

Condition-based maintenance for a two-component system with **stochastic and economic** dependencies

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Abstract

This paper develops a model of a condition-based maintenance policy for a two-component system with both stochastic and economic dependencies. The stochastic dependency is such that the degradation rate of each component depends not only on its own state (degradation level) but also on the state of the other component. The economic dependency is such that combining multiple maintenance activities has lower cost than performing maintenance on components separately. To select a component or components to be preventively maintained, adaptive preventive maintenance and opportunistic maintenance rules are proposed. A cost model is developed to find the optimal values of decision variables. A case study of a gearbox system demonstrates the utility of the proposed model.

Keywords: Condition-based maintenance, maintenance optimization, two-component system, state dependence, stochastic dependence, economic dependence.

1 Introduction

Maintenance involves preventive and corrective actions that are carried out to retain a technical system in or restore it to an operating condition. Maintenance optimization aims to determine effective and efficient maintenance plans for each component of a system in order to meet

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operator requirements for safety, reliability and value. In the literature, many policies have been developed for the maintenance of single-component systems [8, 31]. Such maintenance policies may be applied to multi-component systems when the dependencies between components in these systems are neglected. However, for many technical systems it is not reasonable to assume components are independent, and it is necessary to model component dependencies. Dependencies can be classified into three main types [21, 6, 10]: (i) economic dependence, whereby the cost of joint maintenance of a group of components does not equal the sum of individual maintenance costs for these components; (ii) structural dependence, whereby components structurally form a part, so that maintenance of a failed component implies maintenance (at least the dismantling) of other unfailed components; (iii) stochastic dependence whereby the state of a component influences the lifetime distribution of other components. Recently, in [16], stochastic dependencies have been categorized in three groups: (i) failure-induced damage or failure dependence, whereby the failure of a component can lead to failure of other components; (ii) load sharing, whereby the failure of a component can increase the degradation rate of other components; and (ii) common-mode degradation, whereby an increase in degradation of one component is associated with an increase in degradation of other components. It is important to note that a fourth type of dependence, namely resource or logistical dependence, has been also identified in [16]. Indeed, resource dependence exists when several components are connected through a shared and limited set of spares. For example if a single repairman is responsible for the maintenance activities of various units or systems, or if a single stock of spare parts is used for the replacement of multiple units .

Taking into consideration dependencies between components when modelling maintenance of multi-component systems has recently shown an increase in popularity among researchers [4, 10, 12, 14, 21, 26]. An overview about recent advances on condition-based maintenance for systems with multiple dependent components is given in [16]. In fact, economic dependence has been investigated and integrated in a number of multi-component maintenance models [10, 18, 21, 28]. However, in these works, stochastic and structural dependence are not considered. Failure dependence between components has been studied in the context of inspection by [12] and maintenance and warranty optimization by [26, 35] for two-component systems. In the latter, several block replacement models considering both economic and failure interaction are proposed. Condition-based maintenance (CBM), in which the preventive maintenance decision is based on the observed system condition, has been introduced and has become an important model in maintenance optimization frameworks. Condition-based maintenance has also been

developed for two-component systems, see for example [2, 5, 18]. However, in such maintenance models, again only economic dependence is considered. Recently, a new type of stochastic dependence, called degradation interaction or state dependence, whereby the degradation evolution of a component depends on both its degradation level and that of other components, has been introduced in [3, 4] for prognostics of system lifetime, and in [23] for maintenance optimization. However, this latter work considers neither economic dependence nor intrinsic state dependence (whereby state evolution of a component depends on its own state). Thus, there is a need to consider multiple dependencies in CBM maintenance. With this in mind, this paper develops a model of condition-based maintenance that takes account of both stochastic dependence (intrinsic and extrinsic), through a model of rate-state interactions, and economic dependence that is natural when there is rate-state interaction.

In particular, we propose a CBM model for a two-component system with rate-state interaction, whereby the degradation rate of each component depends not only on its own state but also on the state (degradation level) of the other component. This dependence phenomenon can be found in a number of industrial systems, e.g. the state (quality) of oil may directly impact the degradation process of the crank and vice versa, wear on a pulley may impact the rate of wear of a belt and vice versa (Mark Maher, 2015, “Below the belt”, Maintenance Engineering, July/August, p.17), and likewise for chains and gears. In our model, we suppose that inspections occur at regular time intervals and identify the state of each component. Maintenance actions are then (optimally) planned based on the current, inspected state of the components, and broadly corresponds to a choice of: do nothing; replace component 1 and not component 2; replace component 2 and not component 1; or replace both. An interesting consequence of the rate-state interaction that we study is that when one component is replaced but not the other, obviously the system is not perfectly maintained (i.e. it is not renewed), but more interestingly the new component will deteriorate at a different rate to that when the system was new, because the degradation rate of the new component depends on the state of the old component. This partial replacement, or imperfect maintenance, of the system is then an imperfect “repair” that considers imperfect repair in a different way to the existing approaches in the literature, in which age/hazard reduction models predominate [8, 33, 34]. It is important to note that when considering state dependence between components, existing CBM models may lead to sub-optimal policies. This is because degradation modelling has a significant impact on the determination of optimal maintenance policy in CBM and, in these existing CBM models, state dependence is not yet considered. Therefore, an important contribution of this paper is to propose and develop

a CBM policy in which adaptive preventive maintenance and opportunistic maintenance rules select a component or group of components to be maintained, and in so doing to open a new strand of thinking in the modelling of imperfect maintenance. A cost model is developed to find the optimal maintenance policy. We argue that ignoring stochastic dependence will lead to a maintenance policy that is cost-inefficient. Thus, in our view, our model makes a contribution to the literature that will not only lead to further developments in maintenance optimization for systems with stochastic dependence but also be useful for practical application.

The paper is organized as follows. In the next section, we describe the model of the system and its dependencies. Both state and economic dependencies are herein investigated. Section 3 describes the proposed maintenance policy and the optimization process. To demonstrate the utility of the proposed maintenance policy, a case study of a gearbox system is introduced in Section 4. In the results we include sensitivity analyses. In the final section we presents our conclusions and discuss the managerial implications of the work.

Notation

a	cost-saving factor for joint replacement (when components 1 and 2 are replaced together)
b	duration-saving factor for joint replacement
C^∞	long-run expected maintenance cost per time unit (cost-rate)
$CS_{-,-}$	cost-saving of joint replacement
C_I	cost of an inspection
C_p^i, C_c^i	cost of preventive and corrective replacement of component i respectively
c_d	downtime cost-rate of the system
d_i	duration of a replacement for component i
T_i	the time of the i th inspection of the system
ΔT	inter-inspection interval
$x_{T_k}^i$	state (degradation level) of component i at time T_k
L^i	failure threshold of component i
m_p^i, m_o^i	preventive and opportunistic maintenance thresholds for component i
α^i, β^i	shape and rate parameter of gamma distribution associated with component i

2 System description and dependency modelling

Consider a series system with two dependent components. When one or both components fail the system fails. Each component i is subject to a continuous accumulation of degradation in time that is assumed to be described by a scalar random variable X_t^i . Component i is considered as failed if its degradation level reaches the failure threshold L^i , $i = 1, 2$. When a component is not operating for whatever reason, its degradation level remains unchanged during the stoppage period if no maintenance is carried out. We assume that on replacement of a component, the degradation level of the component is reset to zero. Thus, when the two components are replaced together, the system is returned to the "as new" state (renewal).

In our model, we will use the term replacement of a component to denote the maintenance action whereby the degradation level of the (replaced) component is reset to zero. In reality, such an action may not in fact be a replacement but instead a "repair". Nonetheless, the model will assume a repair and a replacement are synonymous.

2.1 State dependence modeling

Without maintenance interventions, we assume that evolution of the degradation level of component i is denoted by

$$X_{t+1}^i = X_t^i + \Delta X_t^i, \quad (1)$$

where ΔX_t^i is the increment in the degradation level of component i during one time unit (from t to $t + 1$). For two components that are deteriorating in a dependent manner, we suppose that the increment ΔX_t^i has two contributing terms: one that arises intrinsically in the component; and another that is due to (caused by) the degradation level of the other component. In that way, we suggest a general stationary model:

$$\Delta X_t^i = \Delta X_t^{ii} + \Delta X_t^{ji} \text{ with } i, j = 1, 2 \text{ and } (i \neq j), \quad (2)$$

where ΔX_t^{ii} and ΔX_t^{ji} are such that:

- ΔX_t^{ii} is the increment in the degradation level of component i induced by itself during one time unit, namely the intrinsic effect. This means that ΔX_t^{ii} depends only on the state of component i at time t . ΔX_t^{ii} may be specified as deterministic or as a random variable.
- ΔX_t^{ji} is the increment in the degradation level of component i induced by component j during one time unit, called the interaction effect. ΔX_t^{ji} represents the state interaction

between the two components j , i and may be specified as deterministic or as a random variable.

Several variants of the proposed model can be specified:

Case 1 $\Delta X_t^{ii} > 0$ and $\Delta X_t^{ji} = 0$: no interaction effect and the proposed model becomes a basic model describing the degradation behavior of independent components, see for instance [20, 27, 29].

Case 2: $\Delta X_t^{ii} = 0$ and $\Delta X_t^{ji} > 0$, here the two components are stochastically dependent but the increment in the degradation level of component i depends only on the state of the other component. For this case, the proposed model corresponds to the model introduced in [24] where the interaction effect (ΔX_t^{ji}) is described by a normal distribution whose parameters depend on the degradation level of component j .

Case 3: $\Delta X_t^{ii} > 0$ and $\Delta X_t^{ji} > 0$, the two components are stochastically dependent and the increment in the degradation level of component i may depend not only on the state of component i but also on the state of the other component.

As an example, a special case has been studied in [7]. Indeed, therein, the random intrinsic effect i ($i = 1, 2$) ΔX_t^{ii} is assumed to follow a Gamma probability density (pdf) with shape parameter α^i and scale parameter β^i (see Appendix A for more details). The interaction effects (ΔX_t^{ji}) are non-linear functions of the degradation level of other components $\Delta X_t^{ji} = \mu^j \cdot (X_t^j)^{\sigma^j}$ where μ^j, σ^j are non-negative real numbers that quantify the influence of component j on the degradation rate of component i . Figure 1 illustrates the deterioration evolution of the two dependent components. Note that when the deterioration of a component reaches its failure threshold, the component fails. We suppose that the failed component can be immediately replaced, i.e. its deterioration level is reset to zero. The results in Figure 1 show that the interaction effect has an important influence on the degradation process of components

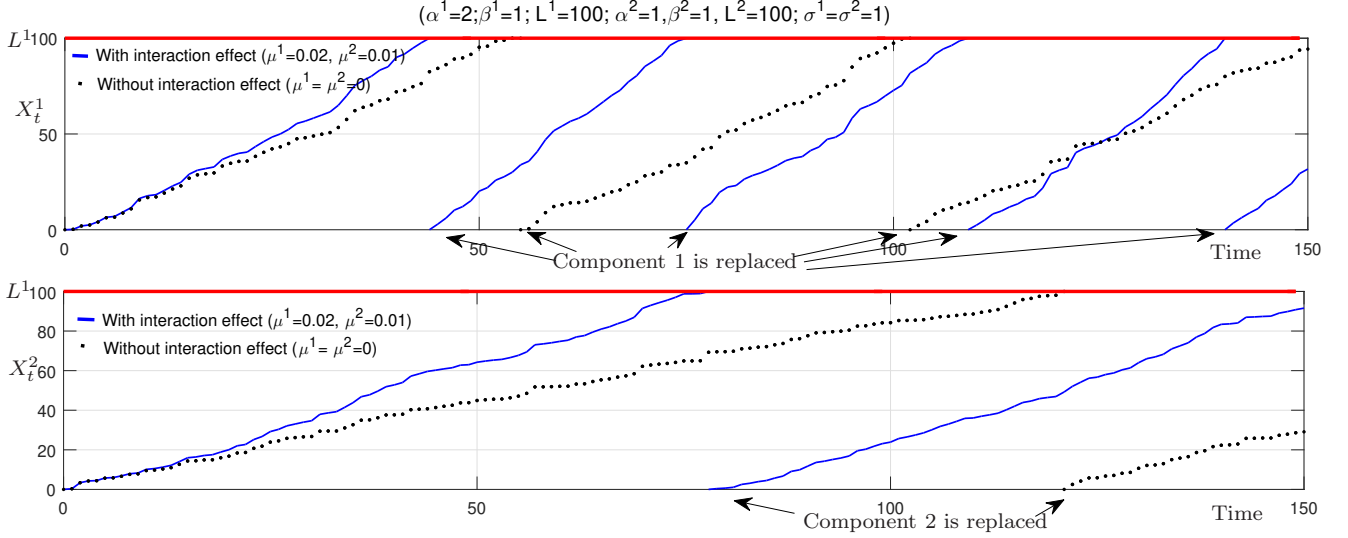


Figure 1: Illustration of the deterioration evolution of two dependent components

It should be noted that the case 3 has been studied in [3] for prognostics of system lifetime, in which a random intrinsic effect is considered and described by a Brownian motion process.

In the present paper, the use of the model (case 3) will be demonstrated and investigated using a case study of a gearbox system consisting of two gears which is partially presented in [1], see Section 4.

2.2 Economic dependence modelling

All necessary maintenance resources (such as spare parts, maintenance tools, repairmen) that are required to execute maintenance actions are assumed always available at a planned inspection time. It is also assumed that maintenance actions (replacements and inspections) are carried out at discrete times. Replacements may be corrective (that is on failure of the system) or preventive (prior to system failure) and that in the standard manner the costs differ in the two cases.

2.2.1 Individual maintenance costs

If a preventive replacement is individually carried out, a preventive cost is then incurred. In a general way, the preventive cost of component i , denoted C_p^i , can be divided into two parts: $C_p^i = c_p^i + c_d \cdot d_i$ where $c_d \cdot d_i$ is the downtime cost due to production loss during replacement that takes d_i time units, and c_p^i includes all other costs (spares, labour, set-up).

In the same manner, the cost of corrective replacement of component i is $C_c^i = c_c^i + c_d \cdot d_i$,

$(c_c^i \geq c_p^i)$.

Note, by preventive replacement of a component, we mean the replacement of a component when it is unfailed, and by corrective replacement of a component, we mean the replacement of a component when it is failed. Full details of the maintenance policy follow in section 3.

2.2.2 Economic dependence and cost saving

When two components are simultaneously replaced, total maintenance cost can be reduced [21, 10, 32]. In our model, this cost saving arises from the sharing of the replacement set-up cost and the reduction of replacement duration. In this way, we define the cost-saving of joint replacement as

$$CS_{-, -} = a \cdot (c_-^1 + c_-^2) + b \cdot (d_1 + d_2) \cdot c_d, \quad (3)$$

where:

- c_-^i ($i = 1, 2$) could be either c_p^i or c_c^i , i.e. preventive or corrective;
- a ($0 \leq a < \min(c_-^1, c_-^2)/(c_-^1 + c_-^2)$) is the cost-saving factor for joint replacement of two components. As is shown in [32], that the cost saving is typically equal to 5% of the total replacement cost of the components ($a = 0.05$);
- b ($0 \leq b \leq \min(d_1, d_2)/(d_1 + d_2)$) is the duration-saving factor for joint replacement.

In this way, a, b express the economic dependence degree between the two components. When $a = 0$ and $b = 0$, the two components are economically independent. The larger are a and b , the stronger is the economic dependence between the two components. Note, the effect of economic dependence on the availability of a system is studied in [9].

It is important to note that, in our paper, the economic dependence is positive ($CS_{-, -} \geq 0$). However, in parallel or complex structure systems where a failure of a component or a group of group of components may not lead to a failure of the system, the economic dependence may be positive or negative, see [19, 30].

In our paper, the elements of the economic dependence are integrated into an opportunistic maintenance model that is described next.

3 Maintenance policy

We assume that the degradation level of each component is measured at an inspection that is instantaneous, perfect, and non-destructive. An inspection incurs a cost c_I . A failure of

a component is assumed to be instantaneously revealed by a self-announcing mechanism, but that replacement can commence only at the next inspection. In this way, the usual practical requirement to prepare for a replacement is modelled while the system downtime due to failure is known.

3.1 Description of the proposed maintenance policy

We assume that the two components of the system are inspected at regular time intervals with inter-inspection interval ΔT . Note that ΔT is a decision variable which is to be optimized. More precisely, for each component i ($i = 1, 2$), the degradation level at inspection times $T_k = k \cdot \Delta T$ ($k = 1, 2, \dots$) is $X_{T_k}^i = x_{T_k}^i$. The maintenance policy is as follows. For $i = 1, 2$:

- if component i fails between (T_{k-1}, T_k) (when its degradation level reaches the failure threshold L^i), then it is replaced at time T_k ;
- if at time T_k , component i is still functioning, it is inspected. Based on the inspection results and the preventive maintenance rules, a decision about whether or not component i should be replaced at time T_k will be taken. We specify rules for individual preventive replacement and for opportunistic preventive replacement.

Individual preventive replacement If the degradation level of component i ($i = 1, 2$) at time T_k is greater or equal to a fixed threshold m_p^i ($x_{T_k}^i \geq m_p^i$), component i is immediately replaced. m_p^i , called the preventive threshold of component i , and is a decision variable to be optimized.

Opportunistic replacement The main idea of the proposed opportunistic replacement model is to capitalize on both the economic dependence and the stochastic dependence between the two components. The economic dependence is manifest in the shared set-up and the cost-saving therein. The stochastic dependence, through the term ΔX^{ij} in equation (2), may also incentivise (depending on the strength of the dependence) joint replacement. To this end, for each component i , an opportunistic threshold, denoted m_o^i ($0 < m_o^i \leq m_p^i$), is introduced. The opportunistic maintenance decision rule is the following. If component j ($j = 1, 2$ and $j \neq i$) is correctively replaced or selected to be preventively replaced at time T_k , component i is preventively replaced together with component j if the degradation level of component i is such that $x_{T_k}^i \geq m_o^i$. The latter implies that the system is renewed at time T_k . m_o^i ($i = 1, 2$) is also a decision variable that must be optimized.

An illustration of the proposed opportunistic maintenance policy is shown in Figure 2.

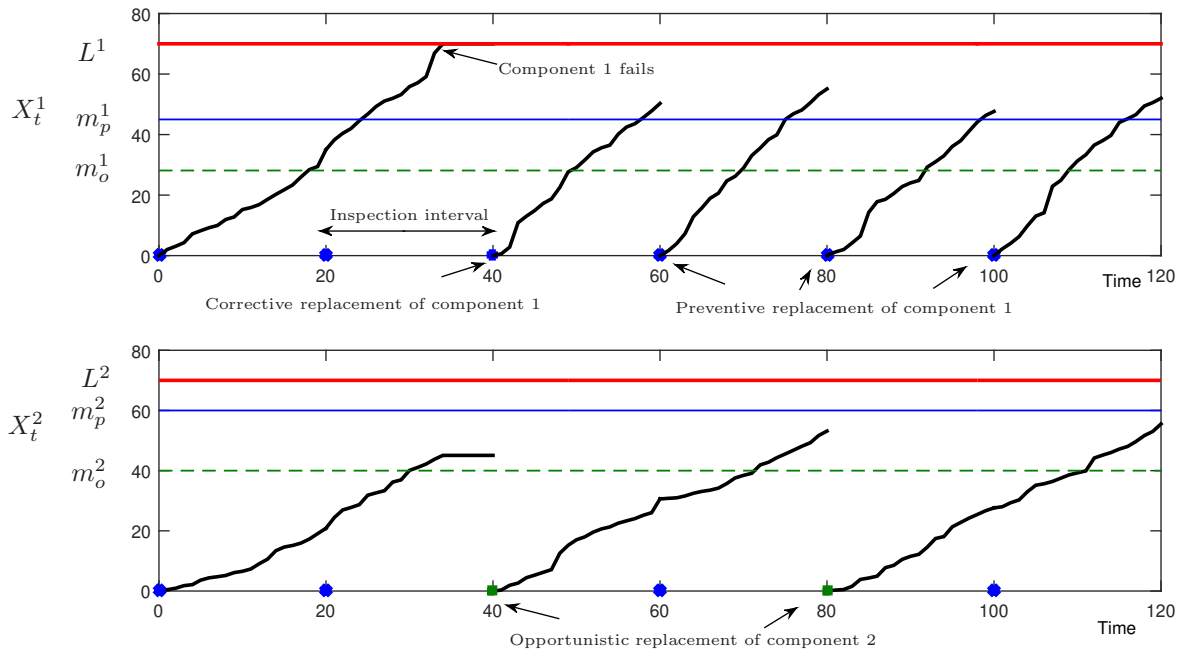


Figure 2: Illustration of components' degradation evolution and the proposed maintenance policy

This general policy we label policy V. To study the impacts of opportunistic replacement, two special cases of this policy are herein considered as follows

- When $m_p^1 = m_o^1$ and $m_p^2 = m_o^2$, there is no opportunistic replacement, the policy becomes a classical condition-based maintenance policy [19] with discrete inspections, which we call policy V1;
- When $m_o^1 = m_o^2 = 0$, two components are jointly replaced together, the proposed policy becomes a joint replacement policy, which we call policy V2.

To investigate the effects of economic and stochastic dependence, we compare the cost-rates of these three policies V, V1 and V2 in Section 4.4.

3.2 Optimization of the proposed maintenance policy

As described, $(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$ are the decision variables of the general opportunistic replacement policy that we study. Their optimal values must be determined, given some suitable criterion. For this purpose, a cost model is developed in this section. In particular, we use the long-run expected cost per unit of time (or cost-rate) including replacement and inspection costs.

The cost-rate is defined generally as:

$$C^\infty(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2) = \lim_{t \rightarrow \infty} \frac{C^t(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)}{t}, \quad (4)$$

where $C^t(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$ is the cumulative total maintenance (replacement and inspection) cost in period $(0, t]$. According to the renewal theory [25], Eq. (4) can be rewritten as follows:

$$C^\infty(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2) = \frac{\mathbb{E}[C^{T_{re}}(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)]}{\mathbb{E}[T_{re}]}, \quad (5)$$

where $\mathbb{E}[\cdot]$ is mathematical expectation and T_{re} is the length of the first renewal cycle of the system, i.e. all components of the system are replaced at time T_{re} . Without losses of generality, we assume that $T_{re} = \Delta T \cdot m$ (m is a positive integer), and so we get:

$$C^{T_{re}}(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2) = \frac{\sum_{k=1}^m (C_{ins}^k + C_{main}^k) + T_{down} \cdot c_d}{m \cdot \Delta T},$$

with:

- $C_{ins}^k = u \cdot c_I$ with u ($u = 0, 1, 2$) being the number of components inspected at T_k , noting that failed components are not inspected;
- $C_{main}^k = C_p^1 + C_p^2 - CS_{p,p}$ if the two components are jointly, preventively replaced; $C_{main}^k = C_p^i$ if only component i is preventively replaced; $C_{main}^k = C_p^i + C_c^j - CS_{p,c}$ if component i is preventively replaced and component j ($j \neq i$) is correctively replaced; $C_{main}^k = C_c^i$ if only component i is correctively replaced and $C_{main}^k = 0$ if no replacement is performed at T_k .

Obtaining a closed-form expression for the cost-rate in Equation (5) is very difficult or even impossible. In [13], an efficient method based on semi-regenerative theory and Markov decision processes is introduced to obtain a closed-form expression for the cost-rate of a single-unit system with time-homogeneous degradation behavior. In [5], a similar technique is developed to calculate the cost-rate of a two-component system with time-homogeneous and independent degradation behavior. However, this analytical method cannot be applied when there is degradation interaction between components. This is because, in presence of degradation interaction between components, the components's degradation process are dependent and non-longer time-homogeneous. As a consequence, semi-regenerative theory and Markov decision processes cannot be applied. Therefore, in our paper, the cost-rate is evaluated, given $\Delta T, m_p^1, m_o^1, m_p^2, m_o^2$, using Monte Carlo simulation. By varying the values of the decision variables and performing an exhaustive, the minimum cost-rate can be identified.

$$C^\infty(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*}) = \min\{C^\infty(\cdot)_{0 < \Delta T, 0 < m_p^1 \leq L^1, 0 < m_o^1 \leq m_p^1, 0 < m_p^2 \leq L^2, 0 < m_o^2 \leq m_p^2}\}. \quad (6)$$

4 Case study

Gearbox systems play an essential role in industrial machinery. They are widely used for torque and speed conversion. Unforeseen gearbox failures cause downtime and production inefficiency, leading to economic losses, and in some cases may have serious implications for safety. With multiple interacting components, we would expect the degradation trajectories of each of the components of a new gearbox, whereby all components are new, to be different to those of a partially new gearbox, whereby some components are new. This stochastic dependence, and the economic dependence arising from shared set-up costs, mean that an opportunistic maintenance policy is appropriate. Therefore, in what follows, we show how the opportunistic replacement policy can be i) optimized and ii) used in practice. In particular, we study an accelerated-life testing platform for a gearbox shown in Figure 3. This platform provides experimental data for modelling the interacting degradation trajectories of two components in this system.

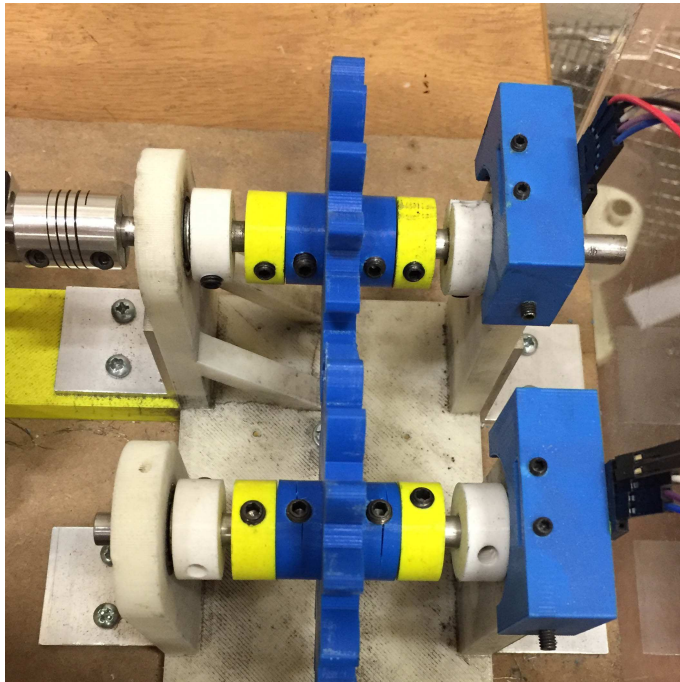


Figure 3: Gearbox system consisting of two interacting gears

4.1 Gearbox experimental scenario

The platform is driven by a DC motor running at 1200 RPM, and the load is provided via a dynamometer system. The vibration signals of the gearbox were collected using accelerometers and a data acquisition card which then transmitted the data to a PC workstation where they were stored and processed.

Data were collected in an experiment with three runs as follows:

Run 1: The first run of the gearbox was conducted until high system vibration was noticed. Destructive pitting occurred on the teeth surface of gear 1. It was therefore replaced with a new gear, while gear 2 showed signs of initial pitting and was not replaced.

Run 2: The second run of the gearbox was then conducted until there was high system vibration. The development of high vibration occurred over a shorter period of time. Destructive pitting occurred in gear 1, so it was replaced with a new gear. The condition of gear 2 showed more significant pitting in the second run but it was not replaced.

Run 3: The third and last run was then conducted until there was high system vibration. This occurred over an even shorter period of time than in the first two runs. The gearbox was then stopped.

The degradation trajectories are shown in Figure 4.

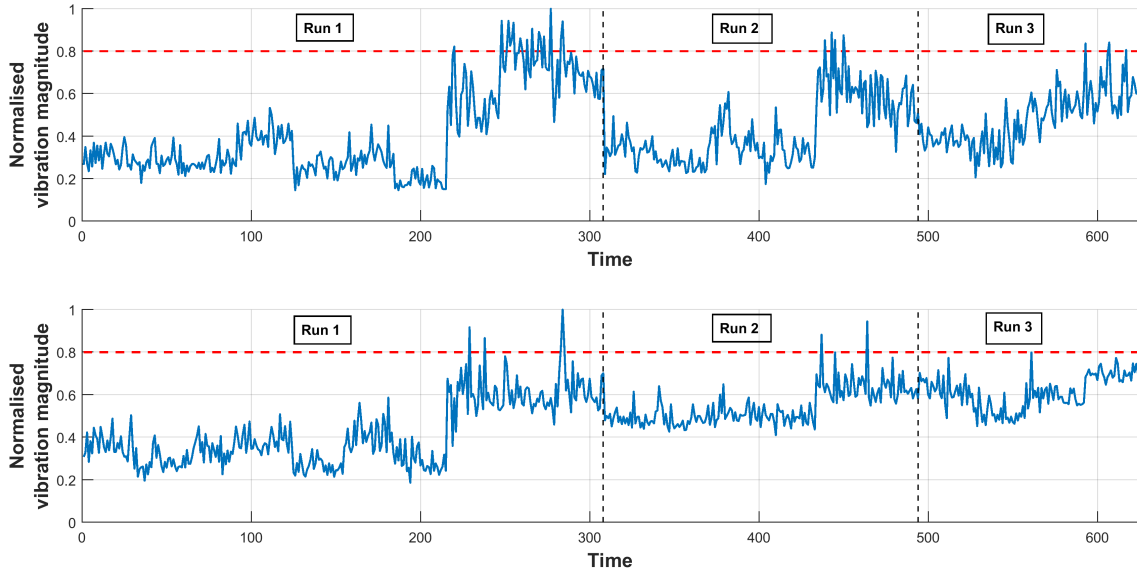


Figure 4: Evolution of degradation of the gears in all three runs, represented by the mesh frequency vibration magnitude

Some data processing of the vibration signals was required to arrive at the data in Figure 4. We refer to gear 1 and 2 as component 1 (C1), and component 2 (C2) respectively. After acquiring the vibration signal, we applied a short time Fourier transform on the time waveform data and obtained spectrograms of the signals for C1 and C2. Then, using a dynamic frequency band placed around the frequency of interest, namely the gear meshing frequency, we obtain

an array of magnitudes for which we compute the root mean square (RMS). The RMS values obtained over the full spectrogram summarise the time series data and are normalized and shown in Figure 4. The RMS values inform us of the vibration energy in the machine that originates from the gears. The higher the vibration energy the more the gears are deteriorated and the more prone the gearbox is to damage. This prones of a gear to damage is the manifestation in reality of the terms ΔX_t^{ii} and ΔX_t^{ij} in the model (equation 2), the former because the gear itself is worn, and the latter because the other gear is worn. We consider a component to be severely worn out or to have failed once it reaches the threshold vibration magnitude of $L^i = 0.8$ with $i = 1, 2$.

It should be noted that the real data are scaled and all parameters are given in arbitrary units, either arbitrary cost unit (acu) or arbitrary time unit (atu).

The inspection cost is 10 acu ($c_I = 10$). When each gear is individually replaced, the replacement cost and the maintenance duration are $c_p^1 = c_c^1 = 500$ acu, $c_p^2 = c_c^2 = 600$ acu and $d_1 = d_2 = 1$ atu. When both gears are replaced together, 5% of the total replacement cost of the components is saved ($a = 0.05$) and the total maintenance duration is reduced by 50% ($b = 0.5$). In addition, when the system fails we have to pay 100 acu per downtime unit ($c_d = 100$). The downtime cost (due to system failure) is taken to be the (negative of the) average of the output performance over the period of observation of the system, although in principle the downtime cost could be specified in other ways.

4.2 State interaction modelling

Due to the physical characteristics of the gears, we know that the degradation level of components C1 and C2 increases with time, and that this degradation level cannot decrease without maintenance intervention. Therefore, both components are considered to have inherent degradation that increases with time. We therefore assume that these degradation increments are gamma-distributed (see Appendix A for more details). These increments are denoted by ΔX_t^{11} and ΔX_t^{22} for C1 and C2 respectively. Thus, $\Delta X_t^{11} \sim \Gamma(\alpha^1, \beta^1)$ and $\Delta X_t^{22} \sim \Gamma(\alpha^2, \beta^2)$.

Next, we model the degradation interactions between the two components. It appears (Figure 4) that the state of C2 affects the rate of degradation of C1. This can be seen when we observe the time to failure of C1 when coupled with a worn out C2 in runs 2 and 3, and that in run 3, where C2 was more worn out, the time to failure of C1 was shorter than run 2. Thus the degradation rate of C1 appears to be dependant on the degradation level of C2 and vice versa. ΔX_t^{21} is used to denote the increment in the degradation level of C1 due to C2, and ΔX_t^{12} the

increment in the degradation level of C2 due to C1.

We denote the degradation states for C1 and C2 at time t by X_t^1 and X_t^2 respectively. Further, since in our model $\Delta X_t^{ii} > 0$ and $\Delta X_t^{ji} > 0$, the state dependence model is as case 3 in section 2.1. Thus the evolution of degradation for C1 is described by:

$$\begin{aligned} X_t^1 &= X_{t-1}^1 + \Delta X_t^1, \\ \Delta X_t^1 &= \Delta X_t^{11} + \Delta X_t^{21}, \\ \Delta X_t^1 &= \Gamma(\alpha^1, \beta^1) + \mu^1 \cdot (X_{t-1}^2)^{\sigma^1}. \end{aligned}$$

and for C2 as:

$$\begin{aligned} X_t^2 &= X_{t-1}^2 + \Delta X_t^2, \\ \Delta X_t^2 &= \Delta X_t^{22} + \Delta X_t^{12}, \\ \Delta X_t^2 &= \Gamma(\alpha^2, \beta^2) + \mu^2 \cdot (X_{t-1}^1)^{\sigma^2}. \end{aligned}$$

Note that there exists four parameters that need to be estimated to describe the degradation of each component. These parameters are denoted by Θ^1 and Θ^2 , where $\Theta^1 = (\alpha^1, \beta^1, \mu^1, \sigma^1)$ and $\Theta^2 = (\alpha^2, \beta^2, \mu^2, \sigma^2)$. In order to fit the degradation model to the data, we use the particle filter (PF) method [11]. PF allows for an online numerical estimation of the parameter values by means of a recursive Bayesian inference approach. The posterior distribution of the model parameters can be then obtained using a number of particles and their corresponding weights. This method is very flexible and can be used for non-linear models where the noise is not necessarily Gaussian. Such an approach has been successfully used in the field of prognostics for model parameter estimation [17, 22, 36]. The choice of PF is further motivated by the fact that it is now considered a state of the art technique for performing diagnostics and prognostics [15].

To estimate Θ^1 and Θ^2 using PF, we start by specifying a number of particles n_p to use. In our case we set $n_p = 1000$. Each particle is then associated with a value for each of the four parameters, by sampling from a prior distribution. We use uniform distributions on the following intervals: $U_\alpha[0,0.1], U_\beta[0,0.1], U_\mu[0,0.1], U_\sigma[0,10]$. The choice of these intervals is made empirically, keeping in mind that a larger n_p means that different combinations of these parameters are more effectively explored. However, this would entail a larger computational cost. After assigning each particle with its parameter values, we then apply these parameters to the degradation model, and use the model to generate a prediction of the next health condition of

the component $\tilde{X}_t^{i,n}$ for $n = 1 : n_p$. Then, after observing the actual health condition y_t^i we can determine the importance weight of each particle by computing the likelihood of that observation given the predicted values of each particle $p(y_t|\tilde{X}_t^{i,n})$. The weights are then normalized and bootstrap-importance sampling (re-sampling with replacement n_p particles from the previous set of particles according to their weights) is performed. This process is repeated using the new set of particles, as shown in Algorithm 1. After 308 iterations, we obtain the mean estimated value of each parameter (Table 1). Note that since the degradation level is normalized between 0 and 1, the greater is the value of the parameter b^i the smaller is the impact of the other component on component i .

Algorithm 1: Particle Filter Algorithm

input : n_p number of particles

Initialisation

$t = 0$

for $i \leftarrow 1$ **to** n_p **do**
 | Sample $x_0^i \sim p(x_0)$
end

for $t \leftarrow 1$ **to** t_{end} **do**
 | *Importance Sampling*
 for $i \leftarrow 1$ **to** n_p **do**
 | Sample $\tilde{x}_t^n \sim p(x_t|x_{t-1}^i)$
 | Set $\tilde{x}_{0:t}^i = (x_{0:t-1}^i, \tilde{x}_t^i)$
 end
 for $n \leftarrow 1$ **to** n_p **do**
 | Evaluate importance weights $\tilde{w}_t^n = p(y_t|\tilde{x}_t^n)$
 end
 Normalise importance weights \tilde{w}_t^n
 | *Particle Selection*
 for $n \leftarrow 1$ **to** n_p **do**
 | Considering \tilde{w}_t^n , re-sample with replacement n_p particles
 end
end

Component	α^i	β^i	μ^i	σ^i
C1	0.0233	0.0425	0.0995	7.6659
C2	0.0125	0.0914	0.0493	9.7375

Table 1: Estimated parameter values

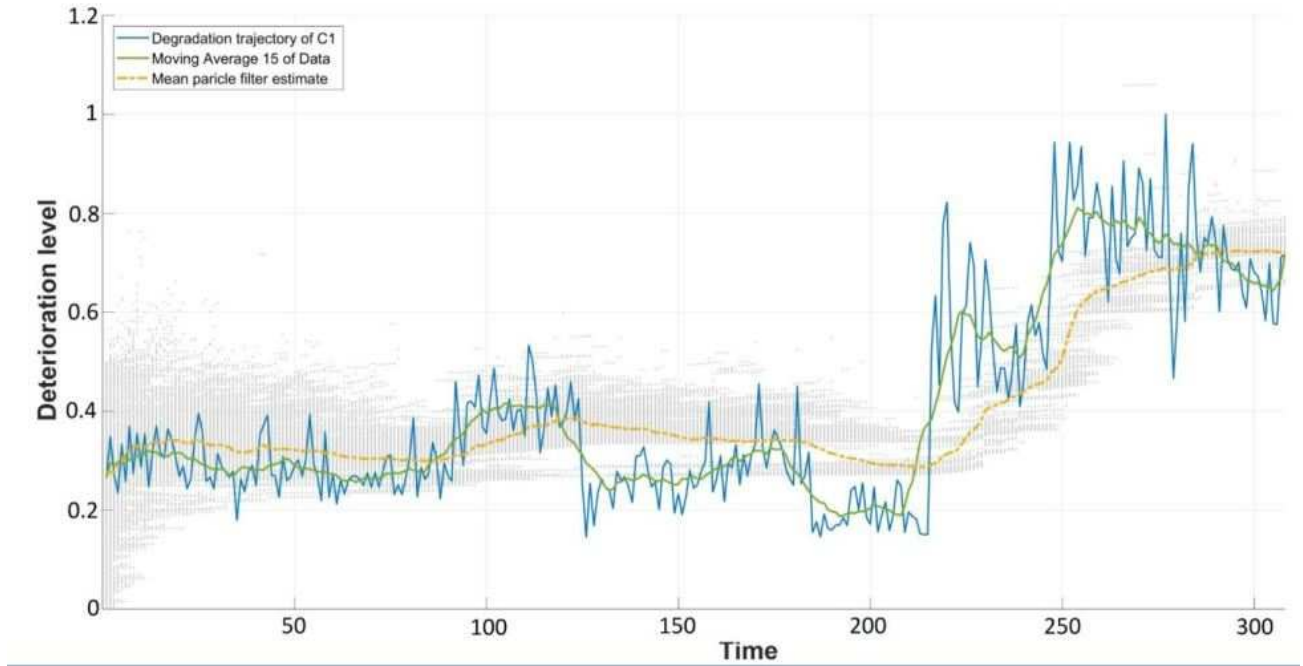


Figure 5: Fit of particle filter estimates to degradation data of component 1 in run 1

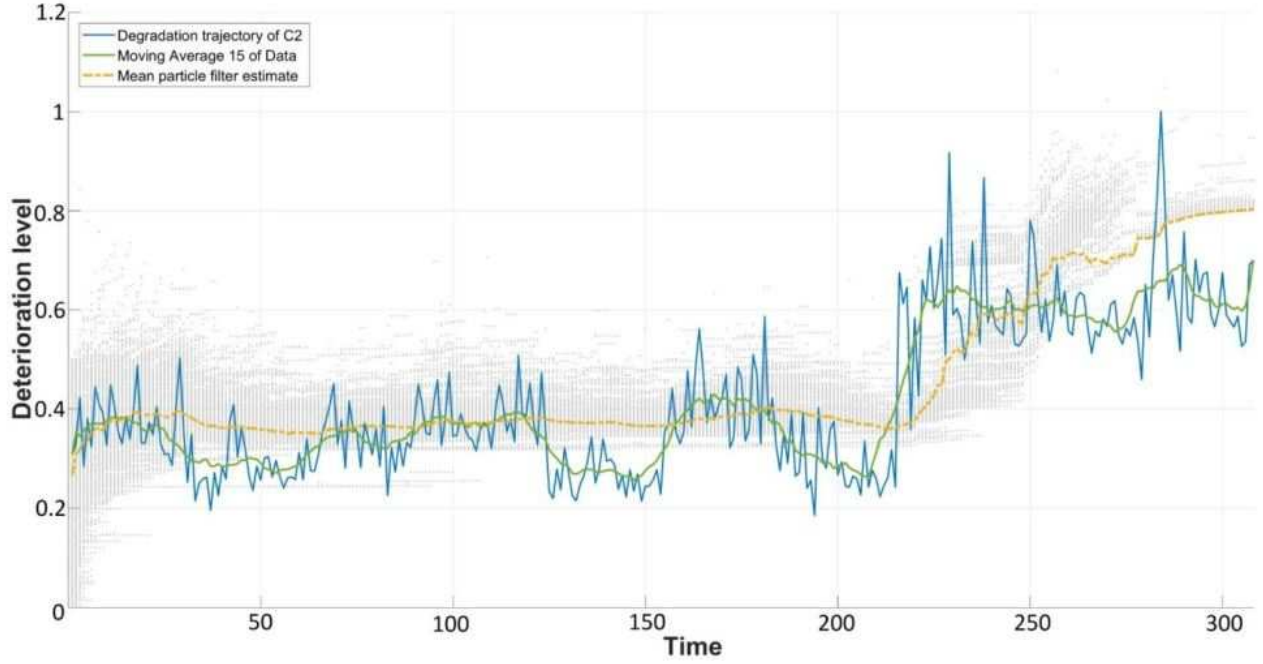


Figure 6: Fit of particle filter estimates to degradation data of component 2 in run 1

Figures 5 and 6 show the particle filter fit to the degradation data of run 1 for components 1 and 2 respectively. The grey dots represent the estimated degradation level at each time step for each of the n_p particles. Therefore these grey dots represent $n_p = 1000$ different degradation trajectories. The yellow dashed line represents the mean value of these trajectories at each time step.

To further validate the parameter values of the degradation model considering the interactions between the 2 components, we compute the R^2 values for the fit of the average estimated degradation trajectory resulting from the particle filter to the real degradation trajectories. For component 1 this is $R_1^2 = 0.792$ and for component 2 it is $R_2^2 = 0.753$. If we were to consider a reduced model whereby no stochastic dependence is considered between the two components and we are left with a gamma process describing the evolution of the degradation level for each component, the average fit of such models results in an $R_1^2 = 0.671$ and $R_2^2 = 0.575$. The further advantage of considering the interactions between components is motivated in section 4.5.

Next, the fitted degradation model is integrated with the proposed maintenance model to find the optimum policy.

4.3 Optimum maintenance policy

To evaluate the cost-rate, a very large number of life cycles of the system were simulated with above data. To find the optimal decision parameters $(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$, the cost-rate $C^\infty(\Delta T, m_p^1, m_o^1, m_p^2, m_o^2)$ is evaluated for different values of ΔT ($\Delta T > 0$), m_p^1 ($0 < m_p^1 \leq L^1$), m_o^1 ($0 < m_o^1 \leq m_p^1$), m_p^2 ($0 < m_p^2 \leq L^2$) and m_o^2 ($0 < m_o^2 \leq m_p^2$) using Equation (5). The step size is 5 for the inter-inspection time, and 0.05 for each preventive or opportunistic threshold. With a precision of 0.010 specified for the cost-rate, the convergence of the cost-rate is reached from 10000 renewal cycles. The optimum values of the decision parameters are $\Delta T^* = 60$, $m_p^{1*} = 0.55$, $m_o^{1*} = 0.50$, $m_p^{2*} = 0.50$ and $m_o^{2*} = 0.40$ with the minimum cost-rate $C^\infty(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*}) = 2.90$ acu. Table 2 reports the proportion of maintenance actions (replacement of C1 or C2; joint replacement of C1 and C2) at maintenance time for each optimal maintenance policy. It should be noticed that two components are always replaced together in policy V2. The proportion of joint replacement in the proposed opportunistic maintenance policy (policy V) is higher than in the non-opportunistic policy (policy V1). This is because the opportunistic thresholds tend towards a joint replacement of C1 and C2.

	Replacement of C1	Replacement of C2	Joint replacement of C1 and C2
Policy V	0.31	0.38	0.31
Policy V1	0.34	0.53	0.12
Policy V2	0	0	1

Table 2: Proportion of maintenance actions

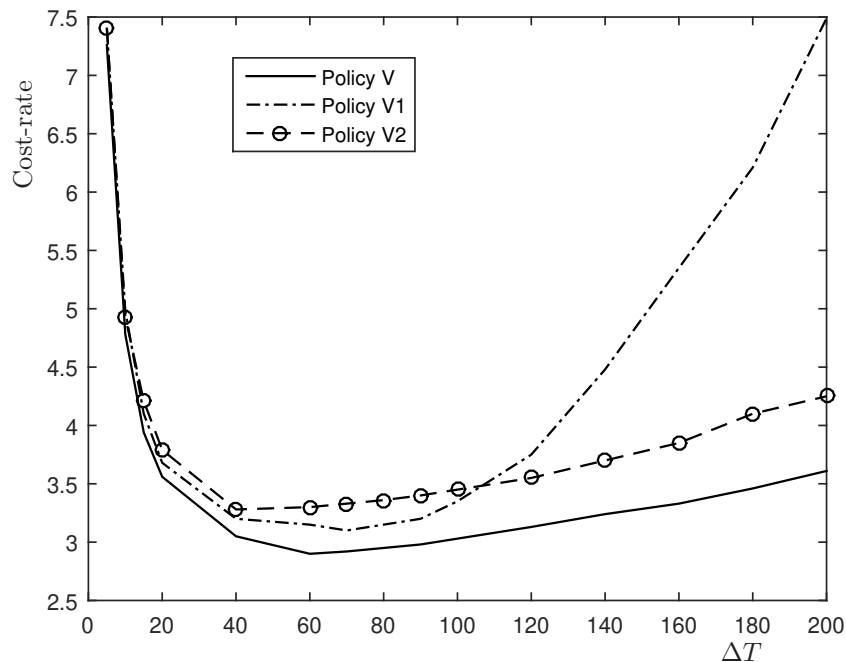


Figure 7: Cost-rate as a function of inter-inspection interval ΔT

Figure 7 shows the relationships between the minimum cost-rate and the inter-inspection interval ΔT for the proposed opportunistic policy (policy V), non-opportunistic policy (policy V1) and the joint replacement policy (policy V2). Each point represents an optimal policy with a given value of ΔT .

This indicates that the proposed opportunistic maintenance policy (policy V) always provides the lowest cost-rate. We observe that when $\Delta T < \Delta T^*$ the maintenance cost increases rapidly and the difference between the three policies reduces with decreasing ΔT . However, when $\Delta T > \Delta T^*$, the cost-rate of the non-opportunistic policy (policy V1) increases rapidly with increasing ΔT , while the cost-rate of policies V and V2 increase slowly with increasing ΔT . This indicates that the opportunistic replacement and the joint replacement can better compensate for a sub-optimally large ΔT .

4.4 Impact of economic dependence on the cost

We now analyze the impact of economic dependence on the opportunistic replacement maintenance policy. This is carried out by analyzing the sensitivity of the minimum cost-rate for three policies V, V1 and V2 to the economic dependence degree (a, b) between the two components.

To study the performance of these three policies, a relative excess-cost in the minimum cost-

rate of the proposed opportunistic policy V compared to policy Vi, denoted ΔC_i ($i = 1, 2$), is used. It is defined as follows:

$$\Delta C_i = \frac{C_{Vi}^\infty - C^\infty(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*})}{C_{Vi}^\infty} \cdot 100\%$$

where C_{Vi}^∞ is the minimum cost-rate of policy Vi with $i = 1, 2$. According to the definition, $\Delta C_i > 0$ means that policy V is more effective than policy Vi and less effective in the opposite case.

4.4.1 Sensitivity analysis to a

We vary a from 0 to 20% while the others parameters remain unchanged. For each value of a the minimum cost-rate of each policy is determined and the excess-cost is then evaluated. Summary results are shown in Figure 8.

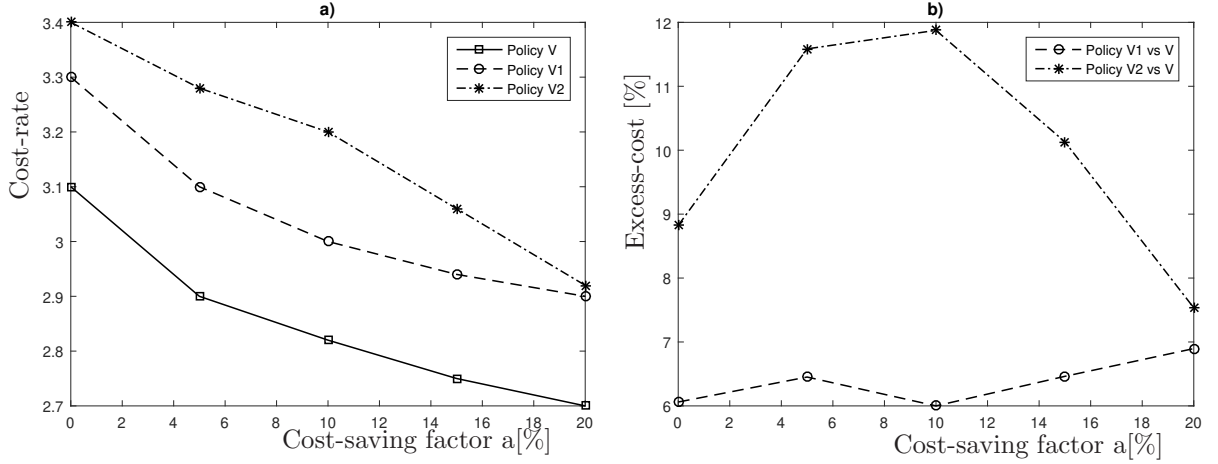


Figure 8: Cost-rate (a) and excess-cost (b) as a function of a

Figure 8(a) shows that the cost-rate decreases with the cost-saving factor a . This can be explained by the fact that the maintenance costs reduce as a increases. It is not surprising that the proposed opportunistic policy V always provides a lowest cost-rate. This is because policies V1 and V2 are two special cases of policy V.

Figure 8(b) shows that when $a < 10\%$ the excess-cost related to policy V2 increases with an increasing of a . This means that the cost-rate of policy V2 decreases more slowly than that of policy V as a increases. However, when $a > 10\%$, the cost-rate of policy V2 decreases more rapidly than the cost-rate of policy V. While the cost-rate of policy V1 decreases more slowly than that of policy V1 with increasing a . This can be explained by the fact that the two components tend to be jointly replaced when the cost-saving factor is high.

To study more the impact of economic dependence degree on the maintenance cost, we consider sensitivity with respect to the duration-saving factor b .

4.4.2 Sensitivity analysis to b

Here we vary b from 0 to 50% while the others parameters remain unchanged. For each value of b the minimum cost-rate of each maintenance policy is determined and the excess-cost is then evaluated. The results obtained are shown in Figure 9.

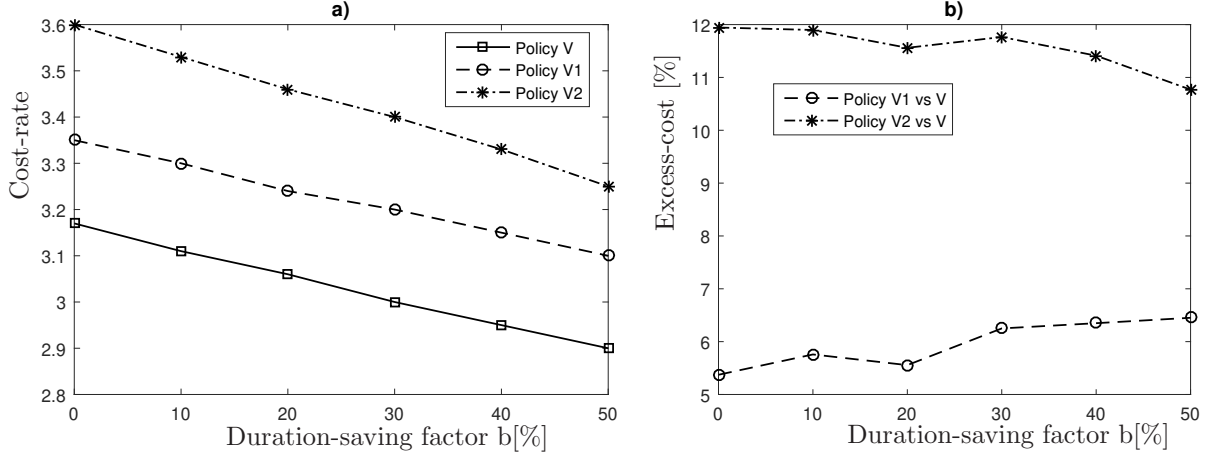


Figure 9: Cost-rate (a) and excess-cost (b) as a function of b

It is not surprising again that an increasing of b (or equivalently a reduction on maintenance duration when two components are replaced together) leads to a decreased cost-rate. However, the effect for both the opportunistic policy (V) and non-opportunistic policy (V1) are broadly the same, in a similar manner to that for varying a . This suggests that for both policies there is a tendency that replacements of components are simultaneous. This is natural for the opportunistic policy because this is its purpose. However we might have expected the non-opportunistic policy to show less dependence on a and b . Our explanation for this is as follows. When there is no opportunistic replacement, the threshold for preventive replacement compensates (for component C1 in this case). It is lower (than with the opportunistic policy) so that more often than not, the replacement of components is simultaneous (and set up cost is saved). If it were not the case that replacements are simultaneous then the cost-rate for policy V1 would not depend on a and b in the way it does. This effect only occurs because of the positive stochastic dependence. If there was no positive stochastic dependence then the simultaneous replacement of the components when one reaches a preventive replacement threshold would be inefficient.

Thus, when there is no stochastic dependence between components, opportunistic policies become more effective as the extent of economic dependence increases. This is well known and obvious. However, it would appear that when there is also positive stochastic dependence this phenomenon is much less apparent. This is because a non-opportunistic policy will then compensate for the absence of opportunities for replacement by lowering the threshold for preventive replacement of the components. The positive stochastic dependence ensures that replacements usually remain simultaneous because components will tend to cross their replacement thresholds together. That said, this “deteriorating together” phenomenon will tend to be more apparent when the lifetimes of the components are broadly similar as is the case for the gearbox system.

4.4.3 Optimum policies when $a = 0$ and $b = 0$

We suppose now that two components are economically independent, i.e. $a = 0$ and $b = 0$. The optimum maintenance policies are given in Table 3 where P_{joint} indicates the probability that two components are jointly replaced at each maintenance.

	Optimum decision variables	P_{joint}	Minimum cost-rate
Policy V	$\Delta T^* = 60, m_p^{1*} = 0.6, m_o^{1*} = 0.55, m_p^{2*} = 0.5, m_o^{2*} = 0.45$	0.18	3.22
Policy V1	$\Delta T^* = 60, m_p^{1*} = 0.6, m_p^{2*} = 0.5$	0.12	3.26
Policy V2	$\Delta T^* = 50, m_p^{1*} = 0.55, m_p^{2*} = 0.65$	1	3.69

Table 3: Optimum maintenance policies when $a = b = 0$

The obtained results show that when two components are economically independent, the proposed opportunistic policy V is still slightly better than the non-opportunistic policy V1. This is due to the opportunistic thresholds, which allow policy V to become more flexible and better able to accommodate the stochastic dependence between components than the non-opportunistic policy V1. However, the joint replacement (policy V2) has a higher cost-rate which means that the joint replacement is not effective for this case.

4.5 Impacts of state dependence on the cost

To study the impact of state dependence between components on the optimum maintenance policy, we assume now that the degradation process of each component evolves independently. In this way, we reduce the degradation model to two independent gamma process for which the shape and scale parameters can be estimated using maximum likelihood estimation (or using the particle filter). The results in the estimates are presented in Table 4.

Component	α^i	β^i
C1	0.1165	0.0100
C2	0.0919	0.0090

Table 4: Estimated parameter values without considering stochastic dependence

The proposed maintenance policy is then applied. We obtained the optimal decision variables $\Delta T^* = 120$, $m_p^{1*} = 0.60$, $m_o^{1*} = 0.45$, $m_p^{2*} = 0.55$ and $m_o^{2*} = 0.40$. When compared with the results obtained in Section 4.3, these optimal values are significantly different. In addition, if we apply these optimal decision variables for the case considering the state dependence between components, the cost-rate is then $C^\infty(\Delta T^*, m_p^{1*}, m_o^{1*}, m_p^{2*}, m_o^{2*}) = 3.75$ acu. This is significantly higher than the one obtained when the state dependence is considered in degradation modeling $((3.75-2.90)/2.90) \times 100 = 29.3\%$ higher). This implies that not considering the state dependence between two components can lead to a sub-optimal maintenance policy. Of course, the difference is itself dependent on the economic “dependence degree” between the components.

5 Conclusions

In this work, a condition-based maintenance policy for a two-dependent component system is studied. Two kinds of dependency are investigated and integrated in the maintenance modeling: state dependence whereby the degradation rate of each component depends not only on its state but on the state of the other component; and economic dependence whereby set-up cost and duration are shared when components are replaced simultaneously. To select the components to be preventively maintained at each inspection epoch, adaptive preventive replacement and opportunistic replacement rules are proposed. A cost model taking into account the economic dependence between components is developed to find the optimal values of the decision variables. The policies are studied in the context of a gearbox system consisting of gears. The results indicate that (i) accounting for the state dependence between components is important, and to ignore it has a significant impact (29.3%) on the cost; (ii) introducing an opportunistic threshold for replacement makes the maintenance policy more flexible and less sensitive to a sub-optimally large inspection interval; and (iii) when there exists positive stochastic dependence between components so that components tend to deteriorate together, introducing an opportunistic threshold for replacement in order to share set-up costs achieves less when there

is positive stochastic dependence between components than when there is not. This is because replacements will tend to be synchronized and this tendency to synchronise arises precisely because of degradation dependence. Thus we might claim a general insight that opportunistic maintenance is less opportune when components tend to deteriorate together than when they do not. It will be very interesting to investigate this claim in a more general context, and this will be the subject of future work.

Finally, the proposed opportunistic maintenance policy might be extended to a larger system, but at the cost of exploring a much larger decision space and a significantly greater computation time. The development of an analytical or efficient heuristic approach for the evaluation of the cost-rate then becomes important. It would also be interesting to apply the proposed maintenance policy in a large-scale industrial case study. Another perspective should be the investigation on real time CBM vs inspection in presence of degradation interaction between components.

Appendix

Appendix A. Gamma distribution

A random variable X which is gamma-distributed with shape α^i and rate β^i is denoted

$$X \sim \Gamma(\alpha^i, \beta^i).$$

The corresponding probability density function (PDF) is

$$f_{\alpha^i, \beta^i}(x) = \frac{1}{\Gamma(\alpha^i)} \cdot (\beta^i)^{\alpha^i} \cdot x^{\alpha^i-1} \cdot e^{-\beta^i \cdot x} \cdot \mathcal{I}_{\{x \geq 0\}},$$

where:

- $\Gamma(\alpha^i) = \int_0^{+\infty} u^{\alpha^i-1} \cdot e^{-u} du$ denotes a complete gamma function;
- $\mathcal{I}_{\{x \geq 0\}}$ is an indicator function. $\mathcal{I}_{\{x \geq 0\}} = 1$ if $x \geq 0$, $\mathcal{I}_{\{x \geq 0\}} = 0$ and otherwise.

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