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NUMERICAL SIMULATION OF VON KARMAN SWIRLING BIOCONVECTION NANOFLUID FLOW FROM A DEFORMABLE ROTATING DISK

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Introduction

Rotating disk bio-reactors are fundamental to numerous medical and chemical engineering processes including oxygen transfer, chromatography, purification and swirl-assisted pumping [1]. The modern upsurge in biologically-enhanced engineering devices has embraced new phenomena including bioconvection of micro-organisms (photo-tactic, magneto-tactic, oxy-tactic, gyrotactic etc). The proven thermal performance superiority of nanofluids i.e. base fluids doped with engineered nano-particles, has also stimulated immense implementation in biomedical designs. Motivated by these emerging applications, in the current study, we study analytically and computationally the time-dependent 3-dimensional viscous gyrotactic bioconvection [2] in swirling nanofluid flow from a rotating disk configuration. The disk is also deformable i.e. able to extend (stretch) in the radial direction. Stefan blowing is included. The nanofluid is assumed to be dilute and the Buongiorno formulation [3] is adopted wherein Brownian motion and thermophoresis are the dominant nanoscale effects. Semi-numerical solutions are developed for the problem using the efficient Adomian decomposition method (ADM) [4]. Validation with earlier Runge-Kutta shooting computations in the literature is also conducted. Extensive computations are presented (with the aid of MATLAB symbolic software) for radial and circumferential velocity components, temperature, nano-particle concentration, micro-organism density number and gradients of these functions at the disk surface (radial local skin friction, local circumferential skin friction, Local Nusselt number, Local Sherwood number, motile microorganism mass transfer rate). Extensive interpretation of the results is included. The work provides a useful benchmark for further computational fluid dynamics simulations of nano-bioconvection rotating disk reactors.

Methods and Materials

Forced bioconvection in three-dimensional time-dependent viscous incompressible nanofluid over a rotating stretchable disk is considered in a cylindrical polar coordinate system Velocity componets in these coordinates are The nanofluid is assumed to be a dilute suspension with a homogenous distribution of gyrotatcic micro-organisms. Mass convective and no-slip boundary conditions are imposed at the disk surface. Stefan blowing is simulated via a w-(axial) velocity condition. The rotating disk flow model is illustrated in **Fig. 1**.

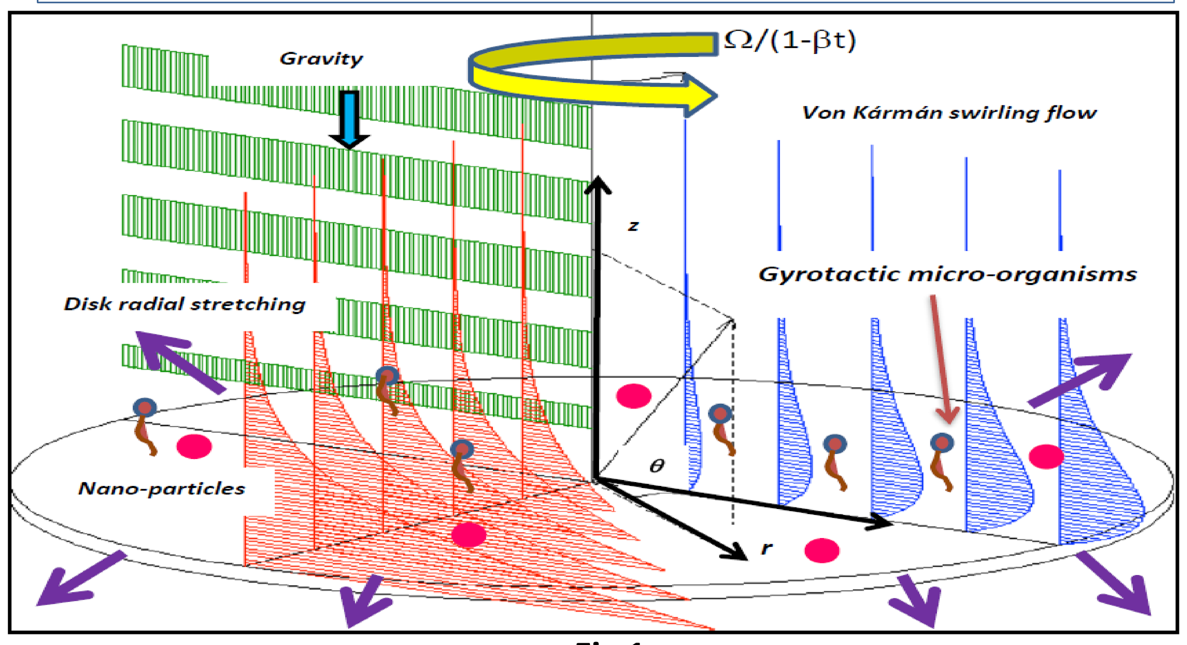

Fig 1

The nano-particle species concentration conditions close to the disk surface follow $c_f > c_w > c_\infty$, in order to simulate mass convective boundary conditions. Intrinsic to this is the presence of a different nano-particle concentration from the neighborhood to the disk concentration, wall concentration and ambient concentration which yields the required mass transfer coefficient h_m. With appropriate similarity variables, the conservation boundary layer equations reduce to the following system of strongly coupled, nonlinear, multi-order, multi-degree ordinary differential equations for **primary and secondary momentum, energy, nano-particle species and micro-organism density species conservation**:

$$f''' + 2\,f\,f'' - f'^2 + g^2 - S\left(f' + \frac{1}{2}\eta\, f''\right) = 0$$

$$g'' + 2\,f\,g' - 2\,g\,f' - S\left(g + \frac{1}{2}\eta\, g'\right) = 0$$

$$\theta'' + Nb\,\phi'\,\theta' + Nt\,\theta'^2 - \frac{1}{2}\,S\,Pr\,\eta\theta' + 2\,Pr\,f\,\theta' = 0$$

$$\frac{1}{Le\,Pr}\phi'' + \frac{1}{Le\,Pr}\frac{Nt}{Nb}\theta'' - \frac{1}{2}\,S\,\eta\,\phi' + 2\,f\,\phi' = 0$$

$$\frac{1}{Lb\,Pr}\psi'' - \frac{1}{2}\,S\,\eta\,\psi' + 2\,f\,\psi' - Pe\left(\psi\varphi'' + \varphi'\psi'\right) = 0$$

The corresponding boundary conditions:

$$\left.\begin{array}{l} \eta = 0: \begin{cases} f'(0) = \alpha,\ g(0) = 1,\ f(0) = \dfrac{f_w}{Le\,Pr}\varphi'(0),\ \theta(0) = 1, \\ \varphi'(0) = -Nd\left[1 - \varphi(0)\right], \quad \psi(0) = 1 \end{cases} \\ \eta \to \infty: \left\{ f'(\infty) = 0,\ g(\infty) = 0,\quad \theta(\infty) = 0,\quad \varphi(0) = 0,\ \psi(\infty) = 0 \right. \end{array}\right\}$$

Here the non-dimensional parameters are defined as follows: S= β/Ω is the unsteadiness (time-dependent) parameter, Pr is Prandtl number, Nb is Brownian motion, Nt is thermophoresis parameter, Le is ordinary Lewis number (ratio of thermal and nano-particle species diffusivities), Lb is bioconvection Lewis number (ratio of thermal and motile micro-organism mass diffusivities), Pe denotes bioconvection Peclet number (ratio of the rate of advection of micro-organisms driven by the flow to the rate of diffusion of micro-organisms under gyrotaxis), f_w is the Stefan blowing parameter, Nd is the Biot number and α is the disk stretching parameter. It is important to note the presence of nanoparticles does not modify the incompressibility or biological nature of the suspension considered since the nanoparticles do not interact with the motile (i.e. self-propelled) microorganisms. They are a separate "species". The micro-organisms possess mean diameters which range from 1mm to 200mm and are therefore significantly larger than nanoparticle dimensions. These microorganisms, which are considered to be gyrotactic in the present study, possess slightly greater densities than the water-based fluid e.g., several percent for algae and no more than 10% for bacteria e.g., *B. subtilis, Bacillus, Chlamydomonas, Volvox and Tetrahymena*. For rotating disk bioreactor design [5], important transport characteristics include the radial and tangential skin friction coefficients, local Nusselt number, local Sherwood number, local wall motile micro-organism number (wall mass flux gradient).

ADM Numerical Solution

The eleventh order system of coupled ordinary differential equations (ODEs) with boundary conditions does not admit exact analytical solutions. Recourse is therefore necessary to computational methods. ADM [6] is one of the most accurate numerical methods. It uses very precise polynomial expansions to achieve faster convergence than many other procedures and has been applied by for example Bég *et al.* [7] (magneto-rheological squeeze films). An advantage of ADM is that it can provide analytical approximation or an approximated solution to a wide class of nonlinear equations without linearization, perturbation closure approximations or discretization methods. ADM deploys an infinite series solution for the unknown functions and utilizes recursive relations. The ***unknown functions f, g, θ, ϕ and ψ*** are expressed as infinite series' and then the linear and nonlinear components are decomposed by an infinite series of Adomian polynomials as follows:

$$\left.\begin{array}{l} \sum_{m=0}^{\infty} A_m = f\,f'',\ \sum_{m=0}^{\infty} B_m = f'^2,\ \sum_{m=0}^{\infty} C_m = g^2,\ \sum_{m=0}^{\infty} D_m = f',\ \sum_{m=0}^{\infty} E_m = f'' \\ \sum_{m=0}^{\infty} F_m = f\,g',\ \sum_{m=0}^{\infty} G_m = f'\,g,\ \sum_{m=0}^{\infty} H_m = g,\ \sum_{m=0}^{\infty} I_m = g', \\ \sum_{m=0}^{\infty} J_m = \theta'\,\phi',\ \sum_{m=0}^{\infty} K_m = \theta'^2,\ \sum_{m=0}^{\infty} L_m = \theta',\ \sum_{m=0}^{\infty} M_m = f\,\theta' \\ \sum_{m=0}^{\infty} N_m = \theta'',\ \sum_{m=0}^{\infty} O_m = \phi',\ \sum_{m=0}^{\infty} P_m = f\,\phi', \\ \sum_{m=0}^{\infty} Q_m = \psi',\ \sum_{m=0}^{\infty} R_m = f\,\psi',\ \sum_{m=0}^{\infty} S_m = \psi\,\phi'',\ \sum_{m=0}^{\infty} T_m = \phi'\,\psi' \end{array}\right\}$$

The Adomian transformed boundary conditions:

$$\left.\begin{array}{l} f(0) = -\beta(1-r),\quad f'(0) = \alpha,\quad f''(0) = p,\quad g(0) = 1,\quad g'(0) = q, \\ \theta(0) = 1,\ \theta'(0) = t,\ \phi(0) = r,\ \phi'(0) = -Nd(1-r),\ \psi(0) = 1, \psi'(0) = u \end{array}\right\}$$

The power-series Adomian solutions are numerically evaluated in the subroutine bvp4c in MATLAB symbolic software. To verify the accuracy of the ADM solutions, comparisons are made with the shooting (Runge-Kutta) quadrature computations of Latiff *et al.* [8] for the general model (i.e. with all parameters included) with a variation in unsteadiness parameter, *S* (deceleration case i.e. *S* < 0) for two different scenarios, namely a *non-deformable disk* (α = 0) and a *radially stretching disk* (α = 1.0). These are documented in **Table 1** for radial local skin friction, local circumferential skin friction, local Nusselt number, local Sherwood number, motile micro-organism wall mass flux for α=0.

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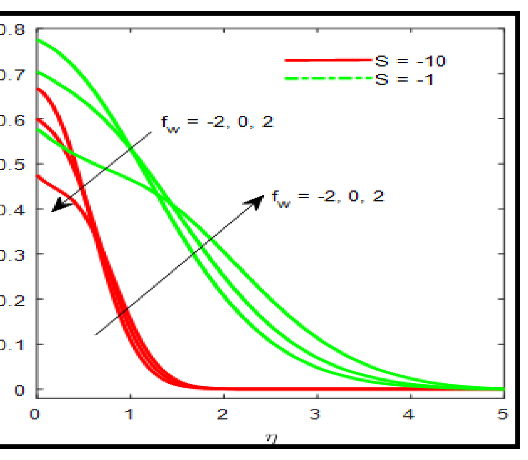
	f//(0)		g/(0)		-θ/(0)	-ϕ/(0)	-ψ/(0)
S	RK45	ADM (present)	RK45	ADM (present)			
-0.1	0.5287	0.527889	-0.5779	-0.57663	0.38045	0.107966	0.444195
-0.2	0.5496	0.54907	-0.5407	-0.53914	0.412973	0.107892	0.476487
-0.5	0.6132	0.613551	-0.4280	-0.42734	0.499995	0.112086	0.564525
-1	0.7196	0.719461	-0.2365	-0.23433	0.616827	0.12128	0.683967
-2	0.9315	1.079269	0.1550	0.282443	0.826726	0.136449	0.897399

Table 1: ADM Validation with RK 45 [8]

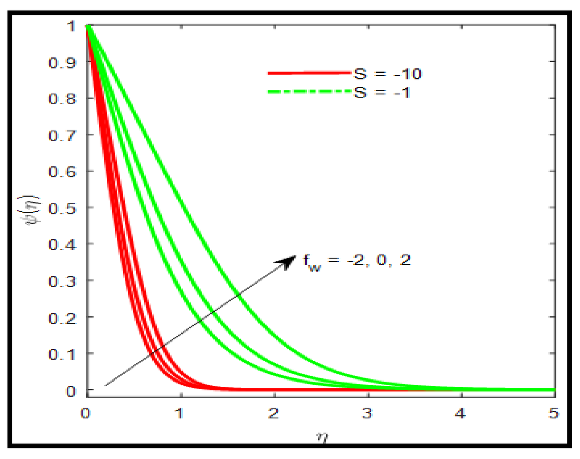
ADM Results

Selected computations are shown below. For brevity we have restricted attention to stretching (*S*), unsteadiness (α), and Stefan blowing/suction (f_w) effects. we note that although a mass transfer Biot number, *Nd* appears in the boundary conditions it is prescribed value of 0.4. This parameter is the solutal analogy to the thermal Biot number. Whereas thermal Biot number represents the ratio of internal thermal conduction resistance to external thermal convection resistance, the mass transfer Biot number symbolizes the relative contribution of internal mass diffusion resistance to the external mass diffusion resistance. Since *Nd* <1, in our simulations the external mass diffusion resistance dominates the internal mass diffusion resistance, and this relates to the nano-particle concentration field, not the micro-organism species.



Effect of unsteadiness (*S*) and suction/injection (f_w) on the nanoparticle concentration

Fig 2



Effect of unsteadiness (*S*) and suction/injection (f_w) on the motile micro-organism density number

Fig 3

Discussion

In **Fig. 2** nano-particle concentration (volume fraction of nano-particle) profiles exhibit a more complex response to unsteadiness and blowing/suction parameters. Two different types of behaviour can be delineated in the zones can be identified, namely the *near disk region* and the *far field region*. In the near disk zone (i.e. 0 < η < 1) initially with strong blowing the nano-particle concentration magnitudes are elevated, and are significantly higher for weak disk deceleration (*S* = -1) as compared with strong disk deceleration (*S* = -10). With further distance from the disk surface, these trends are reversed i.e. blowing is observed to reduce nano-particle concentration whereas suction is observed to slightly enhance it. Again the deviation in profiles for a specific *S* value is markedly greater for the weak disk deceleration situation compared with the strong disk deceleration scenario. The diffusion of nano-particles into the boundary layer is therefore a function of the location from the disk surface and is not a single consistent response throughout the entire regime. **Fig. 3** reveals that a much more controlled response in motile micro-organism density numbers is produced with a change in unsteadiness (*S*) and Stefan blowing/suction (f_w) effects. The quantity of micro-organisms is decreased with suction and elevated with Stefan blowing. Strong deceleration again decreases motile micro-organism density number compared with weak disk deceleration i.e. the motile micro-organism species boundary layer thickness is depleted with faster deceleration of the disk. Inevitably this is associated with the radial and azimuthal momenta fields which are damped simultaneously..

Conclusions

The ADM simulations generally show:

I)Increasing radial stretching parameter decreases radial velocity and radial skin friction, reduces azimuthal velocity and skin friction, decreases local Nusselt number and motile micro-organism mass wall flux whereas it increases nano-particle local Sherwood number.

II)Disk deceleration accelerates the radial flow, damps the azimuthal flow, decreaes temperatures and thermal boundary layer thickness, depletes the nano-particle concentration magnitudes (and associated nano-particle species boundary layer thickness) and furthermore decreases the micro-organism density number and gyrotactic micro-organism species boundary layer thickness.

III)Increasing Stefan blowing accelerates the radial flow and azimuthal (circumferential flow), elevates temperatures of the nanofluid, boosts nano-particle concentration (volume fraction) and gyrotactic micro-organism density number magnitudes whereas suction generates the reverse effects.

IV)Increasing suction effect reduces radial skin friction and azimuthal skin friction, local Nusselt number, and motile micro-organism wall mass flux whereas it enhances the nano-particle species local Sherwood number.

V)ADM achieves very rapid convergence and highly accurate solutions and shows excellent promise in simulating swirling multi-physical nano-bioconvection fluid dynamics problems and offers good promise for bioreactor heat and mass transfer simulations.

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