Nonlinear phenomena in the magnetic pendulum problem: butterflies and chaos, fractals and Wada

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POSTER ABSTRACT

The magnetic pendulum is a tabletop toy that provides a striking demonstration of dissipative chaos. It typically comprises three magnets placed on the vertices of an equilateral triangle in the horizontal base plane, above which is suspended a traditional pendulum bob (another magnet) that may otherwise swing freely. The bob tends to follow complicated and seemingly-irreproducible paths as it orbits the base-plane magnets.

Here, we consider a two-fold generalization of the standard problem. First, regular-polygon arrangements of N base-plane magnets are accommodated beyond the most familiar case of $N = 3$. Particular attention is paid to $N = 4$, which allows interesting new structures to develop in the basins of attraction. Second, a more physically correct formulation of magnetic interactions is incorporated: the magnets are treated as dipoles, and the strength of their interaction force varies with $1/(\text{distance})^4$. An overview of the pendulum's dynamical properties will be explored by studying a range of initial-value problems, wherein the Wada-type fractal basin boundaries (in both real and velocity spaces) are found to possess features across multiple scales. Such a property is tightly connected to the phenomenon of *sensitive dependence on initial conditions*, and we quantify this property by estimating the uncertainty dimension.