Unstable Resonators with Gosper-Island Boundary Conditions: Virtual-Source Computation of Fractal Eigenmodes

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The Gosper island is a well-known fractal belonging to a family of self-similar "root 7" curves constructed from a simple iterative algorithm [1]. One begins with a regular hexagon (the initiator, corresponding to iteration $n = 0$) with sides of reference length l_0, and then breaks each of these straight-edge elements into three equal segments of length $l_n = l_0(1/7^{1/2})^n$ where $n = 1, 2, 3,\ldots$ (the generator stages). If the total number of length elements after applying the generator n times is given by $N_n = 6 \times 3^n$, then the Hausdorff-Besicovich dimension of such a curve is calculated to be $D \equiv \lim_{n\to\infty} -\log(N_n)/\log(l_n) = 2\log(3)/\log(7) \approx 1.1292$.

In this presentation, we report on our latest theoretical results predicting the modes of unstable resonators [2,3] when the small feedback mirror has a shape corresponding to increasing iterations of the Gosper island fractal. A fully two-dimensional generalization of Southwell's virtual source (2D-VS) method [4] (itself an approximation of Horwitz's asymptotic theory [5]) is deployed, whereby the resonator is unfolded into an equivalent sequence of apertures illuminated by a plane wave. Each aperture has a characteristic size (capturing a band of pattern spatial scalelengths), and it acts as a virtual source of diffracted waves that are computed using edge-wave decompositions within a circulation-integral method [6]. The empty-cavity eigenmodes are then constructed from a linear combination of the constituent single-aperture Fresnel patterns.

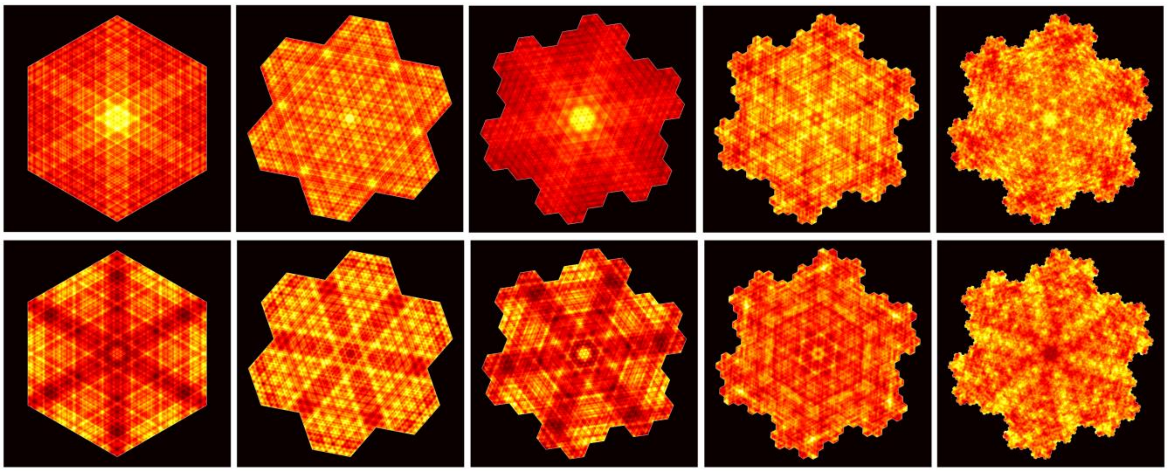

Fig. 1. 2D-VS computations of the mode patterns for an unstable resonator (parameters: $N_{eq} = 30$ and $M = 1.5$) based on the Gosper island curve whose feedback mirror progresses through the first four applications of the generator algorithm (left to right). Top row: lowest-loss modes. Bottom row: next-lowest-loss modes.

Unstable resonators are described by two key physical parameters: the round-trip magnification M and the equivalent Fresnel number N_{eq} [3,4]. A systematic review will be given of fractal eigenmode patterns for Gosper-island resonators. Spectra will be presented for the cavity eigenvalues $\{\alpha_m\}$ with $m = 0, 1, 2,\ldots$, whose magnitudes and arguments prescribe round-trip losses and phase shifts, respectively, as functions of M and N_{eq}. The weighting factors in the 2D-VS expansions are parametrized by the individual eigenvalues, and so our method gives direct access to a hierarchy of higher-order modes after only a single application (see Fig. 1). In contrast, conventional computational schemes based on fast Fourier transforms and ABCD (paraxial) matrix optics tend to yield only the lowest-loss modes (with higher-order modes often proving awkward to excite). We conclude with a summary of results for dimension estimations, where specialist software [7] has been used to quantify various different measures [8] including roughness-length, rescaled range, and variogram.

References

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