Spontaneous patterns from Ablowitz-Ladik equations: cavity boundary conditions, instabilities, and mean-field theory

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TALK ABSTRACT

In physics, the discrete nonlinear Schrödinger (dNLS) equation plays a key role in modelling wave propagation in periodic systems. Optical architectures typically involve light confined to a set of waveguide channels with nearest-neighbour coupling and whose dielectric response has a local cubic nonlinearity. While the widely-used dNLS model is non-integrable, it possesses an exactly-integrable counterpart—the Ablowitz-Ladik (AL) equation—which is often of greater interest in applied mathematics contexts. The trade-off for introducing integrability is a nonlinearity in the AL equation that remains cubic but which becomes nonlocal in a way that eludes straightforward physical interpretations. Despite their subtle differences, both models share common asymptotic properties in the long-wavelength (continuum) limit.

Here, the pattern-forming properties of the AL equation are explored in detail. Linear analysis in conjunction with periodic longitudinal boundary conditions—mimicking feedback in an optical cavity—is deployed. Threshold spectra for static patterns are calculated, and simulations test those theoretical predictions in AL systems with both one and two transverse dimensions. Subject to perturbed plane-wave pumping, we find the emergence of stable cosine-type and hexagon patterns, respectively. We conclude with an excursion into mean-field theory, where the AL equation takes on the guise of a discrete Ginzburg-Landau model.