Diffraction problems on two-dimensional complex domains: dusts, triangles, and carpets

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We consider a class of problem involving the scattering of plane waves by domains that are *complex* in the sense of being rough or (in the limit) scale-free. One instructive example is the von Koch snowflake, where a simple initiator-generator algorithm is used to gradually build-up a self-similar closed shape from a hierarchy of equilateral triangles. This abstraction may be used to model either a multi-scale aperture in an opaque screen (of principal interest in physics), or an opaque multi-scale screen surrounded by otherwise-empty space (often of greater interest in applied mathematics). Alternatively, one might be interested in scatterers that possess an interior structure such as the Cantor dust or the Sierpinski triangle.

Here, two methods will be discussed for attacking two-dimensional complex-domain problems in the classic geometries of optics experiments. Attention is first paid to a Fresnel level of description wherein the diffraction integral over the area of the aperture is transformed into a circulation around its boundary [J. H. Hannay, J. Mod. Opt. **47**, 121 (2000)]. However, the Fresnel approximation tends to break down fairly abruptly when faced with the complementary screen. We thus also propose a more refined technique based on Rayleigh-Sommerfeld (RS) diffraction integrals. A crucial physical feature of this second approach is that it is free from the assumption of paraxiality which ultimately hinders any Fresnel-based solutions. Another is that, after some manipulation, the RS circulation can be expressed as a linear superposition of edge waves that are fully-nonparaxial generalizations of their counterparts from Fresnel optics [J. G. Huang *et al.*, J. Opt. Soc. Am. A **23**, 2768 (2006)]. Both of these highly desirable properties suggest that the RS formulation offers some distinct advantages, particularly in near-field regimes. We conclude with a selection of diffraction patterns from pre-fractal iterations of the dust, triangle, and carpet.