Vector waves with spatiotemporal dispersion and $\chi^{(3)}$ nonlinearity: transformations and relativity, solitons and stability

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Summary

A vector model, fully-second-order in both space and time, is proposed for coupled electromagnetic modes in nonlinear waveguides. Our formalism has strong overlaps with the special relativity. Exact two-component solitons are derived, asymptotic analysis recovers classic solutions, and simulations address wave robustness.

Introduction: *beyond slowly-varying envelopes*

The origin of conventional models for nonlinear optical pulse propagation lies in the ubiquitous slowly-varying envelope approximation (SVEA) in conjunction with a Galilean boost to a local-time frame. While such a near-universal procedure typically results in a simple model of the nonlinear Schrödinger-type, a more general but less well-explored class of wave equation underpins the wider propagation problem.

Menyuk's seminal analysis [1] has undeniably helped lay the foundations of today's understanding of coupled waves in nonlinear optical systems. Formulated in terms of slowly-varying envelopes and Galilean boosts, scores of vectorized Schrödinger-type models have been proposed and studied over nearly three decades. While the SVEA remains a theoretical mainstay of describing wave-based nonlinear systems, Biancalana and Creatore [2] have pointed out that there exist modern contexts (for instance, in condensed-matter physics) where its validity may be reassessed. In particular, they assert that spatial dispersion (related to light-exciton coupling inside superlattice host materials) is not necessarily well-described by the SVEA.

In this paper, we generalize our earlier scalar approach to pulse evolution [3] by accommodating the simultaneous propagation of two coupled optical waves which may represent, for instance, the excitations in two orthogonal polarizations of a fibre waveguide whose core has $\chi^{(3)}$ (Kerr-type) nonlinearity. Moreover, the mathematical context of finding exact solitary solutions to universal hyperbolic or elliptic envelope equations (as generalizations of parabolic models) is both timely and novel.

Spatiotemporal model: *transformations & relativity*

We consider a pair of normalized fully second-order (in space and time) coupled equations describing optical wave envelopes u_j , where j = 1 and 2 and

$$\kappa\frac{\partial^2 u_j}{\partial\zeta^2}+i\left(\frac{\partial u_j}{\partial\zeta}+\alpha_j\frac{\partial u_j}{\partial\tau}\right)+\frac{s_j}{2}\frac{\partial^2 u_j}{\partial\tau^2}+\left(\left|u_j\right|^2+\sigma\left|u_{3-j}\right|^2\right)u_j=0. \tag{1}$$

Here, τ and ζ are the dimensionless time and (longitudinal) space coordinates in the laboratory frame, respectively, α_j is related to the (linear) group velocity, spatial dispersion is quantified by $\kappa_j << O(1)$, group-velocity dispersion (GVD) by s_j [positive and negative values for anomalous- and normal-GVD regimes, respectively, and typically with $| s_j | = O(1)$], and σ determines the strength of cross-phase modulation.

Frame-of-reference considerations take centre stage in our approach, and space-time coordinate transformations dominate much of the analysis [3]. Conventional pulse theory emerges asymptotically from Eq. (1) and its solutions in much the same way that Newtonian mechanics corresponds to the low-speed limit of Einstein's relativistic physics (e.g., the velocity combination rule for spatiotemporal pulses is akin to that for particles in relativistic kinematics).

Operationally, one can recover (a generalized version of) Menyuk's classic vector model [1] alongside all its predictions by: (i) assuming $|\kappa_j \partial^2 u_j/\partial\zeta^2| \ll |\partial u_j/\partial\zeta|$, and (ii) Galilean-boosting to a local-time frame moving at an averaged group speed $1/\alpha$ by introducing new coordinates $\tau_{\mathrm{loc}} = \tau - \alpha\zeta$ and $\zeta_{\mathrm{loc}} = \zeta$, where $\alpha \equiv (\alpha_1 + \alpha_2)/2$. Implementing such a transformation without first making the SVEA hinders rather than helps the analysis of spatiotemporal effects (e.g., by generating mixed-derivative terms that can be awkward to interpret physically) [3].

Dark-bright and dark-dark waves: *solitons & stability*

Exact analytical dark-bright and dark-dark solitons of Eq. (1) will be presented, derived by combining ansatz methods with transformations in the space-time plane. Such phase-topological solutions offer the greatest potential impact in the arena of future optical device designs when their continuous-wave (cw) backgrounds are not susceptible to spontaneous fluctuations. A vector generalization of our scalar linear analysis [4] has been deployed to quantify the modulational instability spectrum for cw solutions (obtained by solving an 8th-degree polynomial characteristic equation) that have been subjected to small disturbances. Simulations with Eq. (1) have tested, and subsequently verified, theoretical predictions for the most-unstable frequency in the system.

Finally, we will report results from numerical perturbative analyses demonstrating instability in some conventional dark-bright and dark-dark solitons [5] when used as initial conditions in Eq. (1) (see Fig. 1). In contrast, we fully expect the more general spatiotemporal dark-type vector solitons to be relatively robust entities when operating in regimes where the constituent cw backgrounds are modulationally stable.

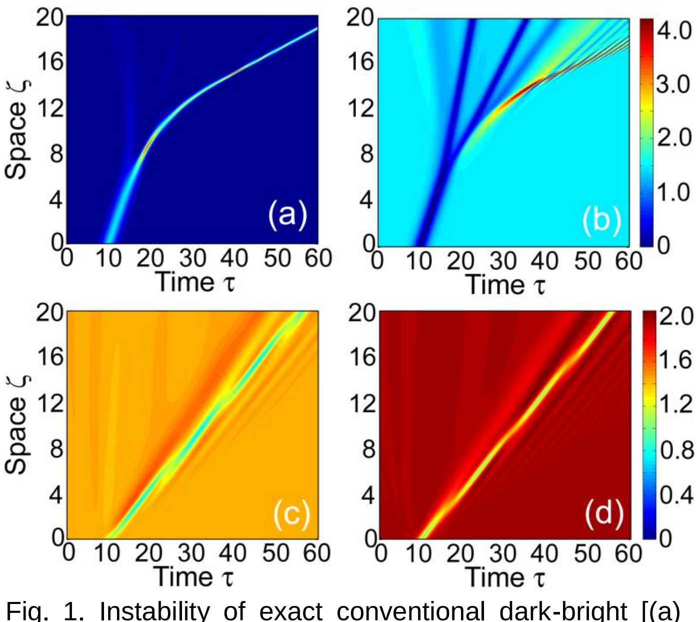

Fig. 1. Instability of exact conventional dark-bright [(a) bright $|u_1|^2$, (b) dark $|u_2|^2$] and dark-dark [(c) $|u_1|^2$, (d) $|u_2|^2$] solitons when used as initial conditions in Eq. (1) [$\kappa_1 = 1.0\times10^{-3}$, $\kappa_2 = 2.5\times10^{-3}$, and $\sigma = 2/3$]. Pulse splitting, snaking, and radiation shedding are observed.

References

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