.

In a distributed EDP involving several DERs, the underlying communication network has a huge effect on the ability of the prosumers to reach a consensus on the optimality of their energy demand and generation cost. Thus, maximizing the economic benefits of P2P energy trading and sharing (P2P-ETS) while respecting sustainability and environmental obligations is crucial to incentivize prosumers' participation in P2P-ETS market. P2P-ETS is a collective term to indicate P2P energy interaction which could include energy trading, energy sharing, energy exchange, etc. We aim to solve the

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EDP among DERs over realistic imperfect communication links. Furthermore, the utility derived based on the optimal distributed EDP is assessed. Fairness is considered in the allocation of network resources to ensure a balanced energy network.

We employ the multi-commodity network flow (MCNF) optimization [18], [19] for optimizing the distributed flow of resources in a distribution network. This is because MCNF optimization offers the opportunity to consider the communication links whilst solving the optimization tasks for energy trading. For instance, MCNF optimization provides the insight into both the communication and the energy transfer between prosumers, which can be modeled simultaneously. The suitability of MCNF for dynamic energy management has recently been assessed by [20] and applied to the smart grid in [21]-[23]. The results show faster convergence and robustness to delay and packet loss in delay-sensitive networks such as smart energy systems.

The main contributions of this paper are summarized as follows.

1) MCNF optimization is presented that allows dual optimization of energy and communication resources in P2P-ETS where the prosumers work in consensus to meet aggregate demand and maximize their utilities.

2) Although EDP is previously investigated in [9], [13], [15], [17] without considering imperfect communication links, the proposed algorithm offers faster convergence with the imperfect communication links characterized by signal delay, signal loss, and asynchronous communication. These imperfections usually result in stringent impacts on the optimal utility of the prosumers due to stale energy prices.

3) In addition, the optimization of utility satisfaction perceived by the prosumers is evaluated considering such imperfect communication links in the smart grid with an interest in fair allocation of the distribution network in terms of supply and demands.

The rest of the paper is organized as follows. The literature review is presented in Section II. The problem formulation including MCNF optimization, and utility maximization among P2P energy traders are presented in Section III. The simulation and results are discussed in Section IV. Section V concludes and identifies the future work.

II. Literature Review

The performance limitation posed by centralized control approaches for energy dispatch among DERs connected at the edges of the power distribution network has birthed the increasing proposals on distributed algorithms [9], [13], [15], [17], [24] for energy control and P2P energy trading.

A P2P energy trading scheme is proposed in [25] using a leader-follower Stackelberg game for the power system to reduce its electricity demand during peak hours. For additional control to reduce the curtailment of renewable generation, [26] proposes a local energy market for distribution systems integrating P2P energy trading with locational marginal pricing. To increase users' participation in P2P energy trading, a game-theoretic design is proposed in [27], which shows the potential in attracting users to participate in the energy trading for more reductions of carbon and cost, through a proposal of a bilateral contract in [28].

A mixed-integer linear programming-based predictive design and a dispatch optimization algorithm are proposed in [29], while [24] utilizes a two-level incremental cost consensus distributed algorithm to solve EDP in smart grid. With the evolving digitization of power grid, communication systems have become an integral component of the smart grid, which poses the problems of stale energy prices due to time delay and packet losses from imperfect communication links. Thus, the influence of time delays on the distributed algorithms is investigated in [13] and [15]. In [13], the influence of time delays over different types of information exchanged among DER is investigated, and it is found that the consensus algorithm either converges to an incorrect value or fails to converge altogether. Further, [17] proposes a distributed algorithm based on push-sum and gradient method to solve the EDP among connected DERs over fixed and time-varying network delays. Reference [9] proposes a robustified extension of [17] using the same method but solves the coordination problem over packet-dropping communication links. Other efforts for reducing the communication delay in DERs are found in [30], [31].

To maximize the social welfare of generators and consumers, [32] proposes an incremental welfare distributed consensus algorithm, which is further extended in [33] to incorporate transmission loss and directed communication topologies. In [34], a social welfare maximization problem using open control law to minimize the generator and load adjustment rates is addressed. In contrast, we present the use of MCNF optimization in solving EDP in smart grid considering the dis-joint electrical and communication variables. The results are then analyzed for maximizing prosumer social welfare in a P2P energy trading network.

III. Problem Formulation

Figure 1 presents the results for ideal case showing the convergence of generated energy and incremental cost. In Fig. 1(a), the physical network represents the physical connectivity of the prosumers depicted as P to a distribution network. The virtual network denotes a nodal representation of the physical nodes to reflect the communication among them. Figure 1 also illustrates the relationships between the assets (physical network and virtual network) and multi-commodity modeling (EDP, and social welfare of prosumers) presented in this paper.

To formalize the relationships, let the connectivity of the prosumers be represented using a strongly connected graph network, which models the pairwise relations between nodes and links. The nodes are called the vertices, and the links connecting the vertices are the edges. In this model, the strongly connected energy network is defined by graph $G=(V,E)$ of $V=\{1,2,\dots,N\}$ interconnected nodes, $E\subseteq V\times V$ sets of bidirectional links of any interconnected prosumers n_p and n_c, and N is the total number of energy prosumers in the network. Note that $\mathcal{P}\triangleq\{1,2,\dots,n_p\}$ represents the set of energy generators with index $i\in\mathcal{P}$ and $\mathcal{C}\triangleq\{1,2,\dots,n_c\}$ is the set of energy consumers with index $j\in\mathcal{C}$. No prosumer has the combined characteristics of generator and consumer simultaneously at a given trading period t, and thus $\mathcal{P} \cap \mathcal{C} = \varnothing$. It also follows that the set of all prosumers is $V = \mathcal{P} \cup \mathcal{C}$ and the total number of prosumers is $N \triangleq n_c + n_p$. The goal of each prosumer is to optimize its energy output and maximize its benefit and to collectively meet the total energy demand in a distributed way.

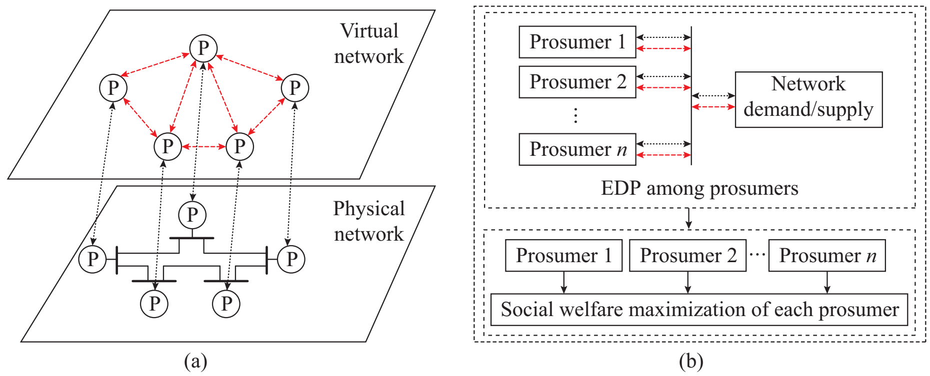

Fig. 1 Results for ideal case showing convergence of generated energy and incremental cost. (a) An IEEE 5-bus system of prosumers showing physical and virtual connectivities. (b) Schematic of EDP and social welfare maximization problems.

A. Communication Network

The power network is overlayed by a communication network that conveys energy trading messages as shown in Fig. 1(a). Let the communication network be represented as a time-varying graph $G(t) = (V, E(t))$ with $E(t)$ links, where each set of links changes over time based on the state of the communication link at time t. A directed link from prosumer n_p to prosumer n_c is denoted by $(i,j) \in E(t)$. Each directed link $(i,j) \in E(t)$ is characterized by its upper bounds of energy trading messages through the link u_{ij}, delay $\bar{k}_{ij}$, and signal loss probability f_{ij} on links connecting n_p to n_c.

B. Energy Generation and Demand

For tractable solutions, we assume that the prosumers are virtually clustered using communication network into M virtual microgrids (VMGs) [35]. We are interested in minimizing the costs of energy generation and maximizing social benefits within the VMG for prosumers. This problem can be approached by minimizing the total aggregated energy cost and assuming small clusters of energy generators. Through clustering, energy demand can be matched with a supplier within the m^{th} VMG in a local P2P energy trading fashion [36]. Let $\mathcal{M} = \{1, 2, \ldots, M\}$ be the set of VMGs such that $m \in \mathcal{M}$. Thus, during the trading period t, there exist $E_m(t)$ sets of links and $D_m(t)$ sets of demands in the m^{th} VMG. Each VMG is thus characterized by $n_p \in \mathcal{P}$ energy generators, producing $x_i(t), i \in n_p$ units of energy. To cluster the prosumers, non-commodity charges could be reduced for optimal node density and the cost of energy trading could also be reduced [35], [36]. The generation cost minimization problem during the trading period $t \in \mathcal{I}_{ij}$ is formulated as:

$$\min_{\{x_i\}} \sum_{t \in \mathcal{I}_{ij}} \sum_{i \in n_p} C_i x_i(t) \quad (i,j) \in E_m(t), m \in \mathcal{M} \tag{1}$$

s.t.

$$\sum_{t \in \mathcal{I}_{ij}} \sum_{i \in n_p} x_i(t) \le D_m(t) \quad (i,j) \in E_m(t), m \in \mathcal{M} \tag{2}$$

$$x_i \ge 0 \quad \forall i = 1, 2, \ldots, n_p \tag{3}$$

where C_i is the cost function of prosumer $i \in n_p$ for generating energy x_i. It is assumed that the cost function follows a convex function model for tractability. The model in (1)-(3) implies that the total generated energy within a cluster m must satisfy the total energy demand for energy conservation to hold. Henceforth, we shall focus on a single period of one hour in the P2P energy market similar to [37] as the single-period problem can be extended to a multi-period problem with temporally coupled constraints. The solution for the single-period problem demonstrates the performance of the proposed algorithm in a more explicit manner [37]. The notation t should be dropped in (1) and (2).

C. Energy as MCNF Problem

The most basic MCNF problem can be represented as:

$$\min_{\{x_{ij,k}\}} \sum_{k \in K} \sum_{(i,j) \in E_m} C_{ij,k}(x_{ij,k}) \quad m \in \mathcal{M} \tag{4}$$

where $x_{ij,k}$ is the energy flow of commodity k on the link between the nodes n_p and n_c; K is the message from different peers in the distribution network; and $C_{ij,k}(\cdot)$ is the cost function of energy flow in the links, which is the convex monotonically increasing function [20]. The decision variable in this model is the energy flow $x_{ij,k}$, which must follow flow conversation criterion for the power network to be balanced, i.e., the energy flow entering the node must be equal to the energy flow leaving the node. In addition, energy flows through the links are limited by lower and upper bounds, which translates to the maximum energy flowing through the link at the trading period t. For consistency, throughout the rest of this paper, the term commodity represents the message flows from different prosumers, which is a communication parameter.

Without the loss of generality, the EDP can be represented as an MCNF optimization of (4), subject to the following constraints:

$$\sum_{k\in K}\sum_{(i,j)\in E_m} x_{ij,k} \le D_m \quad m\in\mathcal{M} \tag{5}$$

$$l_{ij,k} \le x_{ij,k} \le u_{ij,k} \quad \forall (i,j)\in E_m, m\in\mathcal{M} \tag{6}$$

$$x_{ij,k} \ge 0 \quad \forall k\in K, \forall (i,j)\in E_m, m\in\mathcal{M} \tag{7}$$

$$x_{ij,k}^{\min} \le d_{ij} \le x_{ij,k}^{\max} \quad \forall d_{ij}\in D, (i,j)\in E_m \tag{8}$$

where d_{ij} is the demand at each bus, so that $\sum_{(i,j)\in E_m} d_{ij}=D$; $u_{ij,k}$ and $l_{ij,k}$ are the upper and lower bounds of energy flows in the link (i,j), respectively; and $x_{ij,k}^{\max}$ and $x_{ij,k}^{\min}$ are the upper and lower bounds of power generation of n_p, respectively. Constraint (5) is the conservation of energy flow constraint. Constraint (6) is the upper and lower bounds of energy flows in the links, which must not exceed the capacity of the link. Constraint (7) represents non-negativity constraints, i. e., a generation unit must generate energy x_i, satisfying the lower and upper bounds of their generation capacities as shown in (8).

D. Dual Lagrange Problem for EDP

To solve the minimization over $x_{ij,k}$ of (4), we first present its dual Lagrange problem followed by the derivation of the distributed (sub)gradient algorithm. The Lagrangian function $\mathcal{L}(x,\lambda)$ to relax the flow conservation constraints of problem (4) is:

$$\begin{cases}\mathcal{L}(x,\lambda)=\sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k})-\sum_{k\in K}\lambda_{ij,k}D+\sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}x_{ij,k}\\ \text{s.t. (6)-(8)}\end{cases} \tag{9}$$

where $\lambda_{ij,k}\ge 0$ is the Lagrange multiplier and the incremental cost associated with the energy flow constraint. This is usually an optimal parameter, which ensures that the constraint conditions are not violated. Problem (9) and constraints (6)-(8) are summarized as (10) and further expressed as (11).

$$\mathcal{L}(x,\lambda)=\sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k})-\sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}D+\sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}x_{ij,k} \tag{10}$$

$$\mathcal{L}(x,\lambda)=\sum_{k\in K}\sum_{(i,j)\in E_m} C_{ij,k}(x_{ij,k})-\sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}d_{ij,k}+\sum_{k\in K}\sum_{(i,j)\in E_m}\lambda_{ij,k}x_{ij,k} \quad m\in\mathcal{M} \tag{11}$$

where $\sum_{(i,j)\in E_m} d_{ij,k}=D$ and $k\in K$. Note that the model discussed is peculiar to energy trading and may include energy sharing when producers do not charge peers. The model (11) can further be summarized in terms of the energy flows, thus we can obtain:

$$\mathcal{L}(x,\lambda)=\sum_{k\in K}\bar{C}_{ij,k}(x_{ij,k})+\sum_{k\in K}\lambda_{ij,k}\bar{x}_{ij,k}-\sum_{k\in K}\lambda_{ij,k}\bar{d}_{ij,k} \quad m\in\mathcal{M} \tag{12}$$

where $\bar{C}_{ij,k}(\cdot)=\sum_{(i,j)\in E_m} C_{ij,k}(\cdot)$; $\bar{x}_{ij,k}=\sum_{(i,j)\in E_m} x_{ij,k}$; and $\bar{d}_{ij,k}=\sum_{(i,j)\in E_m} d_{ij,k}$. The argument $x_{ij,k}^\star$ that minimizes the Lagrangian given in (12) by following a dual decomposition formulation can be expressed as:

$$\begin{cases}x_{ij,k}^\star=\arg\min\limits_{(6)-(8)}\mathcal{L}(x,\lambda)\\ \text{s.t. } \lambda_{ij,k}\ge 0 \quad k\in K\end{cases} \tag{13}$$

When $C_{ij,k}(\cdot)$ is strictly convex, and the cost function can be investigated for the optimum (minimum) value.

The dual objective function $w(\cdot)$ enables each energy generator in the distribution network to participate in solving the distributed optimization of the energy traded in the distribution network. This is quite scalable and efficient and also could improve the trust level in the distribution network. Besides, the energy trading information of each prosumer is private and thus the optimization problem cannot be solved centrally because the central agent cannot access the private energy information. Thus, the dual objective function is expressed as:

$$w(\lambda_{ij,k})=\min_{x_{ij,k}\ge 0}\mathcal{L}(x,\lambda_{ij,k})=\min_{x_{ij,k}\ge 0}\sum_{k\in K}\bar{C}_{ij,k}(x_{ij,k})+\sum_{k\in K}\lambda_{ij,k}\bar{x}_{ij,k}-\sum_{k\in K}\lambda_{ij,k}\bar{d}_{ij,k}=\sum_{k\in K}\min_{x_{ij}\ge 0}(\bar{C}_{ij,k}(x_{ij,k})+\lambda_{ij,k}\bar{x}_{ij,k}-\bar{\lambda}_{ij,k}d_{ij,k}) \tag{14}$$

Clearly, (14) shows a fully $k\in K$ distributed problems that each energy generator i participates in solving the problem. The optimal dual solution can be estimated in terms of the Lagrange of the dual function problem as:

$$w^\star(\lambda_{ij,k}^\star)=\max_{\lambda_{ij,k}\ge 0} w(\lambda_{ij,k}) \tag{15}$$

where $\lambda_{ij,k}^\star$ is the optimal pricing information, which is required to establish $x_{ij,k}^\star$ transferred by the generator unit to the demand unit. This can be realized through an update of the pricing information in an iterative fashion.

E. Distributed Dual-gradient (DDG) Algorithm for EDP

Problem (15) is solved using the (sub)gradient algorithm, which is a generalization of the gradient descent, using the iterations as:

$$\lambda_{ij,k}(\tau+1)=\left[\lambda_{ij,k}(\tau)-\alpha_\tau g(\tau)\right]^+ \quad k\in K, (i,j)\in E_m \tag{16}$$

where α_τ is the step size at time τ; and $g(\tau)$ is a (sub)gradient to $w(\lambda_{ij,k})$ at $\lambda_{ij,k}(\tau)$. Note that $[s]^+=\max(s,0)$.

Assumption: since the cost function within the dual objective function is strictly convex, the dual function $w(\lambda_{ij,k})$ is continuously differentiable [38].

The (sub)gradient $g(\tau)$ is realized by taking the first derivative of (14) and setting the result equal to zero.

$$g(\tau)=\frac{\partial w(\lambda_{ij,k})}{\partial\lambda_{ij,k}}=0\Rightarrow-\left(\sum_{k\in K}\bar{d}_{ij,k}-\sum_{k\in K}\bar{x}_{ij,k}\right)=0 \tag{17}$$

Substituting (17) into (16), a (sub)gradient update of (15) along each dual variable is obtained and expressed as:

$$\lambda_{ij,k}^{(\tau+1)}=\left[\lambda_{ij,k}^{(\tau)}-\alpha_\tau\left(\sum_{k\in K}\bar{x}_{ij,k}-\sum_{k\in K}\bar{d}_{ij,k}\right)\right]^+ \quad (i,j)\in E_m \tag{18}$$

As can be seen in (18), when the demand is greater than the supply, the generators will increase the price of the excess demand energy units by α_τ. For example, when $\sum_{(i,j)\in E_m} d_{ij,k}>\sum_{(i,j)\in E_m} x_{ij,k}$, $\alpha_\tau\left(\sum_{k\in K}\bar{x}_{ij,k}-\sum_{k\in K}\bar{d}_{ij,k}\right)$ in (18) will be greater than zero which leads to $[s]^+=\max(s,0)>0$. The dual variables are updated bi-directionally and synchronously at discrete time $\tau=\{0,1,\ldots,\infty\}$, and only neighbors can communicate. For instance, each generation unit will wait a random time before transmitting the next update of its generated output. At every time step, there is an upper bound on the optimal value of the Lagrange function (9), which is obtained by evaluating the dual objective function (15). Each link computes its (sub)gradient coordinate using the energy flow variable $x_{ij,k}$. To reduce excess overheads and delay that could result in assigning additional scalar variables to the estimate of each generator unit at each iteration as shown in [9], the information communicated among the generators is completely distributed and limited to the incremental cost $\lambda_{ij,k}$. The novelty lies in that each generator ensures that the price is used as an indicator function to generate the required energy that satisfies the network demand. $\lambda_{ij,k}$ is used by each generator to update its generation output $x_{ij,k}$ at the k^{th} flow. Note that the model (18) is changed to a consensus problem when all the incremental costs $\lambda_{ij,k}$ are identically equal to zero [39].

F. Modeling Communication Delay and Signal Loss

The robustness of an algorithm can be measured, i.e., its ability to converge in the presence of faults which could result from the out of sequence delivery or signal loss. In a consensus network where all peers are minimizing their objectives to achieve a collective goal, the higher the transmission delay in such a network, the longer it takes for the peers to reach the desired agreement. Communication delay is prevalent in distributed networks. Therefore, we observe the robustness of DDG when the communication network is subjected to high signal/transmission delay. In a realistic scenario, there is always a communication delay $\bar{k}_{ij}(\tau)$ on the communication link (i,j) in sending a message from n_p to n_c. Similarly, there exists an end-to-end time delay $\tau+\bar{k}_{ij}(\tau)$ to receive a response from n_c by n_p [40]. The impact of high-signal delay would lead to an outdated link cost in the gradient iteration, which would generate algorithm oscillations without reaching an optimal solution. The gradient update of (18) becomes:

$$\lambda_{ij,k}^{(\tau+\bar{k}_{ij}(\tau)+1)}= \left[\lambda_{ij,k}^{(\tau+\bar{k}_{ij}(\tau))}-\alpha_\tau\left(\sum_{(i,j)\in E_m} x_{ij,k}-\sum_{(i,j)\in E_m} d_{ij,k}\right)\right]^+ \quad \forall (i,j)\in E_m, m\in\mathcal{M} \tag{19}$$

Reference [40] shows that the introduction of communication delay would not affect the convergence speed of the algorithm but would result in convergence to a larger neighborhood of the optimal value. However, the choice of step size determines the algorithm convergence. In this paper, a constant step size is used, which converges to optimal value when the objective function is differentiable [41], [42].

Similarly, a probabilistic approach [9] is employed to model the signal loss on the communication link. A communication between n_p and n_c is viewed to be successful when the information sent by n_p is received by n_c without loss in real time. However, due to signal loss on $(i,j)\in E$, a failure set $f_{ij}(\tau)$ is introduced, where $f_i(\tau)=1$ if the communication from prosumer i at iteration τ is received, otherwise $f_i(\tau)=0$. Thus, $w(\lambda_{ij,k})$ in (14) becomes:

$$\left[\sum_{(i,j)\in E_m} f_i f_j\left(\sum_{k\in K}\min_{x_{ij}\geq 0}\bar{C}_{ij,k}(x_{ij,k})-\bar{C}_{ji,k}(x_{ji,k})+ \lambda_{ij,k}((x_{ij,k}-d_{ij,k})-(x_{ji,k}-d_{ji,k}))\right)\right]^+ \quad \forall (i,j)\in E_m, m\in\mathcal{M} \tag{20}$$

G. Resource Allocation for P2P-ETS

The energy market of prosumers is further considered for a fair allocation of communication resources. The weighted general fairness utility model $U(x_i^*)$ is given by:

$$U(x_i^*)=\omega_i\frac{(x_i^*)^{1-\sigma}}{1-\sigma} \quad i\in n_p, m\in\mathcal{M} \tag{21}$$

where σ is the fairness parameter; ω_i is the weight associated with the utility of prosumer i; and x_i^* is the optimal energy resources obtained from solving the EDP problem using MCNF. Reference [36] shows that the utility model follows the weighted concave function of the energy resources as:

$$U(x_i^*)=\omega_i\ln(x_i^*) \quad \forall i\in n_p, m\in\mathcal{M}. \tag{22}$$

Suppose that $x_i^*=0, \forall i\in n_p$, then $\ln(x_i^*)=-\infty$. To overcome this problem, a constant $\theta_i\geq 1$ is introduced so that the utility becomes $U(x_i^*)=\omega_i\ln(x_i^*+\theta_i), i\in n_p$. We aim to maximize the resources allocated to a prosumer over a finite link capacity. This is approached by maximizing the utility of each prosumer subject to a capacity constraint, considering the optimization variable as the energy resources traded over the finite link. Then, the optimization problem becomes:

$$\begin{cases}\max\limits_{\{x_i^*\}_{i\in n_p}} \sum\limits_{i\in n_p} U(x_i^*) \\ \text{s.t.} \sum\limits_{i:\ell\in i} x_i^*\leq c_\ell \quad \forall \ell\in n_p \\ \quad x_i^*\geq 0 \quad i\in E_m, m\in\mathcal{M}\end{cases} \tag{23}$$

where c_ℓ is the capacity of link ℓ.

Figure 2 demonstrates the utility function of five prosumers in a distribution network with different weights ω_i. The utility increases for varying increasing weights of the energy traders. Physically, the weights may be interpreted to the willingness to trade energy with other peers. Prosumers with a higher willingness to trade the energy achieve higher utility than other prosumers with little or no willingness.

H. Optimal Resource Allocation

Invariably, if the utility is proportional to the willingness, higher energy flow will be experienced in the distribution network, thus the resources must be fairly and optimally allocated to each prosumer so as not to starve other prosumers in the network. We consider the fairness parameter $\sigma=1$ as shown in (21). Next, the optimal resources are considered to be realized and allocated to the link $i\in n_p, m\in\mathcal{M}$ considering the capacity $c_\ell, \forall \ell\in E_m$. By taking the Lagrangian of (23), we can obtain:

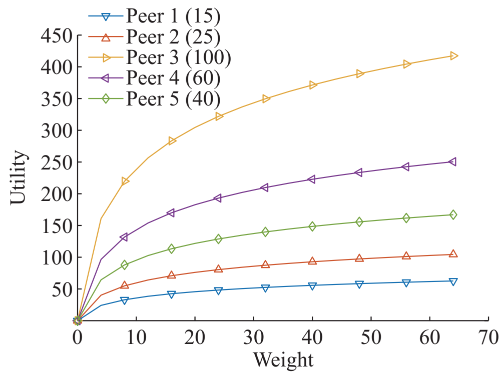

Fig. 2 Relationship between utility function and varying weights of prosumers.

$$\mathcal{F}(x,\eta)=\sum_{i\in n_p}U(x_i^*)-\left(\sum_{i\in n_p}x_i^*\eta_i-\sum_{i\in n_p}\eta_i c_\ell\right)\quad \ell\in E_m \tag{24}$$

where $x=(x_1,x_2,\ldots,n_p)$ and $\eta=(\eta_1,\eta_2,\ldots,n_p)$ are the Lagrangian multipliers. By taking the first derivative of (24) with respect to x_i and setting the result equal to zero, the following is derived as:

$$\frac{\partial\mathcal{F}(x_i^*,\eta_i)}{\delta x_i^*}=0\Rightarrow\sum_{i\in n_p}\frac{\omega_i}{x_i^*+\theta_i}-\sum_{i\in n_p}\eta_i=0\Rightarrow$$
$$\sum_{i\in n_p}x_i^{**}=\frac{\sum_{i\in n_p}\omega_i-\sum_{i\in n_p}\eta_i\theta_i}{\sum_{i\in n_p}\eta_i}\Rightarrow x_i^{**}=\frac{\omega_i-\eta_i\theta_i}{\eta_i}$$
$$\forall i\in n_p, m\in\mathcal{M} \tag{25}$$

From (25), the optimal resource allocation x_i^{**}, $\forall i\in n_p$, $m\in\mathcal{M}$ depends on the congestion price η_i and the number of prosumers on the link ℓ. For example, the reduction of the resource flow on the link ℓ due to prosumer i implies the increase of the congestion price η_i. Similarly, the increase of the resource flow due to prosumer i implies the reduction of the network congestion price η_i. In addition, from (25), increasing the congestion price will be useful in controlling the congestion in the distribution network since a lower amount of data will be sent by each prosumer over the link ℓ.

I. Social Welfare Maximization

Using the foregoing utility function, this subsection introduces a social welfare maximization objective to improve the overall costs and maintain the fairness for all generators and demands. $\mathcal{W}$ represents the total social welfare of prosumers, i. e., producers $W_i(\cdot),\forall i\in n_p$ and consumer $W_j(\cdot)$, $j\in n_c$, and $\bar{p}$ is the price of electricity.

$$\max_{\{x_i^*,d_j\}}\mathcal{W}=\sum_{i\in n_p}W_i(x_i^*,\bar{p}_i)+\sum_{j\in n_c}W_j(d_j,\bar{p}_j) \tag{26}$$

s.t.

$$\sum_{i\in n_p}x_i^*=\sum_{j\in n_c}d_j \tag{27}$$

$$x_i^{*\min}\leq x_i^*\leq x_i^{*\max}\quad i\in n_p \tag{28}$$

$$d_j^{\min}\leq d_j\leq d_j^{\max}\quad j\in n_c \tag{29}$$

Constraint (27) is the conservation of energy flow constraint. The operation constraints (28) and (29) represent the lower and upper bounds of energy generation and consumption, respectively, which are further defined as follows.

1) Welfare for generating prosumer: $\bar{p}_i x_i^*,\forall i\in n_p, m\in\mathcal{M}$ represents the revenue that prosumer i receives from selling x_i^* units of the energy with selling price $\bar{p}_i$, then the social welfare of the prosumer i can be expressed as:

$$W_i(x_i^*,\bar{p}_i)=\bar{p}_i x_i^*-C_i(x_i^*) \tag{27}$$

where $C_i(x_i^*)$ is the cost incurred by i to generate x_i^* units of the energy shown in (2). We model the cost as a convex quadratic function of the form as:

$$C_i(x_i^*)=\frac{1}{2}a_i(x_i^*)^2+b_i x_i^*+c_i \tag{28}$$

where $a_i\geq 0$, $b_i>0$, and $c_i=1,\forall i\in n_p$ are the cost parameters.

2) Welfare for consuming prosumer: at the consumer side, social welfare is the difference between the utility it derives and the costs of procuring $x_j^*, j\in n_c$ units of energy.

$$W_j(d_j,\bar{p}_j)=U_j(d_j)-\bar{p}_j x_j^*\quad j\in n_c \tag{29}$$

where $U_j(d_j)$ is the utility function that defines the amount of satisfaction that prosumer j receives from demanding d_j units of the energy; and $\bar{p}_j$ is the payment made for d_j. As shown in (22), the utility function of the consumer is continuously differentiable and non-decreasing.

Substituting (29) and (27) into the prosumer welfare in (26), respectively, the optimization problem becomes:

$$\max_{\{x_i^*,d_j\}}\mathcal{W}=\sum_{j\in n_c}U_j(d_j)-\sum_{i\in n_p}C_i(x_i^*) \tag{30}$$

s.t.

$$\sum_{i\in n_p}d_j\leq\sum_{j\in n_c}x_i^* \tag{31}$$

$$x_i^{*\min}\leq x_i^*\leq x_i^{*\max}\quad i\in n_p \tag{32}$$

Note that in (30), the power balance criteria defined in (5) enables $\sum_{i\in n_p}\bar{p}_i x_i^*-\sum_{j\in n_c}\bar{p}_j d_j$ to be eliminated. Due to the concave properties of (30), the model in (30)-(32) is a concave maximization problem and can be solved using convex programming algorithms. The model terms in (30)-(32) are individually differentiable. Thus, we involve the use of DDG in solving the welfare maximization problem, which is similarly applied in the literature [36], [43]. Therefore, we start by formulating the Lagrangian of problem (30)-(32) as:

$$\mathcal{J}(d_j,x_i^*,\rho_{ij})=\sum_{j\in n_c}U_j(d_j)-\sum_{i\in n_p}C_i(x_i^*)-\rho_{ij}\left(\sum_{j\in n_c}d_j-\sum_{i\in n_p}x_i^*\right) \tag{33}$$

where ρ_{ij} is the Lagrangian multiplier. In terms of producers and consumers, problem (33) can be decomposed and solved in a distributed fashion as:

$$\mathcal{J}(d_j,\rho_{ij})=\sum_{j\in n_c}U_j(d_j)-\sum_{j\in n_c}d_j\rho_{ij} \tag{34}$$

$$\mathcal{J}(x_i^*,\rho_{ij})=\sum_{i\in n_p}\rho_{ij}x_i^*-\sum_{i\in n_p}C_i(x_i^*) \tag{35}$$

By taking the first derivatives of (34) and (35) with respect to d_j and x_i, and (33) with respect to ρ_{ij}, respectively, and setting the result to be zero, the optimal flow variables can be expressed as:

$$\frac{\partial \mathcal{J}(d_j, \rho_{ij})}{\partial d} = 0 \Rightarrow d_j^* = \frac{\omega_j - \rho_{ij}}{\rho_{ij}} \tag{36}$$

$$\frac{\partial \mathcal{J}(x_i^*, \rho_{ij})}{\partial x_i} = 0 \Rightarrow x_i^* = \frac{\rho_{ij} - b_i}{a_i} \tag{37}$$

$$\frac{\partial \mathcal{J}}{\partial \rho_{ij}} = 0 \Rightarrow \rho_{ij} = -\sum_{j \in n_c} d_j + \sum_{i \in n_p} x_i^* \tag{38}$$

From (36), the demand is inversely proportional to the price. The prices of energy will be assessed by the consumers to buy more energy units at low prices or buy fewer energy units at higher prices. From the producers' side in (37), they are motivated to supply more at higher prices and vice versa. From (38), the update price function $\rho_{ij,n+1}$ is expressed as:

$$\rho_{ij,n+1} = \left[\rho_{ij,n} - \alpha_n \left(\sum_{i \in n_p} x_i^* - \sum_{j \in n_c} d_j\right)\right]^+ \tag{39}$$

In (39), the energy producers assign an additional α_n penalty to the network fees at the n^{th} time step if the total demand d_j exceeds the total supply x_i^* in the distribution network. However, the fees lower than the network fees will not be charged due to $[\cdot]^+$.

IV. Numerical Simulation and Result Analysis

To evaluate the performance of the developed distributed algorithm for EDP, simulations are performed using Java [19], [44]. Five prosumers adopted from [9] are considered for comparison, where an IEEE 5-bus system is applied as shown in Fig. 1. The generation cost function is set to be a value of ±20 kWh of each prosumer's demand. For instance, each prosumer generates ±20 kWh of the energy above or below its demand, which serves as flow bounds for each prosumer. We aim to optimize the generation output of each prosumer to satisfy the aggregated energy demand in the distribution network. Five prosumers are connected by 16 links. A set of energy demands in kWh of $d_1 = 40$, $d_2 = 30$, $d_3 = 100$, $d_4 = 40$, and $d_5 = 90$ is considered. The step size α is set to be a constant value of 1 for most of the cases considered. As stated in [22], with a constant step size of 1, the distribution network achieves lower delay, and the algorithm converges faster.

A. DDG Algorithm: Without Communication Delay

The ideal case without communication delay is the most basic case study that exists in the literature. It is used as a starting point to test the robustness of the proposed algorithm. The stability of an algorithm, defined as the ability to converge to a solution in a finite amount of time, is used to measure the performance and efficiency of the proposed algorithm.

The result is analyzed based on the convergence time of the algorithm. The results for ideal case showing convergence of generated energy and incremental cost are shown in Fig. 3. The optimal generated energies by each of the prosumers are x_i, $i \in M$, where $x_1 = 40$ kWh, $x_2 = 20$ kWh, $x_3 = 115$ kWh, $x_4 = 45$ kWh, and $x_5 = 80$ kWh with a total $\sum_{i=1}^{5} x_i = 300$ kWh. Note that the initial energy demand for prosumer 1 is 40 kWh, and the generation output is 40 kWh. It shows that x_1 generates its own energy, while other prosumers generate the energy below or above their energy demands to satisfy the total demanded energy of 300 kWh in the distribution network.

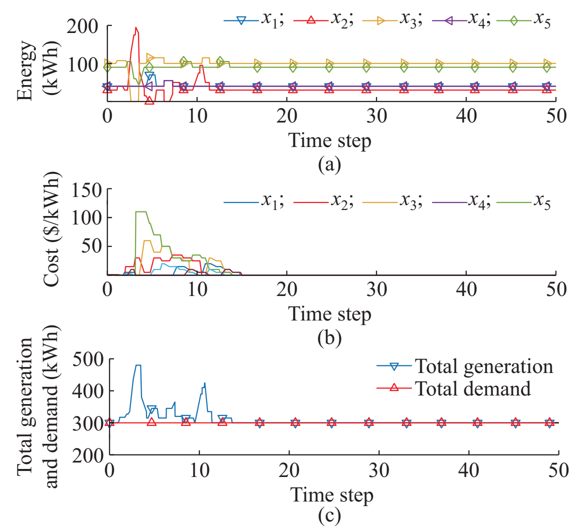

Fig. 3. Results for ideal case without communication delay. (a) Evolution of generated energy from each prosumer. (b) Convergence of incremental costs $\lambda_{ij,k}$. (c) Total generation and total demand by prosumers.

Furthermore, it can be observed from Fig. 3(a) and (c). that at the 3[rd] time step, the total generated energy results in an increase in the cost function, as shown in Fig. 3(b). However, as the output of energy generation descends overtime to meet the demand, the incremental cost equally descends to 0. The increasing cost before the convergence can be interpreted as the need for an additional storage space for the generated energy in excess of demand. Thus, the optimization algorithm minimizes the cost by solving the EDP when the generated energy meets the demand.

To explore the scalability of the optimization algorithm, Fig. 4 shows the convergence of the cost function for 5, 10, 15, 22, and 30 prosumers with the total energy demands of 300 kWh, 600 kWh, 900 kWh, 1300 kWh, and 1800 kWh, respectively. The network with 5 prosumers converge faster than the network with 10, 15, and 30 prosumers. A network consisting of 15 prosumers attains an optimal value of 900 kWh at the 25[th] time step as compared with that of 22 prosumers that attain an optimal value of 1300 kWh at the 42[nd] time step. Note that during the simulation, the computation time and the number of iterations per time step increase as the number of prosumers increases.

In [24], a two-level incremental cost consensus (ICC) algorithm is proposed to solve the EDP in the smart grid. A comparison test of the convergence time in this paper to the ICC algorithm [24] is shown in Fig 5. It can be observed that the DDG algorithm converges faster than the ICC algorithm.

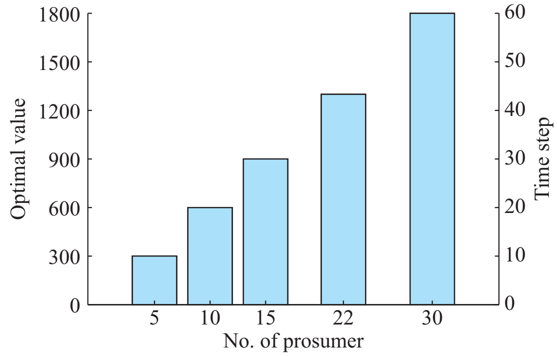

Fig. 4. Scalability results showing convergence of cost function for 5, 10, 15, 22, and 30 prosumers.

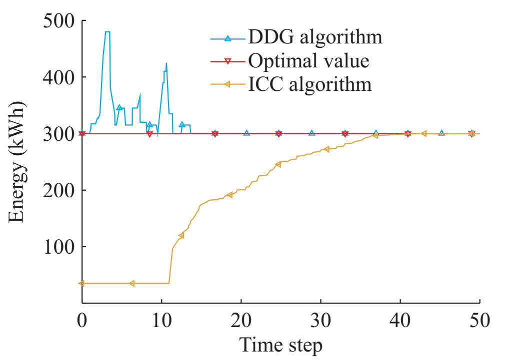

Fig. 5. Comparative analysis of convergence properties for ICC algorithm and DDG algorithm.

For instance, the total energy generation matches the total energy demand at the 14th time step for the DDG algorithm, whereas the ICC algorithm converges at the 38th time step.

It is implied that the EDP can be solved by both consensus algorithms, while the DDG algorithm would be a better choice in a large-scale network.

B. Impact Evaluation of Communication Delay

A communication delay of 10 time steps is adapted from [17] with $\alpha = 1$, and the result is shown in Fig. 6.

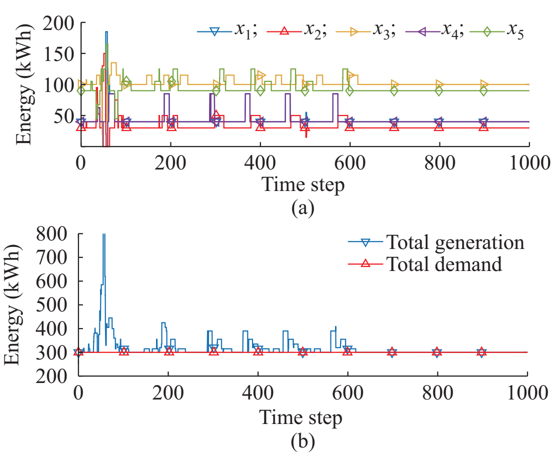

Fig. 6. Results for a network with communication delay showing fast convergence for DDG algorithm. (a) Evolution of generated energy from each prosumer. (b) Total generation and total demand by prosumers.

It can be observed that each of the variables ultimately converges to the optimal value as the ideal case, despite the signal delays, i.e., the convergence occurs at the 610th time step. Compared with the algorithm presented in [17], the DDG algorithm attains its optimal solutions faster, which is shown in Table I for the convergence time analysis. Note that each agent in the algorithm presented in [17] holds a couple of variables that are updated and communicated at each iteration. Whereas, in this paper, the only communicated variable is the incremental cost signifying the time to increase or reduce the generated energy so as to meet the energy demand, which significantly improves the communication delay and leads to faster convergence.

TABLE I
RESULT COMPARISON WITH RELATED WORKS SHOWING CONVERGENCE TIME AND COMMUNICATION-RELATED FACTORS

Description	Ref. [17]	Ref. [9]	This paper
Signal delay	Yes	Yes	Yes
Communication signal loss	No	Yes	Yes
Signal delay and signal loss	No	No	Yes
Asynchronous communication	No	No	Yes
Algorithm convergence time step (result for message delay case)	>900		610
Algorithm convergence time step (result for signal loss probability)		>45	18

Furthermore, Table I compares the contribution of the DDG algorithm with those in the literature by detailing the considered cases and the convergence time. Unlike the research work presented in [9] and [17] that do not consider the cases of both signal loss and signal delay simultaneously.

C. Impact of Communication Signal Loss

In this subsection, an unreliable communication network with a probability of message signal loss on the communication links is considered. Motivated by [9] and for comparison, the signal loss probability is set to be 0.1. The results are shown in Fig. 7.

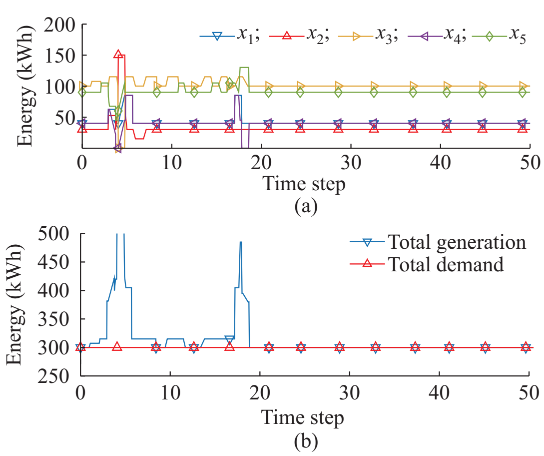

Fig. 7. Results for a network with signal loss probability showing fast convergence for DDG algorithm. (a) Evolution of generated energy from each prosumer. (b) Total generation and total demand by prosumers.

It can be observed that the signal loss probability has a negligible effect on the algorithm convergence which shows a better performance, in terms of convergence time to [9] as shown in Table I. Comparing Figs. 6 and 7, it can be observed that the convergence for the delay is higher than that of signal loss probability. The loss is modeled as a probability function that could occur or otherwise, whereas the delay is a constant value with high significance. For instance, when there are delays in the gradient updates, the prosumers could advertize stale energy prices. Similarly, if the energy flow data is significantly delayed or lost or the bad data are detected, the energy trading information, e.g., price, could be significantly higher or lower than the prices advertized by the neighbors. In such a case, energy producers or consumers could resort to state estimation which elongates the decision and agreement periods.

D. Impact of Communication Delay and Signal Loss

A case is considered, where the communication network is both affected by signal delay of 10 time steps and a signal loss probability of 0.1. The simultaneity of the two network impairments is omitted in [9] and [17], which only consider either a case of delay or packet loss. However, the combined effects of the impairments on the communication network are remarkably different from the effect of each variable in isolation.

As shown in Fig. 8, the algorithm ultimately converges to the optimal value in the ideal case. In addition, the signal delay and signal loss result in the highest communication link cost since the prosumers synchronously transmit their update after the communication delay, thereby oversubscribing the communication links. It is implied that the DDG algorithm is robust against the signal delay and signal loss of the underlying communication link. However, a significant level of signal loss and signal delay might result in algorithm oscillations without being converged.

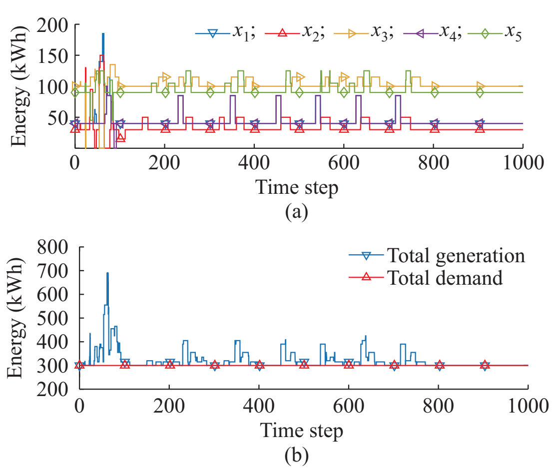

Fig. 8. Results for a network with both communication delay and signal loss probability showing fast convergence for DDG algorithm. (a) Evolution of generated energy from each prosumer. (b) Total generation and total demand by prosumers.

Note that the robustness of the DDG algorithm results from the use of MCNF optimization, as it offers an opportunity to consider the communication links whilst solving the optimization task. In addition, unreliable communication mostly results from link utilization and congestion, which will lead to signal drop and signal delay [22]. By utilizing the MCNF optimization, this paper has already set a limit to the maximal allowed traffic based on the capacity of the communication link at the time, thus reducing the probability of maximal utilization, congestion and signal loss.

E. Numerical Example of Optimal Resource Allocation and Social Welfare

To evaluate the utility model and the optimal resource allocation problem of (23), a linear network topology shown in Fig. 9 is used, where the numbers represent prosumers in the network, C is the capacity constraint of the communication link t defined in (23), and L1-L3 are the communication links connecting the prosumers defined in (23).

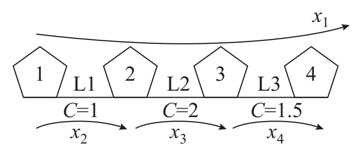

Fig. 9. An example of a linear network topology for resource allocation demonstration.

The results shown in Fig. 10 demonstrate the optimal data flow rates under the σ-fairness condition, which indicates that prosumer 3 has the highest utility function.

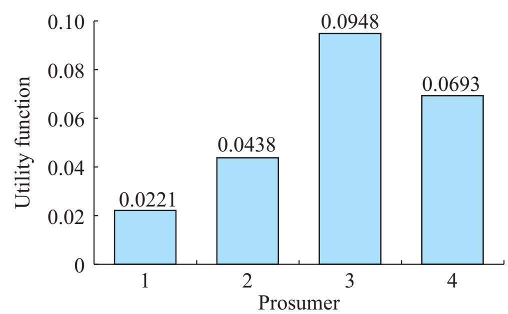

Fig. 10. Optimal data flow rates under σ-fairness condition for prosumers.

Figure 11 shows that prosumers 2, 3, and 4 have more social welfare than prosumer 1.

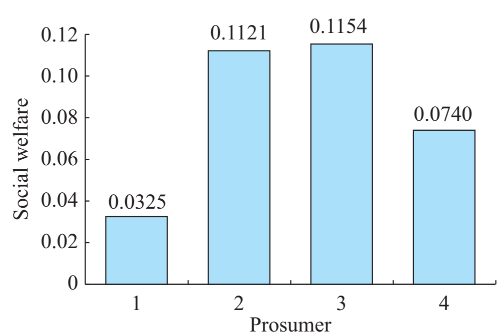

Fig. 11 Optimal social welfare under σ-fairness condition for prosumers.

In addition, Fig. 12 shows a typical run over 24 hours, depicting the relationship with energy demand and supply in the network. The results reflect a reduction in the quantity of energy demanded when the energy supply is at the highest price, obeying the law of demand and supply. However, the ratio of the producers to consumers in the network affects the price paid for the energy bought or sold as shown in Fig 12. For instance, at 40 s, with 20 producers and 8 consumers, the total energy supply is 2.2 kWh and the energy demand is 9 kWh. However, with 10 producers and 20 consumers, the total energy supply is 3.6 kWh, and the energy demand is 7 kWh.

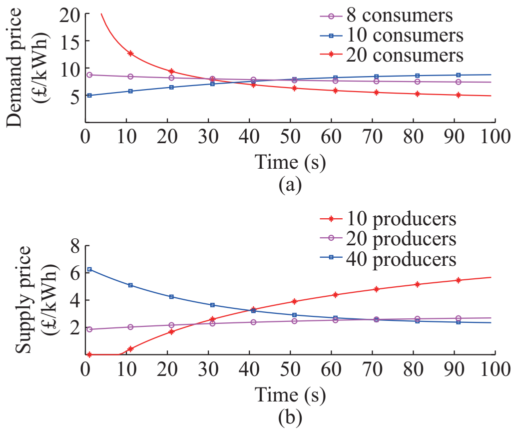

Fig. 12 Optimal social welfare for different numbers of producers and consumers (prosumers). (a) Demand price. (b) Supply price.

V. Conclusion

We present a DDG algorithm based on multi-commodity flow algorithm and a dual-(sub)gradient algorithm for the application of distributed EDP. Specifically, we test the DDG algorithm with an unreliable communication network by considering signal loss probability, message delay, and asynchronous communication of the prosumers. The DDG algorithm converged faster than the previously proposed algorithms, which is a desired feature, especially in a large power network connecting several DERs. The model is further extended to realize the global utility maximization among market-based participants to improve overall costs and maintain the fairness of all generators and demands. The results show a reduction in quantity demanded when the energy supply is at the highest price, but the price paid is dependent on the ratio of producers to the consumers in the network. For instance, the lower the number of the producers, the higher the energy price, and the lower the energy demanded by the consumers. In the future, we shall investigate the flexibility of demands, time-variation, and other time-coupling constraints of the prosumers on the proposed model.

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