Scattering of waves by pre-fractal Cantor-set apertures: nonparaxial formulation and numerical analysis

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The scattering of plane waves by distributions of bars that mimic increasing iterations of the Cantor set has been known for many years in optics [C. Allain and M. Cloitre, Phys. Rev. B **33**, 3566(R) (1986)]. Experiments and analyses have often been conducted in the far field under the conditions of Fraunhofer diffraction, and the observed patterns may then be described quite simply in terms of Fourier transforms. More recent investigations of fractal-type gratings have been based on the Fresnel integral with principal emphasis placed on understanding self-imaging phenomena [J. Wang *et al.*, Appl. Opt. **53**, 2105 (2014)].

Here, we formulate wave scattering in a more elementary way by discarding the paraxial approximation entirely and instead retaining the greater generality of the underlying two-dimensional Helmholtz equation. The scatterer of interest corresponds to pre-fractal (that is, a finite number of) iterations of the Cantor-set initiator-generator algorithm and where the constituent bars may be envisaged as being cut out of an opaque screen of infinite extent. A formal solution in the forward half-plane can be written down which involves a convolution between the incident field and the free Green's function (strictly, its derivative normal to the screen). This approach corresponds to a Rayleigh-Sommerfeld (RS) diffraction integral and it renders subsequent analyses predominantly numerical rather than mathematical.

In addition to the proposed nonparaxial formulation, we will present a selection of computed diffraction patterns and intensity power spectra from new physical regimes of illumination wherein the waveform incident on the pre-fractal scattering obstacle is, itself, also pre-fractal. It will be seen that the RS approach is a natural one to adopt when considering problems involving illumination with multi-scale waves. The broad spatial bandwidth of the outgoing waves is a key feature but it is not always handled satisfactorily by treatments based on Fresnel optics.