

Vibro-acoustic analysis of a domestic product based on experimental measurement and hybrid modelling



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ABSTRACT

The purpose of this paper is to present a case study whereby a hybrid experimental–numerical model is used to analyse the structure-borne radiation from a domestic product (vacuum cleaner head). The passive (including radiative) properties of the structure are modelled using the hybrid FE-SEA method. The product's operational activity, which lies beyond the capabilities of conventional modelling methods, is characterised experimentally using inverse force identification. The identified forces and passive model are combined to form a so called hybrid FE-SEA-eXperimental model of the assembly. The FE-SEA-X model is then used to identify dominant contributions from the assembly's sub-components.

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1. Introduction

Domestic products such as vacuum cleaners, refrigerators, etc. are a major source of disturbance in the home. This is true to the extent that manufacturers are legally required to provide appliance noise ratings (sound power), and sales may be prohibited if set levels are exceeded [1]. Poor acoustic design may even go as far as causing reputation damage [2]. On the other hand, the rewards for good acoustic design are high, with increased sales and brand differentiation [2]. As such, it is no surprise that vibro-acoustic attributes are afforded a very high priority in product design.

Clearly, an appropriate understanding of a product's vibro-acoustic performance would benefit its manufacturer, enabling design modifications that reduce product sound level (or perhaps improve sound quality) whilst retaining an expected performance. This issue is often tackled through physical prototyping. This is not only time consuming but expensive. For this reason computational approaches are in favour. However, the complexities of a product's operation often render standard numerical methods (e.g. the finite element method) unsuitable. These operational features are, however, amenable to experimental characterisation. Hence, a robust

design tool should incorporate both experimental and numerical approaches, gaining from their respective advantages. In the present paper we consider a case study whereby the combined application of experimental and numerical methods (i.e. a hybrid model) are used to analyse the radiated sound pressure from a domestic product.

The appliance considered is a Dyson cordless vacuum cleaner (see Fig. 1a). In particular, we are interested in the sound radiated from its cleaner head (see Fig. 1b). The cleaner head is a shell like structure with an integrated rotating brush bar which is in contact with the floor. A (numerical) hybrid FE-SEA model is set up to describe the propagation and radiation, whilst experimental inverse force identification is used to obtain operational characteristics (which are too complex to model numerically). These approaches are then combined to form a hybrid FE-SEA-X model [3], capable of predicting the radiated sound pressure from the cleaner head and the floor that it is in contact with.

Having outlined the context of this paper, its content will be organised as follows. Section 2 will begin by describing the problem and detailing the modelling strategy adopted. Section 3 then introduces the key methods used in this paper (including hybrid FE-SEA, blocked forces and pseudo forces). The case study results are then presented in Section 4. Finally, some concluding remarks are drawn in Section 5.

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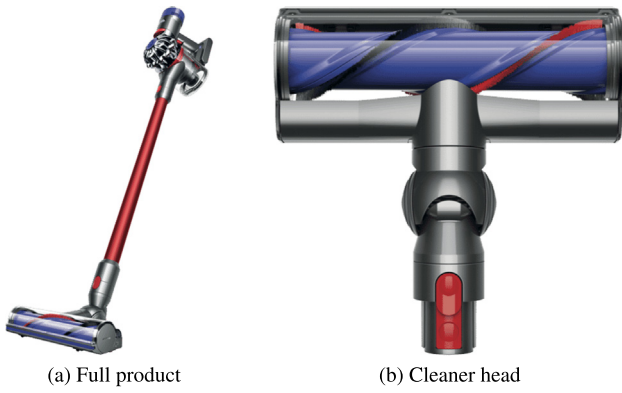


Fig. 1. Dyson vacuum cleaner.

2. Problem description

The cleaner head in question has an integrated brush-bar with carbon fibre and stiff nylon bristles that rotates at approx. 3600 RPM. The stiff bristles agitate debris embedded into carpets while the carbon fibre filaments sweep up fine dust on hard-floor surfaces. Although previous work [4] indicates that the noise emitted by brush-bars of this type is predominantly driven by vibro-acoustic radiation from the housing of the assembly, it is thought that on lightweight hard floors, the brush bar may be able to excite the floor which could then radiate acoustic energy to the far field (see Fig. 2). To investigate this possibility, a hybrid FE-SEA-X model has been used to determine vibro-acoustic contributions of the housing and floor to the total structurally radiated sound pressure level.

2.1. Modelling strategy

The strategy adopted within this work was devised to further understand the sources and transfer paths primarily responsible for emitting the sound associated with vacuum cleaner heads of this type. A schematic of the hybrid FE-SEA-X model considered here is shown in Fig. 3. The problem has been subdivided into 3 distinct domains, or sub-systems: the cleaner head/shell, the floor/plate, and the surrounding room/cavity. These sub-systems are separated by 3 interfaces (orange links in Fig. 3); the cleaner head-room (structural-acoustic) interface, the cleaner head-floor (structural-structural) interface, and floor-room (structural-acoustic) interface. Note that the cleaner head-floor interface, as represented by the dashed link, provides only weak coupling; the

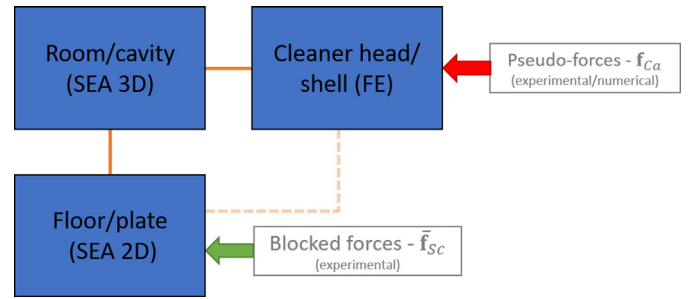


Fig. 3. Schematic of the hybrid FE-SEA-X model.

cleaner head and floor sub-systems have a negligible influence on one another. Hence, this interface is not considered in the current hybrid model.

Owing to their high modal densities, the room and floor are represented by SEA sub-systems, whilst the cleaner head structure is modelled using a simplified finite element shell model.

The operation activity of the cleaner head can similarly be subdivided into two parts: the cleaner head (inc. brush bar)-floor interaction, and the brush bar-cleaner head interaction. The cleaner head-floor interaction occurs at their separating interface, illustrated in Fig. 4a by a series of green markers. In the present paper this interaction will be represented by a series of experimental blocked/contact forces (see Section 3.2). The brush bar-cleaner head interaction occurs over a more complex interface within the cleaner head structure. For experimental convenience a pseudo force-based approach will be adopted, where the underlying interface forces are instead represented by a series of external point-like (pseudo) forces (see Section 3.3). These pseudo forces will be determined using a mixed experimental/numerical method. Some preliminary external force locations considered are shown in Fig. 4b and c by red markers.

The operational (blocked/contact and pseudo) forces are determined by inverse force identification methods, as detailed in Section 3.2 and 3.3, respectively. They will be used to excite the hybrid FE-SEA-X model, as illustrated in Fig. 3 and shown in Section 4.

3. Methods

This section will provide a brief summary of the methods used to construct the hybrid FE-SEA-X model and characterise the structure's operational activity. Where necessary references will be given to more detailed articles on the respective methods.

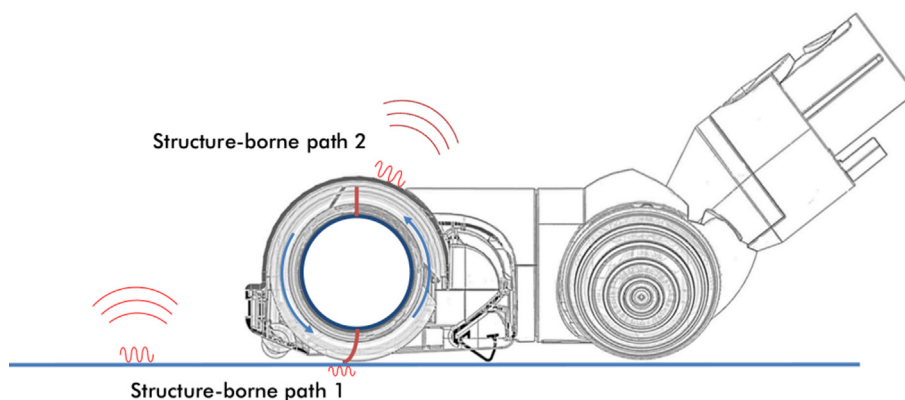


Fig. 2. Diagram of the vibro-acoustic transfer paths considered.

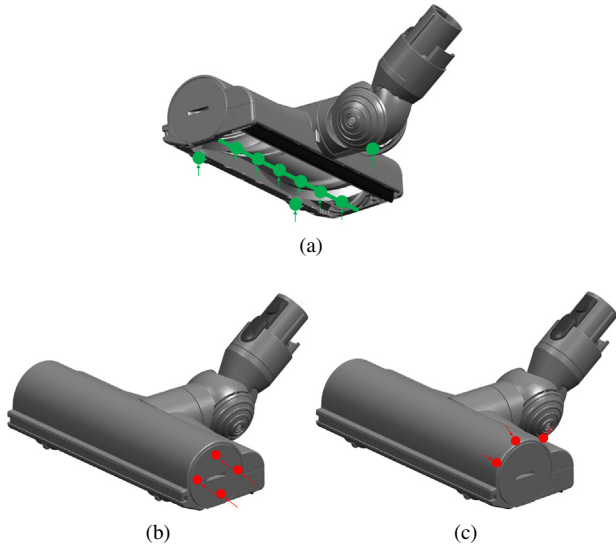


Fig. 4. Location of forces used to represent cleaner head operation. a) Position of blocked/contact forces at the cleaner head-floor interface. b/c) Positions of the external pseudo forces used to represent the brush bar-shell interaction.

3.1. Hybrid FE-SEA-X model

Finite Element (FE) and Statistical Energy Analysis (SEA) are suitable, respectively, for the analysis of long and short wavelength (sub-) systems. However, the components that comprise complex engineering structures often have widely varying characteristic wavelengths. In this case neither FE nor SEA are appropriate to model the complete system.

Based on a diffuse field reciprocity relation [5,6], a general wave-based approach for coupling both theories into a single model has been proposed by Shorter and Langley [7]. Termed the ‘hybrid FE-SEA’ method, this approach has since been validated both numerically and experimentally [8], and has proven itself as a valuable vibro-acoustic prediction tool. The method has been further extended to predict the variance of a response due to both non-parametric (i.e. arising from SEA sub-systems) [9] and parametric (i.e. arising from FE components) [10,11] uncertainties. The use of experimental data to represent complex sub-systems that cannot be modelled directly was recently proposed in [3]. In the present paper we will apply this FE-SEA-X methodology to the vibro-acoustic analysis of a domestic product, namely, a Dyson cordless vacuum cleaner.

The hybrid equations introduced below are done so in the context of the current case study. For a more general treatment of the hybrid FE-SEA(-X) method the reader is referred to [7,3].

The hybrid FE-SEA method requires each component of the vibro-acoustic system under study (see Section 2) to be identified as either deterministic or statistical. The deterministic part is represented by a finite set of DoFs \mathbf{q} , and the statistical part by a set of sub-systems, each represented by a single DoF, their vibrational (or acoustical) energy E . As discussed in Section 2.1, in the present case study the cleaner head structure is considered deterministic, owing to its low modal density. In contrast, given their high modal densities, the floor and room (plate and cavity) sub-systems are considered statistical.

Treatment of the deterministic part follows standard Finite Element procedure; the cleaner head is represented by the simplified shell model shown in Fig. 5. Whilst simplified geometrically, it is expected that this FE model will suitably describe the radiative properties of the cleaner head.

For the statistical sub-systems (i.e. the floor and room), we consider the contained wave fields as combinations of two separate

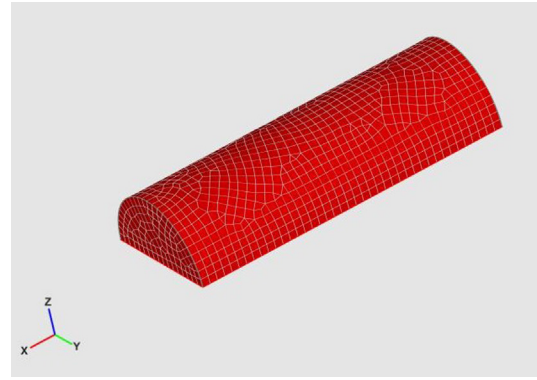


Fig. 5. FE model representing the cleaner head’s outer shell.

fields: the response due to the initially generated waves (termed the direct field), and the contribution from all waves reflected by the sub-system boundaries (termed the reverberant field). These boundaries are considered unknown, and so the reverberant field is considered random (in an ensemble sense). Based on this separation, a direct field dynamic stiffness matrix \mathbf{D}_{dir} is defined for each statistical sub-system. This is the dynamic stiffness of the sub-system in the absence of any reflections, i.e. the infinite sub-system response. Analytical solutions for \mathbf{D}_{dir} are available for many simple cases. For the floor/plate and room/cavity considered here, formulations of the direct field stiffness matrices are described, respectively, in [3,12,13].

The contribution of the k th statistical sub-system to the total system response \mathbf{q} is modelled by first adding its direct field dynamic stiffness $\mathbf{D}_{dir}^{(k)}$ to that of the deterministic stiffness matrix \mathbf{D}_d (here representing the cleaner head structure), and then applying an appropriate reverberant force $\mathbf{f}_{rev}^{(k)}$ to the connecting degrees of freedom (DoFs). This force describes the loading that arises due to the reverberant field in the statistical sub-system. For the vibro-acoustic system considered here, the governing equations of motion are [7],

$$\mathbf{D}_{tot}(\omega)\mathbf{q}(\omega) = \mathbf{f}_d(\omega) + \sum_{k=1}^2 \mathbf{f}_{rev}^{(k)}(\omega) \quad (1)$$

where \mathbf{f}_d represents an external forcing applied directly to the deterministic part, and

$$\mathbf{D}_{tot}(\omega) = \mathbf{D}_d(\omega) + \sum_{k=1}^2 \mathbf{D}_{dir}^{(k)}(\omega) \quad (2)$$

is the total stiffness matrix. In Eq. (2), \mathbf{D}_d represents the dynamic stiffness matrix of the deterministic cleaner head structure, whilst $\mathbf{D}_{dir}^{(1)}$ and $\mathbf{D}_{dir}^{(2)}$ represent, respectively, the direct field stiffness matrices of the statistical floor (plate) and room (cavity) sub-systems. Similarly, in Eq. (1), $\mathbf{f}_{rev}^{(1)}$ and $\mathbf{f}_{rev}^{(2)}$ represent the reverberant forces applied to the cleaner head by, respectively, the floor and room sub-systems.

The hybrid FE-SEA-X methodology considers the case whereby certain components of a system cannot be modelled numerically, so are instead represented by experimental sub-systems. Generally, in the presence of j experimental sub-systems the deterministic dynamic stiffness matrix \mathbf{D}_d is modified as so,

$$\mathbf{D}_d = \mathbf{D}_{d,FE} + \sum_j \mathbf{D}_{d,exp}^{(j)} \quad (3)$$

where $\mathbf{D}_{d,FE}$ and $\mathbf{D}_{d,exp}^{(j)}$ represent the dynamic stiffness matrices of the numerical part and the j th experimental sub-system,

respectively. In the presence of j active experimental sub-systems (i.e. vibration sources) the deterministic force vector \mathbf{f}_d is also modified as,

$$\mathbf{f}_d = \mathbf{f}_{ext} + \sum_j \bar{\mathbf{f}}_{exp}^{(j)} \quad (4)$$

where \mathbf{f}_{ext} represents an external force applied directly to the numerical part, and $\bar{\mathbf{f}}_{exp}^{(j)}$ the blocked force of the j th experimental sub-system. If the j th experimental sub-system is purely passive, then $\bar{\mathbf{f}}_{exp}^{(j)} = \mathbf{0}$.

In the present case study the dynamic stiffness of the cleaner head structure is modelled numerically, so no experimental stiffness matrices are required. As discussed in Section 2, the cleaner head's operational activity is separated into two parts. The cleaner head-floor interaction is characterised by a blocked/contact force $\bar{\mathbf{f}}_{exp}^{(1)}$ (see Section 3.2) at the separating interface. The brush bar-cleaner head interaction is instead represented by a set of external pseudo forces \mathbf{f}_{ext} (see Section 3.3) applied directly to the numerical part.

To obtain a hybrid FE-SEA(-X) model the above equations are not enough. The energetic properties of the statistical sub-systems must be related to their reverberant forces and direct field dynamic stiffness matrices. This is achieved by means of the diffuse field reciprocity relation [5], an identity that relates the cross-spectral matrix of a statistical sub-system's reverberant force, $\mathbf{S}_{ff}^{(k),rev} = E[\mathbf{f}_{rev}^{(k)} \mathbf{f}_{rev}^{(k)*T}]$, where $E[\]$ denotes the ensemble average, to its energy E_k and direct field dynamic stiffness $\mathbf{D}_{dir}^{(k)}$. This relationship, valid when the ensemble response constitutes a diffuse wave field [6], can be expressed as,

$$\mathbf{S}_{ff}^{(k),rev} = \frac{4a_k E_k}{\pi \omega n_k} \text{Im}\{\mathbf{D}_{dir}^{(k)}\} \quad (5)$$

where E_k and n_k are, respectively, the ensemble averaged vibrational energy and modal density of sub-system k , and a_k is a term that takes into account local concentrations in the wave field [14,9].

Converting Eq. (1) into a quadratic form such that $\mathbf{S}_{qq} = E[\mathbf{q}\mathbf{q}^*T]$, and substituting the diffuse field reciprocity relation, leads to,

$$\mathbf{S}_{qq} = \mathbf{D}_{tot}^{-1} \left[\mathbf{S}_{ff} + \sum_{k=1}^2 \frac{4a_k E_k}{\pi \omega n_k} \text{Im}\{\mathbf{D}_{dir}^{(k)}\} \right] \mathbf{D}_{tot}^{-1*T} \quad (6)$$

where, for the present case study, the cross-spectral matrix is given by,

$$\mathbf{S}_{ff} = \mathbf{S}_{ff}^{ext} + \mathbf{S}_{ff}^{exp} \quad (7)$$

In the above; \mathbf{S}_{qq} represents the cross-spectral matrix of the system response \mathbf{q} , \mathbf{S}_{ff}^{ext} and \mathbf{S}_{ff}^{exp} are the cross-spectral force matrices corresponding to the brush bar-cleaner head pseudo force and the cleaner head-floor blocked force, respectively, and E_1 and E_2 represent the vibrational and acoustical energy in the floor and room sub-systems, respectively.

To solve Eq. (6) the energy E_k of each sub-system must first be determined. This is done by solving a power balance equation of the form [7],

$$\mathbf{C}_0 \hat{\mathbf{E}} = \mathbf{P} + \mathbf{P}_{in}^{ext} \quad (8)$$

where $\hat{\mathbf{E}}$ is a vector of ensemble averaged modal energies such that $\hat{E}_k = E_k/n_k$, \mathbf{P} and \mathbf{P}_{in}^{ext} are vectors of input powers due to external forces, and \mathbf{C}_0 is a matrix of coupling loss factors. For further details the reader is referred to [7]. Once computed, the sub-system energies can be related to spatially averaged variables, for example E_2 can be used to compute the sound pressure level in the surrounding acoustic space/room.

Eqs. (6)–(8) form the basis of the hybrid FE-SEA-X model considered here. What is left to consider is the experimental determination of the cross-spectral force matrices \mathbf{S}_{ff}^{ext} and \mathbf{S}_{ff}^{exp} .

It is noted that for the present case study the hybrid equations presented above are solved using the commercial software wave6 [15], hence some more detailed aspects of the model's construction are omitted.

3.2. Inverse (blocked) force identification

To include an experimental description of a vibration source within a hybrid FE-SEA-X model its operational activity must be characterised *independently*, i.e. in such a way that it is invariant to the receiver structure. An appropriate characterisation can be achieved using the blocked force [16].

With reference to Fig. 6, the blocked force describes the force required to constrain the interface DoFs (c) of a vibration source such that their velocity (also displacement and acceleration) is zero,

$$\bar{\mathbf{f}}_{Sc} = \mathbf{f}_{Cc}|_{\mathbf{v}_{Cc}=\mathbf{0}} \quad (9)$$

where capitalised sub-scripts S and C denote the source and the coupled source-receiver assembly, respectively. The rigid constraint of the interface DoFs c removes the dynamic influence of the receiver structure, and so the blocked force is an independent property of the vibration source. As such, it remains a valid source characterisation even if the receiver structure is modified or replaced.

Direct measurement of the blocked force is complicated by the requirement of rigid constraints at the source interface. Fortunately, an indirect approach is available. It has been shown that the blocked force can be obtained from in-situ measurements with the source installed on an arbitrary receiver structure [16]. The equation of note is given by,

$$\mathbf{v}_{Cc} = \mathbf{Y}_{Ccc} \bar{\mathbf{f}}_{Sc} \quad (10)$$

where \mathbf{v}_{Cc} is an operational velocity measured at the interface DoFs c , \mathbf{Y}_{Ccc} is a mobility matrix measured at and between the interface DoFs, and $\bar{\mathbf{f}}_{Sc}$ is the unknown blocked force ($\bar{\mathbf{f}}_{exp}^{(j)}$ in Eq. (4)). The

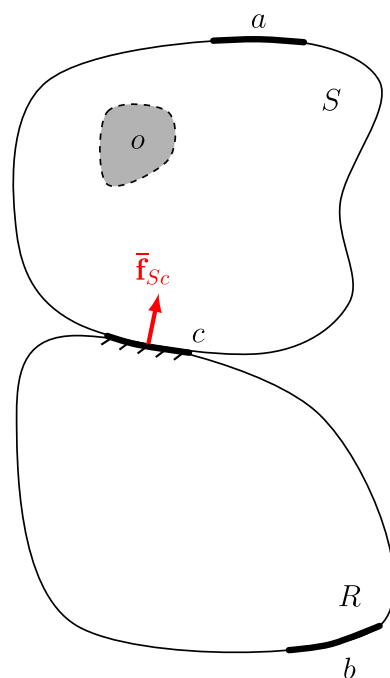


Fig. 6. Representation of a source-receiver (SR) assembly and blocked force.

blocked force is obtained by inversion of the measured mobility matrix,

$$\bar{\mathbf{f}}_{Sc} = \mathbf{Y}_{Ccc}^{-1} \mathbf{v}_{Cc}. \quad (11)$$

Once obtained, the blocked force can be used to predict the operational response at the remote receiver position b as per,

$$\mathbf{v}_{Cb} = \mathbf{Y}_{Cbc} \bar{\mathbf{f}}_{Sc}. \quad (12)$$

The in-situ blocked force characterisation requires a two part measurement procedure. In part one the source is turned off and the mobility matrix \mathbf{Y}_{Ccc} is measured. Then, in part two, the source is operated and the operation velocity \mathbf{v}_{Cc} is measured. For further details regarding practical considerations the reader is referred to [17,18].

To correctly represent a vibration source within a hybrid FE-SEA-X model, it is important that the blocked force is accompanied by a description of the passive source properties. This is achieved as per Eq. (3), by adding its dynamic stiffness matrix \mathbf{D}_{Scc} ($\mathbf{D}_{d,exp}^{(j)}$ in Eq. (3)) to that of the numerical part. Experimentally, the dynamic stiffness can be obtained by inversion of the free source mobility, \mathbf{Y}_{Scc} , according to,

$$\mathbf{D}_{Scc} = i\omega \mathbf{Y}_{Scc}^{-1}. \quad (13)$$

Alternatively, it can be modelled numerically using standard FE methods. In the present case study the blocked force approach is used to characterise operational activity at the cleaner head-floor interface, as described in Section 2. Following a preliminary test it was observed that, due to weak coupling between the floor and cleaner head, the coupled mobility \mathbf{Y}_{Ccc} was approximately equal to uncoupled floor (receiver) mobility \mathbf{Y}_{Rec} (see Fig. 7). Importantly, this weak coupling introduces an equivalence between the blocked force and contact force. Furthermore, it suggests that the floor and cleaner head can be treated independently, i.e. their coupling can be neglected in the hybrid model. Hence, the cleaner head and floor sub-system in Fig. 3 are not connected. This is of course unique to the present case study, and in a more general FE-SEA-X model coupling between all sub-systems should be considered.

For experimental convenience, the receiver mobility is used in place of the coupled mobility to characterise the cleaner head-floor interaction. Hence, the forces determined, strictly speaking, are contact forces. Nevertheless, given their equivalence to the blocked force in the presence of weak coupling, the term blocked force will continue to be used.

Upon discretising the cleaner head-floor interface (see Fig. 4a) a set of 9 blocked forces are determined by inverting the 9×9

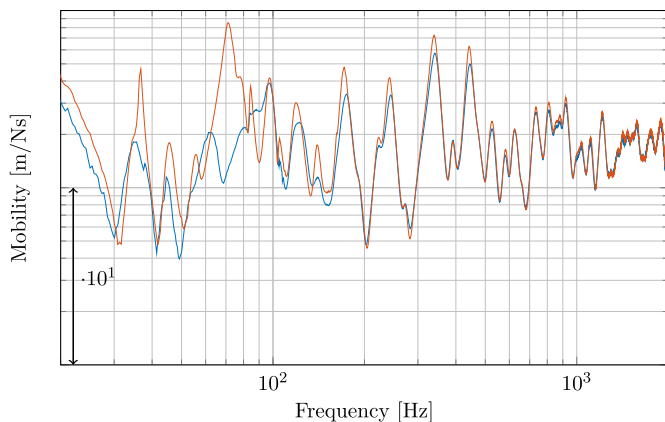


Fig. 7. An example interface point mobility with (blue, \mathbf{Y}_{Ccc}) and without (orange, \mathbf{Y}_{Rec}) the cleaner head present. Their similarity demonstrates a) the equivalence of the contact force and blocked force and b) the lack of direct coupling between cleaner head and floor sub-systems.

mobility matrix \mathbf{Y}_{Rec} measured at the interface. Validation results are presented in Section 4.1.1. The cross-spectral force matrix \mathbf{S}_{ff}^{exp} is then obtained by,

$$\mathbf{S}_{ff}^{exp} = \bar{\mathbf{f}}_{Sc} \bar{\mathbf{f}}_{Sc}^T. \quad (14)$$

This cross-spectral force matrix will be used in the hybrid FE-SEA-X model to represent the cleaner head-floor interaction.

3.3. Pseudo forces

Often the interface between a source and receiver is unclear, or perhaps inaccessible. In this situation, it is not straightforward to obtain the (independent) blocked force as described above. An alternative approach is available however, given some limiting restrictions [19,20].

Suppose some internal forces \mathbf{f}_{so} are developed within a vibration source. These forces will excite the source, which will in turn excite the connected receiver through some set of interface DoFs c . The response at this interface is given by,

$$\mathbf{v}_{Cc} = \mathbf{Y}_{Cco} \mathbf{f}_{so}. \quad (15)$$

The internal forces are generally in accessible for characterisation. However, it is possible to define a new set of external pseudo forces \mathbf{f}_{Ca} that, in place of \mathbf{f}_{so} , generate an identical response field at the interface c ,

$$\mathbf{v}_{Cc} = \mathbf{Y}_{Cca} \mathbf{f}_{Ca}, \quad (16)$$

and hence in the receiver structure also,

$$\mathbf{v}_{Cb} = \mathbf{Y}_{Cba} \mathbf{f}_{Ca}. \quad (17)$$

An exact reproduction of \mathbf{v}_{Cc} (and \mathbf{v}_{Cb}) would require the number of external forces (i.e. DoFs a) to be equal to the number of interface DoFs c . The position of these new pseudo forces are arbitrary (they can be chosen for convenience), provided that they excite a sufficient number of modes. Hence, they provide a convenient alternative to the in-situ blocked force method when the interface is inaccessible.

The pseudo forces \mathbf{f}_{Ca} are obtained by inversion of the measured mobility matrix \mathbf{Y}_{Cca} (or \mathbf{Y}_{Cba}), as per a standard inverse force identification,

$$\mathbf{f}_{Ca} = \mathbf{Y}_{Cca}^{-1} \mathbf{v}_{Cc} = \mathbf{Y}_{Cba}^{-1} \mathbf{v}_{Cb}. \quad (18)$$

Whilst the pseudo forces will reproduce the interface and receiver response fields, they are not unique. Different pseudo force positions will yield different pseudo forces. Nor are they transferable between assemblies (like the blocked force). Nevertheless, they will reproduce the velocity field across a receiver structure, providing they are sufficient in number.

In the present case study the pseudo force method is used to characterise the excitation of the cleaner head's shell-like structure by the operating brush bar. The brush bar-cleaner head interface is complex and hard to define, hence the use of the pseudo force. Given their arbitrary positioning, different sets of excitation DoFs can be considered. As part of a preliminary experimental study two sets of pseudo force locations are considered. These are shown in Fig. 4b and c by red markers, with results presented in Section 4.1.2.

Whilst pseudo forces are typically obtained by experimental means, where both the operational response and mobility matrix are measured, the pseudo forces used to excite the hybrid model considered here are instead determined using a combined experimental/numerical approach. Experimental response measurements are combined with a numerical mobility matrix obtained from the FE shell model representing the cleaner head. The cross-spectral force matrix \mathbf{S}_{ff}^{ext} is then obtained by,

$$\mathbf{S}_{ff}^{ext} = \mathbf{f}_{Ca} \mathbf{f}_{Ca}^T \quad (19)$$

This cross-spectral force matrix will be used in the hybrid FE-SEA-X model to represent the brush bar-cleaner head interaction.

4. Results

4.1. Experimental

In this section we will describe in greater detail the experimental procedures employed to characterise and validate the operational activity of the cleaner head assembly. We will further consider, by experimental means, the radiated structure-borne sound contributions from the floor (represented here by a plate) and cleaner-head shell components. This result will serve as a comparison for the hybrid FE-SEA-X model to be introduced shortly.

Elements of the procedures presented below will be used in Section 4.2 to provide excitation of the hybrid FE-SEA-X model (see Fig. 3). In summary, the SEA floor sub-system will be excited by the blocked forces described in Section 4.1.1. The cleaner head shell will be excited by the set of hybrid experimental/numerical pseudo forces described in Section 4.2.2. Prior to this the pseudo force methodology will be validated experimentally in Section 4.1.2.

4.1.1. Floor response prediction

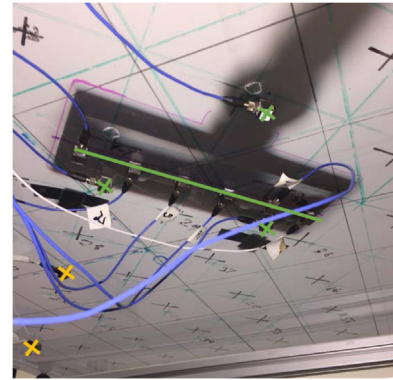
To predict the radiated sound pressure from the floor due to the operational cleaner head, the forces acting at their separating interface must be determined. These forces, when combined with appropriate vibro-acoustic transfer functions, should provide an estimate of the radiated sound pressure due to the interface forces acting on the floor only, i.e. neglecting radiation from the cleaner head's shell structure.

As is often the case in practical scenarios, the interface between the floor and vacuum head is somewhat unclear. Three point like DoFs were identified as the two small front wheels and a rear ball pivot (see Figs. 2 and 4a). The remaining interface is that between the rotating brush bar and the floor. In reality, this may be viewed as a moving interface; the brush bar bristles are only ever in contact with the floor at two positions as it rotates. To simplify the experimental procedure the interface has instead been defined as a series of point-like DoFs (so as to approximate a line junction). Given the rotational speed of the brush bar, all DoFs will experience repeated excitation through a single FFT window. Shown in Figs. 4a and 8a are the chosen interface DoFs.

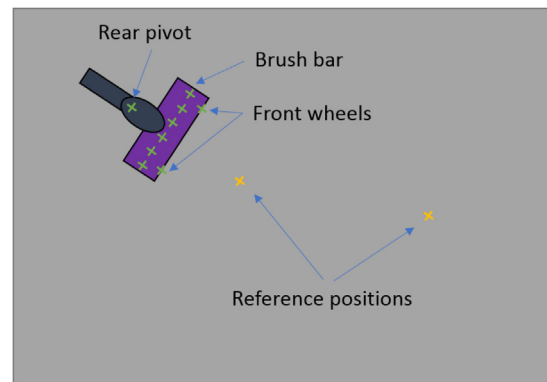
Given the weak coupling between the cleaner head and floor, it was noted that the presence of the cleaner head had little to no effect on the dynamics of the interface (i.e. the coupled and uncoupled mobilities are approximately the same $\mathbf{Y}_{Ccc} \approx \mathbf{Y}_{Rcc}$) above 100 Hz. This result is illustrated in Fig. 7, where the point mobility at one interface DoF is shown for the coupled (blue, cleaner head-floor) and uncoupled (orange, floor only) case. It is clear that from approximately 100 Hz the two are in very close agreement. A similar result was obtained for all remaining interface mobilities. Based on this result we may justly use the interface *contact* force in place of the *blocked* force.

Shown in Fig. 11a is the experimental set-up used. The cleaner head was placed on a perspex plate (representing the floor), to which 11 accelerometers were stuck; 9 at the separating interface *c*, and 2 at remote positions *b*. The 9×9 interface mobility matrix $\mathbf{Y}_{Rcc} \approx \mathbf{Y}_{Ccc}$ was measured using an instrumented force hammer, after which the operational response \mathbf{v}_{Cc} was recorded. The forces were then determined as per Eq. (11).

Before considering the radiated sound pressure, the interface description described above was validated using an on-board validation procedure [21]. The obtained forces were used to predict the



(a)



(b)

Fig. 8. Experimental set-up for validation of blocked/contact force characterisation. Green crosses - interface DoFs *c*, yellow crosses - reference receiver DoFs *b*. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

operational response at the two reference positions away from the interface (see Fig. 8) as per Eq. (12).

For comparison purposes, these reference responses were also measured directly. The on-board validation results are shown in Fig. 9. The predicted velocity response (orange) is in good agreement with the directly measured response (blue) in both cases. This result suggests that the interface description used is appropriate and that the contact forces are able to sufficiently characterise the cleaner head-floor interaction.

Having characterised the forces imparted on the floor by the operational cleaner head, it is possible to perform a contribution analysis (also known as Transfer Path Analysis) to investigate the dominance of a particular set of DoFs. An example of this is shown in Fig. 10 where the contributions arising from the brush bar, the front wheels, and the rear pivot are separated and compared against one another. These results indicate that at low frequencies (below 100 Hz) all DoFs tend to contribute equally, in the mid-frequency range (between 100 and 1500 Hz) the front wheels tend to dominate, and at high frequencies (above 1500 Hz) the brush bar tends to dominate. This sort of information may prove useful, for example in assessing design changes.

Based on the above results it is deemed that the interface description adopted is suitable for characterising the cleaner head-floor interaction, and furthermore that the cleaner head dynamics may be neglected when considering the floor contribution.

In Section 4.1.3 the above procedure is repeated whilst the cleaner head-floor assembly is installed in a semi-anechoic chamber. This will enable the prediction of radiated sound pressure by

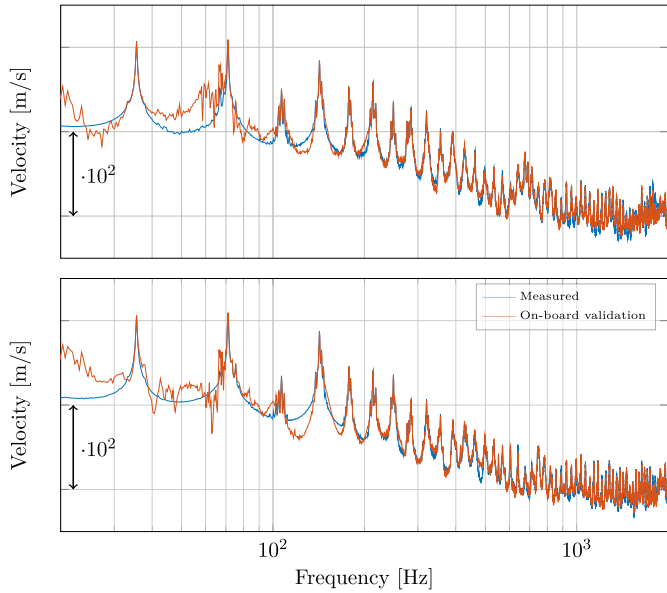


Fig. 9. On-board validation of the preliminary floor study. The acquired forces are used to predict the operational response v_{cb} at two different reference receiver positions.

measuring the vibro-acoustic FRFs from the interface DoFs to a microphone hemi-sphere. In Section 4.2.1 the obtained forces will be used to excite the floor sub-system of the hybrid FE-SEA-X model of the assembly.

4.1.2. Shell response prediction

In Section 4.1.1 we considered the cleaner head-floor interface. In this section we will consider the brush bar-cleaner head (shell) interface, with the intention of predicting its radiated sound pressure contribution. The approach adopted here will be that of the pseudo force described in Section 3.3.

As discussed in Section 3.3, pseudo forces are external forces which, when applied, recreate an identical response field in a receiver structure.

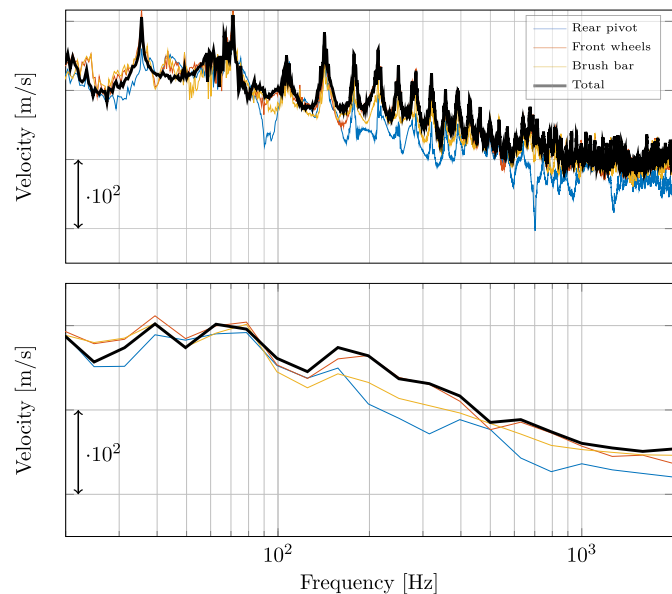


Fig. 10. Contribution analysis of the operational response v_{cb} due to the brush bar, front wheels and pivot. Upper – narrow band; lower – third octave bands.

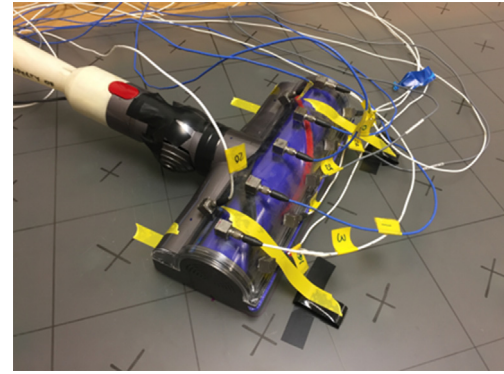


Fig. 11. Experimental set-up for the validation of the pseudo-force method.

An important feature of the pseudo force approach is the arbitrary nature of their positioning. We are interested in adopting the pseudo force approach to characterise the mechanisms that cause the cleaner head shell to vibrate, and consequently radiate sound. To validate the pseudo force method, in this section we consider an entirely experimental application, where both mobilities and operational responses are measured. The pseudo forces used within the hybrid FE-SEA-X model in Section 4.2, however, are determined by using transfer mobilities obtained from a FE shell model, as described in Section 4.2.2.

Shown in Fig. 8 is the experimental set-up used. The cleaner head shell was instrumented with 12 response sensors (accelerometers). Based on the operational responses measured at these positions, two different sets of pseudo forces are determined. The number, and position, of these pseudo forces are shown in Figs. 4b and c. Two sets of pseudo forces were considered given that the brush bar-shell interface is somewhat unclear. The pseudo forces were obtained by first measuring the operational shell response at each sensor position. Then the transfer mobilities between each force position and sensor were measured. The pseudo forces were then obtained as per Eq. (18). Note that when calculating the pseudo forces only 11 response measurements are used. The 12th is retained as a reference point for an on-board validation.

Shown in Fig. 12 are the on-board validation results for the two sets of pseudo forces. Good agreement is obtained in both cases up to 1 kHz, beyond which the two predictions begin to deviate from

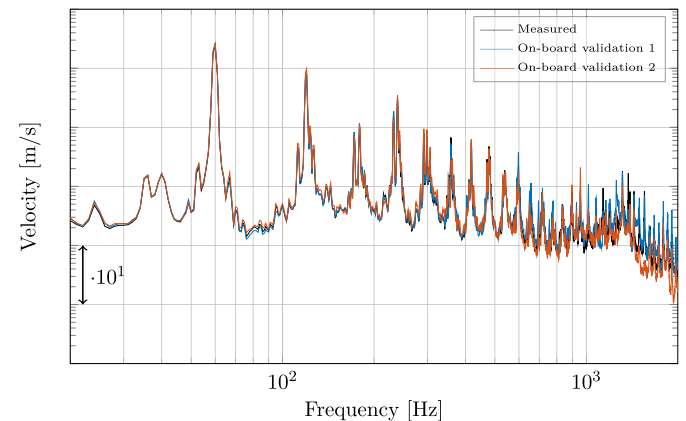


Fig. 12. On-board validation of pseudo forces. The determined pseudo forces are used to predict the operational response v_{cb} at a remote position on the shell.

the measure response. Nevertheless, the general shape of the response is still predicted quite well. The results suggest that the pseudo forces method is appropriate and able to reproduce the response of the cleaner head shell when subject to the brush bar excitation.

4.1.3. Vibro-acoustic contributions

Sections 4.1.1 and 4.1.2 focused on predicting the structural response of the floor and cleaner head, respectively, based on the obtained blocked/contact and pseudo forces. In this section we will consider the contribution of these forces to the total radiated sound.

To predict the radiated sound pressure the blocked/contact and pseudo forces are combined with measured vibro-acoustic FRFs. These FRFs were measured in a semi-anechoic chamber by applying a known force to each interface DoF/pseudo force location, and measuring the resultant pressure level over a hemi-spherical microphone array surrounding the cleaner head-floor assembly.

Shown in Fig. 13 are the (spatially) averaged FRFs from each excitation position to a single microphone. Shown in bold is an averaged FRF across all excitation and response positions. Based on these FRFs the averaged sound pressure contributions of the floor and cleaner head are shown in Fig. 14, alongside the total measured response.

It should be noted only structure-borne contributions are considered here; any air-borne contribution, say due to brush bar-plate interaction, is neglected. Hence, the predictions are simply intended to provide an indication of the relative contribution of the cleaner head and floor vibration to the total radiated sound pressure level.

From Fig. 14 it is clear that the relative contribution of floor and cleaner head varies considerably with frequency. At low frequencies, below approximately 300 Hz their contributions are similar. Above 300 Hz the cleaner head radiation tends to dominate. The general trend of this result appears in agreement with previous studies on a similar brush bar assembly [4]. Furthermore, as expected, there appears to be a general under prediction in the mid to high frequency range. This is most likely due to air-borne contributions that are not accounted for by the structural methods employed here. Nevertheless, the result appear sensible.

In the following section a similar prediction will be made, instead using a hybrid FE-SEA-X model of the assembly, supplemented with the experimental data presented in this section.

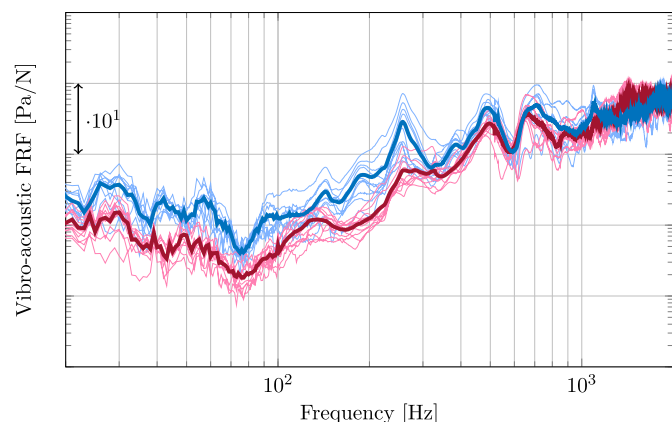


Fig. 13. Vibro-acoustic transfer function from the shell pseudo-force positions (red) and cleaner head-floor interface DoF (blue) to microphone hemi-sphere. Bold plot shows the spatial average. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

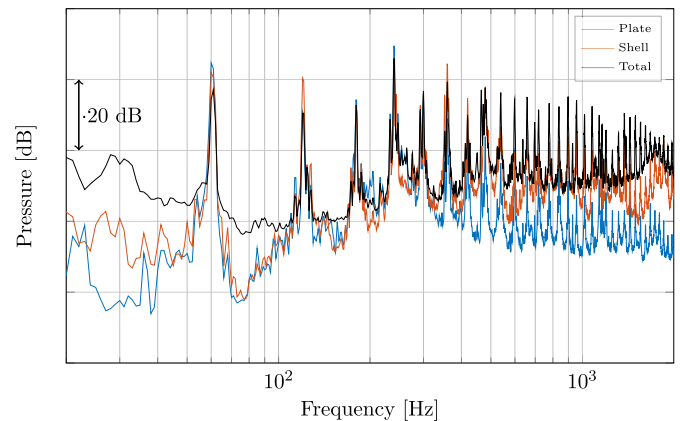


Fig. 14. Comparison of the floor/plate (blue) and shell (orange) contributions to the spatially averaged radiated sound pressure level. Also shown is the total measured pressure level (black). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.2. Hybrid FE-SEA-X model

In this section, we will develop a hybrid FE-SEA-X model to represent the vibro-acoustic behaviour of the vacuum cleaner operating within a room. As for the experimental procedures presented in Section 4.1, the aim is to use this model to compare the radiated structure-borne sound contributions from the cleaner head and floor. In brief, the hybrid model considers the outer shell of the cleaner head as the deterministic part of the system, represented by an FE model, and both the room volume and the floor as statistical sub-systems. As discussed in the following subsections, the operational activity of the cleaner head is characterised using the experimental data obtained through Section 4.1.

A schematic of the hybrid model is presented in Fig. 3. The experimental blocked/contact forces characterising the brush-bar-floor interaction are input to a 2D SEA plate sub-system, itself coupled to a 3D SEA cavity sub-system representing the enclosed room. A set of experimental/numerical pseudo forces (see Section 4.2.2), representing the brush bar-shell interaction, are used to excite the FE model. This FE model is then coupled to the same 3D SEA sub-system. The model has been implemented in the commercial vibro-acoustic software wave6 [15].

The room sub-system is given dimensions 3.5 m × 4.5 m × 3 m, with a floor surface area of 15.75 m² and a room volume of 42 m³. The floor sub-system is specified as concrete with a thickness of 6 cm. The noise radiated into the room up to 2000 Hz will be considered in the calculations. In the frequency range of interest the floor sub-system has around 300 structural modes, while more than 30000 modes are expected in the room sub-system. Hence, an SEA representation is considered appropriate.

4.2.1. Floor response prediction

The developed hybrid FE-SEA-X model assumes that the floor and the room can be considered as statistical sub-systems. This assumption is supported by the large number of modes estimated in each sub-system. On the other hand, the cleaner head geometry and material properties suggest that it should be modelled as a deterministic structure, a suggestion that will be supported in the next subsection.

The results presented in Section 4.1.1 showed that, due to the weak coupling between the cleaner head and the floor, the obtained blocked forces are approximately equal to the interface contact forces. Therefore, the floor contribution to the radiated structure-borne sound can be computed without having to consider the cleaner head. This result is particularly advantageous

here, as it states that there is no need to explicitly couple a detailed FE model of the cleaner head to that of the SEA floor sub-system. Their contributions may be treated separately. Then, as indicated in Fig. 3, the floor contribution to the room response is directly computed by inserting the experimental blocked/contact forces as point forces applied on the SEA floor sub-system.

4.2.2. Shell response prediction

The approach used for predicting the shell contribution to the radiated structure-borne sound resembles the pseudo-force approach presented in Section 4.1.2. In this case, however, the aim is to reproduce the operational response of the cleaner head using a simplified FE model that only considers its outer shell, thus overcoming the challenge of modelling the cleaner head structure in detail.

The proposed approach assumes that, when operating, the cleaner head's outer shell is mainly excited by forces transmitted through both shell ends (see Fig. 15). The outer shell is then modelled as a simplified FE structure excited by a discrete set of point loads applied on its ends. The complex amplitudes of the point loads can be determined by imposing that the FE model response to these excitations is equal to the response measured experimentally, as discussed in Section 3.3. The above amounts to determining a set of 'hybrid' pseudo forces based on experimental response measurements, and FE mobilities. It is proposed that these hybrid pseudo forces, when applied to the simplified FE model, will provide a reasonable approximation of the structural radiation. The FE model of the cleaner head's shell structure has been developed using wave6 software [15]. The shell has been assumed to be made of perspex, with a constant loss factor $\eta = 0.06$. The FE model, shown in Fig. 5, consists of 1293 2D shell elements. The model predicted that the outer shell component has 42 modes up to 2000 Hz, supporting the assumption that it is best represented as a deterministic system.

As in the experimental case, the pseudo forces f_{ca} are obtained using Eq. (18), where v_{cb} are the measured operational shell responses. In this case, however, the structure mobility Y_{cba} has been computed using the shell FE model, instead of being determined experimentally. The pseudo force approach has been applied considering three point forces at each end of the cleaner head. The mobility matrix Y_{cba} was obtained by computing the response to each one of these six excitations at the 12 receiver points representing the measured response positions. These positions have been marked as blue dots in Fig. 16.

As in the experimental case, the pseudo forces are determined using only a subset of the total number of response positions; the remaining positions are used for validation purposes.



Fig. 15. Picture of the cleaner head outer shell component.

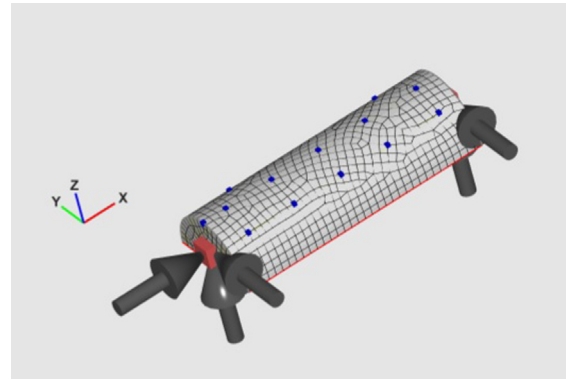


Fig. 16. Pseudo-forces and response positions considered in the FE model.

Shown in Fig. 17 are the on-board validation results at two reference positions for the hybrid pseudo forces; measured responses are compared against those predicted using the FE mobility matrix and the hybrid experimental-numerical pseudo forces. Good agreement is obtained up to around 1500 Hz, after which the computed response seems to over-predict the measured one. Therefore, it can be stated that the outer shell response under operational conditions can satisfactorily be represented using the considered set of pseudo forces for most of the frequency range of interest.

As indicated in Fig. 3, the cleaner head's shell contribution to the radiated response is computed by applying the obtained pseudo forces to the FE shell component of the complete FE-SEA-X model of the cleaner head-floor-room system.

4.2.3. Vibro-acoustic contributions

A comparison between the radiated structure-borne sound contributions is finally obtained by embedding the experimental blocked/contact forces acting on the floor, and the pseudo forces applied on the cleaner head shell, in the hybrid FE-SEA-X model. To take into account the absorption and transmission of energy through the room walls and ceiling (which have not been included

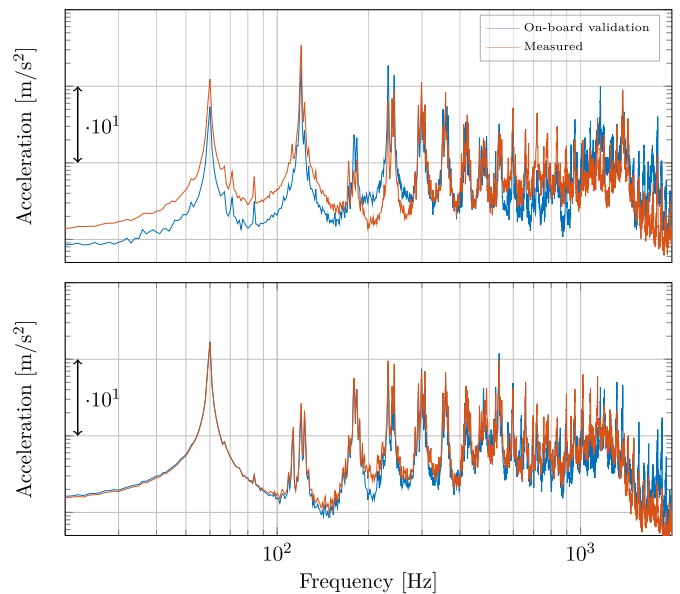


Fig. 17. On-board validation of the hybrid pseudo-forces obtained using experimental response data and the shell FE model. Two different reference positions are considered.

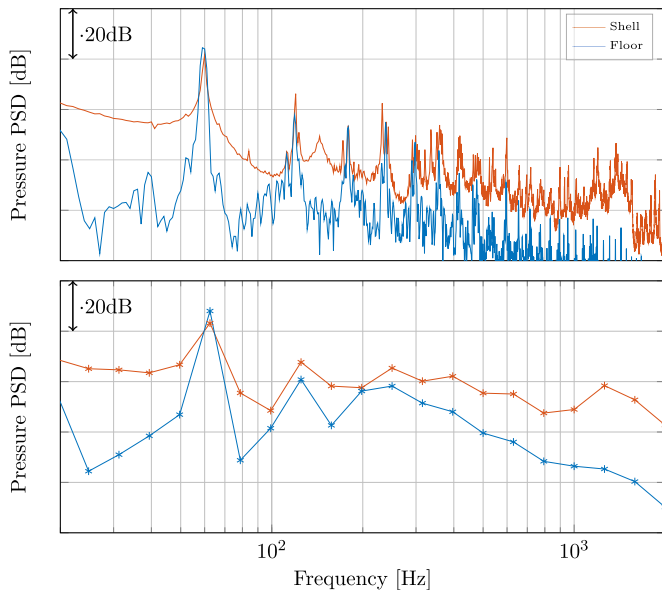


Fig. 18. Comparison between the radiated structure-borne sound contributions from the floor and from the cleaner-head shell components. Results are presented in narrow band (top) and in one-third octave bands (bottom).

as sub-systems in the model) a room damping of 1% has been considered in the calculations. It should be noted, however, that the aim of the calculation is not to compare absolute sound pressure values, but to compare relative contributions.

Shown in Fig. 18 are the relative contributions of the floor and cleaner head, as predicted by the hybrid FE-SEA-X model. The numerical prediction appears in good agreement with the experimental results shown in Fig. 14, following the same general trend. Both experimental and hybrid predictions indicate that the cleaner head tends to contribute more towards radiated sound than the floor, particularly at higher frequencies. This result supports previous studies on similar brush bar assemblies [4].

The hybrid floor contribution is only greater than the cleaner head around the fundamental peak of 60 Hz, i.e. the operational frequency of the brush bar. A similar result was obtained experimentally in Fig. 14. It should be noted that, despite its apparent importance, this peak would be strongly reduced if an A-weighting were used, reducing its overall contribution. Agreement between the general trends of the experimental results and the hybrid model predictions are promising, and certainly supportive of hybrid FE-SEA-X approach to modelling complex operational structures.

5. Conclusions

The purpose of this paper was to present a case study whereby a hybrid experimental-numerical model is used to analyse the vibro-acoustic performance of a domestic product. It was argued that whilst conventional numerical methods are able to predict the propagative and radiative properties of a sub-structure or assembly, the complex mechanisms that generate vibration lay beyond their capabilities. These operational features are however, amenable to experimental characterisation. By combining the in-situ blocked force approach for source characterisation, with the established FE-SEA method for propagation and radiation, complex operational structures can be modelled by exploiting their respective advantages.

The hybrid FE-SEA-X methodology was adopted here to investigate relative vibro-acoustic contributions for a floor/vacuum clea-

ner assembly. The in-situ blocked force approach was used to characterise the activity of the cleaner head-floor interface. The experimental blocked force data was then used to excite an SEA sub-system, representing the floor, as part of a hybrid FE-SEA-X model. To account for the radiated contribution of the cleaner head, a set of pseudo forces were determined by combining experimental response measurements with a numerical mobility matrix. The 'hybrid' pseudo forces were used to excite a simplified FE model of the cleaner head. Using the hybrid FE-SEA-X model, the relative contributions of the SEA floor sub-system and the FE cleaner head, to the level in an enclosing room, were predicted. Comparison against a similar, but experimental, contribution analysis showed the same general trends.

Whilst the case study was not aimed at providing a one-to-one comparison between model and experiment, the results obtained are supportive of the hybrid FE-SEA-X approach to modelling complex operational structures.

CRedit authorship contribution statement

J.W.R Meggitt: Conceptualization, Methodology, Validation, Investigation, Writing - original draft, Writing - review & editing. **A. Clot:** Conceptualization, Methodology, Validation, Investigation, Writing - original draft, Writing - review & editing. **G. Banwell:** Conceptualization, Methodology, Validation, Investigation, Writing - original draft, Writing - review & editing. **A.S. Elliott:** Conceptualization, Methodology, Validation, Investigation. **A.T. Moorhouse:** Supervision, Funding acquisition. **R.S. Langley:** Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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