The effect of coherent coupling nonlinearity on modulation instability and rogue wave excitation

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Abstract: We study modulation instability (MI) in both anomalous and normal dispersion regimes of a coherently coupled system. It is found that there exist three types of MI spectra with distinct characteristics termed baseband, passband, and zero-baseband based on the instability analysis of the in-phase and out-of-phase CW solutions. The coherent coupling nonlinearity is the source of the passband and zero-baseband variants. Guided by analytical predictions, we investigate numerically the excitation of rogue waves on weakly perturbed in-phase and out-of-phase continuous wave solutions in the parameter space where different kinds of MI exist. Simulations provide supporting evidence that rogue waves can only emerge in regimes where baseband or zero-baseband MI occurs. Moreover, the peak intensity of rogue waves in the case of baseband MI is greater than in the zero-baseband case. Finally, a combination of analysis and numerics uncovers the parameter conditions necessary for the generation of rogue waves.

Keyword: modulation instability, rogue wave, coherent coupling nonlinearity, nonlinear Schrödinger equations

1. Introduction

Rogue waves (RWs) – initially observed as giant waves appearing suddenly on the ocean surface – have become important when describing nonlinear phenomena across many physical contexts [1-7]. Although some uncertainty still remains on the exact origins of RWs, there is general agreement that modulation instability (MI) and the collision of breathers are important building blocks [8-12]. Recently, researchers found that MI is a necessary but not sufficient condition for RW generation [13-15]. In other words, some kinds of MI are not responsible for the excitation of RWs. Within the framework of the Manakov and Fokas-Lenells equations, Baronio et al. [13, 14] discovered that baseband MI (defined as the spectral region of MI containing the zero-frequency perturbation as a limiting case) coincides with RW generation, while passband MI (the spectral region of MI not including the zero-frequency perturbation) leads only to a train of nonlinear oscillations. Also importantly, Zhao et al. [15] found that RWs may emerge from MI with a resonance perturbation by investigating the connections between instability and several nonlinear waves governed by nonlinear Schrödinger (NLS) equations. Chen et al. [16, 17] investigated the RW on periodic standing wave and its relation with the MI of periodic standing wave. Therefore, a detailed study of MI and its link to RWs in different nonlinear systems is instructive for uncovering the true origin of RWs.

MI describes the exponential growth of small perturbations on a continuous wave (CW) background. It is related to many important nonlinear processes in addition to RW formation, such as supercontinuum generation [18] and soliton generation [19]. Although linear stability analysis has limitations that have been corrected using the exact weakly nonlinear theory of wave propagation, it still plays a significant role for MI analysis [20, 21]. Based on linear stability analysis, it is found that the MI of freely-propagating waves only occurs in the focusing regime for a nonlinear system governed by the standard NLS equation [20]. However, in multi-component generalizations, the focusing regime is not a necessary condition for MI. For example, MI can exist in the defocusing regime due to the cross-phase modulation between two different waves [22, 23]; subsequent experiments with two polarization modes [24, 25] verified that theoretical prediction. Indeed, MI in multi-component systems is a more complicated problem than in typical single-component systems, and so they potentially yield new and rich RW patterns [26-28]. For example, Chan et al. [26] discovered additional MI regimes and novel RW structures linked to group-velocity mismatch in coupled multi-wave systems. The novel wave-based phenomena found in coherently coupled systems give rise to additional complexity due to the existence of additional mechanisms for energy transfer between the two constituent waves [29, 30].

An obvious question to pose is, what happens to perturbed CW solutions in coherently coupled systems with different parameters? Furthermore, in which parameter spaces can RWs be excited? And what is the difference between the properties of RWs in the various parameter spaces? To answer these questions, in section 2 we investigate MI in a coherently coupled nonlinear system by using the linear stability analysis, and discuss the influence of coherent coupling on MI spectra. Based on that analysis, in section 3 we study numerically the emergence of RWs in parameter spaces where different kinds of MI are present. We also obtain the parameter conditions for RW generation. Conclusions are drawn in section 4.

2. Modulation instability in the coherently coupled nonlinear Schrödinger system

A coherently coupled nonlinear system can be described by the following system of two dimensionless NLS-type equations [29],

$$i\frac{\partial u}{\partial z}+\beta\frac{\partial^2 u}{\partial t^2}+\gamma_1|u|^2u+\gamma_2|v|^2u+\gamma_3v^2u^*=0, \tag{1a}$$

$$i\frac{\partial v}{\partial z}+\beta\frac{\partial^2 v}{\partial t^2}+\gamma_1|v|^2 v+\gamma_2|u|^2 v+\gamma_3 u^2 v^*=0\,. \tag{1b}$$

Here, u and v are the wave envelopes, γ_1 represents the nonlinearity coefficient, while γ_2 and γ_3 denote incoherent and coherent coupling coefficients, respectively. In nonlinear fiber models, z and t are the propagation distance and time respectively; γ_1 and γ_2 are the self- and cross-phase modulation coefficients, respectively, while γ_3 is the four-wave mixing coefficient (which can be neglected in the case of large birefringence) [30, 31]. In the context of Bose-Einstein condensates, z and t are, respectively, the time and the space coordinates; γ_1 and γ_2 are the intra- and inter-component strengths, while γ_3 describes the pair-transition effect caused by the interaction between atoms [32].

System (1) has the CW solution

$$u_{01}(z,t)=A_0\exp\left[i(\alpha z+\omega t)\right], \tag{2a}$$

$$v_{01}(z,t)=\pm A_0\exp\left[i(\alpha z+\omega t)\right]. \tag{2b}$$

Here, A_0, ω and $\alpha=(\gamma_1+\gamma_2+\gamma_3)A_0^2-\beta\omega^2$ are the amplitude, frequency, and wave number, respectively. These waves are in-phase or out-of-phase when the “+” or “-” sign, respectively, is adopted in Eq. (2b).

For $\gamma_3=\gamma_1-\gamma_2$, system (1) possesses the more general CW solution

$$u_{02}(z,t)=A_1\exp\left[i(\alpha z+\omega t)\right], \tag{3a}$$

$$v_{02}(z,t)=A_2\exp\left[i(\alpha z+\omega t)\right], \tag{3b}$$

where $\alpha=\gamma_1(A_1^2+A_2^2)-\beta\omega^2$. Finally, when $\gamma_1=\gamma_2=\gamma$ and $\gamma_3=0$, system (1) reduces to the Manakov equation, for which the well-known CW solution is [13, 33]

$$u_{03}(z,t)=A_1\exp\left[i(\alpha_1 z+\omega_1 t)\right], \tag{4a}$$

$$v_{03}(z,t)=A_2\exp\left[i(\alpha_2 z+\omega_2 t)\right], \tag{4b}$$

where $\alpha_j=\gamma(A_1^2+A_2^2)-\beta\omega_j^2$, $j=1,2$.

The effect of birefringence on the MI of CW solution (3) is studied in Ref. [27], and RWs with ultra-high amplitudes are presented in Ref. [29]. The link between the type of MI and RWs in the Manakov system is revealed in Ref. [13] by comparing the existence condition of the RW solution and the parameter spaces where different MI regimes occur. Here, we are interested in quantifying how novel effects arising from the interactions between parameters γ_1, γ_2 and γ_3 impact their MI spectra and potential for RW generation. Therefore, we consider the CW solution (2) and begin by perturbing it according to

$$u(z,t) = u_{01}(z,t)\left[1 + q_1(z,t)\right], \tag{5a}$$

$$v(z,t) = v_{01}(z,t)\left[1 + q_2(z,t)\right]. \tag{5b}$$

The functions q_1 and q_2 represent weak perturbations (i.e., having magnitudes much less than unity) in the two components. Substituting Eq.(5) into Eq. (1) and ignoring high-order terms of perturbation, one can obtain the following linearized equations

$$iq_{1z} + 2i\beta\omega q_{1t} + \beta q_{1tt} + (\gamma_1+\gamma_3)A_0^2 q_1^* + (\gamma_1-\gamma_3)A_0^2 q_1 + \gamma_2 A_0^2 q_2^* + (\gamma_2+2\gamma_3)A_0^2 q_2 = 0\,, \tag{6a}$$

$$iq_{2z} + 2i\beta\omega q_{2t} + \beta q_{2tt} + (\gamma_1+\gamma_3)A_0^2 q_2^* + (\gamma_1-\gamma_3)A_0^2 q_2 + \gamma_2 A_0^2 q_1^* + (\gamma_2+2\gamma_3)A_0^2 q_1 = 0\,. \tag{6a}$$

Assuming the weak perturbations q_1 and q_2 have the form [20, 22, 34, 35]

$$q_1(z,t) = q_{11}\exp\left[i(Kz-\Omega t)\right] + q_{12}\exp\left[-i(Kz-\Omega t)\right], \tag{7a}$$

$$q_2(z,t) = q_{21}\exp\left[i(Kz-\Omega t)\right] + q_{22}\exp\left[-i(Kz-\Omega t)\right], \tag{7b}$$

where K and Ω are the wave number and frequency of the perturbations, we substitute the weak perturbation (7) into Eq. (6), and then obtain a set of four linear coupled equations as following

$$\begin{bmatrix} \chi+2\beta\omega\Omega-K & A_0^2(\gamma_1+\gamma_3) & A_0^2(\gamma_2+2\gamma_3) & A_0^2\gamma_2 \\ A_0^2(\gamma_1+\gamma_3) & \chi-2\beta\omega\Omega+K & A_0^2\gamma_2 & A_0^2(\gamma_2+2\gamma_3) \\ A_0^2(\gamma_2+2\gamma_3) & A_0^2\gamma_2 & \chi+2\beta\omega\Omega-K & A_0^2(\gamma_1+\gamma_3) \\ A_0^2\gamma_2 & A_0^2(\gamma_2+2\gamma_3) & A_0^2(\gamma_1+\gamma_3) & \chi-2\beta\omega\Omega+K \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{12} \\ q_{21} \\ q_{22} \end{bmatrix} = 0\,, \tag{8}$$

where $\chi = A_0^2\gamma_1 - \beta\Omega^2 - A_0^2\gamma_3$.

In order to admit nontrivial solutions of the set of equations (8), the determinant of the 4×4 matrix in Eq. (8) requires to vanish, which results in the following dispersion relation

$$K^4 + B_3K^3 + B_2K^2 + B_1K + B_0 = 0\,, \tag{9}$$

where

$$B_0 = \beta\Omega^2\left\{16\gamma_3 A_0^6\left[\gamma_1^2 - (\gamma_2+\gamma_3)^2\right] - 4\beta^2\Omega^2 A_0^2(\gamma_1-\gamma_3)(\Omega^2-4\omega^2) + \beta^3(\Omega^3-4\omega^2\Omega)^2\right.$$

$$+4\beta A_0^4\left[\gamma_1^2\Omega^2-\gamma_2^2\Omega^2+\gamma_1\gamma_3(8\omega^2-4\Omega^2)-2\gamma_2\gamma_3(4\omega^2+\Omega^2)-\gamma_3^2(8\omega^2+\Omega^2)\right]\Big\}, \quad (10a)$$

$$B_1=8\beta\omega\Omega\left[4\gamma_3A_0^4(\gamma_1-\gamma_2-\gamma_3)+2\beta\Omega^2A_0^2(\gamma_1-\gamma_3)-\beta^2\Omega^2(\Omega^2-4\omega^2)\right], \quad (10b)$$

$$B_2=8\gamma_3A_0^4(\gamma_1-\gamma_2-\gamma_3)+4\beta\Omega^2A_0^2(\gamma_1-\gamma_3)-2\beta^2\Omega^2(\Omega^2-12\omega^2), \quad (10c)$$

$$B_3=-8\beta\omega\Omega. \quad (10d)$$

Dispersion relation (9) possesses two pairs of roots,

$$K_{1,2}=2\beta\omega\Omega\pm|\beta|\sqrt{\Delta_1}, \quad (11a)$$

$$K_{3,4}=2\beta\omega\Omega\pm|\beta|\sqrt{\Delta_2}, \quad (11b)$$

where the discriminants are

$$\Delta_1=\Omega^2\left(\Omega^2-\frac{2A_0^2(\gamma_1+\gamma_2+\gamma_3)}{\beta}\right), \quad (12a)$$

$$\Delta_2=\left(\Omega^2+\frac{4A_0^2\gamma_3}{\beta}\right)\left[\Omega^2-\frac{2A_0^2(\gamma_1-\gamma_2-\gamma_3)}{\beta}\right]. \quad (12b)$$

Since MI arises from a non-vanishing imaginary part of K, the signs of the discriminants in Eqs. (12a) and (12b) are associated with the existence of any instability. CW solution (2) is thus robust against small perturbations when Δ_1 and Δ_2 are both positive, and susceptible to perturbations when at least one of them is negative. The MI gain spectrum exhibits two pairs of spectral sidebands when Δ_1 and Δ_2 are both negative. Here, we define MI-A and MI-B for parameter regimes where $\Delta_1<0$ and $\Delta_2<0$, respectively. Obviously, the MI characteristics of the CW solution are determined both MI-A and MI-B. It is also worth noting that if pairs of perturbation amplitudes are the same, i.e. $q_{11}=q_{21}$ and $q_{12}=q_{22}$ in Eq. (7), then dispersion relation (9) will reduce to a quadratic equation whose two roots are given in Eq. (11a). In that case, MI is determined solely by Δ_1 [cf. Eq. (12a)].

One may quantify MI by way of dispersion relation (9). For definiteness in the following analysis, we set $\beta=1$ (anomalous dispersion) or $\beta=-1$ (normal dispersion). Schematic plots of the MI-A gain $G_A=2\,\mathrm{Im}(K_{1,2})$ versus the frequency Ω and the coherent coupling nonlinearity γ_3 are shown in Fig. 1. It can be seen that MI-A occurs when γ_3 exceeds the threshold value $\gamma_{\mathrm{th1}}=-(\gamma_1+\gamma_2)$ for anomalous dispersion [Fig. 1(a)], but it occurs when γ_3 is less than the threshold value γ_{th1} for normal dispersion [Fig. 1(b)]. In both regimes, MI-A behaves exactly like baseband MI [Fig. 1(c)].

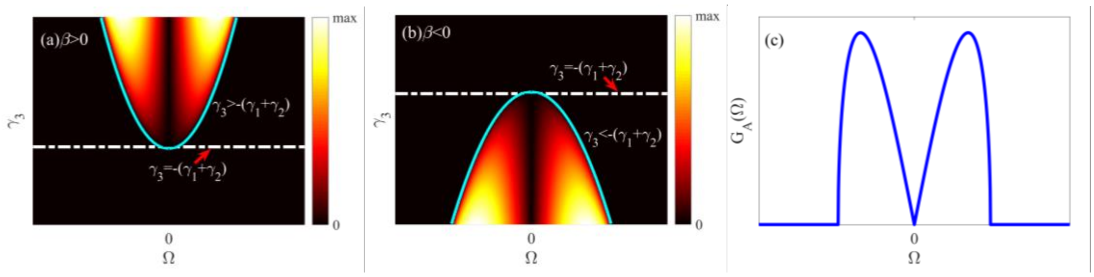

Fig.1. Schematic plots of MI-A gain as functions of frequency Ω and coherent coupling nonlinearity γ_3 in the (a) anomalous and (b) normal dispersion regimes. (c) The cross-sectional views of (a) for $\gamma_3 > -(\gamma_1+\gamma_2)$ and (b) for $\gamma_3 < -(\gamma_1+\gamma_2)$. The cyan solid lines in (a) and (b) represent $\Omega^2 = 2A_0^2(\gamma_1+\gamma_2+\gamma_3)/\beta$. The adopted parameters are $\gamma_1 = 1$, $\gamma_2 = -2$, (a) $\beta = 1$, (b) $\beta = -1$.

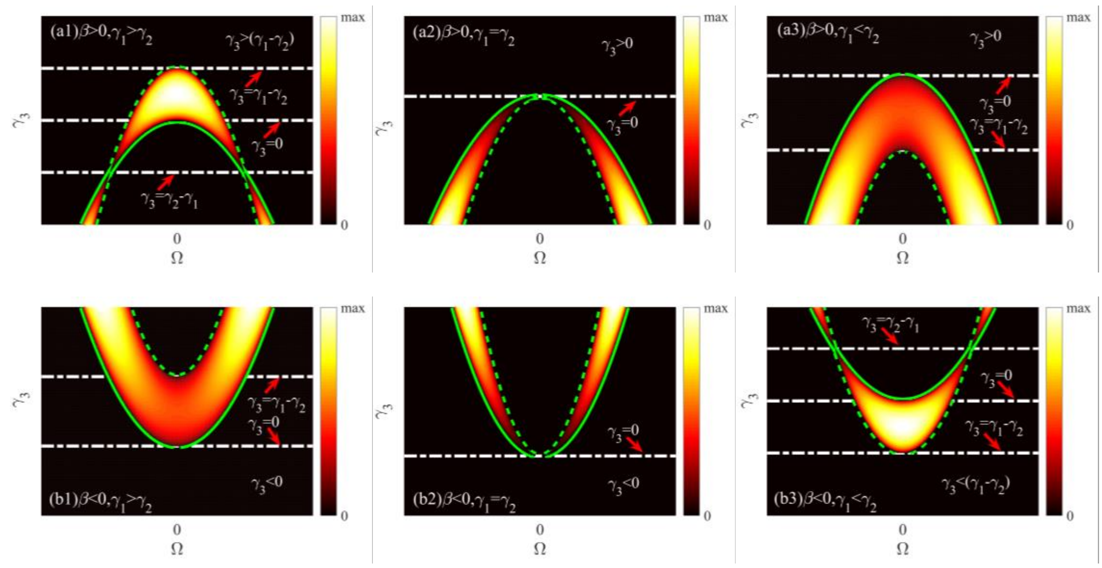

Fig.2. Schematic plots of MI-B gain as functions of frequency Ω and coherent coupling nonlinearity γ_3 in the (a) anomalous and (b) normal dispersion regimes for different relations between γ_1 and γ_2. The green solid line and green dotted line denote $\Omega^2 = -4A_0^2\gamma_3/\beta$ and $\Omega^2 = 2A_0^2(\gamma_1-\gamma_2-\gamma_3)/\beta$, respectively. The adopted parameters are $\gamma_1 = 1$, (a1) $\beta = 1$, $\gamma_2 = -2$, (a2) $\beta = 1$, $\gamma_2 = 1$, (a3) $\beta = 1$, $\gamma_2 = 2$, (b1) $\beta = -1$, $\gamma_2 = -2$, (b2) $\beta = -1$, $\gamma_2 = 1$, (b3) $\beta = -1$, $\gamma_2 = 2$.

Figure 2 depicts the dependence of the MI-B gain $G_B = 2\,\mathrm{Im}(K_{3,4})$ on Ω and γ_3 for different relations between γ_1 and γ_2. There is again a threshold value, $\gamma_{\text{th2}} = \mathrm{sgn}[\beta]\cdot\max\{\mathrm{sgn}[\beta]\cdot(\gamma_1-\gamma_2),0\}$; MI-B occurs when $\gamma_3 < \gamma_{\text{th2}}$ for anomalous dispersion, and when $\gamma_3 > \gamma_{\text{th2}}$ for normal dispersion. It is worth noting that there is a stable critical point $\gamma_{3c} = \gamma_2 - \gamma_1$ for $\gamma_1 > \gamma_2$ in the anomalous dispersion regime, and for $\gamma_1 < \gamma_2$ in the normal dispersion regime. Figure 3 shows cross-sectional views of the MI-B gain spectra in Fig 2 for different γ_3. It is evident that MI-B behaves rather differently from MI-A because two other kinds of spectral structure appear in addition to the familiar baseband contribution (blue solid lines). One kind (black dash-dot lines) behaves exactly like passband MI [13, 14]. The characteristics of the other (red dotted lines) are qualitatively different from those of both baseband and passband MI; this second kind is referred to as zero-baseband MI.

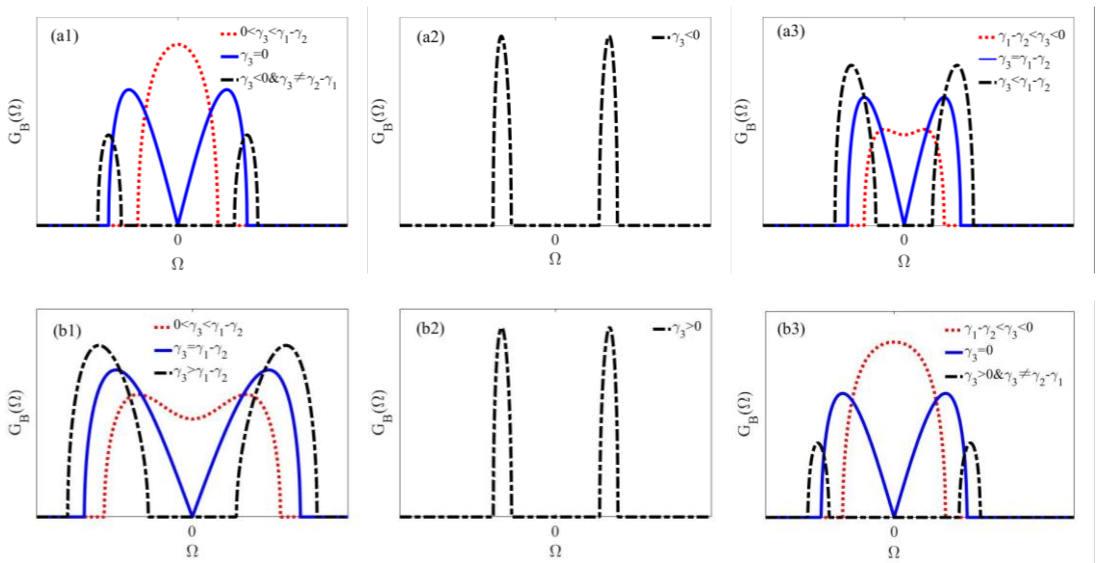

Fig.3. The corresponding cross-sectional views of the MI-B gain spectra in Figs. 2(a1-a3) and (b1-b3) at different γ_3.

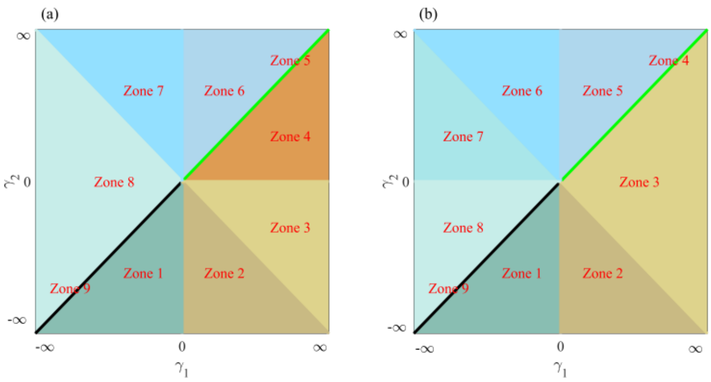

Fig.4. Division of the $[\gamma_1 \quad \gamma_2]$ plane in the (a) anomalous and (b) normal dispersion regimes. Zones 1 to 9 for (a) and (b) are specified in the text.

As mentioned above, the instability of the CW solution is determined by both MI-A and MI-B. By comparing the values of γ_3 corresponding to the boundaries between different MI regimes in Fig. 1 and Fig. 2, the $[\gamma_1 \quad \gamma_2]$ plane in both dispersion regimes can be divided into nine zones. As shown in Fig. 4(a), the zones for anomalous dispersion are labelled Zone 1

$\gamma_2 < \gamma_1 \le 0$, Zone 2 $0 < \gamma_1 \le -\gamma_2$, Zone 3 $0 \le -\gamma_2 < \gamma_1$, Zone 4 $0 < \gamma_2 < \gamma_1$, Zone 5 $\gamma_1 = \gamma_2 > 0$, Zone 6 $0 < \gamma_1 < \gamma_2$, Zone 7 $-\gamma_2 < \gamma_1 \le 0$, Zone 8 $\gamma_1 < \gamma_2 \& \gamma_1 \le -\gamma_2$, and Zone 9 $\gamma_1 = \gamma_2 \le 0$. Similarly, the nine zones for normal dispersion are labelled Zone 1 $\gamma_2 < \gamma_1 \le 0$, Zone 2 $0 < \gamma_1 \le -\gamma_2$, Zone 3 $\gamma_1 > -\gamma_2 \& \gamma_1 > \gamma_2$, Zone 4 $\gamma_1 = \gamma_2 > 0$, Zone 5 $0 < \gamma_1 < \gamma_2$, Zone 6 $-\gamma_2 < \gamma_1 \le 0$, Zone 7 $0 < \gamma_2 < -\gamma_1$, Zone 8 $\gamma_1 < \gamma_2 \le 0$, and Zone 9 $\gamma_1 = \gamma_2 \le 0$, as shown in Fig. 4(b). The dependence of MI gain on Ω and γ_3 in the different zones for anomalous and normal dispersion regions is shown in Figs. 5 and 6, respectively. It can be seen that if MI occurs, only the baseband contribution is present in both anomalous and normal regimes when the coherent coupling nonlinearity is absent (γ_3=0). Therefore, the existence of passband and zero-baseband contributions is due to the presence of γ_3 alone. More detailed analysis reveals

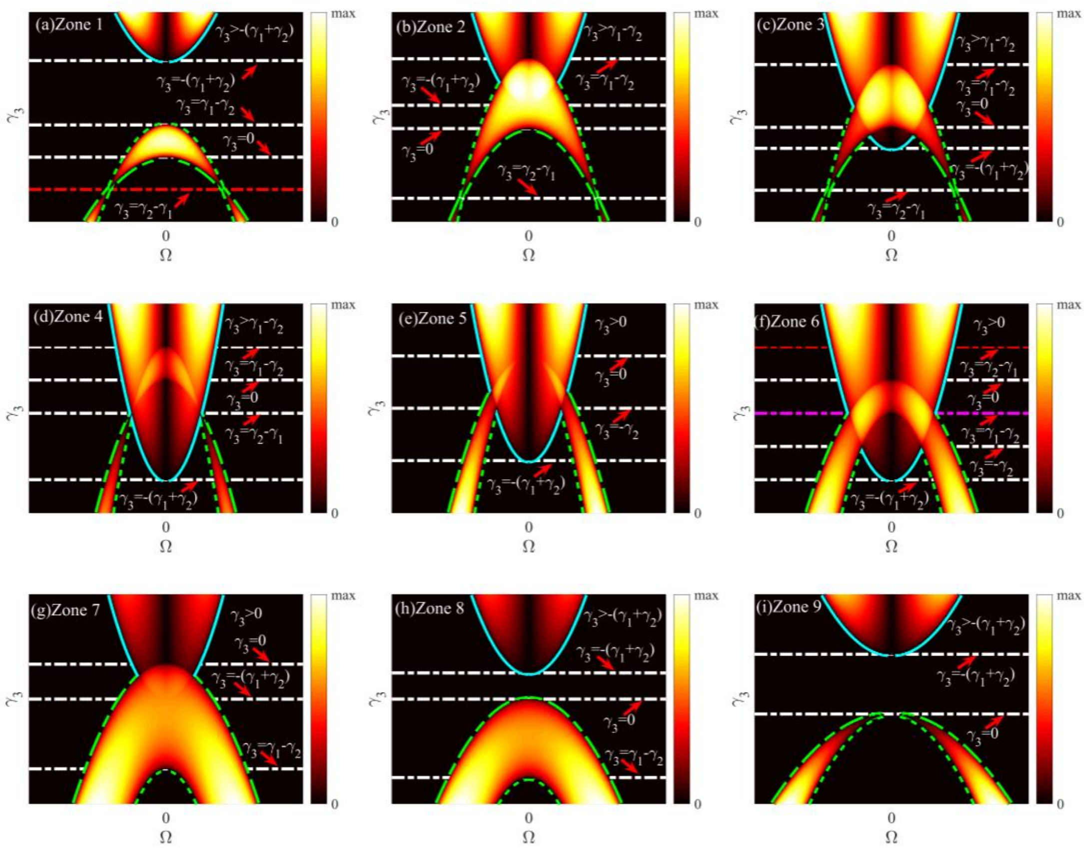

Fig.5. Schematic plots of anomalous-dispersion MI gain as functions of frequency Ω and coherent coupling nonlinearity γ_3 for different parameter spaces located in zones 1 to 9. The adopted parameters are β=1, (a) $\gamma_1 = -1$, $\gamma_2 = -2$, (b) $\gamma_1 = 1$, $\gamma_2 = -2$, (c) $\gamma_1 = 2$, $\gamma_2 = -1$, (d) $\gamma_1 = 2$, $\gamma_2 = 1$, (e) $\gamma_1 = 1$, $\gamma_2 = 1$, (f) $\gamma_1 = 1$, $\gamma_2 = 2$, (g) $\gamma_1 = -1$, $\gamma_2 = 2$, (h) $\gamma_1 = -2$, $\gamma_2 = 1$, (i) $\gamma_1 = -1$, $\gamma_2 = -1$.

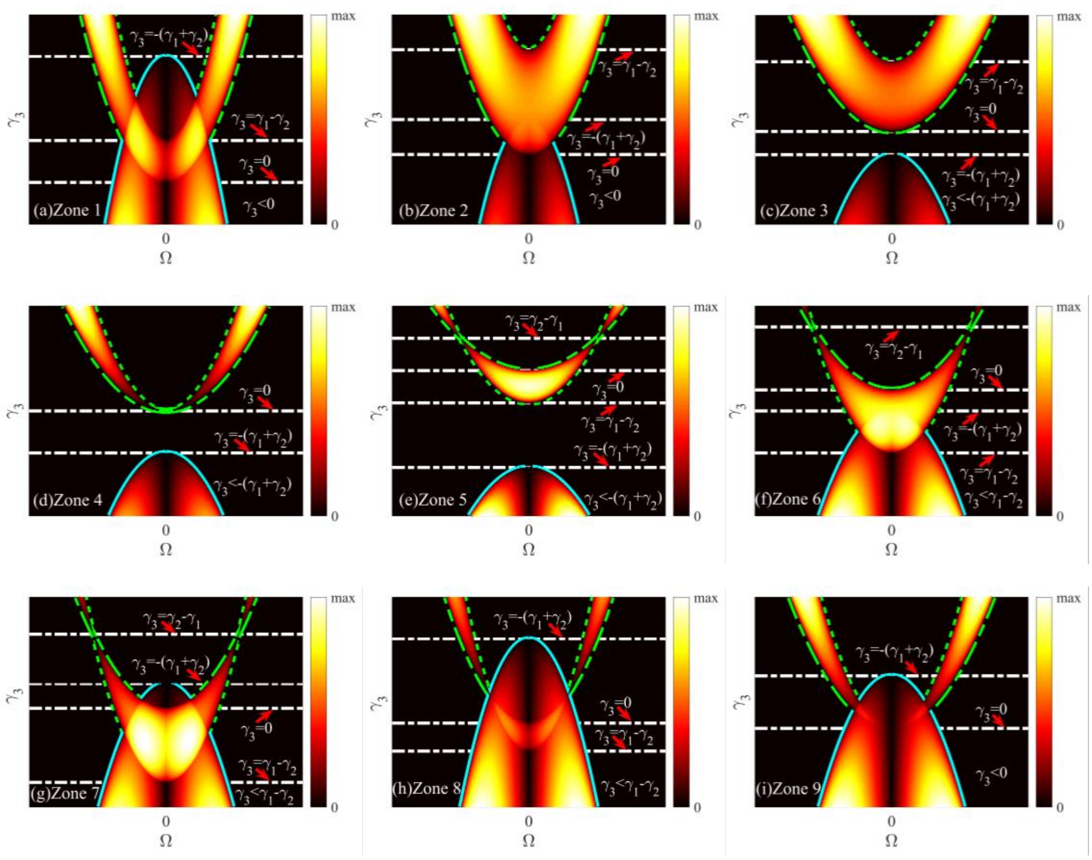

Fig.6. Schematic plots of the normal-dispersion MI gain as functions of frequency Ω and coherent coupling nonlinearity γ_3 for different parameter spaces located in zones 1 to 9. The adopted parameters are $\beta=-1$, (a), $\gamma_1=-1$, $\gamma_2=-2$, (b) $\gamma_1=1$, $\gamma_2=-2$, (c) $\gamma_1=2$, $\gamma_2=-1$, (d) $\gamma_1=1$, $\gamma_2=1$, (e) $\gamma_1=1$, $\gamma_2=2$, (f) $\gamma_1=-1$, $\gamma_2=2$, (g) $\gamma_1=-2$, $\gamma_2=1$, (h) $\gamma_1=-2$, $\gamma_2=-1$, (i) $\gamma_1=-1$, $\gamma_2=-1$.

that for anomalous dispersion, the introduction of $\gamma_3<0$ may lead to passband and zero-baseband MI, while $\gamma_3>0$ can lead only to zero-baseband MI (see Fig. 5). However, for normal dispersion, the introduction of $\gamma_3<0$ can lead only to zero-baseband MI, while $\gamma_3>0$ may lead to passband and zero-baseband MI (see Fig. 6). It should be noted that the above results are based on the analysis of CW solution (2).

3. RW excitation by localized perturbation in different parameter spaces

Having been fairly exhaustive exploring MI in Section 2, it is clear that there are many new possible avenues for potential RW excitation. Using the split-step Fourier method, we now perform simulations in different parameters spaces to solve perturbed initial-value problems of the form

$$u(0,t)=u_{01}(0,t)\left[1+g_1(t)\right], \tag{13a}$$

$$v(0,t)=v_{01}(0,t)\left[1+g_2(t)\right], \tag{13b}$$

where $g_i(t)$ ($i = 1,2$) denotes local disturbances at $z = 0$. Three common forms for perturbations were tested: Gaussian $g_i(t) = \varepsilon_i \exp[-(t - t_{0i})^2 / w_i]$, super-Gaussian $g_i(t) = \varepsilon_i \exp[-(t - t_{0i})^4 / w_i]$, and hyperbolic secant $g_i(t) = \varepsilon_i \mathrm{sech}[(t - t_{0i}) / w_i]$ [36, 37], where in each case the real constants ε_i, w_i, and t_{0i} determine the initial amplitudes, widths, and positions, respectively. The simulations were found to be largely independent of whatever form was used, which is a finding consistent with Ref. [36]. Hence, for illustrative purposes, only a selection of results for Gaussian perturbations is shown here.

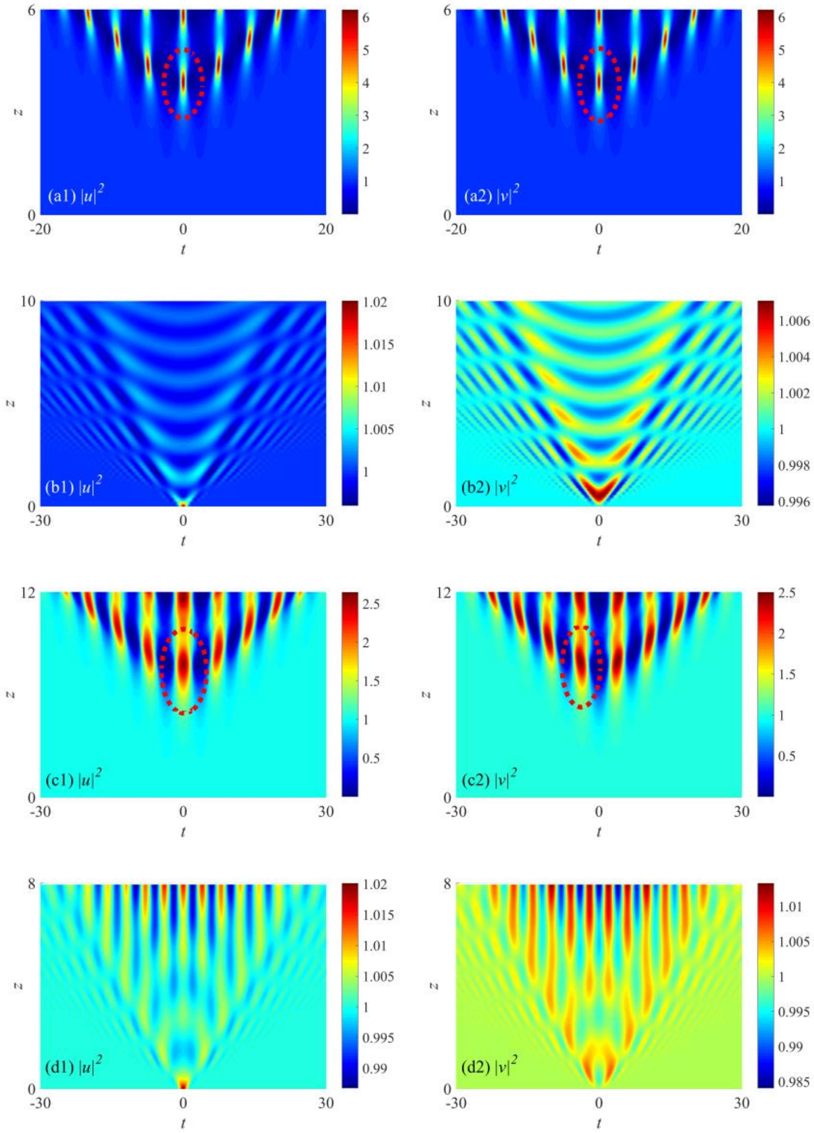

Fig.7. Typical numerical results for the in-phase CW solution with Gaussian perturbations for different parameter spaces in Fig. 5(a) when $g_1(t) \neq g_2(t)$. γ_1, γ_2, and γ_3 are located in the parameter spaces of Fig. 5(a), where (a) baseband MI, (b) modulation stability, (c) zero-baseband, and (d) passband MI occur. The other parameters are ε_1=0.01 , w_1=0.5 , t_{01}=0 , ε_2=0 . A RW is highlighted by a red dashed ellipse in (a) and (c).

We begin by considering the evolution of an in-phase wave when the two initial perturbations differ from one another, i.e. $g_1(t) \neq g_2(t)$. Figures 7(a)-7(d) present typical numerical solutions from the various parameter spaces in Fig. 5(a), where baseband MI, modulation stability, zero-baseband MI and passband MI occur, respectively. These simulations show that a RW (highlighted by a red dashed ellipse) can be excited in the parameter spaces for baseband and zero-baseband MI [Figs. 7(a) and 7(c)]. However, the solution splits into sets of pulses in the parameter spaces for modulation stability and passband MI [Figs. 7(b) and 7(d)]. Moreover, the maximal intensity of the RW in the case of zero-baseband MI is somewhat smaller than that in the case of baseband MI [compare Figs. 7(c) and 7(a)].

We have also performed extensive simulations of the in-phase CW solution in different parameter spaces of Figs. 5(b-i) and Fig. 6, along with the corresponding out-of-phase CW solution. The results were always consistent with those presented in Fig. 7 though, for brevity, they are omitted here. The key physical prediction to emerge is that a RW can be excited from both in-phase and out-of-phase CW solutions in parameter spaces where baseband or zero-baseband MI occur. In contrast, perturbations to an in-phase or out-of-phase CW solution tend to cause a splitting into pulses in parameter spaces where passband MI or modulation stability occurs.

Combining the analysis of MI with supporting simulations, it can be inferred that when MI occurs, a RW can be excited if $\gamma_3 > \gamma_{\text{th3}} = \min\left[0, \gamma_1 - \gamma_2, -(\gamma_1 + \gamma_2)\right]$ in the anomalous dispersion regime, or if $\gamma_3 < \gamma_{\text{th4}} = \max\left[0, \gamma_1 - \gamma_2, -(\gamma_1 + \gamma_2)\right]$ in the normal dispersion regime. Some special cases of these results have been reported elsewhere. Based on system (1) with $\beta = 1/2$, $\gamma_1 = \gamma_3 = \sigma$ and $\gamma_2 = 2\sigma$, Ling et al. [38] obtained the RW solution for $\sigma = 1$ but reported soliton solutions only for $\sigma = -1$; Zhang et al. [39] found that MI occurs when $\sigma = 1$ and that modulation stability appears for $\sigma = -1$. In our work, the parameter space with $\sigma = 1$ corresponds to the red line in Fig. 5(f), where baseband MI appears; the parameter space with $\sigma = -1$ corresponds to the stable critical point shown by red line in Fig. 5(a). Sun et al. [40] took the in-phase CW solutions as a seed and obtained RWs via the Darboux transformation of system (1) with $\beta = 1$, $\gamma_1 = 2$, $\gamma_2 = 4$, and $\gamma_3 = -2$. The parameter space in Ref. [40] corresponds to the magenta line in Fig. 5(f), where baseband MI occurs. Obviously, the conclusions presented here are more general.

Finally, we consider the evolution of CW solutions subject to identical initial perturbations, i.e. $g_1(t) = g_2(t)$. As mentioned in Section 2, when the perturbations in the two components are the same, dispersion relation (9) possesses only one pair of roots $K_{1,2}$, so that MI is completely determined by MI-A. In such a scenario, only baseband MI exists. Figures 8(a)-8(d), respectively, show a set of simulations for perturbed in-phase CW solutions under the condition $g_1(t) = g_2(t)$ for the different parameter spaces of Fig. 5(a). It can be seen that a RW (highlighted by the red dashed ellipse) can be excited from a perturbed in-phase CW solution in the parameter space where baseband MI occurs, while the same solution splits into smaller pulses in other parameter spaces (which is agreement with the analysis in Section 2).

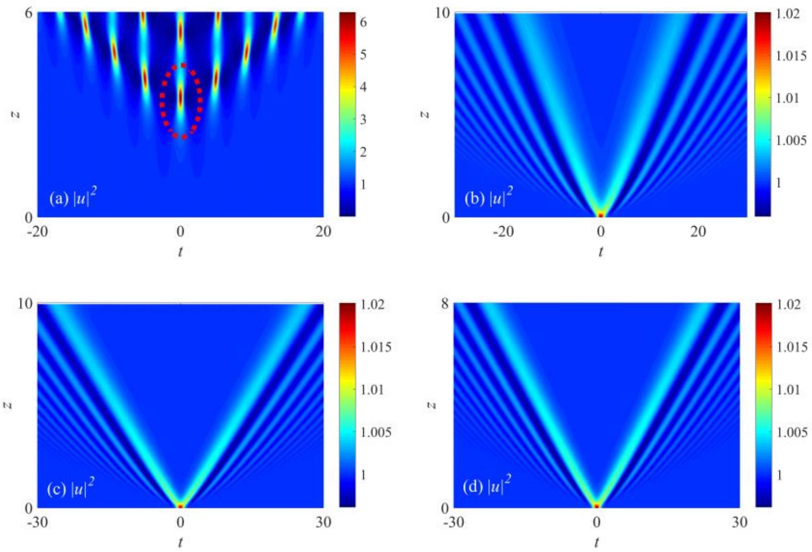

Fig.8. Typical numerical results for the in-phase CW solution with Gaussian perturbations for different parameter spaces in Fig. 5(a) when $g_1(t) = g_2(t)$. The adopted parameters in (a-d) are the same as those in Figs. 7(a-d), respectively, except for ε_2=0.01, w_2=0.5, and t_{02}=0. A RW is highlighted by a red dashed ellipse in (a), and only the *u* component is presented (the *v* component shows very similar behaviour).

4. Conclusions

In the framework of coherently coupled system (1), we have mapped out the MI characteristics of the in-phase and out-of-phase CW solutions in both anomalous and normal dispersion regimes. It has been found that when the initial perturbations in the two components are identical, only baseband MI can occur. However, when the initial perturbations are different (thereby introducing a symmetry-breaking element), there appears a richer and much more intricate spectral structure: baseband, passband, and zero-baseband MI. Moreover, one may attribute coherent coupling nonlinearity directly to the existence of passband and zero-baseband MI phenomena.

Based on the analysis of dispersion relation (9), we have addressed numerically the possibility of exciting RWs from in-phase and out-of-phase CW solutions (subject to Gaussian perturbations) in those parameter spaces where three types of MI are supported. Simulations have revealed that RWs emerge only when baseband or zero-baseband MI occur. Moreover, the peak intensities of RWs in the zero-baseband case are typically less than those in the baseband case.

By combining analysis and numerics, we have found that when MI is present, RWs can be generated by perturbing either in-phase or out-of-phase CW solutions if γ_3 exceeds the threshold γ_{th3} in the anomalous dispersion regime, or if γ_3 falls below the threshold γ_{th4} in normal dispersion regime. Our results, thus, extend over the entire parameter space of system (1) and, in that way, go beyond what has been published to date [27, 39]. Moreover, the thresholds reported here are key research findings that are essential for identifying regions of parameter space capable of supporting RW formation without needing to solve system (1) directly.

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