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BIOMATHEMATICAL MODEL FOR GYROTACTIC FREE-FORCED BIOCONVECTION WITH OXYGEN DIFFUSION IN NEAR-WALL TRANSPORT WITHIN A POROUS MEDIUM FUEL CELL

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ABSTRACT

Bioconvection has shown significant promise for environmentally friendly, sustainable "green" fuel cell technologies. The improved design of such systems requires continuous refinements in biomathematical modelling in conjunction with laboratory and field testing. Motivated by exploring deeper the near-wall transport phenomena involved in bio-inspired fuel cells, in the present article, we examine analytically and numerically the combined free-forced convective steady boundary layer flow from a solid vertical flat plate embedded in a Darcian porous medium containing gyrotactic microorganisms. Gyrotaxis is one of many taxes exhibited in biological microscale transport, and other examples include magneto-taxis, photo-taxis, chemotaxis and geo-taxis (reflecting the response of micro-organisms to magnetic field, light, chemical concentration or gravity, respectively). The bioconvection fuel cell also contains diffusing oxygen species which mimics the cathodic behavior in a proton membrane exchange (PEM) system. The vertical wall is maintained at iso-solutal (constant oxygen volume fraction and motile micro-organism density) and iso-thermal conditions. Wall values of these quantities are sustained at higher values than the ambient temperature and concentration of oxygen and biological micro-organism species. Similarity transformations are applied to render the governing partial differential equations for mass, momentum, energy, oxygen species and micro-organism species density into a system of ordinary differential equations. The emerging eight order nonlinear coupled, ordinary differential boundary value problem features several important dimensionless control parameters, namely Lewis number (*Le*), buoyancy ratio parameter i.e. ratio of oxygen species buoyancy force to thermal buoyancy force (*Nr*), bioconvection Rayleigh number (*Rb*), bioconvection Lewis number (*Lb*), bioconvection Péclet number (*Pe*) and the mixed convection parameter ($\boldsymbol{\varepsilon}$) spanning the entire range of free and forced convection. The transformed non-linear system of equations with boundary conditions is solved numerically by a finite difference method with central differencing, tridiagonal matrix manipulation and an iterative procedure. Computations are validated with the symbolic Maple 14.0 software. The influence of buoyancy and bioconvection parameters on the dimensionless temperature, velocity, oxygen concentration and motile microorganism density distribution, Nusselt, Sherwood and gradient of motile microorganism density are studied. The work clearly shows the benefit of utilizing biological organisms in fuel cell design and presents a logical biomathematical modelling framework for simulating such systems. In particular, the deployment of gyrotactic micro-organisms is shown to stimulate improved transport characteristics in heat and momentum at the fuel cell wall.

KEYWORDS: *Bioconvection; Gyrotactic micro-organisms; Oxygen diffusion; Buoyancy; Lewis number; Boundary layers; Fuel cells; Numerical solution; Nusselt number; Sherwood number.*

NOMENCLATURE

b Chemo taxis constant

C volume fraction of oxygen species

C_w Wall volume fraction of oxygen species

C_∞ Ambient volume fraction of oxygen species

D Mass diffusivity of the porous medium

D_n Diffusivity of gyrotactic motile micro-organisms

g Acceleration due to gravity

k Thermal conductivity

K Permeability of the porous medium

Lb bioconvection Lewis number

Le Lewis number

N_{u_x} Local Nusselt number

n Volume fraction of gyrotactic motile micro-organisms

N_{n_x} Local density number of gyrotactic motile micro-organisms

Pe Bioconvection Péclet number

Ra Rayleigh number for the porous medium

Ra_x local Rayleigh number for the porous medium

Rb Bioconvection Rayleigh number

Sh_x Local Sherwood number

T Temperature

T_w Wall temperature

T_∞ Ambient temperature

W_c Maximum cell swimming speed

$\bar{u}$,$\bar{v}$ Velocity components along $\bar{x}$&$\bar{y}$ axes

$\bar{x}$,$\bar{y}$ Cartesian coordinates

($\bar{x}$-axis is aligned along the horizontal surface and $\bar{y}$ - axis is normal to it)

Greek Symbols

α Effective thermal diffusivity of the porous medium

$\varphi(\eta)$ Dimensionless oxygen species concentration distribution

η Similarity variable

γ Average volume of a micro-organism

$\theta(\eta)$ Dimensionless temperature

ν Kinematic viscosity of fluid

ρ Fluid density

$\sigma(\eta)$ Dimensionless density of gyrotactic motile micro-organisms

Ψ Stream function

1.INTRODUCTION

Mixed convection is a combination of forced and free convection which is determined by both an outer forcing system and inner volumetric forces (thermal, species buoyancy etc). The study of combined convection boundary layer flow is of considerable interest in modern technology owing to diverse and ever-growing applications in nuclear reactor transport, transpiration cooling, materials processing, fire spread and fuel cells. Boundary layer theory is particularly useful in evaluating near-wall fluid dynamic, heat and mass transfer characteristics. Porous media also arise in extensive systems including geothermal reservoirs, insulation, biomechanics, foams, combustion and petrochemical filtration. Many researchers have investigated both free and forced convection heat and mass transfer in porous media in *external boundary layer flows* from vertical surfaces. Many investigations in this regard have been reported with consideration of different multi-physical effects. Lai [1] analyzed the thermo-solutal convection in a porous medium showing that the buoyancy ratio and Lewis number have a profound influence on flow behavior from the asymptotic free convection limit to that of the forced convection limit. Bég *et al.* [2] used a network electrothermal numerical code to study the natural convection Sakiadis flow in a thermally stratified nonlinear permeable regime. Tsai and Huang [3] considered cross diffusion effects in non-isothermal free convection boundary layer flow. Srinivasachary and Reddy [4] considered non-Newtonian mixed convection in porous media with Soret and Dufour diffusional effects. Bég*et al.* [5] used a finite difference method to simulate mixed thermal convection nanofluid boundary layer flow in porous media with Buongiorno’s nanoscale model. Further studies include Srinivasachary and Surender [6] (on double stratification in nanofluid mixed convection in porous media), Bhargava*et al.* [7] (on finite element analysis of pulsating flow and heat in porous biomaterials), Bansod[8] (on heat and mass transfer in isotropic porous media),Bég*et al.* [9] (onunsteady rotating Couette flow in Darcy-Forchheimer porous media),Bég*et al.* [10] (on transientradiative-convective flow in a Darcian porous medium), Postelnicu[11] (on chemically reactive thermo-solutal free convection in porous media with cross diffusion) and Bég [12] (on mixed thermal convection from a rotating cone in orthotropic porous media). In most of these studies the classical Darcy model has been employed which is valid for low-velocity, viscous-dominated transport. Darcy's model relates the effective flow velocity to the pressure drop across a porous medium. It approximates quite accurately many diverse fluid dynamics applications including energy systems. Porous media are frequently exploited in renewable thermal energy installations including geothermics [13] and hybrid solar thermal absorption collectors [14, 15]. The Darcy model has been popularized in such areas since the seminal work of Cheng and Minkowyez[16] who presented similarity solutions for Newtonian free thermal convection from a vertical plane surface in a porous medium as an approximation of geothermal energy transport. Nakayama and Koyama [17] have also suggested similarity transformation for pure, combined and forced convection in Darcian and non- Darcian porous media. Kumari *et al.* [18] extended the Cheng-Minkowyez formulation to rheological power-law fluids. Bég [19] used the Nakayama-Koyama Darcy formulation to investigate magnetohydrodynamic thermo-solutal mixed convection dynamics from an extending sheet in porous media with cross diffusion. Further studies include Tripathi and Bég [20] who employed a generalized Darcy model for studying peristaltic pumping of non-Newtonian Maxwell viscoelastic fluids through a porous medium and Bég*et al.* [21] who deployed the Cheng-Minkowyez Darcy formulationto simulate fluid-particle transport in dialysis filtration systems.

In biomedical systems [22] and also in modern sustainable fuel cell design, oxygen transport is an important consideration [23]. To simulate oxygen diffusion a separate species conservation equation is generally required in addition to the momentum and energy conservation equations associated with thermal convection. Interesting studies in this regard which address proton exchange membrane (PEM) fuel cells include Bradean*et al.* [24] who simulated flow dynamics near the porous cathode of a proton exchange membrane fuel cell and Jeng *et al.* [25] who analyzed numerically the oxygen mass transfer in PEM fuel cell gas diffusion layers. These studies emphasized the importance of utilizing physically viable data for oxygen diffusion to achieve realistic estimates of transport characteristics. Another interesting aspect of modern fuel cell design (microbial and PEM) is bio-enhanced engineering. Biological micro-organisms can enhance significantly the performance of green, ecologically-friendly fuel cell systems, whether solar, hydrogen or of other types. In traditional multiphase flow mechanics, solid particles are either carried by the fluid flow or pushed by external forces. However, in microbiological fluid mechanics, engineers utilize the flow of self-propelled microorganisms, such as motile species of bacteria and algae. A common manifestation of micro-organism propulsion is *bioconvection*. Bioconvection patterns are observed in cultures of swimming micro-organisms which are heavier than water and tend to propel themselves toward the upper surface of their environment in response to external stimuli such as gravity, light, magnetic field, chemical gradient etc. These control mechanisms are known as taxes and the corresponding micro-organisms may be gravitactic, photo-tactic, magneto-tactic, chemo-tactic etc. These microorganisms swim by rotating flagella automated by commutative molecular motors that are embedded in the cell wall [26]. Bioconvection can be classified as the *macroscopic fluid motion* due to the density gradient associated with swimming micro-organisms [27]which intensify the density of the base fluid (water) in a specific direction that generates the bioconvection flow.Mathematical models of bioconvection propulsion are immensely beneficial to designing optimal microbial fuel cell systems. These models robustly simulate the mechanisms which compel microorganisms to swim in specific directions depending on the ambient condition [28, 29]. Nield and Bejan [30] define bioconvection as “the pattern formation in suspensions of microorganisms such as bacteria and algae due to up swimming of the microorganisms”. Gyrotactic microorganisms such as Cnivalis swim upward in still water due to the fact that their center of mass is located behind the center of buoyancy. They respond to torque dynamics as the taxis. In recent years many studies of near-wall boundary layer free/forced/mixed bioconvection flows have been communicated which have featured many different computational approaches due to the inherent nonlinearity in bioconvection boundary value problems. These include Uddin *et al.* [31] (bioconvection nanofluid slip flow from undulating walls using finite elements and Maple quadrature), Latiff*et al.* [32] (microstructural forced bioconvection slip nanofluid flow from extending/contracting elastic sheets using finite difference schemes), Basir*et al.* [33] (coating flow of a stretching cylinder with nano-bioconvection boundary layers using shooting methods) and Bég*et al.* [34] (Jeffery-Hamel nozzle and swirling disk nanofluid bioconvection flows which employed Adomian decomposition methods). Khan and Makinde [35] investigated magnetic nano-bioconvection boundary layer flow with a convective boundary condition and shooting method. Raees *et al.* [36] studied mixed gyrotactic bioconvection nanofluid flow using a homotopy analysis method (HAM). Xu and Pop [37] studied gyrotactic bioconvection nanofluid flows in a horizontal channel using homotopy analysis and the passively controlled nanofluid model. Bég *et al.* [37] used the Keller box finite difference implicit method and Nakamura tridiagonal method to analyze free and forced Newtonian nanofluid oxytactic bioconvection in Darcian porous media, describing the key influences of bioconvection Rayleigh and Lewis numbers on heat, mass and momentum transfer characteristics.

In the present work, we investigate the gyrotactic bioconvection boundary layer flow with oxygen diffusion in the vicinity of the wall of a PEM fuel cell containing a homogenous, isotropic porous medium. The base fluid is water. Laminar and steady state flow is considered, and the Darcy model is adopted. The conservational partial differential boundary layer equations are normalized with appropriate similarity variables and rendered into a system of coupled, nonlinear ordinary differential boundary layer equations with suitable wall and free stream boundary conditions. The transformed ordinary differential boundary value problem is solved with a robust finite difference scheme. Extensive visualization of velocity, temperature, oxygen species concentration and motile micro-organism density number functions is presented for the influence of profile buoyancy parameter, bioconvection Raleigh number, bioconvection Lewis number, Lewis number and biconvection Péclet number. Validation of solutions is included using Maple-based numerical quadrature. Nusselt, Sherwood and gradient of motile microorganism density are also computed. The study is motivated by elucidating in more detail the near-wall transport phenomena in modern biological-inspired microbial fuel cells [39] and plant–microbial fuel cells (PMFCs) [40] which are newly emerging devices utilizing energy generated by microorganisms that use root exudates as fuel. The simultaneous presence of bioconvection and oxygen diffusion is important to developing deeper understanding of the heat and mass transfer characteristics at the cathode wall and biofilm communities in such systems [41-43] wherein the bacteria have been shown to utilize oxygen as an electron acceptor, but not known to have exo-electrogenic activity. Furthermore, the present simulations provide a good benchmark for more advanced computational fluid dynamics analysis of, for example, solar-hybrid photo-microbial fuel cells [44-45].

2. GYROTACTIC BICONVECTION TRANSPORT MODEL

Consider a two-dimensional steady-state boundary layer flow of a Newtonian fluid containing gyrotactic microorganisms. For the survival of the micro-organisms we have considered water as a base fluid. Due to gravity the fluid and micro-organisms both fall downwards along the vertical surface (fuel cell cathode wall). We select the coordinate system such that the *x*-axis is orientated along the wall and the *y*-axis is directed normal to it. The physical flow model is illustrated in **Fig.1**. The vertical wall is maintained at iso-solutal (constant oxygen volume fraction and motile micro-organism density) and iso-thermal conditions. The flow regime consists of four boundary layers- the momentum boundary layer (designated U in **fig. 1**), thermal boundary layer (designated T in **fig. 1**), oxygen species boundary layer (designated c in **fig. 1**) and the motile micro-organism species boundary layer (designated n in **fig. 1**). The vertical wall is kept at constant temperature, T_w, oxygen volume fraction C_w and the motile micro-organism density, n_w which are considered to be of greater magnitude than the ambient temperature and oxygen and motile micro-organism concentrations, T_∞, C_∞ and n_∞.

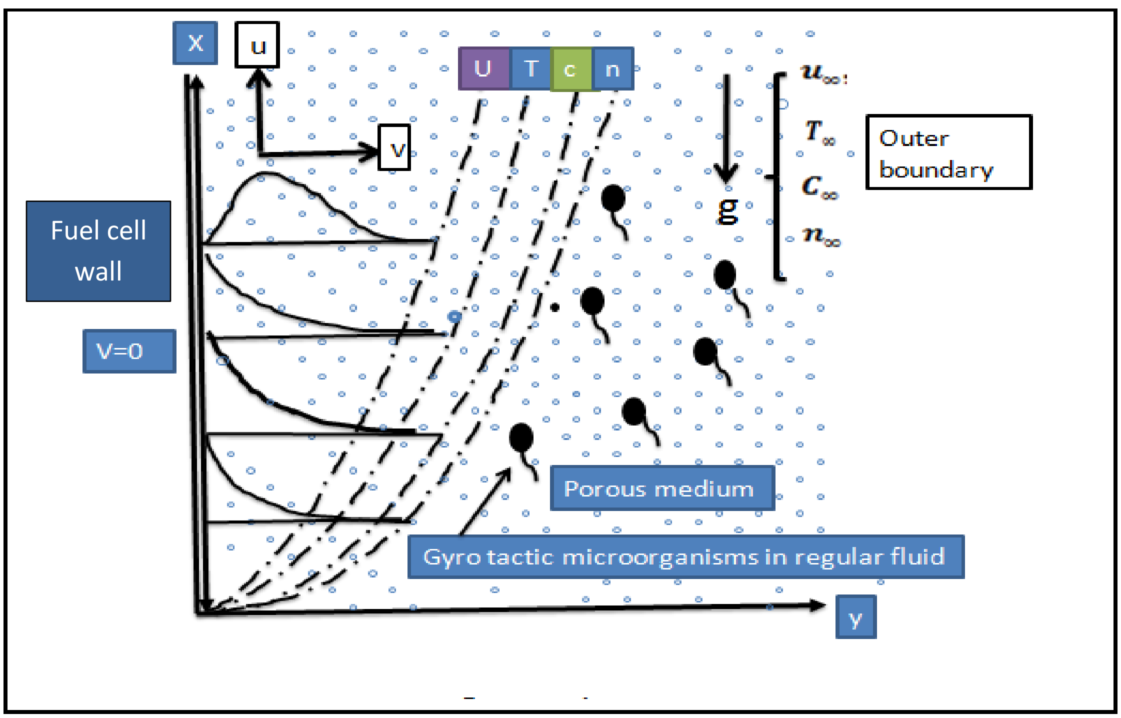


Fig. 1: Physical model for near-wall transport in bioconvection fuel cell

The governing partial differential equations for conservation of mass, momentum, energy, oxygen species and gyrotactic motile micro-organism species, using the Oberbeck-Boussinesq approximation can be written, by neglecting nanoscale effects, following Xu *et al.* [37] and Bég *et al.* [38]:

Mass conservation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum (Darcy Equation):

$$\frac{\partial u}{\partial y}=\frac{(1-C_\infty)\rho g k}{\mu}\frac{\partial T}{\partial y}-\frac{\rho g k}{\mu}\frac{\partial C}{\partial y}-\frac{g\gamma\Delta\rho k}{\mu}\frac{\partial n}{\partial y} \tag{2}$$

Thermal Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Oxygen Conservation Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Motile Micro-organism Conservation Equation:

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_C}{(C_w-C_\infty)}[\frac{\partial}{\partial y}(n\frac{\partial C}{\partial y})]=D_n\frac{\partial^2 n}{\partial y^2} \tag{5}$$

The following boundary conditions are prescribed at the wall and at the edge of the boundary layer:

$$v=0\ ,\ T=T_w\ ,\ C=C_\infty\ ,\ n=n_\infty \quad \text{at} \quad y=0 \tag{6}$$

$$u\to u_\infty,\ T\to T_\infty\ ,\ C\to C_\infty,\ n\to n_\infty \text{ at } y\to\infty \tag{7}$$

Introducing the following dimensionless quantities, as elaborated earlier in [37, 38] (all scaling transformation proofs via Lie algebra symmetry are provided in these works and are omitted here for brevity):

$$\eta = \frac{y}{x} Ra_x^{\frac{1}{3}} \left(1 + \frac{Pe_x^{\frac{1}{2}}}{Ra_x^{\frac{1}{3}}}\right), \ \psi = \alpha \ Ra_x^{\frac{1}{3}} \left(1 + \frac{Pe_x^{\frac{1}{2}}}{Ra_x^{\frac{1}{3}}}\right) f(\eta)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \ , \ \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \ , \ \chi(\eta) = \frac{n - n_\infty}{n_w - n_\infty}$$

$$Ra_x = \frac{(1 - C_\infty)\rho k g \beta (T_w - T_\infty) x}{\mu \alpha} \ , Pe_x = \frac{u_\infty x}{\alpha} \tag{8}$$

The continuity equation is satisfied by stream function such that

$$u = \frac{\partial \psi}{\partial y} \ , v = -\frac{\partial \psi}{\partial x} \tag{9}$$

According to Nakayama and Koyama [16],in the case of *Darcy free convective flow* over a vertical flat plate, we can consider:

$$\frac{\mu}{K} u_\infty (\text{Darcy resistance}) \sim (1 - C_\infty)\rho g \beta (T_w - T_\infty)(\text{Buoyancy force}) \tag{10}$$

In view of the above relation, it follows that:

$$u_\infty = \frac{(1 - C_\infty) k g \beta (T_w - T_\infty)}{\nu}, \text{ where } \quad \nu = \frac{\mu}{\rho} \tag{11}$$

We get the following *ordinary differential equations*:

$$f'' - \varepsilon^2 [\theta' - Nr\varphi' - Rb\chi'] = 0 \tag{12}$$

$$\theta'' + \frac{2+\varepsilon}{6} f\theta' = 0 \tag{13}$$

$$\varphi'' + Le\frac{2+\varepsilon}{6}f\varphi'=0 \tag{14}$$

$$\chi'' + Lb\frac{2+\varepsilon}{6}f\chi' - \mathrm{Pe}\,[\varphi'\chi' + \chi\varphi''] = 0 \tag{15}$$

According to [36], we have set $n_\infty = 0$ which satisfies the boundary conditions at infinity. The transformed boundary conditions now become:

$$\eta = 0\,, f = 0\,, \theta = 1, \varphi = 1, \chi = 1 \quad and \tag{16}$$

$$\eta \to \infty\,, f' \to \varepsilon^2\,, \theta \to 0, \varphi \to 0, \chi \to 0 \tag{17}$$

Here the following definitions apply:

$$\mathrm{Nr}=\frac{C_w-C_\infty)}{(T_w-T_\infty)(1-C_\infty)\beta}, \qquad \mathrm{Rb} = \frac{\gamma\Delta\rho(n_w-n_\infty)}{(1-C_\infty)\rho\beta(T_w-T_\infty)}, \quad \mathrm{Le} = \frac{\alpha}{D},$$

$$\mathrm{Lb}=\frac{\alpha}{D_n} \qquad \mathrm{Pe} = \frac{bW_c}{D_n}, \varepsilon = \frac{1}{1+\frac{Ra_x^{\frac{1}{3}}}{Pe_x^{\frac{1}{2}}}} \tag{18}$$

Le, Nr, Rb, Lb, Pe,Ra_x,Pe_x,$\boldsymbol{\varepsilon}$ denote Lewis number, buoyancy ratio parameter, bioconvection Rayleigh number, bioconvection Lewis number, bioconvection Peclét number, local Darcy-Rayleigh number, local Peclét number and mixed convection parameter. It can be noted that $\varepsilon = 0$ ($Pe_x = 0$) corresponds to *pure free convection* whereas $\varepsilon = 1$ (Ra_x =0) corresponds to *pure forced convection*. Therefore, the values of the entire regime from$\varepsilon = 0$ to $\varepsilon = 1$ correspond to *mixed convection*. Here small values of $\frac{Ra_x^{\frac{1}{3}}}{Pe_x^{\frac{1}{2}}}$ correspond toεbeing close to 1 which indicates the forced convection regime. On the other hand, large values of $\frac{Ra_x^{\frac{1}{3}}}{Pe_x^{\frac{1}{2}}}$correspond to εbeing close to 0 which is associated with the *free convection regime*. In this study, the local Nusselt number Nu_x, the Sherwood number Sh_x and the wall gradient of the local density number of the motile microorganisms Nn_x are the quantities of relevance to fuel cell design, and are defined as:

$$Nu_x = \frac{xq_w}{k\,(T_w-T_\infty)}, Sh_x = \frac{xq_m}{D\,(C_w-C_\infty)}, Nn_x = \frac{xq_n}{D_n\,(n_w-n_\infty)} \tag{19}$$

Whereq_w,q_m, and q_n are the *wall heat, the wall mass and wall motile micro-organisms fluxes*, respectively and are defined mathematically as follows:

$$q_w = -k(\frac{\partial T}{\partial \bar{y}})_{y=0} \ ,q_m = -D_B(\frac{\partial C}{\partial \bar{y}})_{y=0} \ ,q_n = -D_n(\frac{\partial n}{\partial \bar{y}})_{y=0} \tag{20}$$

Using the variables defined earlier in Eqn. (8) and (18):

$$\varepsilon Pe_x^{\frac{-1}{2}} Nu_x = -\theta'(0) \ ,\varepsilon Pe_x^{\frac{-1}{2}} Sh_x = -\varphi'(0) \ ,\varepsilon Pe_x^{\frac{-1}{2}} Nn_x = -\chi'(0) \tag{21}$$

Here $\varepsilon Pe_x^{\frac{-1}{2}} Nu_x$, $\varepsilon Pe_x^{\frac{-1}{2}} Sh_x$, $\varepsilon Pe_x^{\frac{-1}{2}} Nn_x$designate Nusselt number, Sherwood number andgradient of the local density number of the motile micro-organisms, respectively.

3. NUMERICAL METHOD AND VALIDATION

A simple, accurate and efficient technique is applied for numerical solution of the problem. The numerical procedures of this technique are the following: (i) It is based on the common finite difference method with central differencing (ii) on a tridiagonal matrix manipulation and (iii) on an iterative procedure. This numerical method is described in detail in Nakayama [46]. According to the numerical procedure we formulate the transformed similarity momentum, energy, concentration and motile microorganism density equations as follows. The momentum Eqn. (12) can be written as:

$$f'' - \varepsilon^2[\theta' - Nr\varphi' - Rb\chi'] = 0$$

The above equation is a second order linear differential equation. Setting $y(x) = f(n)$we obtain the equation in the form:

$$P(x)y''(x) + Q(x)y'(x) + R(x)y(x) = S(x) \tag{22}$$

Here:

$$P(x) = 1, \ \ Q(x) = 0, \ \ R(x) = 0, \ \ S(x) = \varepsilon^2[\theta' - Nr\varphi' - Rb\chi'] \tag{23}$$

According to the above procedure all equations of the system can be reduced to the form of Eqn. (20).Then for the energy equation:

$$P(x) = 1,\ \ Q(x) = \frac{2+\varepsilon}{6} f,\ \ R(x) = 0,\ \ \ S(x) = 0 \tag{24}$$

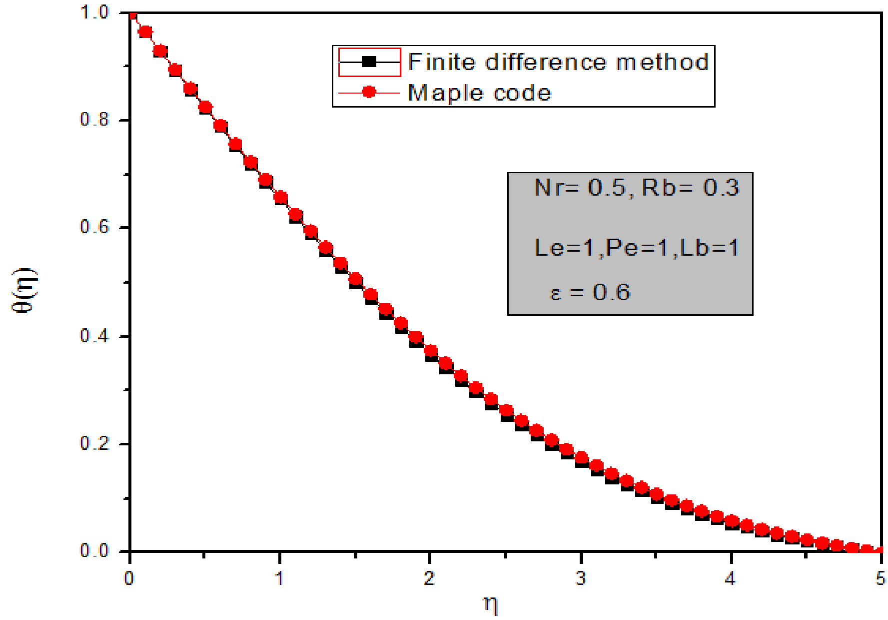
For the oxygen conservation equation:

$$P(x) = 1,\ \ Q(x) = Le\,\frac{2+\varepsilon}{6} f,\ \ R(x) = 0,\ \ \ S(x) = 0 \tag{25}$$

For the motile microorganism density equation:

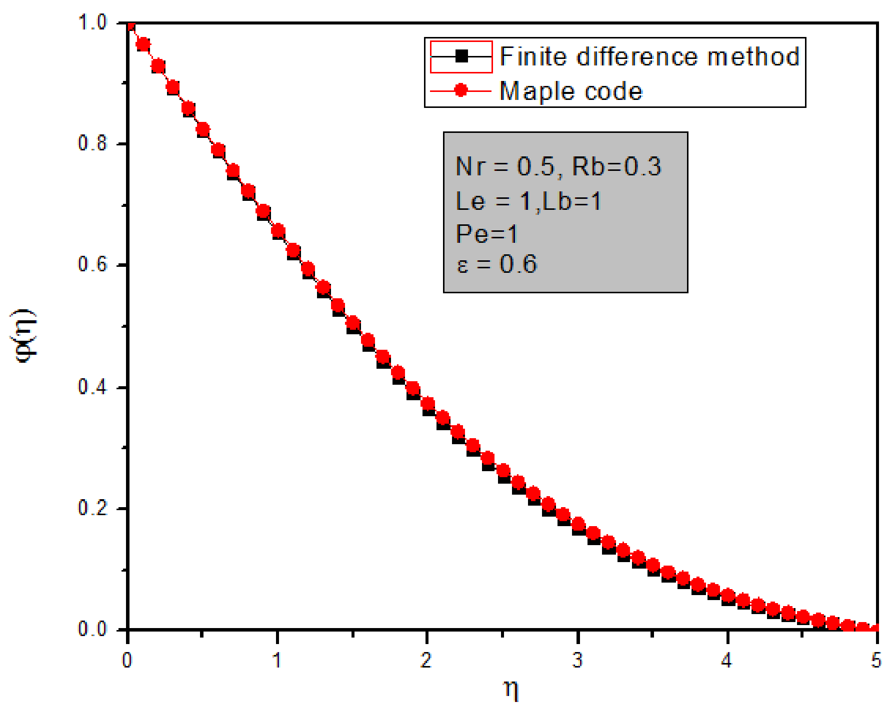
$$P(x) = 1,\ \ Q(x) = \frac{2+\varepsilon}{6} f - Pe.\varphi'\ \ , R(x) = Pe.Le.\frac{2+\varepsilon}{6} f\varphi',\ \ \ S(x) = 0 \tag{26}$$

All the transformed equations can be solved by a common finite difference method, based on central differencing and tridiagonal matrix manipulation.To start the solution procedure, we assume initial guesses $\eta = 0$ and $\eta = \eta_{\infty}(\eta \to \infty)$ which satisfy the boundary conditions (16) and (17). In order to apply to our numerical computation a proper step size $h = \Delta\eta = 0.01$ and appropriate η_{∞} value as $(y \to \infty)$ must be determined. This process is continuing until convergence of the solution is attained. To verify the accuracy of the present solution we solve the equations using **Maple 14.0**. By using the**dsolve** command, this type of BVP or IVP problem is easily identified and appropriate algorithm are applied. The validity and accuracy of the Maplealgorithm have been extensively confirmed in many recent works [47, 48]. The graphical representation of the comparison is shown in **Figs. (2-4)**for temperature, oxygen concentration and microorganism density number function, where excellent correlation is found.



1.0
0.8
0.6
0.4
0.2
0.0
θ(η)
0
1
2
3
4
5
η
Finite difference method
Maple code
Nr= 0.5, Rb= 0.3
Le=1,Pe=1,Lb=1
ε = 0.6

Fig 2



1.0
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ε = 0.6

Fig 3

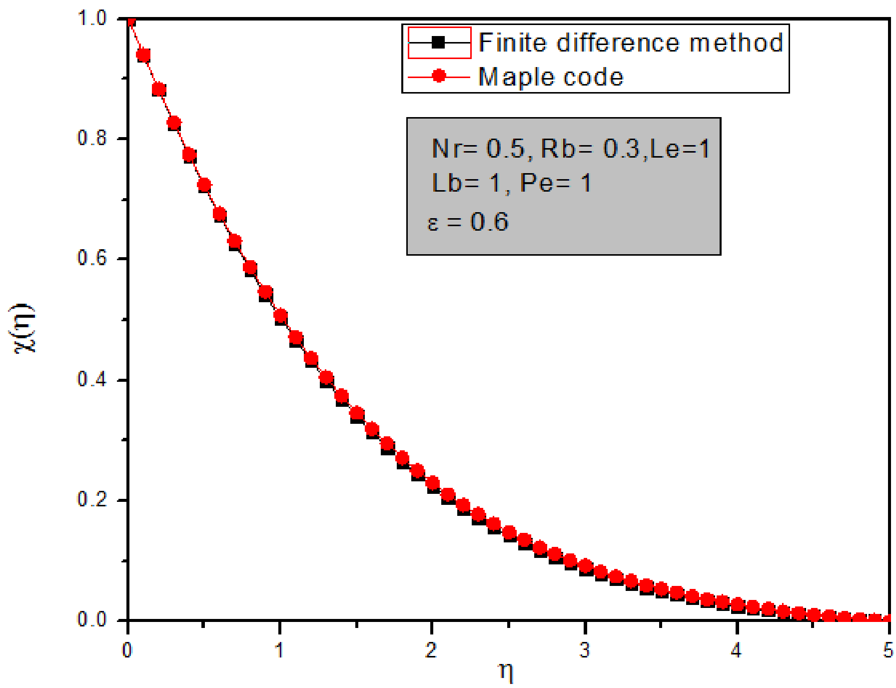

Fig 4

4. NUMERICAL RESULTS AND DISCUSSION

In **Fig 5-7**we have shown effect of different values of mixed convection parameter on temperature, concentration and motile microorganism distribution for $Nr = 0.4$,$Rb = 0.1$, $Pe = 0.2$,$Le = 5$ and $Lb = 1$.As different values of mixed convection parameter indicate the different types of thermal convection regime, so we can easily observe the profiles for the natural and forced convection in **Figs. 5-7.**Temperature, oxygen concentration and micro-organism density number are all depleted with increasing mixed convection parameter. The associated boundary layer thicknesses also all decrease according with greater mixed convection parameter ε.It is also observed that boundary layer thickness of concentration profile is thinner than the temperature and motile microorganism profiles. The relative rates of diffusion (heat, oxygen, micro-organism) are therefore adjusted with mixed convection effects indicating that fuel cell performance is tunable in actual devices.

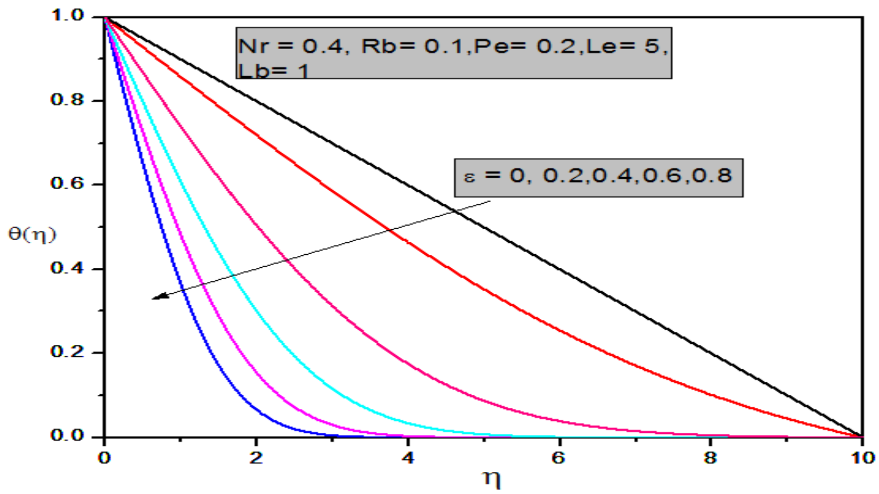

Fig 5:Effects of mixed convection parameter on temperature profile

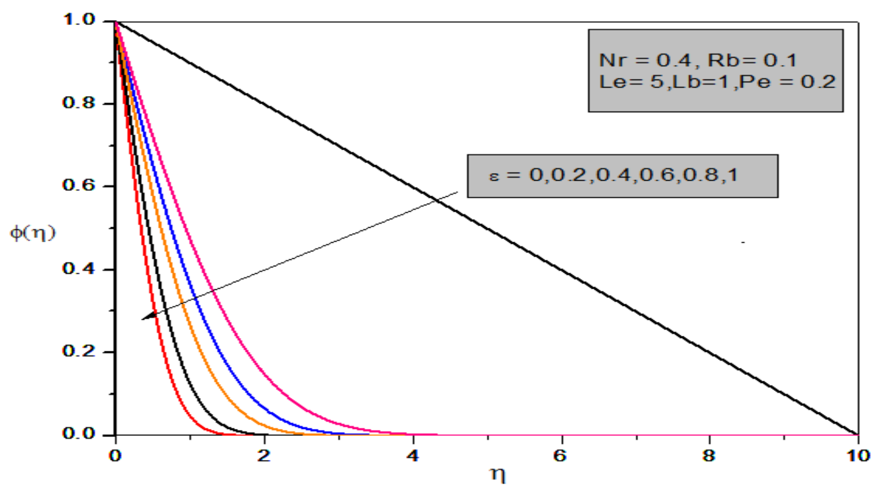

Fig 6:Effect of mixed convection parameter on concentration profile

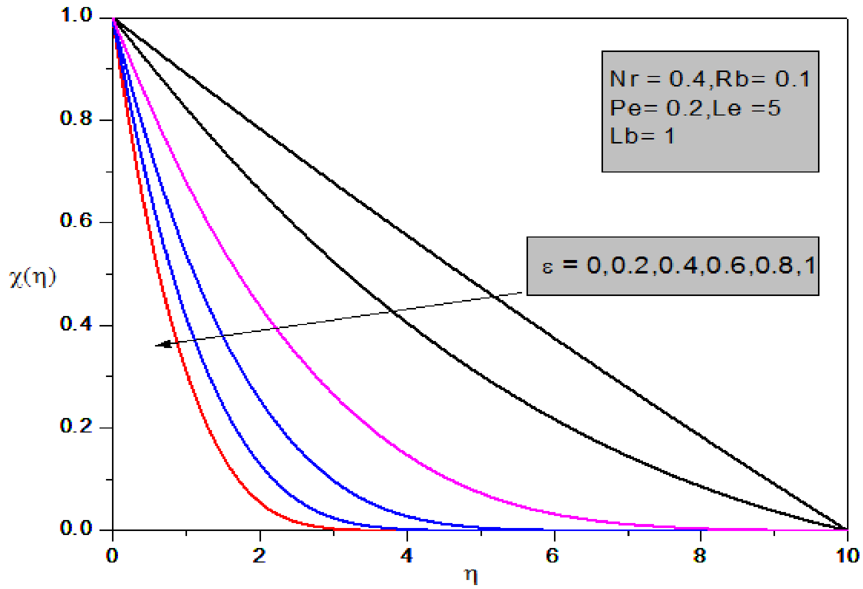

Fig 7:Effect of Mixed convection parameter on motile microorganism profile

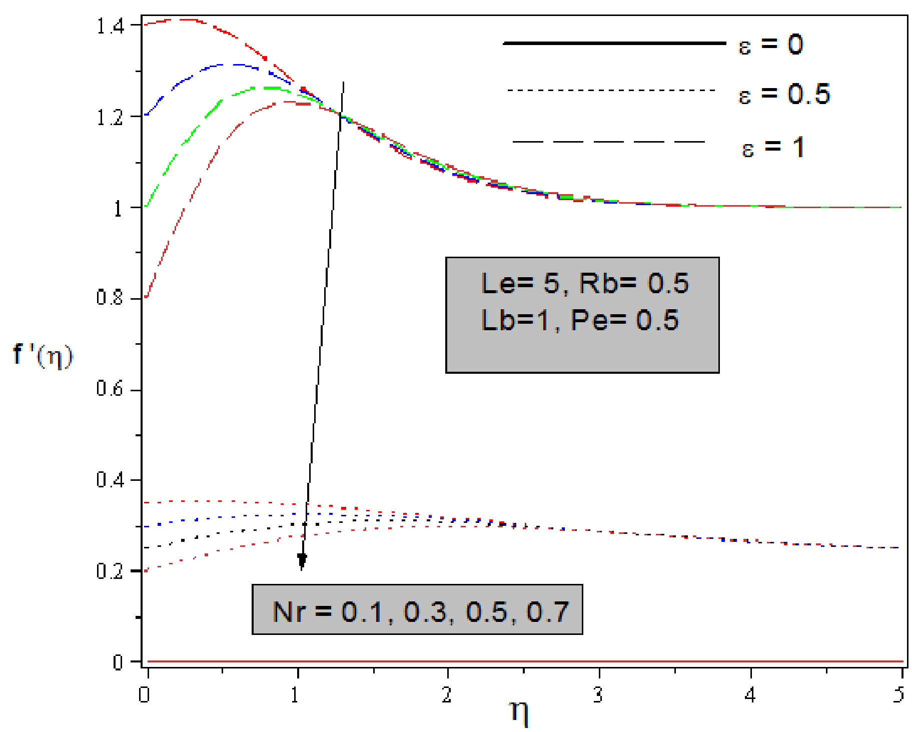

Fig 8: Effect of buoyancy ratio parameter for free forced convection on velocity profile

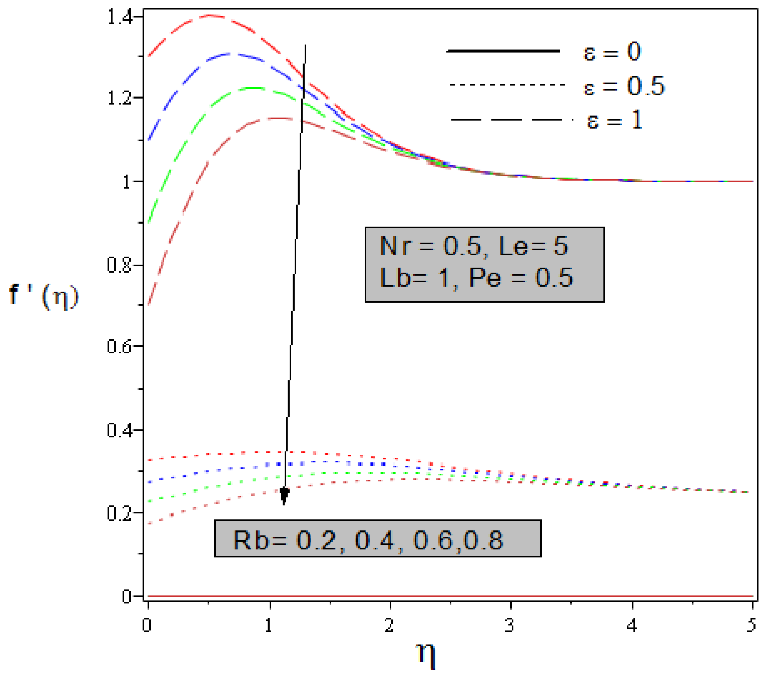

Fig 9: Effect of bioconvection Rayleigh number for free forced convection on velocity profile

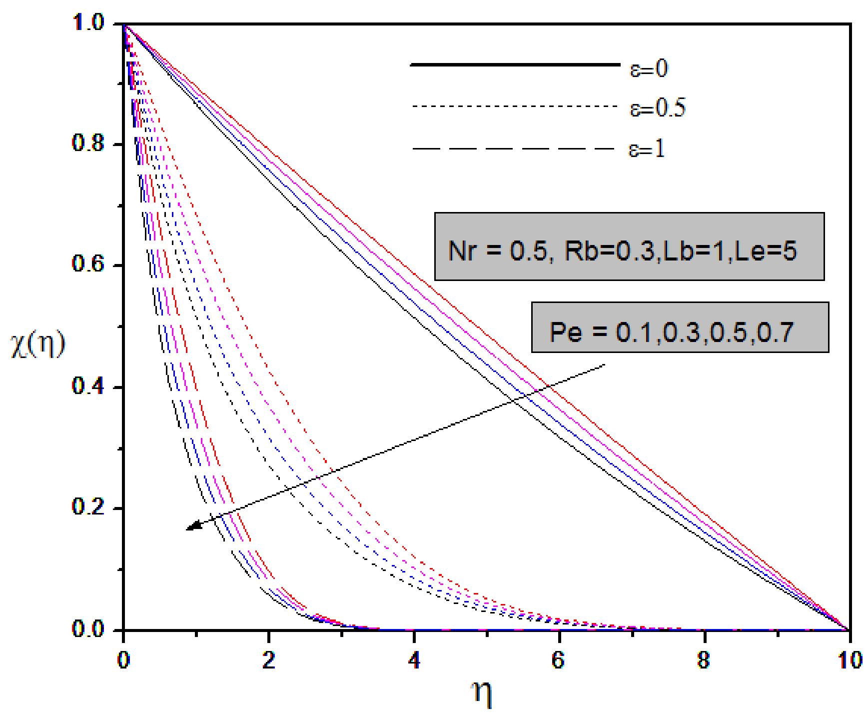

Fig 10:Effect of Bioconvection Peclet number on Microorganism profile

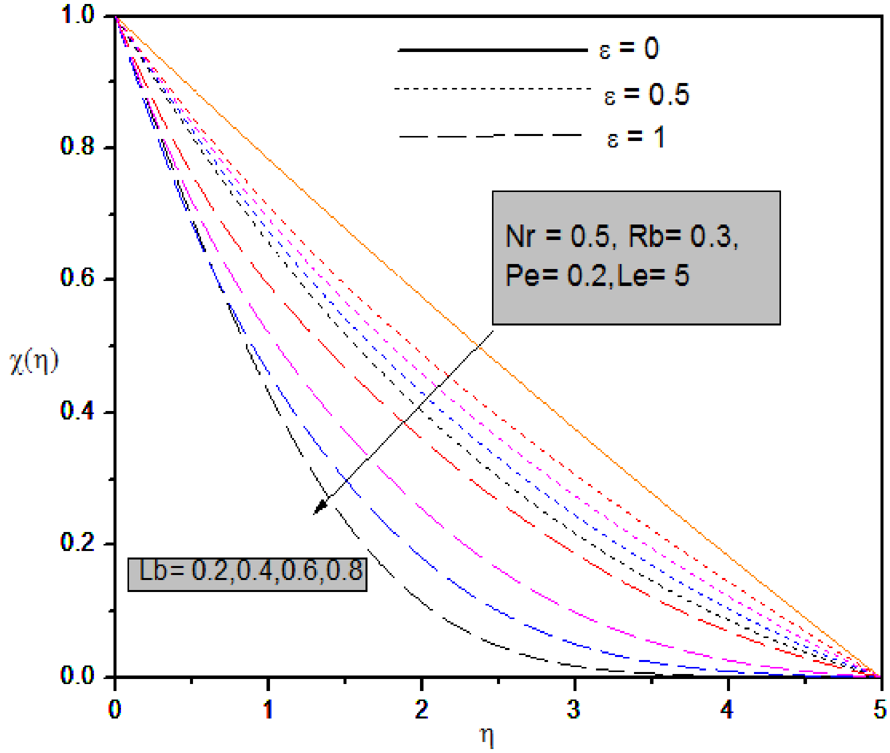

Fig 11: Effect of bioconvection Lewis number on Microorganism profile

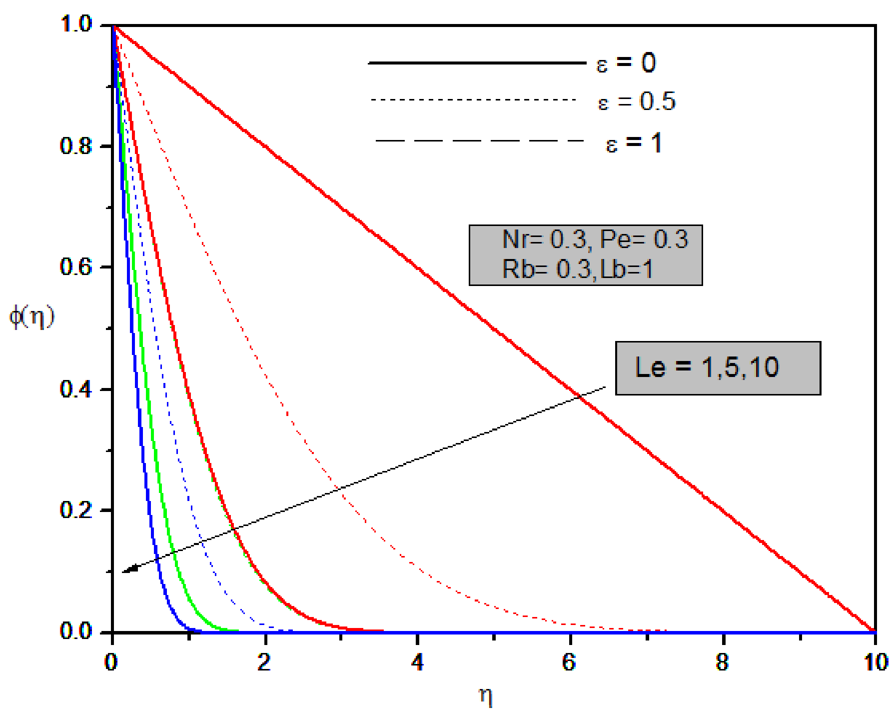

Fig 12: Effect of Lewis number on dimensionless concentration

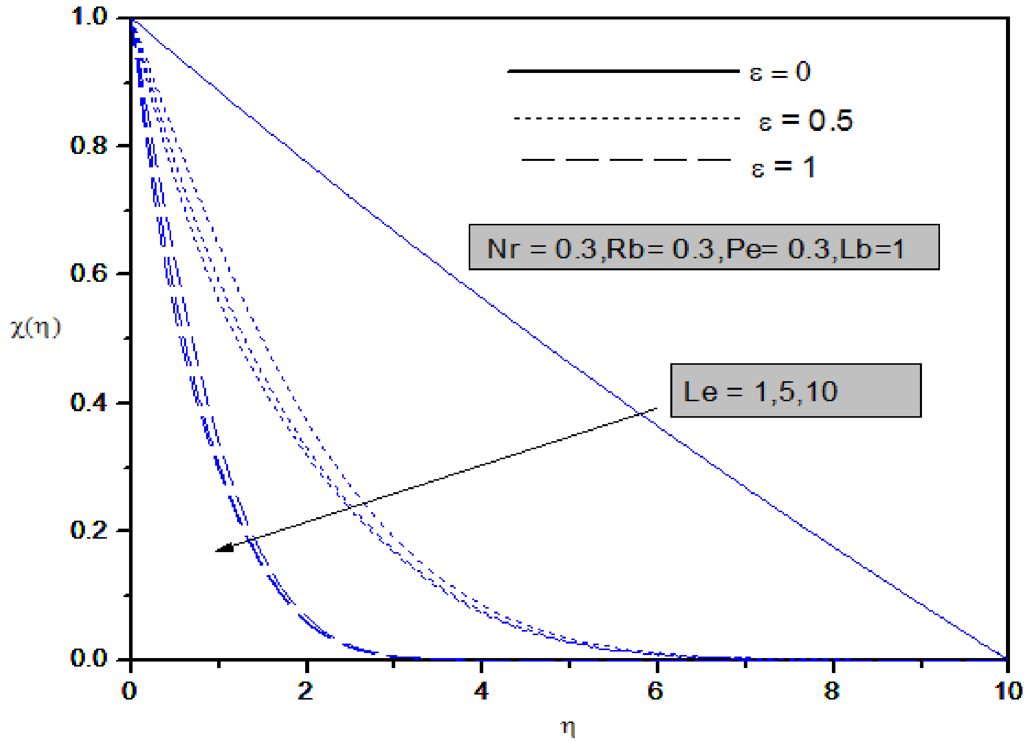

Fig 13: Effect of Lewis number on dimensionless microorganism profile

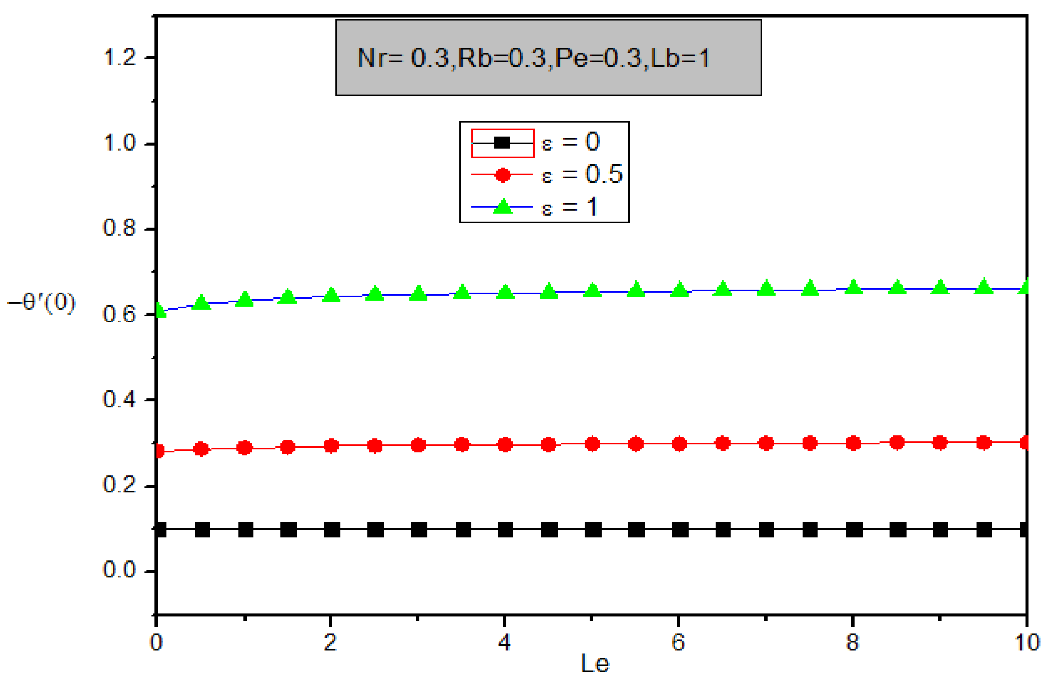

Fig 14 : Effect of Lewis number with free forced convection on nusselt number

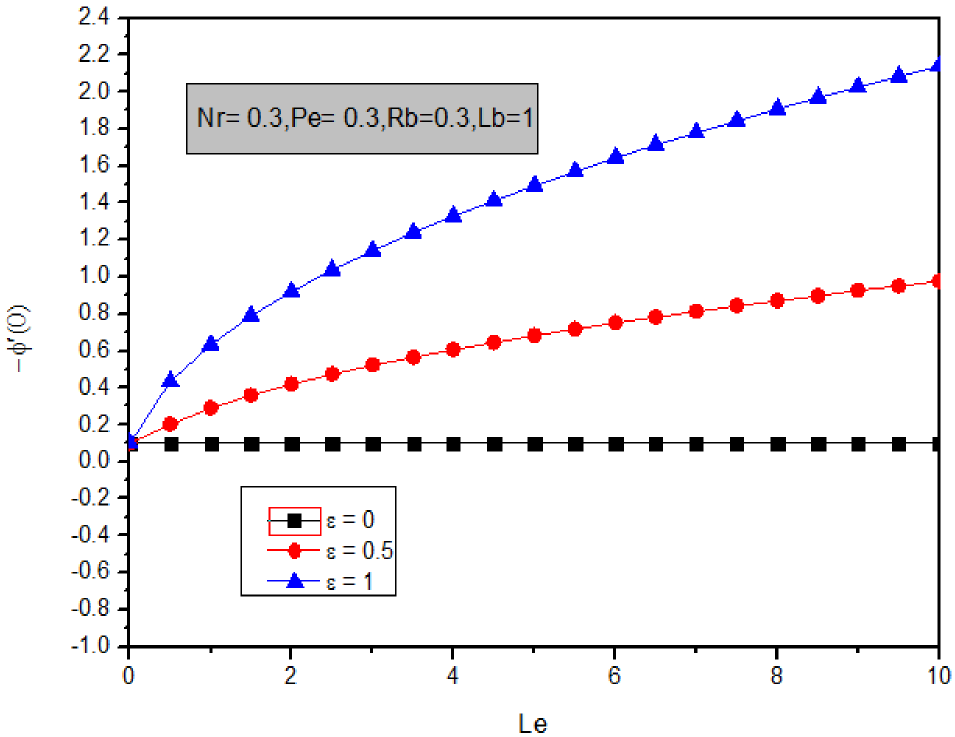

Fig 15 : Effect of Lewis number with free forced convection on Sherwood number

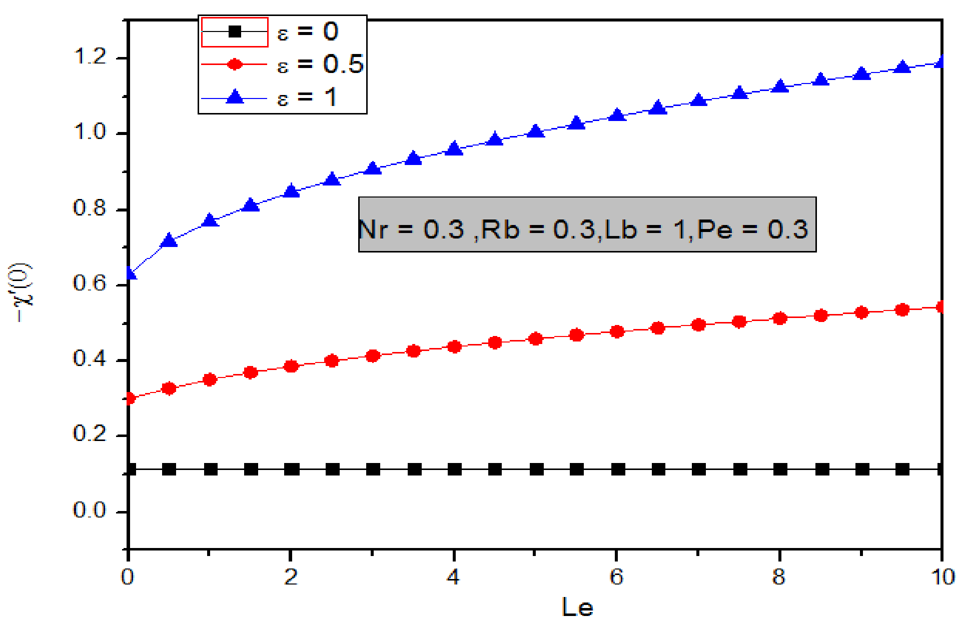

Fig 16 : Effect of Lewis number with free forced convection on density of motile microorganism

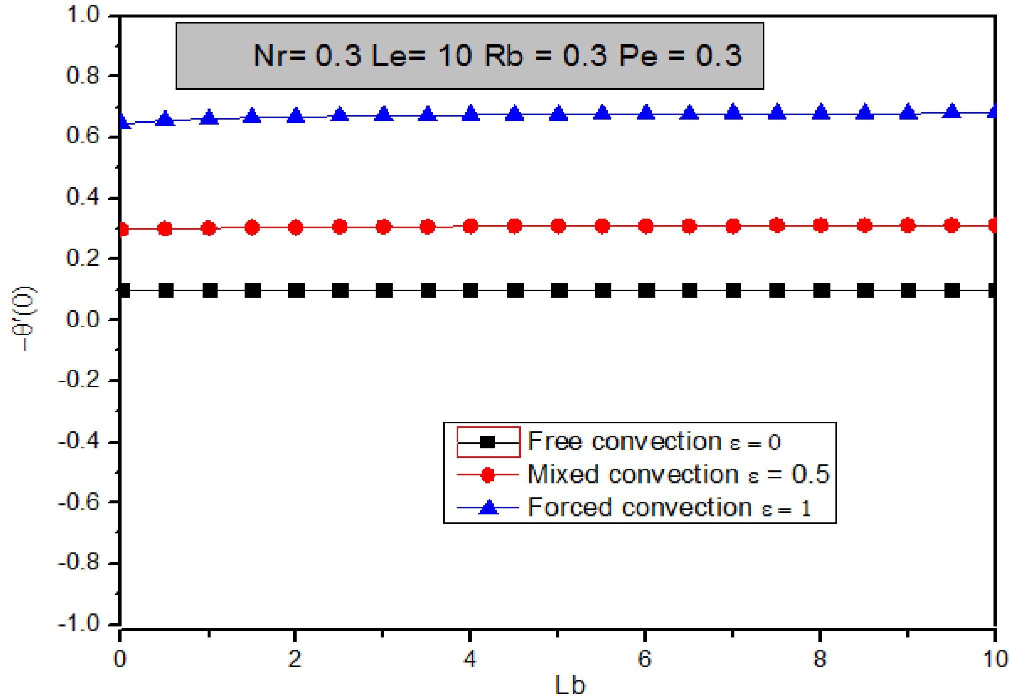

Fig 17 : Effect of bioconvection Lewis number with free forced convection on nusselt number

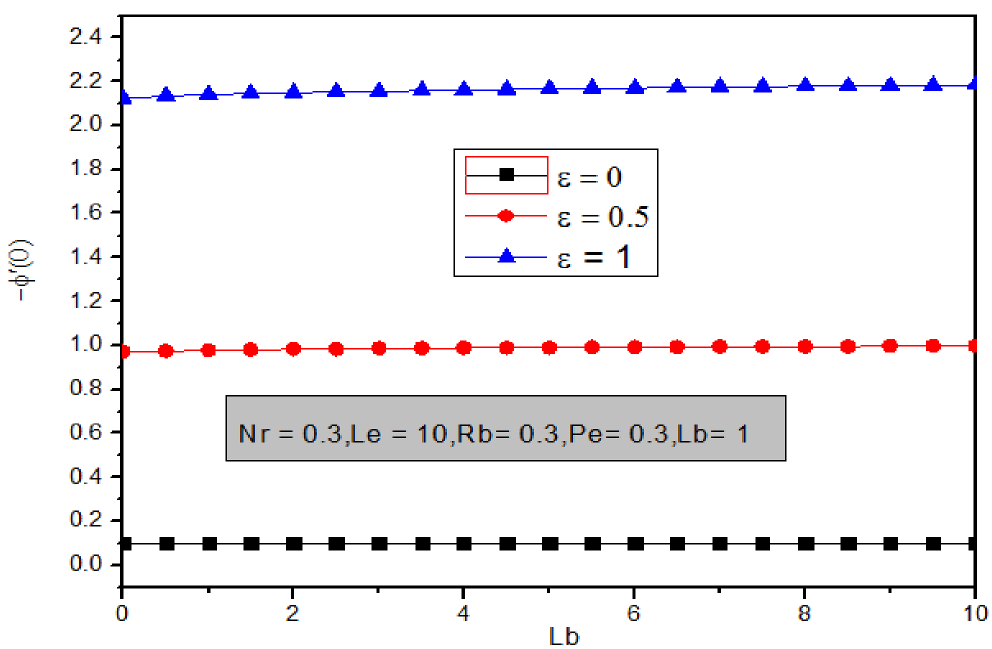

Fig18 :Effect of bioconvection Lewis number with free forced convection on Sherwood number

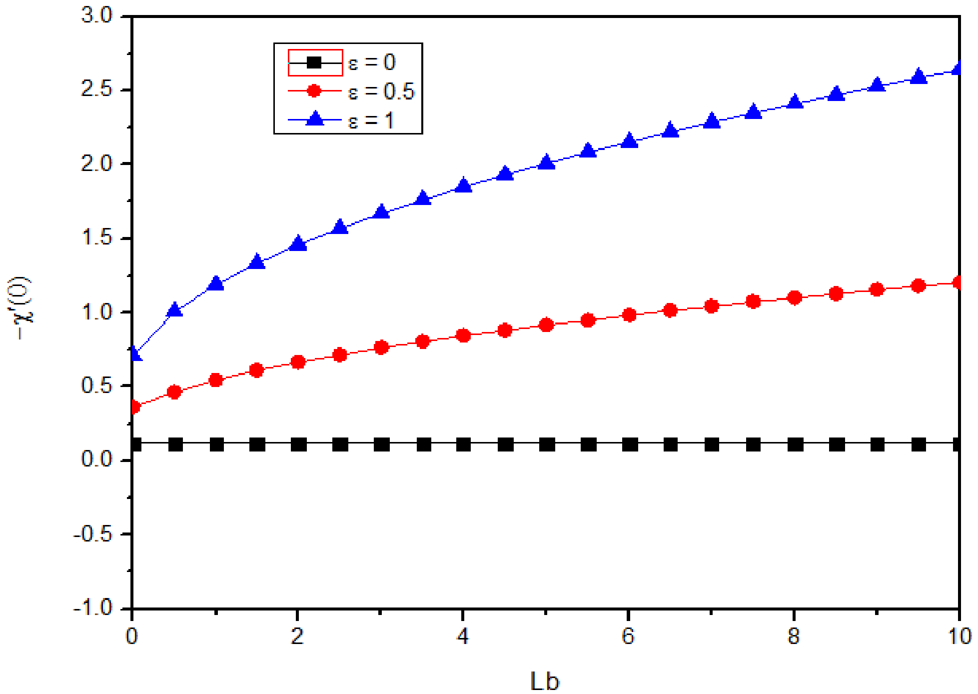

Fig 19 : Effect of bioconvection Lewis number with free forced convection on density of motile microorganism

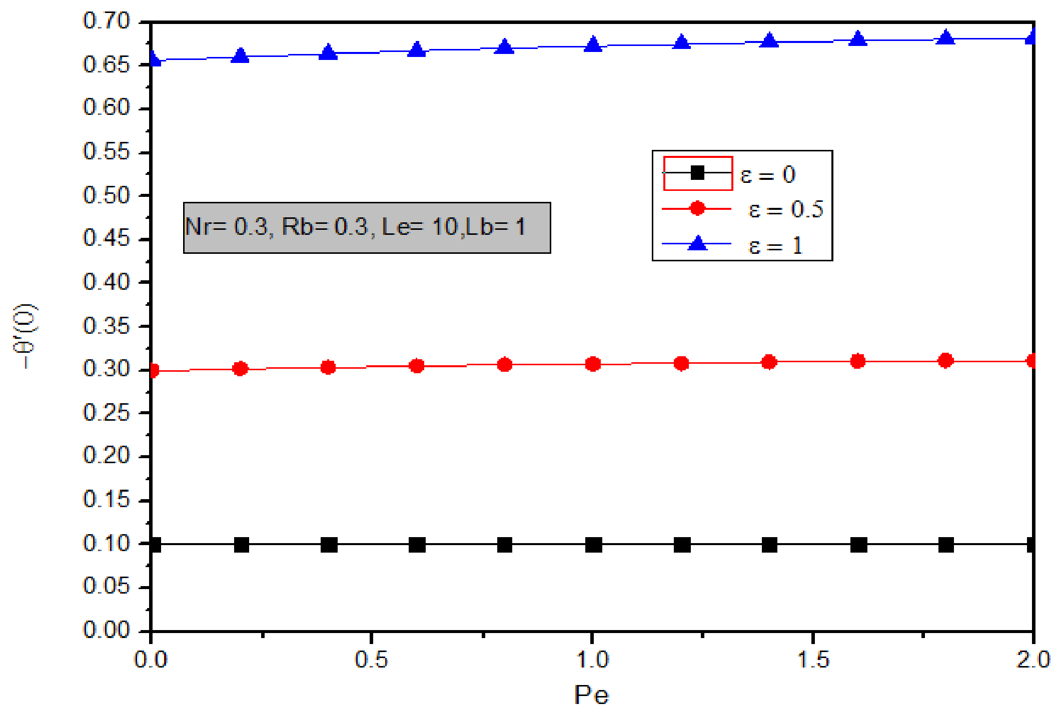

Fig 20 : Effect of bioconvection peclet number with free forced convection on nusselt number

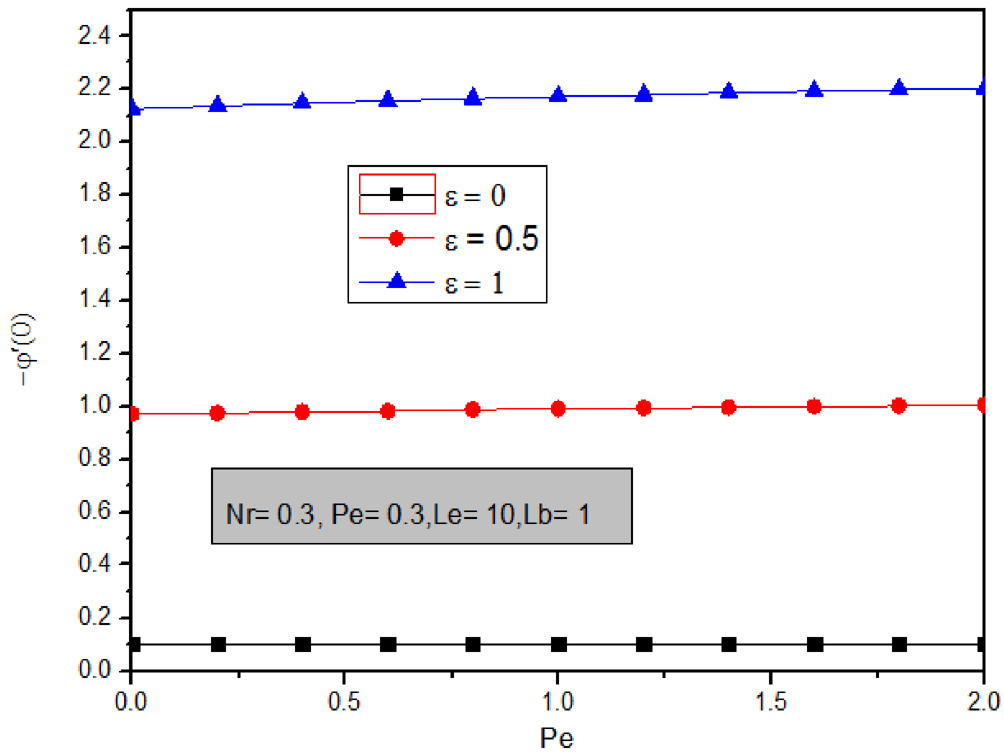

Fig 21 : Effect of bioconvection peclet number with free forced convection on Sherwood number

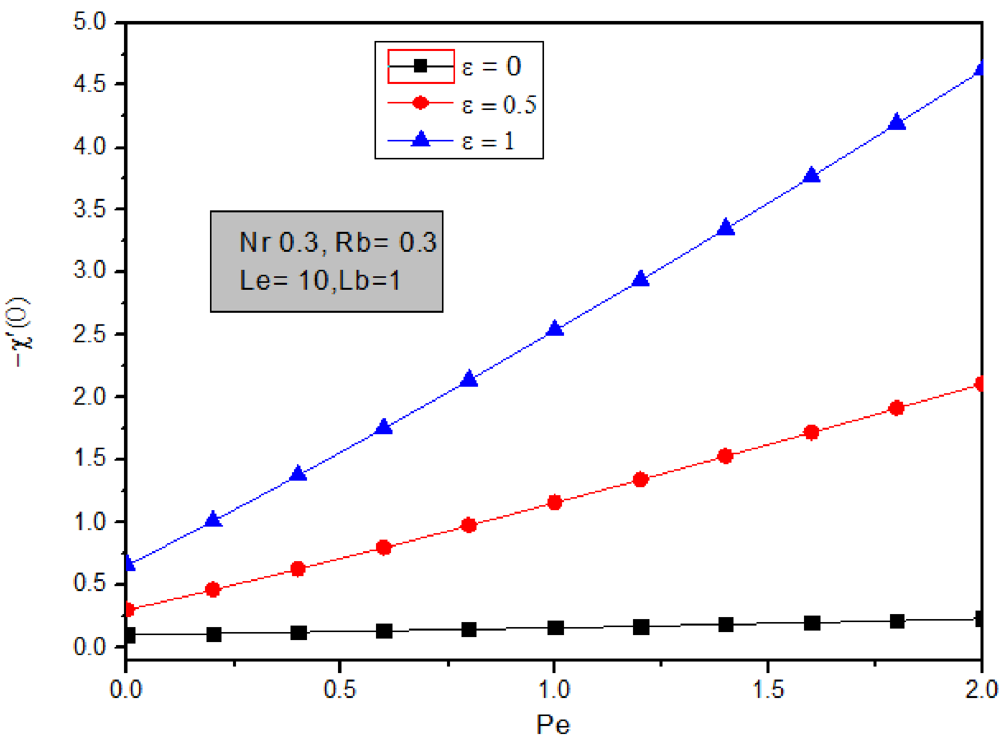

Fig 22 : Effect of bioconvection peclet number with free forced convection on density of motile microorganism

The effects of buoyancy ratio parameter and bioconvection Raleigh number on velocity profiles are shown in **Figs. 8-9**. Velocity profile attains its maximum value for minimum values of buoyancy ratio parameter and bioconvection Raleigh number. As velocity profile is dependent on the mixed convection parameter, it is observed that velocity profile has great impact in the forced convection regime for buoyancy ratio parameter and bioconvection Rayleigh number. In **Figs. 10-13**, the concentration and microorganism boundary layer thickness is found to decrease with increasing Lewis number, bioconvection Péclet number and biconvection Lewis number for the pure free forced and mixed convection. Bioconvection lewis number and bioconvection Péclet number have a tendency to decrease micro-organism profiles as shown in **Figs. 10-11**. As Lewis number reduces rhe oxygen species mass diffusivity, this in turn decreases the penetration depth of the concentration boundary layer as visualized in **Fig. 12.**

In **Figs. 14-16**, we can see the response in Nusselt number, Sherwood number and wall gradient of the density of motile micro-organism. Nusselt number and Sherwood number (with $Nr = 0.3$, $Le=10$, $Rb= 0.3$, $Pe= 0.3$) are both enhanced with increasing value of Lewis number in the mixed and forced convection regime. Lewis number has negligible effect on Nusselt number indicating that the heat transfer rate at the fuel cell wall (boundary) is not modified with the ratio of thermal diffusivity to oxygen diffusivity (Le = $\frac{\alpha}{D}$). It is more sensitive to the micro-organism species diffusivity and this justifies the implementation of gyrotactic species in the fuel cell.

The behavior of Nusselt number, Sherwood number and gradient of motile microorganism density function, with increasing value of bioconvection Lewis number are depicted in **Figs. 17-19** and with increasing values of bioconvection Péclet number are shown in **Figs. 20-22**. In both cases, the density of motile micro-organism increases in both the mixed and forced convection regime with bioconvection Lewis and Péclet number. This indicates that an enhancement in near-wall transport characteristics is inducible with careful manipulation of bioconvection parameters.

5. CONCLUSIONS

A theoretical and computational study has been presented for free-forced convective boundary layer flow from a vertical plate embedded in a Darcian permeable medium, as a model of near-wall PEM fuel cell transport phenomena. The transformed boundary layer equations for mass, momentum, oxygen species and micro-organism density function have been non-dimensionalised with appropriate similarity transformations. The emerging nonlinear ordinary differential boundary value problem has been solved with a finite difference computational method. Verification of solutions has been included using Maple symbolic software quadrature. A graphical analysis of the effects of the mixed convection and bioconvection parameters on velocity, temperature, oxygen concentration and microorganism density profiles has been conducted. The main findings of the study are summarized as follows:

- ❖ The effects of buoyancy parameter, bioconvection Raleigh number, Lewis number, bioconvection Lewis number, bioconvection Péclet number in the forced convection regime are more prominent than in the pure free convection regime. Fuel cell performance can therefore be optimized by careful selection of these parameters.
- ❖ Lewis number, bioconvection Lewis number, bioconvection Péclet number have negligible impact on the Nusselt number.
- ❖ Sherwood number increases with Lewis number indicating that wall oxygen mass transfer rate is enhanced.
- ❖ Density of motile micro-organisms increases with bioconvection Lewis number, Lewis number and bioconvection Péclet number which implies enhanced performance in the near wall zone of the fuel cell.

The present study has been confined to *Newtonian* fluids. Future investigations will consider microstructural working fluids using the Eringen micropolar model [49] which are also of relevance to PEM fuel cell systems and efforts in this regard are currently underway.

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