Chaotic scattering problems with polygons and polyhedra: exit basins and uncertainty fractal dimension

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POSTER ABSTRACT

The Gaspard-Rice (GR) problem provides a paradigm for studying ballistic scattering in a plane, and it exhibits the phenomenon of sensitive dependence on initial conditions. The classic incarnation comprises a point-particle projectile reflecting specularly from three hard-edged discs located at the vertices of an equilateral triangle. Between reflections, the projectile travels at constant velocity. In this presentation, a regular-polygon generalization is proposed and its impact on properties such as exit basins, their boundaries, and time-delay functions are explored through computations.

Consideration is also given to chaotic scattering from regular polyhedra, placing particular emphasis on the tetrahedron and hexahedron with hard spheres centred on their vertices. The ray-tracing algorithm implemented for GR-type systems can be readily adapted to accommodate motion in three spatial dimensions. Our analysis focuses on the calculation of exit basins, and on probing the uncertainty fractal dimension of their boundaries as a function of scatterer configuration. Previous investigations of polyhedra have been primarily concerned with exploring the topology of basin boundaries through the similarity and box-counting dimensions. Here, we measure the fraction of phase space associated with uncertain outcomes, allowing us to establish the susceptibility of these purely deterministic scattering experiments to small fluctuations in their initial conditions.