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SIMULATING ENTROPY GENERATION IN SOLAR MAGNETOHYDRODYNAMIC HEAT DUCTS WITH ERINGEN'S MICROPOLAR MODEL AND BEJAN THERMODYNAMIC OPTIMIZATION

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ABSTRACT

Magnetohydrodynamic (MHD) solar power has recently been developed in the USA and is an exciting novel area in renewable power. In this hybrid solar energy design, high-powered magnets are employed to increase the efficiency of conversion from sunlight to electricity by stripping electrons from high-energy plasma jets and thereby generating power with no moving parts. The significantly higher temperatures generated in solar MHD have been shown to achieve much higher efficiencies than other conventional types of solar thermal technologies that work at a much lower temperature. The working fluids in solar MHD designs may be non-Newtonian and are electrically-conducting and strong thermal convection effects may also be present. To optimize thermal performance, Bejan's entropy generation minimization technique is a powerful approach. In the present poster we describe for the first time a novel analytical and computational model for entropy generation in magnetohydrodynamic non-Newtonian flows due to constant pressure gradient in a vertical-parallel plate channel as a simulation of an MHD solar power system. To more accurately simulate the rheological working fluid, the elegant Eringen thermo-micropolar material model is employed which features gyratory motions of micro-elements (suspended particles). This is a new approach to real fluids in solar MHD pumps. The normalized conservation equations are solved with the powerful Liao homotopy analysis method (**HAM**) with physically viable boundary conditions at the channel (duct) walls. Numerical computations are conducted in **MATLAB** symbolic software. The impact of selected parameters e.g. non-Newtonian couple stress parameter, Eringen micropolar parameter, Reynolds number, Grashof (thermal buoyancy) number, Hartmann magnetic number and Brinkman (viscous heating) number on thermofluid characteristics (velocity, temperature, Nusselt number) and on entropy generation number and Bejan number are studied. The prescribed ranges of parameters are physically representative of real magnetohydrodynamic solar energy systems employing non-Newtonian fluids. The computations show that increasing magnetic field effect reduces the entropy production at the channel walls, whereas the converse behaviour is observed for increasing couple stress parameter, Reynolds number, Grashof number and Brinkman number. Increasing Eringen micropolar parameter and Hartmann number are observed to decrease the entropy generation production in solar MHD systems. This aids designers in achieving thermally more efficient solar MHD duct performance.

INTRODUCTION

In recent years, engineers have verified that the entropy generation analysis via the Second Law of Thermodynamics (SLT) is more robust and accurate than via the first law of thermodynamics. It is established that thermal processes are inherently irreversible. There exists an entropy generation which destroys the available energy of a system. Entropy generation in thermal systems is mainly generated by heat transfer which occurs in different modes i.e., conduction, convection and radiation. In addition to these, additional effects including fluid friction (viscosity), buoyancy and magnetic field may also contribute to this entropy production. Entropy generation or irreversibility in flow systems was pioneered by **Bejan [1]**. The entropy generation analysis in ducts (e.g. parallel-vertical plate systems) have many applications in modern thermal engineering, including the cooling of nuclear reactors, industrial heat exchanger optimization, petroleum equipment performance enhancement, microelectronic devices, etc. micropolar fluid theory, an advanced sub-branch of rheology, has mobilized significant interest due to immense applications in engineering. **Eringen [2, 3]** proposed the theory of non-Newtonian **micropolar fluids** in the mid-1960s, as a simplification of its earlier and more general (and complex) micro-morphic fluid theory. This non-Newtonian fluid model sustains couple stresses, body couples and possesses a non-symmetric stress tensor. The micropolar fluid model has an independent rotational vector in addition to the velocity vector since the fluid particles undergo translational as well as rotational motions. This theory has provided a good model for studying a number of very sophisticated industrial fluids, e.g. polymers, suspension fluids, paints, liquid crystals, colloidal solutions, lubricating oils, propellants, physiological and environmental liquids. This theory can be used to accurately model solar MHD duct working fluids since it realistically captures the suspension nature of the fluent medium. Here we present numerical simulations for the effects of viscous dissipation and magnetic force on entropy generation in incompressible electrically-conducting thermo-micropolar fluid flow in a solar MHD duct. MHD laminar theory is used **[4]**. The governing nonlinear equations are non-dimensionalized and then solved subject to physically realistic boundary conditions have been solved using the Homotopy Analysis Method (HAM) **[5]**. The influence of various thermophysical and rheological flow parameters on linear velocity, microrotation, temperature and entropy related distributions are displayed graphically and interpreted in detail.

MATHEMATICAL MODEL

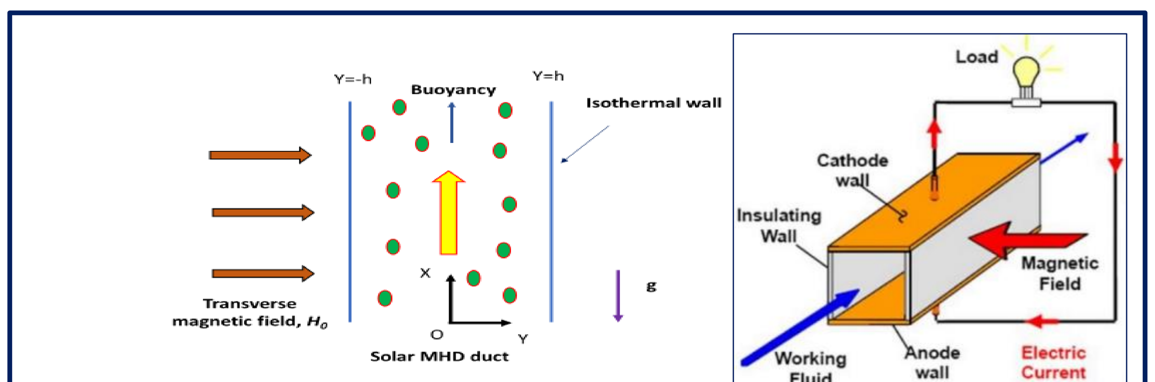

Fig. 1: Physical Model of solar MHD duct

Fig. 1 illustrates the physical model considered. Electrically-conducting incompressible micropolar fluid flows steadily under thermal buoyancy between two vertical isothermal plates of infinite length at temperature T_I and T_{II}. Take a co-ordinate system *(X, Y)*, where the *X*-axis lies along the direction of channel plates and the *Y*-axis is orthogonal to it. Assume that the fluid is flowing due to a constant pressure drop. A uniform static external magnetic field H_0 is applied in *Y*-direction and is orthogonal to the flow direction, which generates a Lorentzian magnetohydrodynamic drag force. Since the length of the channel plates is infinite, the velocity, microrotation and temperature are functions *Y* only. The fluid physical properties are constant except for density variations in the body force term where it is considered as a function of temperature. The governing fluid flow equations take the following form:

Momentum:

$$(\mu+\kappa)\frac{d^2U}{dY^2}+\kappa\frac{dM}{dY}-\frac{dP}{dX}+\rho\bar{g}b(T-T_w)-\sigma H_0^2U=0$$

Micro-rotation:

$$\gamma\frac{d^2M}{dY^2}-\kappa\frac{dU}{dY}-2\kappa M=0$$

Energy:

$$\left[\mu\left(\frac{dU}{dY}\right)^2+\kappa\left(\frac{dU}{dY}+2M\right)^2+\beta\left(\frac{dM}{dY}\right)^2+\sigma H_0^2U^2\right]+k\frac{d^2T}{dY^2}=0$$

Here β*,* γ are gyro-viscosity coefficients and μ, κ are the viscosity coefficients (Newtonian dynamic viscosity and Eringen vortex viscosity, respectively) of thermo-micropolar fluids.

MATHEMATICAL MODEL ctd

The equations are non-dimensionalized and the **non-dimensional ordinary differential equation boundary value problem** (**ODE BVP**) reduces to

$$\frac{d^2u}{dy^2}+c\frac{dM}{dy}+\frac{Gr}{Re}(1-c)\,\theta-Ha^2(1-c)u-\mathrm{Re}\,B(1-c)=0$$

$$\frac{d^2M}{dy^2}-s\frac{du}{dy}-2sM=0$$

$$\frac{d^2\theta}{dy^2}+Br\left[\left(\frac{du}{dy}\right)^2+\left(\frac{c}{1-c}\right)\left(\frac{du}{dy}+2M\right)^2+\delta\left(\frac{dM}{dy}\right)^2+Ha^2u^2\right]=0$$

Velocity: u(y)=0 at y = -1 and y = +1, (no-slip condition)
Micro-rotation: M(y)=0 at y = -1 and y = +1, (hyper-stick condition)
Temperature: θ (y) =0 at y = -1 and θ (y) = 1 at y = 1

Note- *s* is the couple stress parameter, c is the coupling number or Eringen micropolar vortex parameter (for Newtonian viscous flow c=0),Gr is the Grashof (natural convection) number, Re is the Reynolds number, Br=EcPr is the Brinkman number, Ec is the Eckert number, Pr is the Prandtl number, Ha^2 is the square of the Hartmann number (magnetic parameter),δ is rheological parameter, and B=dp/dx is the pressure gradient.

HOMOTOPY NUMERICAL SOLUTION

The coupled non-linear ODE BVP is solved with HAM. This method has garnered exceptional interest from researchers due to its enormous applications in engineering and science. Liao **[5]** introduced HAM in the 1990s and initially applied the method to viscous fluid dynamics problems. HAM has been subsequently utilized successfully to obtain solutions for a diverse range of multi-physical non-linear problems stagnation rotating nanofluid dynamics **[6]**. HAM has the attractive feature of not requiring small or large parameters (as with perturbation methods) and thus it can be adapted to solve non-linear problems without such restrictions. The method is a series based semi-numerical technique which achieves very high accuracy. Power series expansions are evaluated with symbolic software, e.g. MAPLE, MATLAB, MATHEMATICA etc. HAM further provides greater choice to select an auxiliary linear, non-linear operators and initial approximations. This method introduces a parameter known as homotopy embedding parameter (*q*), which assumes values from 0 to 1. When *q* = 0, the problem under study gets a simple form which gives us a closed form analytical solution for an initial guess satisfying boundary conditions. As *q* is increased and finally takes the value one, the exact solution to the actual problem is recovered. A significant advantage of this approach is that it is analytical. Also, this method uses two other parameters, a convergence controlling parameter (*h*) and a function, *H(y)*, the choice of which are selected to achieve an optimum solution. The **homotopy deformation equations** for velocity (u), micro-rotation (M) and temperature (θ) are defined as follows:

$$(1-q)L\left[u(y;q)-u_0(y)\right]=q\,h_u\,N_u\left[u(y;q)\right]$$

$$(1-q)L\left[\mathrm{M}(y;q)-\mathrm{M}_0(y)\right]=q\,h_{\mathrm{M}}\,N_{\mathrm{M}}\left[\mathrm{M}(y;q)\right]$$

$$(1-q)L\left[\theta(y;q)-\theta_0(y)\right]=q\,h_\theta\,N_\theta\left[\theta(y;q)\right]$$

Here *h* terms are non-zero auxiliary parameters and *L* is an auxiliary linear operator chosen as d^2/dy^2. Using power series expansions then the m^{th} order deformation equations are generated and solved to give the desired solutions for velocity, micro-rotation and temperature. Once the base solutions are obtained then gradient functions can be computed at the solar MHD duct wall e.g. **Nusselt number** in non-dimensional form defines the heat transfer characteristics at the walls and is written as:

$$Nu=-\frac{d\theta}{dy}\Big]_{y=\pm1}$$

BEJAN ENTROPY GENERATION ANALYSIS

From the known velocity, micro-rotation and temperature fields, the **volumetric rate of entropy** for a non-Newtonian micropolar fluid in the presence of magnetic field is given as:

$$S_{gen}=\underbrace{\frac{k}{T_0^2}\left(\frac{dT(Y)}{dY}\right)^2}_{\ge0}+\underbrace{\frac{\mu}{T_0}\left(\frac{dU(Y)}{dY}\right)^2}_{\ge0}+\underbrace{\frac{\kappa}{T_0}\left(\frac{dU(Y)}{dY}+2M(Y)\right)^2}_{\ge0}+\underbrace{\frac{\beta}{T_0}\left(\frac{dM(Y)}{dY}\right)^2}_{\ge0}+\underbrace{\frac{\sigma}{T_0}\left(H_0U(Y)\right)^2}_{\ge0}$$

On the RHS of the above equation, the **first term denotes the entropy due to heat conduction effect**, the **next three terms denote the viscous dissipation function** and the **last term denotes the irreversibility due to external magnetic field**. In order to calculate the exergy loss in the heat transfer, the ***entropy generation number Ns*** for micropolar fluid with non-dimensional quantities may be defined as follows (where *Ω* is the temperature difference and *Br/Ω* is the group (or viscous dissipation) parameter :

$$Ns=\frac{S_{gen}}{S_{gen,c}}=\left(\frac{d\theta}{dy}\right)^2+\left(\frac{Br}{\Omega}\right)\left[\left(\frac{du}{dy}\right)^2+c\left(\frac{du}{dy}+2\mathrm{M}\right)^2+\delta\left(\frac{d\mathrm{M}}{dy}\right)^2+Ha^2u^2\right]$$

The above Eqn. can be expressed as the sum of the irreversibilities due to heat conduction(Nh), fluid friction (viscous dissipation) (Nf) and magnetic field (Nm). To understand the entropy generation mechanisms, it is required to analyze the contribution of heat transfer to overall irreversibility. For this purpose, an alternative irreversibility parameter is introduced, known as the **Bejan number (Be)**, which is the ratio of irreversibility due to heat transfer to the total irreversibility and takes values between 0 and 1.

$$Be=\frac{Ny}{Ns}$$

When Be = 1 irreversibility due to heat transfer is dominates and Be = 0 represents the irreversibility due to fluid friction and magnetic field is dominant. It is clear when Be = 0.5, the contribution to entropy due to heat transfer is equal to the sum of magnetic and fluid friction effects. The total entropy generation number is expressed by integrating across the cross-sectional area of the duct, A:

$$S_G'=\int_A S_G\;h\;dA$$

In normalized form, the total entropy generation rate is obtained by integrating Ns over the channel width:

$$N_S'=\int_{-1}^{1}N_S\,dy=\int_{-1}^{1}\left(\left(\frac{d\theta}{dy}\right)^2+\frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2+c\left(\frac{du}{dy}+2\mathrm{M}\right)^2+\delta\left(\frac{d\mathrm{M}}{dy}\right)^2+Ha^2u^2\right]\right)dy$$

SELECTED NUMERICAL RESULTS

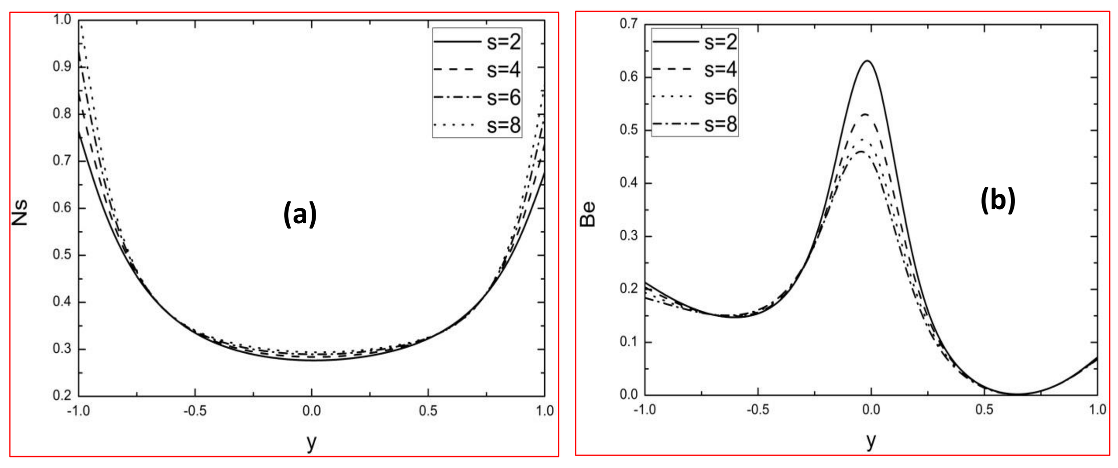

Figure 2: Dimensionless **a)** Entropy generation number and **b)** Bejan number versus *y* for various values of couple stress parameter (s) with B= -0.1, Br = 0.1, c =0.1, Gr = 0.2, Ha =0.8, δ =0.1 and Ω=1.

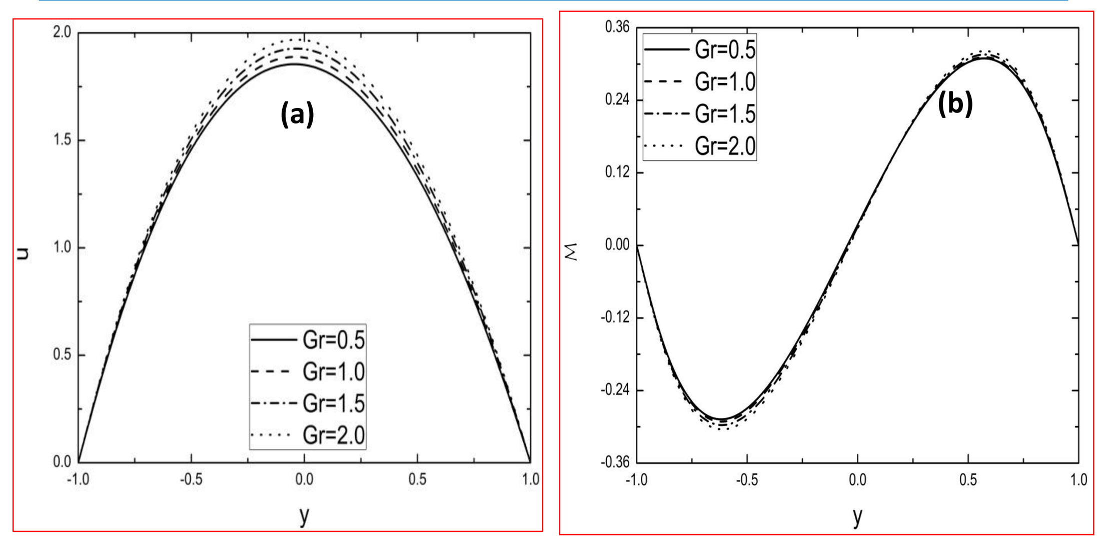

Figure 3: Dimensionless **a)** Velocity and **b)** Microrotation profiles versus position *y* for various values of *Gr* with B= -0.2, Br = 0.1, c =0.3, Ha =0.5, Re = 2, s = 2, δ =0.1

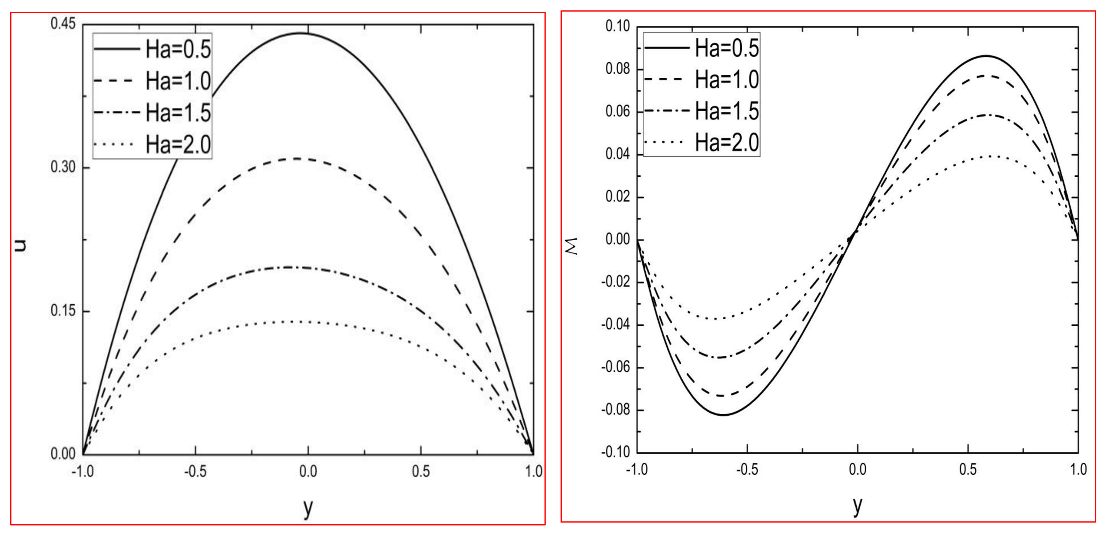

Figure 4: Dimensionless **a** Velocity and **b** Microrotation profiles versus position *y* for various values of *Ha* with B= -0.1, Br = 0.1, c =0.3, Gr =0.4, Re = 1, s =2, δ =0.1

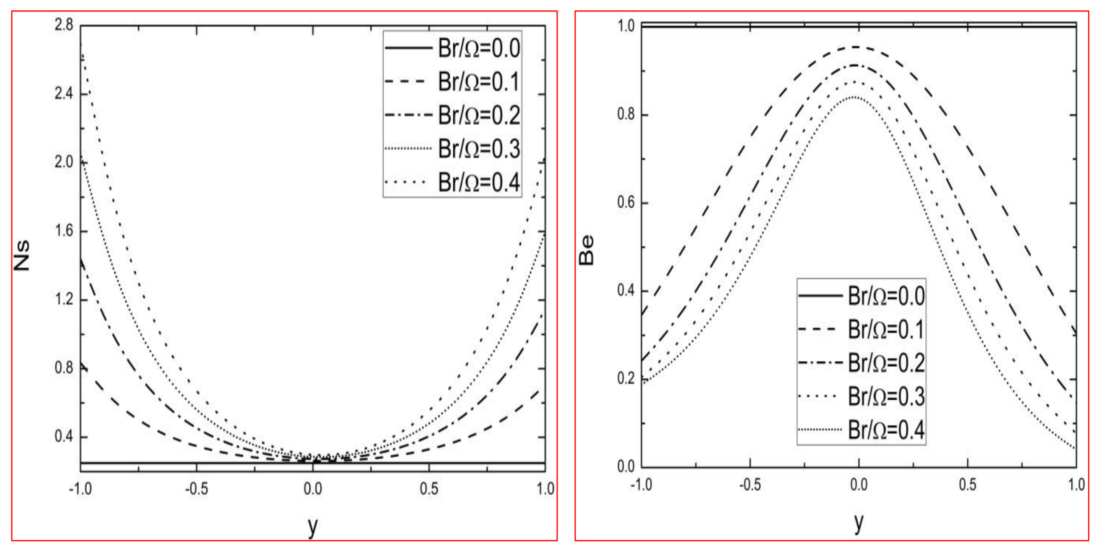

Figure 5: Dimensionless **a)** Entropy generation number and **b)** Bejan number versus position *y* for various values of Br/Ω with B= -0.1, c =0.1, Gr =0.2, Ha = 0.5, Re = 2, s =2, δ =0.1

CONCLUSIONS

Selected **HAM** computations are shown in **Figs 2-5a,b**. The main observations of the present study can be summarized as follows:

i) Increasing couple stress effect in the working fluid decreases the velocity.

ii) An elevation in micropolarity (vortex viscosity) parameter decreases the velocity in comparison with the Newtonian fluid case. Further, it is noticed that micropolarity parameter can be used to control the flow motion.

iii) The entropy generation production is maximum near to the solar duct plates as compared to that of the duct channel centre. This demonstrates that the frictional forces are dominant near the channel plates and these enhance entropy generation. Conversely, Bejan numbers have minimum values near to the plates and maximum values near to the channel centre.

iv) Bejan number is a maximum at the centre point of the channel. This reveals that the amount of available energy for work is more and irreversibility is less.

v) A strong increase in the entropy generation distribution (*Ns*) is noted with an increase in couple stress parameter (*s*), Grashof number (*Gr*), Reynolds number (*Re*) and group parameter (Br/Ω).

vi) The micropolarity parameter (*c*) and magnetic parameter (*Ha*) have decreasing effect on entropy generation production.

The present study has demonstrated the powerful ability of HAM in simulating entropy generation problems in non-Newtonian magnetohydrodynamics solar duct systems. However, it has neglected thermal radiative heat transfer effects **[7, 8]** which are presently under consideration with algebraic flux and differential models (Rosseland, P1, Chandrasekhar etc).

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