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<http://dx.doi.org/10.1108/EC-03-2019-0101>

Title	Numerical investigation into the stability of earth dam slopes considering the effects of cavities
Authors	Al-ateya, H and Ahangar Asr, A
Type	Article
URL	This version is available at: http://usir.salford.ac.uk/id/eprint/53018/
Published Date	2019

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ISSN 0264-4401
Volume 00 Number 00 2018



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Journal:	<i>Engineering Computations</i>
Manuscript ID	EC-03-2019-0101.R2
Manuscript Type:	Research Article
Keywords:	Slope stability, earth dam, cavities, rapid drawdown, PLAXIS 2D

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Numerical investigation into the stability of earth dam slopes considering the effects of cavities

Abstract

This research is an attempt to estimate the influence of the presence of cavities on the stability of slopes in earth dams under rapid drawdown conditions. The aim of the investigation is to study the influence of different factors, such as the diameter and location of cavities. A series of finite element simulations was conducted using PLAXIS 2D to develop models and analyse slope stability in earth dams while considering the effect of cavities in the subsoil. The combined effects of cavities and the strength parameters of slopes on the stability were also investigated and parametrically analysed. The results indicated that presence of cavities and an increase in the diameter of cavities decreased the stability of the upstream face dramatically for all examined locations in a horizontal direction; however, this effect was less on the downstream side. The results also showed that variations in the location of cavities in the horizontal direction have a greater effect on the stability compared to the vertical direction. The results revealed that increasing shear strength parameters of embankment do not reduce the influence of cavities on stability when those cavities are in critical locations.

Keywords

Slope stability; earth dam; cavities; rapid drawdown; PLAXIS 2D

List of notation

γ	is the unit weight of soil
ν'	is the Poisson's ratio
C'	is the cohesion of soil
Ψ	is the angle of dilatancy
ϕ'	is the internal angle of friction
k_x	is the coefficients of permeability in X direction
k_y	is the coefficients of permeability in Y direction
E'	is the Young's modulus.
D	is the diameter of cavity
SRM	is the Strength Reduction method
$ u $	is the total displacement

1. Introduction

The failure of earth structures such as earth dams or natural slopes is one of the most important problems in geotechnical engineering, because such failures could cause significant damage to infrastructure and heavy loss of life (Maula and Zhang, 2011), e.g. the number of deaths due to the failure of Vajont dam in Italy in 1963 about 2600 while the 1976 failure of Teton dam in America resulted economic loss of around 1 billion dollars. (Shahba and Soltani, 2016). (Sherard et al., 1963) conducted a vast survey on the reasons for dam failures and reported they may be caused by overtopping, earthquakes, embankment and foundation piping, embankment sliding, cracks and differential settlement, sliding during construction, wave action, damage as a result of materials dissolved in water, damage caused by animals and damage caused by surface drying. The most common causes of dam failure were reported as leakage and piping 35%, overtopping 25%, spillway erosion 14%, excessive deformation 11%, sliding 10%, gate failure 2%, faulty construction 2% and earthquake instability 2%, (Lukman et al., 2011).

Earth dams sensitive engineering structures that require successful design and construction and need continual analysis of their stability. The stability of a slope is a significant issue for dam designers before, during and after construction. A slight change in the safety factor can lead to a big change in the cost of construction, create the need for expensive repair work or imperil public safety (Khabbaz et al., 2012). The critical periods in the dam's life that must be assessed from the viewpoint of shear failure are: the crucial phase in an upstream side at the end of construction and during rapid drawdown conditions, whilst the stability of downstream side must be ensured at the end of construction and during the steady state phase (Jalil, 2011).

Rapid drawdown is a classic scenario in slope stability that occurs when the water level neighbouring wholly or partly submerged slopes or embankments decreases quickly after a long period of rising, either at the normal operating level for a dam or, in the case of levees, during a prolonged flood. The rapid drawdown of the water level decreases the upstream stability due to the removal of the supporting external hydrostatic pressure and, combined with variations in internal pore water pressure, leads to a decrease in the effective stresses and an increase in shear stresses within the embankment (VandenBerge et al., 2013).

Many studies have been conducted to assess the stability and calculate the safety factor of earth dams and natural slopes. (Sherard et al., 1963) characterized many cases of upstream slope failures and ascribed them to rapid drawdown conditions. In most of the analysed failure cases the drawdown was less than the maximum water depth; in fact, it was roughly half (from the maximum reservoir height to roughly mid-dam level).

According to the Deterioration of Dams and Reservoirs report (ICOLD, 1984) thirty-three cases of upstream failures occurred due to rapid drawdown of reservoir water to that date. (Lowe and Karafiath, 1960) and Baker et al. (1993) carried out undrained analyses to compute the safety factor of slopes during rapid drawdown conditions. (Yan et al., 2010 and Wang et al., 2012) studied the stability of slopes and dams under the influence of drawdown conditions using laboratory tests, while (Viratjandr and Michalowski, 2006) adopted numerical methods to evaluate slope stability; (Gao et al., 2014) used limit analysis to simulate the stability of submerged slopes subjected to water drawdown. (Berilgen, 2007) presented a study of slope stability under drawdown conditions using PLAXIS 2D focusing on soil permeability, drawdown rate and ratio, and taking into consideration the nonlinear constitutive model and loading conditions. (Shivamant et al., 2015) used SLOPE/W and PLAXIS 2D to investigate the dams' stability and to calculate the safety factor at different stages

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3 of construction and under different operating conditions. Dam stability was assessed after the end of the
4 construction stages and rapid drawdown in their study. (Athania et al., 2015) conducted an investigation to
5 simulate the slope stability of earth dams in steady state and transient conditions utilizing PLAXIS 3D. Different
6 values of Young's modulus (E) and the angle of internal friction (ϕ) were considered in the investigation.
7
8 Cavities formed under earth structures are problematic in the field of geotechnical engineering and may lead to
9 structural damage and loss of life and property. This issue requires exceptional attention in case of the presence
10 of cavities under hydraulic structures, because these cavities can become underground channels for water that
11 could potentially expand and then become subject to collapse when they reach critical size (Culshaw and
12 Waltham, 1987).

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16 There are no systematic studies related to the issue of the influence of cavities on the stability of earth dams.
17 Many researchers have performed investigations to study the influences of cavities or tunnels on the bearing
18 capacity of piles, and the performance of footings resting on soil with cavities/voids. Cavity shapes are irregular
19 in the nature however in this research the shapes are idealised for simplification and modelling purposes.
20 Previous research works such as Lavasan et al., (2016), (Jayamohan et al., 2016) and Khattab and Khalil (2009)
21 also had similar approaches to their developed models. Lavasan et al., (2016) conducted a parametric study to
22 examine the bearing capacity of a strip foundation built on soil containing twin circular voids using PLAXIS 2D
23 software. This study involves some parameters related to the location of voids such as depth, position and
24 eccentricity in addition to examining the effect of the size of footing/voids. It is shown that there is a critical
25 horizontal distance between cavities and a critical depth at which the effects of cavities weaken the ultimate
26 capacity of the foundation. (Jayamohan et al., 2016) carried out a numerical investigation using PLAXIS 2D to
27 improve the bearing capacity of a strip footing situated over a weak, clayey soil containing circular voids by
28 adding a reinforced foundation bed. A series of laboratory-scale load tests were performed to validate the finite
29 element analyses results. Based on the experimental and numerical results, it is concluded that the influence of
30 the void is significant when it is situated within a critical depth and critical eccentricity. It is also showed that
31 the behaviour of load-settlement improved considerably with the addition of foundation bed and reinforced
32 foundation bed. It was also noted that there is a great concentration of stress near the void.

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36 Khattab and Khalil (2009) used PLAXIS 2D and PLAXIS 3D software to study the effect of the existence of a
37 cavity on the performance of the foundation. The influence of parameters such as horizontal location, depth,
38 shape, size and the sectional area of a single cavity beneath the foundation base on the displacement and the
39 distribution of stress was evaluated in this search. The results obtained from 2D and 3D analyses indicated an
40 increase in the values of the effective vertical stress in the locations under the foundation when the cavity was
41 located at a depth of 1m below the foundation base level. It was shown that the shape and volume of the cavity
42 has an influence on the settlement and concentration of stress under the foundation when the cavity exists at a
43 depth less than twice the width of strip footing, below the base of the foundation with negligible differences
44 between the 2D and 3D analyses outcomes.

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48 This research is a parametric study on the influence of cavities on the slope stability of earth dams during rapid
49 drawdown conditions using PLAXIS 2D, considering factors such as horizontal position, depth and the diameter
50 of cavities. Furthermore, a simulation was carried out to examine the combined effects of the presence, location
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and diameter of cavities and the variations in the shear strength parameters of the embankment on a dam's stability.

2. Numerical simulation

2.1 Two-dimensional analysis

Currently, slope stability simulations are mostly carried out using 2D models under the supposition of plane strain case, which assumes that the slip surface is infinitely wide, and therefore 3D effects are minimal due to the infinite width of the sliding mass. Of course, slopes are not infinitely wide, and the 3D effects influence on slopes stability (Duncan, 1992). Moreover, 3D analyses are considered more factual in terms of their ability to account for the 3D nature of models inputs (Griffiths and Marquez, 2007). 2D methods remain the most widely used methods in slope stability analysis, 2D methods remain the most widely used methods in slope stability analysis, despite the limitations of 2D methods. The main reason for this obvious omission is that 2D analysis is convenient for assessing slope stability because it gives a conservative estimation for the safety factor (Duncan 1992, Cheng et al., 2005; Nian et al., 2012; Leong and Rahardjo, 2012). This is because the calculation of the 2D safety factor does not include the end effects. It is assumed in general that 2D stability analyses give more a conservative assessment of the 3D slope stability problem. On the condition that 2D analyses are computed for the most crucial 2D section (Duncan 1996), the difference between the safety factor obtained from both analyses did not exceed 15%. (Cavoundis, 1987) reported that 2D safety factor is less than 3D safety factor, and 3D methods that may include simplified assumptions that ignore significant sides give a ratio of two-dimensional safety factor to three-dimensional safety factor greater than 1. Two-dimensional analysis can be used instead of three-dimensional analysis when the difference between them is small (Zhang et al., 2011). Even though the three-dimensional technique vastly utilized in the analyses, the majority of analyses are still conducted in two dimensions because of the easy of the construction of model and completing the simulation in a relatively shorter time (Wines, 2016). In recent decades, the two-dimensional techniques have been increasingly accepted in the analysis of slopes (Griffiths and Lane, 1999; and Cheng et al., 2007).

2.2 Plaxis 2D

PLAXIS 2D is powerful finite element-based geotechnical analysis software designed for 2D analysis of deformations, stability and groundwater flow in geotechnical engineering applications. PLAXIS is widely utilized in different geotechnical engineering applications, from excavations, embankment and foundations to tunnelling, mining and reservoir geomechanics (Brinkgreve et al., 2018).

2.2.1 Stability analysis safety factor (ΣMsf) in PLAXIS

In traditional stability analysis methods, the safety factor is known as a ratio of the available shear strength to the minimum soil shear stress wanted for equilibrium. PLAXIS 2D simulates the stability of earth dams and slopes based on the strength reduction method (Phi (fiction angle of soil)-c (cohesion) reduction). The strength reduction method (SRM) has been used in the FEM to calculate safety factors for slopes since 1975 (Dawson et al., 1999 and Griffith and Lane, 1999). In the SRM the critical sliding surface can be found automatically from decreasing the shear strength (Cheng et al., 2007) and it is not essential to determine the shape of the slide

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3 **surface at first.** In this method, the safety factor against slope failure is calculated through the reduction of the
4 soil strength parameters (cohesion and friction angle) until the slope reaches the critical failure state, and the
5 safety factor is considered equal to the proportion of soil strength and the reduction of soil strength at critical
6 failure (He and Zhangm, 2012). PLAXIS defines the safety factor as the ratio of the available strength to the
7 strength at failure, and indicates the safety factor as ΣMsf , as shown in the equations 1 and 2 (Brinkgreve et al.,
8 2018). In this analysis the sliding surface is set automatically to ensure that the shear surface is kept close to the
9 natural sliding surface (Dawson et al., 1999).

15 **2.2.2 PLAXIS 2D modelling**

16 The creation a 2D FEM in PLAXIS 2D starts with the generation of a geometry model. The geometry model
17 includes a composition of surfaces, lines and points. The soil stratigraphy at various positions is determined by
18 defining multiple vertical boreholes, and the soil layer locations are interpolated between the vertical boreholes.
19 When the geometry model is entirely determined, it must be discretised into finite elements in order to create a
20 finite element mesh to begin the calculations. The generated mesh has to be adequately fine to gain exact
21 numerical findings. Very fine meshes must can be used; however, this will result in increased calculation times.
22 The finite element meshes are fully generated automatically by PLAXIS 2D. The mesh creation depends on a
23 strong triangulation procedure. The mesh creation operation takes into consideration the soil stratigraphy,
24 structural objects, loads and boundary conditions. Plaxis 2D provides two types of triangular elements, 6-node
25 and 15-node, to model the soil and other volume clusters. In this study, 15-node triangles were selected as the
26 hypothetical element. This provides fourth-order interpolation for displacements and the numerical integration
27 encompasses twelve stress points. The 15-node triangle is extremely exact and gives high-quality findings for
28 complicated problems such as collapse calculations for incompressible soils (Nagtegaal et al., 1974; Sloan, 1981
29 and Sloan and Randolph, 1982). 15-node element analysis requires more memory and results in longer
30 calculation times than 6-node triangular element analysis (Brinkgreve et al., 2018). In PLAXIS 2D, the stability
31 of a dam is analysed as a staged construction analysis (Brinkgreve et al., 2018). Details of the model creation
32 stage in this study follows this section. **In the slope analysis processes followed in this research, where looking
33 into effect of variations in various parameters on the stability of the developed model, separate analyses were
34 conducted to search for the failure surfaces as the considered parameters were incremented.**

45 **2.2.3 Earth dam modelling**

46 PLAXIS 2D was adopted to model a numerical earth dam model to evaluate the combined influence of the
47 presence of cavities and rapid drawdown conditions on earth dams' stability. The soil is modelled with a
48 medium elemental mesh with 15-node triangular element plane strain applied to the finite element mesh to
49 implement the deformation and slope stability analysis (Brinkgreve et al., 2018). The height of the
50 homogeneous earth dam model is 15m from the crest to the sub-soil, a crest width of 6m, a subsoil depth of 20m
51 and the inclinations of both the upstream and downstream are 1Vertical: 2.5 Horizontal. The geometric design
52 of the numerical dam model was assumed according to the recommendations of the British Dam Society (1994).
53 The incipient water level of the upstream reservoir was assumed at 12m from the sub-soil; thereafter it was

speedily reduced to level of 4m over 5 days to simulate rapid drawdown conditions. The water can flux through all boundaries except at the bottom boundary of model in PLAXIS 2D Tutorial Manual (Brinkgreve et al., 2018). Figure 1 depicts the geometry of the earth dam model and the schematic finite element mesh.

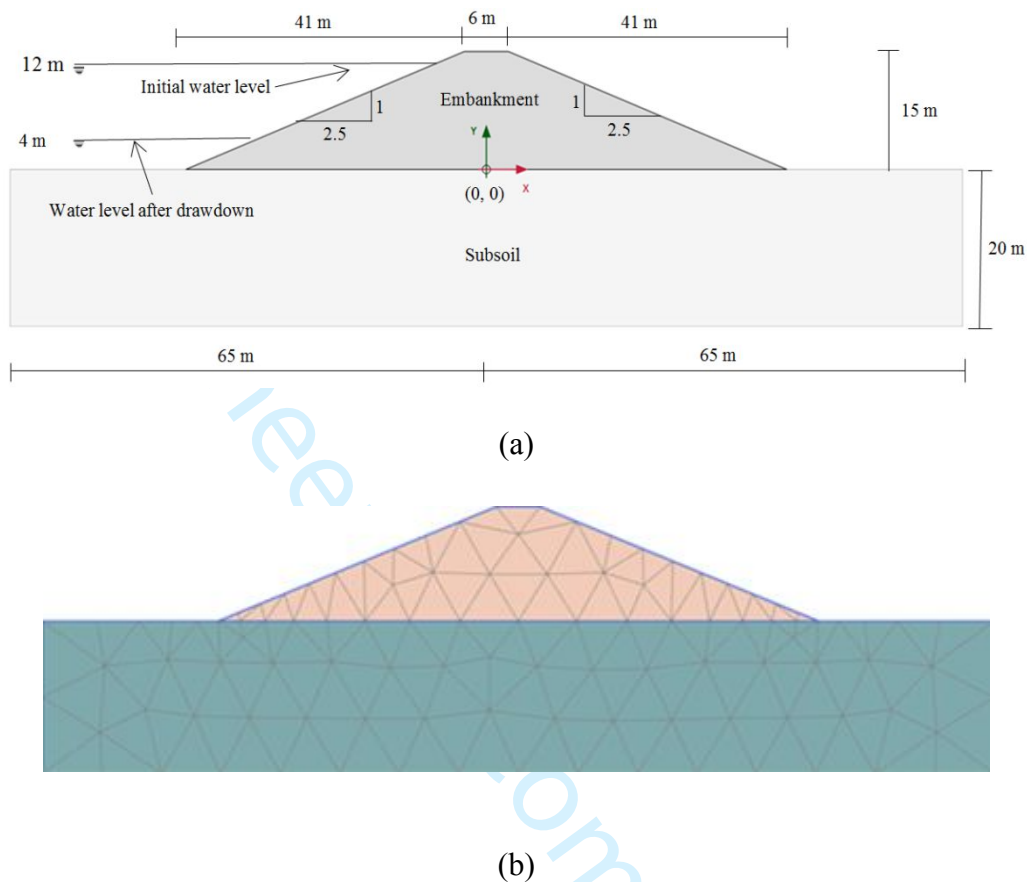


Figure 1. (a) the earth dam geometry; (b) finite element mesh

2.2.4 Cavity modelling

Cavities were modelled using PLAXIS 2D as idealized holes excavated away from the soil mass and assumed to have no lining. According to Aziz (2008), cavities can form in various shapes and diameters underground, and their diameters range between 10 and 300cm. To simplify the modelling process circular cavities with 20, 60 and 100cm diameters were selected in this study to simulate the effect of cavity diameter on the stability of the dam. Figure 2. shows the geometry of the cavity with the parameters, which are defined in section 2.3

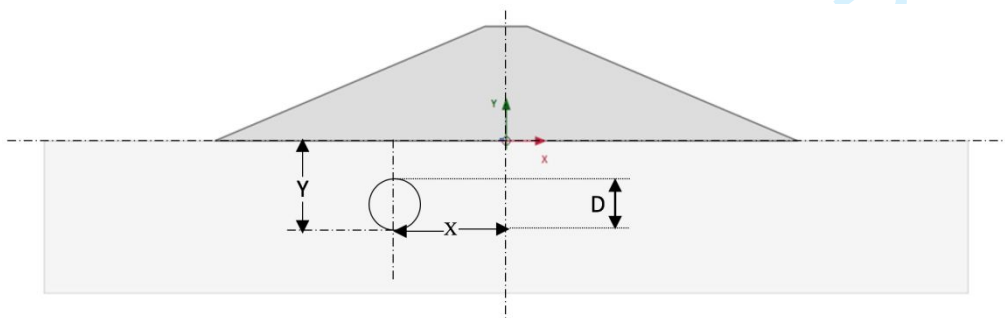


Figure 2. Detailed geometry of the cavity and its location parameters.

2.2.5 Constitutive model

The elastic perfectly-plastic Mohr-Coulomb (MC) failure criterion was chosen to model the behaviour of the embankment materials and sub-soil. This model has been extensively utilized in numerical analysis because of its simplicity, ease of understanding, good accuracy and because it isn't very difficult to extract input parameters compared to other constitutive models, and is recommended by researchers as a preliminary analysis approach (Al-Jazaairry and Sabbagh, 2017). A fixed stiffness average is appreciated for the soil layer. Because of this fixed stiffness, the calculations incline to be comparatively quick and a preliminary estimation of deformations can be obtained (Brinkgreve et al., 2018). In the developed models in this research the stiffness variation in depth was not explicitly taken into account in introducing material properties; however, the applied stiffness value was considered to be the average value to reflect the depth factor as well as satisfying model simplification purposes and was kept closest possible to real stiffness values. The input parameters model involve: γ unit weight of soil; ν' , Poisson's ratio; C' , cohesion; ψ , angle of dilatancy; ϕ' , internal angle of friction; k_x and k_y , coefficients of permeability in the x and y directions; and E' , Young's modulus. The soil model developed was considered to be made of two types of homogeneous materials (one for the body of the dam and the other for the subsoil -base of the dam- in the model). The shear strength parameter (c and Φ) values for the body and base materials were (25 kPa, 22.5 degree) and (5 kPa, 35.0 degree) respectively. Table 1 itemizes the input parameter values required in this model, which are appropriate values for the present analysis (Brinkgreve et al., 2018).

Table 1. Model properties for embankment and subsoil

Parameters	Embankment	Subsoil	Unit
Model	Mohr-Coulomb	Mohr-Coulomb	
Drainage type	Drained	Drained	
γ unsaturated	16.0	17.0	kN/m ³
γ saturated	20.0	21.0	kN/m ³
ν'	0.33	0.3	
c'	25	5.0	kN/m ²
ψ, ϕ'	1.0, 22.5	5.0, 35.0	Degrees
k	1E-4	0.01	m/day
E'	2.0E4	5.0E4	kN/m ²

2.3 Analysis and results

The numerical investigation, involving two primary parts, was conducted to simulate the influence of the presence of cavities in the sub-soil of an earth dam on stability of its slopes. In the first part, the effects of position, depth and size of cavities were studied, where:

- Cavity position (X) is the horizontal distance between the **cavity's centerline (according to PLAXIS software)** and the earth dam's centerline.
- Cavity depth (Y) is the vertical distance between the embankment's base and the **cavity's centerline (according to PLAXIS software)**.
- Cavity size (D) is the diameter of the cavity.
- Cavity location L (x, y) are the coordinates of the cavities on the x-axis and the y-axis.

In the second part, the combined influence of cavities and shear strength parameters (cohesion and internal friction angle) on slope stability were studied.

All stability analyses were performed using the numerical model developed in PLAXIS 2D during rapid drawdown conditions, assuming the presence of a single cavity in the sub-soil of the earth dam model.

2.3.1 Effect of cavity position from the dam centerline (X)

In order to study the influence of a cavity's horizontal distance from the dam's centerline on the dam's stability, sixteen hypothetical horizontal positions in the sub-soil of the upstream and downstream sides were considered. All stability analyses at this stage were performed assuming the presence of a single cavity with a diameter of 60cm located at a depth of 1m **from the base of embankment to cavity centreline**. Table 2 presents the coordinates of the positions of cavities in the X-axis.

Table 2: Coordinates of the locations of cavities in X and Y directions

Cavity location	Cavity depth (Y), m	The coordinates of position in X-axis (X), m	
		Upstream side	Downstream side
L1	1	0	0
L2	1	-8	+8
L3	1	-17	+17
L4	1	-20	+20
L5	1	-24	+24
L6	1	-28	+28
L7	1	-35	+35
L8	1	-45	+40

2.3.1.1 Cavities located under the upstream slope

The influence of cavity position on the values of safety factor and displacement are shown in Figures 3 and 4. As revealed in these figures, existence of cavities reduces the stability of the upstream slope noticeably: the safety factor lowered from 1.992 to 0.899 for models without and with cavities at location L2 (-8, -1) respectively, whilst the value of maximum displacement increased from 24.91 to 29.46mm for models without and with cavities. **There is a reasonable trend in the numerical analysis results showing displacements and factors of safety; however, noticeably bigger larger changes can be seen that happens at location L2 (upstream side of the dam). This behaviour can be attributed to a decrease in the subsoil strength as a consequence of the**

presence of a cavity as well as a possible intersection of the cavity location and a potential failure surfaces of the slop in dam in the upstream with minimum shear resistance. Moreover, in the rapid drawdown of the reservoir water level, deformations occur in the upstream slope due to an abrupt drop in the effective stress values as the soil in the embankment remains saturated immediately after drawdown without availability of the time required for the drainage of water from the upstream slope, and as a result without dissipation in the excess pore water pressures (Tran, 2004). It is clear from Figure 5 that stress concentration area started to expand within the sliding mass concentrated above the cavity at location L2 (-8, -1), where the horizontal distance from the dam's centerline increased to its end (from L2 (-8, -1) to L8 (-40, -1)). Consequently, safety factor values increased from 0.899 to 1.947, while displacement values decreased from 26.38 to 24.83mm for models with cavities at locations L2 and L8 respectively (see Table 2 for details of the positions). It is worth mentioning that the safety factor value for the model with a cavity at L2 was less than the minimum acceptable value (1.2 to 1.3), thus the dam is considered to be not safe according to the criterion for the stability of an earth dam during rapid drawdown conditions (NRCS, 2005; USBR, 2011; ULDC, 2012).

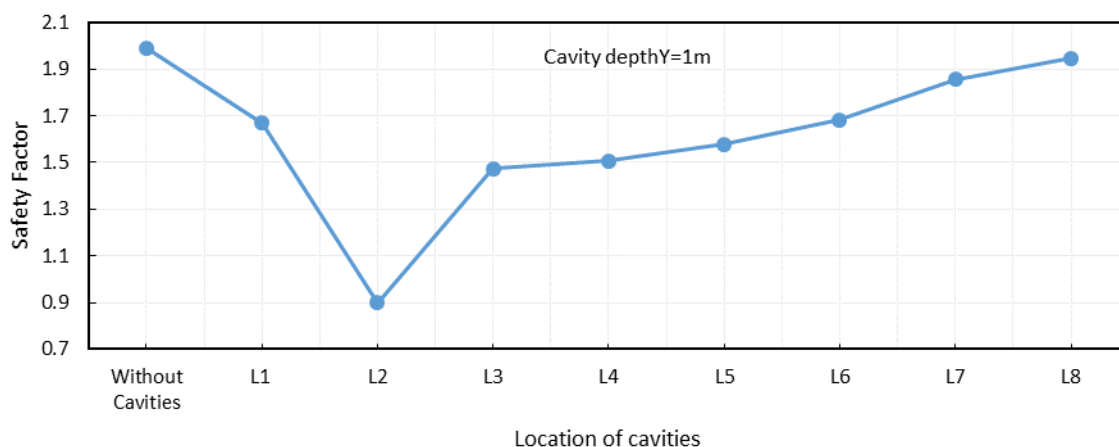


Figure 3. Effect of cavity location on the values of safety factor

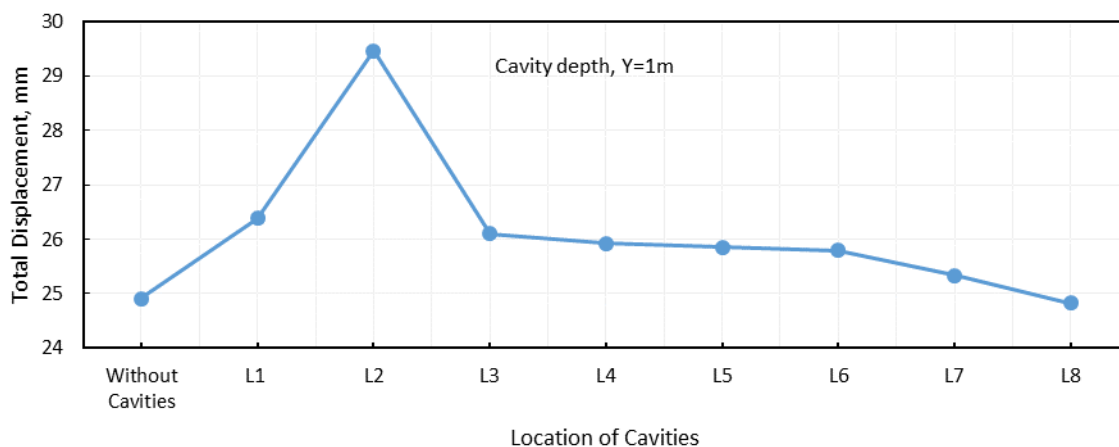
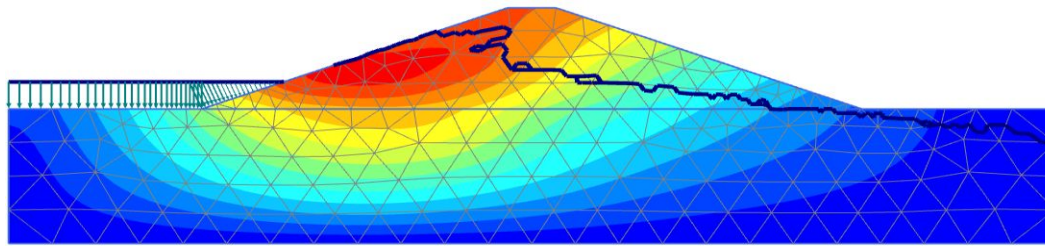
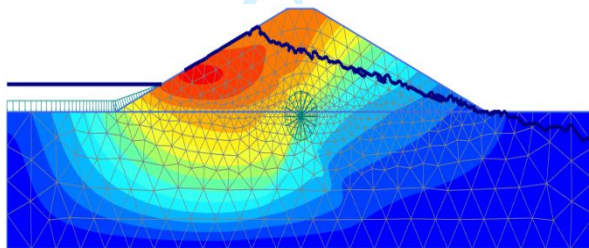


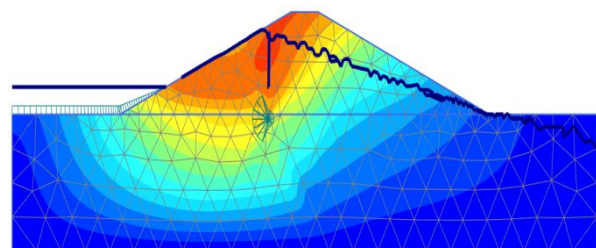
Figure 4. Effect of cavity location on the maximum values of total displacement



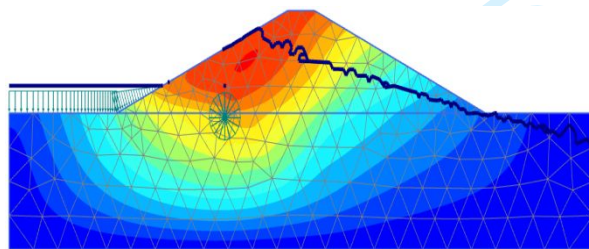
(a) Without cavity, maximum of total displacement $|u| = 24.91\text{mm}$



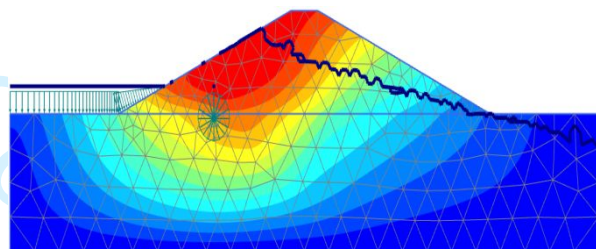
(b) Cavity at location (L1), $|u| = 26.38\text{mm}$



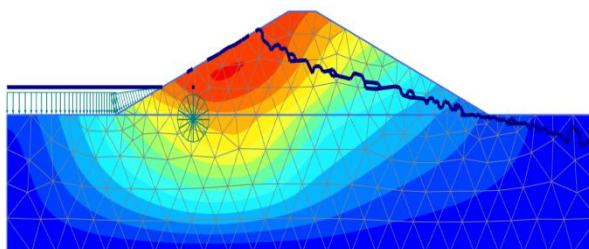
(c) Cavity at location (L2), $|u| = 29.46\text{mm}$



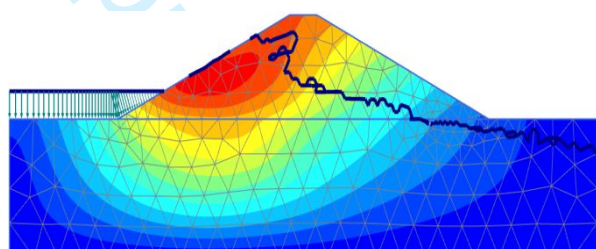
(d) Cavity at location (L3), $|u| = 26.10\text{mm}$



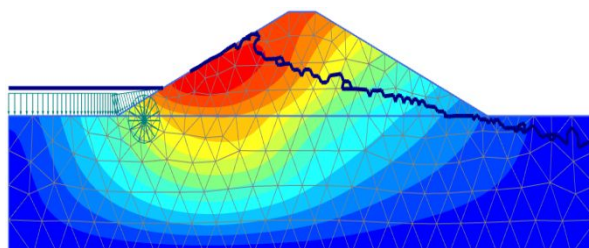
(e) Cavity at location (L4), $|u| = 25.93\text{mm}$



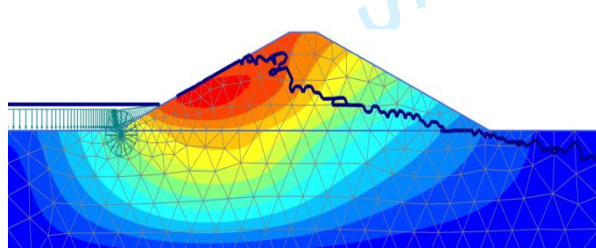
(f) Cavity at location (L5), $|u| = 25.85\text{mm}$



(g) Cavity at location (L6), $|u| = 25.79\text{mm}$



(h) Cavity at location (L7), $|u| = 24.89\text{mm}$



(i) Cavity at location (L8), $|u| = 24.83\text{mm}$

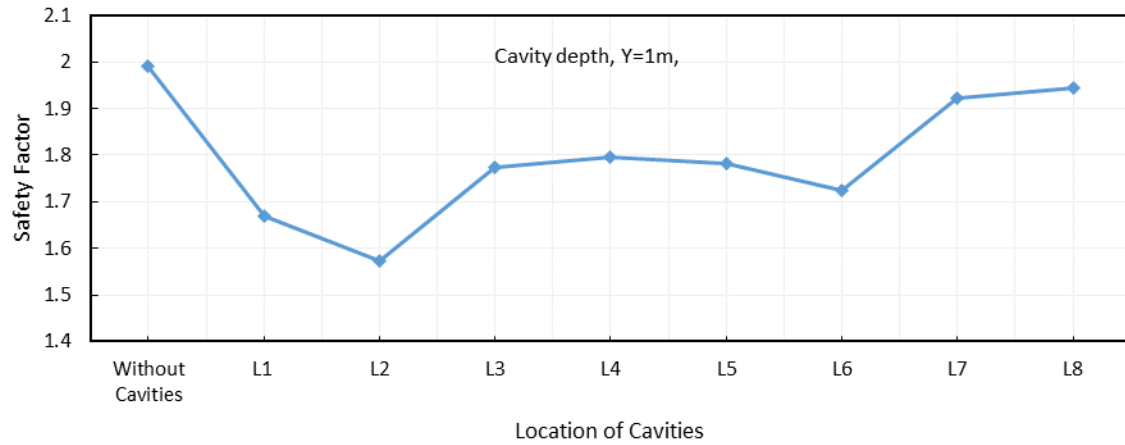
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Figure 5. Contour of total displacement for the effect of cavities locations under upstream

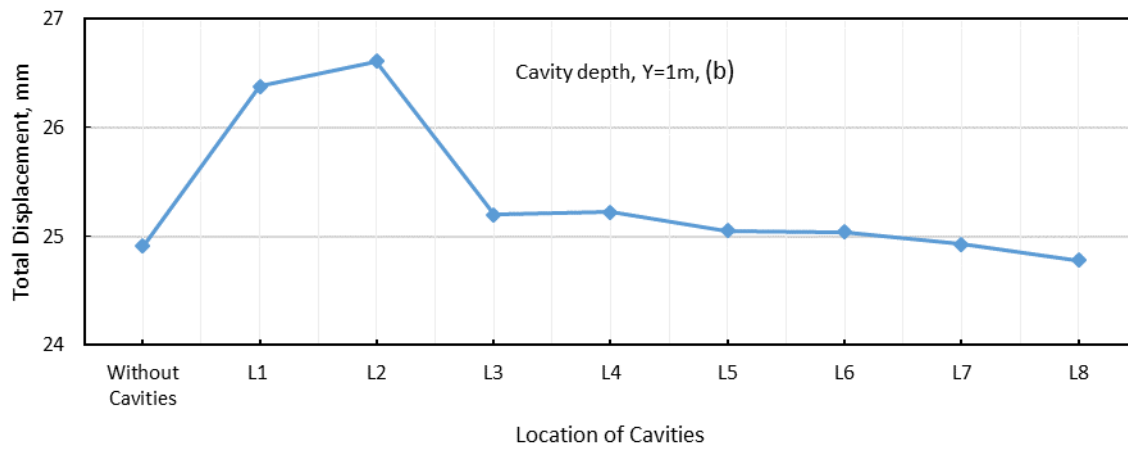
2.3.1.2 Cavities located under the downstream

Figure 6 indicates the effects of cavities located under the downstream side on slope stability (see Table 2 for details of the positions). It can be noted from results that the presence of cavities reduced the stability of the dam in varying proportions: the safety factor decreased from 1.992 for the model without cavities to 1.572 and 1.943 for models with cavities at locations L2 (8, -1) and L8 (40, -1) respectively. However, the safety factor values remained larger than the minimum specified value (1.2-1.3) according to the slope stability criterion mentioned above. This suggests that this earth dam is more stable or safer under rapid drawdown conditions when cavities are located under the downstream side as opposed to the upstream side, where the presence of cavities could significantly damage stability. This behaviour may be attributed to the fact that cavities are further away from potential failure surfaces. Figure 7 shows contours of total displacement for the effect of cavities locations under downstream. As expected, the maximum displacement values slightly increased from 24.91mm for a cavity-free model to 26.61 and 24.93mm for models with cavities at locations L2 and L8 respectively.

Figure 8 shows a comparison between the impact of the existence of cavities in the subsoil of the upstream and downstream sides of the model earth dam for cavities with different horizontal positions and depths. It can be seen that a rapid drop in water level in the reservoir greatly influences the stability of the slopes on the upstream side (inside the reservoir); however, as it is clearly indicated in the Figure, this effect is not as significant on the downstream side of the dam. **This is because of the combined impact of the existence of a cavity and presence of rapid drawdown conditions. In rapid-drawdown conditions the countervailing water pressure of upstream decreases, which leads to a reduction in upstream stability. The soil inside the dam body remains saturated whilst the flow of water begins. Seepage and hydrodynamic pressures create downward forces that negatively affect stability and lead to critical conditions on the upstream side (Fattah et al., 2017).** The Figure shows that on the upstream side, the safety factor values significantly decreased by about 54.9% at location L2, compared to 21.1% on the downstream side of the model dam; whereas the values of displacement increased by about 18.3% on the upstream side, compared to 6.8% on the downstream side.



(a)



(b)

Figure 6. Effect of cavity location on the values of: (a) safety factor; (b) maximum total displacement

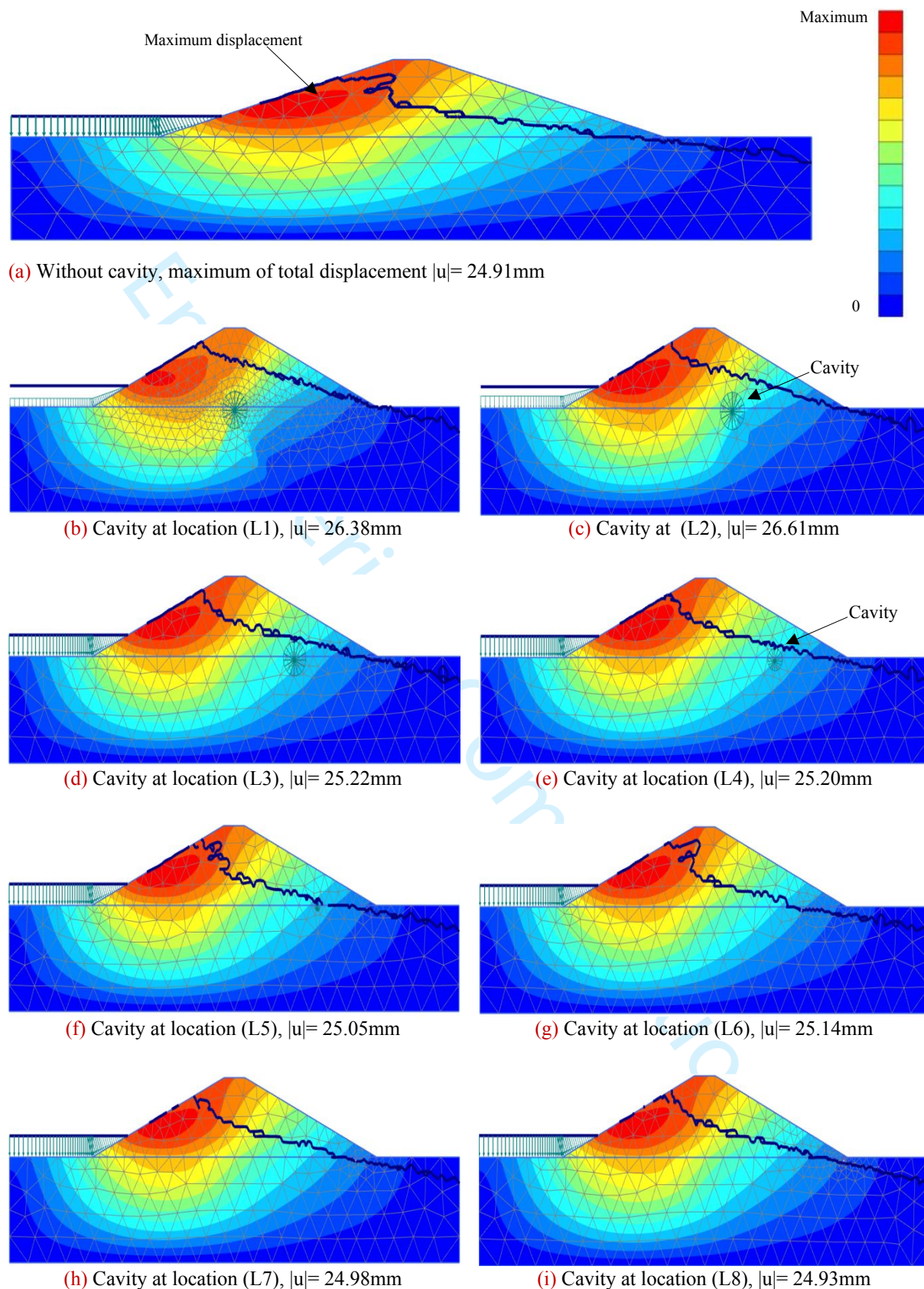


Figure 7. Contour of total displacement for the effect of cavities locations under downstream

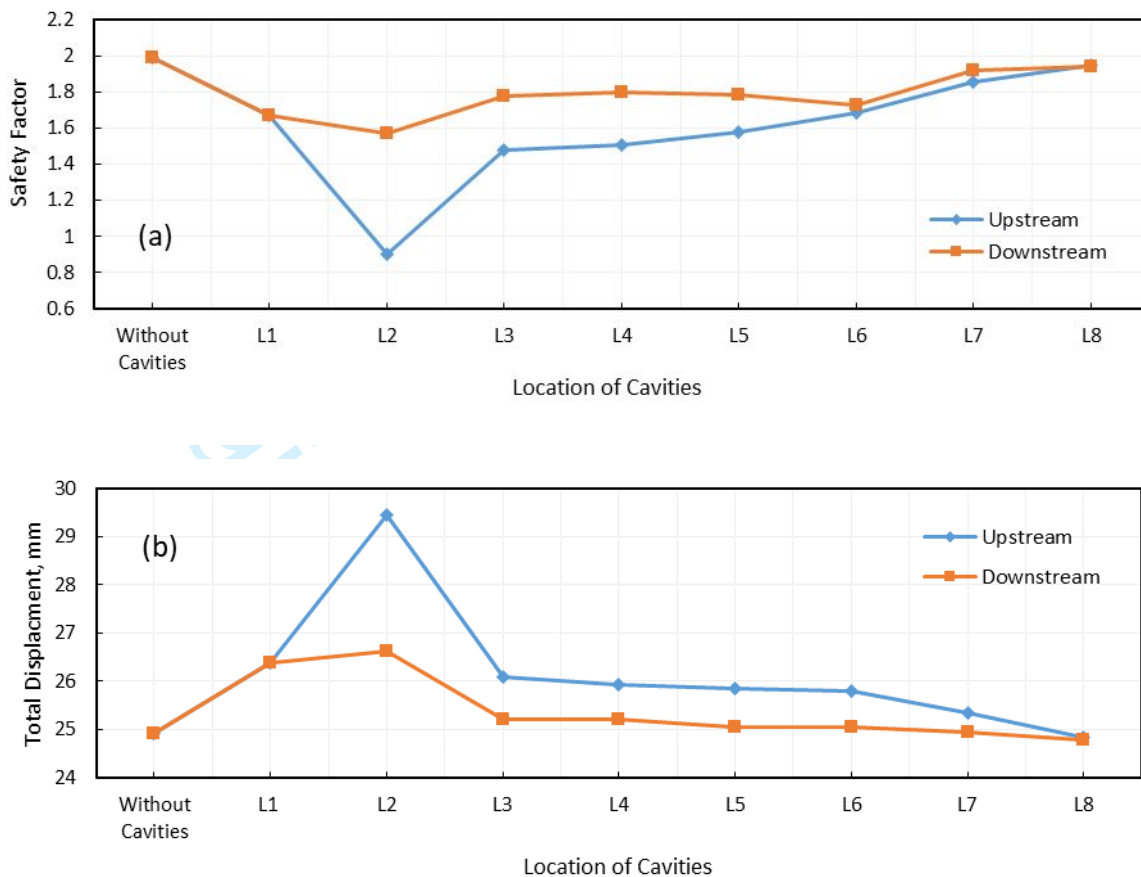


Figure 8. Comparison between the effect of cavities presence under upstream and downstream sides of the model dam: (a) safety factor; (b) total displacement

2.3.2 Effect of cavity depth

To evaluate the effect of cavity depth on the slope stability of earth dams, four hypothetical depths have been selected under the upstream and downstream slopes; $Y = 1, 2, 3$ and 4m (from the base of embankment to cavity's centerline). The simulation was assumed the diameter of cavities was 60cm and the positions of cavities as were previously defined in Table 2.

2.3.2.1 Cavities located under the upstream slope

The effect of cavity depth on slope stability is illustrated in Figure 9 and Figure 10. The stability results proved that cavity depth slightly affects slope stability. The safety factor values slightly fluctuated (increasing or decreasing) with increasing cavity depth for all locations, with the exception of location L5 (-24, -2). E.g. the values of safety factor for models with cavities at locations (-8, -1), (-8, -2), (-8, -3) and (-8, -4) were 0.904, 0.948, 0.979 and 1.077 respectively. This behaviour could be because the cavities were no longer in the zone influenced by the stress distribution as the depth increases. This could also mean that the presence of cavities creates potential weaknesses in the soil; however, if the cavities are located outside of the stress zone of influence or nearer to the boundary of the stress bulb, these effects could be negligible.

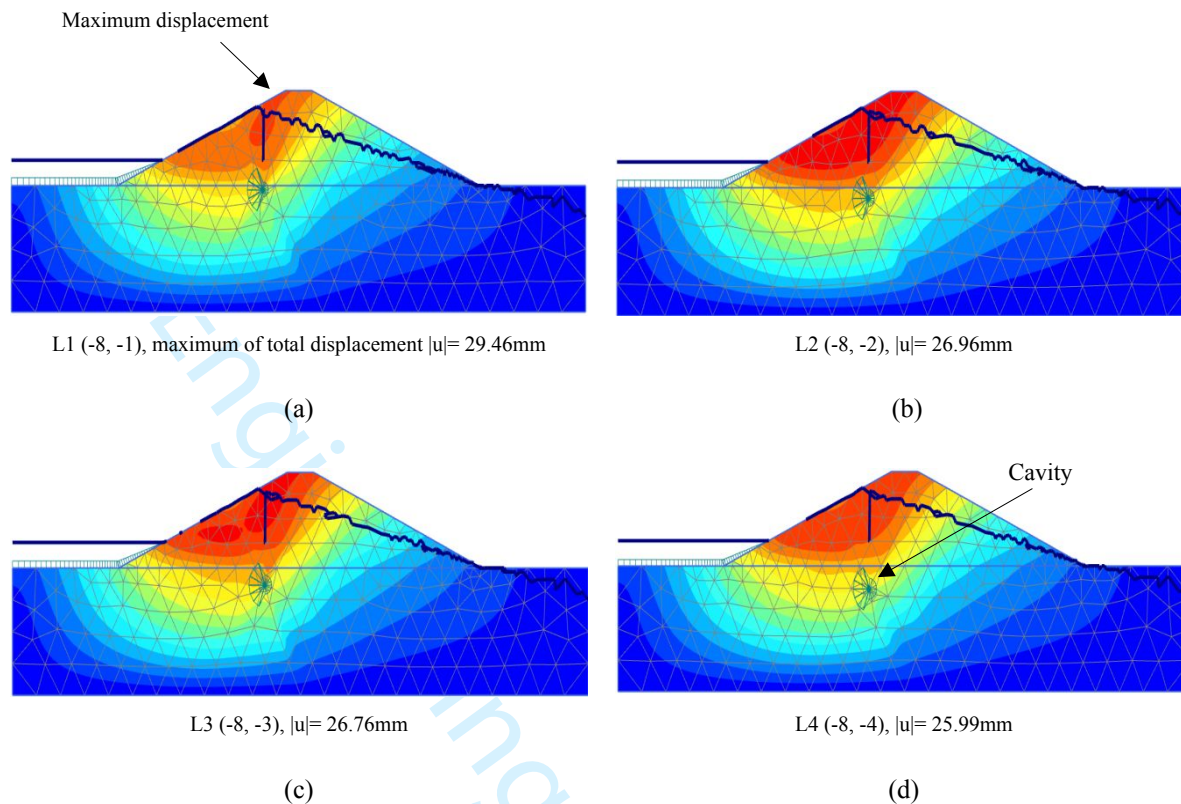


Figure 9. Contour of total displacement for the effect of cavity depth: (a) $Y= 1\text{ m}$, (b) $Y= 2\text{ m}$, (c) $Y= 3\text{ m}$, (d) $Y= 4\text{ m}$ - values range from 0 to maximum in the contours (contour values shown in Figure 7)

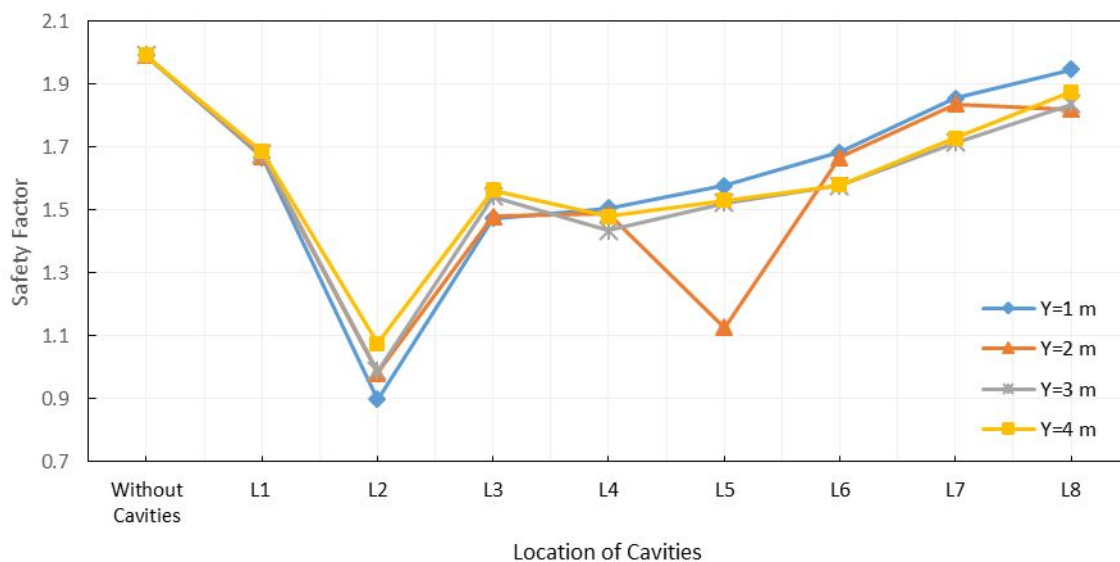


Figure 10. Safety factor versus horizontal positions of cavities for different depths ($Y1$ to $Y4$) under upstream

2.3.2.2 Cavities located under the downstream slope

Figure 11 demonstrates the influence of cavity depth on the safety factor values when cavities are located under the downstream slope. It can be seen from the results that by increasing the cavity depth, the values of safety factor fluctuate somewhat for all locations of cavities, except for position (X2= 8m), where the safety factor value increased from 1.572 to 1.835 as cavity depth increased from 1 to 4m. There is also an exception at (X7= 35m), where the safety factor value decreased from 1.921 to 1.593 as cavity depth increased from 1 to 3m. This could be due to the presence of cavity coinciding with the weakest potential slip surfaces, inducing a further drop in the safety factor.

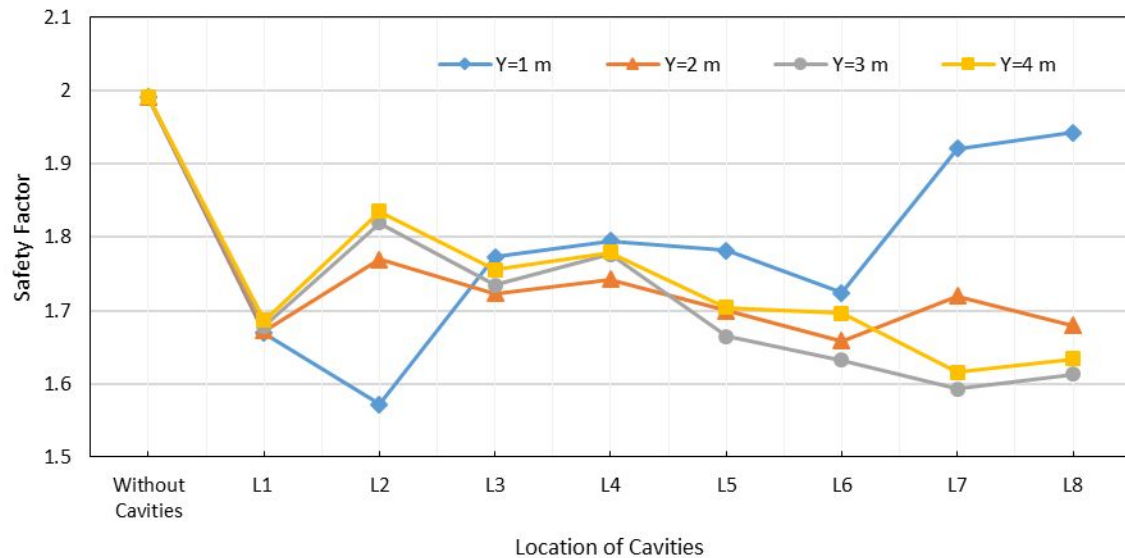


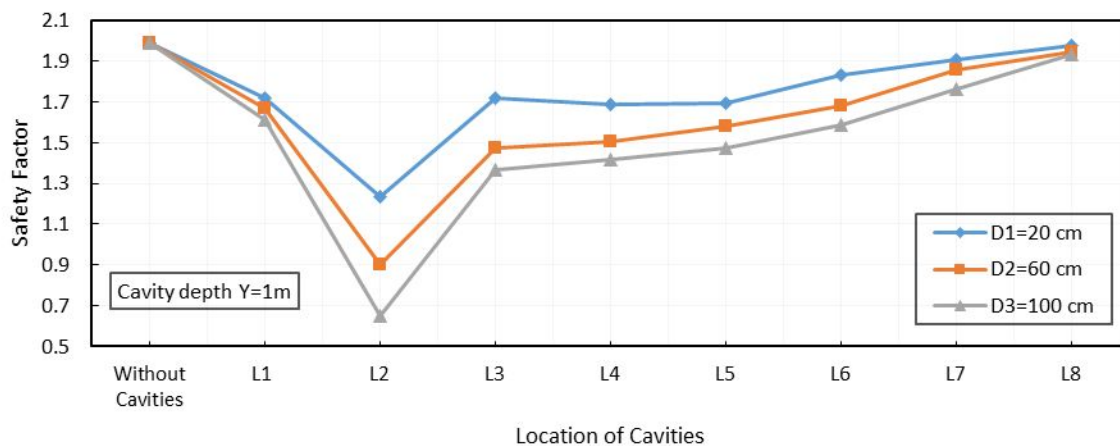
Figure 11. Safety factor versus horizontal positions of cavities for different depths (Y1 to Y4) under downstream

2.3.3 Effect of cavity diameter

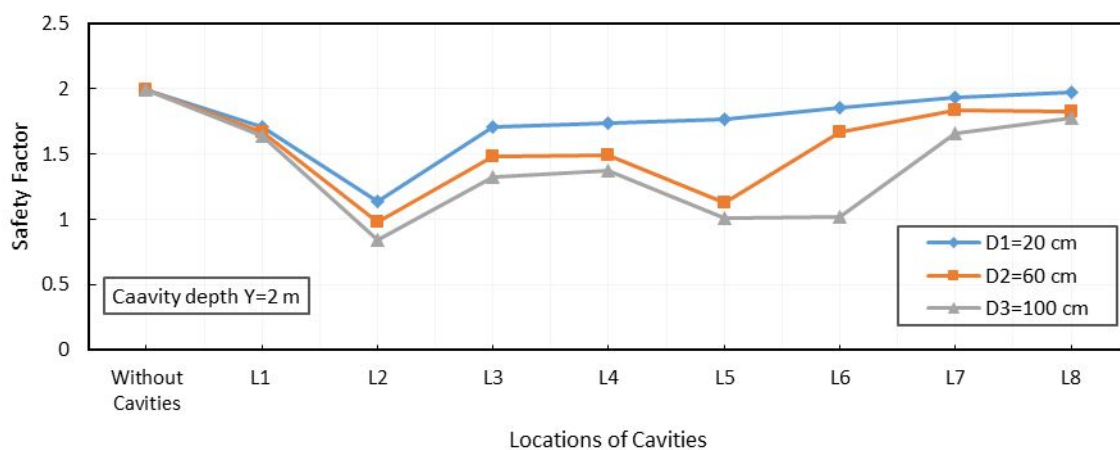
In this part of the study, the effect of cavity diameter on slope stability was investigated. Cavities with diameters 20, 60 and 100cm were considered in this analysis. The same horizontal positions of cavities used in part one analyses were adopted, and the depths of cavities were Y= 1, 2 and 3m (from the base of embankment to cavity's centerline).

2.3.3.1 Cavities located under the upstream slope

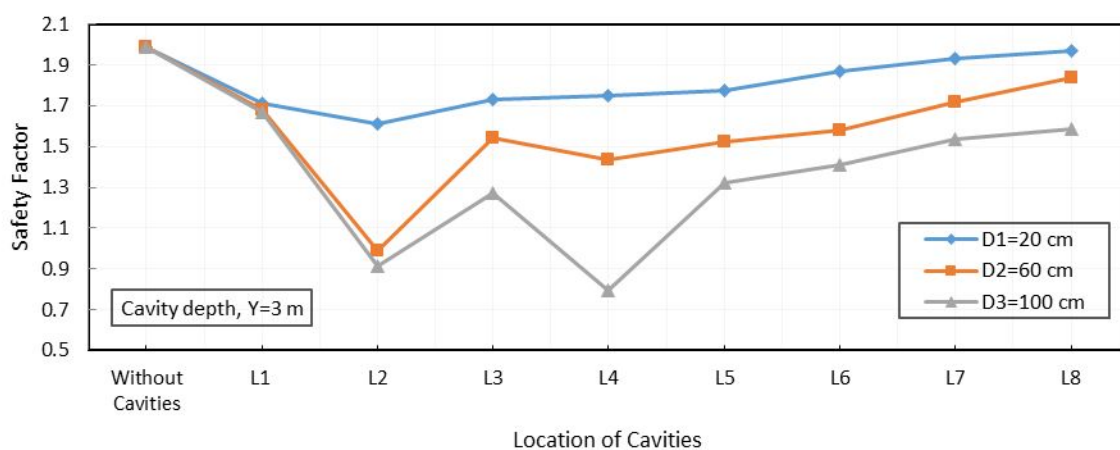
Figure 12 illustrates the combined relations between cavity location and the safety factor values for various cavity diameters. It can be observed that cavity size dramatically influences earth dam stability, as the cavities' diameters increase from 20 cm to 100 cm. This observation was seen to be applicable to all models no matter where the cavities were located, either horizontally or vertically. Safety factors drop from 1.235 to 0.650 and 1.750 to 0.794 for cavities located at L2 (-8, -1) and L4 (-20, -3) respectively. Also, further increasing the cavity diameter to 100 cm makes the earth dam unstable when the cavity is situated in locations L2 (-8, -2), L2 (-8, -3), L4 (-20, -1), L3 (-17, -3), L5 (-24, -2) and L6 (-28, -2), leading to safety factor values smaller than the values recommended by the codes of practice for earth dam stability under rapid drawdown conditions (NRCS, 2005; USBR, 2011; ULDC, 2012).



(a)



(b)



(c)

Figure 12. Effect of cavities diameter on the factor of safety of upstream for different horizontal locations and depths: (a) Y1=1 m, (b) Y2=2 m, (c) Y3=3 m

2.3.2.2 Cavities located under the downstream slope

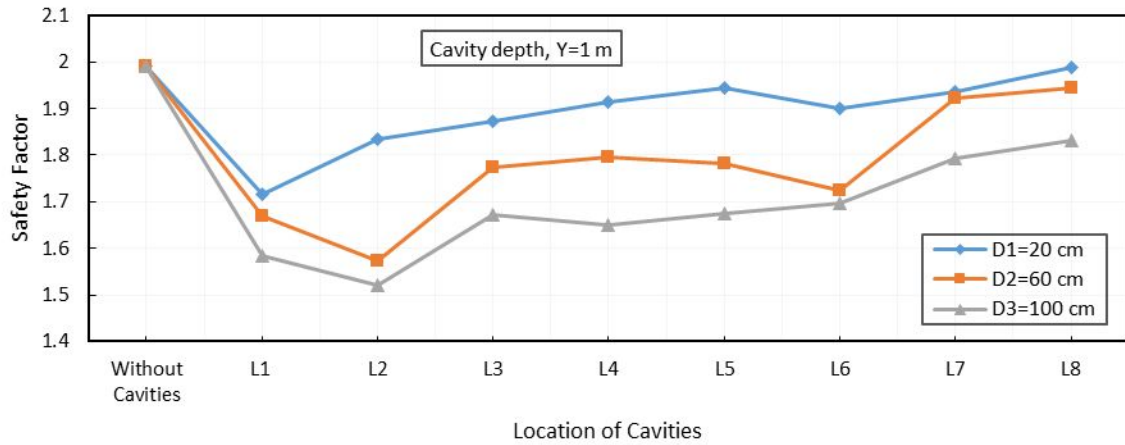
Similar to the upstream side results presented above, it is clear from the results shown in Figure 13 that increasing the cavity diameter from 20 to 100cm decreased the safety factor, as expected for models with cavities under the downstream slope regardless of changes in the locations of the cavities, either horizontally or vertically. The smallest safety factor value obtained, however, was 1.353 (belonging to the model with a cavity diameter of 100cm at location L8 (-40, -2), which is greater than the minimum recommended value (1.2-1.3) by the codes of practice for the stability of dam slopes during rapid drawdown conditions (NRCS, 2005; USBR, 2011; ULDC, 2012). This is unlike the situation in the upstream side, where increasing the cavity diameters led to small values, suggesting the dam is unstable. These results convey the fact that although increasing the diameters of cavities under the downstream side of the earth dam model caused the safety factor values related to the slopes of the dam model to drop, the effect was not sufficient to cause a failure.

The combined effect of cavity diameter and size of the dam (H/D) on slope stability as shown in Table 3. Three heights of the dam (15m, 30m and 45m) and cavities with diameters 20cm, 60cm and 100cm were considered in this analysis. The stability analyses were performed assuming the existence of a single cavity situated at a depth of 1m under the centerline of the dam. Where: H is the depth from the centerline of the cavity to the top of the embankment, D is a diameter of the cavity.

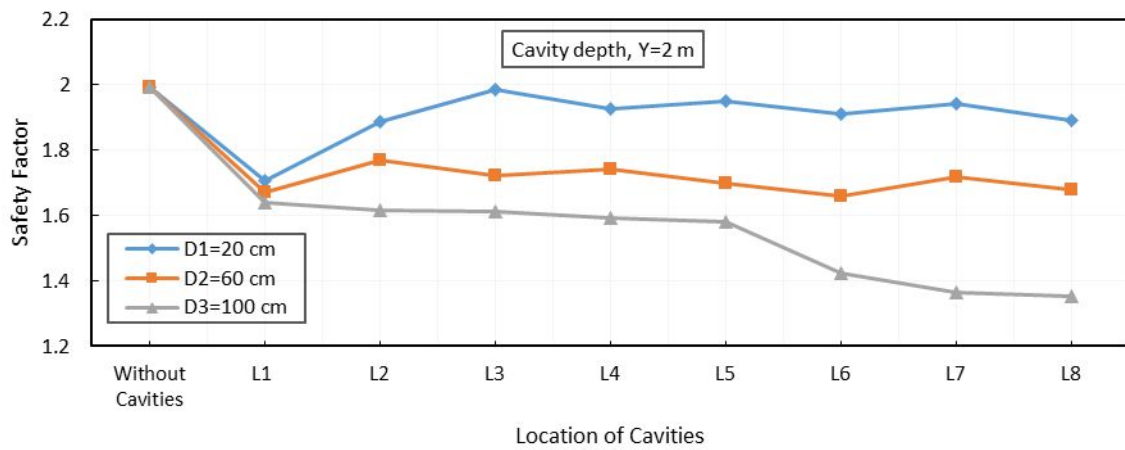
It is clear from Table 3 that safety factor values decreased with increasing the dam height and cavity diameter. The safety factor value decreased from 1.717, 1.425 and 1.192 to 1.610, 1.395 and 1.160 with decreasing the value of H/D for height of dam of 15m, 30m and 45m, respectively.

Table 3: H/D and the corresponding safety factor values

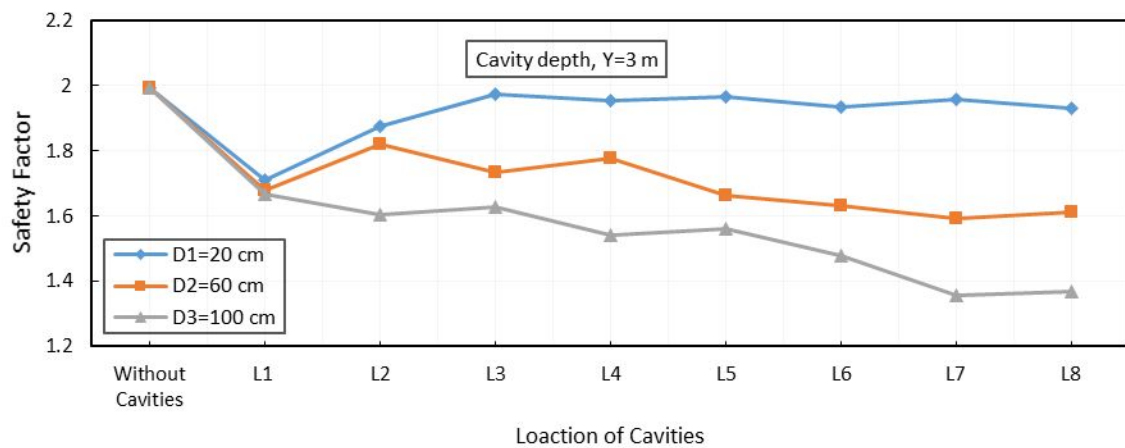
Dam height, cm	Cavity diameter, cm	H/D	Safety factor
1500	20	76	1.717
	60	26	1.669
	100	16	1.610
3000	20	151	1.425
	60	51	1.405
	100	31	1.395
4500	20	226	1.192
	60	76	1.172
	100	46	1.160



(a)



(b)



(c)

Figure 13. Effect of cavities diameter on the safety factor of downstream for different horizontal locations (L1 to L8) and depths: (a) Y1=1 m, (b) Y2=2 m, (c) Y3=3 m

2.3.3 Combined effect of cavities and shear strength parameters on stability

A parametric study was conducted to estimate the combined influence of the presence of cavities and embankment shear strength parameters (c' and ϕ) on the stability of earth dams in rapid drawdown conditions. This section of the study is also divided into two sub-sections. In the first sub-section the combined influence of shear parameters, the presence of cavities and relocation are analysed, whereas in the second sub-section the investigation focuses on the combined effects of cavity diameter and shear strength parameters. These analyses have been performed assuming the presence of a single cavity located under the upstream reservoir. The cavities' positions have been selected based on the previous analysis results, X1 (-8), X2 (-17) and X3 (-24) at depths of $Y=1, 2$ and 3m (from the base of embankment to cavity's centerline).

2.3.3.1 The combined effect of cavity location and the soil's apparent cohesion

The soil cohesion values (c') considered in this simulation were 15, 25, 30, 35, 40, 50, 60 and 80 kPa. The horizontal positions and depths of cavities are the same as in the previous section, and the diameter of cavities was set to 60 cm. The model developed and used in this research is considered to be homogeneous in nature; therefore, the implemented material properties are assigned in a way to reflect this (Abbas, 2015).

Figure 14 displays the safety factor values versus soil cohesion values for models with and without cavities for the positions and depths previously indicated. The results reveal that safety factor values increased from 1.740 to 2.799 by about 60.9%, along with an increase in the cohesion value from 15 to 80 kPa for models without cavities, as expected. The results show that safety factor values for models with a cavity at position X1 stayed more or less the same, with slight increases as the cohesion increased, and were under the minimum recommended safety factor value (1.2-1.3). The safety factor value increased from 0.665 to 0.959 by about 44.2%, as the cohesion value increased from 15 to 80 kPa for models with cavities at location (-8, -1). It is apparent that increasing the soil cohesion does not lessen the effect of cavities on stability when the cavities are located at critical locations (locations that recorded safety factor values less of the allowable limit). It is clear from results that by increasing the soil cohesion, the safety factor values slightly increase for models with cavities located at positions X2 and X3. Results reveal that increasing the cavity depth causes slight changes in the safety factor values as the cohesion increases. For example, the safety factor values were 0.959, 0.996 and 0.971 for models with cavities at locations (-8, -1), (-8, -2) and (-8, -3) respectively and a cohesion of 80 kPa.

2.3.3.2 The combined effect of cavity diameter and the soil's apparent cohesion

The combined effect of cavity diameter and the embankment's soil cohesion on stability has been analysed in this section. Models with a single cavity of diameter 20, 60 and 100cm located at locations X1 (-8m), X2 (-17m) and X3 (-24m) and at a depth of 1m were utilized. Figure 15 shows that increasing the cavity diameter from 20cm to 100cm results in considerable drops in the safety factor values. The decrease in safety factor values was from 1.244 to 0.738, 1.734 to 1.374 and 1.741 to 1.493 for models with cavities at positions X1, X2 and X3 respectively and a cohesion of 30 kPa.

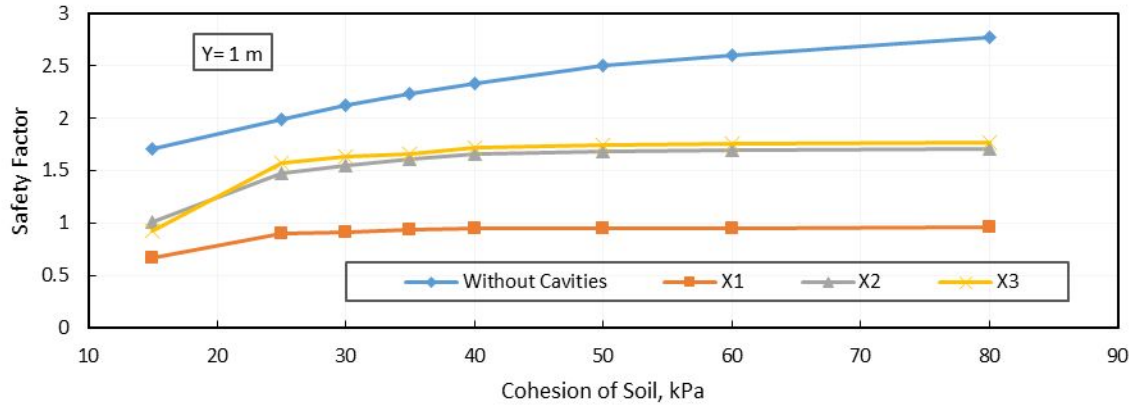
2.3.3.3 Combined effect of cavity location and angle of internal friction (ϕ')

This section focuses on the combined influence of the angle of internal friction and varying cavity location horizontally and vertically. These analyses were conducted assuming the values of ϕ' were 10° , 15° , 22.5° , 27° , 30° , 33° and 35° , and the cavities locations were X1 (-8m), X2 (-17m) and X3 (-24m) at depths of Y= 1, 2 and 3m for models with a single cavity with a diameter of 60cm.

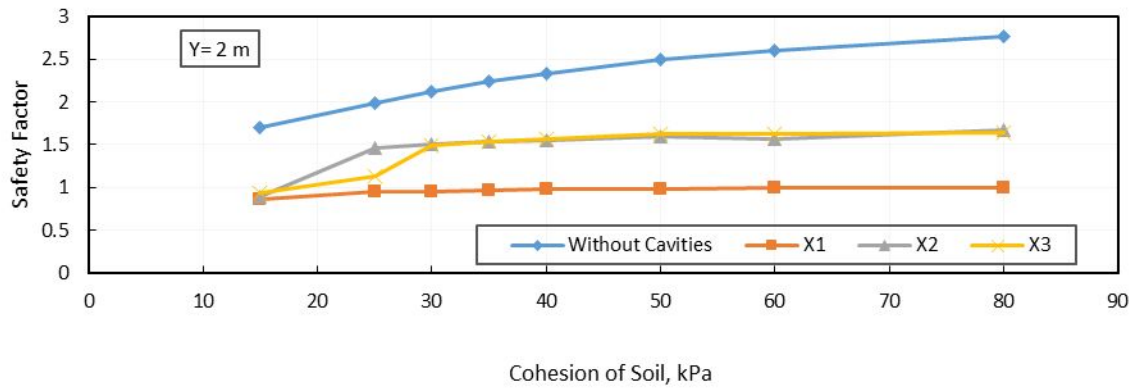
It can be observed from Figure 16 that the safety factor value increases from 1.503 to 2.399 by increasing ϕ' from 10° to 35° for models without cavities. As the value changed from 10° to 35° , the safety factor value increased from 0.667 to 1.103 by about 65.4 % for models with a cavity located at (-8, -1), which are smaller than the limit required for slope safety under rapid drawdown conditions; whereas the safety factor values increased from 0.710 to 1.614 by about 127.3 % and 0.853 to 1.669 by about 95.7 % along with an increase in the cohesion for locations (-17, -1) and (-24, -1) respectively. As revealed in the previous section, increasing the value of ϕ' does not reduce the effect of cavities on stability when they are situated at critical locations. The results also showed that increasing cavity depth results in a slight change in the safety factor with an increase of ϕ' for all locations of cavities considered in the analyses.

2.3.3.4 Combined effect of cavity diameter and angle of internal friction (ϕ')

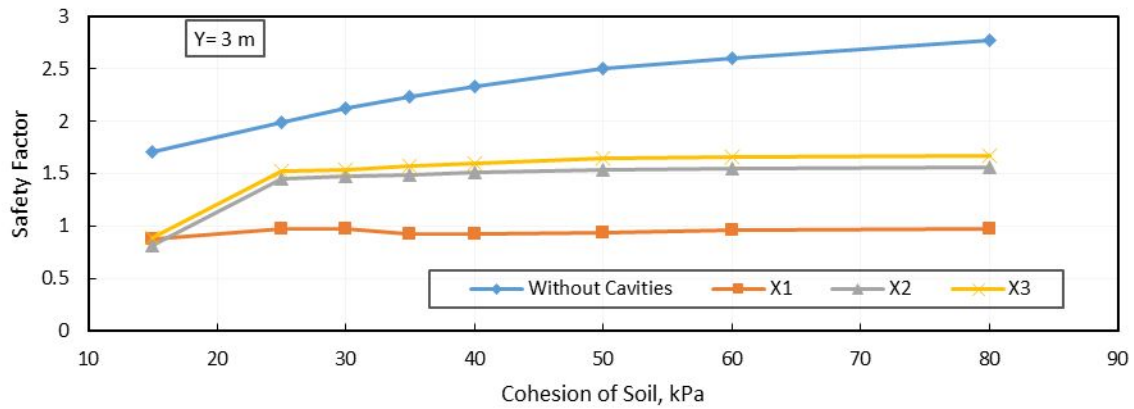
The combined influence of cavity diameter and ϕ' has been analysed for models with cavities with diameters of 20, 60 and 100cm. Cavities were created in upstream subsoil at positions X1, X2 and X3 and at a depth of 1m. The results of the simulations are shown in Figure 17, which shows that increasing the cavity diameter from 20 to 100cm causes a reduction in the safety factor values for models with cavities at these positions despite increasing the value of ϕ' from 10° to 35° . This outcome reveals the fact that the cavity diameter can have much more significant disturbing effect on slope stability in earth dams under rapid drawdown conditions; compare this to the milder improving effects resulting from increasing the internal friction angle of the dam material. This could be due to a more significant drop in the shear strength of the soil due to the presence of a bigger cavity, which cannot be compensated for by using a more frictional material.



(a)

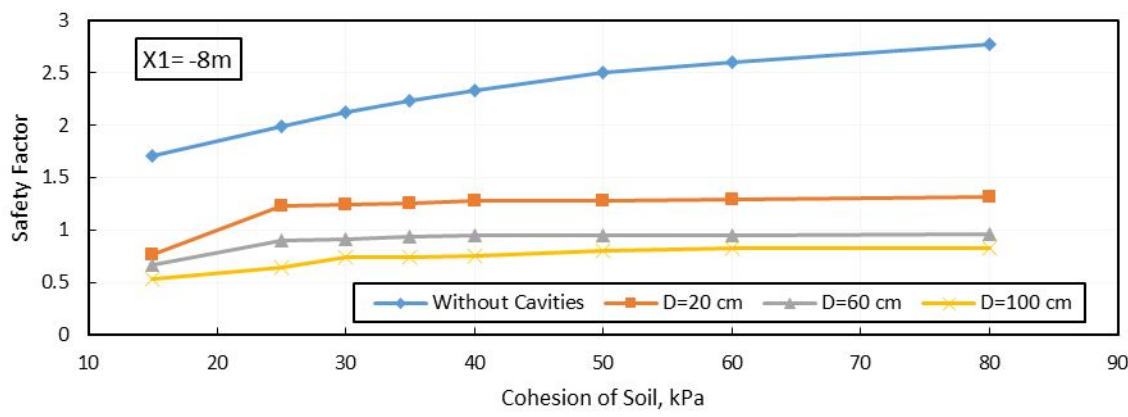


(b)

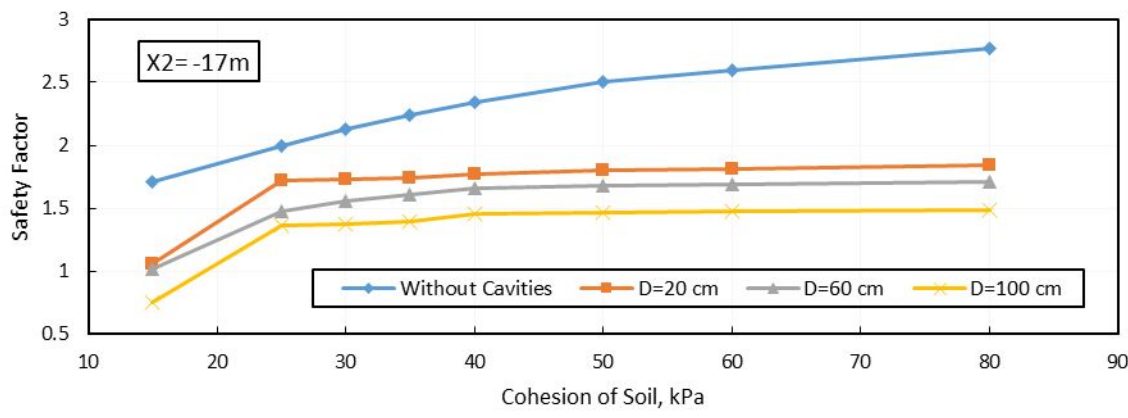


(c)

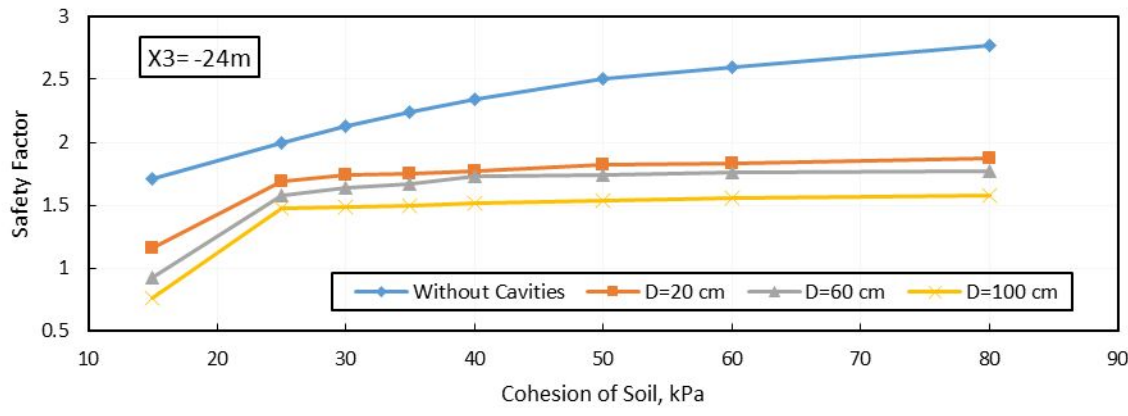
Figure 14. Combined effect of cavity location and soil cohesion on the factor of safety of upstream for various depths: (a) Y=1 m, (b) Y=2 m, (c) Y=3 m



(a)

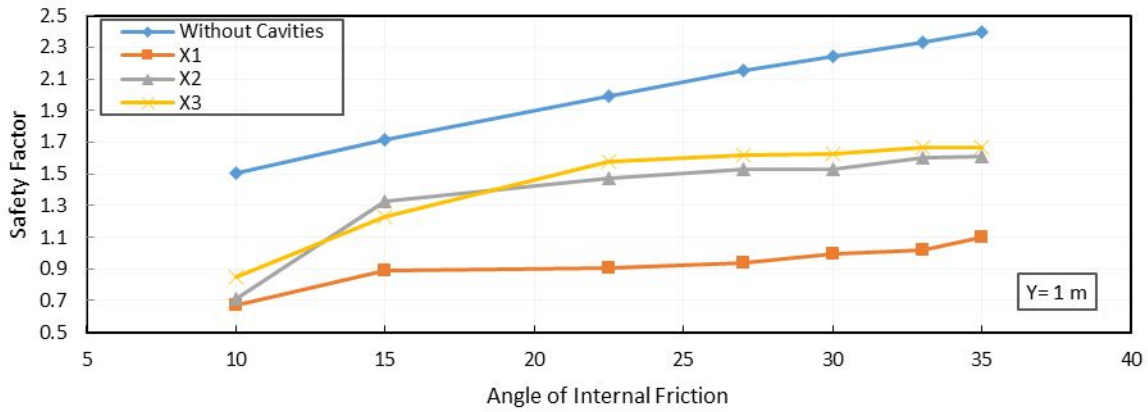


(b)

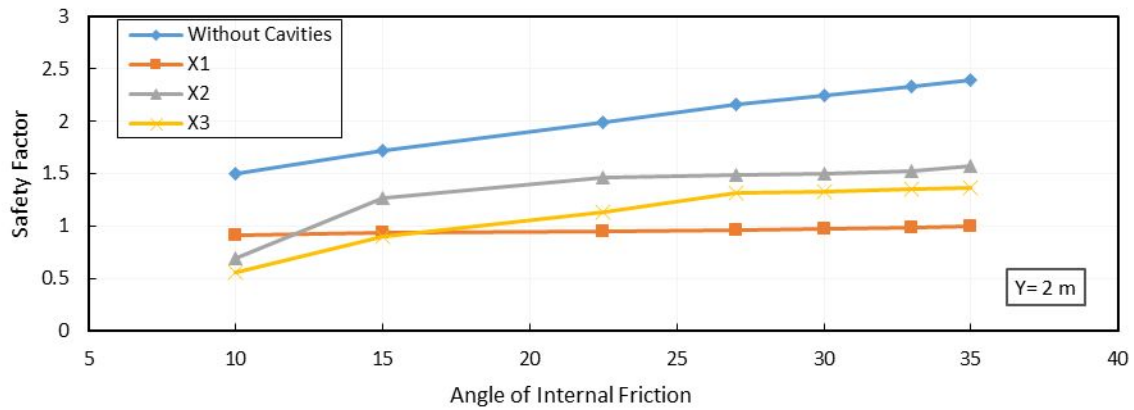


(c)

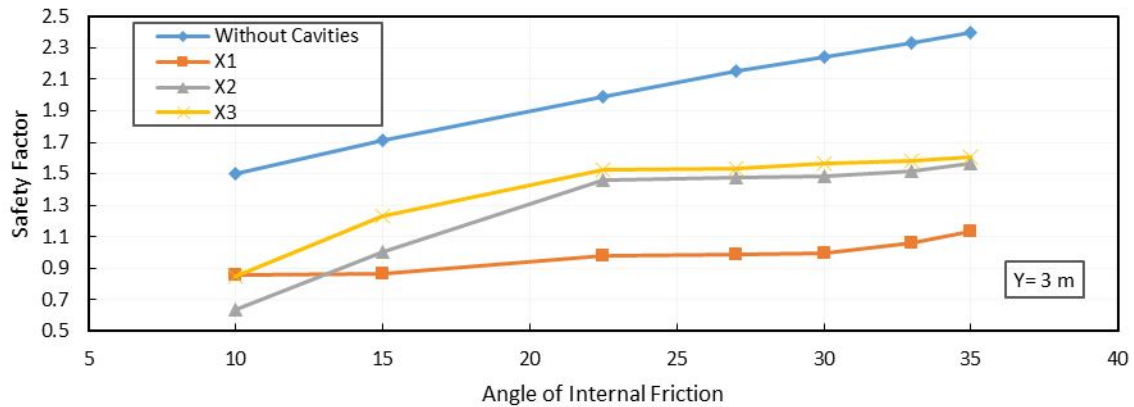
Figure 15. Combined effect of cavity diameter and soil cohesion on the factor of safety of upstream for various horizontal locations of cavities: (a) L 1, (b) L 2, (c) L



(a)

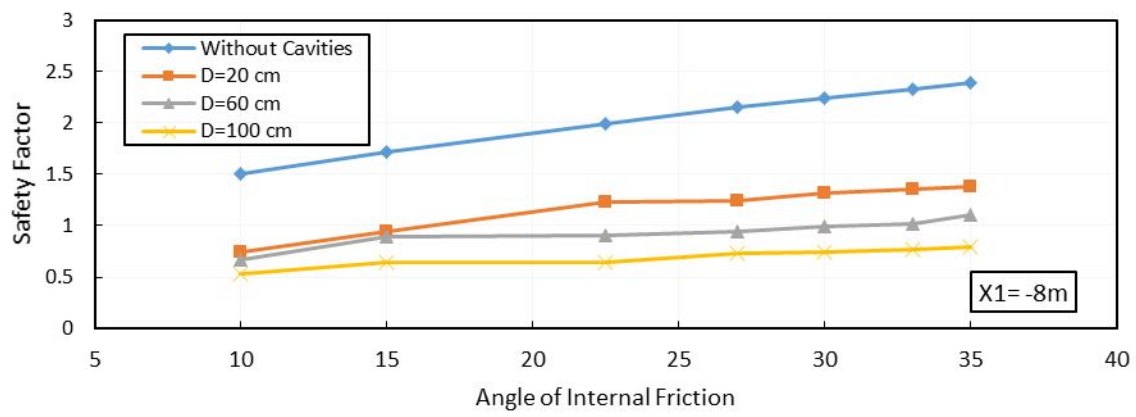


(b)

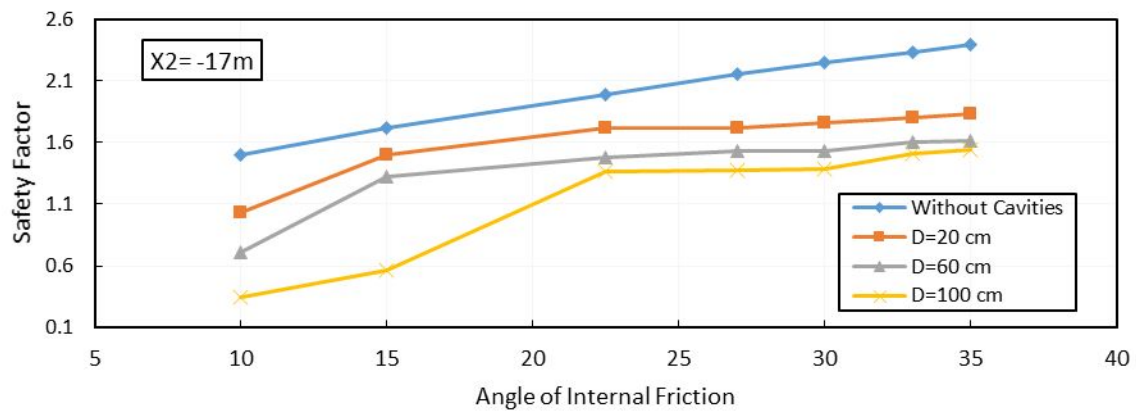


(c)

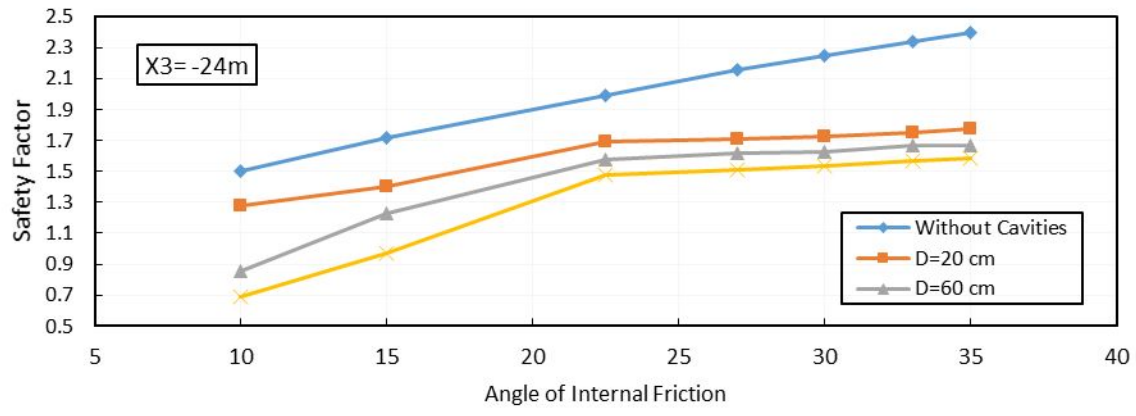
Figure 16. Combined effect of cavity location and angle of internal friction (unit: degrees) on the factor of safety of upstream for various depths: (a) Y=1 m, (b) Y=2 m, (c) Y=3 m



(a)



(b)



(c)

Figure 17. Combined effect of cavity diameter and angle of internal friction (unit: degrees) on the factor of safety of upstream for various horizontal locations of cavities: (a) L 1, (b) L 2, (c) L 3

4. Conclusions

The influence of cavity existence on earth dam slope stability under rapid drawdown condition was investigated and factors such as cavity horizontal position, depth and size were considered. In addition, a further analysis was conducted to look into the combined effects of presence, location and **diameter** of cavities with the variations in the shear strength parameters of the embankment on the stability. Below are the key conclusions from this study:

- Presence of cavities under the upstream side in the subsoil dramatically affects the stability of the slopes; however, this effect is not as significant when the cavities show under the downstream subsoil and does not seem to seriously threat the overall stability of the structure. The safety factor values significantly decreased by about **55.6%** at location L2 (upstream), compared to **22.8%** on the downstream side; whereas the values of maximum displacement increased by about **18.3%** on the upstream side, compared to **6.8%** on the downstream side as a result of cavity presence.
- Variations in the location of cavities in the horizontal direction is more influential on the safety factors than their location variation in the vertical direction, whether cavities are situated in the sub-soil of either upstream or downstream.
- In general, stability of the dam increases as the horizontal distance from the dam centreline to cavity centreline increases.
- Regardless of the location of the cavities, any increase in the **diameters** of available cavities significantly decreases the safety factors and drives the structure towards imminent failure.
- Slopes stability in the earth dam under rapid drawdown condition increases with increasing the soil cohesion and angle of internal friction; however, these increases are minimal compared to the considerable effects of the cavities on the stability and whilst introducing improvements, are unable to compensate for the significant disturbing effects of the cavities. This effect is even greater when the cavities are situated in locations with maximum effects on the stability.
- The safety factor in earth dam under rapid drawdown condition is more sensitive to the changes in the horizontal position of the cavities rather than the depth or the **diameter** with cavities closer to the geometrical centre line of the dam causing most significant stability challenges.

Acknowledgements

The research has been financed by the Government of Iraq via Kufa University as part of a PhD scholarship for the lead author.

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