Synergies between the high-frequency Boundary Element Method and Geometric Acoustics

Jonathan A. Hargreaves [1]
[1] Acoustics Research Group, University of Salford, UK
j.a.hargreaves@salford.ac.uk

ABSTRACT

The audible frequency range covers many octaves in which the wavelength changes from being large with respect to dominant features of a space to being comparatively much smaller. This makes numerical prediction of a space's acoustic response, e.g. for auralisation, extremely challenging if all frequencies are to be represented accurately. Different classes of algorithm give the best balance of accuracy to computational cost in different frequency bands. At low frequencies, wave effects such as diffraction and interference are essential, but methods modelling the underlying PDEs directly have computational cost that scales with problem size and frequency, rendering them inefficient or intractable at high frequencies. At high frequencies, geometric ray descriptions are more efficient, but the accuracy they can achieve is limited by how well the geometric assumption represents sound propagation in a given scenario; this comprises accuracy at low frequencies in particular. It is therefore often necessary to operate two algorithms in parallel handling different bandwidths but, combining their output data can be an awkward process due to their differing formulations. This is particularly important for early reflections, which give crucial spatial perceptual cues – for late time the wave field becomes chaotic at high frequencies and the benefits are less clear. There is therefore a need for a unified full audible bandwidth algorithm for early reflections.

This paper will describe ongoing research to develop such an algorithm by exploiting synergies between Boundary Element Method (BEM) and Geometric Acoustics (GA). It will describe how appropriately chosen oscillatory basis functions in BEM can produce leading-order GA behavior at high frequencies, and how these might be assembled into a full. It will introduce the 'Wave Matching' BEM, a new formulation that can be solved by marching-on-in-reflection, a property shared with GA that makes it inherently suitable for modelling early reflections.

1. EXTENDED ABSTRACT

Prediction models are at the heart of modern acoustic engineering and are used in a diverse range of applications, from refining the acoustic design of classrooms and concert halls to predicting how noise exposure varies through an urban environment. They also allow Auralisation to be performed for buildings and spaces before they are built or long after they are lost.

Room acoustic simulation is currently dominated by Geometrical Acoustic (GA) solvers. These models are efficient and have been widely used for approaching 30 years [1], but it is also widely known that they are not accurate in all scenarios. This is usually at lower frequencies or in smaller rooms, where the modal density remains low up to a higher frequency.

'Wave-based' methods such as Finite Difference Time Domain [2] or Boundary Element Method (BEM) are extremely accurate [3] but their computational cost scales badly with frequency, meaning there is an upper limit to their practical use. Moreover, it is well known that at late time, the sound field in rooms at high frequencies becomes diffuse and chaotic at high frequencies. There is therefore little to be gained from running a computationally expensive wave method in this region; an energy-based algorithm that gives an ensemble average of likely responses is more appropriate [4].

Figure 1 illustrates different regions in a room impulse response spectrogram. GA is currently the de-facto method for the orange and red regions, with high modal density, but it is common to apply different algorithms in each block. Often for the orange block, with its sparse reflections that are important for spatial cues, a deterministic algorithm such as the Image Source Method is employed. Later in the red block, it is common to switch to a stochastic ray tracing method. This is less accurate, but its computational cost for higher reflection orders is very favourable compared to Image Source, and the chaotic nature of the field negates the benefits of a more accurate method.

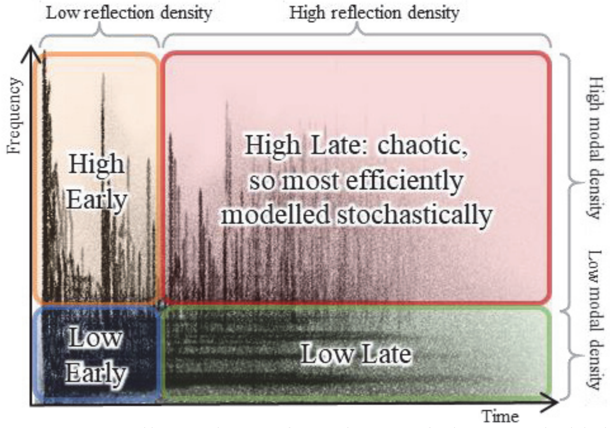

Figure 1: Illustration of regions of low and high reflection density (left to right) and modal density (bottom to top) in a room impulse response spectrogram.

Notable here is an emerging thrust of research on what Svensson & Savioja term “Surface-based” GA [1]. The variants of these that consider wave direction are the most sophisticated, and these ideas have been suggested independently by several groups [4]–[7]. Being based on a boundary element mesh, these methods appear particularly suitable for coupling to BEM, likely via stochastic energy statements [8].

The blue and green regions of Figure 1, with their low modal density, are much better handled by a ‘wave’ solver such as BEM. ‘Hybrid’ approaches have therefore been suggested that use such a solver for this region while GA is used for the higher frequencies [3], [9]–[11]. But while efficient, this is not as straightforward as it seems, due to the requirement for a crossover between the impulse responses generated by two algorithms. For late time, a conventional crossover filter is adequate, but for early time more care is required to merge the data correctly. This is because of the sparse reflection patterns and the importance of these subjectively. Aretz et al [12] suggested a non-linear method that avoids cancellations at the crossover frequency when the algorithms predict different phases for reflections, but in unpublished work by this author it has produced ‘pre-ringing’ artefacts.

This paper addresses these issues through a vision where a unified full-audible bandwidth algorithm would address early-time prediction for all frequencies i.e. the blue and the orange blocks. While ambitious, this is substantially more tractable than producing a single efficient algorithm to cover the entire impulse response, because it is limited in reflection order. In particular, this demarcation would allow the efficiency gains seen in Hybrid Numerical Asymptotic BEM [13] to be leveraged. For specific classes of scattering problem, this has been shown to reduce the number of degrees of freedom necessary so as to scale with log frequency [14], or even be frequency independent [15]. But such problems are characterized by a small number of dominant wave directions, and this only holds for rooms during the early reflection regime. After that, energy must be transferred to another algorithm optimized for late time, likely a surface-based GA algorithm [4]–[7] at high frequencies, and a time domain BEM algorithm at low frequencies [16], [17].

This presentation instead aims to explore the physical interpretation of such algorithms and their relation to GA, which is well understood within the acoustics community, rather than focus on the detailed mathematical exposition of such methods, as is more common. It also aims to showcase a new formulation ‘The Wave-Matching Boundary Integral Equation’ [18], which has not yet been disseminated at an acoustics conference. This contains several features congruent with the espoused vision, notably the ability to be solve by marching on in reflection order. Such progress is early steps towards a unified full-audible bandwidth solver for early time, but is progress in that direction, nonetheless.

2. REFERENCES

[1] L. Savioja and U. P. Svensson, “Overview of geometrical room acoustic modeling techniques,” *J. Acoust. Soc. Am.*, vol. 138, no. 2, pp. 708–730, Aug. 2015, doi: 10.1121/1.4926438.

[2] S. Bilbao, “Passive Volumetric Time Domain Simulation for Room Acoustics Applications,” *J. Acoust. Soc. Am.*, 2018.

[3] J. A. Hargreaves, L. R. Rendell, and Y. W. Lam, “A framework for auralization of boundary element method simulations including source and receiver directivity,” *J. Acoust. Soc. Am.*, vol. 145, no. 4, pp. 2625–2637, Apr. 2019, doi: 10.1121/1.5096171.

[4] R. S. Langley, “A wave intensity technique for the analysis of high frequency vibrations,” *J. Sound Vib.*, vol. 159, no. 3, pp. 483–502, Dec. 1992, doi: 10.1016/0022-460X(92)90754-L.

[5] S. Siltanen, T. Lokki, S. Kiminki, and L. Savioja, “The room acoustic rendering equation,” *J. Acoust. Soc. Am.*, vol. 122, no. 3, pp. 1624–1635, Sep. 2007, doi: 10.1121/1.2766781.

[6] D. J. Chappell, G. Tanner, and S. Giani, “Boundary element dynamical energy analysis: A versatile method for solving two or three dimensional wave problems in the high frequency limit,” *J. Comput. Phys.*, vol. 231, no. 18, pp. 6181–6191, Jul. 2012, doi: 10.1016/j.jcp.2012.05.028.

[7] L. P. Franzoni, D. B. Bliss, and J. W. Rouse, “An acoustic boundary element method based on energy and intensity variables for prediction of high-frequency broadband sound fields,” *J. Acoust. Soc. Am.*, vol. 110, no. 6, p. 3071, Dec. 2001, doi: 10.1121/1.1416201.

[8] J. W. Rouse, “A boundary element based method for accurate prediction of the surface pressure cross-spectral density matrix,” *J. Acoust. Soc. Am.*, vol. 141, no. 5, pp. 3697–3697, May 2017, doi: 10.1121/1.4988058.

[9] M. Aretz and M. Vorländer, “Combined wave and ray based room acoustic simulations of audio systems in car passenger compartments, Part I: Boundary and source data,” *Appl. Acoust.*, vol. 76, pp. 82–99, Feb. 2014, doi: 10.1016/j.apacoust.2013.07.021.

[10] M. Aretz and M. Vorländer, “Combined wave and ray based room acoustic simulations of audio systems in car passenger compartments, Part II: Comparison of simulations and measurements,” *Appl. Acoust.*, vol. 76, pp. 52–65, Feb. 2014, doi: 10.1016/j.apacoust.2013.07.020.

[11] J. E. Summers, K. Takahashi, Y. Shimizu, and T. Yamakawa, “Assessing the accuracy of auralizations computed using a hybrid geometrical-acoustics and wave-acoustics method,” *J. Acoust. Soc. Am.*, vol. 115, no. 5, p. 2514, 2004, doi: 10.1121/1.4809339.

[12] M. Aretz, R. Nöthen, M. Vorländer, and D. Schröder, “Combined Broadband Impulse Responses Using Fem and Hybrid Ray-Based Methods,” in *EAA Symposium on Auralization*, 2009, pp. 1–6, [Online]. Available: http://auralization.tkk.fi/EAAsymposium09.

[13] I. Graham, E. Spence, S. Chandler-Wilde, and S. Langdon, “Numerical-asymptotic boundary integral methods in high-frequency scattering,” *Acta Numer.*, vol. 21, pp. 89–305, Apr. 2012, doi: 10.1017/S0962492912000037.

[14] S. Langdon and S. N. Chandler-Wilde, “A Galerkin boundary element method for high frequency scattering by convex polygons,” *SIAM J. Numer. Anal.*, vol. 45, no. 2, pp. 610–640, 2007, doi: 10.1137/06065595X.

[15] O. P. Bruno and C. A. Geuzaine, “An O (1) integration scheme for three-dimensional surface scattering problems,” *J. Comput. Appl. Math.*, vol. 204, no. 2 SPEC. ISS., pp. 463–476, Jul. 2007, doi: 10.1016/j.cam.2006.02.050.

[16] S. A. Sauter and M. Schanz, “Convolution quadrature for the wave equation with impedance boundary conditions,” *J. Comput. Phys.*, vol. 334, pp. 442–459, Apr. 2017, doi: 10.1016/J.JCP.2017.01.013.

[17] L. Banz, H. Gimperlein, Z. Nezhi, and E. P. Stephan, “Time domain BEM for sound radiation of tires,” *Comput. Mech.*, vol. 58, no. 1, pp. 45–57, Jul. 2016, doi: 10.1007/s00466-016-1281-3.

[18] J. A. Hargreaves and Y. W. Lam, “The Wave-Matching Boundary Integral Equation — An energy approach to Galerkin BEM for acoustic wave propagation problems,” *Wave Motion*, vol. 87, pp. 4–36, Jul. 2018, doi: 10.1016/J.WAVEMOTI.2018.07.003.