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Multi-period Market Risk Estimation and Performance Evaluation: Evidence from Univariate, Multi-variate and Options Data

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Abstract

There are different risk management approaches available, as different firms have different risk goals. Value at risk (VaR) is the most frequently used risk measure for asset or portfolio risk and certainly, per the Basel framework, is a preferred measure for market risk for banks and financial institutions.

VaR is still the most popular method for performing financial risk, although it has been criticized on many grounds by academic researchers. A coherent measure of financial risk referred to as expected shortfall (hereinafter ES) was proposed by Artzner et al. (1999) to overcome problems associated with VaR.

In the first part of the thesis we evaluate expected shortfall (ES) with a new 6-parameter heavy tailed distribution by Baker (2014) alongside recent generalizations of the asymmetric Student *t* by Zhu and Galbraith (2010) and exponential power distributions by Zhu and Zinde-Walsh (2009). This is allowing separate parameters to control skewness and tail thickness for both stocks and indexes. The results suggest that GAT of Baker (2014) outperforms both AST of Zhu and Galbraith (2010) and APED by Zhu and Zinde-Welsh (2009) for both 1-day and multi-day ES forecasts.

In the second part of the thesis, we present and discuss the use of copulas and vine copulas for financial risk management, also introduce the term structure of risk for bivariate and multivariate data. To the best of our knowledge, this study is the first to explore multivariate term structure of risk with both static and dynamic conditional correlation. The results suggest that copula models for two-dimensional data and vine copula models for five, seven and fifteen-dimensional data provide a good fit and accurately and efficiently forecast the expected shortfall as compared to DCC-norm and DCC-t.

In the third part of the thesis, we compare the performance of the Heston option pricing model, Bates option pricing model, Merton jump diffusion option pricing model, Kou option pricing model and variance gamma option pricing model with a traditional Black-Scholes option pricing model. We also evaluate expected shortfall estimates for European options for 1-day and 10-days at a range of confidence levels with full Monte Carlo and Monte Carlo delta and Monte Carlo delta gamma derived from option pricing models tested in our research. The results indicate that full valuation appears to be one of the top models for both 1-day ahead and multi-days ahead ES. This gives us clear implications for calculation of ES beyond 10-days.

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Finally, I would like to pay my gratitude to the Almighty for giving me enough courage to stay strong, and the opportunity to complete this thesis. Without Him, nothing is possible.

Dedication

To my family

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LIST OF ABBREVIATIONS

VaR	Value at Risk
ES	Expected Shortfall
GARCH	Generalized Autoregressive Conditional Heteroskedastic
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedastic
NGARCH	Non-linear Asymmetric GARCH
TGARCH	Threshold GARCH
GAT	Generalized Asymmetric *t* Distribution
AEPD	Asymmetric Exponential Power Distribution
SEPD	Skewed Exponential Power Distribution
AST	Asymmetric Student *t* Distribution
SST	Skewed Student *t* Distribution
ST	Student *t* Distribution
TTD	Twin *t* Distribution
MCS	Monte Carlo Simulation
FHS	Filtered Historical Simulation
BSM	Black Scholes Model
VG	Variance Gamma Model
HS	Heston Model
Bat	Bates Model
MJD	Merton Jump Diffusion Model
Kou	Double Exponential Jump Diffusion Model

Chapter One: General Introduction

Over the years, financial risk management has become a popular discipline among academic researchers, market analysts and regulators. Cassar and Gerakos (2013) define risk management 'as procedure and mechanism used to monitor and manage an organization's exposure to risk'.

There are different risk management approaches available, as different firms have different risk goals. Some companies use cash-flow volatility, while others use the volatility in the firm's value as risk measurement object. The size of the company is an important factor when considering risk management, as large companies manage risk more effectively than smaller companies (Christoffersen, 2012).

The effectiveness of risk management practices is an important research issue. The theory of corporate risk management indicates that shareholders are better off if a firm maintains smooth cash flows (Rountree et al., 2008).

According to Stulz (1996), risk management has the capability both to increase the size of debt and to facilitate equity stakes for management by reducing the possibility of financial distress. Risk management also eliminates risk of bankruptcy effectively by reducing the direct cost of administration and reorganization (lawyers and court cost), indirect cost of interference from the bankruptcy court on investment and the operating cost of the firm to zero, as well as increasing the company's value.

The academic literature has concentrated on volatility reduction as the primary objective of risk management and on variance as the main measure of risk. However, the main purpose of most corporate risk management is to avoid lower tail outcomes rather than aiming to reduce variance. Many commercial banks and their financial institutions are determined to reduce the probability of lower tail outcomes by using the measure known as value at risk (hereinafter VaR). VaR is the maximum potential loss over a given time at a certain confidence level.

VaR is the most frequently used risk measure for asset or portfolio risk and is a preferred measure for market risk for banks and financial institutions per the Basel framework.

The lack of adequacy of traditional risk measures such as VaR explains by the recent financial crises. VaR made the financial crises worse by giving wrong security to bank executives and regulators. VaR is still the most popular method for assessing financial risk, ass Basel II and Basel III regulatory frameworks for banking supervision and risk management still prefer VaR as the market risk measure.

VaR has been criticized on many grounds by academic researchers, as VaR does not identify any loss beyond VaR level and is not a coherent measure of risk as it cannot satisfy the property of subadditivity (Artzner et al., 1997, 1999). Yamai and Yoshiba (2002) mentioned two more issues that a rational investor who wants to maximize expected utility may be misled by information provided by VaR. VaR is also difficult to use when the investor wants portfolio optimization. Prause (1999) criticized VaR because to avoid bankruptcy one should forecast the distribution of the maximum expected loss.

Artzner et al. (1999) introduced a new coherent measure of financial risk referred to as expected shortfall (hereinafter ES) to overcome problems associated with VaR. ES satisfies the sub-additivity property, as observed by Acerbi and Tasche (2002). The VaR provides information only if the loss is above a certain level, but does not consider the magnitude of the given loss; ES does provide this information.

VaR and ES estimation frequently assume that long returns are normally distributed. However, the assumption of normality of returns is inadequate for most of the time series. The most important deviations from normality are the heavy tails.

The Student *t*-distributions have played a particularly significant role as a long-tailed distribution. However, the Student *t* distribution that allows for heavier tails than the normal assumes that the distribution is symmetric around zero. The skew *t* distribution proposed by Hansen (1994) allows modelling of skewness in conditional distributions of financial returns.

Since Hansen's (1994) introduction of the skew *t* distribution, many skew *t* distributions have been introduced in the financial literature (Fernandez et al., 1995; Fernandez and Steel, 1998; Theodossiou, 1998; Branco and Dey, 2001; Bauwens and Laurent, 2002; Jones and Faddy, 2003; Sahu et al., 2003; Azzalini and Capitanio, 2003; Ayebo and Kozubowski, 2004; Aas and Haff, 2006; Komunjer, 2007; Zhu and Galbraith, 2010; Su et al, 2014 and Tolikas, 2014).

Unlike the skew *t* distribution, the generalized asymmetric student *t* distributions constrain the modelling of asymmetry and tails by using two parameters, which together control skewness and thickness of the left and right tails. Zhu and Zinde-Walsh (2009) proposed as asymmetric exponential power distribution (AEPD) that in addition to skewness suggests different decay rates of density in the left and right tails. Baker (2014) proposed a 6-parameter generalized *t* distribution (GAT) that allows asymmetry of scale and tail power. The GAT distribution generalized the *t*-distribution through two types of skewness (parameters c and r), and how soon tail behaviour starts (parameter α).

In Chapter 2, we explore a 6-parameter fat tailed distribution by Baker (2014) alongside the recent generalizations of the asymmetric Student-*t* by Zhu and Galbraith (2010) and exponential power distribution by Zhu and Zinde-Walsh (2009). We also include the Student *t*-distribution as used by Bollesslev (1987), skewed *t* distribution by Zhu and Galbraith (2010), skewed exponential distribution by Zhu and Zinde-Walsh (2009) and twin- *t* distribution by Baker and Jackson (2014) as benchmark models.

The empirical analysis focuses on two different groups of data. We include the world's five major indices Standards and Poor's 500, FTSE-100, NASDAQ 100, Nikkei -225, and DAX-30 indexes for the period 1995-2014. We also include individual stock from Standards and Poor's 500 (Adobe, Bank of America, J P Morgan, Pfizer and Starbucks).

When portfolio variance is time varying, going from 1-day-ahead to h-days-ahead ES without knowing the detail for the distribution of returns can be found using simulation based methods. We consider Monte Carlo simulation (MCS) with GAT, AEPD, SEPD, AST, SSTD, ST and TTD as standardized distributions of returns and filtered historical simulation (FHS) for 5-days and 10-days ES in this research.

We explore different ES methods i.e. EGARCH-GAT by Baker (2014), EGARCH-TTD by Baker and Jackson (2014) with EGARCH-AST, EGARCH-SST by Zhu and Galbraith (2010), EGARCH-AEPD, EGARCH-SEPD by Zhu and Zinde-Welsh (2009) and Standardized Student *t* distribution as used by Bollesslev (1987). The longer period ES forecasts are estimated by using Monte Carlo simulation with GAT, AEPD, SEPD, AST, SSTD, ST and TTD as standardized distributions of returns and filtered historical simulation (FHS) for the world's major five stock indices (S & P 500, FTSE100, NASDAQ, Nikkei and DAX30).

Due to the financial crisis of 2007-2009 and increasing volatility at international financial markets, an active risk management is important for any financial organization. It is also mandatory by Basel II and III for the banking sector to encourage the use of sophisticated internal models. However, a critical concern of those models is in the handling of dependence among different assets.

Sklar (1959) introduced the copula as a statistical function that links together univariate distributions to form multivariate distributions. According to Sklar's Theorem, any multivariate joint distribution can decompose into univariate marginal distribution functions and a copula which describes the dependence part of the distribution.

The copula has become a popular multivariate modelling tool mainly due to easy modelling and estimation of marginal distributions and copula separately. The approach is to model the well-known stylized facts of financial returns using marginal distributions (Cherubini et al., 2004 and McNeil et al., 2005).

Constructing higher-dimensional copulas is considered as a difficult problem. There are a huge number of parametric bivariate copulas, but the set of known higher-dimensional copulas is rather limited. The pair-copula construction can be a straightforward and powerful tool for model building and extending bivariate copulas to higher dimensions (Aas et al., 2009).

There are a significant number of feasible pair-copula decompositions available for high-dimensional distributions. Bedford and Cooke (2001, 2002) presented a graphical model denoted the regular vine (R-vine). The R-vines are very common and include many possible pair-copula decompositions.

In Chapter 3, we calculate VaR and ES for different GARCH-Copula models with various marginals. They are implemented and tested on both bivariate and multivariate data. The forecasts from copula models are then compared to DCC-GARCH-models, as both dynamic conditional correlations models (DCC) by Engle (2002) and copulas models by Sklar (1959) allow for two steps modelling of portfolio returns. Marginal return distributions are specified in the first step. In the second step, the marginal is linked to a joint distribution either via time variant correlations or a time invariant link function (Berger, 2013).

Zhang et al. (2014) and Brechmann and Czado (2011) provides strong evidence of the superiority of the VaR model calculating with the vine copula over historical simulation, mean-variance and DCC-GARCH models. For multivariate analysis, all developed models and methods are used to analyse the five, seven and fifteen companies from DAX 30 index, a major market indicator for the Eurozone.

The accurate valuation of options is critical for financial market analysts. Since Black and Scholes derived their formula on option pricing in the early 1970s, there has been a significant amount of theoretical and empirical work on the subject. The fundamental assumption underlying the Black-Scholes model is that the underlying asset return dynamics are captured by the normal distribution. However, over the last three decades, many pricing models have been presented as an alternative to the classic Black-Scholes approach as the underlying assumptions by Black-Scholes are violated by observed asset returns.

The rejection of constant variance Brownian motion results in a new class of stochastic volatility models introduced by Hull and White (1987), which suggest that volatility is stochastic, varying both for time and for the price level of the underlying security. Since then, many other stochastic volatility models have been developed, including Heston (1993), Duffie and Kan (1996), Ghysels et al.(1996), Diffie et al.(2000) and Balajewicz and Toivanen (2017).

In finance, all types of models belong to a class of Levy processes called exponential Levy processes. Exponential Levy models generalize the classical Black and Scholes formulation and enable jumps in the stock prices, while the independence and stationarity of returns are maintained.

Exponential Levy models are helpful in finance and can be divided into two classes. The first class is called jump-diffusion models, in which the normal change of prices is given by a diffusion process, interrupted by jumps at irregular breaks. The second class is called infinite activity models and consist of models accompanying an absolute number of jumps in each time interval.

Over the last few years several kinds of jump diffusion models have been developed. Two important jump-diffusion models are proposed by Merton (1976) and Kou (2002) respectively. The variance-gamma process and the normal-inverse Gaussian process are two examples of infinite activity processes. These models can represent both insignificant and persistent jumps as well as substantial and exceptional ones.

Merton's and Heston's models of option pricing were combined by Bates (1996), who suggested a stock price model with stochastic volatility and jumps. The Bates model ignores interest rate risk, while Scott (1997) introduced another model that supports interest rates to be stochastic.

There are several methods to price options. A lot of numerical methods need to implement the partial differential equations (PDE). The fast fourier transform (FFT) pricing method is very useful to efficiently price derivatives under any model with a known characteristic function, some of which are only expressible in this form (Hirsa and Neftci, 2014; Carr and Madan ,2002; Duffie et al., 2000; Bakshi and Madan, 2000; Lewis, 2000; Schoutens, 2003; Chourdakis, 2005; Fang and Oosterlee, 2008; Gong and Zhuang, 2016a and Deelstra and Simon, 2017).

The purpose of chapter 4 is to compare option pricing models, which are based on the stochastic volatility model, jump diffusion model, infinite activity model and combined stochastic volatility and jump diffusion model. We compare the performance of Heston's (1993) stochastic volatility model, Bates's (1996) as combined stochastic volatility model, Merton jump diffusion model, the Kou model as jump diffusion model and the variance gamma as infinite activity model with traditional the Black-Scholes model. We measure the mean absolute error relative to observed option prices.

We have computed VaR and ES estimates for the short position of the reference option, at -day and 10-days horizon, different ranges of significance levels, and for in sample and out of sample.

We evaluated various ES models based on partial Monte Carlo and full Monte Carlo methods. For partial Monte Carlo, we have calculated delta based and delta gamma based models. The preceding deltas and gammas were derived from the Black Scholes model (BSM), variance gamma model (VG), Heston model (HS), Bates model (Bat), Merton jump diffusion model and double exponential jump diffusion model (Kou). We evaluate 1-day and 10-days expected shortfall (ES) for options based on the minimum mean absolute error (MAE).

This dissertation has contributed to the knowledge in several ways. For univariate data in Chapter 2, our study provides further support for the usefulness and superiority of heavy tailed distributions especially asymmetric distributions in the US, UK, Japanese and German stock markets. Moreover, it proposes the use of heavy tailed distributions to measure financial risk for a longer horizon.

Chapter 3 not only presents and discusses the use of copulas and vine copulas for financial risk management, but also shows the term structure of risk for bivariate and multivariate data. To the best of our knowledge, this study is the first to explore multivariate term structure of risk with both static and dynamic correlation.

In Chapter 4, we not only compare non-normal option pricing with Black Scholes but also calculate ES with partial Monte Carlo and full Monte Carlo. Per our knowledge, this study is the first to derived delta and gamma from the Black Scholes model (BSM), variance gamma model (VG), Heston model (HS), Bates model (Bat), Merton jump diffusion model and double exponential jump diffusion model(Kou) for both 1-day and 10-days ES forecasts.

Chapter Two: Dynamic Expected Shortfall with Non-Normal Distributions: A Univariate Analysis

1. Introduction

Over the years, financial risk measurement has become a popular discipline among researchers, market analysts and regulators. The lack of adequacy of traditional risk measures was demonstrated by recent financial crises. Recently, value-at-risk (hereinafter VaR) and expected shortfall (hereinafter ES) have become the most common risk measures used in financial risk management practice. VaR is the maximum loss given a confidence level during a specific time interval, while ES is the average loss once this loss overcomes VaR. VaR has become the benchmark measure for financial market risk and is endorsed by the Basel Committee. Regardless of its simplicity and ease of implementation, VaR has been criticized for not being a coherent measure of risk (Artzner etal., 1999). Artzner et al. (1999) introduced a new coherent measure of financial risk referred to as ES, to overcome problems associated with VaR.

Various methods can be used to calculate VaR and ES. The choice of method depends on portfolio type, availability of computational resources and time constraints. Parametric, historical simulation and Monte-Carlo simulation are the main three approaches. Kim and Lee (2016) examined nonlinear regression models to calculate ES and VaR.

In this chapter, we mainly focus on the parametric approach. The parametric approach assumes that the asset returns follow a specific probability distribution (for example, the normal). The VaR and ES measures are based on the estimated parameters of the specific distribution.

Several models have been suggested in the literature to account for stylized facts in financial returns. GARCH models of volatility that were introduced by Engle (1982) and Bollerslev (1986) are particularly designed to capture the volatility clustering of financial returns (Engle and Patton, 2001; Jondwau and Rockinger, 2003; Poon and Granger, 2003). There are many different GARCH models available. The standard GARCH model does not consider the possibility of leverage effect, where volatility increases more by a negative shock than by a positive shock of the same significance. The leverage effect is noticeable in equity markets, where it exists as a strong negative correlation between the equity returns and the change in volatility (Alexander, 2008). In this chapter, we captured the asymmetric volatility response by an exponential GARCH (E-GARCH) model, threshold GARCH (TGARCH) model and nonlinear GARCH (NGARCH) model.

Precise modelling of the empirical distribution of financial returns is critical for estimating financial risk measures such as value at risk (VaR) and expected shortfall (ES). It is assumed in the financial risk management that returns follow a normal distribution. There is evidence that the empirical distribution of returns has fatter tails. The normality assumption first was criticized by Mandelbrot (1963) and Fama (1965). As observed from the finance literature, extreme events follow heavier tails than normal distribution, especially for high frequency data (McNeil, 1997; Da Silva et al., 2003; Jondeau and Rockkinger, 2003).

Historical simulation employs recent historical data so it allows the presence of heavy tails without assuming the probability distribution. However, McNeil and Frey (2000) argued that extreme quantiles were especially difficult to estimate under HS because extrapolation beyond past values is impossible. When any unusual value enters the sample, quantile estimates obtained by HS tend to be very volatile. Moreover, HS is unable to distinguish between high and low volatility periods, especially if a long data sample is used to mitigate the influence of the first two problems on the quality of tail-estimate.

Filtered historical simulation (FHS) proposed by Barone-Adesi et al. (1999) has an advantage of combining historical simulation with the power and adaptability of conditional volatility models. In the view of Zikovic and Aktan (2009), Angelidis et al. (2007), Kuester et al (2006) and Marimoutous et al. (2009) FHS perform better than other models for estimating VaR.

VaR and ES estimation frequently assume that long returns are normally distributed (Angelidis et al. 2004; Bellini and Figa-Talamanca, 2007; Chen and Liao, 2009; So and Yu, 2006). However, the assumption of normality of returns is inadequate for most of the time series of returns.

The most important deviations from normality are the heavy tails. For financial significance, the return distribution displays heavy tails because of the chance of an extremely large negative return. Distributions with heavier tail likelihood compared to a normal distribution are called heavy tailed. Heavy-tailed distributions are essential models in finance because equity returns and other changes in market prices have fat tails.

Extreme value theory (hereinafter EVT) is also an important method that focuses on the tail behavior of the distribution of returns. Stochastic volatility is also an important issue, as it has been observed that financial returns are not independent over time. In the literature, both stationary (unconditional) return distributions and conditional return distributions with stochastic volatility assumption are considered. However, EVT emphasizes only big prices changes and their related probabilities by directly examining the tails of a probability distribution (Tolika, 2014).

The Student *t* distributions have played a particularly significant role in financial research as models for the distribution of heavy-tailed phenomena such as financial markets data. However, the Student *t* distribution allows for heavy tails than the normal, but assumes that the distribution is symmetric. The Student *t* distribution can permit for kurtosis in the conditional distribution but not for skewness.

Hansen (1994) was the first to propose a generalization of the Student *t* distribution that allows modelling of skewness in conditional distributions of financial returns. Since then, several skew extensions of the Student-*t* distribution have been proposed in the financial risk management literature (Fernandez et al., 1995; Fernandez and Steel, 1998; Theodossiou, 1998; Branco and Dey, 2001; Bauwens and Laurent, 2002; Jones and Faddy, 2003; Sahu et al., 2003; Azzalini and Capitanio, 2003; Ayebo and Kozubowski, 2004; Aas and Haff, 2006, Komunjer, 2007; Zhu and Galbraith, 2010; Su et al., 2014 and Tolikas, 2014). Zhu and Zinde-Walsh (2009) suggested an alternative to the skewed t distribution that is skewed exponential power distribution.

However, for finance applications, skew extensions of the Student *t* distribution may not be able to capture all the asymmetries of distributions of financial returns, especially asymmetry in the tails. The asymmetric *t* distribution is more complex but allows for skewness as well as kurtosis. The generalized version of Student *t* distributions constrains the modelling of asymmetry and tails by using two parameters together control skewness and thickness of the left and right tails. Asymmetric generalizations of the skewed *t* distribution allow a separate parameter to control skewness and thickness of the left and right tails (Zhu and Galbraith, 2010). Zhu and Zinde-Walsh (2009) proposed an asymmetric exponential power distribution (AEPD) that in addition to skewness suggests different decay rates of density in the left and right tails.

Baker (2014) proposed a 6-parameter generalized *t* distribution(GAT) that allows asymmetry of scale and tail power. The GAT distribution generalizes the *t* distribution through two types of skewness (parameters c and r), and how soon tail behaviour starts (parameter α). The GAT distribution avoids the discontinuity of the second derivative of the AST distribution that is problematic for the estimation of standard errors on fitted model parameters because of the reliance on the second derivative of the likelihood. The GAT distribution can fit at least as well as the AST distribution. Moreover, varying the parameter α can sometimes improve the fit, although this option is not available with the AST distribution.

In the financial risk management literature, long tailed and asymmetric conditional distributions are studied substantially. Researchers extensively applied the fat tailed and asymmetric class of returns distributions to calculate VaR and ES. Nadarajah et al. (2014) developed a detailed survey of well-known techniques for expected shortfall calculation. Abad et al. (2014) explored the existing literature, especially new approaches on VaR estimation and back-testing approaches to evaluate different VaR methods, performances.

Baker and Jackson (2014) introduced another distribution known as the twin-*t* distribution(TTD). The TTD is heavy-tailed like the t distribution, but is closer to normality in the central part of the curve. This property has an implication that the distribution could be helpful when one requires a normal distribution, but with robustness to outliers.

We only include long tailed distributions in this chapter for the analysis of ES, as previous studies strongly recommend that those models which allow fat tails or skewness estimate VaR and ES correctly (Abad et al., 2014; Keller and Rosch, 2016).

Furthermore, we explore a new 6-parameter heavy tailed distribution by Baker (2014) alongside recent generalizations of the asymmetric Student *t* by Zhu and Galbraith (2010) and exponential power distribution by Zhu and Zinde-Walsh (2009) to allow separate parameters to control the skewness and the thickness of each tail for modelling the conditional distribution of asset returns and downside risk through expected shortfall. We also include the Student *t*-distribution by Bollesslev (1987), skewed *t* distribution by Zhu and Galbraith (2010), skewed exponential distribution by Zhu and Zinde-Walsh (2009) and twin- *t* distribution by Baker and Jackson (2014) as benchmark models.

The empirical analysis focuses on two different groups of data. Initially, we use the same data as used by Zhu and Galbraith (2011), which includes Standard and Poor's index, Adobe, Bank of America, JP Morgan, Pfizer and Starbuck. Furthermore, we extend this study to a new dataset of world four major world indexes. We include FTSE-100, NASDAQ 100, Nikkei -225 and DAX-30 indexes for the period 1995-2014.

Calculation of 1-day ahead ES follows a two-stage procedure. In the first step, a asymmetric GARCH-type stochastic volatility model is fitted to the historical data by maximum likelihood (ML). From this model, the so-called standardized residuals are extracted. The asymmetric GARCH-type model is used to calculate 1-step predictions of conditional mean (μ_{t+1}) and conditional standard deviation (σ_{t+1}). In the second step, various long-tailed and asymmetric distributions are applied to the standardized residuals and calculate $F^{-1}(p)$ with estimated parameters of distributions. Finally, one day ahead conditional ES_{t+1} calculated.

For the situation where the portfolio variance is time varying, going from 1-day-ahead to h-days ahead ES is complicated. As in the case of GARCH, scale by the horizon h is not attainable as variance means revert. Additionally, the returns over the next h days are not normally distributed. The answer of computing VaR and ES for longer horizons without knowing the detail for the distribution of returns can be found using simulation based methods. We consider Monte Carlo simulation (MCS) with GAT, AEPD, SEPD, AST, SSTD, ST and TTD as standardized distributions of returns and filtered historical simulation (FHS) for 5-days and 10-days ES in this research.

The contribution of this chapter is as follows. First, our study provides further support for the usefulness and superiority of fat tailed distributions especially asymmetric distributions in the US, UK, Japanese and German stock markets. Second, it proposes the use of fat tailed distribution to measure financial risk for a longer horizon. This is in contrasts with the current literature that mainly focuses on the one day ahead ES, our approach considers the usefulness of fat tail distribution for calculation of ES beyond 1-day.

To the best of our knowledge, our research is the first to consider two new distributions and compare them with other previous distributions for expected shortfall calculation. Moreover, our study is also first to calculate VaR and ES with Monte Carlo simulation with selective numbers of distributions and Filter Historical simulation for longer horizons. As in this manner, the contribution of our research to the literature is many fold.

The remainder of this paper is organized as follow: Section 2 addresses the previous studies; Section 3 addresses the methodological framework; Results are discussed in section 4; Section 5 concludes the findings.

2. Review of Literature

2.1. Introduction

The academic literature has concentrated on volatility reduction as the primary objective of risk measurement, and on variance as the main measure of risk. However, the main purpose of most corporate risk measurement is to avoid lower tail outcome rather than aiming to reduce variance. Many commercial banks and their financial institutions are determined to reduce the probability of lower tails outcome by VaR. VaR is maximum potential loss over a given period at a certain confidence level. Value at Risk is the most frequently used risk measure for asset or portfolio risk, and certainly, per the Basel framework is a preferred measure for market risk for banks and financial institutions.

McNeil (2000) define VaR as a high quantile of the distribution of losses, usually the 95th or 99th percentile. VaR is criticized for not a being coherent risk measure with no indication of the potential size of the loss that exceeds it. While using the VaR for risk management the optimizing behaviour of investor may results in extreme loss, because VaR comes up with confusing results. Consequently, such investor behaviour end with higher volatility in equilibrium security prices (Basak and Shapiro, 2001).

Artzner et al. (1999) suggest a new coherent risk measure the expected shortfall or tail conditional expectation instead of VaR. The tail conditional expectation is the expected size of a loss that exceeds VaR. Expected shortfall needs a larger size sample than VaR for the same level of accuracy (Yamai and Yoshiba, 2002b).

Yami and Yoshiba (2002b) claim that widespread use of VaR for risk management could lead to market instability. So, use of VaR and expected shortfall should not dominate financial risk management. The assumption of normal distribution in standard VaR models disregards the fat tailed properties of actual returns and underestimates the likelihood of extreme price movement. To capture the information disregarded by VaR and expected shortfall, it is necessary control to diverse aspects of the profit/loss distribution, such as tail fatness and asymptotic dependence.

2.2. Value at Risk and Expected Shortfall Techniques

Value at risk and expected shortfall is well presented in previous literature (Hull, 2006; Jorion, 2001; McNeil et al., 2005; Dowd, 2005 and Christoffersen, 2012). VaR is defined as a high quantile of the distribution of losses and expected shortfall is defined as the conditional expectation of loss for losses beyond VaR as a coherent measure of risk. Expected Shortfall is the expected size of a return exceeding VaR.

Perignon et al. (2008) reinforce the debate on the accuracy of the VaR models used by commercial Banks. They find very substantial evidence of overstatement of VaR by commercial banks. This empirical outcome against the common understanding that banks purposely under report their risk to reduce market risk capital charge. Due to overstating of VaR there is a high cost borne by the banks, but there are also other negative implications. Banks appear riskier than perceived; exaggeration about VaR reporting may also result in inefficient portfolio allocation; and redundant regulatory capital may restrict some attractive projects being funded, which has a harmful effect on the economy's growth.

Righi and Ceretta (2015) explore the criterion with regard to the value of various models and methods in ES estimation, taking into account well defined asset categories, estimation windows and significance levels. Unconditional, conditional and quantile regression based models have been used. To evaluate ES a new test based on the dispersion truncated by VaR has proposed alongside with the usual ES back-test.

Kim and Lee (2016) perform a simulation study for the expectile regression models generated from various GARCH classes of models and back-test the performance of the VaR and ES. Wied et al. (2016) proposed new test evaluating VaR for assessing the systemic risk of the banking sector.

There are a few stylized facts of financial returns that must be considered while selecting a risk method. It is widely agreed that financial asset return volatilities are time-varying. The generalized autoregressive conditional heteroskedasticity (GARCH) type of volatility specification can capture the stylized facts existing in the volatility of financial market returns. The GARCH family of models has been extensively used in financial research for estimating volatility and financial risk (Engle & Patton, 2001; Jondeau & Rockinger, 2003; Poon & Granger, 2003 and Leeves, 2007).

Previous research shows that risk models that perform poorly can work better by just apply volatility models like GARCH, exponential smoothing and Historical Simulation. GARCH volatility models are advantageous and frugal framework for modeling key dynamic features of returns, including volatility (Andersen and Bollerslev, 2006 and Bin Su et al., 2014).

Ardia and Hoogerheide (2014) explore the effect of the estimated frequency of frequently used GARCH models on one day ahead forecasts of Value at risk(VaR) and Expected Shortfall(ES). Practically the revising frequency is used for the extensive computational significance of substantial risk management system that includes thousands of models expected to be estimated and updated.

Even though generalized autoregressive conditional heteroscedastic (GARCH) models have been extensively used over the years, they are not free of limitations. Another stylized fact of financial volatility is that the large change in asset prices tend to be followed by further large changes and the small changes are followed by smaller changes known as volatility clustering (Brooks, 2008).

Daily returns have very little autocorrelation, meaning that the returns are almost impossible to predict from their past. The stock market exhibits occasional, very large drops but not equally large up- moves. Consequently, the return distribution is asymmetric or negatively skewed. Black (1976) associates this impact to the evidence as increase in leverage of the asset (i.e., the debt equity ratio), that causing the asset to be more volatile. Pagan and Schwert (1990), Engle and Ng (1993) defined the concept of news impact curve that associates past return shocks to current volatility. The asymmetric news impact on volatility is commonly referred to as the "leverage effect" (Christie, 1982; French et al., 1987; Pagan and Schwert, 1990) and Christophersen, 2012).

However, GARCH model assumes only significance of unanticipated excess return determines the conditional variance without considering the positive and negative changes. Several parameterizations have been proposed for a model in which the conditional variance responds asymmetrically to positive and negative residuals. The exponential GARCH or EGARCH model of Nelson (1991), the threshold GARCH model or TGARCH of Zakoian (1991), The GJR model of Jagan-Nathan and Runkle (1993) and Absolute GARCH model or AGARCH model of Hentschel (1995) have been proposed to present potential improvements over the conventional GARCH models.

In risk management practice the selection of the appropriate distribution is very critical. As it directly affects the measurement processes. Any drawback of statistical distribution can produce inaccurate estimation of financial risk and result in significant flaws in financial risk management. For instance, an inadequate capital provided to reduce the probability of extreme losses. Therefore, finding a statistical distribution that captured the extreme events in financial returns for VaR and ES calculation remains a serious research issue.

Non-normality of the asset return is also an important stylized fact, the heavy tailless is the most important deviations from normality and the Student *t* distribution captures this feature. In financial literature, it is common to use Student *t* distribution to capture the heavy tail-ness when modelling VaR and ES (Baillie& Bollerslev, 1992; Beine, Laurent, & Lecourt, 2002; Bollerslev, 1987; Angelidis et al., 2004; Huang & Lin, 2004; Ane 2006 and So & Yu, 2006). Even though the Student *t* distribution capture the feature of fat tails in financial returns. However, the problem of the *t* distribution is that it can allow for kurtosis in the conditional distribution but not for skewness because it is symmetrical around zero.

Baker and Jackson (2014) developed a new distribution, heavy-tailed like the *t* distribution that is closer to normality in the main body of the distribution. Hanssen (1994) was the first who addresses the lack of skewness in the Student *t* distribution and suggested a skew extension to the *t* distribution for modelling financial returns. Since then, many studies have examined the application of the Skew Student *t* distribution to modelling financial returns.

Hansen (1994) extends the Student *t* distribution with skewness parameter in the distribution. Fernandez and Steel (1998) introduce a distribution that account both skewness and fat tailless in the same distribution, which allow for very flexible modelling of skewness and fat tail features of the data. The skewness parameter controls the mass and degree of freedom account for the fat tailless.

There are many extensions of skewed *t* distribution are available in the literature (Hansen, 1994; Fernandez and Steel, 1998; Theodossiou, 1998; Branco and Dey, 2001; Bauwens and Laurent, 2002; Jones and Faddy, 2003; Sahu et al., 2003; Azzalini and Capitanio, 2003; Aas and Haff, 2006; Zhu and Zinde-Welsh, 2009 and Zhu and Gilbraith, 2010).

These skewed *t* distributions capture the asymmetry and tails by using two parameters which together control skewness and thickness of the left and right tails. For this reason, generalized skewed Student *t* distributions with separate skewness parameter and tail parameters can improve the fit and forecast of empirical data in the tail area are important to risk management practice.

Zhu and Galbraith (2010) proposed an asymmetric *t* distribution(AST) a three-parameter generalized *t* distribution. In their view, the generalizations of the Student *t* that allows asymmetry are potentially valuable in empirical modelling and forecasting. When one of the tail parameters goes to infinity, the AST behaves as a Student *t* on the left side and as a Gaussian on the right side, implying one heavy tail and one exponential tail. With two tail parameters, the AST can favour empirical distributions of daily returns of financial returns that are often skewed and have one heavy tail and one relatively thin tail.

Zhu and Zinde-Welsh (2009) proposed Asymmetric Exponential Power Distribution (AEPD) and suggest a generalizes of Skewed Exponential Power Distributions (SEPD) in a way that in addition to skewness, introduces different decay rates of density in the left and right tails.

Baker (2014) proposed a 6- parameter fat tailed distribution(GAT) that allows asymmetry of scale and tail power. The advantage of GAT by Baker (2014) over AST by Zhu and Galbraith (2010) is that it avoids the discontinuity of the second derivative of AST distribution. The discontinuity of second derivative cause no problem in fitting the distribution by likelihood maximisation, but the calculation of standard errors on fitted model parameters is troublesome because of reliance on the second derivative of the log-likelihood. The GAT distribution generalises the t- distribution through two types of skewness (parameter c and r) and how soon tail behaviour starts (parameter α). The GAT distribution can fit the financial data as correctly as the AST, and sometime even improves the fit by letting the parameter α to vary.

2.3. Extreme Value Theory (EVT)

McNeil (1997) consider EVT as an easily implemented method for specific risk measurement problems such as market risk measurement, and has an important role for future risk measurement developments. While in the view of Diebold et al. (1999), EVT provides convenience for the sub areas of risk management, but it will not transform the discipline of risk management. There is need to use EVT with caution as very low frequency events with small sample size are filled with pitfalls. However, it helps to draw smooth curves through extreme tails of empirical functions.

McNeil (2000) combines quasi maximum likelihood fitting of GARCH models to estimate the current volatility and extreme value theory (EVT) to estimate the tail of the distribution of GARCH model. The results indicate that conditional distribution of asset returns with current volatility performs better for VaR estimation than unconditional approach. As an alternative risk measure the ES has better theoretical properties than quantile, and should be modeled by a fat tailed distribution preferably EVT.

The EVT provides critical view on skewness, fat tails, rare events and stress scenarios. However, to estimate tails beyond or at the limit of mathematical data assumptions is needed, which are difficult to verify in practice. Other issues should be considered like multiple dependent risk factors unresolved, maximum likelihood estimators are not granted, there is need to consider other estimation procedure like methods of moments, Monte Carlo simulation or parametric bootstrapping can be considered when applied EVT theory to portfolios (Embrechts, 2000). Empirical literature proves that techniques based on the EVT and FHS are superior and feasible methods for forecasting VaR (Abad et al., 2014).

Extreme theory captures tail areas that are very different and possibly more advantageous than the tails obtained with the standard approach. For risk management techniques, extreme value theory provides a more accurate approach and tail estimation procedure for value at risk calculation. With extreme value distribution VaR calculations are more rigorous, and tails allow more authentic estimates of the occurrence rate and the extreme observations size (Bali, 2003).

In EVT block maxima and Peaks over Threshold (POT) are two extensively used estimation approaches. POT employs the data more effectively than the block maxima method when individual data points are available, like with high frequency financial data. Independence of financial data is an important requirement for the application of POT, but most of the financial data show external clustering. However, this problem can be overcome by combining autoregressive (AR) and GARCH with POT and estimate VaR and ES (Chen et al., 2010).

2.4. Longer Horizon Value at Risk and Expected Shortfall

For the computation of longer horizon VaR and ES longer, we need to depend on simulation based methods instead of closed form solution. We can employ the simulation based dynamic risk models to estimate VaR and ES at any horizon of interest and therefore to calculate the entire term structure of risk (Christoffersen and Diebold, 2000; Dowd et al, 2003; Asai and McAleer, 2009; Pesaran et al. 2009; Wang et al., 2011 and Christoffersen, 2012).

Dowd et al (2003) conducted a study to calculate the long term VaR. They proposed a simple technique to the calculation of long-term VaR to avoid issues related to the square-root procedure for hypothesizing VaR and anticipation of day-to-day volatility forecasts over longer horizons.

In the view of Wang (2011) VaR calculation over a short horizon, square root time rule (SRTR) scaling to transform to longer-term tail risks, is probably to be unsuitable and ambiguous. It is necessary to apply the SRTR carefully.

Degiannakis et al. (2014) estimated multi-day Value-at-Risk (VaR) and Expected Shortfall (ES) through the Monte Carlo simulation technique for computing multi-period volatility to a Fractionally Integrated Generalized Autoregressive Conditional Heteroscedasticity (FIGARCH) structure for leptokurtic and asymmetrically distributed portfolio returns.

Considering the discussion above, there is no obvious consensus in the financial literature on which is the most suitable 1-day ahead Value at risk (VaR) and Expected Shortfall (ES) model. Moreover, there is no extensive literature available on VaR and ES forecasting based on longer horizons.

Nadarajah et al. (2014) develop a detail investigation of notable techniques for expected shortfall computation. Their research contains 140 references that emphasis on recent developments in calculation of expected shortfall. The survey performs as a source of reference for further research for financial risk measures.

To be more precise, in our research we studied different 1-day ES methods by using different non-normal distributions. Another objective of this research is to examine the predictive ability of Monte Carlo Simulation (MCS) and Filtered Historical Simulation (FHS) in longer forecasting horizons. There is no extensive literature on VaR and ES forecasting based on longer horizons.

3. Methodological Framework

In the section, we explain in detail the procedure for the calculation and comparison of different models for the matter of VaR and ES estimation. We split this section into seven parts and then into further sub parts for best understanding: (1) asset returns; (2) stylized facts of returns; (3) stochastic volatility models; (4) dynamic risk models; (5) calculating VaR and ES models;(6) term structure of risk for univariate models; (7) back-testing ES models.

3.1. Asset Returns

The market risk is explained by the asset prices movements, and we observe prices in the financial market. However, most empirical studies involve asset returns for the risk analysis. The reason is that asset returns produces more attractive statistical properties than price series. There are many return definitions; our study involves log returns.

3.1.1. Simple Returns

P_t is the price of an asset for the period t (with assumption of no dividend). Simple gross return is:

$$1 + r_t = \frac{P_t}{P_{t-1}}$$

$$P_t = P_{t-1}(1 + r)$$

One period simple return:

$$r_t = \frac{P_t}{P_{t-1}} - 1$$

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

Holding the asset for k-period from t to t-k, k-period simple gross returns:

$$1 + r_t[k] = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \ldots \times \frac{P_{t-k+1}}{P_{t-k}}$$

$$1 + r_t[k] = (1 + r_t)(1 + r_{t-1}) \ldots (1 + r_{t-k+1})$$

$$= \prod_{j=0}^{k-1} (1 + r_{t-j}) \tag{2}$$

The k-period simple return:

$$r_t[k] = \frac{P_t - P_{t-k}}{P_{t-k}} \tag{3}$$

3.1.2. Log-Returns

The natural log of simple gross returns is called log return:

$R_t = ln(1 + r_t) = ln\frac{P_t}{P_{t-1}} = p_t - p_{t-1}$ (4)

where

$p_t = lnP_t$

An advantage of the log-return is that multi-period return is easy to calculate, as it is simply the sum of one period return.

$R_t[k] = ln(1 + r_t)[k] = ln[(1 + r_t)(1 + r_{t-1}) \ldots . (1 + r_{t-k+1})]$

$R_t[k] = R_t + R_{t-1} + \cdots + R_{t-k+1}$

$R_t[k] = \sum_{j=0}^{k-1} R_{t-j}$ (5)

3.2. Stylized Facts of Financial Returns

Many empirical studies identified common statistical properties of financial returns that are known as stylized facts. To gain better intuition of the stylized facts of financial data, we will look at a sample at Standard and Poor's 500 for the period 1995-2013.

Daily returns have very little autocorrelation. We can write autocorrelation function as:

$Corr(R_{t+1}, R_{t+1-\tau}) \approx 0,$ for $\tau = 1,2,3 \ldots \ldots \ldots \ldots ,40$ (6)

In other words, returns are almost impossible to predict from their past. Figure 1 shows the correlation of daily S&P 500 returns with returns lagged from 1 to 40 days. This is an evidence of very low conditional mean of returns.

The unconditional distribution of daily returns does not follow the normal distribution. Figure 2 shows a histogram of the daily S&P 500(1) return data with the normal distribution curve. Notice how the histogram is more peaked around zero than the normal distribution. Although the histogram is not an ideal graphical tool for analyzing extremes, extreme returns are also more common in daily returns than in the normal distribution. We say that the daily return distribution has fat tails. Fat tails mean a higher probability of large losses (and gains) than the normal distribution would suggest. Appropriately capturing these fat tails is crucial in risk management.

The stock market exhibits occasional, very large drops but not equally large up- moves. Consequently, the return distribution is asymmetric or negatively skewed. Some markets such as that for foreign exchange tend to show less evidence of skewness.

The standard deviation of returns completely dominates the mean of returns at short horizons such as daily. It is not possible to statistically reject a zero-mean return. Our S&P 500 data have a daily mean of 0.0002% and a daily standard deviation of 1.0346%.

Variance, measured, for example, by squared returns, displays positive correlation with its own past. This is called volatility clustering. This is most evident at short horizons such as daily or weekly. Observations of this type in financial time series have led to the use of GARCH models in financial forecasting and derivatives pricing.

Figure 3 shows the autocorrelation in squared returns for the S&P 500 data, that is:

$$Corr(R_{t+1}^2, R_{t+1-\tau}^2) > 0,\ for\ small\ \tau \tag{7}$$

Equity and equity indices display negative correlation between variance and returns. This is often called the leverage effect, arising from the fact that a drop in a stock price will increase the leverage of the firm if debt stays constant. This increase in leverage might explain the increase in variance associated with the price drop. The leverage effect can be captured by incorporating the EGARCH, NGARCH or TGARCH models. For news impact curve see figure 4.

Even after standardizing residuals by a time-varying volatility measure, they still have fatter than normal tails. We will refer to this as evidence of conditional non -normality (see, Figure 5).

3.3. GARCH Type Models

The volatility of financial asset returns changes over time, with periods when volatility is atypically high as compared with periods when volatility is unusually low. This volatility clustering behaviour depend on the frequency of the data and is very common in daily data. Volatility clustering has important implications for financial risk measurement. The generalized autoregressive conditional heteroscedasticity (GARCH) models of volatility that were introduced by Engle (1982) and Bollerslev (1986) are particularly designed to capture the volatility clustering of financial returns (Alexander, 2008).

Based on the above stylized facts our returns take the form:

$$R_{t+1} = \mu_{t+1} + \sigma_{t+1} z_{t+1} \qquad with\ z_{t+1} \sim i.i.d.\ D(0,1) \tag{8}$$

where σ_{t+1} is the conditional standard deviation of the return, so the conditional variance is σ_{t+1}^2 and μ_{t+1} is the conditional mean of the return. These values are considered to depend in a deterministic way on the past behavior of return. The innovation term $z_{t+1}$is assumed to be an independent with an identical unknown distribution. We assume that unknown distribution has a mean zero and variance 1 i.e. D (0, 1). In the time series language R_{t+1} is assumed to be a stationary process. We include GARCH (1, 1), EGARCH (1, 1), NGARCH (1, 1) and TGARCH (1, 1) with GAT, AEPD, SEPD, AST, STT, ST, and TTD as unknown distributions in this research.

The simplest generalized autoregressive conditional heteroskedasticity (GARCH) model of dynamic variance can be define as:

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 R_t^2 + b_1 \sigma_t^2 \tag{9}$$

where $\alpha_0 > 0, \alpha_1 \geq 0, b_1 \geq 0$

The symmetric GARCH model suppose the response of the conditional variance to negative market shocks is just the same as its response to positive market shocks of the same significance (Alexander, 2008). As we know from the stylized facts bad news or negative shocks have more impact on volatility than good news or positive shocks. Per Black (1976) this is because bad shocks lower the stock price, thus result in increased leverage (the debt and equity ratio) and stock become riskier. Asymmetric news impact on the volatility is referred as leverage effect. We introduce EGARCH (p, q), TGARCH (p, q) and NGARCH (p, q) to capture the asymmetric effect.

The EGARCH is an asymmetric GARCH model that specifies not only the conditional variance but the logarithm of the conditional volatility. It is widely accepted that EGARCH model gives a better in-sample fit than other types of GARCH models (Alexander, 2008).

The exponential GARCH model or EGARCH by Nelson (1991) captures the leverage effect and is defined as:

$$ln\sigma_{t+1}^2 = \alpha_0 + \alpha_1 z_t + \gamma\big(|z_t| - E(|z_t|)\big) + b_1 ln\sigma_t^2 \tag{10}$$

α_i capture the sign effect, so when $\alpha_i < 0$ there is leverage effect and γ_i represent the size effect, so when $\gamma_i > 0$ the leverage effect present.

The threshold GARCH referred as TGARCH model by Zakoian (1994) specification is:

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 R_t^2 + \gamma S_t R_t^2 + b_1 \sigma_t^2 \tag{11}$$

$$S_t = \begin{cases} 1 & if \quad R_t < 0 \\ 0 & if \quad R_t \geq 0 \end{cases}$$

The leverage implies that $\gamma > 0$.

The model specification for nonlinear GARCH (NGARCH) model by Engle and Ng (1993) is:

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 \sigma_{t+1}^2 (z_t - \gamma)^2 + b_1 \sigma_t^2 \tag{12}$$

$\gamma > 0$ implies present of leverage effect, indicates a negative correlation between the innovations in the asset return and conditional volatility of the return.

3.4. Financial Risk Measures

3.4.1. Value at Risk (VaR)

Value at risk is generally defined as possible maximum loss over a given holding period within a fixed confidence level. Let R be the random variable whose unknown cumulative distribution function is F_R. In this research, we calculate VaR based on log return. Let the P_t be the price of financial asset R for day t, R_t is defined as:

$$R_t = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{13}$$

VaR is defined as:

$$P(-R_t > VaR) = p$$

$$P(R_t < -VaR) = p$$

Christopherson (2012) defined "VaR as the number so that we would get a worse log return only with probability p. That is, we are *(1−p)* 100% confident that we will get a return better than VaR".

The dynamic of R_t is given by:

$$R_{t+1} = \mu_{t+1} + \sigma_{t+1}Z_{t+1}$$

$$Z_{t+1} = \frac{R_{t+1} - \mu_{t+1}}{\sigma_{t+1}}$$

one day ahead VaR_{t+1}^p:

$$P(R_{t+1} < -VaR_{t+1}^p) = p \tag{14}$$

$$= P(\mu_{t+1} + \sigma_{t+1}Z_{t+1} < -VaR_{t+1}^p) = p$$

$$= P\left(Z_{t+1} < \frac{-VaR_{t+1}^p - \mu_{t+1}}{\sigma_{t+1}}\right) = p$$

$$= F\left(\frac{-VaR_{t+1}^p - \mu_{t+1}}{\sigma_{t+1}}\right) = p$$

$$\frac{-VaR_{t+1}^p - \mu_{t+1}}{\sigma_{t+1}} = F^{-1}(p)$$

$$VaR_{t+1}^p = -\mu_{t+1} - \sigma_{t+1}F^{-1}(p) \tag{15}$$

Note that $F^{-1}(p)$ is the standardized quantile function, which measures risk in terms of the number of standard deviations from zero.

3.4.2. Problems associated with VaR Models

3.4.2.1. The Problem of Tail Risk

VaR main drawback is that it ignores extreme losses. The VaR number only tells us that 1% of the time we will get a return below the reported VaR number, but it says nothing about what will happen in those 1% worst cases.

3.4.2.2. VAR is not Coherent

Per Artzner et al. (1999) a risk measure that satisfies these conditions is called a coherent risk measure:

I. Monotonicity:

For all $X \leq Y$ for each outcome, then $\rho(X) \leq \rho(Y)$

II. Subadditivity:

For all X and Y,$\rho(X+Y) \leq \rho(X) + \rho(Y)$

III. Positive homogeneity.

For a positive constant k, $\rho(kx) = k\rho(x)$

IV. Translation invariance.

For a constant k,$\rho(k+x) = k + \rho(x)$

The VaR is not considered a coherent measure of risk, because it fails to satisfy the subadditivity property, i.e., the VaR of a two assets portfolio can be greater than the sum of the two individual VaR's.

3.4.3. Expected Shortfall

We previously discussed a key shortcoming of VaR, namely that it is concerned only with the percentage of losses that exceed the VaR and not the magnitude of these losses. The magnitude, however, should be of serious concern to the risk manager. Extremely large losses are of course much more likely to cause financial distress, such as bankruptcy, than are moderately large losses; therefore, we want to consider a risk measure that accounts for the magnitude of large losses as well as their probability of occurring. The challenge is to come up with a portfolio risk measure that retains the simplicity of the VaR, but conveys information regarding the shape of the tail. Expected Shortfall (ES), or Tail-VaR as it is sometimes called, is one way to do this.

Mathematically ES is defined as:

$$ES_{t+1}^{p} = -E_t\left[R_{t+1} | R_{t+1} < -VaR_{t+1}^{p}\right] \qquad (16)$$

where the negative signs in front of the expectation and the VaR are needed because the ES and the VaR are defined as positive numbers. The Expected Shortfall tells us the expected value of tomorrow's loss, conditional on it being worse than the VaR. Expected Shortfall. (ES) is coherent measure of risk, (Artzner et al., 1999).

The distribution tail gives us information on the range of possible extreme losses and the probability associated with each outcome. The Expected Shortfall measure aggregates this information into a single number by computing the average of the tail outcomes weighted by their probabilities. So, where VaR tells us the loss so that only 1% of potential losses will be worse, the ES tells us the expected loss given that we actual get a loss from the 1% tail.

3.5. Calculating Value at Risk and Expected Shortfall

Following by Christofersen (2012) and others the calculation of VaR and ES follows a two-stage procedure:

1. A GARCH-type volatility model is fitted to the historical data by maximum likelihood (ML). From this model, the so-called standardized residuals are extracted. The GARCH-type model is used to calculate 1-step predictions of conditional mean (μ_{t+1}) and conditional standard deviation (σ_{t+1}) .
2. Various long tail and asymmetric distributions are applied to the standardized residuals and calculate$F^{-1}(p)$ with estimated parameters of distributions. Finally, one day ahead Conditional VaR_{t+1} and conditional ES_{t+1} calculated.

3.5.1. Normal Distribution

In the simple model, it is assumed that returns are normally distributed. The normal density function is:

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} \tag{17}$$

The standard normal distribution is:

$$f(x) = \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{x^2}{2}\right\}$$

The log-likelihood of the normal distribution is express as:

$$L(\mu,\sigma^2;x_1,\dots x_n) = -\frac{1}{2}\sum_{i=1}^{n}\left[ln(2\pi) + ln(\sigma^2) + \frac{(x-\mu)^2}{\sigma^2}\right] \tag{18}$$

$$= -\frac{n}{2}ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x-\mu)^2$$

The maximum likelihood estimators of mean variance are:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\sigma_i^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})$$

The cumulative normal distribution function is:

$$\phi(x) = \frac{1}{2}\left[1 - erf\left(\frac{x-\mu}{2\sigma^2}\right)\right]$$

The cumulative of standard normal distribution function is:

$$\phi(z) = \frac{1}{2}\left[1 - erf\left(\frac{x}{2}\right)\right]$$

Value at risk when returns are normally distributed:

$$VaR_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1} z_q$$

where $z_q = \Phi_p^{-1}$ is

$$VaR_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1}\Phi_p^{-1} \quad (19)$$

Expected shortfall with normal distribution:

$$ES_{t+1}^{p} = -E_t\left[R_{t+1} | R_{t+1} \le -VaR_{t+1}^{p}\right]$$

$$= -\sigma_{t+1}E_t\left[z_{t+1} | z_{t+1} \le -VaR_{t+1}^{p}/\sigma_{t+1}\right]$$

$$= \sigma_{t+1}\frac{\phi\left(-VaR_{t+1}^{p}/\sigma_{t+1}\right)}{\Phi\left(-VaR_{t+1}^{p}/\sigma_{t+1}\right)} \quad (20)$$

By putting the value of VaR_{t+1}^{p} of normal distribution we get:

$$ES_{t+1}^{p} = \mu_{t+1} + \sigma_{t+1}\frac{\phi(\Phi_p^{-1})}{p} \quad (21)$$

3.5.2. Generalized Asymmetric *t* Distribution (GAT)

A 6-parameter asymmetric fat-tailed distribution (GAT) is proposed by Baker (2014). The pdf of the GAT is:

$$f_{GAT}(x|\mu,\phi,\alpha,r,c,v) = \frac{\alpha(1+r^2)}{r\phi}\frac{\left\{\left(cg\left((x-\mu)/\phi\right)\right)^{\alpha r} + \left(cg\left((x-\mu)/\phi\right)\right)^{-\alpha/r}\right\}^{-v/\alpha}}{B\left(\frac{v/\alpha}{1+r^2},\frac{r^2 v/\alpha}{1+r^2}\right)}\left(1+\left((x-\mu)/\phi\right)^2\right)^{-1/2} \quad (22)$$

where B is the beta function, ν > 0 controls tail power, μ is a center of location (not necessarily the mean), ϕ > 0 is a measure of scale (but not the variance, which may not exist), r > 0 controls tail power asymmetry, c > 0 controls the scale asymmetry, and α > 0 controls how early ‘tail behaviour’ is apparent.

The cdf of the GAT distribution is:

$$F_{GAT}(x|\mu,\phi,\alpha,r,c,v) = B\left(\frac{v}{\alpha(1+r^2)}, \frac{vr^2}{\alpha(1+r^2)}; q(x)\right) \tag{23}$$

where

$$q(x) = \frac{1}{1 + c^{-\alpha(1+r^2)/r}\left\{\frac{(x-\mu)}{\phi} + \sqrt{1 + \frac{(x-\mu)^2}{\phi^2}}\right\} - \alpha(1+r^2)/r}$$

Value at risk for GAT is:

$VaR_{GAT}(p|\mu,\phi,\alpha,r,c,v) = F_{GAT}^{-1}(p|\mu,\phi,\alpha,r,c,v)$

where $F_{GAT}^{-1}$is the inverse of cdf F_{GAT}.

Conditional Value at risk of GAT is:

$$VaR_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1} VaR_{GAT}(p|\mu,\phi,\alpha,r,c,nu) \tag{24}$$

The expected shortfall of the GAT is:

$$ES_{GAT}(p|\mu,\phi,\alpha,r,c,v) = -E_t[R|R < -VaR_{GAT}(p|\mu,\phi,\alpha,r,c,v)] \tag{25}$$

Conditional Expected shortfall for the GAT is:

$$ES_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1} ES_{GAT}(p|\mu,\phi,\alpha,r,c,v) \tag{26}$$

3.5.3. The Asymmetric Exponential Power Distribution

The asymmetric exponential power distribution is proposed by Zhu and Zinde-Walsh (2009).

$$f_{AEP}(x|\beta) = \begin{cases} \left(\frac{\alpha}{\alpha^*}\right)\frac{1}{\sigma}K_{EP}(d_1)exp\left(-\frac{1}{d_1}\left|\frac{x-\mu}{2\alpha^*\sigma}\right|^{d_1}\right), & x \le \mu \\ \left(\frac{1-\alpha}{1-\alpha^*}\right)\frac{1}{\sigma}K_{EP}(d_2)exp\left(-\frac{1}{d_2}\left|\frac{x-\mu}{2(1-\alpha)^*\sigma}\right|^{d_2}\right), & x > \mu \end{cases} \tag{27}$$

where $\beta = (\alpha, d_1, d_2, \mu, \sigma)^T$ is parameter vector, $\mu \in R \; and \; \sigma > 0$ is still location and scale parameters respectively, $\alpha \in (0,1)$ is skewness parameter. $d_1 > 0$ and $d_2 > 0$ are left and right tail parameters respectively, $K_{EP}(d)$ is the normalizing constant of exponential power distribution(EPD):

$$K_{EP}(d) \equiv \frac{1}{\left[2p^{1/d}\Gamma\left(1 + {1}/{d}\right)\right]}$$

and α^* is:

$\alpha^* = \alpha K_{EP}(d_1)/[\alpha K_{EP}(d_1) + (1-\alpha)K_{EP}(d_2)]$

Note that:

$$\left(\frac{\alpha}{\alpha^*}\right)K_{EP}(d_1) = \left(\frac{1-\alpha}{1-\alpha^*}\right)K_{EP}(d_1) = [\alpha K_{EP}(d_1) + (1-\alpha)K_{EP}(d_2)]$$

The AEPD density function is still continuous at every point and unimodal with mode at μ. The parameter α^* in the AEPD density provides scale adjustments respectively to the left and right parts of the density to ensure continuity of the density under changes of shape parameters(α, d_1, d_2).
The value at risk and expected short fall is computed analytically for the AEPD distribution in Zhu and Galbraith (2011).

Value at risk (VaR) of the AEPD distribution is:

$$VaR_{AEP}(p|\alpha, d_1, d_2) = \begin{cases} -2\alpha^* \left[d_1 Q^{-1}\left(\frac{p}{\alpha}, \frac{1}{d_1}\right)\right]^{\frac{1}{d_1}}, & p \leq \alpha \\ 2(1-\alpha^*) \left[d_2 Q^{-1}\left(\frac{1-p}{1-\alpha}, \frac{1}{d_2}\right)\right]^{1/d_2}, & p > \alpha \end{cases} \quad (28)$$

$Q(\alpha, x)$ denotes the regularized complementary incomplete gamma function:

$$Q(\alpha, x) = \int_x^\infty t^{\alpha-1} exp(-t)dt/\Gamma(\alpha)$$

Q^{-1} denotes the inverse of $Q(\alpha, x)$ and Γ is gamma function:

$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} exp(-t)dt$Dynamic Value at risk for AEPD is:

$$VaR_{t+1}^p = -\mu_{t+1} - \sigma_{t+1} VaR_{AEP}(p|\alpha, d1, d2) \quad (29)$$

The expected shortfall of AEPD is:

$$ES_{AEP}(p|\alpha, d_1, d_2) = -\frac{2\alpha^*}{p}\int_0^p \left[d_1 Q^{-1}\left(\frac{p}{\alpha}, \frac{1}{d_1}\right)\right]^{\frac{1}{d_1}} dp + \frac{2(1-\alpha^*)}{p}\int_0^p \left[d_2 Q^{-1}\left(\frac{1-p}{1-\alpha}, \frac{1}{d_2}\right)\right]^{\frac{1}{d_2}} dp \quad (30)$$

Dynamic expected shortfall for AEPD is:

$$ES_{t+1}^p = -\mu_{t+1} - \sigma_{t+1} ES_{AEP}(p|\alpha, d_1, d_2) \quad (31)$$

3.5.4. Skewed Exponential Power Distribution (SEPD)

Skewed is the special case of AEPD proposed by Zhu and Zinde-Walsh (2009), if $d_2 = d_1 = d$ implying $\alpha = \alpha^*$ The AEPD reduced to SEPD:

$$f_{SEP}(x|\beta) = \begin{cases} \frac{1}{\sigma} K_{EP}(d) exp\left(-\frac{1}{d}\left|\frac{x-\mu}{2\alpha\sigma}\right|^d\right), & x \leq \mu \\ \frac{1}{\sigma} K_{EP}(d) exp\left(-\frac{1}{d}\left|\frac{x-\mu}{2\alpha\sigma}\right|^d\right), & x > \mu \end{cases} \quad (32)$$

where $\beta = (\alpha, d, \mu, \sigma)^T$. The SEPD density is skewed to the right for α < 1/2 and to the left for α > ½.

The VaR for SEPD is:

$$VaR_{SEP}(p|\alpha,d) = \begin{cases} -2\alpha^* \left[d_1 Q^{-1}\left(\frac{p}{\alpha},\frac{1}{d}\right)\right]^{\frac{1}{d}}, & p \le \alpha \\ 2(1-\alpha^*)\left[dQ^{-1}\left(\frac{1-p}{1-\alpha},\frac{1}{d}\right)\right],^{1/d} & p > \alpha \end{cases} \quad (33)$$

Dynamic value at risk for SEPD is:

$$ES_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1} VaR_{SEP}(p|\alpha,d) \quad (34)$$

The Expected shortfall for the SEPD is:

$$ES_{SEP}(p|\alpha,d) = -\frac{2\alpha^*}{p}\int_0^p \left[dQ^{-1}\left(\frac{p}{\alpha},\frac{1}{d}\right)\right]^{1/d} dp + \frac{2(1-\alpha^*)}{p}\int_0^p \left[dQ^{-1}\left(\frac{1-p}{1-\alpha},\frac{1}{d}\right)\right]^{1/d} dp \quad (35)$$

Dynamic expected shortfall for SEPD is:

$$ES_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1} ES_{SEP}(p|\alpha,d) \quad (36)$$

3.5.5. Asymmetric Student *t* Distribution (ASTD)

ASTD proposed by Zhu and Galbraith (2010) and density function is defined as:

$$f_{AST}(x|\beta) = \begin{cases} \left(\frac{\alpha}{\alpha^*}\right) K(v_1)\left[1+\frac{1}{v_1}\left(\frac{x}{2\alpha^*}\right)^2\right]^{-\frac{v_1+1}{2}}, & x \le 0 \\ \left(\frac{1-\alpha}{1-\alpha^*}\right) K(v_2)\left[1+\frac{1}{v_2}\left(\frac{x}{2\alpha^*}\right)^2\right]^{-\frac{v_2+1}{2}}, & x > 0 \end{cases} \quad (37)$$

Where $\beta = (\alpha, v_1, v_2)$, $\alpha \in (0,1)$ is skewness parameter. $v_1 > 0$ and $v_2 > 0$ are left and right tail parameters respectively.

$K(v) \equiv \Gamma(v+1)/\sqrt{\pi v^3}$

where $\Gamma(.)$ is gamma function and α^* is:

$\alpha^* = \alpha(v_1)/[\alpha K(v_1) + (1-\alpha)K(v_2)]$

Denoting by μ and σ the location (center) and scale parameters, respectively, the general form of the AST density is expressed as $\frac{1}{\sigma} f_{AST}\left(\frac{x-\mu}{\sigma}; \alpha, v_1, v_2\right)$.

Note that

$$\left(\frac{\alpha}{\alpha^*}\right) K(v_1) = \left(\frac{1-\alpha}{1-\alpha^*}\right) K(v_2) = \alpha(v_1)/[\alpha K(v_1) + (1-\alpha)K(v_2)] \equiv B_{AST}$$

The value at risk of Asymmetric *t* distribution is:

$$VaR_{AST}(p|\alpha,v_1,v_2) = 2\alpha^* S_{v_1}^{-1}\left(\frac{min(p,\alpha)}{2\alpha}\right) + 2(1-\alpha^*) S_{v_2}^{-1}\left(\frac{max(p,\alpha)+1-2\alpha}{2(1-\alpha)}\right) \quad (38)$$

where $S_v(.)$ is the cumulative distribution function of the standard Student *t* distribution with ν degrees of freedom and S_v^{-1} is its inverse.

Dynamic value at risk for ASTD is:

$$VaR_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1} VaR_{AST}(p|\alpha,v_1,v_2) \quad (39)$$

The Expected shortfall function of the ASTD is:

$$ES_{AST}(p|\alpha, v_1, v_2) = -\frac{4B}{p}\left\{\frac{(\alpha^*)^2 v_1}{v_1 - 1}\left(1 + \frac{1}{v_1}\left[\frac{min(q-\mu,0)}{2\alpha^*}\right]^2\right)^{\frac{1-v_1}{2}} - \frac{(1-\alpha^*)^2 v_2}{v_2 - 1}\left(1 + \frac{1}{v_2}\left[\frac{min(q-\mu,0)}{2\alpha^*}\right]^2\right)^{\frac{1-v_2}{2}}\right\}$$

where $q = VaR_{AST} \equiv F_{AST}^{-1}$ (40)

Dynamic Expected shortfall for ASTD is:

$$ES_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1} ES_{AST}(p|\alpha, v_1, v_2) \quad (41)$$

3.5.6. Skewed Student *t* Distribution

By letting $v_2 = v_1 = v$ and $\alpha^* = \alpha$ in ASTD by Zhu and Galbraith (2010), we get new parameterization of skewed student *t* distribution (SSTED):

$$f_{SST}(x|\beta) = \begin{cases} \frac{1}{\sigma}K(v)\left[1 + \frac{1}{v}\left(\frac{x-\mu}{2\alpha\sigma}\right)^2\right]^{-\frac{v+1}{2}}, & x \le \mu \\ \frac{1}{\sigma}K(v)\left[1 + \frac{1}{v}\left(\frac{x}{2\alpha\sigma}\right)^2\right]^{-\frac{v+1}{2}}, & x > \mu \end{cases} \quad (42)$$

where $\beta = (\alpha, v)$.

The value at risk for the skewed student *t* distribution is:

$$VaR_{SST}(p|\alpha, v) = 2\alpha^* S_v^{-1}\left(\frac{min(p,\alpha)}{2\alpha}\right) + 2(1-\alpha^*)S_v^{-1}\left(\frac{max(p,\alpha)+1-2\alpha}{2(1-\alpha)}\right) \quad (43)$$

where $S_v(.)$ is the cumulative distribution function of the standard Student *t* distribution with ν degrees of freedom and S_v^{-1} is its inverse.

Dynamic value at risk for SSTD is:

$$VaR_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1} VaR_{SST}(p|\alpha, v) \quad (44)$$

The Expected shortfall function of the SSTD is:

$$ES_{SST}(p|\alpha, v) = -\frac{4B}{p}\left\{\begin{array}{c} \frac{(\alpha^*)^2 v}{v-1}\left(1 + \frac{1}{v}\left[\frac{min(q-\mu,0)}{2\alpha^*}\right]^2\right)^{\frac{1-v}{2}} \\ -\frac{(1-\alpha^*)^2 v}{v-1}\left(1 + \frac{1}{v}\left[\frac{min(q-\mu,0)}{2\alpha^*}\right]^2\right)^{\frac{1-v}{2}} \end{array}\right\} \quad (45)$$

where $q = VaR_{SST} \equiv F_{SST}^{-1}$

Dynamic Expected shortfall for SSTD is:

$$VaR_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1} ES_{SST}(p|\alpha, v) \quad (46)$$

3.5.7. Standardized *t* Distribution

The density function for the student *t* distribution with *v* degree of freedom is:

$$f_t(x;v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\frac{v}{2}\sqrt{v\pi}}\left(1+\frac{x^2}{v}\right)^{-\left(\frac{1+v}{2}\right)} \tag{47}$$

where Γ is the gamma function. When random variable *x* has student *t* distribution, the distribution has zero mean and skewness but for *v* >2 the variance of the student *t* distribution is not one.

$$\mu = E(x) = 0$$

$$Var(x) = \frac{v}{v-2}$$

Kurtosis and excess Kurtosis are:

$$k = \frac{3(v-2)}{v-4}, \qquad \zeta = k-3 = \frac{6}{v-4}$$

When $v = 1$ the Student density function is the Cauchy density function and when $v \to \infty$ the Student distribution converges to the normal distribution.

By standardizing *x* define *z* as:

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{x-\mu}{\sqrt{var(x)}} = \frac{x}{\sqrt{\gamma^2\, v/v-2}} = \frac{x\sqrt{v-2}}{\gamma\sqrt{v}}$$

Bollesslev (1987) proposes using the standardized *t* distribution with $v > 2$. The standardized *t* distribution density with v > 2 is then:

$$f_{\tilde{t}}(z,v) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}}\left(1+\frac{z^2}{v-2}\right)^{-\left(\frac{1+v}{2}\right)} \tag{48}$$

where $\Gamma(v) = \int_0^\infty e^{-x}x^{v-1}dx$ is the gamma function. v is the parameter that describe the thickness of tails.

The log-likelihood function of standardized-*t*-distribution is:

$$L_{\tilde{t}}(z;v) = \sum_{i=1}^{n} ln\big(f(z;v)\big)$$

$$L_{\tilde{t}}(z;v) = n\, ln\left(\frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\left(\sqrt{\pi(v-2)}\right)}\right) - \frac{1}{2}\sum_{i=1}^{n}(v+1)ln\left(1+\frac{z^2}{v-2}\right)$$

$$= n\left\{ln\left(\Gamma\left(\frac{v+1}{2}\right)\right) - ln\left(\Gamma\left(\frac{v}{2}\right)\right) - ln\left(\frac{\pi}{2}\right) - ln\left(\frac{v-2}{2}\right)\right\} - \frac{1}{2}\sum_{i=1}^{n}(v+1)ln\left(1+\frac{z^2}{v-2}\right) \tag{49}$$

In the standardized *t* distribution, random variable *z* has mean equal to zero and a variance equal to 1. The parameter *v >2* for standardized distribution to be well defined.

The standardized *t* distribution is symmetric around zero, and the mean μ, variance σ^2, skewness ζ_1, and excess kurtosis ζ_2 of the distribution are:

$$\mu = E(z) = 0$$

$$\sigma^2 = E\left[\left(z - E(z)\right)^2\right] = 1$$

$$\zeta_1 = \frac{E(z)^3}{\sigma^3} = 0$$

$$\zeta_2 = \frac{E(z)^4}{\sigma^4} - 3 = \frac{6}{v-4}$$

Under the standardized *t* distribution VaR is calculated as:

$$VaR_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1}\tilde{t}_p^{-1}(v) \tag{50}$$

where $\tilde{t}_p^{-1}$ is the *p*th quantile of standardized *t* distribution.

VaR for the student t distribution is calculated as:

$$P\left(R_t \leq t_q^{-1}\right) = p$$

$$as\ R_t = z_t\sqrt{\frac{v}{v-2}}$$

$$P\left(z_t\sqrt{\frac{v}{v-2}} \leq t_p^{-1}\right) = p$$

$$P\left(z \leq \sqrt{\frac{v}{v-2}}\,t_p^{-1}\right) = p$$

$$\tilde{t}_p^{-1} = \sqrt{\frac{v-2}{v}}\,t_p^{-1}$$

VaR for standardized *t* distribution is:

$$VaR_{t+1}^{p} = -\mu_{t+1} - \sigma_{t+1}\sqrt{\frac{v-2}{v}}\,t_p^{-1}(v) \tag{51}$$

where t_p^{-1} is the *p*th quantile of student *t* distribution.

The expected shortfall for the standardized *t* distribution is:

$$ES_{t+1}^{p} = \mu_{t+1} - \sigma_{t+1}ES_{\tilde{t}(v)}(p) \tag{52}$$

$$ES_{\tilde{t}(v)}(p) = \frac{C(v)}{p}\left[\left[1 + \frac{1}{v-2}t_p^{-1}(v)\right]^{\frac{1-v}{2}}\frac{v-2}{1-v}\right] \tag{53}$$

where $C(v) = \frac{\Gamma\left((v+1)/2\right)}{\Gamma(v/2)\sqrt{\pi(v-2)}}$

The main drawback of the Student *t* distribution is that it is symmetrical while financial time series can be skewed.

3.5.8. Twin *t* Distribution

Baker and Jackson (2014) applied Johnson's transformation to statistical modelling, and constructs a new long tailed distribution that is like the *t*-distribution. The *t* like distribution is useful for fitting data, it is more normal in the body of the distribution but has the same power law tail behaviour.

The probability density function is:

$$f(x) = \frac{2^{5/2}\Gamma(v/4+3/2)}{\sqrt{\pi v}\Gamma(v/4)(v+1)}\left(x^2/v + \sqrt{1+\left(1+(x^2/v)\right)^2}\right)^{-(v+1)/2} \tag{54}$$

As $v \to \infty$ the distribution becomes standard normal. The distribution function for $x > 0$ is:

$$F_{TTD}(x) = 1/2 + \frac{2^{3/2}x(S+C)^{-(v+1)/2}}{\sqrt{v}(v+1)B(v/4,3/2)} + \left(\frac{1}{2}\right) I(1-(C+S)^{-2}); v/4\,, 3/2 \tag{55}$$

where $S = \frac{x^2}{v}, and\ C = \sqrt{1+S^2}$

B is the beta function and I the regularized incomplete beta function.

Value at risk for TTD is:

$$VaR_{TTD}(p|v) = F_{TTD}^{-1}(p|v) \tag{56}$$

where $F_{TTD}^{-1}$is the inverse of cdf F_{TTD}.

Conditional Value at risk of TTD is:

$$VaR_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1}VaR_{TTD}(p|v) \tag{57}$$

The expected shortfall of the TTD is:

$$ES_{TTD}(p|v) = -E_t[R|R < -VaR_{TTD}(p|v)] \tag{58}$$

Conditional Expected shortfall for the TTD is:

$$ES_{t+1}^{p} = -\,\mu_{t+1} - \sigma_{t+1}ES_{TTD}(p|v) \tag{59}$$

3.6. Term Structure of Risk for a Univariate Model

The literature has relatively little to say on longer-term VaR and ES as compare to one-day risk. The most popular method is the square-root rule, and is usually applied to short time horizons.

If we consider a simple case of normal distribution with a constant variance σ_{PF}^2, per square–root rule, the VaR for returns over the next h days calculated on day t, as:

$$VaR_{t+1;h}^{p} = -\sqrt{h}\sigma_{PF}\Phi_p^{-1} = \sqrt{h}VaR_{t+1}^{p} \tag{60}$$

In the same way ES can be calculated as:

$$ES_{t+1,h}^{p} = \sqrt{h}\sigma_{PH}\frac{\phi(\Phi_p^{-1})}{p} = \sqrt{h}ES_{t+1}^{p} \tag{70}$$

However, when we consider a situation where the portfolio variance is time varying, going from 1-day-ahead to h-days-ahead VaR is not so straightforward.

As in the case of GARCH, scaling by the horizon h is not possible as variance does mean revert, and again the returns over the next h days are not normally distributed, although 1-day returns are supposed to be normally distributed. The question of computing VaR and ES for longer horizons without knowing the detail for the distribution of returns can be found with simulation based methods.

Following the work of Christoffersen (2012), in this research, we will consider Monte Carlo simulation (MCS) and filtered historical simulation (FHS) for h days VaR and ES.

3.6.1 Monte Carlo Simulation (MCS)

Consider GARCH (1, 1) with normal model of returns:

$R_{t+1} = \sigma_{t+1} z_{t+1}$ with $z_{t+1} \sim N(0,1)$

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta \sigma_t^2 \quad (71)$$

We get R_t at the end of day t and we can calculate σ_{t+1}^2 that is tomorrow's variance in the GARCH model.

In order to calculate term structure of risk with Monte Carlo simulation as described in Christoffersen (2012), we need the following steps:

1. Draw a set of artificial random numbers $\check{z}_{i,1}$ with zero mean and one variance by using random number generator from the standard normal distribution:

$$\check{z}_{i,1}, \quad i = 1,2 \ldots\ldots, MC \quad (72)$$

where *MC* denotes the number of draws.

2. Now, from these pseudo random numbers calculate a set of hypothetical returns for tomorrow:

$$\check{R}_{i,t+1} = \sigma_{t+1} \check{z}_{i,1} \quad (73)$$

3. Update the variances to get a set of hypothetical variances for the day after tomorrow, $t+2$:

$$\check{\sigma}_{i,t+2}^2 = \omega + \alpha \check{R}_{i,t+1}^2 + \beta \sigma_{i,t+1}^2 \quad (74)$$

4. Given a new set of random number generated from $N(0,1)$,

$$\check{z}_{i,2}, \quad i = 1,2 \ldots\ldots, MC$$

We can calculate the hypothetical return on day $t+2$:

$$\check{\sigma}_{i,t+3}^2 = \omega + \alpha \check{R}_{i,t+2}^2 + \beta \sigma_{i,t+2}^2 \quad (75)$$

We end up with MC sequences of pseudo future daily returns for day $t+1$ to day $t+h$. From these hypothetical future daily returns, we can easily calculate the hypothetical h day return from each Monte Carlo path:

$$\breve{R}_{i,t+1:t+h} = \sum_{k=1}^{K} \breve{R}_{i,t+h}\,, \qquad for\ i = 1,2,\dots,MC \tag{76}$$

5. If we collect these MC hypothetical *h*-days returns in a set $\left\{\breve{R}_{i,t+1:t+h}\right\}_{i=1}^{MC}$, then we can calculate the *h*-day value at risk simply by calculating the 100pth percentile as in:

$$VaR^{p}_{t+1:t+h} = -Percentile\left\{\left\{\breve{R}_{i,t+1:t+h}\right\}_{i=1}^{MC}, 100p\right\} \tag{77}$$

We can also use Monte Carlo to compute the expected shortfall at different horizons:

$$ES^{p}_{t+1:t+h} = -\frac{1}{p.MC}\sum_{i=1}^{MC} \breve{R}_{i,t+1:t+h} \,.\, 1\left(\breve{R}_{i,t+1:t+h} < -VaR^{p}_{t+1:t+h}\right) \tag{78}$$

where $1(\cdot)$ takes the value 1 if the argument is true and zero otherwise. The key advantage of the MCS technique is its flexibility. We can use MCS for any assumed distribution of standardized returns-normality is not required.

3.6.2. Filtered Historical Simulation

If we like that past returns data to tell us about the distribution directly without making specific distribution assumptions, FHS approach is the appropriate method. FHS combines model-based methods of variance with model-free methods of the distribution of shocks.

Consider a simple GARCH (1,1) model:

$$R_{t+1} = \sigma_{t+1} z_{t+1} \tag{79}$$

where

$$\sigma^2_{t+1} = \omega + \alpha R^2_t + \beta \sigma^2_t$$

Given a sequence of past returns, $\{R_{t+1-\tau}\}_{\tau=1}^{m}$, we can estimate the GARCH model and calculate past standardized returns from the observed returns and from the estimated standard deviation as:

$$\check{z}_{t+1-\tau} = R_{t+1-\tau} \quad for\ \tau=1,2,\dots,m \tag{80}$$

We will refer to the set of standardized return as $\{z_{t+1-\tau}\}_{\tau=1}^{m}$. The number of historical observations, *m*, should be as large as possible.

At the end of day *t* we obtain R_t and we can calculate σ^2_{t+1}, which is day *t*+1's variance in the GARCH model. Instead of drawing random $\check{z}s$ from a random number generator, which relies on a specific distribution, we can draw with replacement from our own database of past standardized residuals, $\{z_{t+1-\tau}\}_{\tau=1}^{m}$.

We end up with a sequence of pseudo future daily returns for day $t+1$ to day $t+h$. From these hypothetical future daily returns, we can easily calculate the hypothetical h day return as:

$$\breve{R}_{i,t+1:t+h} = \sum_{k=1}^{K} \breve{R}_{i,t+h}\,, \qquad for i = 1,2,\ldots,FH \tag{81}$$

where *FH* is the number of times we draw from the standardized residuals on each future date.

If we collect these FH hypothetical *h*-days returns in a set $\left\{\breve{R}_{i,t+1:t+h}\right\}_{i=1}^{FH}$, then we can calculate the *h*-day value at risk simply by calculating the 100pth percentile as in:

$$VaR_{t+1:t+h}^{p} = -Percentile\left\{\left\{\breve{R}_{i,t+1:t+h}\right\}_{i=1}^{FH}, 100p\right\} \tag{82}$$

We can also use Monte Carlo to compute the expected shortfall at different horizons:

$$ES_{t+1:t+h}^{p} = -\frac{1}{p.FH}\sum_{i=1}^{FH} \breve{R}_{i,t+1:t+h} \,.\, 1\left(\breve{R}_{i,t+1:t+h} < -VaR_{t+1:t+h}^{p}\right) \tag{83}$$

where $1(\cdot)$ takes the value 1 if the argument is true and zero otherwise.

3.7. Back-testing Risk Models

Lopez (1999) proposed a forecast evaluation framework based on loss function. By specifying a utility function and ranking the risk models, loss function satisfies the specific need of the risk manager. Let consider a vector of variables x_t known. Lopez (1999) loss function take the following specific form:

$$\Psi_{t+1} = \begin{cases} 1 + \left(VaR_{t+1|t} - x_{t+1}\right)^2 & \text{if violation occurs} \\ 0 & else, \end{cases} \tag{84}$$

which accounts for the magnitude of the tail losses $\left(VaR_{t+1|t} - x_{t+1}\right)^2$ and adds a score of one whenever a violation is observed. The model that minimizes the total loss $\sum_{t=1}^{T} \Psi_t$ outperforms other models.

This approach has a main drawback that the return x_{t+1} should be better compared with ES measure not with the VaR, as VaR does not give any evidence of the size of the expected loss. Therefore, the proposed loss function for the Expected shortfall can be proposed as:

$$\Psi_{1|t+1}^{(i)} = \begin{cases} \left|x_{t+1} - ES_{t+1|t}^{(i)}\right| & if\ voilation\ occurs \\ 0 & else, \end{cases} \tag{85}$$

$$\Psi_{2|t+1}^{(i)} = \begin{cases} \left(x_{t+1} - ES_{t+1|t}^{(i)}\right)^2 & if\ voilation\ occurs \\ 0 & else, \end{cases} \tag{86}$$

To judge the models by loss functions we calculate MAE and MSE:

$$MAE = \tilde{T}^{-1} \sum_{t=1}^{T} \Psi_{1|t+1}^{(i)} \tag{87}$$

$$MSE = \tilde{T}^{-1} \sum_{t=1}^{T} \Psi_{2|t+1}^{(i)} \tag{88}$$

The best model is preferred with the lowest MAE and MSE error.

4. Empirical Results

In this section, we explore the two different groups of data sets to estimate different non-normal distributions. Then, we evaluate different ES models from different returns of distribution for one-day head. Finally, we compare two simulation based methods to evaluate long term ES.

4.1. Data Analysis and Preliminary Tests

As the purpose of this chapter is to study the long tail *t* distributions to give a complete view of the ES estimation, we select the world's five major stock indices consisting of Standard and Poor's 500, FTSE 100, NASDAQ 100, Nikkie 225 and DAX 30 and several individual stocks from Standard and Poor's 500 including Adobe, Bank of America, J P Morgan, Pfizer and Starbucks.

We select the data of the daily closing prices of the S&P500, FTSE100, Nikkei225 and DAX 30 indices representing American, British, Japanese and German markets, while NASDAQ -100 index consists of non-American and non-financial top 100 companies on the NASDAQ exchange. These markets mainly represent developed economies from America, Europe and Asia with relevant trading volume for the period of 1995-2014. We also include another sample of S& P 500 to exclude the financial crises of 1998 for the period 1999-2014.

The individual companies stock of Standard and Poor's 500 represent different industries and lengths of data history. Adobe information represents information technology for the period 1986-2013, Bank of America and J P Morgan represent the bank and financial services for the period 1973-2013, Pfizer represents health care sector for the period 1973-2013 and Starbucks represent consumer discretionary for the period 1993-2013. Two individual companies represent the banking financial services sector because of the effects of the recent financial crises on the financial sector. All daily prices data has been taken from DataStream database. Adobe index consists of 7042 observations, Bank of America, J P Morgan, Pfizer all consist of 10666 values and Starbucks contains 5583 values. Table 1 shows the start date, end date and number of observations of the data analyzed in this chapter. Figure 6 plots daily prices, returns, squared returns and absolute returns for each analyzed data set. Each plot of each time series exhibits the typical empirical time series properties.

The plots of the closing prices of each data set are not stationary that mean the data does not revert around mean and it changes throughout the time series. On the other hand, the plot for the returns does fluctuate around mean. It is the desirable property of time series to have a stationary data set because the characteristics of a stationary time series allow handling models that are independent of a specific starting point, practically which may be difficult to obtain. The squared daily returns exhibit evidence of volatility clustering that large changes tend to be followed by large changes and suggests the presence of heteroskedasticity.

The summary statics are presented in table 2 and 3. The value of skewness is negative for all return series and indicating an asymmetry in the distribution of return. A negatively skewed distribution or skewed to the left has a long-left tail. Our all data series are characterized by many small gains and a few extreme losses.

As positive kurtosis indicates a relatively peaked distribution and negative kurtosis indicates a relatively flat distribution. A normal distribution has a kurtosis of 3. The kurtosis of our all data sets is greater than 3 reflecting heavy tails. We reject the null hypothesis of the normal distribution as the *p* value for the Jarque-Bera (1980) test is less than 0.05. The non-normality of the data is also apparent from the normal QQ plot figure 7 and figure 8. The Jarque-Bera Test confirms that all return series have non-normal distributions.

The Ljung-Box (1978) Q-statistics reported in Table 2 and 3 for both returns and squared returns for all data series also reject the null hypothesis of no autocorrelation through 20-lags at a 5% significance level.

Plots of log-returns show the so-called phenomenon of volatility clustering (i.e. large changes in returns are likely to be followed by large changes). Moreover, volatility seems to react differently to a big increase in asset price or a big drop in asset price, sometimes referred as the leverage effect. In application, volatility plays an important role in calculation of VaR and ES.

Before performing VaR and ES analysis, we first estimate different GARCH (1, 1) models. For the first group of data, we applied GARCH (1, 1) model for each data series to model the fluctuations of the variances of the time series data. The GARCH model also considers volatility clustering and tail behavior that is important features of financial time series. To imply leverage effect, we also applied three asymmetric GARCH models i.e. EGARCH (1,1), TGARCH (1,1) and NGARCH (1,1). We estimated the parameters of each model by Maximum Likelihood (ML) and then calculated the model diagnostic tests (ARCH LM Test, Q statistic, and Q2 statistics) and model comparison criteria (AIC, BIC, SIC, and SH). As we can see from the results EGARCH model outperform other models in almost all the case (see Appendix A for results).

We performed a Ljung-Box test and ARCH-LM test on each of the standardized residuals of world top indices and individual companies. The test statistics shown in tables 103, 104, 105, 106, 107, 108 and 109 in Appendix A suggest that the null hypothesis of no ARCH effect is accepted for all data series and for all GARCH models except for few cases. The Ljung-Box Q-statistic is applied on the standardized residual and standardized square for all data set and for all GARCH models accept the null hypothesis of no auto correlation except for few cases. We can now see that the both Ljung-Box Q-statistic and ARCH-LM test is satisfied reasonably well, which means that the purpose of our GARCH models in filtering the returns by autocorrelation was accomplished. Ljung-Box Q-statistic for standardized residuals for FTSE, NASDAQ, NIKKIE and DAX in table 110 also satisfied the no auto correlation.

The better model per these criterions is the one with lower AIC, BIC, SIC and SH. According to AIC, BIC, SIC and SH model chosen for SP (1) AB, PF and ST is EGARCH and TGARCH, for SP (2), Adobe model chosen is EGARCH, and NGARCH for JP. It is clear that EGARCH (1,1) is the best model for almost all data sets. We have chosen EGARCH model standardized residuals to calculate the parameters of distributions of return, conditional mean and conditional standard deviation for both groups of data.

4.2. Parameter Estimation

We can estimate the parameters of EGARCH and parameters of the distribution of returns together. However, we first estimate the parameters of all GARCH models with normal innovation and then choose the best GARCH model with Akaike Information Criterion (hereinafter AIC), Bayesian Information Criterion (hereinafter BIC), Schwarz Information Criterion (hereinafter SIC) and Shibata Criterion (hereinafter SH), based on minimum value of AIC, BIC, SIC and SH. EGARCH is turned best model for almost for all datasets. We have followed two step procedure because of several parameters involved in estimation. In the second step, we extract standardized residuals of EGARCH (1,1) model and estimate parameter of all distribution by maximum likelihood estimation.

Table 4 gives parameter estimates, log-likelihood values, AIC and BIC values for models fitted to Standard and Poor’s 500(1). Table 5 presents parameter estimates, log-likelihood values, AIC and BIC values for models fitted to FTSE-100. Table 6 gives parameter estimates, log-likelihood values, AIC and BIC values for models fitted to NASDAQ-100. Table 7 presents parameter estimates, log-likelihood values, AIC and BIC values for models fitted to Nikkei-225. Table 8 gives parameter estimates, log-likelihood values, AIC and BIC values for models fitted to DAX-30.

Table 9 presents parameter estimates, log-likelihood values, Akaike information criterion (AIC) and Bayesian Information criterion values for models fitted to S and P500(2). Table 10 presents parameter estimates, log-likelihood values, Akaike information criterion (AIC) and Bayesian Information criterion values for models fitted to Adobe. Table 11 presents parameter estimates, log-likelihood values, Akaike information criterion (AIC) and Bayesian Information criterion values for models fitted to Bank of America. Table 12 presents parameter estimates, log-likelihood values, Akaike information criterion (AIC) and Bayesian Information criterion values for models fitted to J P Morgan. Table 13 presents parameter estimates, log-likelihood values, Akaike information criterion (AIC) and Bayesian Information criterion values for models fitted to Pfizer. Table 14 presents parameter estimates, log-likelihood values, Akaike information criterion (AIC) and Bayesian Information criterion values for models fitted to Starbucks.

The bold values of AIC and BIC criteria in all tables represent top three best models for the specific data set. Per AIC and BIC values in Table 4, the best-fitting models for Standard and Poor's 500(1) data are the generalized asymmetric *t* distribution (GAT), Student *t* distribution (ST) and double *t* distribution (TTD). All above models have lowest AIC and BIC while, asymmetric t distribution (AST) and skewed exponential power distribution (SEPD) have highest AIC and BIC value respectively.

If we look carefully at the values of AIC and BIC in tables 5, 6, 7 and 8 for the indexes FTSE, NASDAQ, Nikkei and DAX, we observed that the generalized asymmetric *t* distribution (GAT), Student *t* distribution (ST) and double *t* distribution (TTD are the top three models. When we compare GAT and AST, we concluded that GAT clearly outperforms AST. AEPD distribution as an alternative to AST and GAT performs better that AST but under performs GAT.

Now we will discuss the estimated parameter results of induvial stocks. Table 9 shows that the best fitting models for Standard and Poor's 500(2) data are the GAT, ST and TTD distribution. This also indicates that when we avoid financial crises of 1998, the best-fitted models remain the same. As reported by AIC and BIC values in Table 10, the best-fitting model for Adobe data is the ST, GAT and TTD. Per AIC and BIC values in Table 11, ST, GAT and TTD are the best fitted models for the Bank of America, For J P Morgan in Table 12, Pfizer in Table 13 and for Starbucks in table 14 GAT, ST and TTD remain the top three models.

We see that the best fitting models for all the eleven data sets are our two new distributions (GAT and TDD) and standardized student *t* distribution. Overall the GAT distribution is the best model, as it has many advantages over standardized student *t* distribution. Standardized *t* distribution does not support asymmetry. None of Zhu-Welsh (2009) asymmetric exponential power distribution and Zhu and Galbraith (2010) asymmetric t distribution is the best-fitted model. Per AIC and BIC, the new twin *t* distribution also performs better than asymmetric *t* distribution and exponential power distribution for all data sets.

With respect to model fit, as we have noted, both AIC and BIC favour GAT, ST and TTD instead of AST and AEPD and their skewed versions.

4.3. One day ahead Expected Shortfall Back-testing

Once all the parameters of all the distributions are calculated, we can calculate the VaR and ES for 5%, 2.5%,1% and 0.5% significance levels. As we know from the previous literature that there are many problems associated with VaR, so, we only evaluate different ES models as a better measure of risk.

The competing risk models included in this study are, the GAT model, the AEPD model, the SEPD model, the AST model, the SST model, the ST model and TTD model.

We compare ES by a loss function that calculated the difference between the actual and the expected losses. Model ranking by MAS and MSE provided in the table 4. The mean absolute error and the mean squared errors appear small enough to suggest that the best fitting models are reasonable. In table 4 we show the predictive performance for expected shortfall risk on world major indexes and five individual companies of S and P 500. The entries in the table are the mean absolute error and the mean square error of the expected shortfall predictions for one day ahead. The values in parenthesis show the ranks of the model.

Table 15 shows the result for Standard and Poor's(SP), and per MAE and MSE the best model is EG-GAT and EG-TTD, as both having the least MAE and MAE at 5% significance level. The EG-SST is the second-best model. We also observe that for the significance levels of 1% and 0.5% the performance of EG-GAT is like EG-AEP and EG-SEP and MAE while MSE of EG-TTD are less than that of EG-GAT. It is noticeable that both EG-AST and EG-ST are the poorer models with highest MAE and MSE respectively. When we compare asymmetric *t* distribution of Baker (2014) GAT and Zhu and Galbraith (2010) AST, the result clearly indicates that MAE of GAT is significantly greater than AST.

Tables 16, 17, 18 and 19 present the ES evaluation results for FTSE, NADAQ, NIKKIE and DAX. ES evaluation results for FTSE in table 16 shows that at 5%, 2.5% and 1% significance levels EG-TTD is the best model based on MAE and MSE, and for the 0.5% significance level it is the second best model. EG-SEPD came second for 5%, 2.5% and 1% significance levels and come first at the 0.5% significance level. EG-GAT remains in the top three models for all significance levels for both evolution measures.

Table 17 shows that MAE and MSE for NASDAQ of EG-TDD are slightly greater than of EG-GAT at the 5% significance level. At 2.5%, 1% and 0.5% significance levels EG-SEPD has slightly less MAE and MSE than of EG-GAT. The MAE and MSE of EG-AST is significantly higher than of EG-GAT and EG-AEPD. ES evaluation results for NIKKIE in table 18 shows almost the same results of NASAQ. In table 19 for DAX-30 index the MAE and MSE indicated that EG-TTD is lowest at 5% and 2.5% significance levels.

The MSE of EG-SSTD is the lowest at 1% significance level. For all other confidence level EG-SSTD have the second highest values for MAE and MSE followed by the EG-AST with highest error values.

From the table 20 again EG-GAT and EG-TTD are best models, and EG-SST is second best model at 5% significance level. At 2.5% significance level, EG-TTD and EG-SST are the top models and EG-GAT is second best model. EG-SST, EG-SEP and EG-TTD are the first, second and third top models respectively at 1% and 0.5% significance level. Again, at lower significance level EG-GAT and EG-AEP behave similarly. EG-AST have the highest MAE and MSE values at all the significance levels.

Table 21 compares the MAE and MSE results of Adobe and it indicates that per MAE EG-SST and EG-GAT are the best models and per MSE EG-GAT and EG-SST are best model at 5% significance level. EG-TTD and AEPD are third and fourth better models respectively. Although EG-GAT remains in the top three models at 5%, 2.5% and 1% significance levels, at 0.5% significance level EG-AEP and EG-SEP performed better than EG-GAT.

Table 22 present the MAE and MSE values of Bank of America and 23 represent J P Morgan. For both data sets EG-SST, EG-GAT and EG-TTD are top models at 5% and 2.5% significance levels with lower MAS and MSE, and EG-AST and EG-ST have highest MAS and MSE respectively.

The values of MAE and MSE of Pfizer are present in table 24. The EG-GAT MAE and MSE values are slightly higher than of EG-SST, however, lower than all other models at the 5% significance level. EG-TTD is third best model at the 5% significance level. The Starbucks MAE and MSE value in table 25 shows that again EG-SST and EG-GAT are the top two models and EG-TTD is the third best model. The results for the predicted expected shortfall can be summarized as follows:

1. In the empirical prediction of expected shortfall, GAT model and TTD models are in the top three models at 5% and 2.5% significance levels in almost all cases.
2. AST model have highest values of MAE and MSE for almost all datasets and significance levels.
3. The skewed version of AST model (SSTD model) has the second highest MAE and MSE in all cases except few exceptions.
4. AEPD model as alternative to asymmetric distributions performs better than the AST, but GAT model clearly outperforms AEPD.
5. The skewed version of AEPD model (SEPD model) performs better than of the skewed version of AST model (SSTD model).
6. When we compare two samples of Standards and Poor's 500, we observed for both SP and SP (2) the GAT and TTD models are ranked as the top models and EG-SSTD is ranked as the second-best model at 5% and 2.5% significance values. However, as significance values decrease the result may slightly differ.
7. The results of MSE and MAE indicate different model ranking for the same significance level. However, the top three models ranking remain the same.
8. The result gave us a strong indication that new parameterization of generalized asymmetric distribution provides valuable improvement in the results. As discussed, when we compare ES back-testing for two asymmetric *t* distributions, MAE and MSE of GAT are significantly lower than of AST. These results indicate strong implication for further research for use of asymmetric *t* distribution as expected shortfall measure.

Based on the expected shortfall back-tests conducted through MAE and MSE, we conclude that the GAT model by Baker (2014) outperforms the competing AST by Zhu and Galbraith (2010) model by a significant margin. As an alternative to asymmetric t distribution AEPD model also underperforms GAT model.

4.4. Longer Horizon Expected Shortfall Back-testing

In this section, we evaluate expected shortfall for 5- days and 10- days horizon. We have the Monte Carlo simulation and filtered historical simulation for h days ES calculation. As mentioned earlier, the key advantage of the MCS technique is flexibility. We can use MCS for any assumed distribution of standardized returns, normality is not required. We used GAT, AEPD, SEPD, AST, SSTD, ST and TDD as distribution of standardized return. While, FHS combines model-based methods of variance with model-free methods of the distribution of shocks.

ES evolution results for longer time horizon for univariate data present in tables 26, 27, 28, 29, 30, 31, 32, 33, 34, and 35. The objective of this analysis has thus been to compare Monte Carlo simulation and Filtered Historical Simulation approaches that can be used to calculate long term risk in the univariate risk models.

Table 26 indicates the expected shortfall back-test results for S & P 500 for 5 days ahead. When we compare FHS model with different Monte Carlo models that result indicates that FHS has lowest MAE at 1%, 2.5% and 5% significance levels for 5-day ES. While, at the 10% significance levels MC-GAT has lowest MAE and MSE values. When we compare different Monte-Carlo models, the results show that MC-SEPD model has the lowest MAE and MSE at the 1% significance level, at 2.5%, 5% and 10% significance levels MC-GAT has lowest the MAE and MSE values.

For FTSE in table 27 FHS remain the best models for all significance levels by both evaluation measures MAE and MSE. MC-SST has the second the lowest MAE and MSE at 1% and 2.5 % significance levels, but at 5% and 10% significance levels MC-GAT has lower MAE and MSE values. At 1% and 2.5 % significance levels, MC-SEPD has lower MAE and MSE than MC-GAT. In table 28 MAE and MSE values for NASDAQ illustrate that FHS and SST is the first and second-best model respectively.

For both Nikkei and DAX in tables 29 and table 30 FHS has minimum MAE and MSE values at 1% and 2.5%, while at 5% and 10% significance levels MC-GAT has minimum MAE and MSE values. MC-AST has highest MAE and MSE for all most all the data sets. MC-AEPD, MC-TTD and MC-ST give us mix results.

Tables 31, 32, 33, 34 and 35 present 10-day ES evolution results at 1%, 2.5%, 5% and 10% significance levels. It is clear from the MAE values that MC-GAT has smallest MAE and MSE values for Standards & Poor's 500 in table 31, FTSE 100 in table 32, Nikkei in table 34 and DAX-30 in table 35 for all significance levels. Only for NASDAQ in table 33, MC-TTD has slightly lower MAE and MSE than of MC-GAT.

Both MC-AEPD and MC-SEPD have highest MAE and MSE for all the datasets and significance level. MAE and MSE values for MC-AST remain higher than FHS, MC-SST, MC-ST and MC-TTD for all the data set, except for Nikkei where AST has lower MAE and MSE than of MC-SST.
The results for the predicted expected shortfall for 5-days and 10-days can be summarized as follows:

1. The ES evolution results in table 5 and 6 results suggest different best models across the horizon.
2. As for 5-days FHS is the best model for 1% and 2.5% significance levels and MC-GAT is the best model at 5% and 10% significance levels.
3. However, when we increase the number of horizon to 10-days, MAE and MSE values clearly suggest MCS-GAT as best model.
4. FHS is no longer the best model even for a single data set at any significance level for 10- days horizon. However, it performs better than MC-AEPD, MC-SEPD and MC-AST.
5. Both MC-AEPD and MC-SEPD perform very poorly to forecast ES for 10-days horizon at various significance levels.

To conclude, after checking the ES evolution results for 5-days and 10- days horizons, we can infer that results of ES models are not similar across different time horizons. However, the satisfactory predictions of the MC-GAT are in accordance with the findings of 1-day ahead ES evaluation. Again, like 1-day ahead ES, the MC-GAT model out performs MC-AST model and gives a clear implication for the use of the GAT distribution for multi-days risk forecasting.

5. Concluding Remarks

As we discussed earlier, expected shortfall (ES) is a superior measurement of market risk. However, measuring and forecasting market ES for financial assets are challenging because of the asymmetry and heavy tailed-ness of return distributions. Recently, good progress has been made in the academic literature to deal with features of asymmetry and heavy tailed-ness for different financial assets.

Findings of Kellner and Rosch (2016) recommend that only models which allow for heavy tailed-ness or skewness can accurately estimate both VaR and ES.

Our research has tried to make additional progress on this important research issue. We have compared a new asymmetric *t* distribution (GAT) by Baker (2014) with asymmetric exponential power distribution by Zhu and Zinde-Welsh (2009) and asymmetric *t* distribution by Zhu and Galbraith (2010). These distributions allow separate parameters to control the skewness and the thickness of each tail, that may result in potential improvements in forecasting ability of expected shortfall because of better estimates of the left tail thickness. In addition, the GAT distribution adds another parameter of how soon tail behaviour starts.

In this chapter, ES for only long tailed distributions are evaluated for world five major indexes and five individual companies of S and P 500 for 1-day ahead returns. We compare ES from EGARCH (1, 1) of various non-normal distributions with two new distributions proposed by Baker (2014) and Baker and Jackson (2014). For longer horizon ES, we have used filter historical simulation as model-free methods of the distribution of shocks and Monte Carlo simulation with GAT, AEPD, SEPD, AST, SSTD, ST and TTD as standardized distribution of returns.

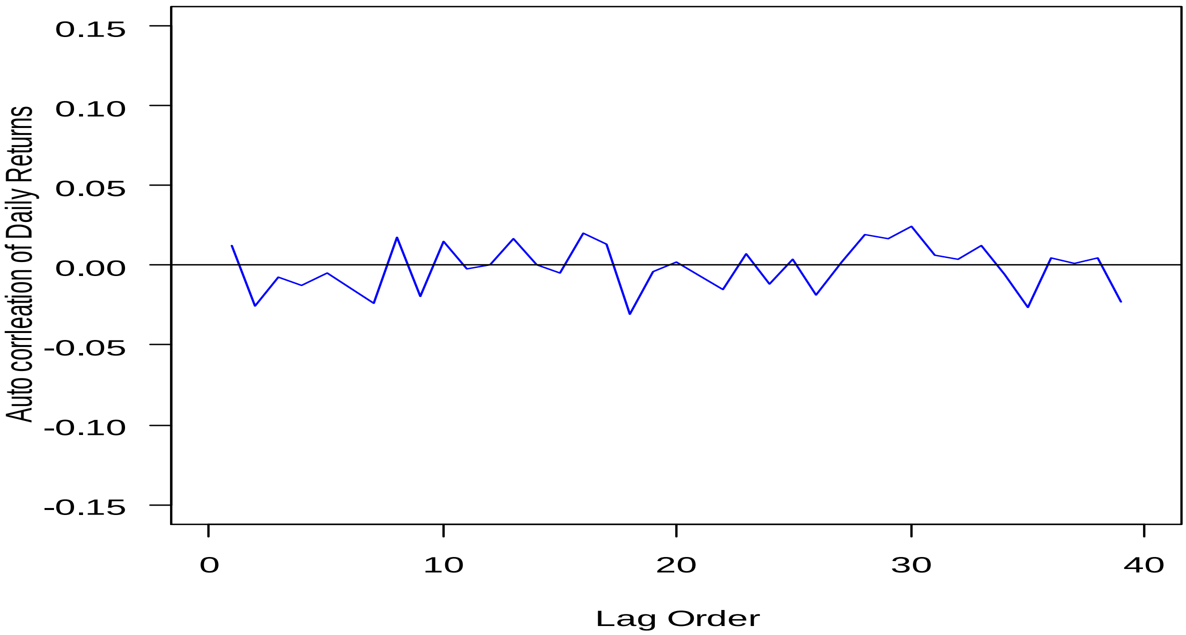
The empirical results indicate that the asymmetric *t* distribution of Zhu and Galbraith (2011) is not the best distribution for 1-day head ES. We find that GAT outperforms both AST of Zhu and Galbraith (2010) and APED by Zhu and Zinde-Welsh (2009). Our results also indicate that AEPD and AST perform like the results of Zhu and Galbriath (2011).

Our results suggest that generalized distributions that account for both asymmetry and fat tails are important for risk analysis and produce good results. As also suggested by Abad et al. (2014), asymmetric extensions of parametric methods of VaR and ES estimation present promising results.

Simulation based MCS and FHS methods for calculating ES indicate the importance of long term risk. For both 5 days and 10 days ES results show that MCS with GAT distribution of return performs better than AEPD and AST.

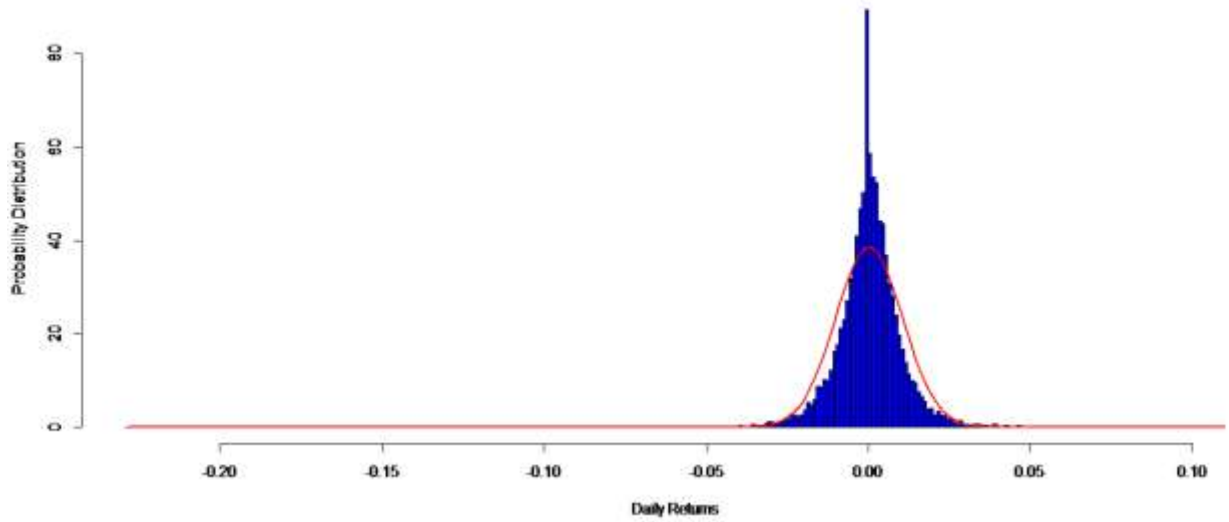
One of the limitations of applying Monte Carlo with GAT, AEPD, SEPD, AST, SSTD, ST and TTD as standardized distribution of returns rather than square-root-of-time rule to calculate longer horizon ES is that the Monte Carlo method is a lengthy procedure.

Figure 1: Autocorrelation of Daily S&P 500 for the Period 1995-2013.



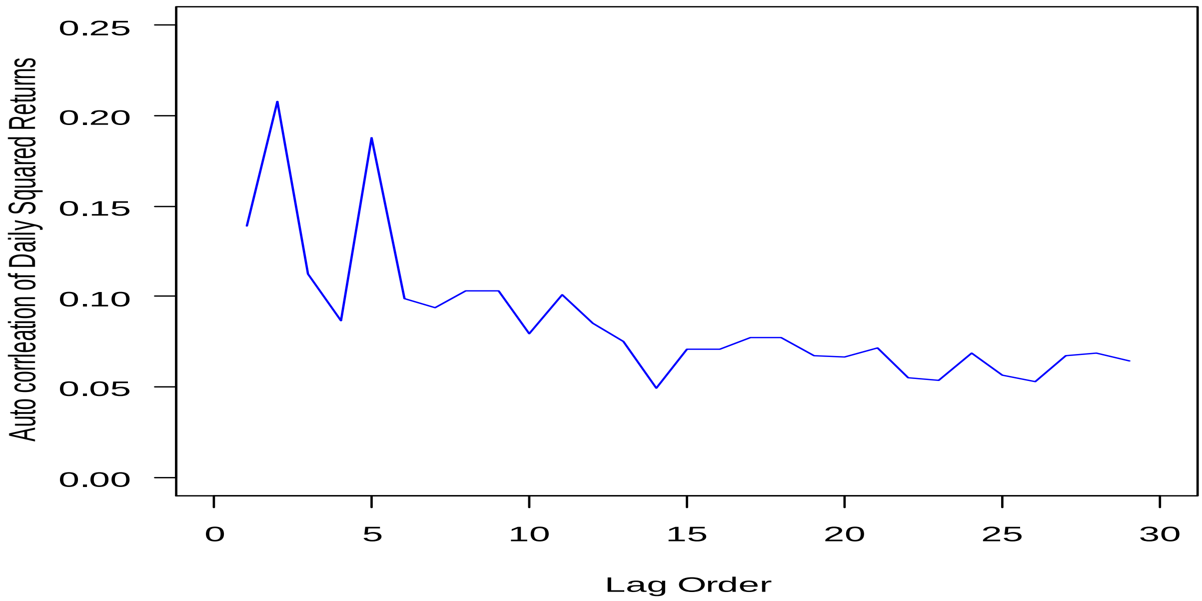
Notes: Using daily returns on the S&P 500 index from 1995-2013, the figure shows the autocorrelations for the daily returns. The lag order on the horizontal axis refers to the number of days between the return and the lagged return for a specific autocorrelation.

Figure 2: Histogram of Daily S&P 500 Returns and the Normal Distribution.



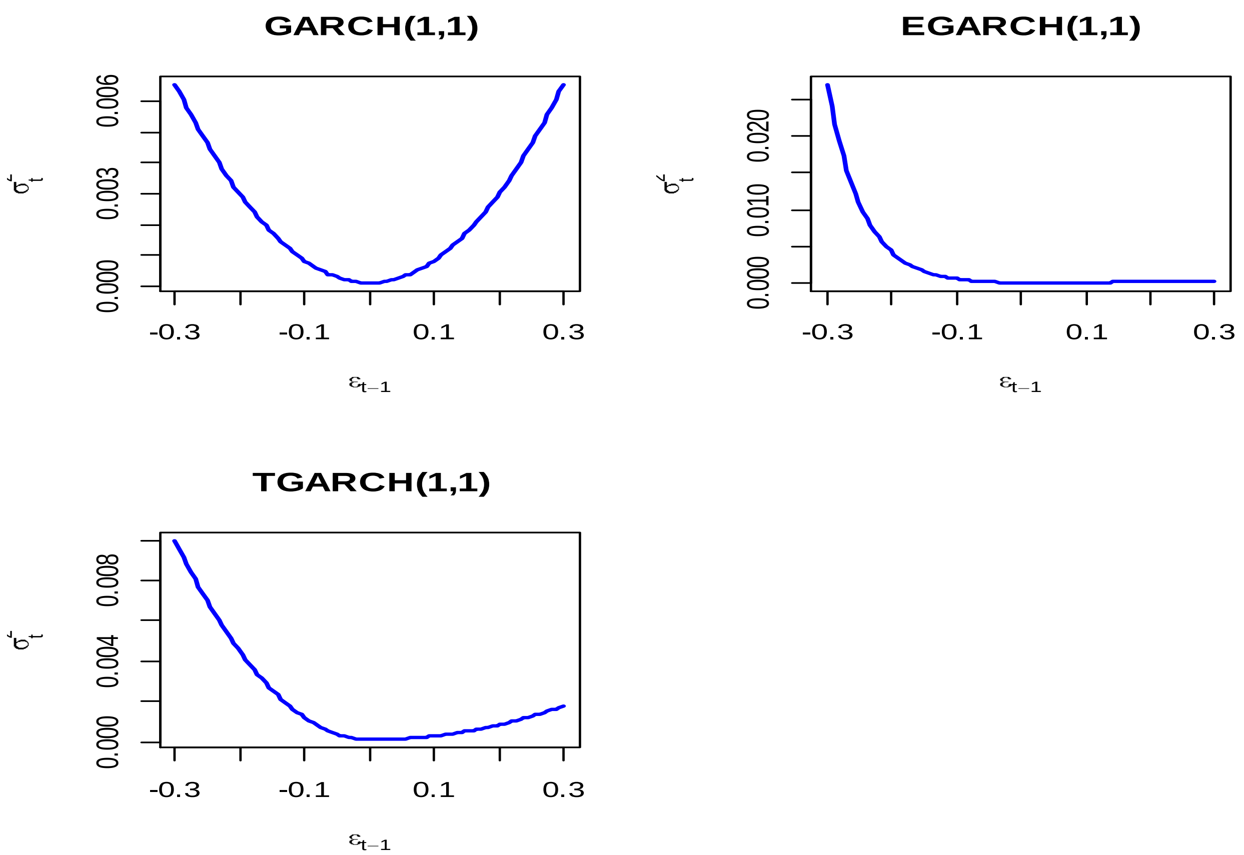
Notes: The daily S&P 500 returns from 1995-2013 are used to construct a histogram shown by bars. A normal distribution with the same mean and standard deviation as the actual returns is shown using the curve.

Figure 3: Autocorrelation of Squared Daily S&P 500 Returns 1995– 2013.



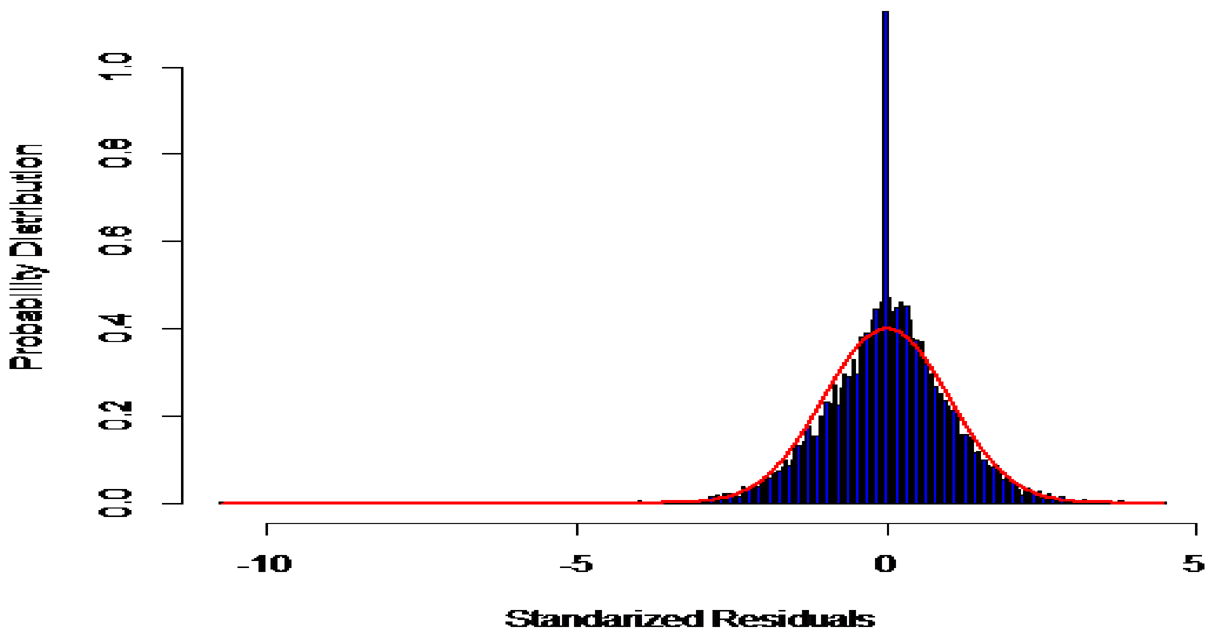
Notes: Using daily returns on the S&P 500 index from 1995-2013 the figure shows the autocorrelations for the squared daily returns. The lag order on the horizontal axis refers to the number of days between the squared return and the lagged squared return for a specific autocorrelation.

Figure 4: News Impact curve of Different GARCH Models.



Notes: Using daily returns on the S&P 500 index from 1995-2013 the figure shows the news impact news curves with different GARCH models.

Figure 5: Histogram with Normal Density and QQ-Plot of Standardized Residuals.



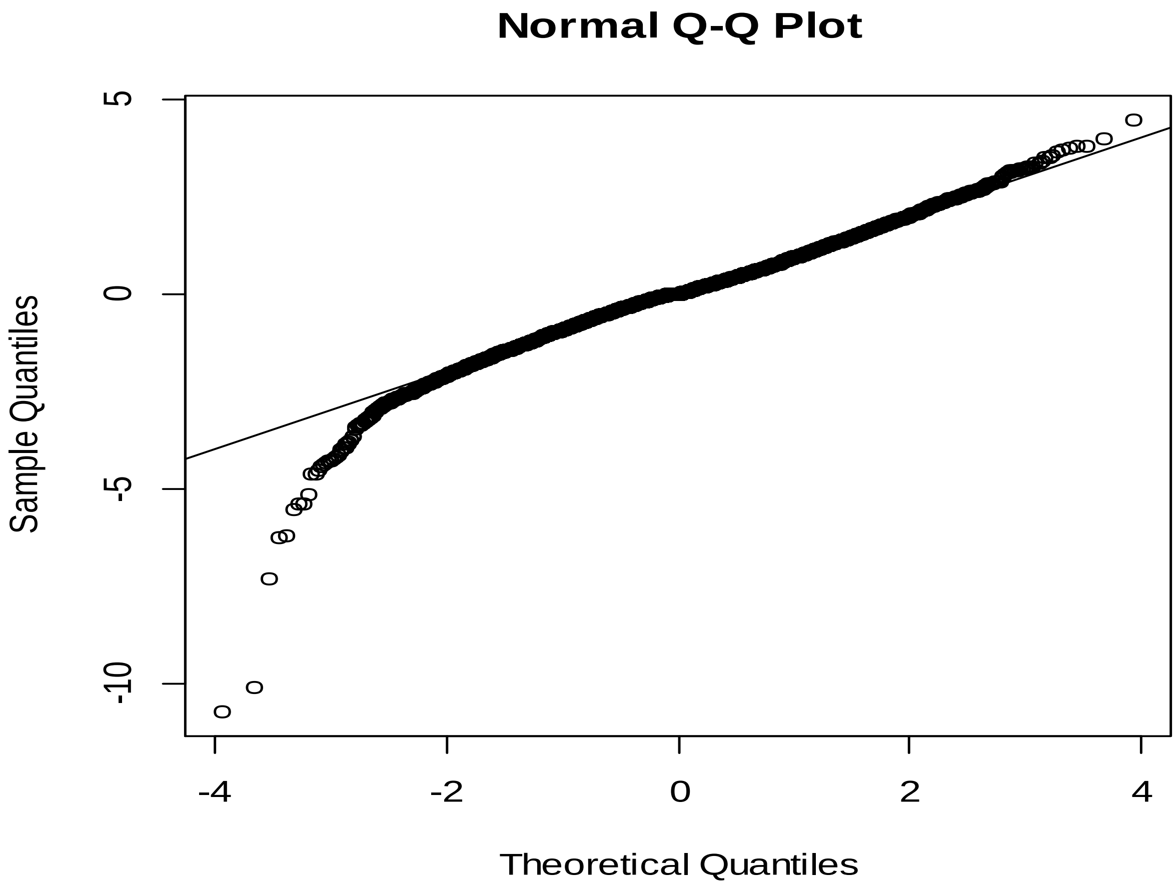
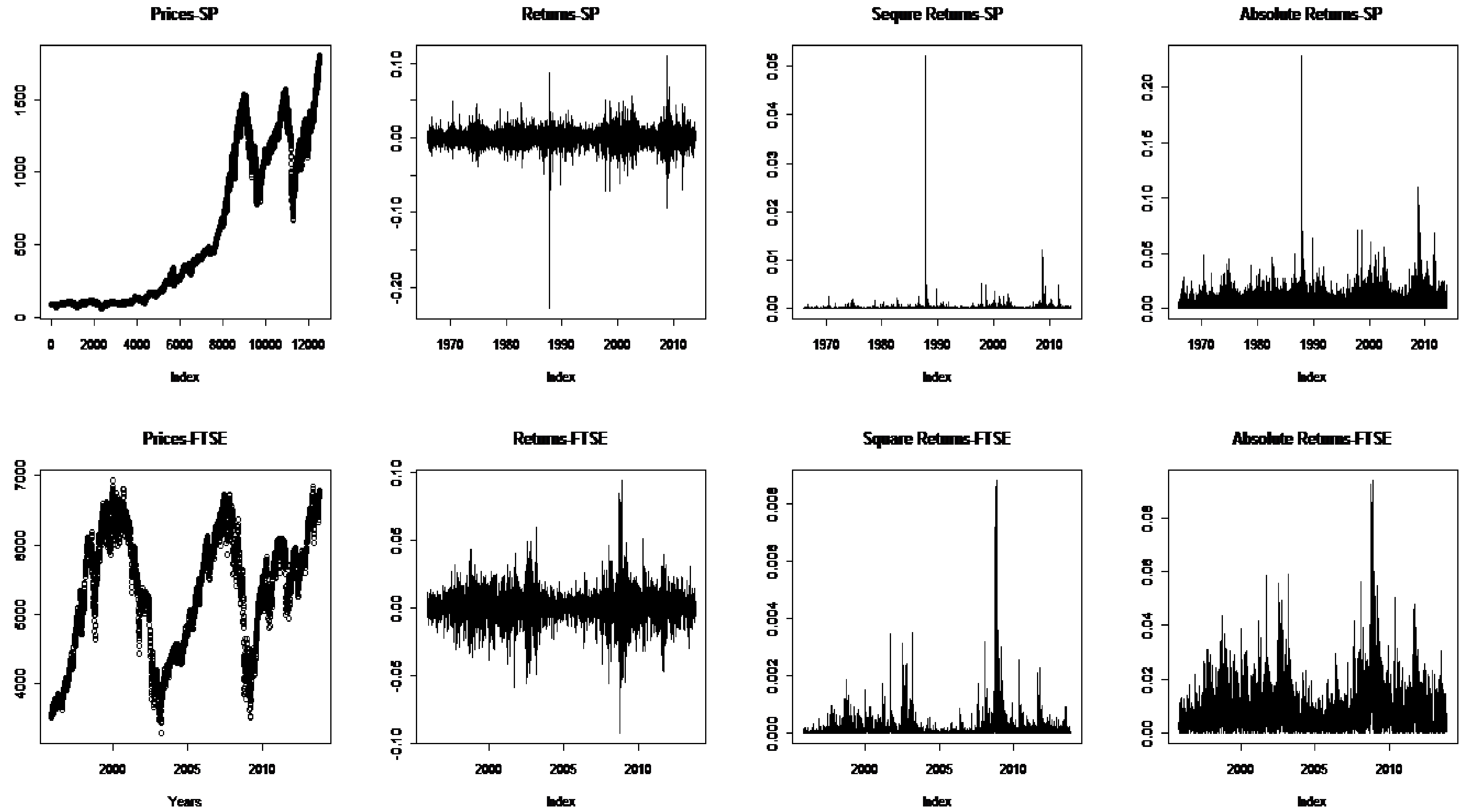
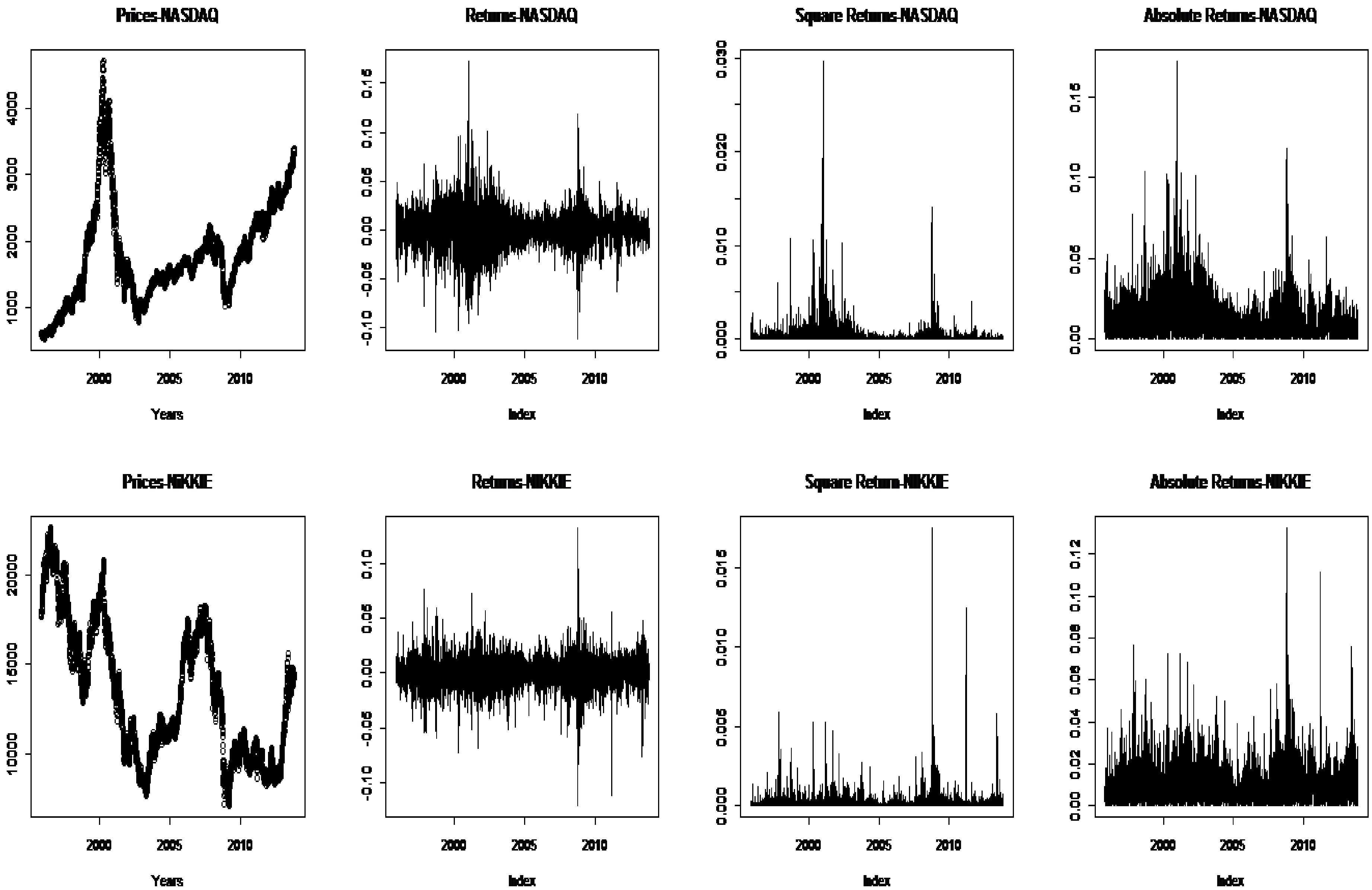


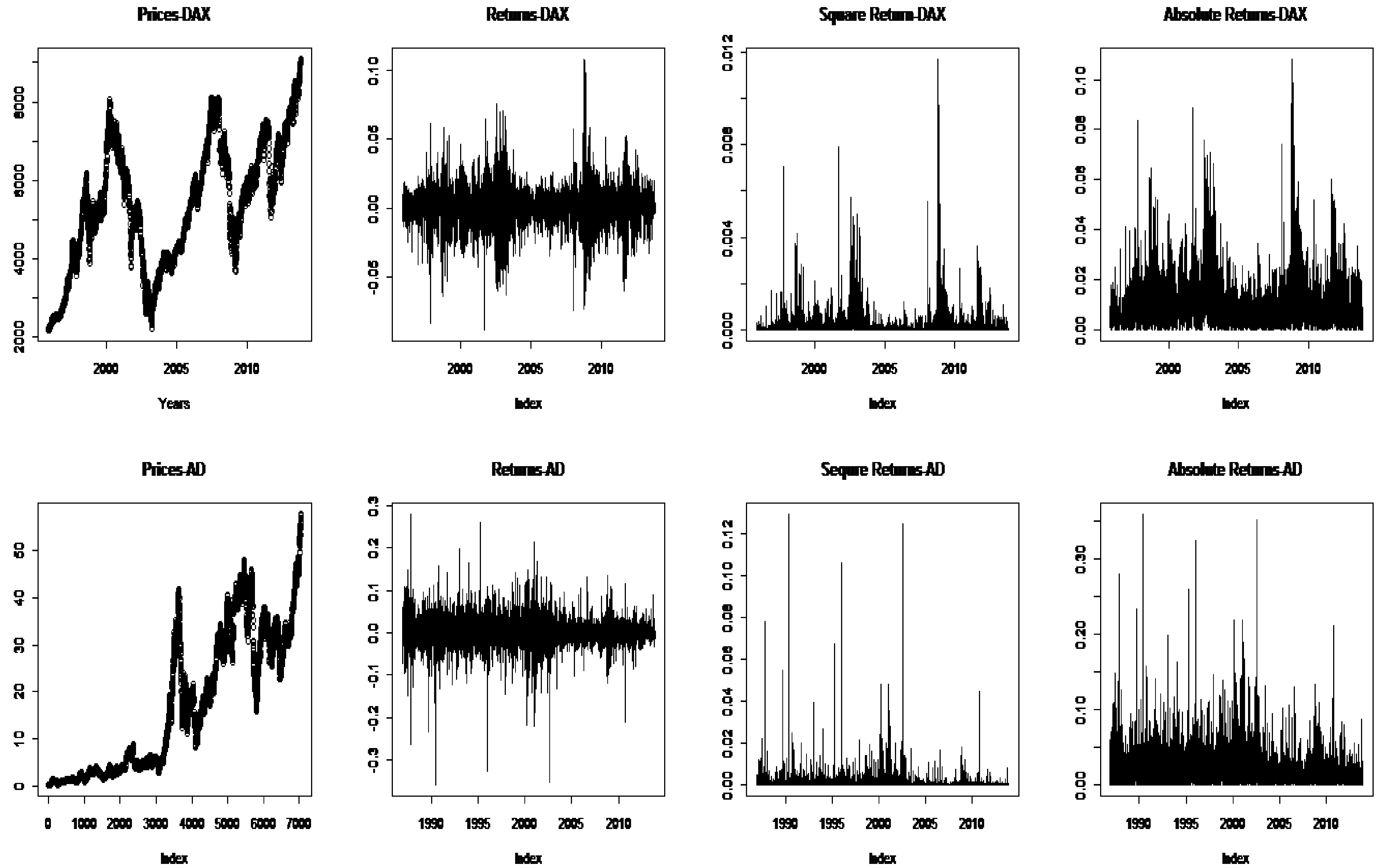
Table 1: Data Analyzed.

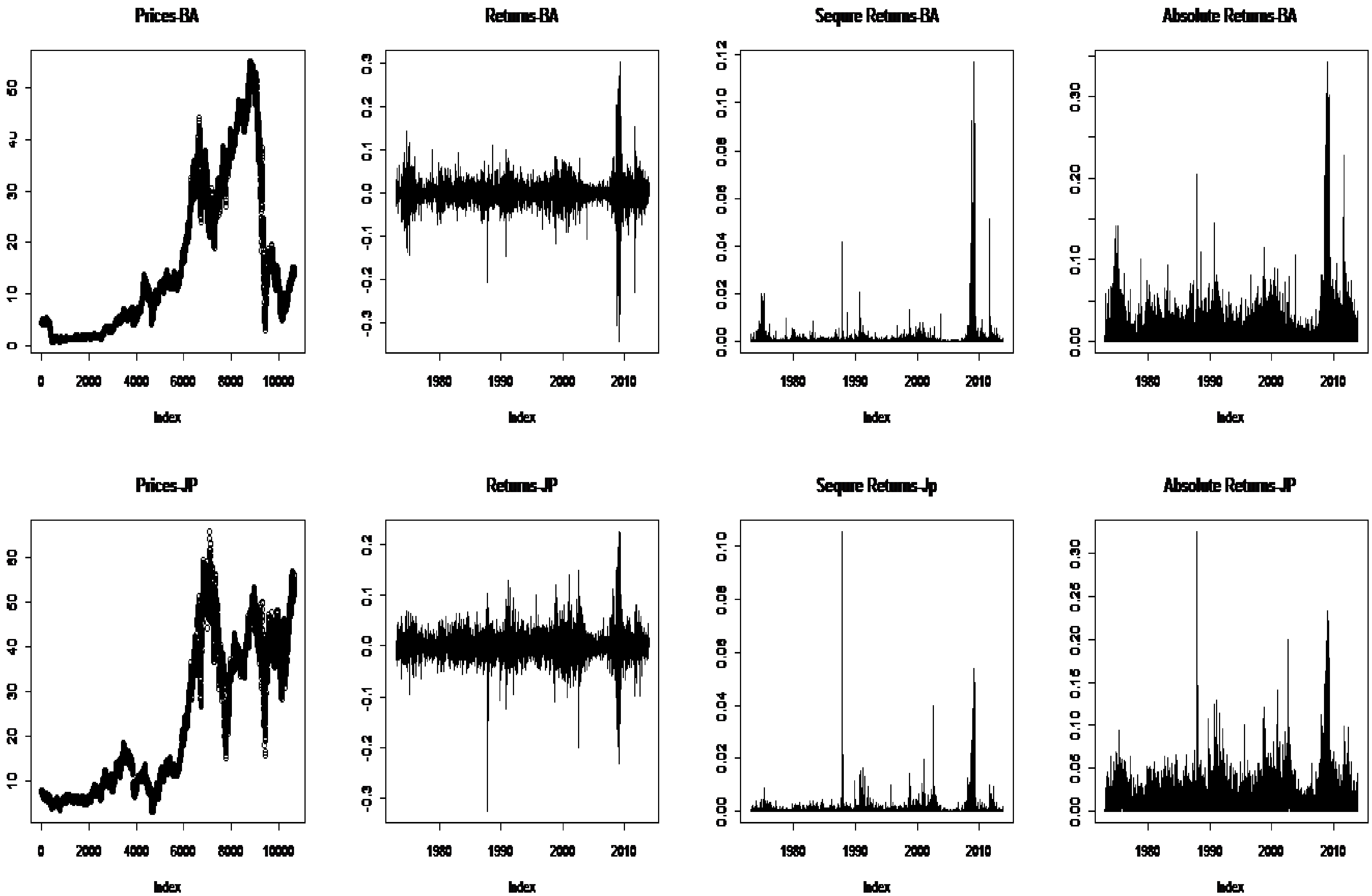
No	Stock	Ticker	Start	End	Observations
1	**S & P 500**	SP	07/01/1995	07/11/2013	4698
2	**FTSE-100**	FTSE	07/01/995	07/11/2013	4698
3	**NASDAQ -100**	NAS	07/01/1995	07/11/2013	4698
4	**NIKKIE-225**	NIK	07/01/1995	07/11/2013	4698
5	**DAX30**	DAX	07/01/1995	07/11/2013	4698
6	**S & P 500(2)**	SP2	01/01/1999	20/11/2013	3874
7	**Adobe**	AD	24/11/1986	20/11/2013	7042
8	**Bank of America**	BA	02/01/1973	20/11/2013	10666
9	**J P Morgan**	JP	02/01/1973	20/11/2013	10666
10	**Pfizer**	PF	02/01/1973	20/11/2013	10666
11	**Starbucks**	ST	20/11/1993	20/11/2013	5583

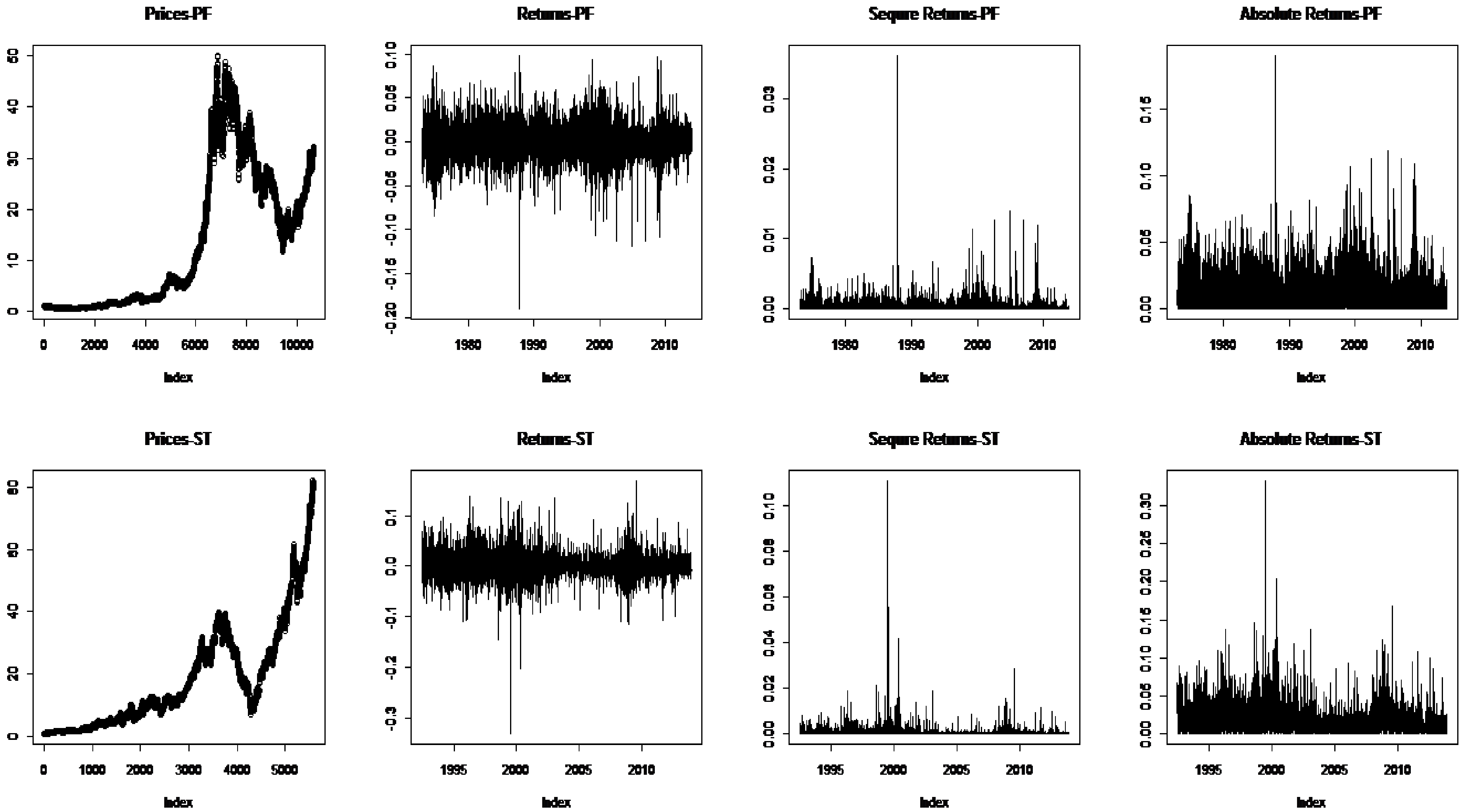
Figure 6: Prices, Returns, Squared Return, and Absolute Returns.




Prices-NASDAQ
Returns-NASDAQ
Square Returns-NASDAQ
Absolute Returns-NASDAQ
Years
Index
Index
Index
Prices-NIKKIE
Returns-NIKKIE
Square Return-NIKKIE
Absolute Returns-NIKKIE
Years
Index
Index
Index


Prices-DAX
Years
Returns-DAX
Index
Square Return-DAX
Index
Absolute Returns-DAX
Index
Prices-AD
Index
Returns-AD
Index
Sequre Returns-AD
Index
Absolute Returns-AD
Index


Prices-BA
Returns-BA
Sequre Returns-BA
Absolute Returns-BA
Index
Index
Index
Index
Prices-JP
Returns-JP
Sequre Returns-Jp
Absolute Returns-JP
Index
Index
Index
Index



**SP:Standard and Poor 500),SP2: standard and Poor 500(2), AD: Adobe, BA: Bank of America, JP: JP Morgan, PF: Pfizer, ST: Starbucks.*

Table 2: Summary Descriptive Statistics.

	SP	FTSE	NASDAQ	NIKKIE	DAX
Mean	0.0002	0.0001	0.0004	-0.0001	0.0003
Median	0.0001	0.0001	0.0005	0.0000	0.0007
Min	−0.2283	-0.0927	-0.1111	-0.1211	-0.0887
Max	0.1096	0.0938	0.1720	0.1323	0.1080
Std.dev.	0.0103	0.0120	0.0192	0.0151	0.0153
Skewness	−1.0212	-0.1562	-0.1083	-0.3329	-0.1238
Kurtosis	27.1950	5.9081	5.1532	6.1275	4.3434
Jarque-Bera Test	38077.34	6850.3	5206.39	7434.82	3704.07
P-Values	0.0000	0.0000	0.0000	0.0000	0.0000
ADF-Unit Root	−23.536	-16.795	-15.699	-16.336	-16.011
P-values	0.01	0.01	0.01	0.01	0.01
Phillips-Perron Unit Root Test	-110.59	-70.746	-74.035	-71.213	-69.362
p-Values	0.01	0.01	0.01	0.01	0.01
KPSS Test	0.13	0.0965	0.1684	0.1204	0.1335
p-values	0.01	0.01	0.01	0.01	0.01
Auto-Corr-r**					
Lag 1	0.013	-0.024	-0.070	-0.035	-0.010
Lag 5	-0.005	-0.051	-0.016	0.009	-0.031
Lag 10	0.015	-0.012	0.003	0.001	-0.001
Lag 20	0.002	-0.008	-0.020	0.009	0.012
Ljung-Box (20)	145.55	129.88	132.97	63.017	64.015
p-value	0.0000	0.0000	0.0000	0.0000	0.0000
Auto-Corr- r2					
Lag 1	0.139	0.231	0.235	0.164	0.187
Lag 5	0.188	0.338	0.211	0.152	0.248
Lag 10	0.079	0.264	0.274	0.264	0.192
Lag 20	0.066	0.123	0.184	0.114	0.124
Ljung-Box (20)	3486.21	6329.1	3888.3	5777.8	4512.2
p-value	0.000	0.000	0.000	0.000	0.000

*Note: SP: Standard and Poor 500, FTSE: FTSE, NAS: NASDAQ, NIK: NIKKIE, DAX: DAX. **The Lag orders are selected to examine a range of possible autocorrelations.*

Table 3: Summary Descriptive Statistics.

	SP2	AD	BA	JP	PF	ST
Mean	0.0003	0.0007	0.0001	0.0002	0.0003	0.0009
Median	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
Min	−0.0947	-0.3595	−0.3421	−0.3246	-0.1899	-0.3325
Max	0.1096	0.2792	0.3021	0.2239	0.0975	0.1687
Std.dev.	0.0114	0.0331	0.0244	0.0221	0.0176	0.0260
Skewness	−0.2367	-0.4161	−0.3254	−0.0939	-0.1938	-0.1549
Kurtosis	8.9912	10.7099	27.2005	15.1951	4.3278	8.4168
Jarque-Bera Test	21049.96	33858.79	328998.43	102627.36	102627.35	16502.05
P-Values	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ADF-Unit Root	−18.38	-18.81	−20.52	−21.05	-21.052	-18.28
P-values	0.01	0.01	0.01	0.01	0.01	0.01
Phillips-Perron Unit Root Test	-84.54	-84.51	-100.87	-105.51	-100.81	-79.21
p-Values	0.01	0.01	0.01	0.01	0.01	0.01
KPSS Test	0.14	0.12	0.13	0.045	0.17	0.12
p-values	0.01	0.01	0.01	0.01	0.01	0.01
Auto-Corr-r**						
Lag 1	-0.060	-0.006	0.023	-0.020	0.027	0.013
Lag 5	-0.027	-0.016	-0.044	-0.009	-0.001	-0.005
Lag 10	0.028	-0.009	0.046	0.014	0.003	0.015
Lag 20	-0.003	0.001	0.014	-0.005	-0.002	0.002
Ljung-Box (20)	144.28	49.34	227.84	57.11	7.11	50.01
p-value	0.0000	0.0003	0.0000	0.0000	0.0000	0.0002
Auto-Corr-r2**						
Lag 1	0.211	0.121	0.280	0.236	0.130	0.061
Lag 5	0.311	0.104	0.235	0.192	0.162	0.054
Lag 10	0.250	0.054	0.219	0.109	0.071	0.074
Lag 20	0.216	0.067	0.190	0.112	0.047	0.072
Ljung-Box (20)	8611.888	866.698	11479.74	5244.616	2513.462	359.06
p-value	0.000	0.000	0.000	0.000	0.000	0.000

*Note: SP2: Standard and Poor 500(2), AD: Adobe, BA: Bank of America, JP: JP, Morgan, PF: Pfizer, ST: Starbucks. **The Lag orders are selected to examine a range of possible autocorrelations.*

Figure 7: Normal QQ Plot for the Individual Stocks Log-Returns Data.

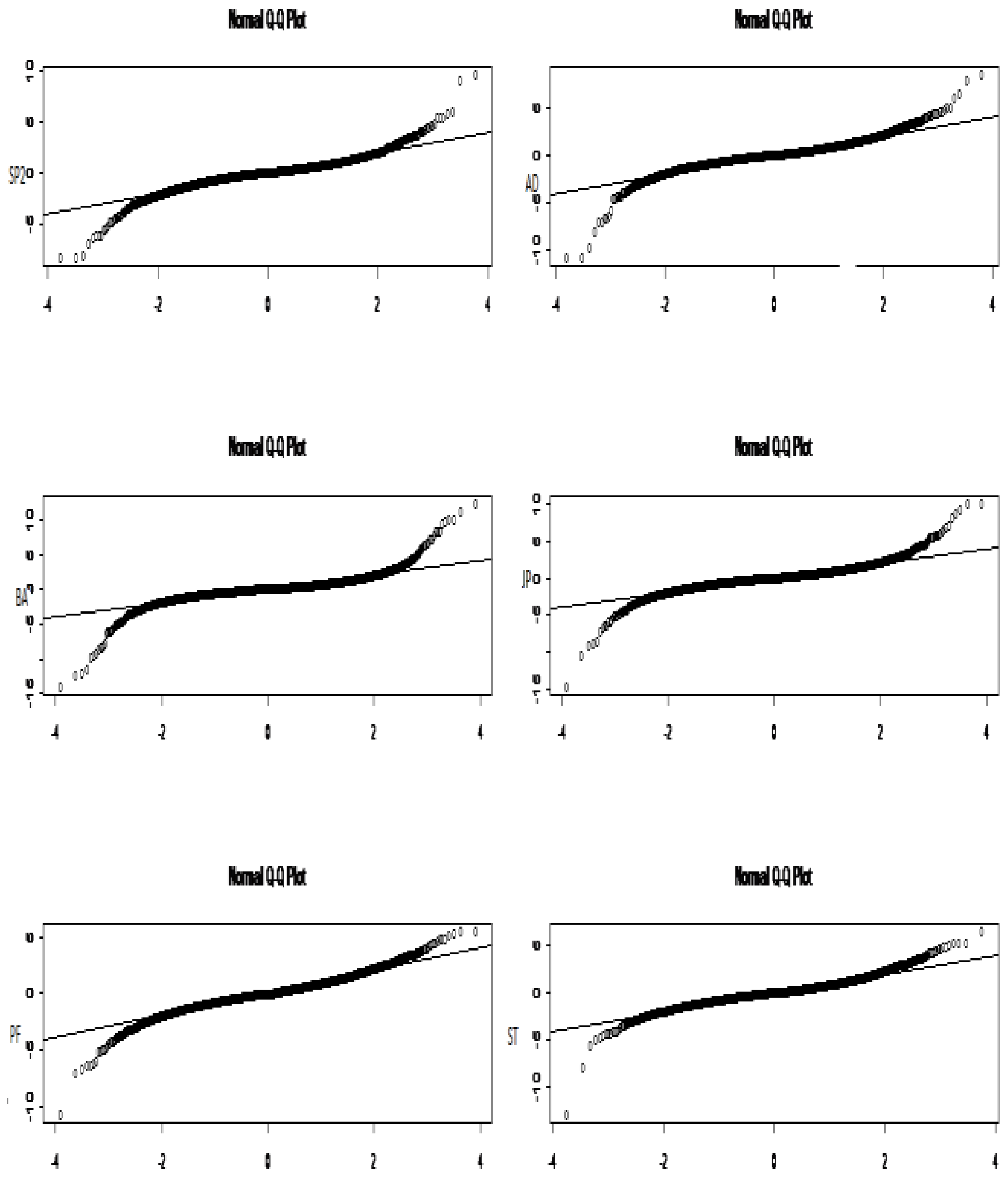


Figure 8: Normal QQ Plot for the Induces Log-Returns.

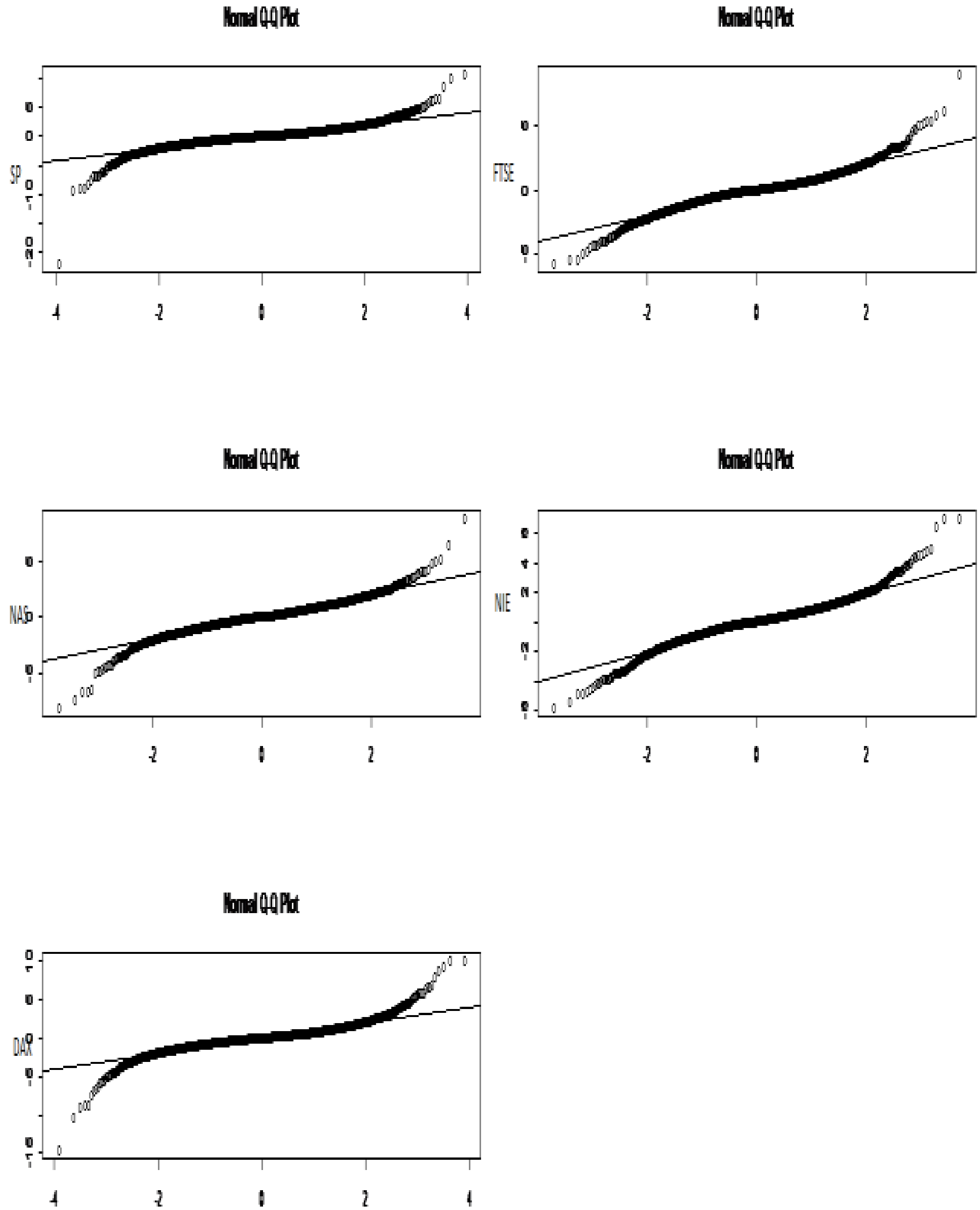


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Table 4: Estimated Parameters and Goodness of Fit Tests for S and Poor's 500(1) for the Period 1995-2013.

Models	Estimated Parameters						Goodness of fit Tests		
							Log L	**AIC**	**BIC**
EG-GAT	μ	ϕ	α	r	c	v			
	0.010	2.724	1.568	1.104	0.926	6.947	-17459.5	**34930.9**	**34975.5**
	(0.242)	(0.121)	(1.64)	(0.082)	(0.116)	(0.426)			
EG-AEPD	α	d_1	d_2						
	0.4874	1.4834	1.8409				-17583.4	35172.8	35195.1
	(0.0037)	(0.0278)	(0.0384)						
EG-SEPD	α	d							
	0.5056	1.623					-17613.7	35231.4	35246.3
	(0.0029)	(0.0227)							
EG-AST	α	v_1	v_2						
	0.493	4.999	5.000				-17846.3	35698.67	35721.0
	(0.0531)	(0.401)	(0.5319)						
EG-SST	α	v							
	0.5016	17.0969					-17608.4	35220.9	35235.7
	(0.0038)	(1.390)							
EG-ST	v								
	6.8787						-17468.2	**34964.8**	**34942.5**
	(0.4200)								
EG-TTD	v								
	5.1375						17505.27	**35012.53**	**35012.0**
	(0.2499)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 5: Estimated Parameters and Goodness of Fit Tests for FTSE for the Period 1995-2013.

Models	**Estimated Parameters**						**Goodness of fit Tests**		
							Log L	**AIC**	**BIC**
EG-GAT	μ	ϕ	α	r	c	v			
	1.3136	3.3876	1.2460	0.9720	1.4802	12.616	-6608.1	**13228.3**	**13267.1**
	(1.5299)	(1.5616)	(4.8008)	(0.288)	(0.013)	(0.220)			
EG-AEPD	α	d_1	d_2						
	0.6329	0.5007	0.4974				-14386.1	28778.3	28797.6
	(0.0068)	(0.0095)	(0.0104)						
EG-SEPD	α	d							
	0.5112	1.8038					-6640.9	13285.9	13298.8
	(0.0045)	(0.0403)							
EG-AST	α	v_1	v_2						
	0.5121	1.3248	1.4625				-6757.8	13521.6	13541.0
	(0.6357)	(0.3305)	(0.8476)						
EG-SST	α	v							
	0.5112	1.8038					-6640.98	13285.9	13298.8
	(0.0045)	(0.0403)							
EG-ST	v								
	11.8647						-6626.7	**13259.4**	**13278.8**
	(1.8486)								
EG-TTD	6.7625								
	(0.7503)						-6629.012	**13260.02**	**13266.48**

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 6: Estimated Parameters and Goodness of Fit Tests for NASDAQ for the Period 1995-2013.

Models	Estimated Parameters						Goodness of fit Tests		
							Log L	AIC	BIC
EG-GAT	μ	ϕ	α	r	c	v			
	0.1935	3.2526	1.3627	1.1799	0.9224	9.9581	-6609.9	13231.9	13270.6
	(1.1715)	(0.717)	(0.5605)	(0.407)	(0.584)	(1.511)			
EG-AEPD	α	d_1	d_2						
	0.4883	1.5454	1.9685				-6624.95	13255.91	13275.2
	(0.0061)	(0.0508)	(0.0658)						
EG-SEPD	α	d							
	0.5090	1.7316					-6638.07	13280.1	13293.1
	(0.0046)	(0.0399)							
EG-AST	α	v_1	v_2						
	0.5087	5.3240	5.4629				-6757.8	13521.6	13541.0
	(0.2880)	(0. 208)	(0. 645)						
EG-SST	α	V							
	0.5090	1.7316					-6638.07	13280.1	13293.0
	(0.0046)	(0.0399)							
EG-ST	v								
	9.5721						-6619.9	**13246**	**13265.3**
	(1.2906)								
EG-TTD	v								
	6.1750						-6626.2	**13254.5**	**13260.9**
	(0.6239)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 7: Estimated Parameters and Goodness of Fit Tests for Nikkei for the Period 1995-2013.

Models	**Estimated Parameters**						**Goodness of fit Tests**		
							Log L	**AIC**	**BIC**
EG-GAT	μ	ϕ	α	r	c	v			
	0.2707	2.5566	1.1257	1.0566	1.0400	7.8906	-6584.44	**13180.9**	**13219.6**
	(0.5765)	(4.547)	(5.731)	(0.266)	(0.486)	(0.508)			
EG-AEPD	α	d_1	d_2						
	0.4858	1.4992	1.8425				-6612.17	13230.34	13249.7
	(0.0062	(0.0501)	(0.0622)						
EG-SEPD	α	d							
	0.5034	1.6517					-6621.73	13247.4	13260.3
	(0.0048)	(0.0396)							
EG-AST	α	v_1	v_2						
	0.5277	2.6048	1.4390				-6707.46	13420.9	13440.2
	(0.8428)	(0.000)	(0. 825)						
EG-SST	α	v							
	0.5034	1.6517					-6634.97	13273.9	13286.8
	(0.0048)	(0.0396)							
EG-ST	v								
	7.7844						-6589.9	**13185.8**	**13205.2**
	(0.8681)								
EG-TTD	v								
	5.5062						-6599.7	**13201.4**	**13207.9**
	(0.4703)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 8: Estimated Parameters and Goodness of Fit Tests for DAX 30 for the Period 1995-2013.

Models	Estimated Parameters						Goodness of fit Tests		
							Log L	AIC	BIC
EG-GAT	μ	ϕ	α	r	c	v			
	1.8020	1.1944	1.9037	0.6323	3.3817	7.4726	**-6594.1**	**13200.2**	**13238.9**
	(1.734)	(1.5266)	(1.4019)	(0.195)	(3.329)	(6.179)			
EG-AEPD	α	d_1	d_2						
	0.4872	1.5238	2.0245				-6616.14	13238.28	13257.64
	(0.0060)	(0.0504)	(0.0639						
EG-SEPD	α	d							
	0.5114	1.7530					-6634.977	13273.95	13286.86
	(0.0046)	(0.0397)							
EG-AST	α	v_1	v_2						
	0.5196	3.5690	2.5628				-6739.868	13485.74	13505.1
	(0.8032)	(0.6909)	(0.3975)						
EG-SST	α	v							
	0.5148	1.7530					-6629.466	13262.93	13275.84
	(0.0063)	(0.0397)							
EG-ST	v								
	9.6497						**-6614.9**	**13235.8**	**13255.2**
	(1.2851)								
EG-TTD	v								
	6.1000						**-6619.6**	**13241.3**	**13247.8**
	(0.5909)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 9: Estimated Parameters and Goodness of Fit Tests for S and Poor 500(2) for the Period 1999-2013.

Models	Estimated Parameters						Goodness of Fit Tests		
							Log L	AIC	BIC
EG-GAT	μ	ϕ	α	r	c	v			
	0.040	1.715	0.317	1.564	0.274	12.635	-8701.331	**17414.66**	**17455.09**
	(0.146)	(0.171)	(0.119)	(0.298)	(0.173)	(3.786)			
EG-AEPD	α	d_1	d_2						
	0.484	1.439	1.938				-8747.885	17501.77	17521.98
	(0.005)	(0.040)	(0.058)						
EG-SEPD	α	d							
	0.509	1.636					-8775.031	17554.06	17567.54
	(0.004)	(0.033)							
EG-AST	α	v_1	v_2						
	0.517	3.001	5.999				-8900.742	17807.48	17827.7
	(0.009)	(0.035)	(0.021)						
EG-SST	α	v							
	0.514	18.124					-8783.127	17570.25	17583.73
	(0.005)	(2.326)							
EG-ST	v								
	6.499						-8718.412	**17442.82**	**17463.04**
	(0.550)								
EG-TTD	v								
	4.994						-8740.973	**17483.95**	**17490.68**
	(0.340)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 10: Estimated Parameters and Goodness of Fit Tests for Adobe for the Period 1986-2013.

Models	Estimated Parameters						Goodness of Fit Tests		
							Log L	AIC	BIC
EG-GAT	μ	ϕ	α	r	c	v			
	1.518	1.816	3.718	0.699	2.545	4.280	-9560.43	**19023.7**	**19089.3**
	(0.165)	(0.307)	(0.562)	(0.039)	(0.395)	(0.438)			
EG-AEPD	α	d_1	d_2						
	0.500	1.452	1.444				-9757.6	19521.2	19541.7
	(0.005)	(0.033)	(0.039)						
EG-SEPD	α	d							
	0.499	1.448					-9757.6	19519.2	19532.9
	(0.004)	(0.023)							
EG-AST	α	v_1	v_2						
	0.454	1.707	4.054				-9778.1	19562.1	19582.7
	(0.011)	(0.585)	(0. 216)						
EG-SST	α	v							
	0.452	11.022					-9694.2	19392.4	19406.1
	(0.006)	(0.804)							
EG-ST	v								
	4.614						-9494.7	**18995.4**	**19016.0**
	(0.255)								
EG-TTD	v								
	4.056						-9574.93	**19151.9**	**19158.7**
	(0.193)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 11: Estimated Parameters and Goodness of Fit Tests for Bank of America for the Period 1973-2013.

Models	Estimated Parameters						Goodness of Fit Test		
							Log L	AIC	BIC
EG-GAT	μ	ϕ	α	r	c	v			
	0.010	1.788	1.302	0.954	1.067	4.451	-14601.96	**29215.92**	**29259.6**
	(0.128)	(0.148)	(0.039)	(0.064)	(0.148)	(0.209)			
EG-AEPD	α	d_1	d_2						
	0.507	1.461	1.444				-14854.67	29715.35	29737.17
	(0.004)	(0.029)	(0.036)						
EG-SEPD	α	d							
	0.506	1.455					-14854.75	29713.9	29728.04
	(0.003)	(0.024)							
EG-AST	α	v_1	v_2						
	0.425	2.075	4.974				-14886.85	29779.70	29801.53
	(0.079)	(0. 94)	(0.989)						
EG-SST	α	v							
	0.436	10.502					-14788.2	29580.40	29594.95
	(0.005)	(0.669)							
EG-ST	v								
	4.545						-14601.79	**29209.58**	**29231.4**
	(0.216)								
EG-TTD	v								
	4.066						-14712.17	**29426.33**	**29433.61**
	(0.164)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 12: Estimated Parameters and Goodness of Fit Tests for J P Morgan for the Period 1973-2013.

Models	Estimated Parameters						Goodness of Fit Test		
							Log L	**AIC**	**BIC**
EG-GAT	μ	ϕ	α	r	c	v			
	0.010	1.509	0.443	0.947	1.148	7.716	-14727.64	**29467.27**	**29510.92**
	(0.258)	(0.344)	(0.579)	(0.289)	(0.851)	(6.349)			
EG-AEPD	α	d_1	d_2						
	0.499	1.523	1.571				-14929.15	29864.29	29886.12
	(0.004)	(0.030)	(0.035)						
EG-SEPD	α	d							
	0.501	1.543					-14929.7	29863.41	29877.95
	(0.003)	(0.024)							
EG-AST	α	v_1	v_2						
	0.442	3.114	5.952				-15021.47	30048.95	30070.77
	(0.075)	(0.436)	(0.956)						
EG-SST	α	v							
	0.456	13.145					-14873.98	29751.97	29766.52
	(0.004)	(0.965)							
EG-ST	v								
	5.5456						-14729.19	**29464.39**	**29486.21**
	(0.3032)								
EG-TTD	v								
	4.481						-14789.01	**29580.02**	**29587.29**
	(0.199)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold*.

Table 13: Estimated Parameters and Goodness of Fit Tests for Pfizer for the Period 1973-2013.

Models	**Estimated Parameters**						**Goodness of Fit Tests**		
							Log L	**AIC**	**BIC**
EG-GAT	μ	ϕ	α	r	c	v			
	0.010	1.691	0.533	0.944	1.128	7.732	-14804.14	**29620.28**	**29663.93**
	(0.037)	(0.342)	(0.285)	(0.074)	(0.261)	(0.631)			
EG-AEPD	α	d_1	d_2						
	0.496	1.560	1.630				-14977.44	29960.89	29982.71
	(0.004)	(0.030)	(0.037)						
EG-SEPD	α	d							
	0.499	1.590					-14978.56	29961.12	29975.67
	(0.003)	(0.024)							
EG-AST	α	v_1	v_2						
	0.447	1.896	2.7380				-15080.1	30166.2	30188.03
	(0.075)	(0.580)	(0.289)						
EG-SST	α	v							
	0.459	13.664					-14921.63	29847.26	29861.81
	(0.004)	(1.020)							
EG-ST	v								
	6.068						-14804.35	**29614.7**	**29636.52**
	(0.348)								
EG-TTD	v								
	4.613						-14843.87	**29689.73**	**29697.01**
	(0.211)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 14: Estimated Parameters and Goodness of Fit Tests for Starbucks for the Period 1993-2013.

Models	Estimated Parameters						Goodness of Fit Test		
							Log L	AIC	BIC
EG-GAT	μ	ϕ	α	r	c	v			
	0.0100	1.374	0.417	0.753	2.026	7.204	-7667.92	**15347.84**	**15387.6**
	(0.076)	(1.050)	(2.042)	(0.149)	(1.012)	(1.469)			
EG-AEPD	α	d_1	d_2						
	0.507	1.626	1.403				-7801.192	15608.38	15628.27
	(0.006)	(0.044)	(0.043)						
EG-SEPD	α	d							
	0.494	1.518					-7808.251	15620.5	15633.76
	(0.005)	(0.032)							
EG-AST	α	v_1	v_2						
	0.455	1.991	7.001				-7847.263	15700.53	15720.41
	(0.102)	(0.041)	(0.870)						
EG-SST	α	v							
	0.450	11.968					-7776.151	15556.3	15569.56
	(0.006)	(1.104)							
EG-ST	v								
	4.852						-7674.449	**15354.9**	**15374.78**
	(0.322)								
EG-TTD	v								
	4.088						-7718.06	**15438.12**	**15444.75**
	(0.226)								

Note: Standard errors are reported in parenthesis. Log L is the maximum value of log likelihood function. AIC is Akaike information criterion, BIC is *Bayesian information criterion. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. The lowest AIC and BIC given in bold.*

Table 15: Back-testing Results for 1-day Ahead Expected Shortfall for Standards and Poor's 500.

P	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0203 (1)	0.0005 (1)	0.0242 (3)	0.0008 (2)	0.0299 (4)	0.0011 (2)	0.0346 (5)	0.0015 (3)
EG-AEPD	0.0225 (4)	0.0007 (3)	0.0262 (5)	0.0009 (3)	0.0310 (5)	0.0012 (3)	0.0340 (4)	0.0015 (3)
EG-SEPD	0.0218 (3)	0.0007 (3)	0.0251 (4)	0.0009 (3)	0.0292 (2)	0.0011 (2)	0.0320 (2)	0.0014 (2)
EG-AST	0.0257 (6)	0.0009 (5)	0.0314 (6)	0.0012 (5)	0.0397 (7)	0.0020 (5)	0.0468 (7)	0.0028 (5)
EG-SSTD	0.0206 (2)	0.0006 (2)	0.0237 (2)	0.0007 (1)	0.0277 (1)	0.0010 (1)	0.0307 (1)	0.0012 (1)
EG-ST	0.0238 (5)	0.0008 (4)	0.0284 (7)	0.001 (4)	0.0346 (6)	0.0016 (4)	0.0399 (6)	0.0020 (4)
EG-TTD	0.0203 (1)	0.0005 (1)	0.0239 (1)	0.0007 (1)	0.0293 (3)	0.0011 (2)	0.0338 (3)	0.0014 (2)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 16: Back-testing Results for 1-day Ahead Expected Shortfall for FTSE.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAR**	**MSE**	**MAR**	**MSE**
EG-GAT	0.0242 (3)	0.0008 (2)	0.0284 (3)	0.0011 (3)	0.0340 (3)	0.0015 (2)	0.0382 (3)	0.0019 (3)
EG-AEPD	0.1013 (6)	0.0125 (4)	0.1327 (6)	0.0214 (5)	0.1800 (5)	0.0392 (3)	0.2200 (5)	0.0586 (4)
EG-SEPD	0.0240 (2)	0.0008 (2)	0.0273 (2)	0.0010 (2)	0.0314 (2)	0.0013 (1)	0.0342 (1)	0.0015 (1)
EG-AST	0.1944 (7)	0.0458 (5)	0.3288 (7)	0.1307 (6)	0.6571 (7)	0.5220 (5)	1.1090 (7)	1.4870 (6)
EG-SSTD	0.0814 (5)	0.0081 (3)	0.1206 (5)	0.0177 (4)	0.2014 (6)	0.0491 (4)	0.2962 (6)	0.1061 (5)
EG-ST	0.0249 (4)	0.0008 (2)	0.0290 (4)	0.0011 (3)	0.0343 (4)	0.0015 (2)	0.0383 (4)	0.0019 (3)
EG-TTD	0.0226 (1)	0.0007 (1)	0.0263 (1)	0.0009 (1)	0.0312 (1)	0.0013 (1)	0.0351 (2)	0.0018 (2)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 17: Back-testing Results for 1-day Ahead Expected Shortfall for NASDAQ.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0384 (2)	0.0022 (2)	0.0454 (3)	0.0029 (3)	0.0550 (4)	0.0041 (2)	0.0626 (4)	0.0052 (4)
EG-AEPD	0.0404 (4)	0.0024 (3)	0.0467 (4)	0.0031 (4)	0.0546 (3)	0.0041 (2)	0.0603 (3)	0.0049 (3)
EG-SEPD	0.0387 (3)	0.0022 (2)	0.0443 (2)	0.0028 (2)	0.0511 (2)	0.0036 (1)	0.0559 (1)	0.0042 (1)
EG-AST	0.1397 (7)	0.0248 (5)	0.2102 (6)	0.0557 (7)	0.3585 (7)	0.1614 (5)	0.5357 (7)	0.3600 (7)
EG-SSTD	0.0487 (6)	0.0033 (4)	0.0590 (5)	0.0047 (6)	0.0740 (6)	0.0072 (4)	0.0868 (6)	0.0098 (6)
EG-ST	0.0410 (5)	0.0024 (3)	0.0480 (5)	0.0032 (5)	0.0574 (5)	0.0045 (3)	0.0648 (5)	0.0056 (5)
EG-TTD	0.0363 (1)	0.0020 (1)	0.0424 (1)	0.0026 (1)	0.0508 (1)	0.0036 (1)	0.0593 (2)	0.0043 (2)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 18: Back-testing Results for 1-day Ahead Expected Shortfall for Nikkei.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0320 (2)	0.0013 (2)	0.0380 (3)	0.0018 (2)	0.0464 (3)	0.0026 (3)	0.0531 (3)	0.0033 (4)
EG-AEPD	0.0342 (4)	0.0015 (4)	0.0396 (4)	0.0019 (3)	0.0464 (3)	0.0026 (3)	0.0513 (2)	0.0031 (3)
EG-SEPD	0.0329 (3)	0.0014 (3)	0.0377 (2)	0.0018 (2)	0.0437 (1)	0.0023 (2)	0.0479 (1)	0.0027 (1)
EG-AST	0.1264 (7)	0.0180 (7)	0.1935 (7)	0.0420 (6)	0.3379 (6)	0.1277 (5)	0.5145 (6)	0.2959 (7)
EG-SSTD	0.0690 (6)	0.0055 (6)	0.0922 (6)	0.0097 (5)	0.1331 (5)	0.0200 (1)	0.1747 (5)	0.0343 (6)
EG-ST	0.0357 (5)	0.0016 (5)	0.0422 (5)	0.0022 (4)	0.0510 (4)	0.0031 (4)	0.0580 (4)	0.0039 (5)
EG-TTD	0.0310 (1)	0.0012 (1)	0.0363 (1)	0.0016 (1)	0.0440 (2)	0.0023 (2)	0.0513 (2)	0.0030 (2)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 19: Back-testing Results for 1-day Ahead Expected Shortfall for DAX.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0136 (1)	0.0003 (1)	0.0156 (1)	0.0004 (1)	0.0186 (1)	0.0006 (1)	0.0213 (1)	0.0007 (1)
EG-AEPD	0.0331 (4)	0.0015 (4)	0.0384 (4)	0.0020 (4)	0.0449 (4)	0.0026 (3)	0.0496 (3)	0.0032 (3)
EG-SEPD	0.0314 (3)	0.0014 (3)	0.0358 (3)	0.0017 (3)	0.0413 (2)	0.0022 (2)	0.0451 (2)	0.0026 (2)
EG-AST	0.1125 (7)	0.0154 (7)	0.1685 (6)	0.0344 (7)	0.2855 (7)	0.0983 (6)	0.4246 (7)	0.2171 (7)
EG-SSTD	0.0500 (6)	0.0032 (6)	0.0633 (5)	0.0050 (6)	0.0845 (6)	0.0088 (5)	0.1042 (6)	0.0132 (6)
EG-ST	0.0370 (5)	0.0018 (5)	0.0443 (5)	0.0026 (5)	0.0549 (5)	0.0038 (4)	0.0635 (5)	0.0050 (5)
EG-TTD	0.0295 (2)	0.0012 (2)	0.0345 (2)	0.0016 (2)	0.0414 (3)	0.0022 (2)	0.0586 (4)	0.0041 (4)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 20: Back-testing Results for 1-day Ahead Expected Shortfall for Standers and Poor's 500(2).

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0221 (1)	0.0007 (1)	0.0269 (2)	0.0010 (2)	0.0336 (4)	0.0015 (3)	0.0391 (4)	0.0020 (4)
EG-AEPD	0.0244 (4)	0.0008 (3)	0.0285 (3)	0.0011 (3)	0.0336 (4)	0.0015 (3)	0.0373 (3)	0.0018 (3)
EG-SEPD	0.0234 (3)	0.0008 (3)	0.0269 (2)	0.0010 (2)	0.0313 (2)	0.0013 (2)	0.0343 (2)	0.0016 (2)
EG-AST	0.0397 (6)	0.0020 (4)	0.0518 (5)	0.0033 (5)	0.0716 (6)	0.0064 (5)	0.0912 (6)	0.0103 (6)
EG-SSTD	0.0227 (2)	0.0007 (2)	0.0261 (1)	0.0009 (1)	0.0305 (1)	0.0012 (1)	0.0337 (1)	0.0015 (1)
EG-ST	0.0262 (5)	0.0009 (4)	0.0313 (4)	0.0013 (4)	0.0384 (5)	0.0019 (4)	0.0443 (5)	0.0025 (5)
EG-TTD	0.0221 (1)	0.0007 (2)	0.0261 (1)	0.0009 (1)	0.0321 (3)	0.0013 (2)	0.0373 (3)	0.0018 (3)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 21: Back-testing Results for 1-day Ahead Expected Shortfall for Adobe.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0671 (2)	0.0060 (1)	0.0819 (2)	0.0086 (2)	0.1041 (3)	0.0133 (2)	0.1231 (4)	0.0182 (4)
EG-AEPD	0.0779 (4)	0.0078 (3)	0.0904 (4)	0.0103 (4)	0.1062 (4)	0.01385 (3)	0.1170 (2)	0.0167 (2)
EG-SEPD	0.0782 (5)	0.0079 (4)	0.0908 (5)	0.0103 (4)	0.1066 (5)	0.0139 (4)	0.1180 (3)	0.0168 (3)
EG-AST	0.2072 (7)	0.0497 (6)	0.3136 (7)	0.1126 (6)	0.5386 (7)	0.3290 (7)	0.8090 (7)	0.7430 (7)
EG-SSTD	0.0655 (1)	0.0060 (1)	0.0762 (1)	0.0075 (1)	0.0905 (1)	0.0103 (1)	0.1014 (1)	0.0127 (1)
EG-ST	0.0931 (6)	0.0108 (5)	0.1142 (6)	0.0158 (5)	0.1459 (6)	0.0252 (6)	0.1736 (6)	0.0352 (6)
EG-TTD	0.0741 (3)	0.0072 (2)	0.0895 (3)	0.0101 (3)	0.1134 (2)	0.0156 (5)	0.1351 (5)	0.0217 (5)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 22: Back-testing Results for 1-day Ahead Expected Shortfall for Bank of America.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0454 (2)	0.0033 (2)	0.0550 (2)	0.0046 (2)	0.0696 (4)	0.0072 (4)	0.0823 (4)	0.0098 (4)
EG-AEPD	0.0494 (4)	0.0039 (4)	0.0573 (4)	0.0050 (3)	0.0673 (2)	0.00679 (2)	0.0744 (2)	0.0081 (2)
EG-SEPD	0.0512 (5)	0.0041 (5)	0.0593 (5)	0.0054 (4)	0.0694 (3)	0.0071 (3)	0.0768 (3)	0.0086 (3)
EG-AST	0.08903 (7)	0.0114 (7)	0.1262 (7)	0.02238 (6)	0.1982 (7)	0.0543 (7)	0.2777 (7)	0.1061 (7)
EG-SSTD	0.0414 (1)	0.0028 (1)	0.0481 (1)	0.0037 (1)	0.0572 (1)	0.0050 (1)	0.06420 (1)	0.0062 (1)
EG-ST	0.0601 (6)	0.0055 (6)	0.0736 (6)	0.0079 (5)	0.0941 (6)	0.0127 (6)	0.1120 (6)	0.0177 (6)
EG-TTD	0.0473 (3)	0.0036 (3)	0.0570 (3)	0.0050 (3)	0.0722 (5)	0.0077 (5)	0.0859 (5)	0.0106 (5)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 23: Back-testing Results for 1-day Ahead Expected Shortfall for J P Morgan.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0445 (2)	0.0028 (2)	0.0532 (2)	0.0039 (2)	0.0657 (4)	0.0057 (4)	0.0760 (4)	0.0075 (4)
EG-AEPD	0.0481 (5)	0.0032 (3)	0.0555 (5)	0.0042 (5)	0.0649 (3)	0.0056 (3)	0.0716 (3)	0.0067 (3)
EG-SEPD	0.0475 (4)	0.0032 (3)	0.0548 (4)	0.0041 (4)	0.0639 (2)	0.0054 (2)	0.0705 (2)	0.0065 (2)
EG-AST	0.0606 (7)	0.0049 (6)	0.0786 (7)	0.0080 (7)	0.1084 (7)	0.01486 (7)	0.1372 (7)	0.0235 (7)
EG-SSTD	0.0415 (1)	0.0025 (1)	0.0480 (1)	0.0032 (1)	0.0565 (1)	0.0043 (1)	0.0630 (1)	0.0053 (1)
EG-ST	0.0547 (6)	0.0041 (5)	0.0659 (6)	0.0057 (6)	0.0823 (6)	0.0087 (6)	0.0960 (6)	0.0117 (6)
EG-TTD	0.0452 (3)	0.0040 (4)	0.0540 (3)	0.0040 (3)	0.0673 (5)	0.0060 (5)	0.0791 (5)	0.0081 (5)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 24: Back-testing Results for 1-day Ahead Expected Shortfall for Pfizer.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0369 (2)	0.0017 (2)	0.0439 (2)	0.0023 (2)	0.0541 (3)	0.0034 (3)	0.0618 (4)	0.0044 (4)
EG-AEPD	0.0407 (5)	0.0020 (4)	0.0469 (5)	0.0026 (4)	0.0547 (4)	0.0035 (4)	0.0603 (3)	0.0042 (3)
EG-SEPD	0.0401 (4)	0.0020 (4)	0.0461 (4)	0.0025 (3)	0.0536 (2)	0.0033 (2)	0.0590 (2)	0.0040 (2)
EG-AST	0.0953 (7)	0.0100 (6)	0.1392 (7)	0.0211 (6)	0.2277 (7)	0.0560 (7)	0.3293 (7)	0.1169 (7)
EG-SSTD	0.0355 (1)	0.0016 (1)	0.0411 (1)	0.0021 (1)	0.0483 (1)	0.0028 (1)	0.0538 (1)	0.0034 (1)
EG-ST	0.0458 (6)	0.0025 (5)	0.0548 (6)	0.0035 (5)	0.0678 (6)	0.0052 (6)	0.0785 (6)	0.0069 (6)
EG-TTD	0.0386 (3)	0.0018 (3)	0.0460 (3)	0.0025 (3)	0.0572 (5)	0.0038 (5)	0.0669 (5)	0.0051 (5)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 25: Back-testing Results for 1-day Ahead Expected Shortfall for Starbucks.

p	5%		2.5%		1%		0.5%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
EG-GAT	0.0525 (2)	0.0036 (1)	0.0626 (2)	0.0049 (2)	0.0761 (2)	0.0071 (2)	0.0874 (3)	0.0091 (3)
EG-AEPD	0.0581 (3)	0.0043 (2)	0.0666 (3)	0.0055 (3)	0.0772 (3)	0.0072 (3)	0.0847 (2)	0.0086 (2)
EG-SEPD	0.0587 (5)	0.0044 (3)	0.0678 (4)	0.0057 (4)	0.0792 (4)	0.0076 (4)	0.0874 (3)	0.0091 (3)
EG-AST	0.1282 (7)	0.0190 (5)	0.1841 (7)	0.0385 (7)	0.2942 (7)	0.0974 (7)	0.4179 (6)	0.1959 (6)
EG-SSTD	0.0514 (1)	0.0036 (1)	0.0596 (1)	0.0045 (1)	0.0704 (1)	0.0061 (1)	0.0786 (1)	0.0075 (1)
EG-ST	0.0721 (6)	0.0064 (4)	0.0879 (6)	0.00929 (6)	0.1115 (6)	0.0145 (6)	0.1319 (5)	0.0201 (5)
EG-TTD	0.0584 (4)	0.0044 (3)	0.0704 (5)	0.0061 (5)	0.08917 (5)	0.0095 (5)	0.1060 (4)	0.0132 (4)

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD. Values in parenthesis represent model raking. Models rankings are based on lowest value of MAE and MSE.

Table 26: Back-testing Expected Shortfall for 5-days Horizon for Standard & Poor 500.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	0.4369	0.1909	0.3733	0.1394	0.3110	0.0967	0.2554	0.0652
MC-GAT	0.7194	0.5175	0.5258	0.2764	0.3527	0.1244	0.2188	0.0479
MC-AEPD	1.3723	1.8833	1.1230	1.2611	0.8836	0.7808	0.6747	0.4553
MC-SEPD	0.7148	0.5110	0.5978	0.3574	0.4743	0.2250	0.3610	0.1303
MC-AST	0.9187	0.8441	0.6978	0.4870	0.4904	0.2405	0.3234	0.1046
M-SST	0.5515	0.3042	0.4495	0.2021	0.3385	0.1146	0.2324	0.0540
MC-ST	0.7851	0.6165	0.6119	0.3745	0.4495	0.2021	0.3056	0.0934
MC-TTD	0.8732	0.7626	0.6891	0.4749	0.4991	0.2491	0.3380	0.1142

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 27: Back-testing Expected Shortfall for 5-Days Horizon FTSE.

P	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	0.2758	0.0761	0.2393	0.05731	0.2005	0.0402	0.1650	0.0272
MC-GAT	0.7312	0.5347	0.5490	0.3014	0.3614	0.1306	0.2027	0.0411
MC-AEPD	1.9403	3.7651	1.5153	2.2964	1.1398	1.2993	0.8334	0.6947
MC-SEPD	0.7131	0.5086	0.5912	0.3496	0.4646	0.2159	0.3428	0.1175
MC-AST	1.2032	1.4478	0.8713	0.7593	0.5758	0.3316	0.3483	0.1213
M-SST	0.6666	0.4444	0.5219	0.2724	0.3697	0.1367	0.2284	0.0522
MC-ST	0.7235	0.5235	0.5579	0.3113	0.3876	0.1503	0.2368	0.0561
MC-TTD	1.0673	1.1393	0.7859	0.6177	0.5371	0.2885	0.3370	0.1136

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 28: Back-testing Expected Shortfall for 5-Days Horizon NASDAQ.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	0.5086	0.2586	0.4319	0.1865	0.3603	0.1298	0.2978	0.0886
MC-GAT	0.8480	0.7191	0.5091	0.2592	0.2099	0.0440	0.0187	0.0043
MC-AEPD	0.7298	0.5326	0.5450	0.2970	0.3460	0.1197	0.1578	0.0249
MC-SEPD	0.7155	0.5119	0.5324	0.2834	0.3471	0.1204	0.1672	0.0279
MC-AST	1.3309	1.7713	0.9054	0.8197	0.5023	0.2523	0.1809	0.0327
M-SST	0.5995	0.3594	0.3999	0.1599	0.1879	0.0353	0.0268	0.0072
MC-ST	0.7559	0.5714	0.4974	0.2474	0.2459	0.0604	0.0210	0.0054
MC-TTD	0.6753	0.4560	0.3758	0.1412	0.1198	0.0143	0.0789	0.0092

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 29: Back-testing Expected Shortfall for 5-Days Horizon Nikkei.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	0.4792	0.2296	0.4046	0.1637	0.3360	0.1129	0.2781	0.0773
MC-GAT	0.7555	0.5708	0.5447	0.2967	0.3585	0.12855	0.2095	0.0439
MC-AEPD	0.78494	0.6161	0.6488	0.4210	0.5066	0.2566	0.3759	0.1413
MC-SEPD	0.8225	0.6765	0.6729	0.4528	0.5190	0.2694	0.3800	0.1444
MC-AST	1.1231	1.2614	0.8437	0.7119	0.5783	0.3344	0.3673	0.1349
M-SST	0.6342	0.4022	0.5022	0.2522	0.3627	0.1315	0.2325	0.05407
MC-ST	0.8915	0.7948	0.6646	0.4417	0.4663	0.2174	0.2972	0.0883
MC-TTD	0.7278	0.5297	0.5136	0.2638	0.3369	0.1135	0.1985	0.0394

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 30: Back-testing Expected Shortfall for 5-Days Horizon DAX 30.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	0.6171	0.3808	0.5400	0.2916	0.4641	0.2154	0.3916	0.1533
MC-GAT	0.8623	0.7436	0.5869	0.3445	0.3435	0.1180	0.1606	0.0258
MC-AEPD	0.8044	0.6471	0.6497	0.4221	0.4854	0.2356	0.3324	0.1105
MC-SEPD	0.8685	0.7543	0.6842	0.4682	0.5003	0.2503	0.3347	0.1120
MC-AST	1.2209	1.4907	0.8735	0.7631	0.5610	0.3147	0.3145	0.0989
M-SST	0.8685	0.7543	0.6842	0.4682	0.5003	0.2503	0.3347	0.1120
MC-ST	0.9372	0.8784	0.6653	0.4427	0.4324	0.1870	0.2392	0.0572
MC-TTD	0.7630	0.5821	0.5101	0.2602	0.3017	0.0910	0.1403	0.0196

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 31: Back-testing Expected Shortfall for 10-Days Horizon for Standard & Poor's 500.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	6.8677	47.166	5.8807	34.5832	4.9057	24.066	4.0287	16.230
MC-GAT	2.5373	6.4381	2.2350	4.9954	1.8978	3.6018	1.5805	2.4981
MC-AEPD	7.7892	60.6725	6.9783	48.6974	6.1953	38.3824	5.5263	30.5406
MC-SEPD	4.8601	23.6211	4.5360	20.5758	4.1805	17.4770	3.8526	14.8429
MC-AST	3.4506	11.9070	2.9242	8.5512	2.4181	5.8474	1.9764	3.9063
M-SST	2.6956	7.2665	2.3987	5.7540	2.0632	4.2570	1.7303	2.9941
MC-ST	3.0945	9.5762	2.7020	7.3011	2.2874	5.2324	1.9062	3.6338
MC-TTD	3.5624	12.6910	3.0697	9.4234	2.5517	6.5114	2.0822	4.3357

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 32: Back-testing Expected Shortfall for 10-Days Horizon for FTSE.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	4.2325	17.914	3.6735	13.495	3.1085	9.6633	2.5875	6.6956
MC-GAT	4.1078	16.8747	3.4962	12.2240	2.8686	8.2293	2.3178	5.3726
MC-AEPD	19.2447	370.36	16.4167	269.51	13.6825	187.21	11.5432	133.24
MC-SEPD	7.8319	61.3400	7.1241	50.7540	6.4061	41.0392	5.7822	33.4348
MC-AST	6.2635	39.2325	5.0524	25.5276	3.9767	15.8148	3.1279	9.7843
M-SST	4.2264	17.8632	3.6306	13.1819	3.0361	9.2184	2.5062	6.2814
MC-ST	4.3563	18.9781	3.7082	13.7514	3.0785	9.4777	2.5376	6.4398
MC-TTD	5.8011	33.6538	4.8290	23.3201	3.9335	15.4731	3.1511	9.9299

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 33: Back-testing Expected Shortfall for 10-Days Horizon for NASDAQ.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	7.7942	60.750	6.6632	44.398	5.5942	31.295	4.6322	21.457
MC-GAT	5.3060	28.1540	4.5456	20.6628	3.7986	14.4296	3.1338	9.8209
MC-AEPD	10.7972	116.580	9.8462	96.9484	8.8832	78.9119	8.0142	64.2280
MC-SEPD	10.496	110.18	9.618	92.516	8.722	76.0739	7.9352	62.9680
MC-AST	8.0139	64.223	6.7039	44.942	5.5176	30.444	4.4977	20.229
M-SST	5.6600	32.036	4.9177	24.184	4.1569	17.280	3.4592	11.966
MC-ST	6.3367	40.154	5.4656	29.873	4.5377	20.591	3.7198	13.837
MC-TTD	5.0608	25.612	4.3995	19.355	3.7222	13.855	3.0969	9.5910

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 34: Back-testing Expected Shortfall for 10-Days Horizon for Nikkei.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	7.4654	55.733	6.4814	42.009	5.4834	30.068	4.5504	20.707
MC-GAT	3.7239	13.8608	3.1290	9.7850	2.5739	6.6203	2.0986	4.4003
MC-AEPD	7.888	62.208	7.123	50.735	6.348	40.2921	5.677	32.225
MC-SEPD	7.795	60.759	6.964	48.494	6.2220	38.702	5.6093	31.454
MC-AST	5.4822	30.044	4.5637	20.819	3.7405	13.984	3.0373	9.2197
M-SST	3.8502	14.817	3.3227	11.034	2.7929	7.7953	2.3151	5.3555
MC-ST	4.4081	19.423	3.7571	14.109	3.0968	9.5846	2.5361	6.4272
MC-TTD	3.7941	14.388	3.1916	10.180	2.6313	6.9190	2.1548	4.6393

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Table 35: Back-testing Expected Shortfall for 10-Days Horizon for DAX 30.

p	1%		2.5%		5%		10%	
Models	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**	**MAE**	**MSE**
FHS	6.3029	39.726	5.4049	29.213	4.5049	20.294	3.6919	13.630
MC-GAT	6.1159	37.404	4.9464	24.467	3.8678	14.960	3.0336	9.2028
MC-AEPD	15.397	237.09	13.353	178.32	11.413	130.27	9.7809	95.666
MC-SEPD	15.000	225.02	13.098	171.55	11.209	125.65	9.6993	94.076
MC-AST	9.9253	98.511	7.9223	62.763	6.1506	37.830	4.7915	22.958
M-SST	10.257	105.21	8.7580	76.702	7.3938	54.668	6.2205	38.694
MC-ST	7.7133	59.495	6.2608	39.197	4.9442	24.445	3.8701	14.977
MC-TTD	6.2353	38.879	5.0692	25.697	4.0363	16.292	3.2092	10.299

Note: MAE: mean absolute error and MSE: mean square error for ES at 5%, 2.5%, 1% and 0.5% significance level. MC: Monte Carlo Simulation; FHS: Filtered Historical Simulation. Generalized asymmetric t distribution: GAT; exponential power distribution: AEPD; skewed exponential power distribution: SEPD; asymmetric t distribution: AST; skewed student t distribution: SST; student t distribution: ST and Twin t distribution: TTD as distributions of standardized returns in MC. Models rankings are based on lowest value of MAE and MSE.

Chapter Three: The Multivariate Modelling Approach and Risk Measurement

1. Introduction

Sklar (1959) introduced the copula as a statistical function that linked together univariate distribution to form multivariate distributions. According to Sklar's Theorem, any multivariate joint distribution can be decomposed into a univariate marginal distribution functions and a copula, which describes the dependence part of the multivaraite distribution.

The copula has become a popular multivariate modelling tool mainly due to easy implementation and separate estimation of marginal distribution and copula separately. The approach is model to the well-known stylized facts of financial returns using marginal distributions (Cherubini et al., 2004 and McNeil et al., 2005).

Constructing a higher-dimensional copula is considered as a difficult problem. There are a large number of parametric bivariate copulas, but the set of higher-dimensional copula is rather limited (Aas et al, 2009).

There are a significant number of feasible pair-copula decompositions are available for high-dimensional distributions. Bedford and Cooke (2001, 2002) have presented a graphical model denoted the regular vine (R-vine). The R-vines are very common and include many possible pair-copula decompositions. We construct two special cases of R-vines; the canonical vine (C-vine) and the D-vine (Kurowicka and Cooke, 2004) in our study. Each model represents a technique of decomposing the density (Joe, 1996; Bedford and Cooke, 2001, 2002; Vrac et al. 2005; Kurowicka and Cooke, 2006 and Aas et al., 2009).

Copulas are also suited for risk measurement by allowing the modelling of the marginal and dependence structures of a multivariate probability model separately (Pourkhanali et al., 2016). For computation of portfolio VaR and ES from both copula models and vine copulas as risk management measures, we need to rely on Monte Carlo simulation. Monte Carlo simulation essentially reverses the steps taken in copula model building (Christoffersen, 2012).

Vine copulas account for a multivariate distribution that combines three or more marginal distributions in a joint distribution. Furthermore, conditional VaR is calculated using copulas as these allowed separate modelling of the marginal and the dependence structures. Tail dependence information from copulas generally supports a measure of CoVaR (Reboredo and Ugolini, 2015).

In this chapter, we calculate VaR and ES for different GARCH-Copula models with various marginal implemented and tested on both bivariate and multivariate data. The forecasts from copula models are then compared to DCC-GARCH-models, as both dynamic conditional correlations models (DCC) by Engle (2002, 2009) and copulas models by Sklar (1959) allows for two steps modelling of portfolio returns. Marginal return distributions are specified in the first step and in the second step the marginal is linked to a joint distribution either via time variant correlations or a time invariant link function (copula) (Berger, 2013).

Zhang et al. (2014) and Brechmann and Czado (2011) provide strong evidence of the superiority of VaR models calculating with vine copula over historical simulation, mean-variance and DCC-GARCH models. For multivariate analysis, all developed models and methods are used to analyse the five, seven and fifteen companies from DAX 30 index, a major market indicator for the Eurozone.

Although quite a few studies have applied vine copula modelling to calculate VaR and ES, the main purpose of these studies is to measure one-day ahead risk. Risk managers require risk across many different horizons rather than just one specific horizon. The mutiperiod risk is referred as the term structure of risk. The focus of our paper is not only to explore one-day VaR and ES but also longer than one-day VaR and ES for multivariate data. We calculated multi-day VaR and ES up to 10 days as the Basel Committee requires financial institutions compute VaR at least 10-days ahead to determine their minimum capital risk requirements (Basel Committee on Banking Supervision, 2009).

Degiannakis et al. (2012) and Degiannakis and Potamia (2016) compare multi-days VaR and ES for univariate data with simulation based methods. Monte Carlo simulation has been employed by Dionne et al. (2009) to calculate multi-day VaR for univariate data. Huang (2010) uses an iterative Monte Carlo simulation approach instead of simple Monte Carlo simulation to calculate VaR forecast. These studies employ Monte Carlo simulation to forecast multi-day VaR and ES for single asset. This chapter present empirical application of 5-days and 10-days VaR and ES forecasts for multivariate data.

The main contribution of this chapter is to suggest the adaptation of the Monte Carlo simulation technique of Christoffersen (2012) for forecasting multiple-step-ahead VaR and ES for multivariate data. At present, to the best of our knowledge, no study explores multi-period VaR and ES for multivariate data using either a static and dynamic correlation with Monte Carlo simulation method. The Monte Carlo simulation method allows us to calculate VaR and ES for multivariate data at any horizon of interest and hence to calculate the entire term structure of risk.

This chapter is organized as follows. Section 2 discusses a review of the literature. Section 3 introduces the methodological framework. Our empirical results are presented and discussed in Section 4. Section 5 concludes the chapter.

2. Review of Literature

The use of copulas in financial literature is very common (see, Cherubini et al., 2004; Aas et al., 2009; Fischer et al., 2009; Berg and Aas, 2009; Min and Czado, 2010; Brechmann et al., 2011; Czado et al., 2012 and So and Yeung, 2014). Mendes et al. (2010) use a D-vine copula with four different bivariate copula families for a six-dimensional data set for portfolio management.

Brechmann et al. (2011) employ R-vine structures on high-dimensional data to complex financial applications, to issues of financial risk management. The multivariate copulas obtained from C- and D-vine structures create very flexible models since bivariate copulas can contain complex dependence structures such as asymmetric dependence or strong joint tail behaviour (Joe et al., 2010).

Czad et al. (2012) provide a broad evidence of the selection of C-vines by describing an appropriate C-vine structure and selecting a fitting pair-copula family. For this purpose, a subsequent approach is developed based on the cardinality of the conditioning variables in association with individual options for each pair-copula as a best fitting pair-copula family from a large category of families.

Schepsmeier (2015) introduced a new goodness of fit test for regular vine copulas. Kim et al. (2013) suggest a mixture of D-vine copulas which includes multiple parameters for examining the different dependencies inherent in multivariate data and can be extended to a multivariate copula function. By incorporating D-vine copulas into a finite mixture model, one can not only create dependence patterns that may not belong to actual copula families, but also manage a comprehensive study of complex and hidden dependence structure in multivariate data.

R-vine copulas are multivariate copulas based on a pair-copula construction (PCC) which decompose the *d*-dimensional density into unconditional and conditional bivariate copulas. The high flexibility is attained from the independently chosen copula families and the choice of the decomposition itself (Schepsmeie, 2013).

Dissmann et at (2013) evaluated a large simulation study and applied it to a 16-dimensional financial data set of international equity, fixed income and commodity indices. They developed a strategy of simultaneously searching for an appropriate R-vine tree framework, the pair-copula families and the parameter values of the chosen pair-copula families. It is a subsequent method starting by describing the first tree, its pair-copula families and estimating their parameters. Based on this the specification of the second tree utilizes transformed variables. The applied transformations depend on the choices made in the first tree. In this manner, all trees together with their choice of pair-copula families and corresponding parameters are made.

Allen et al. (2013, 2014) use Regular Vine copula(R-vine) in an analysis of the co-dependencies of 10 major European Stock Markets. Their empirical results indicate that the dependencies among different assets behave in a complicated process, and are conditional to change in different economic positions. One of the main advantages of this approach is the flexibility in the preference of different distributions to model co-dependencies in calculation of VaR of a portfolio.

In the view of Nagler and Cazado(2016) practical applications of nonparametric density estimators in higher than three dimensions undergo a considerable deal from the notable curse of dimensionality, when dimension increases convergences slows down. They show that one can avoid the curse of dimensionality by presuming an easy vine copula model for the dependence between variables. They specify a general nonparametric estimator for such a model and show under high-level assumptions that the speed of convergence is independent of dimension.

Reboredo and Ugolini (2015) calculated conditional value-at-risk using copulas and vine copulas for systemic sovereign debt distress affecting European financial systems. To report the impact of possible Greek sovereign debt distress on the financial systems of other European markets, it is important to describe dependence between Greek sovereign debt and another country's debt market and financial sector. For this purpose, they consider vine copula as these account for a multivariate distribution that combines three or more marginal distributions in a joint distribution. Furthermore, CoVaR is calculated using copulas as these allowed separate modelling of the marginal and the dependence structure. Tail dependence information from copulas generally supports a measure of CoVaR.

There is great empirical evidence that correlations increase during financial unrest and as a result financial risk increases even further. Therefore, modelling correlation dynamics is essential to a risk manager (Christoffersen, 2012). The Dynamic Conditional Correlation (DCC) model developed by Engle (2002) allows the conditional correlation matrix to vary parsimoniously over time. Hakim et al. (2007), Hakim and McAleer (2009), Palaro and Hotta (2006), Ozun and Cifter (2007) and Aloui et al. (2011) investigate DCC and compare with BEKK models for several bivariate portfolios.

The important practical advantage of the DCC model is that only very few parameters are estimated simultaneously using numerical optimization. In the first step, all the individual variances are estimated by GARCH models. Then, the returns are standardized and the unconditional correlation matrix is estimated. Thirdly, the correlation persistence parameters are calculated. This element makes the DCC easily manageable for risk management of large portfolios (Christoffersen, 2012).

Zhang et al. (2014) provide strong evidence that VaR forecasts with all three vine copula models (R-vine, C-vine and D-vine) are sufficiently accurate as compared to historical simulation, mean-variance and DCC-GARCH models. Moreover, the vine copula methods can correctly forecast the ES of the portfolio based on VaR calculations, and the D-vine copula model performs better than other vine copulas. Aloui and Aissa (2016) find that the C-vine copula model leads to more accurate VaR forecasts than the traditional VaR approaches.

"It is evident that calculating VaR over a short horizon, followed by square root time rule (SRTR) scaling to convert to longer-term tail risks, is likely to be inappropriate and misleading, particularly for markets in Eastern Europe, Central and South America and the Asia Pacific. Caution is necessary in applying the SRTR" (Wang et al., 2011).

The aim of this chapter is to present and discuss the use of copulas and vine copulas for financial risk measurement. As mentioned in previous literature, many problems are associated with SRTR for calculating longer horizon VaR and ES. Inspired by the work of Christoffersen (2012), we developed framework for term structure of risk for bivariate and multivariate data.

For active risk management, a multivariate multi-days model is required (Christoffersen, 2012). Most of the previous studies on multi-days of VaR and ES forecasting were based on only one asset. Multi-days VaR can be calculated based on different techniques, variance-covariance, historical simulation, standard quantile formula, Monte Carlo simulation and square root of time rule (see; Christoffersen and Diebold, 2000; Dowd et al., 2003; Hartz et al., 2006; Dionne et al., 2009; Semenov, 2009; Asai and McAleer 2009; Pesaran et al., 2009; Huang, 2010; Hoogerheide and van Dijk, 2010; Wang et al., 2011 and Christoffersen (2012). Square root of time rule is criticized by Dowd et al. (2003), Engle (2004), Danielsson (2002), Daníelsson and Zigrand (2006) and Wang *et al*. (2011).

The main advantage of the Monte Carlo simulation method is its flexibility. Monte Carlo simulation can be used for any assumed distribution of standardized returns. However, there is no an extensive literature on Monte Carlo simulation for multi-day VaR and ES forecasting. Moreover, these studies focus on only univariate multi-days VaR and ES. Degiannakis et al. (2012) and Degiannakis and Potamia (2016) compare multi-days VaR and ES for univariate data with simulation based method.

For the computation of muti-days VaR and ES, we need to use Monte Carlo simulation methods rather than close form solution. We can use the Monte Carlo simulation based dynamic risk models to compute VaR and ES at any horizon of interest and therefore to compute the entire term structure of risk.

3. Methodological Framework

In this section, we introduce the concept and some basic properties for copulas and risk measures.

3.1. Copula Theory

There are several possibilities to construct two-dimensional distribution families proposed in the literature, one of them being the so-called copula approach, where copulas play the central part.

A copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform. Copulas are used to describe the dependence between random variables (Sklar, 1959; Cherubini et al., 2004; McNeil et al., 2005 and Christoffersen, 2012).

Consider *n* assets with possibly $f_i(z_i)$ is the marginal distributions, *CDFs* are the cumulative density functions $u_i = F_i(z_i)$ for *I* = 1, 2…*n*. Where u_i is simply the probability of observing a value below z_i for asset *i*.

3.1.1 Sklar's Theorem

Sklar's theorem by Sklar (1959), provides us with the theoretical foundation we need for the application of copulas. Sklar's theorem states that every multivariate cumulative density function of $(z_1, \dots, z_n)$ with marginal CDFs $F_1(z_1), \dots, F_n(z_n)$, has a unique copula function $C(.)$, joining the marginals to form the joint distribution

$$F(z_1, \dots, z_n) = C(F_1(z_1), \dots, F_n(z_n)) \tag{1}$$

$$=C(v_1, \dots, v_n)$$

$\mathrm{C}(v_1, \dots, v_n)$ is known as CDF of the copula function. If $F_1, \dots, F_n$ are continuous then C is unique. Otherwise, C is uniquely defined on Range F_1×…× Range F_n. Conversely, if C is a copula and F_1 and F_2 are univariate dfs, then F_n define in above equation is joint df with margins F_1 and F_2.

The multivariate probability density function (PDF) implied by Sklar's theorem is

$$f(z_1, \dots, z_n) = \frac{\partial^n C(F_1(z_1), \dots, F_n(z_n))}{\partial z_1 \dots \partial z_n} \times$$

$$=\frac{C(v_1,\dots,v_n)}{\delta v_1,\dots,\delta v_n} \times \prod_{i=1}^{n} f(z_i)$$

$$=g(v_1, \dots, v_n) \times \prod_{i=1}^{n} f(z_i) \tag{2}$$

where the copula PDF is defined in the previous equation as:

$$g(v_1, \dots, v_n) = \frac{\delta^n C(v_1,\dots,v_n)}{\delta v_1,\dots,\delta v_n} \tag{3}$$

The PDF algorithm is:

$$\ln f(z_1, \dots, z_n) = \ln g(v_1, \dots, v_n) + \sum_{i=1}^{n} \ln f_i(z_i) \tag{4}$$

The above decomposition represents that we can build a complicated multivariate density in a few easier steps:

1. We build and estimate *n* potentially different marginal distribution models.

2. We decide on the copula PDF and estimate it using the probability outputs from the marginal as the data.

The log likelihood function corresponding to the entire copula distribution model is constructed by summing the log PDF over the T observations in the sample:

$$\ln L = \sum_{t=1}^{T} \ln g\left(v_{1,t} \dots, v_{n,t}\right) + \sum_{t=1}^{T} \sum_{i=1}^{n} \ln f_i\left(z_{i,t}\right) \quad (5)$$

But if we have estimated the *n* marginal distributions in a first step then the copula likelihood function is simply:

$$\ln L_g = \sum_{t=1}^{T} \ln g\left(v_{1,t}, \dots v_{n,t}\right) \quad (6)$$

3.2. Dependence Measures and Copulas

To understand the dependence structure between two random variables Z_1 and Z_2 (Embrechts et al., 2002) construct a scalar dependence measure between Z_1 and Z_2 that statisfied four properties. Desirable properties of dependence $\rho(Z_1, Z_2)$ for two random variables Z_1 and Z_2 are:

1. $\rho(Z_1, Z_2) = \rho(Z_2, Z_1)$
2. $-1 \leq \rho(Z_1, Z_2) \leq 1$
3. $\rho(Z_1, Z_2)$ = 1 if Z_1 and Z_2 are co-monotonic, and $\rho(Z_1, Z_2)$ = -1 if Z_1 and Z_2 are counter-monotonic.
4. If T is strictly monotonic, then

$$\rho(T(Z_1), Z_2) = \begin{cases} \rho(Z_1, Z_2) & T \text{ is increasing} \\ -\rho(Z_1, Z_2) & T \text{ is decreasing} \end{cases} \quad (7)$$

3.2. 1.Pearson's Linear Correlation

The traditional concept of linear correlation or Pearson's correlation:

$$\rho = \frac{cov(Z_1, Z_2)}{\sigma_{Z_1} \sigma_{Z_2}} \quad (8)$$

where cov is covariance, $cov(Z_1, Z_2) = E(Z_1, Z_2) - E(Z_1)E(Z_2)$ and $\sigma_{Z_1,}\sigma_{Z_2}$ are the standard deviations of Z_1 and Z_2.

The Pearson's correlation is a measure of linear dependence. If $Z_2 = \alpha + \beta Z_1$ then $\rho = \pm 1$ and if Z_1 and Z_2 are independent then $\rho = 0$.

Even though linear correlation is quite popular due to straightforward variance and covariance calculations, it has several shortcomings:

1. Linear correlation requires existence of both σ_{Z_1} and σ_{Z_2}.

2. Linear correlation is not ideal for financial time series, which display the property of fat tails and nonexistence of higher moments.

3. Independence between two random variables implies that $\rho = 0$, but only the converse is true for the multivariate normal distribution, as explained by Embrechts et al. (2002). For example, $\rho(Z_1, Z_2)$=0 if $Z_1 \sim N(0,1)$ and $Z_2 = Z_1^2$, even though Z_1 and Z_2 are clearly dependent. This is because $cov(Z_1, Z_2)$=0.

4. ρ is not invariant under nonlinear strictly increasing transformations $T: R \rightarrow R$. That is $\rho\big(T(Z_1), T(Z_2)\big) \neq \rho(Z_1, Z_2)$

5. Marginal distributions and correlation do not determine the joint distribution. This is only true for the bivariate normal distribution.

6. For given marginal distributions F_1 and F_2, $\rho \in [\rho_{min}, \rho_{max}]$ and it may be the case that $\rho_{min} > -1$ and $\rho_{max} < 1$.

The limitations of the linear correlation coefficient have motivated statisticians to the consideration of concordance measures of dependence.

3.2.2. Concordance Measures

The random variables Z_1 and Z_2 are labelled as being concordant if large values of Z_1 are associated with large (small) values of Z_2 , and small values of Z_1 are associated with small (large) values of Z_2. The concept of concordance has led to the use of Kendall's τ and Spearman's ρ_s as measures of dependence.

3.2.2.1. Kendall's Tau Statistic

Let F be a continuous bivariate CDF, and let (Z_1, Z_2) and $\big(\tilde{Z}_1, \tilde{Z}_2\big)$ are two pair of independent random variables of this distribution. Then Kendall's τ measure of dependence between two random variables defined as the probability of concordance minus the probability of discordance.

Kendall's tau statistic for the distribution F.

$$\tau = Pr\big\{\big(Z_1 - \tilde{Z}_1\big)\big(Z_2 - \tilde{Z}_2\big)\big\} - Pr\big\{\big(Z_1 - \tilde{Z}_1\big)\big(Z_2 - \tilde{Z}_2\big) > 0\big\}$$

$$= E\big[sign\big\{\big(Z_1 - \tilde{Z}_1\big)\big(Z_2 - \tilde{Z}_2\big)\big\}\big] \qquad (9)$$

The vector of (Z_1, Z_2) and $\big(\tilde{Z}_1, \tilde{Z}_2\big)$ is said to be concordant if $Z_1 > \tilde{Z}_1$whenever $Z_2 > \tilde{Z}_2$, and they are said to be discordant if $Z_1 < \tilde{Z}_1$whenever $Z_2 < \tilde{Z}_2$.

If $C(.)$ is the copula for the continuous random variable (Z_1, Z_2) with function F i.e.
$F(z_1, z_2) = C\big(v_1 = F_1(z_1), v_2 = F_2(z_2)\big)$
then according to Nelsen (2006);
$$\tau = 4E[C(v_1, v_2)] = 4 \iint_{I^2} C(v_1, v_2)\, f(v_1, v_2) dv_1 dv_2 \tag{10}$$
where $f(v_1, v_2)$ is the copula density.

3.2.2.2. Spearman's Rho Statistic

Spearman's ρ_s for two random variables (Z_1, Z_2) with joint density function F and marginal distributions F_1 and F_2 , is defined as the (Pearson) correlation between $F_1(Z_1)$ and $F_2(Z_2)$.

In terms of copula between continuous random variables Z_1 andZ_2, Spearman's $\rho_s$can be shown as:
$$\rho_s(Z_1, Z_2) = cor\big(F_1(Z_1), F_2(Z_2)\big) = 12 \iint_{I^2} C(v_1, v_2) d v_1 d v_2 - 3 = 12E(v_1, v_2) - 3 \tag{11}$$
Kendall's tau and Spearman's rho satisfied all the four properties of dependence.

3.2.3. Tail Dependence Measures

Tail dependence measures are used to capture dependence in the joint tail of bivariate distributions.

The bivariate upper tail dependence represents as:
$$\lambda_u(Z_1, Z_2) = \lim_{q \to 0} Pr\left(Z_2 > VaR_q(Z_2) | Z_1 > VaR_q(Z_1)\right) = \lim_{q \to 1} \frac{1-2q+C(q,q)}{q} \tag{12}$$
where $VaR_q(Z_1)$ and $VaR_q(Z_2)$ are the 100.qth percent quantile of Z_1 and Z_2 , respectively.

Similarly, the bivariate upper tail dependence shown as:
$$\lambda_l(Z_1, Z_2) = \lim_{q \to 0} Pr\left(Z_2 \leq VaR_q(Z_2) | Z_1 \leq VaR_q(Z_1)\right) = \lim_{q \to 0} \frac{C(q,q)}{q} \tag{13}$$
C is said to be lower (upper) tail dependence if $\lambda_u \neq 0 (\lambda_l \neq 0)$

3.3. Elliptical Copulas

Let F be the multivariate CDF of an elliptical distribution and F_i be the CDF of i^{th} margins and F_i^{-1} be its quantile function $i = 1,2, \dots n$.The elliptical copula determined by F is:
$$C(v_1, \dots v_n) = F[F_1^{-1}(v_1), \dots, F_n^{-1}(v_n)] \tag{14}$$

The two most common elliptical copulas are the normal and the Student *t*.

3.3.1. The Normal Copula

One of the most frequently used copulas for financial modelling is the copula of a standard bivariate normal distribution with correlation parameter ρ defined by:

$$\mathrm{C}(v_1, \dots v_n; \rho) = \Phi_\rho\left(\Phi^{-1}(v_1), \Phi^{-1}(v_2)\right) \tag{15}$$

$$= \int_{-\infty}^{\Phi^{-1}(v_1)} \int_{-\infty}^{\Phi^{-1}(v2)} \frac{1}{2\pi\sqrt{1-\rho^2}} exp\left\{-\frac{z_1^2 - 2z_1 z_2 + z_2^2}{2(-\rho^2)}\right\}$$

$$= \Phi_\rho\left(\Phi^{-1}(z_1), \Phi^{-1}(z_2)\right)$$

$$= \Phi(\boldsymbol{z_1}, \boldsymbol{z_2}) \tag{16}$$

where $\Phi^{-1}(\cdot)$the quantile is function of the standard normal distribution or inverse CDF, and Φ_ρ is joint cumulative distribution function of $\Phi^{-1}(v_1) and\, \Phi^{-1}(v_2)$ with correlation coefficient ρ. The normal copula is much more flexible than mutivaraite normal distribution because the normal copula allows for the marginals to be no-normal.

3.3.2. The *t* Copula

As the normal copula does not allow for enough dependence between the tails of the distribution of different assets, the *t* copula derived from *t* distribution allow more flexible financial risk application. The Student t copula with correlation parameter ρ and degree of freedom parameter d is defined by:

$$C(v_1, v_2; \rho, d) = t_{\rho,d}\left(t_d^{-1}(v_1) t_d^{-1}\,(v_2)\right) \tag{17}$$

where t_d^{-1} is inverse CDF of student t distribution parameter d degree of freedom.

The bivariate *t* copula density is:

$$g(v_1, v_2; \rho, d) = \frac{t_{\rho,d}\left(t_d^{-1}(v_1) t_d^{-1}\,(v_2)\right)}{t_{t(d)}\left(t_d^{-1}(v_1) t_{t(d)} t_d^{-1}\,(v_2)\right)}$$

$$= \frac{\Gamma\left(\frac{d+2}{2}\right)}{\sqrt{1-\rho^2}\,\Gamma\left(\frac{d}{2}\right)} \left(\frac{\Gamma\left(\frac{d}{2}\right)}{\Gamma\left(\frac{d+1}{2}\right)}\right)^2 \times$$

$$\frac{\left(1+\frac{\left(t_d^{-1}(v_1)\right)^2+\left(t_d^{-1}(v_2)\right)^2-2\rho t^{-1}t_d^{-1}(v_1)t_d^{-1}(v_2)}{d(1-\rho^2)}\right)^{-\frac{-d+2}{2}}}{\left(1+\frac{\left(t_d^{-1}(v_1)\right)^2}{d}\right)^{-\frac{d+1}{2}}\left(1+\frac{\left(t_d^{-1}(v_2)\right)^2}{d}\right)^{-\frac{d+1}{2}}} \tag{18}$$

3.4. Archimedean Copulas

Most common Archimedean copulas allow an explicit formula that the Gaussian copula doesn't allow. Archimedean copulas are favoured because by controlling the strength of dependence they allow modelling dependence in randomly high dimensions with only one parameter.

Archimedean copulas defied as:

$$C(v_1, v_2; \theta) = \Psi^{-1}\left(\Psi_\theta(v_1) + \Psi_\theta(v_2)\right) \tag{19}$$

The function Ψ is called Archimedean generator, Ψ^{-1} is its inverse function and θ is parameter.

3.4.1. Gumbel Copula

The Gumbel is defined as:

$$C(v_1, v_2; \theta) = exp\left\{-\left[-ln(v_1)^\theta + \left(-ln(v_2)\right)^\theta\right]^{1/\theta}\right\}, \theta \geq 1 \tag{20}$$

and its generator function $\Psi(t) = \left(-ln(t)^\theta\right)$. The parameter θ measure the power of dependence, $\theta \in (1, \infty)$.When θ=1, there is no depended and when $\theta = +\infty$ there is perfect dependence.

3.4.2. Clayton Copula

Clayton copula has the following form:

$$C(v_1, v_2; \theta) = \left[max\{v_1^\theta + v_2^\theta - 1; 0\}\right]^{-1/\theta} \tag{21}$$

where $\theta \in (-1, \infty)/\{0\}$ and generator function is $\Psi(t) = t^{-\theta} - 1$. The parameter θ measure the power of dependence. When θ=0, there is no depended and when $\theta = +\infty$ there is perfect dependence.

3.4.3. Frank Copula

The Frank Copula defined as:

$$C(v_1, v_2; \theta) = -\frac{1}{\theta}\log\left[1 + \frac{(exp(\theta v_1)-1)(exp(\theta v_2)-1)}{exp(-\theta)-1}\right] \tag{22}$$

where $\theta \in R\backslash\{0\}$ and its generator function is $\Psi(t) = -\ln\frac{exp(-\theta t)-1}{exp(-\theta)-1}$.

The dependence parameter θ can assume any real value in $(-\infty, \infty)$. Values of $-\infty$, 0, and ∞ approximate the Fréchet lower bound, independence, and Fréchet upper bound, respectively.

3.4.4. Joe Copula

The Joe Copula defined as:

$$C(v_1, v_2; \theta) = 1 - \left[(1 - v_1)^{\theta} + (1 - v_2)^{\theta} - (1 - v_1)^{\theta}(1 - v_2)^{\theta}\right]^{1/\theta} \quad (23)$$

where $\theta \in (1, \infty)$ and its generator function is $\Psi(t) = -ln\left[1 - (1 - t)^{\theta}\right]$

3.5. Copula Estimation

Several approaches have been proposed in the literature to estimate the parameter of copula models. In addition to MLE (maximum likelihood estimation), a two-step procedure of IFM (inference functions for margins estimation) developed by Joe and Xu (1996) is easy to implement.

3.5.1. Maximum Likelihood Estimation

Let (Z_1, Z_2) represent two random variables from a bivariate distribution F with marginal distributions F_1 and F_2(with density function) f_1 and f_2 and copula C with density c. Each unknown parameters associated with the marginal densities f_1 and f_2 are ϑ_{z_1} *and* ϑ_{z_2}, and unknown parameter of the copula function c is denoted by θ. We denote the unknown vector of parameters by $\vartheta = \left(\vartheta_{z_1}, \vartheta_{z_2}, \theta\right)$.The bivariate density function of (z_1, z_2) may be represented as

$$f(z_1, z_2; \vartheta) = c\left(F_1\left(z_1; \vartheta_{z_1}\right), F_2\left(z_2;\ \vartheta_{z_2}\right); \theta\right) f_1\left(z_1; \vartheta_{z_1}\right) f_2\left(z_2;\ \vartheta_{z_2}\right) \quad (24)$$

where

$$c(v_1, v_2) = \frac{\partial^2 C(v_1, v_2)}{\partial v_1 \partial v_2} \quad (25)$$

The maximum likelihood estimation (MLE) of a model is obtained by maximizing the log likelihood function, as:

$$l(z_1, z_2; \vartheta) = \sum_{i=1}^{n} ln\left(c(F_1\left(z_1; \vartheta_{z_1}\right), F_2\left(z_2; \vartheta_{z_2}\right); \theta) \times f_1\left(z_1; \vartheta_{z_1}\right) \times f_2\left(z_2; \vartheta_{z_2}\right)\right) \quad (26)$$

The exact maximum likelihood estimator is defined as:

$$\hat{\vartheta}_{MLE} = \arg\max_{\vartheta} l(z_1, z_2; \vartheta) \quad (27)$$

The exact maximum likelihood estimation (MLE) estimate the marginal distribution parameters $\vartheta_{z_1}$and ϑ_{z_2} jointly with copula parameter θ. For high dimensional data, MLE may be difficult and for the complicated structure of time-varying dependence, an analytical expression for the gradient vector of the likelihood might not exist.

To solve the optimization problem numerical methods may be adopted, implying dramatic slows down of computation procedure.

3.5.2. Inference Functions for Margins Estimation (IFM)

Instead of maximizing the exact likelihood function of ϑ_i, copula parameter can be estimated by two stage procedure proposed by Shih and Louis (1995) and Joe and Xu (1996), the inference function for margin ns (IFM) method.

In the first step, marginal distribution F_1 and F_2 are estimated.

$\hat{\vartheta}_{z_1} \in \arg\max_{\vartheta_{z_1}} \sum_{i=1}^{n} ln\left(f_1\left(z_1, \vartheta_{z_1}\right)\right),$

$\hat{\vartheta}_{z_2} \in \arg\max_{\vartheta_{z_2}} \sum_{i=1}^{n} ln\left(f_2\left(z_2, \vartheta_{z_2}\right)\right).$

In the second step, the copula parameter θ estimated conditioned on the previous marginal distributions estimates $\hat{F}_1\ and\ \hat{F}_2$.

$$\hat{\boldsymbol{\theta}} \in \boldsymbol{arg} \max_{\theta} \sum_{i=1}^{n} ln\left[c\left(F_1\left(z_1, \hat{\vartheta}_{z_1}\right), F_2\left(z_2, \hat{\vartheta}_{z_2}\right); \theta\right)\right]$$

Per Patton (2006b) if the model is correctly specified then IFM estimators $\hat{\vartheta}_{z_1}, \hat{\vartheta}_{z_2} and\ \boldsymbol{\theta}$ are consistent and asymptotically Normal.

The models were ranked using Akaike's information criterion and defined as:

$$AIC(M) = -2\ln\left(\hat{L}\right) + 2M \qquad (28)$$

where M is no of estimated parameter and $\ln\left(\hat{L}\right)$ is maximum log likelihood value. Smaller the AIC value betters the fit.

Figure 9 show the properties of different bivariate Elliptical and Archimedean Copulas, figure 10 represents contour plots for Normal, t, Clayton, Gumbel, Frank and Joe copula and figure 11 shows pdf plots for Normal, t, Clayton, Gumbel, Frank and Joe copula.

3.6. GARCH Models for the Marginal Distributions

We follow the inference function for margins (IFM) method, two step procedure. In the first step, we estimate the marginal models. A univariate AR (1,1)-GARCH (1,1) with different innovations is usually chosen to model the marginal distributions of return data. AR (1)-GARCH (1,1) with *t* distribution innovation is described as:

$$r_t = \mu + ar_{t-1} + \varepsilon_t$$
$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2$$
$$\varepsilon_t . \sqrt{\frac{v}{\sigma_t^2(v-2)}} \sim iidt_v \tag{29}$$

where ε_t is the innovation process.

3.6.1. Risk Management for Bivariate Copula Models

For the computation of portfolio value at risk and expected shortfall, we have to rely on Monte Carlo simulation (Christoffersen, 2012). To calculate VaR and ES, Monte Carlo simulation reverses the steps taken in estimation of copula model building:

1. First, we estimate dynamic volatility models, σ_t, for each asset and calculate standardised returns.
2. Second, estimate a density model for each asset to get the probabilities $v_{i,t} = F_i(z_{i,t})$ for each asset.
3. Estimate the copula model's parameters using $\ln l = \sum_{t=1}^{T} ln(v_{1,t}, v_{2,t})$.
4. Simulate the probabilities $(v_{1,t}, v_{2,t})$ from copula models.

Now after simulation of data, we need to reverse the steps of estimation of copula models.

5. Now create shock from the copula probabilities by using the inverse of marginal CDF's on each asset, $z_{i,t} = F_i^{-1}(v_{i,t})$.
6. By using the dynamic volatility models, $r_{i,t} = z_{i,t}\sigma_{i,t}$ for each asset we create portfolio returns from the shocks.
7. Now we can calculate VaR and ES for equally weighted portfolio as a $100\alpha th - Percentile$(Christoffersen ,2012)

3.7. Dynamic Conditional Correlation (DCC)

Modelling correlation dynamics is crucial to risk management. Empirical evidences suggested that during financial uncertainty correlation increases, as a result increasing risk even further (Christoffersen, 2012).

Univariate and multivariate dynamics are separated in two stage method to estimate DCC models. Standardized residuals extracted from estimated univariate GARCH models are used to compute the correlation matrix (Engle 2002).

From covariance and volatility correlation is defined as:

$\rho_{ij,t+1} = \sigma_{ij,t+1}/(\sigma_{i,t+1}\sigma_{j,t+1})$

(30)

The definition of correlation allows the decomposition of covariance into volatility and correlation:

$$\sigma_{ij,t+1} = \rho_{ij,t+1}\sigma_{i,t+1}\sigma_{j,t+1} \tag{31}$$

In metrics form, we can write as:

$$H_{t+1} = D_{t+1}\Upsilon_{t+1}D_{t+1} \tag{32}$$

where D_{t+1} is a matrix of standard deviations, $\sigma_{i,t+1}$, on the *i*th diagonal and zero everywhere else, and Υ_{t+1} is a matrix of correlations, $\rho_{ij,t+1}$, with ones on the diagonal.

In the two-asset case:

$$H_{t+1} = \begin{bmatrix} \sigma_{1,t+1}^2 & \sigma_{12,t+1} \\ \sigma_{12,t+1} & \sigma_{2,t+1}^2 \end{bmatrix} = \begin{bmatrix} \sigma_{1,t+1} & 0 \\ 0 & \sigma_{2,t+1} \end{bmatrix} \begin{bmatrix} 1 & \rho_{12,t+1} \\ \rho_{12,t+1} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{1,t+1} & 0 \\ 0 & \sigma_{2,t+1} \end{bmatrix} \tag{33}$$

The volatilities of each asset will estimate through NGARCH. To get the standardized returns ($z_{i,t+1} =, i = 1,2,\dots,n$) dividing the returns by their conditional standard deviation. The conditional covariance of the $z_{i,t+1}$ variables equals the conditional correlation of the raw returns:

$$E_t(z_{i,t+1}z_{j,t+1}) = E_t\left((R_{i,t+1}/\sigma_{i,t+1})(R_{j,t+1}/\sigma_{j,t+1})\right) \tag{34}$$

$= E_t\,(R_{i,t+1}R_{j,t+1})/(\sigma_{i,t+1}\sigma_{j,t+1})$

$= \sigma_{ij,t+1}/(\sigma_{i,t+1}\sigma_{j,t+1})$

$= \rho_{ij,t+1,}$for all i, j

Modelling the conditional correlation of the raw returns is equivalent to modelling the conditional covariance of the standardized returns.

In metrics notation DCC model can be written as:

$$Q_{t+1} = E[z_t z_t'](z_t z_t') + \beta Q_t \tag{35}$$

In the two-asset case:

$$Q_{t+1} = \begin{bmatrix} q_{11,t+1} & q_{12,t+1} \\ q_{12,t+1} & q_{22,t+1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{bmatrix}(1-\alpha-\beta) + \alpha \begin{bmatrix} z_{1,t}^2 & z_{1,t}z_{2,t} \\ z_{1,t}z_{2,t} & z_{2,t}^2 \end{bmatrix} + \beta \begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{bmatrix} \tag{36}$$

where $\rho_{12}$is the unconditional correlation between the two assets and estimated as:

$$\bar{\rho}_{12} = \frac{1}{T}\sum_{t=1}^{T} z_{1,t}z_{2,t}$$

For bivariate normal distribution, the log-likelihood function is expressed as:

$$LL_{12} = -\frac{1}{2}\sum_{t=1}^{T}\left(ln\left(1-\rho_{12,t}^2\right) + \frac{\left(z_{1,t}^2+z_{2,t}^2-2\rho_{12}z_{1,t}z_{2,t}\right)}{\left(1-\rho_{12,t}^2\right)}\right) \quad (37)$$

$$\rho_{12,t+1} = \frac{q_{12,t+1}}{\sqrt{q_{11,t+1}q_{22,t+1}}}$$

$$q_{11,t+1} = 1 + \alpha\left(z_{1,t}^2 - 1\right) + \beta\left(q_{11,t} - 1\right)$$

$$q_{12,t+1} = \bar{\rho}_{12} + \alpha\left(z_{1,t}z_{2,t} - \bar{\rho}_{12}\right) + \beta\left(q_{12,t+1} - \bar{\rho}_{12}\right)$$

$$q_{22,t+1} = 1 + \alpha\left(z_{2,t}^2 - 1\right) + \beta\left(q_{22,t} - 1\right)$$

In the general case of *n* assets:

$$ln(LL) = -\frac{1}{2}\sum_t\left(log|\Upsilon_t| + z_t'\Upsilon_t^{-1}z_t\right) \quad (38)$$

where $|\Upsilon|$denotes the determinant of the correlation matrix, Υ_t(Christoffersen, 2012)

3.8. The Risk Term Structure with Constant Correlations

The *n* asset returns in vector form is:

$$r_{t+1} = D_{t+1}z_{t+1} \quad (39)$$

where D_{t+1} is an $n \times n$ dignal metrics containing the dynamic standard deviations on the diagonal, and zeros on the off diagonal. z_{t+1} is an $1 \times n$ vector contains the shocks from the dynamic volatility model for each asset(see, Christoffersen , 2012).

The conditional covariance matrix of the returns is defined as:

$$Var_t(r_t + 1) = H_{t+1} = D_{t+1}\Upsilon D_{t+1} \quad (40)$$

The $n \times n$ metrics Υ contains the base asset correlations on the off diagonals and ones on the diagonal.

For two uncorrelated bivariate shocks, we have:

$$E\left[z_{t+1}^u\left(z_{t+1}^u\right)'\right] = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \quad (41)$$

To create correlated shocks with the correlation matrix:

$$E\left[z_{t+1}\left(z_{t+1}\right)'\right] = \Upsilon = \begin{bmatrix}1 & \rho_{1,2}\\\rho_{1,2} & 1\end{bmatrix}$$

$$E\left[z_{t+1}(z_{t+1})'\right] = E\left[\Upsilon^{1/2} z^u_{t+1}(z^u_{t+1})'\left(\Upsilon^{1/2}\right)'\right] = \Upsilon \qquad (42)$$

For two assets case:

$$\Upsilon^{1/2} = \begin{bmatrix} 1 & 0 \\ \rho_{1,2} & \sqrt{1-{\rho_{1,2}}^2} \end{bmatrix}$$

$$z_{t+1} = \Upsilon^{1/2} z^u_{t+1}$$

$$z_{1,t+1} = z^u_{1,t+1}$$

$$z_{2,t+1} = \rho_{1,2} z^u_{1,t+1} + \sqrt{1-{\rho_{1,2}}^2}\, z^u_{2,t+1} \qquad (43)$$

This implies:

$$E\left[z_{1,t+1}\right] = E\left[z^u_{1,t+1}\right] = 0$$

$$E\left[z_{2,t+1}\right] = \rho_{1,2} E\left[z^u_{1,t+1}\right] + \sqrt{1-{\rho_{1,2}}^2}\, E\left[z^u_{2,t+1}\right] = 0$$

and

$$Var\left[z_{1,t+1}\right] = Var\left[z^u_{1,t+1}\right] = 1$$

$$Var\left[z_{2,t+1}\right] = {\rho_{1,2}}^2\, Var\left[z^u_{1,t+1}\right] + \left(1-{\rho_{1,2}}^2\right) Var\left[z^u_{2,t+1}\right] = 1$$

We can check the correlation is:

$$E\left[z_{1,t+1} z_{2,t+1}\right] = \rho_{1,2}\, E\left[z^u_{1,t+1} z^u_{2,t+1}\right] + \sqrt{1-{\rho_{1,2}}^2}\, E\left[z^u_{1,t+1} z^u_{2,t+1}\right] = \rho_{1,2} \qquad (44)$$

In order to calculate term structure of risk with constant correlation, we rely on Monte Carlo simulation as described in Christoffersen (2012):

1. Draw a vector of uncorrelated random normal variables $\check{z}^u_{i,1}$ with zero mean and one-variance.

2. To correlate the random variables, use the matrix square root $\Upsilon^{1/2}$ and get $\check{z}^u_{i,1} = \Upsilon^{1/2} z^u_{i,1}$.

3. Update the variances for each asset, as:

$$\check{\sigma}^2_{i,t+2} = \omega + \alpha \check{R}^2_{i,t+1} + \beta \sigma^2_{i,t+1} \qquad (45)$$

where

$$\sigma^2_{i,t+1} = \sum_{i=1}^{n} \sigma^2_i$$

4.Compute returns for each asset:

$\breve{R}_{i,t+1:t+k} = \sum_{k=1}^{K} \breve{R}_{i,t+k}$ for $i = 1,2,\ldots,MC$ (46)

Loop through above steps from day t+1 until day t+k. compute the portfolio return using the equal portfolio weights and the vector of simulated returns on each day. Repeating these steps $i = 1,2,\ldots MC$ times gives a Monte Carlo distribution of portfolio returns. From these MC portfolio returns we can compute VaR and ES from the simulated portfolio returns (see, Christoffersen , 2012):

$$VaR_{t+1,t+k}^{p} = -Percentile\left\{\left\{\breve{R}_{t+1,t+k}\right\}_{i}^{MC}, 100p\right\} \quad (47)$$

$$ES_{t+1,t+k}^{p} = -\frac{1}{p.MC}\sum_{i=1}^{MC.1} \breve{R}_{t+1,t+k} \,.\, 1\left(\breve{R}_{t+1,t+k} < -VaR_{t+1,t+k}^{p}\right) \quad (48)$$

3.8.1. The Risk Term Structure with Dynamic Correlations

Now we consider dynamic correlation as in DCC models instead of constant correlation: at the end of day t we get D_{t+1} and Υ_{t+1} without simulation form GARCH and DCC models (Christofferse, 2012).

$$\breve{r}_{i,t+1} = D_{t+1}\Upsilon_{t+1}^{1/2}\breve{z}_{i,t+1}^{u} = D_{t+1}\breve{z}_{i,t+1} \quad (49)$$

$$\breve{z}_{i,t+1} = \Upsilon_{t+1}^{1/2}\breve{z}_{i,1}^{u} \quad (50)$$

Using the new simulated shock vector, $\breve{z}_{i,t+1}$, we can update the volatilities and correlations using the GARCH models and the DCC model. We thus obtain simulated $\breve{D}_{t+2}$ and $\breve{\Upsilon}_{t+1}$ .Now, draw a new vector of uncorrelated shocks $\breve{z}_{i,2}^{u}$ enables us to simulate the return for the second day ahead as:

$$\breve{r}_{i,t+2} = D_{t+2}\Upsilon_{t+2}^{1/2}\breve{z}_{i,t+2}^{u} = D_{t+2}\breve{z}_{i,t+2}$$

$$\breve{z}_{i,t+2} = \Upsilon_{t+2}^{1/2}\breve{z}_{i,2}^{u}$$

We continue this simulation from day t+1 until day t+k. compute the portfolio return using the equal portfolio weights and the vector of simulated returns on each day. Repeating these steps $i = 1,2,\ldots MC$ times gives a Monte Carlo distribution of portfolio returns. From these MC portfolio returns we can compute VaR and ES from the simulated portfolio returns (see, Christoffersen (2012):

$$VaR_{t+1,t+k}^{p} = -Percentile\left\{\left\{\breve{R}_{t+1,t+k}\right\}_{i}^{MC}, 100p\right\} \quad (51)$$

$$ES^p_{t+1,t+k} = -\frac{1}{p.MC}\sum_{i=1}^{MC.1}\breve{R}_{t+1,t+k}\,.\,1\left(\breve{R}_{t+1,t+k} < -VaR^p_{t+1,t+k}\right) \quad (52)$$

3.9. Multivariate Copulas

Copulas are simply multivariate distribution functions with uniform margins. A d-dimensional copula is defined as multivariate distribution function $C(v_1, \dots, v_d)$ on the$C: [0,1]^d \to [0,1]$. Let F be distribution function for a random vector $Z = (Z_1, \dots, Z_d)$ with margins$F_1, \dots, F_d$. Then according to Sklar's Theorem (Sklar, 1959) there exist a copula C such that for all $z = (z_1, \dots, z_d) \in [-\infty, \infty]^d$:

$$F(z_1, \dots, z_d) = C\left(F_1(z_1), \dots, F_d(z_d)\right) \quad (53)$$

where C is d-dimensional copula, and if F is continuous with strictly continuous increasing marginal distributions $F_1, \dots F_d$:

$$f(z_1, \dots, z_d) = c\left(F_1(z_1), \dots F_2(z_d)\right).\left[\prod_{i=1}^{d} f_i(z_i)\right] \quad (54)$$

where c denotes the density of the copula and given by:

$$c(v_1, v_2) = \frac{\partial^2 C(v_1, v_2)}{\partial v_1 \partial v_2} \quad (55)$$

Detailed copula analysis available in Joe (1996) and Nelson (2006).

3.9.1 Vine Copula/Pair Copula Construction (PCC)

Joe (1996) is the first who introduced selective construction of pair copula. Later Aas et al. (2009) introduced graphical representation to specify pair copula constructions (PCCs), also called vine copula.

We can represent PCC for two, three, more and general multivariate case.

For two random variables(Z_1, Z_2) $d = 2$:

$$f(z_1, z_2) = c_{12}\left(F_1(z_1), F_2(z_2)\right) f_1(z_1) f_2(z_2) \quad (56)$$

For the transformed variables $F_1(z_1)$ and $F_2(z_2)$, c_{12} is the appropriate pair copula density.

The conditional density can be calculated as:

$$f(z_2|z_1) = \frac{f(z_1, z_2)}{f_1(z_1)} = c_{12}\left(F_1(z_1), F_2(z_2)\right) f_2(z_2) \quad (57)$$

For three random variables$(Z_1, Z_2$ and$Z_3)$, $d = 3$

$$f(z_1, z_2, z_3) = f_1(z_1) f_{2|1}(z_2|z_1) f_{3|12}(z_3|z_1, z_2) \quad (58)$$

$$f_{2|1}(z_2|z_1) = \frac{f(z_1, z_2)}{f_1(z_1)} = \frac{= c_{12}\left(F_1(z_1), F_2(z_2)\right) f_1(z_1) f_2(z_2)}{f_1(z_1)} = c_{12}\left(F_1(z_1), F_2(z_2)\right) f_2(z_2), \quad (59)$$

$$f_{3|12}(z_3|z_1,z_2) = \frac{f_{13|2}(z_1,z_3|z_2)}{f_{2|1}(z_2|z_1)} = \frac{c_{13|2}\left(F_{1|2}(z_1|z_2),F_{3|2}(z_3|z_2)\right)f_{2|1}(z_2|z_1)f_{3|2}(z_3|z_1)}{f_{2|1}(z_2|z_1)}$$

$$= c_{13|2}\left(F_{1|2}(z_1|z_2),F_{3|2}(z_3|z_2)\right)f_{3|2}(z_3|z_2)$$

$$f_{13|2}(z_1,z_3|z_2) = c_{13|2}\left(F_{1|2}(z_1|z_2),F_{3|2}(z_3|z_2)\right)f_{1|2}(z_1|z_2)f_{3|2}(z_3|z_2)\ ,$$

$$f_{3|2}(z_3,z_2) = c_{23}\big(F_2(z_2),F_3(z_3)\big)f_3(z_3)$$

$$f_{3|12}(z_3|z_1,z_2) = c_{13|2}\left(F_{1|2}(z_1|z_2),F_{3|2}(z_3|z_2)\right)c_{23}\big(F_2(z_2),F_3(z_3)\big)f_3(z_3) \tag{60}$$

The 3-dimension full decomposition is:

$$= c_{13|2}\left(F_{1|2}(z_1|z_2),F_{3|2}(z_3|z_2)\right)c_{23}\big(F_2(z_2),F_3(z_3)\big)$$

$$\times\, c_{12}\big(F_1(z_1),F_2(z_2)\big)\times f_3(z_3)f_2(z_2)f_1(z_1) \tag{61}$$

Per Czado (2010) a multivariate density can be calculated as a product of pair-copulas, acting on several different conditional marginal distributions.

$$f(z_1,\dots,z_d) = \prod_{t=2}^{d} f(z_t|z_1,\dots,z_{t-1})\times f_1(z_1) \tag{62}$$

For the arbitrary distinct $i,j,i_1,\dots,i_k$ with $i<j$ and $i_1<\cdots<i_k$, let:

$$c_{i,j|}i_1,\dots,i_k = c_{i,j|}i_1,\dots,i_k\left(F(z_i|z_{i1},\dots,z_{ik}),\left(F\big(z_j|z_{i1,\dots,}z_{ik}\big)\right)\right)$$

By expressing$(z_t|z_1,\dots,z_{t-1})$:

$$f(z_t|z_1,\dots,z_{t-1}) = c_{1,t|2,\dots t-1}\times\ f(z_t|z_1,\dots,z_{t-1})$$

$$= \prod_{s=1}^{t-2} c_{s,t|s+1,\dots,t-1}\times c_{(t-1),t}\times f_t(z_t) \tag{63}$$

Czado (2010) showed that for$t=2,\dots,d$, the joint distribution is:

$$f(z_1,\dots,z_d) = \prod_{j=1}^{d-1}\prod_{i=1}^{d-j} c_{i,(i+j)|(i+1),\dots,(i+j-1)}\times\prod_{k=1}^{d} f_k(x_k) \tag{64}$$

This decomposition called pair copula decomposition (PCC). The conditional distributions needed as copula arguments at level j are obtained as partial derivatives of the copula at level $j-1$. Under regular condition, according to Joe (1996):

$$F(z|V) = \frac{\partial C_{zV_j;V_{-j}}\big(F(z|V_{-j}),F(V_j|V_{-j})\big)}{\partial F(V_j|V_{-j})} \tag{65}$$

where $C_{zV_j;V_{-j}}$ is a bivariate copula function, V_j is a arbitrary component of V and V_{-j} denotes the vector V excluding V_j.When V is univariate, z and V are distributed uniformly on the intervals [0,1],then

$$F(z|V) = \frac{\partial C(z,V)}{\partial V}$$

We can assume a parametric specification for $C_{i,j|i_1,\ldots,i_k}$ with given parameter vector θ, pair copula densities and univariate condition ac be simplifying as:

$$h(v_1|v_2;\theta) := F(v_1|v_2;\theta) = \frac{\partial C_{v_1,v_2}(v_1,v_2;\theta)}{\partial\theta} \tag{66}$$

where θ is the parametric vector for C_{v_1,v_2}.

3.9.2. Vine Structure

There are many ways to decompose multivariate density function into PCC. Bedford and Cooke (2001) have specified graphical structure, called a regular vine tree structure, that helped to organize all decompositions.

A *d*-dimensional vine tree structure is sequence of *d-1* trees. Tree *j d+1-j* nodes and *n-j* edges. The edges in tree *j* become nodes in tree *j+1*.Two nodes in tree *j+1* are joined by an edge if the corresponding edges in tree j share a node. The density of a regular vine distribution is defined as by the multiple of pair copula density over the $\left(\frac{d(d-1)}{2}\right)$ edges identified by the regular vine tree structure and the product of the marginal densities.

Regular vine decomposition includes many possible pair copulas. In this research, we construct R-vine and two special cases of R-vine called D-vines and C-vines (canonical vines. C-vine are regular vine distribution for which each tree has a unique node that is connect to *d-j* edges, and D-vine are regular vine distribution for which no node in any tree is connected to more than two edges.

For C-vine copula each tree has a unique node that is connected to all other nodes.

$$f_{1234567} = \begin{pmatrix} f_1.f_2.f_3.f_4.f_5.f_6.f_7 \\ \text{nodes in } \boldsymbol{T_1} \end{pmatrix} \begin{pmatrix} c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17} \\ edges\ in\ \boldsymbol{T_1} \\ nodes\ in\ \boldsymbol{T_2} \end{pmatrix} \begin{pmatrix} c_{23|1}, c_{24|1}, c_{25|1}, c_{26|1}, c_{27|1} \\ edges\ in\ T_2 \\ nodes\ in\ T_3 \end{pmatrix}$$

$$\begin{pmatrix} c_{34|12}, c_{35|12}, c_{36|12}, c_{37|12} \\ edges\ in\ T_3 \\ nodes\ in\ T_4 \end{pmatrix} \begin{pmatrix} c_{45|123}, c_{46|123}, c_{47|123} \\ edges\ in\ T_4 \\ nodes\ in\ T_5 \end{pmatrix}$$

$$\begin{pmatrix} c_{56|1234}, c_{57|1234} \\ edges\ in\ T_5 \\ nodes\ in\ T_6 \end{pmatrix} \begin{pmatrix} c_{67|12345} \\ edges\ in\ T_6 \end{pmatrix}$$

$$M = \begin{bmatrix} 6 & & & & & & \\ 7 & 5 & & & & & \\ 1 & 7 & 4 & & & & \\ 2 & 1 & 7 & 3 & & & \\ 3 & 2 & 1 & 7 & 2 & & \\ 4 & 3 & 2 & 1 & 7 & 1 & \\ 5 & 4 & 3 & 2 & 1 & 7 & 1 \end{bmatrix}$$

1.$(1,2), (1,3), (1,4), (1,5), (1,6), (1,7)$

2.$(2,3|1), (2,4|1), (2,5|1), (2,6|1), (2,7|1)$

3.$(3,4|12), (3,5|12), (3,6|12), (3,7|12)$

4.$(4,5|123), (4,6|123), (4,7|123)$

5. $(5,6|1234), (5,7|1234)$

6. $(6,7|12345)$

Figure 12 represent the decomposition of a four-dimensional C-vine joint density function into pair-copulas and marginal densities.

The density $f(z_1, \dots, z_d)$ of d-dimension for C-vine copula can be written as (Aas et al. 2009):

$$f(z_1, \dots, z_d) = \left[\prod_{k=1}^{d} f(z_k)\right] \times \left[\prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{j,j+1:1,\dots,j-1}\left(F\left(z_i|z_1, \dots z_{j-1}\right), F\left(z_{j+1}|z_{1,t}, \dots z_{j-1}\right)\right)\right] \quad (67)$$

where index j identifies the trees, while i runs over the edges in each tree.

In D-vine no node is connected to more than 2 edges.

$$f_{1234567} = \begin{pmatrix} \{f_1 . f_2 . f_3 . f_4 . f_5 . f_6 . f_7 \\ \text{nodes in } \boldsymbol{T_1} \end{pmatrix} \begin{pmatrix} c_{12}, c_{23}, c_{34}, c_{45}, c_{56}, c_{67} \\ edges\ in\ \boldsymbol{T_1} \\ nodes\ in\ \boldsymbol{T_2} \end{pmatrix} \begin{pmatrix} c_{13|2}, c_{24|3}, c_{35|4,} c_{46|5}, c_{57|6} \\ edges\ in\ T_2 \\ nodes\ in\ T_3 \end{pmatrix}$$

$$\begin{pmatrix} c_{14|23}, c_{25|34}, c_{36|45}, c_{47|56} \\ edges\ in\ T_3 \\ nodes\ in\ T_4 \end{pmatrix} \begin{pmatrix} c_{15|234}, c_{26|345}, c_{37|456} \\ edges\ in\ T_4 \\ nodes\ in\ T_5 \end{pmatrix} \begin{pmatrix} c_{16|2345}, c_{27|3456} \\ edges\ in\ T_5 \\ nodes\ in\ T_6 \end{pmatrix}$$

$$\begin{pmatrix} c_{17|23456} \\ edges\ in\ T_6 \end{pmatrix}$$

$$M = \begin{bmatrix} 1 & & & & & & \\ 7 & 2 & & & & & \\ 2 & 7 & 3 & & & & \\ 3 & 3 & 7 & 4 & & & \\ 4 & 4 & 4 & 7 & 5 & & \\ 5 & 5 & 5 & 5 & 7 & 6 & \\ 6 & 6 & 6 & 6 & 6 & 7 & 1 \end{bmatrix}$$

1.$(1,2), (2,3), (3,4), (4,5), (5,6), (6,7)$

2.$(1,3|2), (2,4|3), (3,5|4), (4,6|5), (5,7|6)$

3.$(1,4|23), (2,5|34), (3,6|45), (4,7|56)$

4.$(1,5|234), (2,6|345), (3,7|456)$

5. $(1,6|2345), (2,7|3456)$

6. $(1,7|23456)$

Figure 13 represent the decomposition of a four-dimensional D-vine joint density function into pair-copulas and marginal densities.

The density $f(z_1, \dots, z_d)$ of d-dimension for D-vine copula can be written as (Aas et al. 2009):

$$f(z_1, \dots, z_d) = \left[\prod_{k=1}^{d} f(z_k)\right]$$

$$\times \left[\prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j;i+1,\dots,i+j-1}\, F\big(z_i|z_{i+1}, \dots z_{i+j-1}\big), F\big(z_{i+j}|z_{i+1}, \dots z_{i+j-1}\big)\right] \quad (68)$$

3.9.3. Estimation of Pair-Copula Decompositions

There is a different method available for parametric estimation for given tree structure and copula families for pair copulas. In sequentially estimation parameters are estimated starting from the top tree until the last, however stander errors are difficult to estimate (Aas et al., 2009 and Czado et al., 2012).

Maximum likelihood estimation is asymptotically efficient, but for high dimensions it is not appropriate and calculation of standard errors is also challenging, Stoeber and Schepsmeier (2012). In our research parameters are estimated by the Inference Function for Margins (IFM) method, where the estimation of the parameters is done in two steps:

1. The parameters in the marginal distributions are estimated.

2. The copula parameters are estimated conditioned on the previous marginal distributions estimates.

For a R- vine decomposition, the log-likelihood is given by:

$$l(z:\theta) = \sum_{t=1}^{T}\sum_{j=1}^{1}\sum_{i=1}^{j+1} ln\left(c_{m_{j,j},m_{i,j};m_{i+1},\dots,m_{n,j}}\left(F\left(z_{mj,j,t}|z_{mj+1,t},\dots z_{1,j,t}\right), F\left(z_{mi,j,t}|z_{mi+1,j,t},\dots z_{m1j,t}\right)\right)\right) \tag{69}$$

For a C- vine decomposition, the log-likelihood is given by:

$$l(z:\theta) = \sum_{t=1}^{T}\sum_{j=1}^{d-1}\sum_{i=1}^{d-j} ln\left(c_{j,j+1:1,\dots,j-1}\left(F\left(z_{i,t}|z_{1,t},\dots z_{j-1,t}\right), F\left(z_{j+1,t}|z_{1,t},\dots z_{j-1,t}\right)\right)\right) \tag{70}$$

For a D- vine decomposition, the log-likelihood is given by:

$$l(z:\theta) = \sum_{t=1}^{T}\sum_{j=1}^{d-1}\sum_{i=1}^{d-j} ln\left(c_{i,i+j;i+1,\dots,i+j-1}\left(F\left(z_{i,t}|z_{i+1,t},\dots z_{i+j-1,t}\right), F\left(z_{i+j,t}|z_{i+1,t},\dots z_{i+j-1,t}\right)\right)\right) \tag{71}$$

Akaike or (Schwarz) Bayesian information criterions are the straightforward approaches to select a copula between non-nested parametric copulas as estimated by maximum likelihood. The Akaike's information criterion (AIC) is defined as:

$$AIC(M) = -2\ln(\hat{L}) + 2M \tag{72}$$

3.9.4. Value-at-Risk and Expected Shortfall Calculation for Pair-Copula

To forecast the Value-at-Risk (VaR) and expected shortfall (ES) of the equally weighted portfolio for one day ahead for pair copula, we need to use Monte Carlo simulate from the estimated pair-copula decomposition followed by the method suggested in Aas et al. (2009). From the fitted copula model, we simulate a sample of random numbers of each copula class. The simulated vine copulas observations are then converted using the inverse skewed *t* distribution cumulative distribution function (CDF) which is an assumption of the marginal distribution in the NGARCH model.

The standardized residuals calculated from inverse skewed *t* CDF along with the estimated parameters of the NGARCH model are later used to forecast the log returns of each asset in the portfolio. We distribute equal weights to each stock log return, and then we get the returns after the weighting; finally, we calculate the value of the portfolio for each of the simulation and use the empirical quantile function to calculate one-day Value-at-Risk and Expected Shortfall at different significance levels.

3.10. Back- testing Risk Models

According to Lopez (1999), to assess the validity of VaR models the statistical tests suggested by Kupiec(1995) and Christoffersen(1998) can have relatively low power against inaccurate VaR models. Lopez (1999) proposed a forecast evaluation framework based on loss function rather than on a statistical testing framework. By specifying a utility function and ranking the risk models, loss function satisfies the specific need of the risk manager.

Lopez (1999) loss function takes the following specific form:

$$\Psi_{t+1} = \begin{cases} 1 + \left(VaR_{t+1|t} - x_{t+1}\right)^2 & \text{if violation occurs} \\ 0 & else, \end{cases} \tag{73}$$

which accounts for the magnitude of the tail losses $\left(VaR_{t+1|t} - x_{t+1}\right)^2$ and adds a score of one whenever a violation is observed. The model that minimizes the total loss $\sum_{t=1}^{T} \Psi_t$ outperforms other models.

This approach has a main drawback that the return x_{t+1} should be better compare with ES measure not with the VaR, as VaR does not give any evidence of the size of the expected loss. Therefore, the proposed loss function for the Expected shortfall can be proposed as:

$$\Psi_{1|t+1}^{(i)} = \begin{cases} \left|x_{t+1} - ES_{t+1|t}^{(i)}\right| & if\ voilation\ occurs \\ 0 & else, \end{cases} \tag{74}$$

$$\Psi_{2|t+1}^{(i)} = \begin{cases} \left(x_{t+1} - ES_{t+1|t}^{(i)}\right)^2 & if\ voilation\ occurs \\ 0 & else, \end{cases}$$

To judge the models by loss functions we calculate MAE:

$$MAE = \tilde{T}^{-1} \sum_{t=1}^{T} \Psi_{1|t+1}^{(i)} \tag{75}$$

The best model is preferred with the lowest MAS error. To evaluate VaR we use loss function of Gonz´alez-Rivera et al. (2004) which is especially suited to assess quantile risk measures, such as the VaR. Gonzalez-Rivera et al. (2004) suggest a loss function to forecast the Value-at-Risk (VaR) of a portfolio of financial assets and describe as for given α:

$$Q = P^{-1} \sum_{i=1}^{T} (\alpha - d_{t+1}^{\alpha})(y_{t+1} - VaR_{t+1}^{\alpha}) \tag{76}$$

where $d_{t+1}^{\alpha} = 1(y_{t+1} < VaR_{t+1}^{\alpha})$. Smaller Q indicates a better goodness of fit.

3.11. Implementation

The methods were implemented in the open-source software R by using the following packages; xlsx (Dragulescu, 2015) , rugarch (Ghalanos, 2015a), rmgarch (Ghalanos 2015b), CDVine (Schepsmeier and Brechmann 2015), VineCopula (Schepsmeier et at.,2016) ,PerformanceAnalytics (Peterson and Carl, 2014), quantmod (Ryan et al.,2015) , car(Fox and Weisberg,2016),FinTS(Graves,2015),hydroGOF(Bigiarini,2014),Metrics (Hamner,2015),fGrch(Wuertz and Chalabi,2015),zoo (Zeileis et al.,2016), fBasics (Wuertz et al.,2015), tseries (Trapletti et al.,2016).

4. Empirical Results

In this section, we model the dependence among the returns of 15 of fifteen companies from DAX -30 index from June 1995 to June 2015. The selection of 15 companies mainly depends on the availability of data for all companies from June 1995. Further we investigate dependence among 7 and 5-dimension data. For the five-dimension data, we have chosen five companies mainly form steel and engineering sector. All the data has been taken from DataStream database.

Table 37 shows the start date, end date and number of observation of the data analysed in this chapter.

4.1. Data Description and Preliminary Analysis

Figure 14 plots daily prices, returns, squared returns for each analysed data set. Each plot of each time series exhibits the typical empirical time series properties.

The plots of the closing prices of each data set are not stationary in other words the data does not revert around mean and it changes throughout the time series. Whereas, the plot for the returns does fluctuate around mean. It is the desirable property of time series to have a stationary data set because the characteristics of a stationary time series allow handling models that are independent of a specific starting point, practically it may be difficult to obtain. The squared daily returns exhibit evidence of volatility clustering that large changes tend to be followed by large changes and suggests the presence of heteroscedasticity.

The summary statics are presented in table 38. The value of skewness is negative for some return series and positive for some returns indicating an asymmetry in the distribution of return. A negatively skewed distribution or skewed to the left has a long-left tail and a positively skewed distribution or skewed to the right has a long right tail. Our all data series are characterized by many small gains and a few extreme losses.

As positive kurtosis indicates a relatively peaked distribution and negative kurtosis indicates a relatively flat distribution. A normal distribution has a kurtosis of 3. The kurtosis of our all data sets is greater than 3 reflect fat tails. We reject null hypothesis of the normal distribution as the *p* value for Jarque-Bera (1980) test is less than 0.05. Jarque-Bera test confirms that all return series have non-normal distributions. Among all stocks, the VOL is the most volatile as it has highest Standard deviation and Standard and HEN is the least volatile asset. The Ljung-Box (1978) Q-statistics reported in Table 2 for both returns and squared returns for all data series also reject the null hypothesis of no autocorrelation through 20-lags at a 5% significance level.

4.2. Marginal Models for Univariate Data

As indicated earlier that we divide our sample into 15, 7, 5 and 2 dimensions. For our entire sample, we adopt the two-step estimation method in this chapter due to the large number of parameters in the time-varying models. As Haff (2012) provides evidence that the performance of the stepwise estimator is rather valid compared to the full log likelihood method.

The first step is to estimate the marginal distribution using GARCH specification. To imply leverage effect, we applied asymmetric GARCH models, i.e. NGARCH with normal, student *t*, skewed *t* and GED innovations. We choose the best specifications for the marginal based on the information criteria AIC and BIC**.** The parameter estimates and standard errors for the marginal distribution models for bivariate data are presented in table 115, table 116, table 117 and table 118 in Appendix. As indicated before our data exhibit non-normal characteristics, so we only compare NGARCH-*t*, NGARCH-skewed *t* and NGARCH-GED. Based on AIC criteria NGARCH-skewed *t* for has lower AIC for all datasets. Moreover, our data sets display clear signs of asymmetry and excess kurtosis, NGARCH-skewed *t* is the best marginal model for our data set.

To set an initial view on the correlation and dependence relationship among five stocks, we present Pearson Correlation, Spearman’s Correlation and Kendall’s Tau Correlation measures in table 39, table 40 and table 41. As expected, the unconditional correlation measure matrices present a high dependence between among all fifteen stocks.

After choosing the NGARCH-skewed *t* marginal models and obtaining the standardized return, we now examine the dependence between the filtered returns. Figure 15 shows the scatter plots of a pair of filtered returns. The scatter plots emerge to be clouded circles with dispersions in lower and upper tails exhibit the disappearing of linear correlation between return series.

It is evident, that fitting of the marginal models did not remove the dependence between each series. Table 42, table 43 and table 44 summarizes the conditional correlation measures. Like the unconditional correlation measures, the conditional correlation matrices indicate that there exists a rather high dependence between standardized return series. In addition, the unconditional correlation measures are slightly higher than the conditional ones.

4.3. Estimation Results for Copula Models

Visual observation of bivariate plots in figure15 confirms that the filtered returns are correlated, to describe this dependence, we will fit copula models on bivariate data sets. The following copula families are fitted to the normal, Student *t*, Gumbel, Clayton, Frank, and Joe copulas. Table 111 in Appendix reports estimated parameter for bivariate data sets alongside with AIC and BIC value. We use the AIC for selection criterion because of evidence that it performs especially well in a simulation study (Brechmann, 2010).

The results in table 111 indicate that all estimated parameters are statistically significant**.** All models are ranked on minimization of AIC. The results indicate that Frank, Clayton and student *t* remained top three models for most of the portfolio. Although in few cases normal copula is in top three models but we don't prefer it because of its properties. The Kendal's tau estimation for student *t* copula and Frank Copula remain higher.

Table 112 in Appendix shows parameter estimation of bivariate DCC-GAECH models. We compared DCC-Normal and DCC-*t*, the results indicates that for all portfolios DCC-*t* model outperform DCC-Normal.

Table 45 and table 46 report the estimated parameter for C and D vine copula for 5-dimension data of engraining and steel sector for selected models. These tables also report the estimated Kendall's tau computed based on the estimated pair-copulas. For C-Vine copula in table 45 Frank copula is preferred model for tree 1, 3 and 4. Three 2 for D-vine copula all three-pair preferred student *t* distribution. For C-vine copula pair BMW.VOL and BMW.LIN have highest Kendell's tau, while for D-vine BMW.VOL/SEI and VOL.LIN/THY have highest Kendell's tau.

For seven-dimension data, we estimate the parameters of C-vine and D-vine for Normal, Student *t*, Gumbel, Clayton, Frank, Joe, BB1, BB6, BB7, and BB8 models. Table 47 and 48 represent that student *t*, Frank, Gumbel, Clayton, BB1 are selected but student t, and Frank and BB1 appear more frequently. For D-vine copula it is the student-t that appears most frequently and then BB1, BB8 and Frank. Pair CON.FRES/ SAP.SIE.BAY.BASF.BMW have the highest Kendell's Tau for C –vine and pair SAP.BASF/SIE.BAY for D-vine copula. For 15-dimensions data we estimate 105 parameters for both C-vine and D-vine for Norma, *t*, Gumbel, Clayton, Frank, Joe, BB1, BB6, BB7 and BB8 models. Copula vine structure, estimated parameters, family selection and Kendell's tau for 14 trees are presented in tables 49, 50, 51, 52, 53, 54, 55, 56, 57 and 58 for both C-vine and D-vine copulas. For C- vine student *t* and Frank copula appeared more frequently in table 53 while for D-vine copula student *t*, Frank and BB8 appeared frequently.

The general conclusion from our estimated parameters for bivariate, 5-dimesion,7-dimension and 15-dimension data is that student *t*-copula and Frank copula are the best choices among the traditional copulas in most of the cases. Moreover, the advantage of vine copulas does not mainly for the flexible tree structure, but also for the flexibility of mixing different bivariate families.

4.4. Expected Shortfall (ES) Back-testing

We forecast the one day ahead VaR and ES for the different copula models and compare them with DCC models by Engle (2001) and Engle (2002) with Normal and Student *t* innovations. The DCC model is popular and thus established the most suitable benchmark for the vine copula models (Brenchman and Cazado, 2013). We have calculated Value at risk (VaR) and Expected Shortfall (ES) for 2- dimensions, 5- dimensions, 7- dimensions and 15- dimensions data. Moreover, we have also estimated VaR and ES for longer horizon for bivariate data and 7-dimensions data by Monte-Carlo (Static) and Monte-Carlo (DCC) models suggested by Christoffersen (2011).

The evaluated different VaR and ES models for bivariate data are presented in table 113 in Appendix. We ranked our ES models based on smallest value of MAE. For the pair BMW/SEI Student *t* rank first while Frank and Joe ranked second and third for 1% significant level. It is evident from the result that student *t* and Frank copula remain the top models and in few cases Clayton and Gumbel copula. Moreover, results indicate that copula-based methods also have better results than the DCC-norm and DCC-*t*.

Table 59 represents the back-test results for VaR and ES for C-vine, D-vine C and DCC models for 5-dimension data as representing the engineering and steel sector from DAX-30. C-Vine Clayton Copula ranked first for all significance levels, C-vine copula ranked second for all significance level. D-vine student *t* copula ranked third for 1%, fifth for 2.5%, and 4th for 5% significance. Again, for multivariate data copula models performed better than DCC-norm and DCC-*t* models.

Table 60 represent the back-test results for VaR and ES for C-vine, D-vine C and DCC models for 7- dimension data. For 7- dimensions data we calculate VaR and ES for C-vine and D-vine for Normal, Student *t*, Gumbel, Clayton, Frank, Joe, BB1, BB6, BB7, and BB8 models. D-Vine Joe Copula and C-Vine Joe Copula ranked first and second for all four significance levels (1%,2.5%,5% and 10%). C-Vine BB6 appeared third for all significance levels and D-vine BB6 as fifth for 1%, 2.5% and 10% while forth for 5% significance level.

Table 62 presents results of Back-testing VaR and ES for 15 dimensions' data. D-vine copula ranked first based on minimum MAE and C-Vine BB6 ranked second for all significance level. D-vine BB6 ranked third for 1%, 2.5%, 5% and forth for 10% significance level and D-Vine Gumbel copula ranked forth for 1%, 2.5%, 5% and third for 10% significance level.

If the main purpose of the risk models is the allocation of optimal portfolio instead of just risk measurement, the multivariate term structure of risk is required (Christoffersen, 2012). Thus, for active risk management we need to consider multivariate term structure of risk. We are applying Monte Caro simulation rather than square-root-of-time rule to calculate multi-days ahead VaR and ES (see Dowd et al., 2004 and Christopherson, 2012). Under the square-root-of-time rule the long-term risk measures such as VaR, is obtained by multiplying the one-day risk measure by the square root of the number of days in the holding period. The simulation-based methods allow to calculate VaR and ES for multivariate data at any horizon of interest and hence to calculate the entire term structure of risk. Monte Carlo simulation method is very flexible as can assume any distribution of return with Mote Carlo simulation based method. We don't need to rely on the assumption of normality of return.

We suggest a new adaptation of Christoffersen (2012) method for calculating multiple VaR and ES with Monte Carlo simulation method using many steps. We estimate NGARCH-Skew *t* at the end of day one and obtain day one returns and tomorrow's variance in the NGARCH model. To simulate the model forward in time using Monte Carlo we need to assume a multivariate distribution of the random shocks. We generate random numbers from the multivariate skewed student-*t* distribution. Then, we use both static and dynamic correlation to correlate these random variables. From correlated random number create the hypothetical returns for tomorrow for each asset. Given these hypothetical returns, we update the variance to get a set of hypothetical variances for the day after tomorrow. Repeat all above steps for multi-days. Now, we compute the portfolio return using the equal portfolio weights and the vector of simulated returns on each day. We compute multi-days VaR and ES from the simulated portfolio returns.

The 5-days and 10-days loss function for VaR indicates in table 61 that Monte Carlo(Static) perform significantly than of Monte Carlo(DCC). The best model is highlighted with bold. Again, for ES, Monte Carlo(Static) has lower MAE than of Mote Carlo(DCC).

It is important to note that that for one-day measure by vine copula models perform better than of DCC-norm and DCC-t. Our results reinforce the findings of Brechmann et al. (2011) that vine copula based models with static correlation are potential alternative to the DCC model for multivariate risk measurement. Our results also indicate that for multi-days VaR and ES model with static correlation perform better than of model with DCC correlation.

5. Concluding Remarks

The aim of this chapter is not only to present and discuss the use of vine copula for financial risk management but also to present the term structure of risk for multivariate data.

We follow the Inference Function for Margins (IFM) method, two step method. In the first step, we estimate the marginal models. A univariate AR (1,1)-GARCH (1,1) with different innovations usually chosen to model the marginal distributions of return data. We employ a NGARCH model with skewed-*t* distribution to filter the return series and construct their marginal distributions.

The C- and D-vine copulas are then estimated and chosen based on minimum AIC. The optimal choices of the copula for both C-vine and D-vine copula are Student *t*, BB1, BB8, Frank.

We finally show the implications of the empirical findings for risk management and calculate VaR and ES. We calculated VaR and ES with both static and dynamic correlation with Monte Carlo simulation rather than square-root-of-time rule for 5-days and 10-days.

The developed methodology is used to analyse the dependence structure among five stocks, seven stocks and fifteen as represented in the DAX 30 index. In these analyses, our models are critically compared to relevant benchmark models such as the DCC models. It turns out that vine copula models for our data provide good fit and accurately and efficiently forecast the expected shortfall as compare to DCC-norm and DCC-*t* (Zhang et al.,2014; Brechmann and Czado ,2011; and Aloui and Aissa, 2016). Among vine copula models it is Joe copula class and BB1 copula class with both C-vine construction and D-vine construction among the top five models. For term structure of risk for multivariate data Monte Carlo simulation with static correlation outperformed Monte Carlo simulation with dynamic correlation.

An innovation of this paper is the estimation of multiple-step-ahead VaR and ES for multivariate stock data for the NGARCH-Skew *t* specification with both static and dynamic correlation. The new methodology has been adapted from Christoffersen (2012).

This research constitutes one of the first applications of multi-days multivariate VaR and ES measure with Monte Carlo simulation method. Our results indicate that for 1-day ahead VaR and ES measure by vine copula with static correlation perform better than of DCC model with both normal and student *t* innovations. Moreover, for both 5-days and 10-days ahead again Monte Carlo with static correlation significantly perform better than of dynamic correlation(DCC).

The present work constitutes one of the first applications of multivariate term structure of risk. An important direction of future research is the consideration of term structure of risk for longer horizon than 5-days and 10-days as Monte Carlo simulation is time consuming for longer horizon as compare to simple rule of square-root-of-time. However, in the view of Wang et al. (2011) square root time rule (SRTR) scaling to convert the longer-term tail risks is inappropriate and misleading.

An important direction of future research is the consideration of filtered historical simulation (FHS) approach for multi-days ahead multivariate VaR and ES measurement. FHS combines model-based methods of variance with model-free methods of the distribution of shocks. As Monte Carlo simulation based models are good if the selected model for distribution of returns is a sufficiently correct specification of reality.

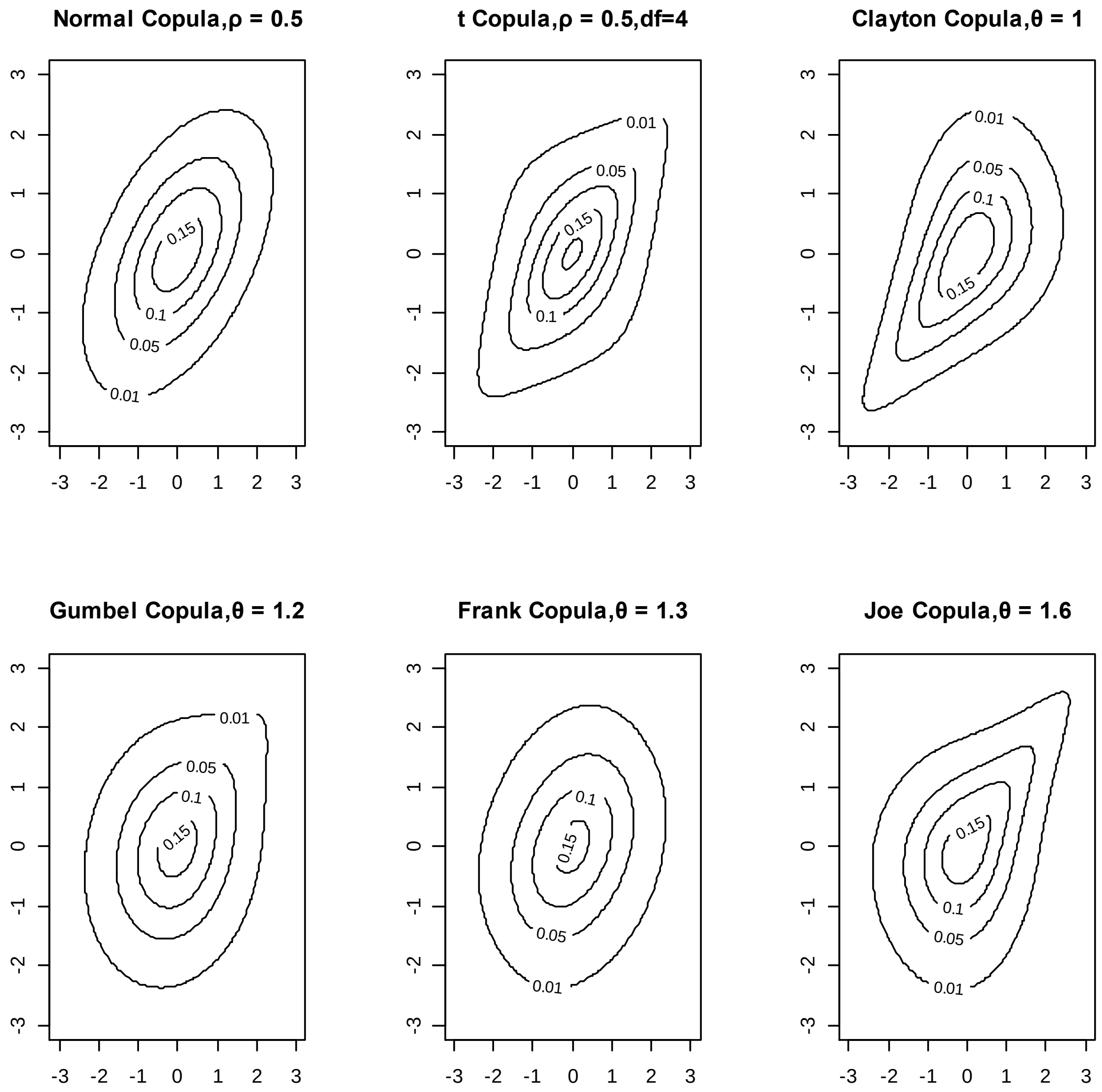
Figure 9: Notation and Properties of Bivariate Elliptical and Archimedean Copulas.

No	Elliptical Distribution		Parameter Range	Kendell's Tau	Tail Dependence
1	Gaussian		$\rho \in (-1,1)$	$\frac{2}{\pi} acrsin(\rho)$	0
2	Student-t		$\rho \in (-1,1)$, $v > 2$	$\frac{2}{\pi} acrsin(\rho)$	$2t_{v+1}\left(-\sqrt{v+1}\sqrt{\frac{1-}{1+}}\right.$
No	**Name**	**Generator Function**	**Parameter Range**	**Kendell's Tau**	**Tail Dependence (lower, upper)**
3	Clayton	$\frac{1}{\theta}\left(t^{-\theta}-1\right)$	$\theta > 0$	$\frac{\theta}{\theta+2}$	$\left(2^{-1/\theta},0\right)$
4	Gumbel	$(-logt)^{\theta}$	$\theta \geq 1$	$1-\frac{1}{\theta}$	$\left(0,2-2^{1/\theta}\right)$
5	Frank	$-log\left[\frac{e^{\theta}-1}{e^{\theta}-1}\right]$	$\theta \in R\backslash\{0\}$	$1-\frac{4}{\theta}+4\frac{D_1(\theta)}{\theta}$	(0,0)

6	Joe	$-log\left[1-(1-t)^{\theta}\right]$	$\theta > 1$	$1-\frac{4}{\theta^2}\int_0^1 tlog(t)(1-t)\theta^{\frac{2(1-\theta)}{\theta}}dt$	$\left(0,2-2^{1/\theta}\right)$
7	BB1	$\left(t^{-\theta}-1\right)^{\delta}$	$\theta > 0, \delta \geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$\left(2^{-1/(\theta\delta)}, 2-2^{1/\delta}\right)$
8	BB6	$\left(-log\left[1-(1-t)^{\theta}\right]\right)^{\delta}$	$\theta \geq 1, \delta \geq 1$	$1+4\int_0^1 \left(-log\left(-(1-t)^{\theta}+1\right) \times \frac{\left(1-t-(1-t)^{-\theta}+t(1-t)^{-\theta}\right)}{\delta\theta}\right)dt$	$\left(0,2-2^{1/(\theta\delta)}\right)$
9	BB7	$\left[1-(1-t)^{\theta}\right]^{-\delta}$	$\theta \geq 1, \delta > 0$	$1-\frac{2}{\delta(2-\theta)}+\frac{4}{\theta^2\delta}B\left(\frac{2-\theta}{\theta}, \delta+2\right)$	$\left(2^{-1/\delta}, 2-2^{1/\theta}\right)$
10	BB8	$-log\left[\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}\right]$	$\theta \geq 1, 0 < \delta \leq 1$	$1+4\int_0^1 \left(-log\left(\frac{(1-t\delta)^{\theta}-1}{(1-\delta)^{\theta}-1}\right) \times \frac{\left(1-t\delta-(1-t\delta)^{-\theta}+t\delta(1-t\delta)^{-\theta}\right)}{\delta\theta}\right)dt$	$(0,0)$

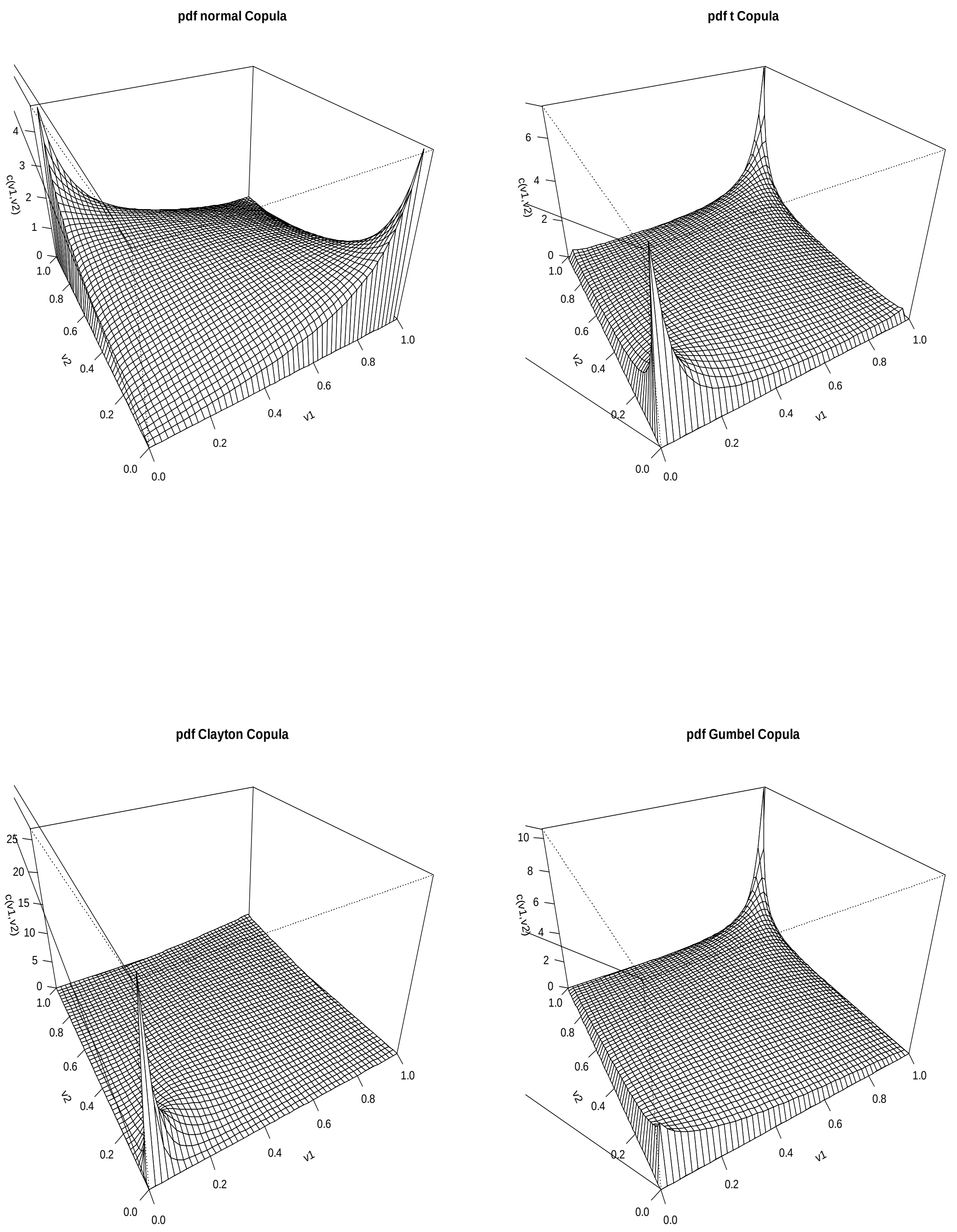
Source: Breckmann and Cazado (2013)

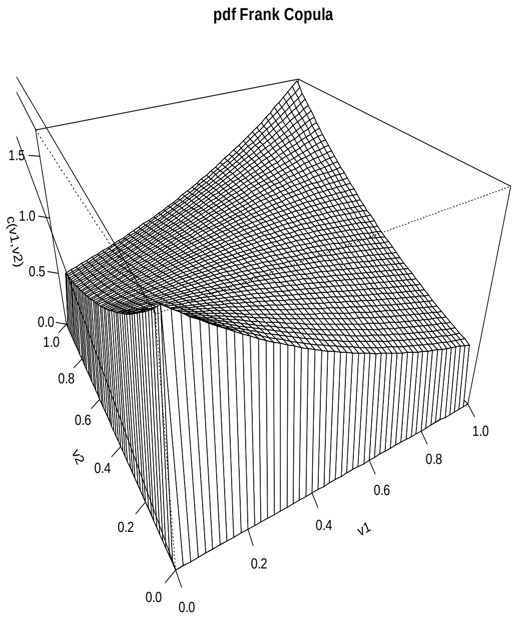
Figure 10: Contour Plots for Normal, Student *t*, Clayton, Gumbel, Frank and Joe Copula.

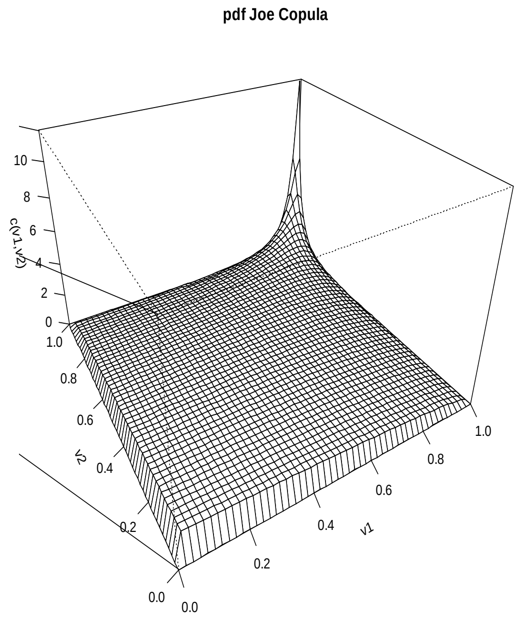


Note: We plot the contour plot for Normal , Student t, Clayton, Gumbel, Frank and Joe Copulas. The marginal distributions are assumed to be normal.

Figure 11: pdf Plots for Normal, Student *t*, Clayton, Gumbel, Frank and Joe Copula.



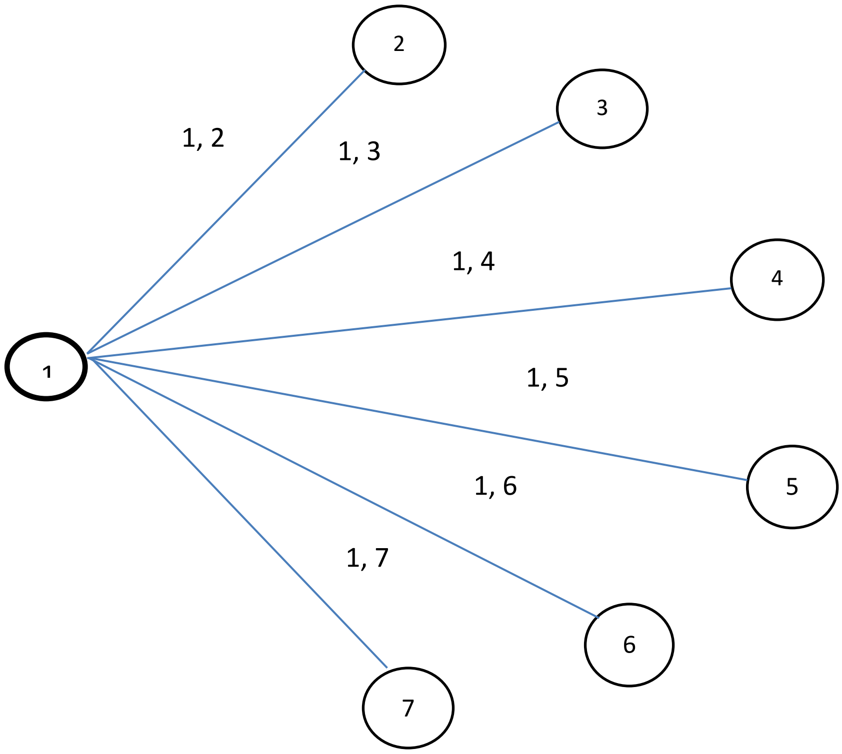




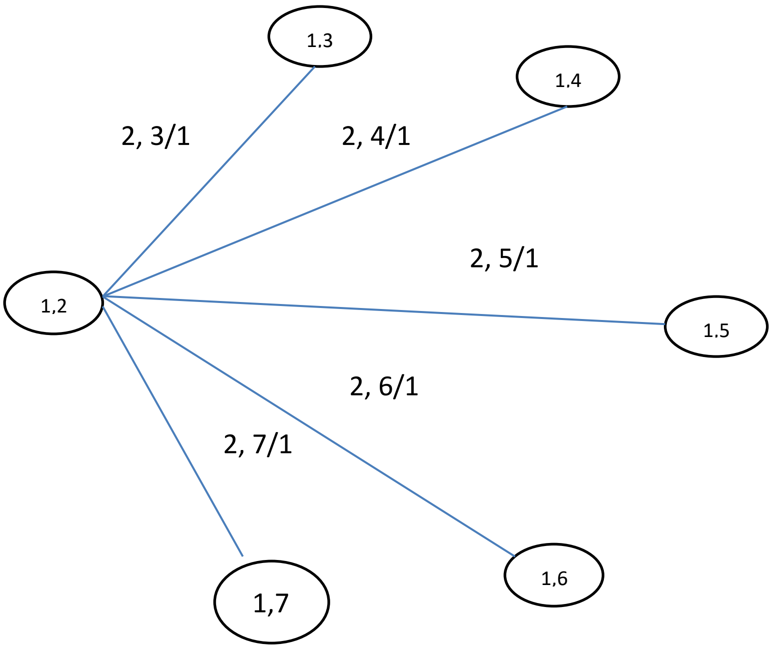
Note: We plot the PDF plot for Normal, t, Clayton, Gumbel, Frank and Joe Copulas. The marginal distributions are assumed to be normal.

Figure 12: C-vine Decomposition for 7 Dimensions.

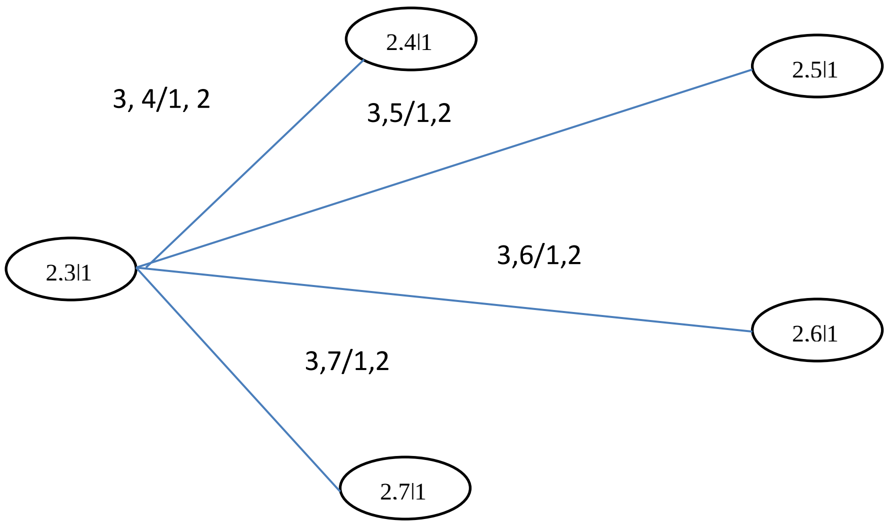
T_1



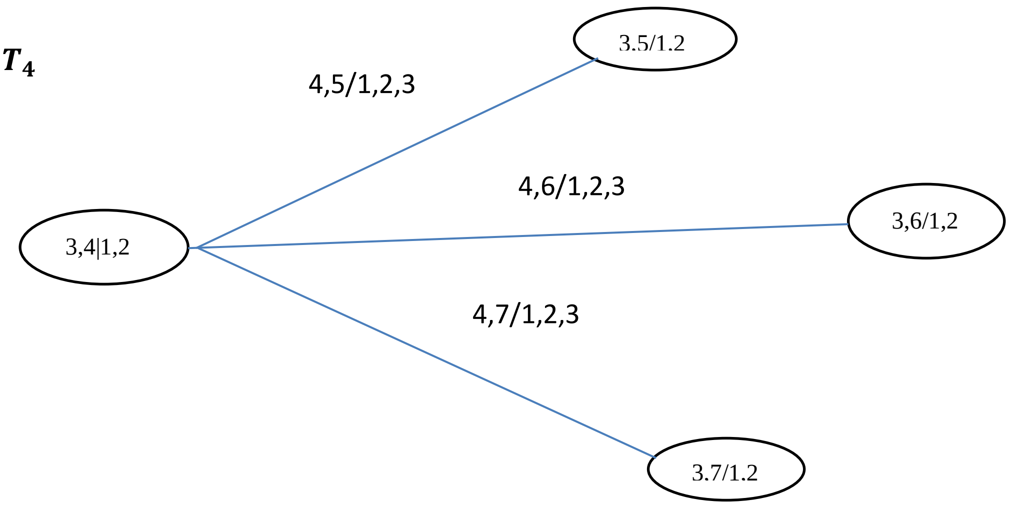
T_2



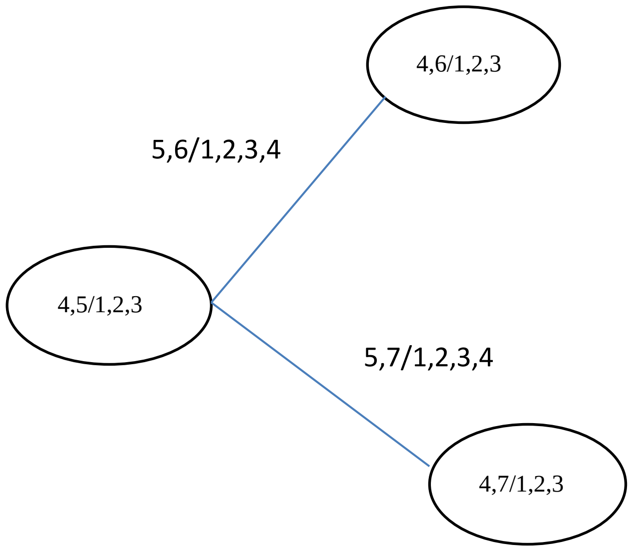
T_3


2.3|1
2.4|1
2.5|1
2.6|1
2.7|1
3, 4/1, 2
3,5/1,2
3,6/1,2
3,7/1,2

T_4


3,4|1,2
3.5/1.2
3,6/1,2
3.7/1.2
4,5/1,2,3
4,6/1,2,3
4,7/1,2,3

T_5


4,5/1,2,3
4,6/1,2,3
4,7/1,2,3
5,6/1,2,3,4
5,7/1,2,3,4

T_6

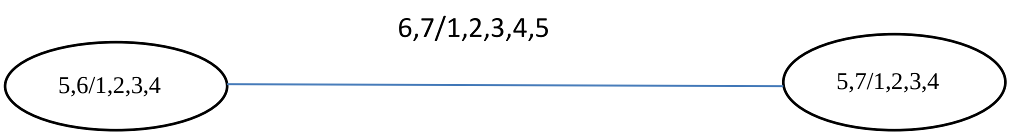

5,6/1,2,3,4
5,7/1,2,3,4
6,7/1,2,3,4,5

Figure 13: D-vine Decomposition for 7 Dimensions.

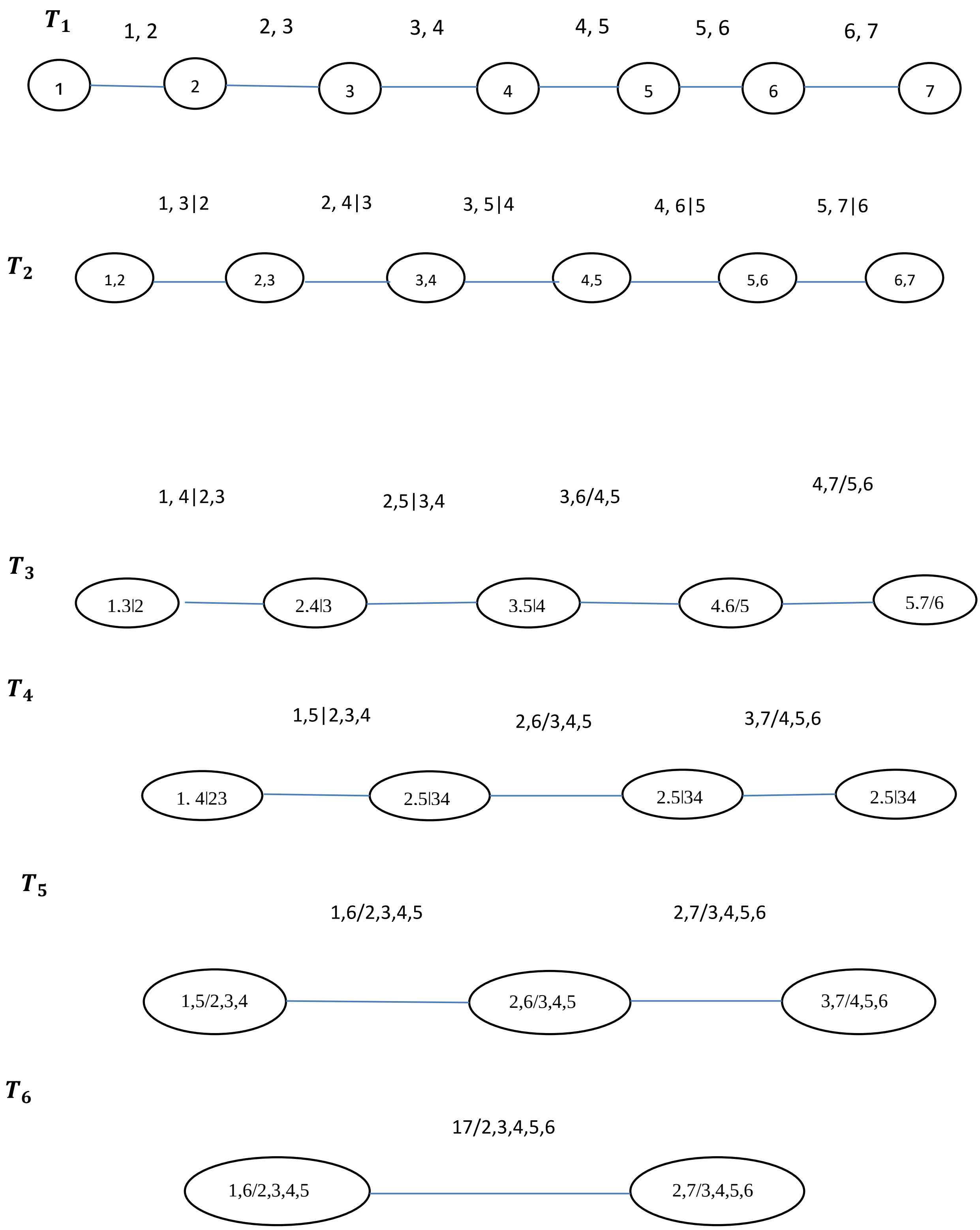
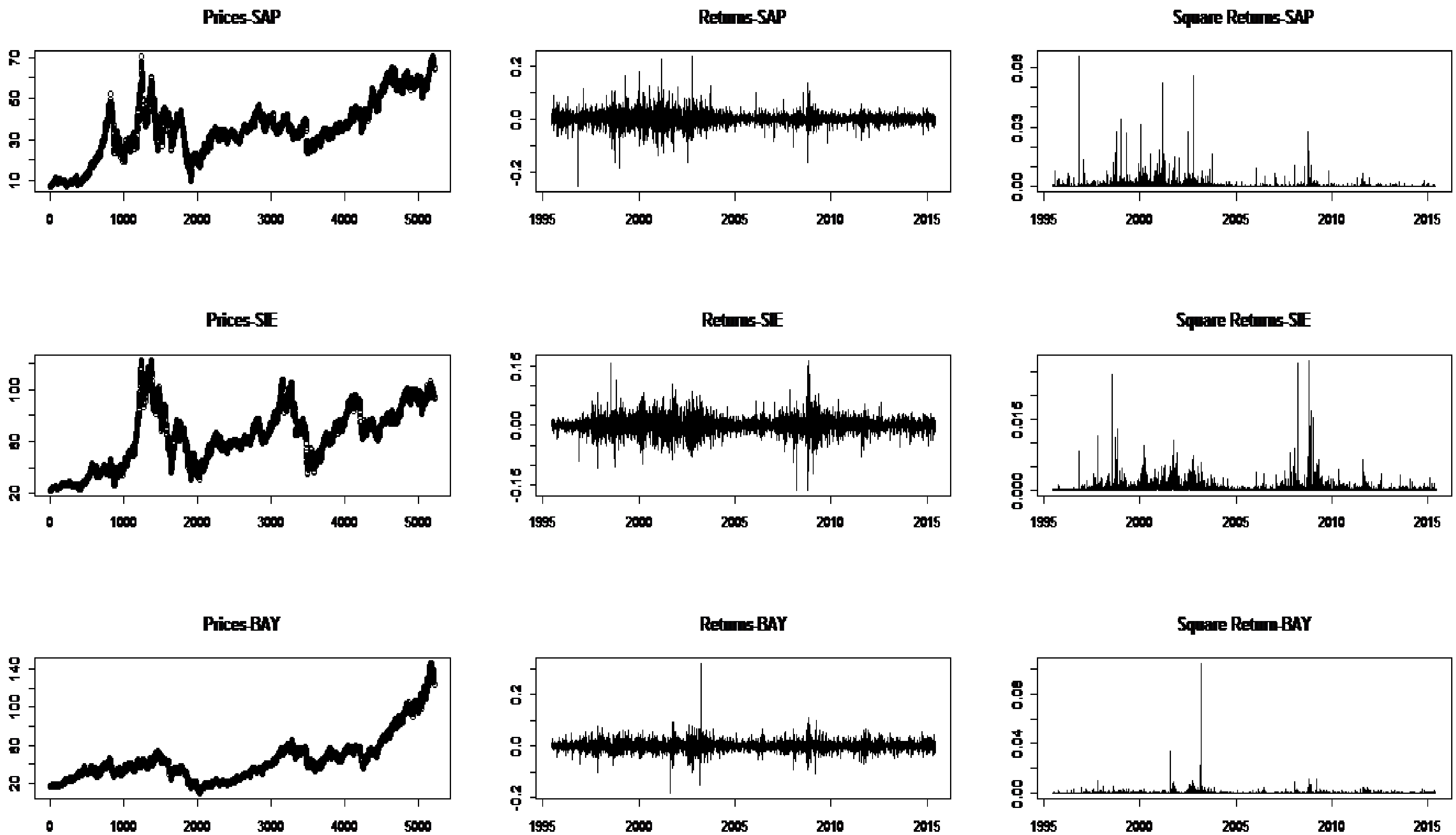
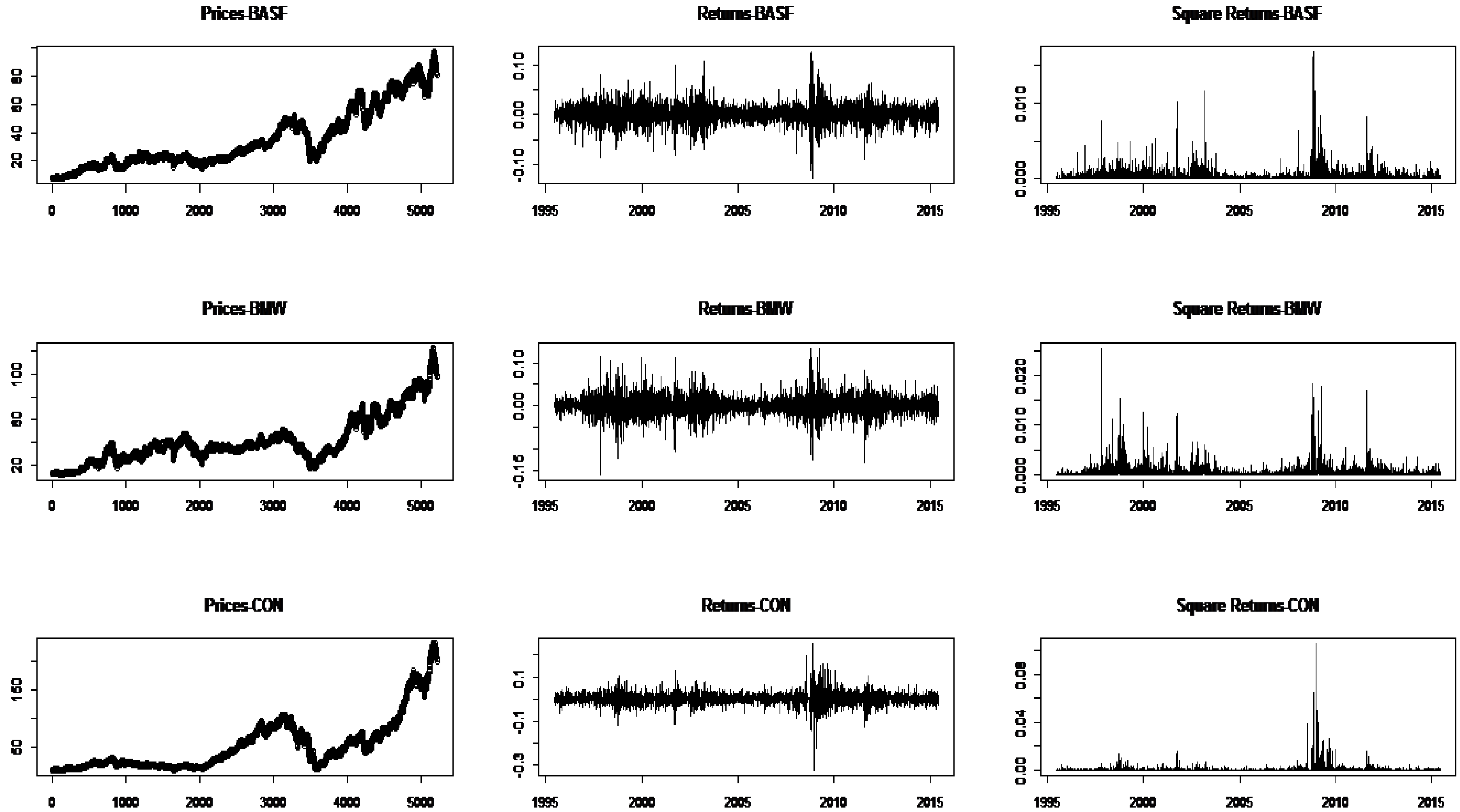


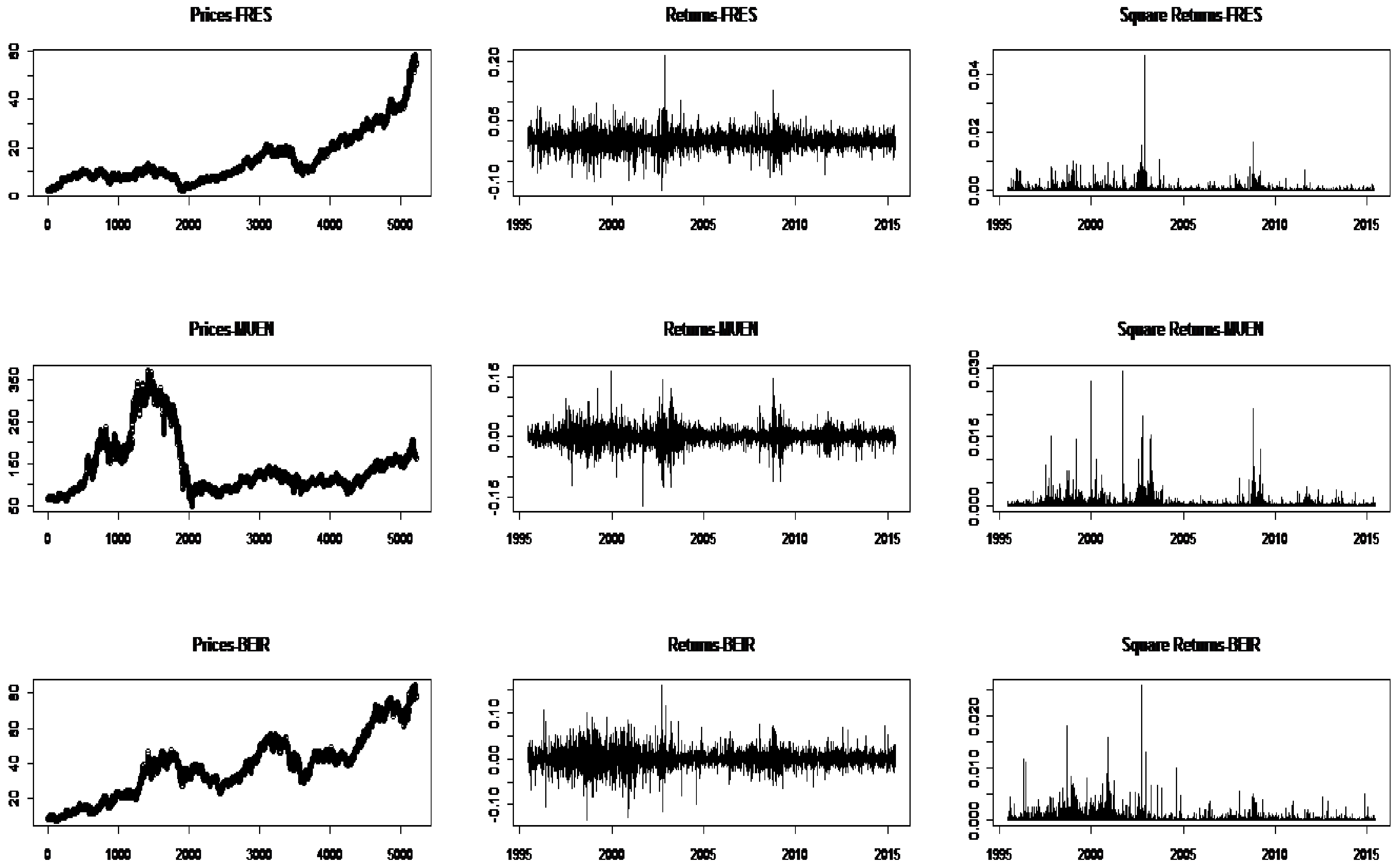
Table 36: Data Analysed

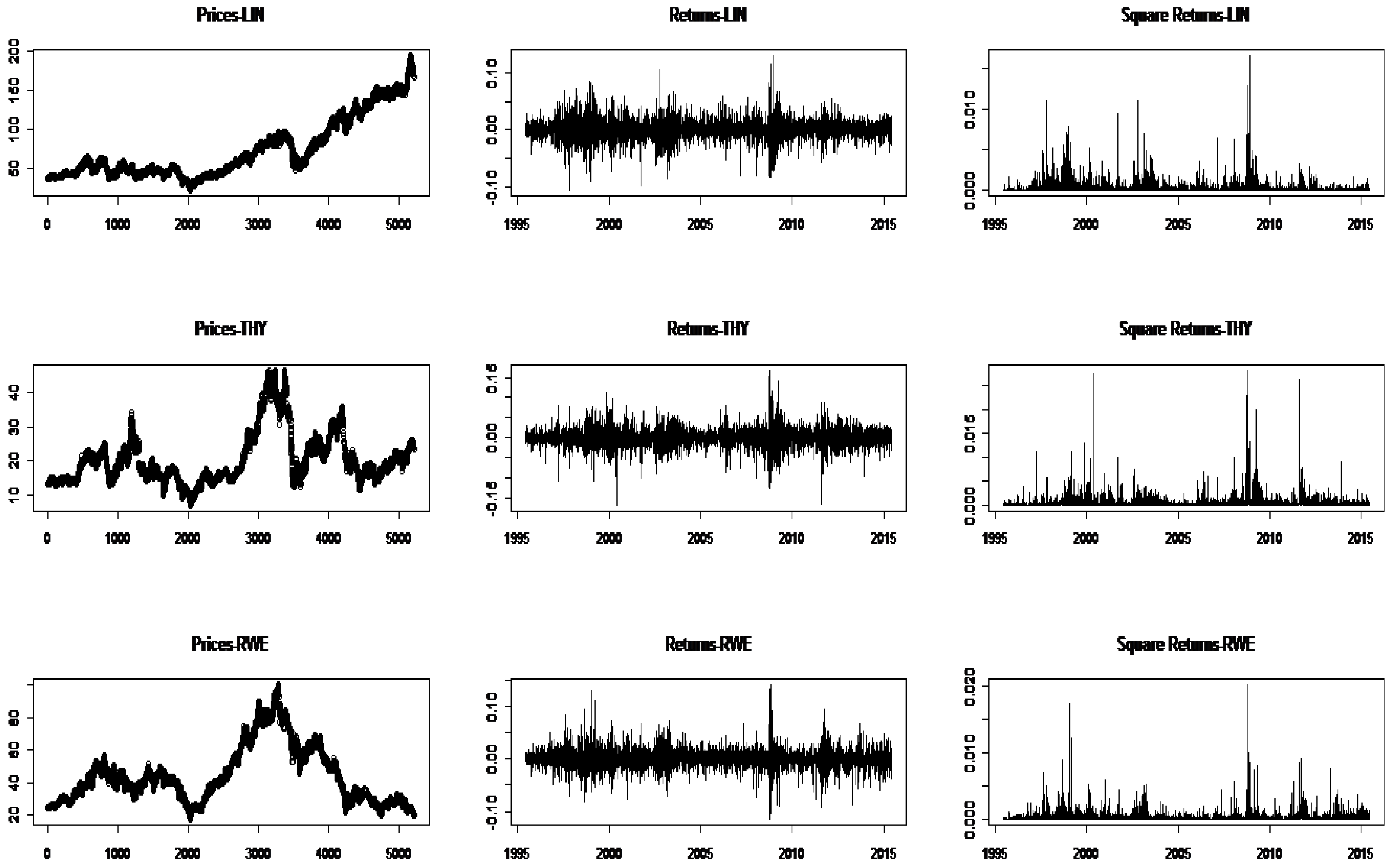
Stock	Ticker	Start	End	Observations
SAP	SAP	09/06/1995	09/06/2015	5217
SIEMENS	SIE	09/06/1995	09/06/2015	5217
BAYER	BAY	09/06/1995	09/06/2015	5217
BASF	BASF	09/06/1995	09/06/2015	5217
BMW	BMW	09/06/1995	09/06/2015	5217
CONTINENTAL	CON	09/06/1995	09/06/2015	5217
FRESENIUS	FRES	09/06/1995	09/06/2015	5217
MUENCHENER RUCK.	MUEN	09/06/1995	09/06/2015	5217
BEIERSDORF	BEIR	09/06/1995	09/06/2015	5217
LINDE	LIN	09/06/1995	09/06/2015	5217
THYSSENKRUPP	THY	09/06/1995	09/06/2015	5217
RWE	RWE	09/06/1995	09/06/2015	5217
DEUTSCHE LUFTHANSA	DEU	09/06/1995	09/06/2015	5217
HENKEL PREF.	HEN	09/06/1995	09/06/2015	5217
VOLKSWAGEN PREF.	VOL	09/06/1995	09/06/2015	5217

Figure 14: Prices, Return and Square Retune Plot.




Prices-BASF
Returns-BASF
Square Returns-BASF
Prices-BMW
Returns-BMW
Square Returns-BMW
Prices-CON
Returns-CON
Square Returns-CON
0 1000 2000 3000 4000 5000
1995 2000 2005 2010 2015


Prices-FRES
Returns-FRES
Square Returns-FRES
Prices-MUEN
Returns-MUEN
Square Returns-MUEN
Prices-BEIR
Returns-BEIR
Square Returns-BEIR


Prices-LIN
Returns-LIN
Square Returns-LIN
Prices-THY
Returns-THY
Square Returns-THY
Prices-RWE
Returns-RWE
Square Returns-RWE

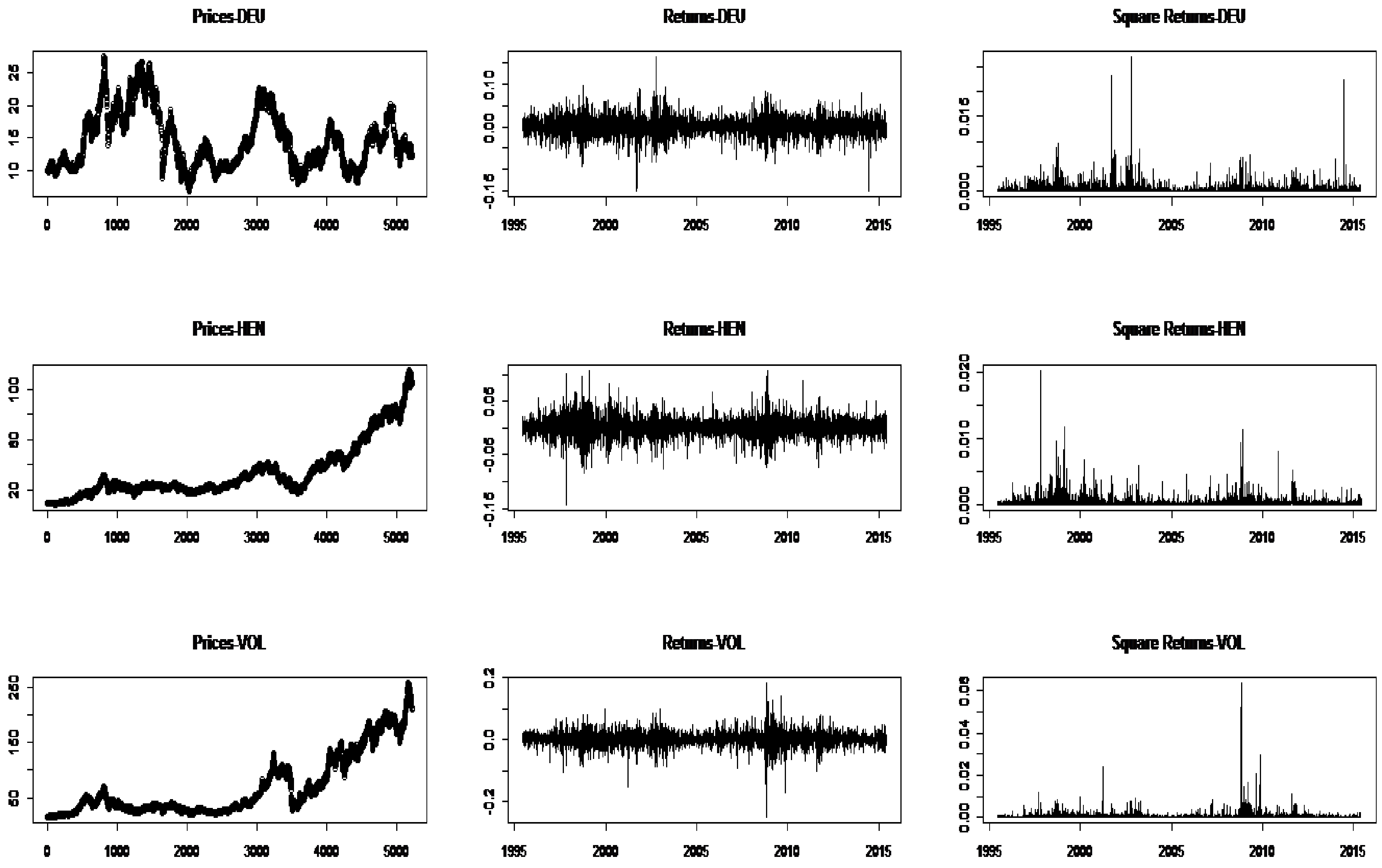

Prices-DEU
Returns-DEU
Square Returns-DEU
Prices-HEN
Returns-HEN
Square Returns-HEN
Prices-VOL
Returns-VOL
Square Returns-VOL

Table 37: Summary Statistics.

Tests	BMW	SEI	VOL	THY	LIN	SAP	BAY	BASF	CON	FRES	MUEN	BEIR	RWE	DEU	HEN
Mean	0.0004	0.0003	0.0005	0.0001	0.0003	0.0004	0.0004	0.0004	0.0006	0.0006	0.0002	0.0004	0.0003	0.0001	0.0005
Median	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Min	-0.159	-0.163	-0.252	-0.165	-0.105	-0.255	-0.184	-0.129	-0.325	-0.124	-0.171	-0.134	-0.105	-0.152	-0.142
Max	0.1352	0.1660	0.1834	0.1679	0.1285	0.2351	0.3230	0.1269	0.2533	0.2147	0.1653	0.1610	0.1285	0.1639	0.1080
Std.dev	0.0218	0.0216	0.0240	0.0230	0.0177	0.024	0.0194	0.0179	0.0245	0.0202	0.0203	0.0186	0.0171	0.0210	0.0171
Skewness	-0.011	-0.069	-0.567	-0.063	0.0513	0.037	0.5412	-0.055	-0.223	0.1249	-0.049	0.1046	0.0408	-0.122	0.0837
Kurtosis	4.3723	5.6496	8.9493	4.4606	4.0742	10.584	19.095	4.5699	14.986	5.7657	7.3939	5.8818	4.3330	3.6096	4.1021
Jarque-Bera Test*	3741	6250	15926	3897	3250	24354	79521	4542	48866	7239	11885	7529	4082	2845	3663
p-values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

ADF-Unit Root*	-17.90	-15.80	-14.67	-15.80	-17.20	-17.34	-18.52	-18.21	-15.67	-17.30	-18.12	-17.96	-17.94	-17.04	-19.40
p-values	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Phillips-Perron Unit Root Test*	-65.4	-67.09	-65.61	-66.24	-72.45	-69.28	-73.83	-72.34	-70.04	-73.48	-67.81	-81.49	-76.42	-70.21	-73.26
p-values	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
KPSS Test	0.0712	0.0657	0.0758	0.0576	0.0850	0.1121	0.1013	0.0543	0.0882	0.1168	0.1542	0.1302	0.0810	0.0655	0.0898
p-values	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Auto-Corr-r**															
Lag 1	0.043	0.045	0.021	0.033	-0.050	0.040	-0.021	-0.001	0.030	-0.018	0.061	-0.103	-0.051	0.028	-0.012
Lag 5	-0.019	-0.023	-0.016	-0.002	-0.025	-0.004	-0.020	-0.025	-0.029	0.002	-0.062	-0.020	-0.023	-0.026	-0.036
Lag 10	-0.010	-0.009	0.036	0.014	0.010	-0.041	-0.005	0.005	0.023	-0.004	-0.003	-0.006	0.010	0.005	0.026
Lag 20	0.037	-0.015	0.037	0.003	0.012	0.001	0.009	0.014	-0.002	-0.027	-0.009	-0.010	0.012	0.012	-0.010
Ljung-Box (20)*	79.39	71.01	74.65	36.62	63.37	53.099	67.222	77.929	90.125	67.779	108.68	108.52	67.644	37.108	54.256

p-values	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Auto-Corr- r2**															
Lag 1	0.220	0.139	0.373	0.135	0.181	0.135	0.074	0.209	0.152	0.140	0.232	0.188	0.186	0.094	0.242
Lag 5	0.123	0.156	0.184	0.160	0.183	0.133	0.034	0.160	0.049	0.084	0.191	0.138	0.187	0.120	0.127
Lag 10	0.097	0.233	0.135	0.142	0.148	0.092	0.026	0.186	0.089	0.092	0.153	0.057	0.153	0.087	0.104
Lag 20	0.097	0.084	0.081	0.136	0.087	0.069	0.019	0.137	0.112	0.049	0.127	0.073	0.093	0.086	0.094
Ljung-Box (20)*	2115.8	2733.4	5537.5	1773	2289.4	1177.8	298.28	4236	1113	1061	4272	1319.4	2695	1443.7	2042.6
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa: DEU ,Henkel Pref: HEN, Volkswagen: VOL. * Indicates significant at the 5% level ,**The Lag orders are selected to examine a range of possible autocorrelations.*

Table 38: Unconditional Correlation Measures Matrix (Linear Correlation).

	Linear Correlation														
	SAP	**SIE**	**BAY**	**BASF**	**BMW**	**CON**	**FRES**	**MUEN**	**BEIR**	**LIN**	**THY**	**RWE**	**DEU**	**HEN**	**VOL**
SAP	1														
SIE	0.51	1													
BAY	0.35	0.49	1												
BASF	0.38	0.57	0.67	1											
BMW	0.34	0.51	0.47	0.56	1										
CON	0.28	0.42	0.36	0.44	0.47	1									
FRES	0.19	0.23	0.24	0.24	0.22	0.18	1								
MUEN	0.36	0.49	0.47	0.51	0.46	0.35	0.22	1							
BEIR	0.20	0.25	0.26	0.28	0.23	0.20	0.19	0.23	1						
LIN	0.32	0.45	0.48	0.54	0.45	0.36	0.21	0.41	0.23	1					
THY	0.33	0.50	0.46	0.56	0.51	0.42	0.20	0.42	0.23	0.48	1				
RWE	0.28	0.42	0.44	0.47	0.39	0.30	0.16	0.45	0.21	0.39	0.40	1			
DEU	0.36	0.47	0.42	0.50	0.48	0.40	0.19	0.44	0.22	0.41	0.45	0.36	1		
HEN	0.25	0.36	0.41	0.44	0.41	0.34	0.19	0.35	0.27	0.41	0.36	0.34	0.36	1	
VOL	0.32	0.42	0.39	0.44	0.53	0.41	0.19	0.37	0.20	0.37	0.41	0.31	0.41	0.29	1

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL.*

Table 40: Unconditional Correlation Measures Matrix (Spearman's Correlation).

	Spearman's Correlation														
	SAP	**SIE**	**BAY**	**BASF**	**BMW**	**CON**	**FRES**	**MUEN**	**BEIR**	**LIN**	**THY**	**RWE**	**DEU**	**HEN**	**VOL**
SAP	1														
SIE	0.53	1													
BAY	0.40	0.49	1												
BASF	0.42	0.55	0.64	1											
BMW	0.38	0.50	0.45	0.50	1										
CON	0.33	0.42	0.37	0.42	0.47	1									
FRES	0.20	0.22	0.22	0.23	0.21	0.18	1								
MUEN	0.38	0.48	0.45	0.48	0.42	0.36	0.20	1							
BEIR	0.24	0.27	0.29	0.30	0.26	0.24	0.21	0.25	1						
LIN	0.35	0.43	0.46	0.49	0.41	0.36	0.20	0.38	0.24	1					
THY	0.36	0.49	0.44	0.51	0.45	0.40	0.18	0.40	0.25	0.44	1				
RWE	0.32	0.41	0.42	0.44	0.36	0.30	0.17	0.42	0.21	0.35	0.36	1			
DEU	0.37	0.46	0.40	0.46	0.44	0.39	0.16	0.42	0.23	0.38	0.42	0.36	1		
HEN	0.28	0.36	0.40	0.41	0.36	0.33	0.19	0.34	0.31	0.37	0.33	0.31	0.34	1	
VOL	0.38	0.45	0.41	0.45	0.54	0.44	0.18	0.38	0.23	0.38	0.42	0.32	0.41	0.32	1

Note:SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:TH Y,RWE:RWE,Deutsche Lufthansa :DEU, Henkel Pref: HEN, Volkswagen: VOL.

Table 40: Unconditional Correlation Measures Matrix (Kendall’s Tau).

	Kendall’s Tau														
	SAP	**SIE**	**BAY**	**BASF**	**BMW**	**CON**	**FRES**	**MUEN**	**BEIR**	**LIN**	**THY**	**RWE**	**DEU**	**HEN**	**VOL**
SAP	1														
SIE	0.39	1													
BAY	0.28	0.36	1												
BASF	0.30	0.40	0.47	1											
BMW	0.27	0.36	0.32	0.36	1										
CON	0.23	0.30	0.26	0.30	0.34	1									
FRES	0.14	0.15	0.15	0.16	0.14	0.12	1								
MUEN	0.27	0.35	0.32	0.34	0.30	0.25	0.14	1							
BEIR	0.17	0.19	0.20	0.21	0.18	0.16	0.14	0.18	1						
LIN	0.25	0.31	0.33	0.35	0.29	0.25	0.14	0.27	0.17	1					
THY	0.25	0.35	0.31	0.36	0.32	0.28	0.12	0.28	0.17	0.31	1				
RWE	0.22	0.29	0.30	0.31	0.25	0.21	0.11	0.30	0.14	0.25	0.25	1			
DEU	0.26	0.33	0.28	0.32	0.31	0.27	0.11	0.30	0.16	0.26	0.29	0.25	1		
HEN	0.20	0.25	0.28	0.28	0.25	0.23	0.13	0.24	0.21	0.26	0.23	0.22	0.24	1	
VOL	0.27	0.32	0.29	0.32	0.39	0.31	0.12	0.27	0.16	0.27	0.30	0.23	0.29	0.22	1

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:TH Y,RWE:RWE,Deutsche Lufthansa: DEU, Henkel Pref: HEN ,Volkswagen: VOL.*

Table 41: Conditional Correlation Measures Matrix (Linear Correlation).

	Linear Correlation														
	SAP	**SIE**	**BAY**	**BASF**	**BMW**	**CON**	**FRES**	**MUEN**	**BEIR**	**LIN**	**THY**	**RWE**	**DEU**	**HEN**	**VOL**
SAP	1														
SIE	0.48	1													
BAY	0.37	0.47	1												
BASF	0.40	0.54	0.65	1											
BMW	0.36	0.50	0.46	0.52	1										
CON	0.32	0.42	0.37	0.43	0.47	1									
FRES	0.19	0.22	0.23	0.23	0.21	0.18	1								
MUEN	0.34	0.46	0.44	0.48	0.43	0.36	0.21	1							
BEIR	0.36	0.47	0.45	0.48	0.43	0.36	0.21	0.93	1						
LIN	0.34	0.44	0.48	0.52	0.44	0.37	0.22	0.38	0.39	1					
THY	0.34	0.47	0.43	0.50	0.45	0.40	0.19	0.39	0.40	0.44	1				
RWE	0.29	0.39	0.40	0.42	0.36	0.29	0.16	0.42	0.43	0.36	0.36	1			
DEU	0.35	0.43	0.40	0.46	0.45	0.38	0.17	0.41	0.41	0.38	0.41	0.33	1		
HEN	0.28	0.35	0.40	0.41	0.37	0.33	0.21	0.35	0.35	0.39	0.33	0.32	0.34	1	
VOL	0.35	0.42	0.40	0.44	0.54	0.45	0.18	0.36	0.37	0.39	0.40	0.31	0.40	0.32	1

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa: DEU, Henkel Pref: HEN, Volkswagen: VOL.*

Table 42: Conditional Correlation Measures Matrix (Spearman's Correlation).

	Spearman's Correlation														
	SAP	**SIE**	**BAY**	**BASF**	**BMW**	**CON**	**FRES**	**MUEN**	**BEIR**	**LIN**	**THY**	**RWE**	**DEU**	**HEN**	**VOL**
SAP	1														
SIE	0.53	1													
BAY	0.41	0.50	1												
BASF	0.44	0.56	0.64	1											
BMW	0.40	0.51	0.46	0.50	1										
CON	0.35	0.43	0.37	0.43	0.48	1									
FRES	0.21	0.22	0.23	0.23	0.21	0.18	1								
MUEN	0.39	0.49	0.46	0.49	0.43	0.37	0.21	1							
BEIR	0.39	0.50	0.46	0.49	0.43	0.37	0.21	0.98	1						
LIN	0.36	0.44	0.47	0.50	0.42	0.36	0.21	0.38	0.38	1					
THY	0.37	0.49	0.44	0.50	0.45	0.40	0.18	0.40	0.40	0.43	1				
RWE	0.33	0.42	0.42	0.44	0.36	0.30	0.17	0.43	0.43	0.35	0.36	1			
DEU	0.38	0.46	0.40	0.45	0.44	0.38	0.16	0.42	0.43	0.38	0.41	0.36	1		
HEN	0.31	0.36	0.40	0.41	0.36	0.33	0.20	0.35	0.35	0.38	0.33	0.31	0.35	1	
VOL	0.39	0.46	0.41	0.44	0.54	0.45	0.17	0.38	0.38	0.39	0.42	0.32	0.41	0.33	1

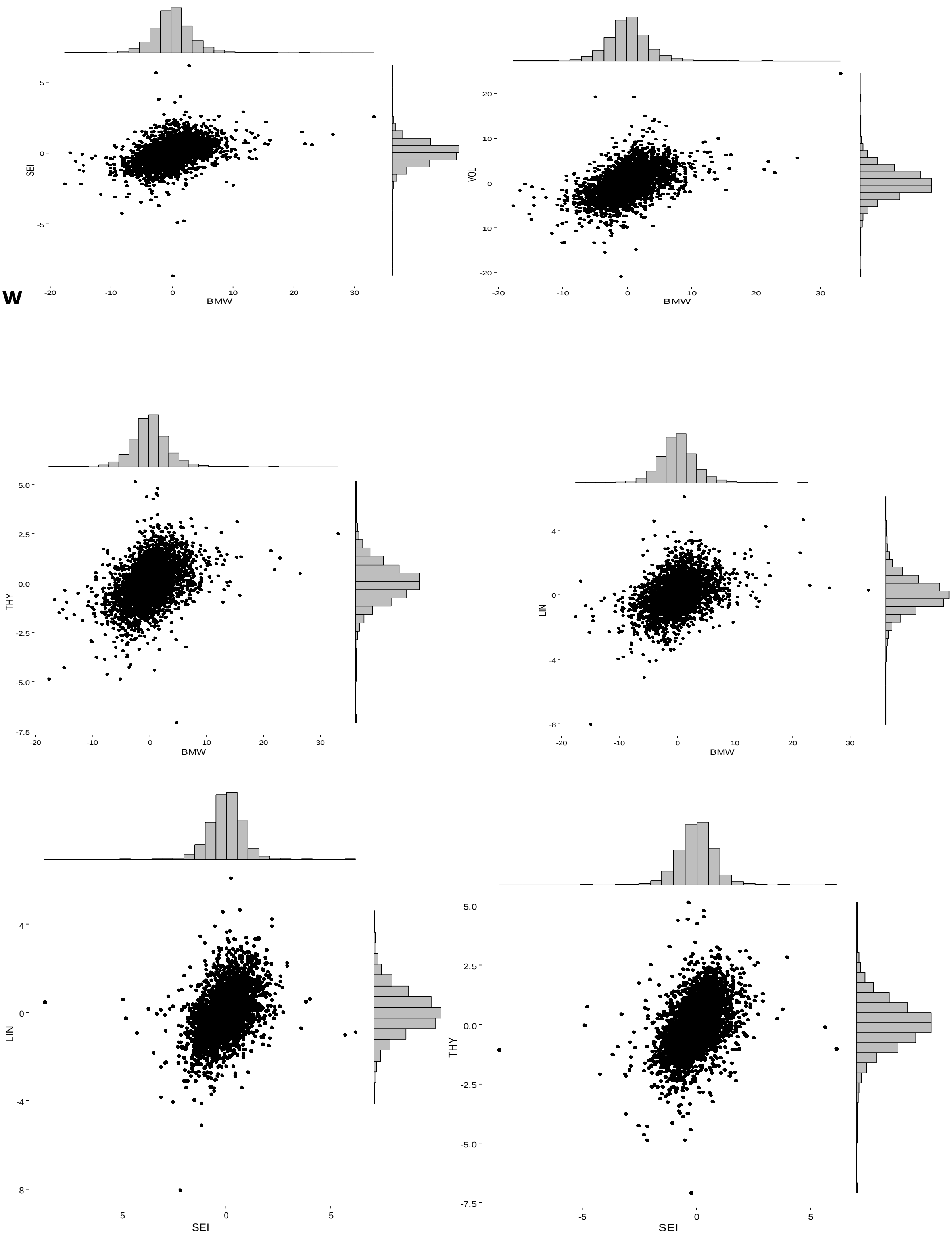
*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa: DEU, Henkel Pref: HEN, Volkswagen: VOL.*

Table 43: Conditional Correlation Measures Matrix (Kendall's Tau).

	Kendall's Tau														
	SAP	**SIE**	**BAY**	**BASF**	**BMW**	**CON**	**FRES**	**MUEN**	**BEIR**	**LIN**	**THY**	**RWE**	**DEU**	**HEN**	**VOL**
SAP	1														
SIE	0.38	1													
BAY	0.29	0.36	1												
BASF	0.31	0.40	0.46	1											
BMW	0.28	0.36	0.32	0.35	1										
CON	0.24	0.30	0.26	0.30	0.34	1									
FRES	0.14	0.15	0.16	0.16	0.14	0.12	1								
MUEN	0.27	0.35	0.32	0.34	0.30	0.25	0.14	1							
BEIR	0.27	0.31	0.32	0.34	0.30	0.25	0.14	0.89	1						
LIN	0.25	0.35	0.33	0.35	0.29	0.25	0.14	0.27	0.27	1					
THY	0.26	0.35	0.30	0.35	0.31	0.28	0.12	0.28	0.28	0.30	1				
RWE	0.23	0.29	0.29	0.31	0.25	0.21	0.11	0.30	0.30	0.24	0.25	1			
DEU	0.26	0.32	0.28	0.32	0.30	0.27	0.11	0.30	0.30	0.26	0.29	0.24	1		
HEN	0.21	0.25	0.28	0.28	0.25	0.23	0.13	0.24	0.24	0.26	0.22	0.21	0.24	1	
VOL	0.27	0.32	0.28	0.31	0.39	0.31	0.12	0.26	0.26	0.27	0.29	0.22	0.34	0.22	1

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:TH Y,RWE:RWE,Deutsche Lufthansa: DEU, Henkel Pref: HEN, Volkswagen: VOL.*

Figure 15: Scatter Plot of Bivariate Standardized Returns.



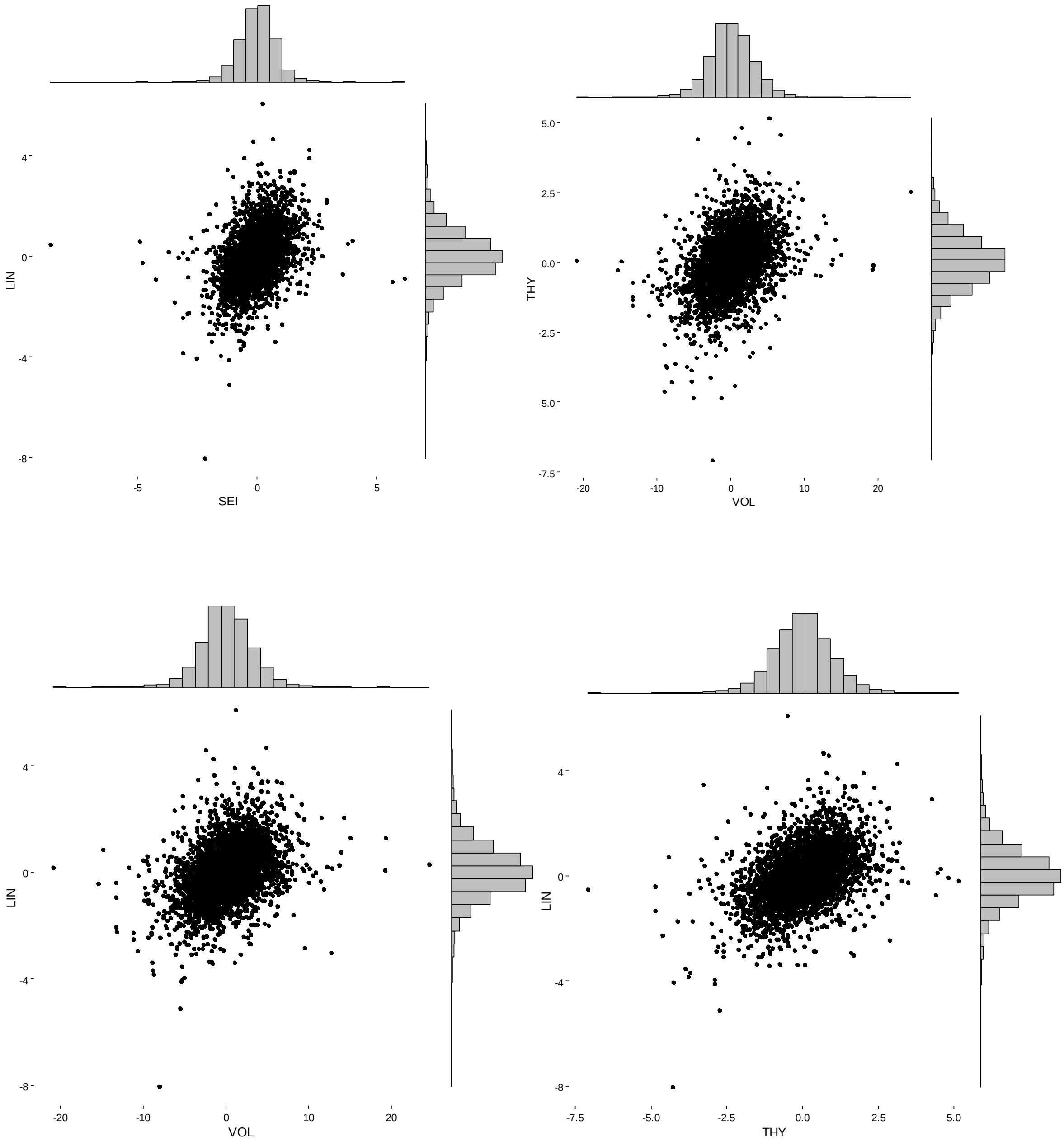

LIN
SEI
THY
VOL
LIN
VOL
LIN
THY

Table 44: Estimation Results of the Static C-vine Copula for 5-Dimensional Data.

	Bivariate Copula	θ_1	θ_2	τ	AIC	BIC
			Tree 1			
BMW.SEI	Frank	1.7739 (0.0803)		0.1912	-671.29	-664.959
BMW.VOL	Frank	3.8739 (0.1208)		0.3786	-1178.7	-1172.45
BMW.THY	Frank	2.0195 (0.0827)		0.2158	-820.93	-814.606
BMW.LIN	Frank	3.3648 (0.1179)		0.3382	-936.68	-930.354
			Tree2			
SEI.VOL/BMW	Student t	0.2592 (0.0074)	8.8941 (0.2718)	0.1669	-5898.8	-5886.14
SEI.THY/BMW	Normal	0.1076 (0.0066)		0.0686	-270.87	-264.546
SEI.LIN/BMW	Normal	0.1088 (0.0066)		0.0694	-277.34	-271.014
			Tree3			
VOL.THY/BMW.SEI	Frank	0.6503 (0.0728)		0.0719	-82.855	-76.5218
VOL.LIN/BMW.SEI	Student t	0.2606 (0.0155)	9.6682 (1.6486)	0.1678	-303.56	-290.900
			Tree4			
THY.LIN/BMW.SEI.VOL	Frank	0.4144 (0.0722)		0.0459	-33.122	-26.7887

*Note***:***Siemens:SIE,BMW:BMW,Linde:LIN,Thyssenkrup:THY,Volkswagen:VOL. Standard errors are presented in parenthesis. All models are ranked on the basis of smallest AIC. AIC is Akaike information criterion, and BIC is* Bayesian information criterion. *τ is Kendell's Tau measure of dependence.*

Table 45: Estimation Results of the Static D-vine Copula for 5- Dimensional Data.

	Bivariate Copula	θ_1	θ_2	τ	AIC	BIC
		Tree 1				
BMW.SEI	Normal	0.1032 (0.0066)		0.0658	-237.66	-231.33
SEI.VOL	Normal	0.0815 (0.0076)		0.0519	-111.04	-104.71
VOL.THY	Student t	0.2910 (0.0170)	9.4489 (1.6754)	0.1880	-269.22	-256.55
THY.LIN	Frank	0.6061 (0.0707)		0.0671	-71.749	-65.416
		Tree2				
BMW.VOL/SEI	Student t	0.4616 (0.0124)	7.5897 (0.9899)	0.3054	-1005.3	992.732
SEI.THY/VOL	Student t	0.2592 (0.0074)	8.8941 (0.2718)	0.1669	-5898.8	-5886.14
VOL.LIN/THY	Student t	0.4833 (0.0145)	6.1854 (0.7919)	0.3211	-697.60	-684.94
		Tree3				
BMW.THY/SEI.VOL	Frank	1.7739 (0.0702)		0.1912	- 671.292	-664.959
SEI.LIN/VOL.THY	Clayton	0.1820 (0.0065)		0.1165	- 747.301	-740.967
		Tree4				
BMW.LIN/SEI.VOL.THY	Frank	2.0195 (0.0728)		0.2158	-820.93	-814.606

Note:*Siemens:SIE,BMW:BMW,Linde:LIN,Thyssenkrup:THY,Volkswagen:VOL. Standard errors are presented in parenthesis. All models are ranked on the basis of smallest AIC. AIC is Akaike information criterion and BIC is* Bayesian information criterion. τ *is Kendell's Tau measure of dependence.*

Table 46: Estimation Results of the Static C-vine Copula for 7-Dimensional Data.

	Bivariate Copula	θ_1	θ_2	τ	AIC	BIC
			Tree 1			
SAP.SIE	Student t	0.0207 (0.0973)	9.9518	0.0132	-184.67	-171.55
SAP.BAY	Frank	0.5389 (0.0117)		0.0597	-349.86	-343.30
SAP.BASF	Gumbel	1.1214 (0.0015)		0.0225	-354.77	-348.21
SAP.BMW	Student t	0.1702 (0.0112)	5.3895	0.0775	-536.58	-523.46
SAP.CON	BB1	0.0804 (0.2875)	1.7729	0.0890	-535.30	-523.46
SAP.FRES	Student	0.3639 (0.0276)	9.0369	0.0234	-125.54	-118.98
			Tree 2			
SIE.BAY/SAP	Clayton	0.0354 (00027)		0.1089	-763.75	-750.63
SIE.BASF/SAP	Student	0.1393 (0.3745)	8.7661	0.1672	-1044.0	-1037.5
SIE.BMW/SAP	Frank	0.3510 (0.1081)		0.1186	-841.63	-835.07
SIE.CON/SAP	Student t	0.4163 (0.0488)	7.8961	0.0291	-199.11	-192.55
SIE.FRES/SAP	BB1	0.5405 (0.3221)	1.1430	0.1981	-987.06	-973.94
			Tree 3			
BAY.BASF/SAP.SIE	Gaussian	0.0367 (0.4991)		0.2283	-268.32	-255.20
BAY.BMW/SAP.SIE	Frank	1.0804 (0.0846)		0.1494	-1081.8	-1068.6

BAY.CON/SAP.SIE	Student t	0.2326 (0.4160)	6.3553	0.1747	-674.2	-661.14
BAY.FRES/SAP.SIE	Student t	0.4497 (0.0042)	6.2048	0.0277	-91.127	-84.567
			Tree 4			
BASF.BMW/SAP.SIE.BAY	Frank	0.2620 (0.3485)		0.2371	341.08	354.20
BASF.CON/SAP.SIE.BAY	Student t	0.2710 (0.0047)	8.2445	0.2733	423.53	436.65
BASF.FRES/SAP.SIE.BAY	Student t	0.4437 (0.2087)		0.2969	605.66	618.78
			Tree 5			
BMW.CON/ SAP.SIE.BAY.BASF	Gumbel	1.0285 (0.0799)		0.2927	-670.85	-657.73
BMW.FRES/ SAP.SIE.BAY.BASF	Student	0.4049 (0.3579)	4.2640	0.2654	-407.01	-393.89
			Tree 6			
CON.FRES/ SAP.SIE.BAY.BASF.BMW	Frank	0.8698 (0.0208)		0.8906	6799.0	6196.1

*Note***:***SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES. Standard errors are presented in parenthesis. All models are ranked on the basis of smallest AIC. AIC is Akaike information criterion and BIC is* Bayesian information criterion. τ *is Kendell's Tau measure of dependence*

Table 47: Estimation Results of the Static D-vine Copula for 7-Dimensional Data.

	Bivariate Copula	θ_1	θ_2	τ	AIC	BIC
		Tree 1				
SAP.SIE	Student t	0.5571 (0.009)	6.3451 (0.5965)	0.3762	-1854.6	-1841.5
SIE.BAY	Student t	0.5404 (0.0103)	4.7926 (0.3763)	0.3634	-1806.0	-1792.9
BAY.BASF	Student t	0.4409 (0.0117)	7.9497 (0.8854)	0.2907	-1052.2	-1039.1
BASF.BMW	Student t	0.46571 (0.0112)	7.7538 (0.8543)	0.3084	-1198.9	-1185.8
BMW.CON	BB1	0.4690 (0.0317)	1.2461 (0.0195)	0.3499	-1813.9	-1800.8
CON.FRES	BB1	0.5194 (0.0332)	1.1814 (0.0183)	0.3280	-1596.8	-1583.7
		Tree2				
SAP.BAY/SIE	BB8	2.1306 (0.5190)	0.6214 (0.1462)	0.1372	-220.73	-207.62
SIE.BASF/BAY	Frank	2.9614 (0.0934)		0.3038	-1000.6	-994.04
BAY.BMW/BASF	Student t	0.5863 (0.0092)	5.6288 (0.1437)	0.3989	-2313.6	-2300.5
BASF.CON/BMW	BB8	3.1031 (0.9745)	0.4885 (0.1437)	0.1687	-305.71	-292.59
BMW.FRES/CON	BB8	3.1085 (0.1077)	0.4902 (0.1634)	0.1699	-341.58	-328.46
		Tree3				
SAP.BASF/SIE.BAY	student t	0.9837 (0.0004)	6.1828 (0.4748)	0.8852	-1802.4	-18011.5
SIE.BMW/BAY.BASF	student t	0.2943 (0.0130)	10.5573 (0.4780)	0.1901	-501.31	-488.19

BAY.CON/BASF.BMW	BB8	1.6939 (0.3019)	0.6939 (0.1328)	0.1059	-139.20	-126.08
BASF.FRES/BMW.CON	Frank	0.7548 (0.0877)		0.0833	-72.019	-65.459
			Tree4			
SAP.BMW/SIE.BAY.BASF	Frank	0.3944 (0.0833)		0.0437	-20.439	-13.879
SIE.CON/BAY.BASF.BMW	Student t	0.2252 (0.0136)	11.9838 (0.0136)	0.1446	-300.22	-287.10
BAY.FRES/BASF.BMW.CON	Frank	0.3729 (0.0839)		0.0413	-17.738	-11.178
			Tree 5			
SAP.CON/SIE.BAY.BASF.BMW	Clayton	0.0496 (0.0152)		0.0242	-9.8432	-3.2835
SIE.FRES/BAY.BASF.BMW.CON	Student t	0.1158 (0.0138)	2.6448 (0.6380)	0.0739	-84.689	-71.570
			Tree 6			
SAP.FRES/SIE.BAY.BASF.BMW. CON	Gaussian	0.0202 (0.0133)		0.0129	-2.974	-6.2622

Note:SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES. Standard errors are presented in parenthesis. All models are ranked on the basis of smallest AIC. AIC is Akaike information criterion and BIC is Bayesian information criterion. τ *is Kendell's Tau measure of dependence.*

Table 48: C-Vine Copula Structure for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	MEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	14														
SIE	15	13													
BAY	1	15	12												
BASF	2	1	15	11											
BMW	3	2	1	15	10										
CON	4	3	2	1	15	9									
FRES	5	4	3	2	1	15	8								
MUEN	6	5	4	3	2	1	15	7							
MEIR	7	6	5	4	3	2	1	15	6						
LIN	8	7	6	5	4	3	2	1	15	5					
THY	9	8	7	6	5	4	3	2	1	15	4				
RWE	10	9	8	7	6	5	4	3	2	1	15	3			
DEU	11	10	9	8	7	6	5	4	3	2	1	15	2		
HEN	12	11	10	9	8	7	6	5	4	3	2	1	15	1	
VOL	13	12	11	10	9	8	7	6	5	4	3	2	1	15	1

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL.*

Table 49: C-Vine Copula Parameter Estimation and Standard Error for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEW	HEN	VOL
SAP	0														
SIE	0.0066	0													
	(0.0089)														
BAY	1.0016	1.0082	0												
	(0.0165)	(0.0147)													
BASF	0.0061	0.01712	0.0325	0											
	(0.0112)	(0.0252)	(0.0251)												
BMW	0.3586	0.1361	0.01896	0.2246	0										
	(0.0196)	(0.0848)	(0.4299)	(0.0116)											
CON	1.0094	0.01657	1.0217	0.0905	0.0091	0									
	(0.0904)	(0.0136)	(0.0114)	(0.0131)	(0.0140)										
FRES	0.6331	0.2034	0.0046	0.0130	0.3954	0.01415	0								
	(0.0208)	(0.0963)	(0.0895)	(0.0131)	(0.0168)	(0.0840)									
MUEN	1.0201	0.0062	1.0075	0.1018	0.0629	0.0259	0.6885	0							
	(0.0145)	(0.0086)	(0.0118)	(0.0111)	(0.0124)	(0.0134)	(0.0008)								
BEIR	0.0508	1.0545	0.0893	0.0851	0.0123	0.0919	0.1527	0.2219	0						
	(0.0203)	(0.0842)	(0.0879)	(0.0094)	(0.0130)	(0.0135)	(0.0872)	(0.0145)							
LIN	0.1056	0.7208	0.10397	0.1605	0.0067	0.4778	0.0937	0.0944	0.0291	0					
	(0.0198)	(0.0138)	(0.0854)	(0.0847)	(0.0138)	(0.0833)	(0.0910)	(0.0131)	(0.082)						

THY	0.1244	0.0348	1.0489	0.2902	0.0997	0.0136	0.1799	4.0183	0.0871	1.0069	0				
	(0.0856)	(0.0143)	(0.0152)	(0.0875)	(0.0135)	(0.0130)	(0.0140)	(0.0141)	(0.0142)	(0.0142)					
RWE	0.2408	0.9973	1.3180	1.5432	0.8821	1.2250	1.2453	0.4632	0.6704	0.5088	0.2469	0			
	(0.0185)	(0.0136)	(0.0139)	(0.0071)	(0.0135)	(0.0135)	(0.0145)	(0.0139)	(0.0129)	(0.0136)	(0.0162)				
DEU	0.4471	0.4618	0.1074	0.2716	0.0859	1.1535	0.1275	0.3119	0.1684	0.1011	0.1099	1.0788	0		
	(0.0144)	(0.0861)	(0.0139)	(0.0883)	(0.0135)	(0.0911)	(0.0875)	(0.0852)	(0.0849)	(0.0867)	(0.0135)	(0.0137)			
HEN	0.2041	0.1985	0.2161	0.1718	0.1394	1.0657	1.4553	1.6197	0.9918	1.2792	0.1511	1.3400	1.5159	0	
	(0.0862)	(0.0898)	(0.0878)	(0.0853)	(0.0839)	(0.0837)	(0.0899)	(0.0848)	(0.0908)	(0.0865)	(0.0847)	(0.0875)	(0.0142)		
VOL	2.7334	1.3172	1.5643	1.5723	1.0601	0.5263	0.4769	0.5974	0.5301	0.5282	0.4964	0.4708	0.4308	0.5310	0
	(0.0138)	(0.0142)	(0.0140)	(0.0136)	(0.0141)	(0.0140)	(0.0143)	(0.0135)	(0.0141)	(0.0139)	(0.0135)	(0.0142)	(0.0135)	(0.0005)	

Note:SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:TH Y,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL

Table 50: C-vine Copula Second Parameter Estimation and Standard Error foe 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	0.000	0													
BAY	0.000	0.000	0												
BASF	9.9899	0.0000	0.0000	0											
	(0.0021)														
BMW	0.0000	0.0000	0.0000	1.8790	0										
				(0.0740)											
CON	0.0000	0.0000	0.0000	0.0000	2.4326	0									
					(0.0010)										
FRES	0.0000	0.0000	0.0000	2.9803	0.0000	0.0000	0								
				(0.2140)											
MUEN	0.0000	0.0000	0.0000	1.6933	5.8733	0.3508	0.0000	0							
				(0.0014)	(0.0782)	(0.0001)									
BEIR	0.0000	0.0000	0.0000	1.9991	3.5790	0.0000	8.5059	0.0000	0						

				(0.0000)			(0.0127)								
LIN	0.0000	3.5183	0.0000	0.0000	2.6601	0.0000	0.0000	1.5684	0.0000	0					
		(0.0063)			(0.0049)			(0.0041)							
THY	0.0000	3.6760	0.0000	0.0000	5.0058	0.0000	6.2863	5.3690	3.4733	0.0000	0				
		(0.0253)			(0.0004)		(0.0007)	(0.0210)	(0.0032)						
RWE	0.0000	1.6923	0.0000	0.0000	4.8128	7.669	4.6182	2.699	5.9754	7.0984	11.7225	0			
		(0.0031)			(0.0154)	(0.0004)	(0.0823)	(0.0023)	(0.0816)	(0.004)	(0.5061)				
DEU	0.0000	7.1966	0.0000	2.8043	0.0000	10.7440	0.0000	0.0000	0.0000	0.0000	0.0000	9.7657	0		
		(0.3140)		(0.2140)		(0.0579)						(0.1240)			
HEN	10.0885	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0	
	(0.5612)														
VOL	8.5986	5.4200	5.2647	10.2816	3.8579	15.5995	15.0892	6.6730	4.6895	5.5414	11.539	6.7186	7.0284	6.459	0
	(0.0418)	(0.0112)	(0.4617)	(0.1840)	(0.0026)	(0.9463)	(0.5473)	(0.0247)	(0.0631)	(0.0029)	(0.4321)	(0.0056)	(0.0293)	(0.0001)	

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa :DEU, Henkel Pref: HEN, Volkswagen: VOL.*

Table 51: C-vine Copula Kendall's Tau for 15- Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	0.00422	0													
BAY	0.0016	0.0082	0												
BASF	0.0258	0.0030	0.0084	0											
BMW	0.0160	0.0397	0.0151	0.0093	0										
CON	0.0249	0.0093	0.0105	0.0212	0.0577	0									
FRES	0.0045	0.0700	0.0226	0.0029	0.0064	0.0438	0								
MUEN	0.0070	0.0197	0.0040	0.0075	0.0649	0.0401	0.0127	0							
BEIR	0.0761	0.0056	0.0516	0.0569	0.0542	0.0061	0.0586	0.0169	0						
LIN	0.0246	0.0673	0.0796	0.0115	0.1026	0.0033	0.0529	0.0529	0.0602	0					
THY	0.0143	0.0794	0.0221	0.0466	0.1874	0.0636	0.0067	0.1151	0.8448	0.0555	0				
RWE	0.1107	0.1548	0.9538	0.1439	0.1675	0.0972	0.1341	0.1362	0.3066	0.4678	0.3398	0			
DEU	0.1588	0.2951	0.3056	0.0685	0.0301	0.1265	0.0814	0.0346	0.1077	0.0645	0.0701	0.1184	0		
HEN	0.1308	0.1272	0.1387	0.1099	0.0890	0.1170	0.1584	0.1754	0.1091	0.1398	0.0965	0.1462	0.1647	0	
VOL	0.2835	0.1438	0.1697	0.1705	0.3528	0.3165	0.4076	0.3557	0.3542	0.3307	0.1164	0.3120	0.2835	0.0061	0

Note:SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa :DEU, Henkel Pref: HEN, Volkswagen: VOL.

Table 52: C-Vine Copula Family Selection for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	1	0													
BAY	4	4	0												
BASF	2	3	3	0											
BMW	3	5	5	2	0										
CON	5	4	1	4	2	0									
FRES	3	5	5	2	3	5	0								
MUEN	3	4	1	4	2	2	2	0							
BEIR	3	5	5	4	2	2	5	2	0						
LIN	3	2	5	5	2	5	5	2	5	0					
THY	5	2	3	5	2	1	2	2	2	5	0				
RWE	3	2	1	4	2	2	2	2	2	2	2	0			
DEU	3	2	5	2	5	2	5	5	5	5	5	2	0		
HEN	2	5	5	5	5	5	5	5	5	5	5	5	5	0	
VOL	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0

Note: *SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL*

Table 53: D-Vine Copula Structure for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	1														
SIE	15	2													
BAY	2	15	3												
BASF	3	3	15	4											
BMW	4	4	4	15	5										
CON	5	5	5	5	15	6									
FRES	6	6	6	6	6	15	7								
MUEN	7	7	7	7	7	7	15	8							
BEIR	8	8	8	8	8	8	8	15	9						
LIN	9	9	9	9	9	9	9	9	15	10					
THY	10	10	10	10	10	10	10	10	10	15	11				
RWE	11	11	11	11	11	11	11	11	11	11	15	12			
DEU	12	12	12	12	12	12	12	12	12	12	12	15	13		
HEN	13	13	13	13	13	13	13	13	13	13	13	13	15	14	
VOL	14	14	14	14	14	14	14	14	14	14	14	14	14	15	1

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL*

Table 54: D-Vine Copula Parameter Estimation and Standard Errors for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	0.5571	0													
	(0.0097)														
BAY	0.5404	0.4409	0												
	(0.0103)	(0.0117)													
BASF	0.4656	0.4690	0.5194	0											
	(0.0112)	(0.0317)	(0.0332)												
BMW	0.2602	0.2227	4.8545	0.3946	0										
	(0.0196)	(0.0138)	(0.4299)	(0.0116)											
CON	0.4610	0.3840	0.3707	0.3661	2.1290	0									
	(0.0113)	(0.0121)	(0.0123)	(0.0123)	(0.5179)										
FRES	2.9614	0.5863	3.0991	3.1084	0.1385	0.3462	0								
	(0.0934)	(0.0092)	(0.9706)	(1.1075)	(0.0141)	(0.0123)									
MUEN	1.0092	0.0488	1.8550	1.5168	0.3454	0.2344	0.9837	0							
	(0.0063)	(0.0145)	(0.0860)	(0.0863)	(0.0125)	(0.0134)	(0.0003)								
BEIR	0.2943	1.6945	0.7553	1.8452	0.2996	0.0358	0.1521	0.0126	0						
	(0.0130)	(0.3023)	(0.0876)	(0.6533)	(0.0128)	(0.0131)	(0.0142)	(0.0143)							
LIN	0.2824	0.2033	0.1704	0.3943	0.2252	0.3728	0.6283	1.9980	0.2033	0					

	(0.0127)	(0.0137)	(0.0138)	(0.0833)	(0.0136)	(0.0839)	(0.0904)	(0.0869)	(0.082)						
THY	0.2478	0.4453	0.3234	1.5258	1.4129	0.0495	0.1158	0.5530	1.0618	0.0569	0				
	(0.0137)	(0.0854)	(0.0858)	(0.0860)	(0.0868)	(0.0152)	(0.0138)	(0.0863)	(0.0886)	(0.0132)					
RWE	0.2310	0.2286	0.0396	0.0373	1.4378	-0.0203	1.0787	0.1449	0.0325	0.2870	0.1923	0			
	(0.0134)	(0.0135)	(0.0148)	(0.0137)	(0.1708)	(0.0133)	(0.0551)	(0.0140)	(0.0126)	(0.0129)	(0.0134)				
DEU	0.4393	0.0351	0.0151	-0.0316	1.1571	0.0400	0.7404	0.2152	0.0930	0.1572	0.0927	0.0141	0		
	(0.0851)	(0.0148)	(0.0146)	(0.0843)	(0.0865)	(0.0144)	(0.0909)	(0.0137)	(0.0141)	(0.0136)	(0.0138)	(0.0143)			
HEN	0.0394	0.1533	0.5842	1.7922	0.1398	0.10834	-0.1379	0.0818	1.1072	0.0651	0.1128	0.0711	-0.0880	0	
	(0.0131)	(0.0141)	(0.0898)	(0.4131)	(0.0139)	(0.0140)	(0.0812)	(0.0144)	(0.0468)	(0.0149)	(0.0141)	(0.0144)	(0.0849)		
VOL	2.0222	0.7003	0.6014	0.0974	0.0126	0.5019	1.0086	0.2154	0.0270	0.0862	1.3109	0.1235	0.1317	1.0040	0
	(0.6504)	(0.0868)	(0.0894)	(0.0143)	(0.0141)	(0.0875)	(0.0058)	(0.0892)	(0.01393)	(0.0141)	(0.1674)	(0.0836)	(0.0871)	(0.0039)	

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:TH Y,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL.*

Table 55: D-Vine Copula Second Parameter Estimation and Standard Errors for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	6.3426	0													
	(0.5965)														
BAY	4.7927	7.9518	0												
	(0.3763)	(0.8854)													
BASF	7.7549	1.2461	1.1814	0											
	(0.8543)	(0.0195)	(0.0183)												
BMW	0.000	11.5553	0.000	12.1587	0										
		(0.1483)		(0.8773)											
CON	6.9099	8.9819	9.1480	10.493	0.6218	0									
	(0.7610)	(0.2420)	(0.2658)	(0 .7114)	(0.1462)										
FRES	0.0000	5.6279	0.4892	0.4901	20.699	8.8965	0								
		(0.5316)	(0.1437)	(0.1634)	(0.7231)	(0 .256)									
MUEN	0.0000	27.0312	0.0000	0.0000	9.9359	19.896	6.1832	0							
		(0.3825)			(0.4565)	(0 .591)	(0.4748)								
BEIR	10.556	0.6937	0.0000	0.5609	11.336	0.0000	29.768	29.230	0						
	(0.4780)	(0.1328)		(0.0043)	(0.2066)		(0.2440)	(0.2191)							

LIN	15.935	15.130	21.566	0.0000	0.0000	0.0000	0.0000	10.001	0.0000	0					
	(0.0927)	(0.1571)	(0.2333)					(0.0094)							
THY	0.0000	0.0000	0.0000	0.0000	21.648	0.0000	0.0000	0.0000	20.379		0				
					(0.5000)				(0.638)						
RWE	12.2814	22.487	0.0000	0.7517	0.0000	0.9042	16.6112	0.0000	20.612	22.516	0.0000	0			
	(0.7613)	(0.1721)		(0.7762)		(0.108	(0.1158)		(0.698)	(0.613)					
DEU	0.0000	21.2256	0.0000	0.0000	0.0000	0.0000	13.3576	23.0898	20.824	0.0000	17.096	0.0000	0		
		(0.0698)					(0.4489)	(0.4670)	(0.900)		(0.622)				
HEN	19.997	0.0000	0.6508	17.8632	20.891	0.0000	23.552	0.9300	18.4992	18.1072	17.7037	0.0000	0.5779	0	
	(0.8384)		(0.0640)	(0.1650)	(0.3322)		(0.5758)	(0.9573)	(0.060)	(0.005)	(0.283)		(0.36)		
VOL	0.0000	0.0000	19.9614	28.970	0.0000	0.0000	0.0000	0.0000	22.872	0.7760	0.0000	0.0000	0.0000		0
			(0.2021)	(0.3888)					(0.101)	(0.771)					

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:TH Y,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL*

Table 56: D-Vine Copula Kendall’s Tau for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	0.3762	0													
BAY	0.3634	0.2907	0												
BASF	0.30839	0.3499	0.3280	0											
BMW	0.1151	0.1430	0.8906	0.2582	0										
CON	0.3050	0.2509	0.2417	0.2386	0.1372	0									
FRES	0.3038	0.3989	0.1688	0.1699	0.0884	0.2251	0								
MUEN	0.0091	0.0311	0.1994	0.1648	0.2245	0.1506	0.8852	0							
BEIR	0.1901	0.1059	0.0834	0.0877	0.1937	0.0228	0.0972	0.0080	0						
LIN	0.1823	0.1303	0.1090	0.0437	0.1446	0.0413	0.0695	0.2137	0.0225	0					
THY	0.1594	0.0493	0.0358	0.1657	0.1539	0.0241	0.0739	0.0612	0.1166	0.0362	0				
RWE	0.1484	0.1468	0.0252	0.0183	0.0806	0.0129	0.0252	0.0926	0.0160	0.1853	0.1232	0			
DEU	0.0487	0.0172	0.0096	0.0035	0.1268	0.0196	0.0818	0.1381	0.0593	0.1005	0.0591	0.0090	0		
HEN	0.0251	0.0980	0.0646	0.1064	0.0893	0.0691	0.0153	0.0521	0.0380	0.0415	0.0720	0.0453	0.0097	0	
VOL	0.1105	0.0774	0.0665	0.0621	0.0080	0.0556	0.0085	0.0239	0.0133	0.0549	0.0623	0.0137	0.0146	0.0040	0

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL.*

Table 57: D-Vine Copula Family Selection for 15-Dimensional Data.

	SAP	SIE	BAY	BASF	BMW	CON	FRES	MUEN	BEIR	LIN	THY	RWE	DEU	HEN	VOL
SAP	0														
SIE	2	0													
BAY	2	2	0												
BASF	2	7	7	0											
BMW	3	2	5	2	0										
CON	2	2	2	2	10	0									
FRES	5	2	10	10	2	2	0								
MUEN	4	2	5	5	2	2	2	0							
BEIR	2	10	5	10	2	1	2	2	0						
LIN	2	2	2	5	2	5	5	5	5	0					
THY	2	5	5	5	5	3	2	5	5	1	0				
RWE	2	2	2	3	10	1	10	2	3	2	2	0			
DEU	5	3	2,	5	5	3	5	2	2	2	1	2	0		
HEN	1	2	5	10	2	2	5	2	10	2	2	2	5	0	
VOL	10	5	5	2	2	5	4	5	3	2	10	5	5	4	0

*Note**:**SAP:SAP,Siemens:SIE,Bayer:BAY,BASF:BASF,BMW:BMW,Continental:CON,Fresenius:FRES,MuenchenerRuck:MUEN,Beiersdorf:BEIR,Linde:LIN,Thyssenkrup:THY,RWE:RWE,Deutsche Lufthansa:DEU,Henkel Pref:HEN,Volkswagen: VOL.*

Table 58: Back-testing Value at Risk (VaR) and Expected Shortfall (ES) for Pair Copulas for 5-Dimensional Data.

Model	VaR-Loss Function				ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
D-Vine Normal	0.2802	0.7044	1.4164	2.8498	0.27858	0.28018	0.2813	0.2828
D-Vine *t* Copula	0.2794	0.7039	1.4149	2.8488	**0.2762 (3)**	**0.2788 (5)**	**0.2805 (4)**	0.2822
D-Vine Clayton	0.2791	0.7024	1.4129	2.8528	**0.2766 (4)**	**0.2787 (4)**	**0.2802 (3)**	0.2821
D-Vine Gumbel	0.2801	0.7057	1.4179	2.8538	0.2785	0.2800	0.2815	0.2829
D-Vine Frank	0.2795	0.7032	1.4119	2.8438	0.2775	0.2793	0.2805	**0.2819 (4)**
D-Vine Joe	0.20448	0.7077	1.4204	2.8548	0.2805	0.2816	0.2825	0.2836
C-Vine Normal Copula	0.27908	0.7012	1.4119	2.8458	**0.2767 (5)**	**0.2785 (3)**	**0.2800 (2)**	**0.2818 (3)**
C-Vine *t* Copula	0.2795	0.7039	1.4129	2.8468	0.2779	0.2795	0.2807	0.2822
C-Vine Clayton	0.2782	0.7022	1.4119	2.8488	**0.2741 (1)**	**0.2774 (1)**	**0.2795 (1)**	**0.2817 (1)**
C-Vine Gumbel	0.27978	0.7022	1.4134	2.8448	**0.2752 (2)**	**0.2784 (2)**	**0.2800 (2)**	**0.2818 (2)**
C-Vine Frank	0.27918	0.7039	1.4139	2.8408	0.2773	0.2792	0.2807	0.2821
C-Vine Joe	0.2805	0.7074	1.4189	2.8538	0.2791	0.2807	0.2820	0.2833
DCC-norm	13.178	6.475	0.8561	0.5708	0.3736	0.3554	0.3335	0.3077
DCC-*t*	26.107	14.947	7.164	0.0010	0.7402	0.6049	0.5086	0.4147

Note: All models are ranked based on the minimum of MAE for ES on 1%,2.5%,5% and 10% significance level. The best models are highlighted by bold.

Table 59: Back testing value at Risk(VaR) and Expected Shortfall(ES) for Pair Copulas for 7-Dimensional Data.

Models	VaR-Loss Function				ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
D-Vine t Copula	2.1685	1.5311	1.1784	0.7293	0.0385	0.0235	0.0192	0.0149
D-Vine Clayton	2.2279	1.7261	1.1974	0.7023	0.0392	0.0313	0.0252	0.0192
D-Vine Gumbel	1.5844	1.2874	0.9694	0.6753	0.0267	0.0218	0.0183	0.0147
								(5)
D-Vine Frank	1.0003	0.8194	0.6464	0.4593	0.0311	0.0251	0.0211	0.0173
D-Vine Joe	2.3071	1.6286	1.1309	0.7203	0.0188	0.0163	0.0140	0.0115
					(1)	(1)	(1)	(1)
D-Vine BB1	1.6339	1.1801	1.0169	0.6843	0.0302	0.0256	0.0212	0.0168
D-vine BB6	2.1982	1.6384	1.1499	0.6483	0.0258	0.0209	0.0178	0.0146
					(5)	(5)	(4)	(5)
D-vine BB7	1.7329	1.4336	1.0264	0.7383	0.0373	0.0293	0.0240	0.0187
D-Vine BB8	2.0299	1.7066	1.3399	0.9723	0.0228	0.0206	0.0180	0.0152
						(4)	(5)	
C-Vine *t*	2.7130	1.9114	1.4729	0.8913	0.0309	0.0262	0.0217	0.0169
C-Vine Clayton	2.8813	2.1844	1.5869	0.9363	0.0383	0.0303	0.0249	0.0194
C-Vine Gumbel	2.0200	1.6286	1.3209	0.9093	0.0255	0.0212	0.0182	0.0152
					(4)			
C-Vine Frank	2.2180	1.7554	1.3209	0.9813	0.0272	0.0230	0.0194	0.0160
C-Vine Joe	1.6141	1.2679	1.0169	0.6933	0.0194	0.0165	0.0142	0.0116
					(2)	(2)	(2)	(2)
C-Vine BB1	2.4061	1.8431	1.4064	0.9093	0.0290	0.0241	0.0204	0.0163
C-Vine BB6	1.7527	1.4726	1.1404	0.7923	0.0243	0.0194	0.0165	0.0134
					(3)	(3)	(3)	(3)
C-Vine BB7	2.7823	1.9601	1.3684	0.9183	0.0350	0.0277	0.0223	0.0171
C-Vine BB8	2.3467	1.7359	1.3969	1.0173	0.0290	0.0237	0.0200	0.0166
DCC-norm	5.8513	4.8071	3.8859	2.8083	0.5893	0.5755	0.5637	0.5501
DCC.*t*	8.3362	6.3769	4.9974	3.6453	0.4368	0.4133	0.3972	0.3822

Note: All models are ranked based on the minimum of MAE for ES on 1%,2.5%,5% and 10% significance level. The best models are highlighted by bold.

Table 60: Back-testing Term Structure of Risk for 7-Dimensional Data.

Model	5- Day VaR-Loss Function				10 -Days VaR-Loss Function			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	4.7029	3.1789	2.1569	1.1883	9.7618	6.5816	4.6079	2.6013
Monte-Carlo(DCC)	9.2965	7.5371	6.0804	4.3923	17.6323	14.1086	11.5619	8.1993
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	0.0588	0.0489	0.0377	0.0274	0.1362	0.1026	0.0788	0.0575
Monte-Carlo(DCC)	0.1098	0.0949	0.0826	0.0691	0.2042	0.1771	0.1542	0.1292

Note: All models are ranked based on the minimum of MAE for ES on 1%,2.5%,5% and 10% significance level. The best models are highlighted by bold.

Table 61: Back- testing Value at Risk(VaR) and Expected Shortfall(ES) for Pair Copulas for 15-Dimensional.

Model	VaR				ES			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
D-Vine Student t	1.9747	1.3403	0.9925	0.5532	0.0245	0.0196	0.0158	0.0121
D-Vine Clayton	2.0341	1.5353	1.0115	0.5262	0.0293	0.0226	0.0178	0.0127
D-Vine Gumbel	1.3906	1.0966	0.7835	0.4992	0.0166	0.0143	0.0119	0.0093
					(4)	(4)	(4)	(3)
D-Vine Frank	1.5886	1.2136	0.9260	0.5982	0.0177	0.0155	0.0133	0.0107
					(5)	(5)		
D-Vine Joe	0.8065	0.6286	0.4605	0.2832	0.0101	0.0084	0.0070	0.0054
					(1)	(1)	(1)	(1)
D-Vine BB1	2.1133	1.4378	0.9450	0.5442	0.0260	0.0210	0.0165	0.0121
D-vine BB6	1.4401	0.9893	0.8310	0.5082	0.0163	0.0138	0.0116	0.0094
					(3)	(3)	(3)	(4)
D-vine BB7	2.0044	1.4476	0.9640	0.4722	0.0248	0.0203	0.0161	0.0119
D-Vine BB8	1.5391	1.2428	0.8405	0.5622	0.0182	0.0156	0.0131	0.0104
							(5)	
C-Vine *t*	1.7866	1.3306	0.9450	0.5532	0.226	0.0183	0.0150	0.0114
C-Vine BB1	1.9945	1.3891	0.9450	0.5622	0.0245	0.0199	0.0160	0.0121
C-Vine BB6	1.2817	0.9893	0.7360	0.4632	0.0152	0.0128	0.0107	0.0084
					(2)	(2)	(2)	(2)
C-Vine BB7	2.0539	1.5061	1.0495	0.6522	0.0256	0.0210	0.0170	0.0129
C-Vine BB8	1.4698	1.1161	0.8690	0.5892	0.0188	0.0152	0.0128	0.0103
DCC-norm	11.072	9.3986	7.933	5.4242	2.496	1.859	1.701	1.515
DCC.*t*	15.642	12.464	9.876	7.842	2.186	1.809	1.544	1.289

Note: All models are ranked based on the minimum of MAE for ES on 1%, 2.5%, 5% and 10% significance level. The best models are highlighted by bold.

Chapter Four: Value at Risk and Expected Shortfall for Options.

1. Introduction

Options play a major role in the financial markets as they can be used by the investors for hedging, speculative, spreading and synthetic positions. The accurate valuation of the option is critical for financial market analysts. Black and Scholes (1973) and Merton (1973) derived a formula to price a European call option based on Black-Scholes-Merton(BSM) option pricing. The Black Scholes Model (BSM) is one of the most effective approaches in modern financial theory and has become the basic benchmark for pricing equity and commodity options.

Several of the assumptions used in the Black-Scholes method are considered unrealistic. The BSM assumes that the price of an asset traded reflect a geometric Brownian motion with constant drift and volatility. The geometric Brownian motion model indicates that the series of first differences of the log prices of an asset must be uncorrelated. However, it was noticed that there are small but statistically significant correlations in the differences of the logs at short time lags.

Another key assumption underlying the Black-Scholes model is that the underlying asset return dynamics are captured by the normal distribution. However, empirical results have shown that assets returns are not normal but have leptokurtic distribution (heavy tailed).

One of inappropriate assumption of the Black-Scholes model is that the volatility of the underlying is constant. However, it is usually observed that for financial time series the level of volatility appears to change with time. However, it is observed by Mandelbrot (1963) that large changes are being followed by large changes, and small changes are being followed by small changes in the level of the initial time series. This kind of pattern is often referred to as volatility clustering. Because of violation of underlying assumptions of the Black-Scholes model the computed options prices may be misleading.

Over the last three decades, a vast number of pricing models have been presented as an alternative to the classic Black-Scholes approach as the underlying assumptions by Black-Scholes are clearly violated by observed asset returns.

Since the stock market crash of 1987, the inconsistency of stock index option prices from the Black-Scholes model has been phenomenal. For different option strikes and maturities, it is required to use different volatilities (implied volatilities), as the Black-Scholes model demand a constant volatility build on the underlying historical volatility.

In many markets, the implied volatilities often represent a smile or skew instead of a straight line. A volatility smile reflects when implied volatilities plotted against strike prices tend to vary in a U-shape relationship resembling a smile. Volatility smirk indicates that market implied volatility for options of lower strike prices is usually higher than for higher strike prices. When the implied volatilities for options at the lower strikes are lower than those at higher strikes is known as forward skew. The stochastic volatility models have been proposed to model the irregularities of volatility.

The rejection of constant variance Brownian motion result in a new class of Stochastic volatility models introduced by Hull and White (1987) which suggest that volatility is stochastic, varying both for time and for the price level of the underlying security. Since then many other stochastic volatility models have been developed (see Heston, 1993; Duffie and Kan, 1996; Ghysels et al., 1996; Duffie et al., 2000 and Balajewicz and Toivanen, 2017).

The Hull and White (1987) model is one of the first stochastic volatility model proposed after the market crash of 1987. It is a simple type of the stochastic volatility models developed later. However, the major disadvantages are the assumption of the zero correlation and the absence of an incorporated mean-reverting part for the volatility dynamics. Moreover, unlike other models we cannot compute the characteristic function in closed-form for Hull-White model.

Stein and Stein (1991) suggested a model with spot and volatility dynamics. The model contributes to closed-form option pricing solutions. Schobel and Zhu (1999) expanded the Stein and Stein's (1991) formation to a general case and derived an analytic solution for option prices. Heston's Model (1993) emerged from other stochastic volatility models as there prevails an analytical solution for European options that deals with correlation between stock price process and volatility process. The advantage of the Heston (1996) model is that it can be solved in closed-form, while other stochastic volatility models require complex numerical methods.

Lévy processes become very popular in option pricing academic research due to the flaws of the classical geometrical Brownian motion. In Lé-vy process the evolution of prices is given by a diffusion process and occurred by jumps at random distance or pure jumps type. Many previous studies introduce the Lévy processes into option pricing (see, Agliardi, 2011; Hsu and Chen, 2012; Ornthanalai, 2014; Fajardo, 2015; Kleinert and Van Schaik, 2015; Jiang et al., 2016; Xiao and Ma, 2016; Gong and Zhuang, 2016; Gong and Zhuang, 2016b; Balajewicz and Toivanen, 2017 and Deelstra and Simon, 2017).

In finance, all types of models belong to a class of Levy processes called “exponential Levy processes”. Exponential Levy models generalize the classical Black and Scholes formation that enable jumps into the stock prices, while the independence and stationarity of returns maintained.

Exponential Levy models are helpful in finance and can be divided into two classes. The first class called jump-diffusion models, in which the “normal” change of prices is given by a diffusion process, interrupted by jumps at irregular breaks. The second-class is known as infinite activity models that consists of models accompanying absolute number of jumps in each time interval.

Over the last few years several kinds of jump diffusion models have been developed. Two important Jump-diffusion models proposed by Merton (1976) and Kou (2002) respectively. For our research, we considered the Merton model and Double Exponential Jump Diffusion Model (Kou Model).

Both Merton and Kou models have certain characteristics that they share with known asset prices. Those models feature are missing in the classical Black Scholes model, like the characteristic of the leptokurtic. However, Kou's model is superior to Merton’s model in various aspects. As per Kou and Wang (2004) one of the features of Kou model is that the memoryless property of the exponential distribution makes it feasible to attain explicit formulas for substantial categories of options.

The jump-diffusion models allow for a finite number of jumps in a finite time interval (Merton ,1976; Ball and Torous, 1983 and Bates, 1991). More recently, infinite-activity models have been proposed that allowed an infinite number of jumps in a finite time interval (Madan and Seneta, 1990; Madan et al., 1998; Eberlein and Keller, 1995; Carr et al., 2002 and Carr and Wu

,2003). To form an infinite activity Lévy process, a Brownian process can be subordinated in time to a pure jump process.

The variance-gamma process and the normal-inverse Gaussian process are two examples of infinite activity processes. These models can represent both insignificant and persistent jumps, as well as substantial and exceptional ones.

Merton's and Heston's models of option pricing were combined by Bates (1996), that suggested a stock price model with stochastic volatility and jumps. The Bates model ignores interest rate risk, while the Scott (1997) introduced another model that supports interest rates to be stochastic.

The primary purpose of Bates (1996) and Scott (1997) option pricing models were to represent two characteristics of asset returns that conditional volatility grows over time in a stochastic but mean-reverting fashion, and the existence of irregular but important deviations in asset returns. These two models combined the Heston (1993) model of stochastic volatility with the Merton (1976) model of independent normally distributed jumps in the log asset price. The studies of Bakshi et al (1997), Bakshi and Madan (2000), Bates (2000,2003,2006), Lee (2004), Sgarra and Miglio (2011), Salami et al. (2013), Ballestra and Cecere (2016) and Balajewicz and Toivanen (2017) further extent and apply the models of Bates (1996) and Scott (1997) on both European and American options.

There are several methods to price options. Numerical methods need to implement to solve partial differential equation (PDE). The Monte-Carlo method easy to implement, however, this method is computationally heavy and needed lot of paths to ensure a good approximation. In this study, we discuss another pricing technique based on the characteristic function, the characteristic function of the asset prices distribution is simply the Fourier transform of its probability distribution function. The probability distribution function can be recovered from the characteristic function through Fourier inversion.

The Fast Fourier Transform (FFT) pricing method is very useful to efficiently price derivatives under any model with a known characteristic function, some of which are only expressible in this form (Hirsa and Neftci, 2013; Carr and Madan, 1999; Duffie et al., 2000; Bakshi and Madan, 2000; Lewis, 2000; Schoutens, 2003; Chourdakis, 2005; Fang and Oosterlee (2008), Gong and Zhuang, 2016a) and Deelstra and Simon, 2017).

The purpose of this chapter is to compare option pricing models, which are based on stochastic volatility model, jump diffusion models, infinite activity model and combined stochastic volatility model. We compare the performance of the Heston (1993) as stochastic volatility model, Bates (1996) as combined stochastic volatility model, Merton Jump Diffusion model and Kou model as jump diffusion model and Variance Gamma as infinite activity model with traditional Black-Scholes model. We measure the mean absolute error relative to observed option prices.

Backus et al. (1997) contributes the formula for delta in the Gram-Charlier Model. Pritsker (1997) examines the critical point of accuracy against computation speed in the full valuation, delta and gamma approaches. Risk measurement for options portfolios has also been calculated in Gibson (2000), Alexander et al. (2006), Sorwar and Dowd (2010), and Simonato (2011). Hull (2011) examine and suggest the delta, gamma and other risk measures in detail.

For a given level of volatility delta based VaR method is accurate only over short periods of time and gamma based VaR under estimate the actual risk. A basic problem associate with the delta and the delta-gamma way of calculating VaR is for a longer horizon is that delta and gamma calculation may not be steady approximations to the risk of the option position because they are expected to be constant over time but they are not.

Full valuation consists of simulating future hypothetical underlying asset prices and using the option pricing model to calculate the corresponding future hypothetical option prices. For each hypothetical future asset price, every option written on that asset must be priced. While full valuation is precise, it is unfortunately also computationally intensive (Christoffersen, 2012).

For risk analysis purpose, we evaluated various ES models based on partial Monte Carlo and full Monte Carlo method. For partial Monte Carlo, we calculated Delta based and Gamma based. The preceding deltas and gammas were derived from the Black Scholes model(BSM), Variance Gamma model(VG), Heston model (HS), Bates model (Bat), Merton Jump diffusion model and Double Exponential Jump diffusion model(Kou). We evaluate 1-day and 10-days ES for options based on the minimum mean absolute error (MAE).

We evaluated ES estimates for European options for all combinations of the following cases: 1-day and 10 days, at a range of confidence levels, and delta and gamma derived from various option models. For longer horizon, ES relies on the typical shortcut to estimating the risk over various time horizons is to scale by the square root of the ratio of the time horizons. Per our knowledge, this study is first to derived Delta and Gamma from the Black Scholes model(BSM), Variance Gamma model(VG), Heston model (HS), Bates model (Bat), Merton Jump diffusion model and Double Exponential Jump diffusion model(Kou) for both 1-day and 10 days ES forecast.

The code written to perform pricing and calibration is an important part of the thesis. The code is written with R programming.

Rest of the chapter is organized as follows. Section 2 discusses theoretical consideration. Section 3 introduces risk management for options. Section 4 looks at data and calibration. Our empirical results are presented and discussed in Section 5. Section 6 concludes the chapter.

2. Theoretical Considerations

European options are the simplest type of options contract that gives the owner the right but not the obligation to buy or sell an underlying asset at the price X on a specific date *T*, depending on the form of the option. The European option only allows exercising the option before the exercising date. On the other hand, American options can be exercised any time before the maturity date.

The holder of a call option gives the owner the right but not the obligation to buy a certain underlying asset (usually a stock) at the price X at pre-determined date *T*. A European put option gives the owner of the option the right to sell a certain of the underlying asset at the specific price X at pre-determined date *T*.

The today's price of the underlying asset is denoted by S_t; and at maturity of the option by S_{t+T} . If the underlying S_{t+T} is worth more than X then the holder of the option would exercise the option and make a profit $S_{t+T} - T$. Alternatively, if S_{t+T} is less than X , then the holder of the option would not exercise, resulting in the option expiring worthless. Mathematically, the value of the call option at maturity of the option T is:

$Max\{S_{t+T} - X, 0\}$

The holder of a put option has the right to sell the underlying for the exercise price X and result in the put option price:

$Max\{X - S_{t+T}, 0\}$

2.1. Stochastic Processes and Mathematical Finance for Options

This section presents a theoretical background to stochastic processes and stochastic calculus. Bjork (2009), Karatzas and Shreve (1991), Mikosch (1999), Oksendal (2010), Shreve (2004) and Zhu (2009) provides detailed introduction to stochastic processes and mathematical finance.

2.1.1 Brownian Motion

Robert Brown in 1828 first introduced Brownian motion. Louis Bachelier in 1900 brought it into finance. It was Norbert Wiener in 1923 that proved Brownian motion mathematically.

Brownian motion is physical phenomenon, but also plays an important role in mathematical finance. It is zig-zagging motion showed by a small fragment, such as a grain of pollen, involved in a liquid or a gas.

Definition (Wiener Process) A stochastic process$X(t)$, for $t \geq 0$, is called a Brownian motion or a Wiener, with following properties:

(i) $X(t) = 0$

(ii) $X(t)$ is continuous for all t.

(iii) $X(t)$ has independent increments. In other word $X(t) - X(s)$ over an interval of length $t - s$ is normally distributed with 0 mean and $t - s$ variance:
$X(t) - X(s) \sim N(0, t - s)$

(iv) If the intervals $[t_1, t_2]$ and $[t_3, t_4]$ don't overlap, then random variables $X(t_2) - X(t_1)$ and $X(t_4) - X(t_3)$ are independent.

As we know from property (iii) that if $0 \leq t_0 < t_1 \ldots < t_n$, then Markov property of the Wiener process is:

$$\mathrm{P}[X(t_0) = x_0, X(t_1) = x_1, \ldots X(t_n) = x_n] = P\left[X(t) \geq x \,\middle|\, X(t_n) = x_n\right] \tag{1}$$

The sum of two independent variables that normally distributed with mean μ_1 and μ_2 and variance σ_1^2 and σ_2^2 is also a random variable with mean $\mu_1 + \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$.In the same way for increments $X(t_2) - X(t_1)$ and $X(t_4) - X(t_3)$ the sum $X(t_2) - X(t_1) + X(t_4) - X(t_3)$ is normally distributed with mean 0 and variance $t_2 - t_1 + t_4 - t_3$.

The property of independent variable is consistent with properties of normal random variables. The probability density function for a random variable with $N(0, t)$ is:

$$p(x, t) = \frac{1}{\sqrt{2\pi t}} exp\left(-x^2/(2t)\right) \tag{2}$$

Now derive the joint density of an event:

$X(t_1) = x_1, X(t_2) = x_2, \ldots X(t_n) = x_n$

It is very much like the joint probability density of another similar event.

$$X(t_1) - X(t_0) = x_1, X(t_2) - X(t_1) = x_2 - x_1, \ldots X(t_n) - X(t_{n-1}) = x_n - x_{n-1} \quad (3)$$

We can get the expression for joint probability density function of $X_{t_1}, \ldots X_{t_n}$ as:

$$f(x_1, t_1; x_2, t_2; \ldots x_n, t_n) = p\left(x_{1,} t\right) p(x_2 - x_1, t_2 - t_1) \ldots p(x_n - x_{n-1}, t_n - t_{n-1}) \quad (4)$$

where $t_0 = 0, x_0 = 0$.

2.1.2. Stochastic Integral -Ito Lemma

Stochastic calculus is one of the important instruments in modern Mathematical Finance. In this section, we now explain the stochastic integral. For that purpose, we consider as given a Brownian motion $X(t)$ and stock prices is of the form $S_t = f(X_t)$. By using Taylor's Theorem:

$$f(X_{t+\delta t}) - f(X_t) = (X_{t+\delta t} - X_t) f'(X_t) + \frac{1}{2!}(X_{t+\delta t} - X_t)^2 f''(X_t) + \cdots \quad (5)$$

The differential equation $S_t = f(X_t)$ will take the form:

$$S_t = f'(X_t) dX_t + \frac{1}{2} f''(X_t) dt$$

In integration form:

$$S_t = S_t + \int_0^t f'(X_s) dX_s + \int_0^t \frac{1}{2} f''(X_s) ds \quad (6)$$

Thus, the Brownian motion has finite quadratic variation.

Definition 1. Suppose π be a partition of $[0, T]$, the variation of f is

$$\lim_{\delta \to 0} \left[\sup_{\pi:\delta(\pi)=\delta} \sum_1^{N(\pi)} \left| f(t_{j+1}) - f(t_j) \right| \right] \quad (7)$$

where $N(\pi)$ is number of intervals that make up π and $\delta(\pi)$ is length of biggest interval of the partition.

Definition 2. The quadratic variation of function f is: X_{t_j}

$$q.v.(f) = \lim_{\delta \to 0} \left[\sup_{\pi:\delta(\pi)=\delta} \sum_1^{N(\pi)} \left| f(t_{j+1}) - f(t_j) \right|^2 \right] \quad (8)$$

Definition 3. Let X_t be Brownian motion and for s partition π of $[0, T]$ is:

$S(\pi_n) = \sum_{j=1}^{N(\pi)} \left|X_{t\,j} - X_{t\,j-1}\right|^2$ (9)

$\delta(\pi_n) \to 0$, then

$\mathbb{E}[|S(\pi_n) - T|^2] \to 0$ as $n \to \infty$

The two further limits as $\delta(\pi_n) \to 0$:

$$\sum_{j=0}^{N(\pi)-1} X_{t\,j+1}\left(X_{t\,j+1} - X_{t\,j}\right)$$

and

$$\lim_{\delta(\pi_n)\to 0} \sum_{j=0}^{N(\pi)-1} \left(\frac{X_{t\,j} + X_{t\,j+1}}{2}\right)\left(X_{t\,j+1} - X_{t\,j}\right)$$

We obtained different integral by selecting different points within every subinterval of the partition. The Ito integral is defined as:

$\int_0^T f(X_s)dX_s = \lim_{\delta(\pi_n)\to 0} \sum_{j=1}^{N(\pi)} f\left(X_{t\,j}\right)\left(X_{t\,j+1} - X_{t\,j}\right)$ (10)

This is on especial case of Ito integral. We will now consider the value on simple function in classical settings.

Definition 4. A simple function is:

$f(X_s) = \sum_{i=1}^{n} \alpha_i(X_s)\chi_{I_i}(s)$ (11)

where $I_i = (s_i, s_{i+1}), \cup_{i=1}^{n} I_i = [0, T], I_i \cap I_j = \{\acute{\emptyset}\}\ if\ i \neq j\ , \alpha_i\ satisfied\ \mathbb{E}[\alpha_i(X_s)^2] < \infty$

By definition:

$\int_0^T f(X_s)dX_s = \sum_{i=1}^{n} \alpha_i(X_s)\left(X_{s_{i+1}} - X_s\right)$ (12)

Definition 5: Let f is a simple function:

1. $\int_0^t f_s(X_s)dX_s$ is a continuous $\mathcal{F}_t - martingale.$
2. $\mathbb{E}\left[\left[\int_0^T f(X_s)dX_s\right]^2\right] = \int_0^T \mathbb{E}\,[f(X_s)^2]ds$

The above statement is famous Ito isometry. It indicates that definition of the integral to functions extended such that $\int_0^t \mathbb{E}[f_s(X_s)^2]ds < \infty$

3. $\mathbb{E}\left[\sup_{t\le T}\left(\int_0^T f(X_s)dX_s\right)^2\right] \le 4\int_0^T \mathbb{E}[f(X)^2]ds$

The above statement observed from the second statement by an implication of a famous result of Doob’s inequality.

Definition 6 (Doob’s inequality): If $\{M_t\}0 \leq t \leq T$ is a continuous martingale:

$$\mathbb{E}\left[\sup_{0\leq t\leq T} M_t^2\right] \leq 4\mathbb{E}[M_T^2] \tag{13}$$

Definition 7: Let $\mathcal{F}_t$ denote the natural filtration producing by Brownian motion. J is a special linear mapping from Ω to the space of continuous $\mathcal{F}_t - martingale$ defined on $[0,T]$ as:

1. If f is simple:

   $J(f)_t = \int_0^t f_s(X_s)dX_s$ 2.
2. If $t \leq T$,

   $$\mathbb{E}[J(f)_t^2] = \int_0^t \mathbb{E}[f_s(X_s)^2]ds$$
3. $\mathbb{E}\left[\sup_{0\leq t\leq T} J(f)_t^2\right] \leq 4\int_0^T \mathbb{E}[f_s(X_s)^2]ds$

Definition 8: (Ito’s formula) For f such that $\frac{\partial f}{\partial x} \in \boldsymbol{\mathcal{H}}$

$$f(t,X_t) - f(0,X_0) = \int_0^t \frac{\partial f}{\partial x}(s,X_s)dX_s + \int_0^t \frac{\partial f}{\partial s}(s,X_s)ds + \frac{1}{2}\int_0^t \frac{\partial^2 f}{\partial^2 x^2}(s,X_s)ds$$

$$df_t = f_t' dX + \dot{f}_t dt + \frac{1}{2}f_t^{"} dt \tag{14}$$

Definition 9: We use Ito’s formula to compute $\mathbb{E}[X_t^4]$

We define$Z_t = Z_t^4$. Then by using Ito’s formula

$dZ_t = 4X_t^3 dX_t + 6X_t^2 dt$

$Z_0 = 0$

In integrated form,

$$Z_t - Z_0 = \int_0^t 4X_s^3 dX_s + \int_0^t 6X_s^2 ds$$

$$\mathbb{E}(Z_t) = \int_0^t 6\mathbb{E}[X_s^2]ds = \int_0^t 6sds = 3t^2 \tag{15}$$

Geometric Brownian motion is the most common model of stock price movement, defined by:
$S_t = exp(vt + \sigma X_t)$
Now applying Ito formula:

$$\begin{cases} dS_t = \sigma S_t dX_t + \left(v + \frac{1}{2}\sigma^2\right) S_t dt \\ S_0 = 1 \end{cases} \tag{16}$$

The equation 16 is called the stochastic differential equation for S_t.

2.1.3. Geometric Brownian Motion

We have explained both Brownian motion $X(t)$ and Itˆo's Lemma, as BM can take on negative values, using BM directly for modelling stock prices is uncertain. Therefore, we introduce an important stochastic process, a non-negative variation of BM called geometric Brownian motion.

A stochastic process $S(t)$ is said to follow a Geometric Brownian Motion if it satisfies the following stochastic differential equation:

$$S(t) = \mu dt + \sigma S(t) X(t) \tag{17}$$

where $X(t)$ is a Wiener process (Brownian Motion) and μ, σ are constants.
By apply the technique of separation of variables:

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dX(t)$$

Apply integral on both side:

$$\int \frac{dS(t)}{S(t)} = \int \left(\mu dt + \sigma dX(t) \right) dt \tag{18}$$

Now involve and Itˆo's Lemma and get the following solution:

$$ln\left(\frac{dS(t)}{S(t)}\right) = \left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma X(t) \tag{19}$$

The analytical solution of this geometric Brownian motion is given by:

$$S(t) = S(0) e^{\left(\left(\mu - \frac{\sigma^2}{2}\right) + \sigma X(t)\right)} \tag{20}$$

If μ and σ are constant, we have the normal geometric Brownian motion model $dS(t) = S(t)\big(\mu dt + \sigma dX(t)\big)$, and the distribution of $S(t)$ is log-normal.

2.2. The Black-Scholes (1973) Model

Black-Scholes model is simple and popular as a benchmark model for pricing and trading in the financial market. We assume the stock price follows a geometric Brownian motion. Under

BMS trading takes place in continuous time (in the absence of arbitrage opportunities), the risk-free rate *r* is known and constant over time, the stock pays no dividend until the maturity of the option. BSM only use to calculate option pricing for European options (it can only be exercised at the expiration date). The stock price follows a geometric Brownian motion over time that generate a log-normal distribution for stock price between any two points in time, the volatility is constant for any strike and maturity. Because of its simplicity, the Black-Scholes formula is extensively used among experts for pricing and hedging options.

Under the BMS model, the stock price, S, follows Geometric Brownian Motion,

$$dS(t) = \mu S(t)dt + \sigma S(t)dX(t) \tag{21}$$

where μ and σ are known constant, $X(t)$ is a standard Brownian motion. The analytical solution of this geometric Brownian motion is given by,

$$S(t) = S(0)e^{\left(\left(\mu-\frac{\sigma^2}{2}\right)+\sigma X(t)\right)} \tag{22}$$

An important part in the BSM methodology is the construction of risk free portfolio. A partial differential can be determined for the price of call option based on the no-arbitrage debate. To get a close form solution for BSM partial differential equation can be easily solve.

Let *C* is the price of a call option.

$$dC = \left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial C}{\partial S}\sigma S dX(t) \tag{23}$$

Now consider a portfolio consisting of short position in a call option and a long position,

$$VP = -C + \Delta S$$

where VP is value of portfolio and Δ is units of stock.

The change in the VP over small intervals,

$$dVP = -dC + \Delta dS$$

By substituting $dS(t)$ and dC into dVP yields,

$$dVP = -\left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt - \frac{\partial C}{\partial S}\sigma S dX(t) + \Delta\mu S dt + \Delta\sigma S dX(t)$$

$$= \left(-\frac{\partial C}{\partial S}\mu S + \frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt + \left(-\frac{\partial C}{\partial S}\sigma S + \Delta\sigma S\right)dX(t) \tag{24}$$

We put $\Delta = \frac{\partial C}{\partial S}$ to make the portfolio risk free,

$$dVP = \left(-\frac{\partial C}{\partial t} - \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt \tag{25}$$

It is clear from above expression that to make the portfolio risk free, the Brownian motion $X(t)$ has been removed. In the absence of arbitrage opportunities, the risk-free portfolio need a risk-free rate,r,

$dVP = rdVPdt$

By substituting VP into expression 25:

$$\left(-\frac{\partial C}{\partial t}-\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt = r\left(-C+\frac{\partial C}{\partial S}S\right)dt$$

$$\frac{\partial C}{\partial t}+rS\frac{\partial C}{\partial S}+\frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 = rC \qquad (26)$$

The expression 26 is the Black Scholes partial differential equation.

Let $C\big(t;r,K,T,\sigma,S(t)\big)$ is the time t price of a European call, K is exercise price, and T is time of maturity on the underlying asset S(t) based on BSM model. We have,

$$C\big(t;r,K,T,\sigma,S(0)\big) = S(0)\Phi(d_1)-exp(-rT)K\Phi(d_2) \qquad (27)$$

where $\Phi()$ is the cumulated normal distribution function,

$$d_1 = \frac{ln\left(\frac{S(0)}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

Proof: For European vanilla call option, the option price is simply its payoff at maturity.

$C = max(S-K,0)$ t=T

For time $t = 0$,

$$C\big(0;r,K,T,\sigma,S(0)\big)$$

$$= E\big[exp(-rT)C\big(T;r,K,T,\sigma,S(T)\big)|\mathcal{F}_0|\big]$$

$$= \int_{-\infty}^{\infty} exp(-rT)max\left\{S(0)exp\left[\left(r-\frac{1}{2}\sigma^2\right)T+\sigma y\right]-K,0\right\}\frac{exp\left(-\frac{1}{2}\frac{y^2}{T}\right)}{\sqrt{2\pi T}}dy$$

$$= \int_{-d_2}^{\infty}\left\{S(0)exp\left[-\frac{1}{2}\sigma^2 T+\sigma y\sqrt{T}\right]-Kexp(-rT)\right\}\frac{exp\left(-\frac{1}{2}\frac{y^2}{T}\right)}{\sqrt{2\pi T}}dy$$

$$= S(0)\Phi(d_1)-exp(-rT)K\Phi(d_2) \qquad (28)$$

where $\Phi()$ is the cumulated normal distribution function,

$$d_1 = \frac{ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

2.3. Implied Volatility

Even though the Black-Scholes formula is very popular and dominant among market practitioners to price stock options and very simple to use. However, Black-Scholes model presents unrealistic assumption of constant volatility that result in the issue of most well-known phenomenon of volatility smile or skew. The implied volatilities from the market prices of options tend to vary by various strike prices, different maturities and underlying assets.

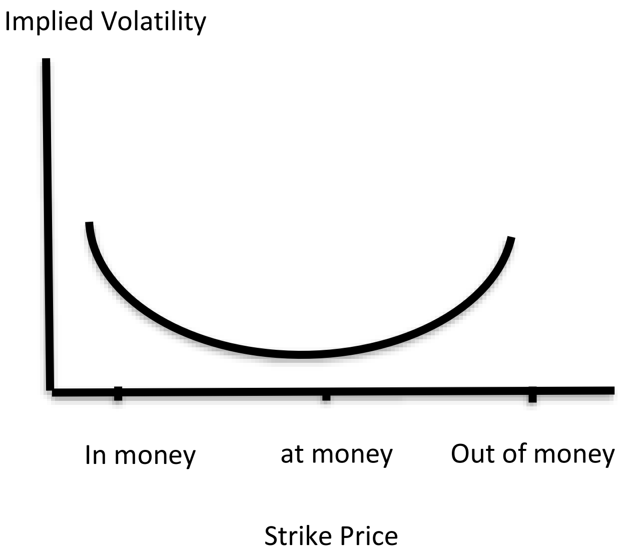
In Black-Scholes option pricing model, the price of a call option is the function of the current price $S(0)$, interest rate r , the strike price K , the constant volatility σ and the maturity T. All the other variables are known except volatility σ. Since the quoted option price C^{obs} is observable, the implied volatility is the volatility used in the Black-Scholes model such that the observed market price of the option equals the model price.

$$\sigma_{BSM}^{iv} = C_{BSM}^{-1}(0; r, K, T, S(0), C^{mkt}) \tag{29}$$

Theoretically, options price under Black-Scholes model should have a flat implied volatility surface, because of assumption of constant volatility. However, practically, the implied volatility surface is not flat, as volatility change with strikes and maturity. This phenomenon is known as the volatility skew. In some market, Implied volatilities plotted against strike prices form a U-shape, which is called the volatility smile. This arrangement is usually notice in options in the foreign exchange market.

The implied volatilities for options at the lower strikes are higher than those at higher strikes. Generally, the shape of the volatility smile is not symmetric. It is called reverse skew or volatility smirk. The reverse skew pattern commonly observes for longer term equity options and index options. The implied volatilities for options at the lower strikes are lower than those at higher strikes. It is called reverse skew. The forward skew pattern is usually seen in the commodities market option.

Figure 16: Volatility Smile



2.4. Stochastic Volatility Models

As the Black- Scholes model fails to model volatility, models based on Black-Scholes presume that the variance is constant over the maturity of the derivative, but empirical evidence indicates that volatility of the stock returns is not constant. The relation between the volatility and the stock or time changes over time.

To overcome these problems, many models have been proposed in the financial literature. Stochastic volatility models are one approach to resolve a shortcoming of the Black-Scholes model. Stochastic volatility models are important because they describe that options with different strikes and expirations have different Black-Scholes implied volatilities in a rational way. Stochastic volatility models assume that the volatility of the option price is a stochastic process rather than a constant.

In stochastic volatility models,

$$dS(t) = \mu S(t)dt + \sigma(t)S(t)dX_1(t) \tag{30}$$

$$\sigma^2(t) = f\big(v(t)\big)$$

$$dv(t) = a\big(t, v(t)\big)dt + b\big(t, v(t)\big)dZ(t)$$

$$dX_1(t)dZ(t) = \rho dt$$

where μ is constant, $\sigma(t)$ is the volatility of the stock price, $f(.)$ is some positive function, $X_1(t)$ and $Z(t)$ are two correlated Brownian motions with correlation ρ, and $v(t)$ is some underlying process which determines the volatility. We can have defined $Z(t)$ as:

$$Z(t) = \rho X_1(t) + \sqrt{1-\rho^2} X_2(t) \quad (31)$$

where $X_2(t)$ is standard Brownian motion independent of $X_1(t)$.

2.4.1. Hull-White Model

Hull and White (1987) introduce a stochastic volatility model for option pricing. Hull-White model assumes a geometric Brownian motion for the variance,

$$dS(t) = dS(t)dt + \sigma(t)S(t)dX(t)$$

$$dv(t) = \alpha v(t)dt + \beta v(t)dZ(t) \quad (32)$$

where volatility is calculated by $\sigma^2(t) = f\big(Y(t)\big)$, α and β are constant, $X(t)$ is uncorrelated to $Z(t)$.

2.4.2. Heston Model

The Heston model proposed by Heston (1993) is very important stochastic volatility model that provides closed-form formula for the European option. In Heston model, the randomness of the variance process varies as the square root of variance,

$$dS(t) = \mu(t)S(t)dt + \sqrt{v(t)}S(t)dZ_1(t)$$

$$dv(t) = k\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}dZ_2(t)$$

$$dZ_1(t)dZ_2(t) = \rho dt \quad (33)$$

where θ is the long-run variance, k is the rate of mean reversion, σ is called volatility of volatility, and dZ_1 and dZ_2 are correlated with the constant correlation value ρ. $v(t)$ is strictly positive when $k\theta \geq \sigma^2$ and is non-negative when $0 \leq 2k\theta < \sigma^2$.

2.5. Levy Process for Financial Modelling

Levy processes are a category of stochastic processes with discontinuous paths. Exponential Levy models generalize the classical Black and Scholes structure that enable jumps into the stock prices, while the independence and stationarity of returns maintained.

As a generalization of Brownian motion, Lévy process is a refined stochastic process which has stationary and independent increments that can keep any category of distribution only if it is infinitely divisible, Xiao and Ma (2016).

In finance, all the models belong to a family of Levy processes called “exponential Levy processes”. Exponential Levy models are very useful in finance and can be divide into two classes. The first class called jump-diffusion models, in which the “normal” change of prices is given by a diffusion process, interrupted by jumps at irregular breaks. The second class called infinite activity models, that consists of models accompanying absolute number of jumps in each time interval.

2.5.1. Jump Diffusion Models

In jumps diffusion models the jumps represent unusual events, crashes. These changes can be defined by a Levy process with a nonzero Gaussian element and a jump part with finitely many jumps,

Let $L = \big(L(t)\big)_{t>0}$ is a Levy Jump diffusion,

$$L(t) = \gamma t + \sigma X(t) + \textstyle\sum_{i=1}^{N_t} Y_i\,, \gamma \in \mathbb{R}, \sigma \geq 0 \tag{34}$$

where $X = \big(X(t)\big)_{t\geq 0}$ is a standard Brownian motion, (Y_i) are i.i.d. sequence of random variables and N is a Poisson process.

Over the last few years several kinds of jump diffusion models have been developed. Two important Jump-diffusion models proposed by Merton (1976) and Kou (2002) respectively.

For this chapter, we considered the Merton model and Double Exponential Jump Diffusion Model (Kou Model).

2.5.1.1. Merton Model

The Merton model (1976) which was the first model in the jump diffusion class to use a discontinuous price process to model asset returns.

Merton's jump-diffusion model tries to capture the negative skewness and excess kurtosis of log stock prices encountered in the Black-Scholes model by a simply inclusion of a compound Poisson process. Addition of the compound Poisson process include three extra parameters to the original Black Scholes model to control skewness and excess kurtosis.

Merton's jump-diffusion model is an exponential Lévy model of the form,

$$S(t) = S(0)e^{L(t)} \tag{35}$$

where the stock price process $S(t); 0 \leq t \leq T$ is modelled as an exponential of a Levy process $L(t); 0 \leq t \leq T$.

Merton's option of the Lévy process is a Brownian motion with drift addition a compound Poisson process. In the Merton, the Levy process $\{L(t)\}_{t\geq 0}$ is given by,

$$L(t) = \mu t + \sigma X(t) + \textstyle\sum_{i=1}^{N_t} Y_i \tag{36}$$

where $\left(X(t)\right)_{t\geq0}$is a standard Brownian motion, $\mu t + \sigma X(t)$ is a Brownian motion with drift, and $\sum_{i=1}^{N_t} Y_i$ is a compound Poisson process. The distinctness between the Black-Scholes and the Merton model is the inclusion of a compound Poisson process $\sum_{i=1}^{N_t} Y_i$. A compound Poission process $\sum_{i=1}^{N_t} Y_i$ consists of two sources of randomness. The first randomness is a Poisson process dN_t with intensity parameter λ which result in random asset price jumps. The second randomness is once the asset price jumps; how much it jumps. Merton's jump-diffusion model assume that log-price jump size follows a Gaussian distribution, i.e. $Y_i \sim N(\alpha, \delta^2)$. Hence, the distribution of the jump size has the density,

$$f(x; \alpha, \delta) = \frac{1}{\delta\sqrt{2\pi}} exp\left[\frac{(x-\alpha)^2}{2\delta^2}\right] \tag{37}$$

Moreover, it assumes that two sources of randomness are independent of each other. Therefore, three extra parameters λ, α and δ are introduce to Black Scholes models to capture the skewness and excess kurtosis.

Hence, the Levy density is,

$$v(x; \lambda, \alpha, \delta) = \lambda f(x; \alpha, \delta) = \frac{\lambda}{\delta\sqrt{2\pi}} exp\left[\frac{(x-\alpha)^2}{2\delta^2}\right] \tag{38}$$

Thus, the Merton model has four parameters excluding drift μ; the diffusion volatility σ, the jump intensity λ, the mean jump size α and the standard deviation of jump size δ .

2.5.1.2. Double Exponential Jump Diffusion Model (Kou Model)

Merton's model assumed that the jump sizes are normally distributed. Kou (2002) proposed another jump diffusion model, where the distribution of jump sizes is an asymmetric exponential.

Both Merton and Kou models have certain characteristics that they share with known asset prices. Those models feature are missing in the classical Black Scholes model, like the characteristic of the leptokurtic. However, Kou's model is superior to Merton's model in various aspects. As per Kou and Wang (2004) one of the features of Kou model is that the memoryless property of the exponential distribution makes it feasible to attain explicit formulas for substantial categories of options.

In the Kou model the Levy process $\{L(t)\}_{t\geq0}$ is given by,

$$L(t) = \mu t + \sigma X(t) + \sum_{i=1}^{N_t} Y_i \tag{39}$$

where $\left(X(t)\right)_{t\geq 0}$is a standard Brownian motion, $\mu t + \sigma X(t)$ is a Brownian motion with drift, and $\sum_{i=1}^{N_t} Y_i$ is a compound Poisson process. The difference between Merton and Kou model is the assumption of log-price jump size that follows a double exponential distribution in Kou's jump diffusion model i.e. $Y_i \sim DbEx(p_1, p_2, \eta_1, \eta_2)$. Thus, the distribution of the jump size has the density,

$$f(x; p_1, p_2, \eta_1, \eta_2) = p_1 \eta_1 e^{-\eta_1 x} 1_{x\geq 0} + p_2 \eta_2 e^{\eta_2 x} 1_{x<0}\,, \eta_1, \eta_2 > 1 \quad (40)$$

After multiplying by λ the Levy density,

$$v(x; p_1, p_2, \eta_1, \eta_2) = \lambda\, f(x; p_1, p_2, \eta_1, \eta_2) = \lambda(p_1 \eta_1 e^{-\eta_1 x} 1_{x\geq 0} + p_2 \eta_2 e^{\eta_2 x} 1_{x<0}) \quad (41)$$

So, five extra parameters p_1, p_2, η_1, η_2 and λ excluding drift μ are introduced to the Black Scholes model: the diffusion volatility σ, the jump intensity λ, the probability of an upward jump p_1 , the probability of a downward jump p_2, and the decay of the tails for positive and negative jump sizes are controlled by η_1 and η_2 respectively.

The five independent parameters in the Kou model make it more flexible and simple to compute asset prices than the Merton model, which has only four parameters.

2.5.2. Infinite Activity Models

The jump-diffusion models allow for a finite number of jumps in a finite time interval (Merton ,1976); Ball and Torous, 1983 and Bates, 1991). More recently, infinite-activity models have been proposed that allowed an infinite number of jumps in a finite time interval (Madan and Seneta, 1990; Madan et al., 1998; Eberlein and Keller; 1995; Carr et al., 2002 and Carr and Wu, 2003). To form an infinite activity Lévy process, a Brownian process can be subordinated in time to a pure jump process.

The variance-gamma process and the normal-inverse Gaussian process are two examples of infinite activity processes. These models can represent both insignificant and persistent jumps, as well as substantial and exceptional ones.

It is noticeable that the variance-gamma process has finite variation, while the normal-inverse Gaussian process has infinite variation (Merton, 2013). In our research, we are interested in only investigation the Variance Gamma model by Carr and Madan (1998) that combines a Brownian process and a jump component.

Carr and Madan (1998) proposed the first major development in the pricing of derivatives using Fourier techniques with variance gamma process. Variance Gamma not only control the volatility but also the skewness and kurtosis of the return distribution. Madan and Seneta (1990) present a symmetric version of the variance gamma process. Madan et al. (1998) extend the model to allow for an asymmetric form and present a formula to price European options under the variance gamma process.

2.5.2.1. Variance Gamma Process

The Variance Gamma process nest the Brownian process on top of a Gamma process that explains the time-unit. Particularly, in situation of the attainment of the random process of time-unit, the price has a Brownian distribution.

The Variance Gamma process is realized by calculating Brownian motion with drift at a random time given by a gamma process,

$$\beta(t;\mu,\sigma) = \mu t + \sigma X_{G(t)} \tag{42}$$

where $G(t)$ is a gamma process with parameters $\alpha = \beta = {}^{1}/_{\xi}$ with ξ is the volatility of the time change. The process $\beta(t;\mu,\sigma)$ is a Brownian motion with drift θ and volatility σ.

As we know that the Lévy measure has finite dimension, the variance-gamma process has an infinite number of jumps in any finite time spell (Schoutens, 2003). The distribution of the variance gamma process is extremely separable and has stationary and independent increments. Moreover, when $\mu < 0$ it is negatively skewed and $\mu > 0$ it is positively skewed.

The moments of the variance-gamma process are:

Mean: μ; variance:$\sigma^2 + \xi\mu^2$; skewness:$\frac{\mu\xi(3\sigma^2+2\xi\mu^2)}{(\sigma^2+\xi\mu^2)^{3/2}}$, and Kurtosis: $3\left(1 + 2\xi - \frac{\xi\sigma^4}{(\sigma^2+\xi\mu^2)^2}\right)$.

2.6. Combining Stochastic Volatility with Jumps

One of most well-known models for option pricing is the classical Black Scholes model (BMS) by Black and Scholes (1973) and Merton (1973). According to BMS the price of the underlying asset is illustrated by a geometric Brownian motion with constant volatility. However, many empirical studies have explained that the volatility is not constant, and that the asset prices are usually subject to jumps. An evident generalization of the Black–Scholes model support one to combined both non-constant(stochastic)volatility and jumps in the asset price has been introduced by Bates (1996) (Ballestra and Cecere, 2016).

The studies of Bakshi et al (1997), Bakshi and Madan (2000), Bates (2000,2003,2006), Lee (2004), Sgarra and Miglio (2011), Salami et al. (2013) and Ballestra and Cecere (2016) further extent and apply the models of Bates (1996) and Scott (1997) on both European and American options.

Merton's and Heston's models of option pricing was combined by Bates (1996), that suggested a stock price model with stochastic volatility and jumps.

2.6.1. The Bates Model

Dynamic of $S(t)$ under historical measure,

$$\frac{dS(t)}{S(t)} = \left(\mu - \lambda\bar{J}\right)dt + \sqrt{v(t)}dX_1(t) + JdY(t)$$

$$dv(t) = \kappa\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}dX_2(t) \tag{43}$$

where μ is spot interest rate, λ is frequency of jump, J is the random percentage jump condition on jump occurring, $v(t)$ is value of spot volatility, θ is long run volatility, σ is volatility of volatility and$X_1(t)$ and $X_1(t)$ are two stochastic process correlated by ρ i.e. $\mathbb{E}^{\mathbb{P}}[dX_1(t)dX_2(t)] = \rho dt.$

$Y(t)$ is compound Poisson process with intensity λ i.e. $prob(dY(t) = 1) = \lambda dt$ and independent jumps J with,

$$ln(1 + J) \sim Y\left(ln\big(1 + \bar{J}\big) - \frac{1}{2}\alpha^2, \alpha^2\right)$$

The parameters J and α determine the distribution of the jumps and the Poisson process $Y(t)$ is consider to independent of the Wiener processes.

Now change measure $\mathbb{P} \to \mathbb{Q}$,

$$\frac{dS(t)}{S(t)} = \left(r - q - \lambda^*\bar{J}^*\right)dt + \sqrt{v(t)}dX_1^{\mathbb{Q}}(t) + J^*dY^*(t)$$

$$dv(t) = \kappa^*\big(\theta^* - v(t)\big)dt + \sigma\sqrt{v(t)}dX_2^{\mathbb{Q}}(t) \tag{44}$$

where

$$E^{\mathbb{Q}}\left[dX_1^{\mathbb{Q}}(t)dX_2^{\mathbb{Q}}(t)\right] = \rho dt$$

$$\kappa^* = \kappa + \xi$$

$$\theta^* = \frac{\kappa\theta}{\kappa + \xi}$$

such that ξ is volatility market price and,

$$J^* = J + JE^{\mathbb{P}}\left[\frac{\Delta J_\omega}{J_\omega}\right]$$

$$\bar{J}^* = \bar{J} + \frac{cov\left(J, \frac{\Delta J_\omega}{J_\omega}\right)}{1 + E^{\mathbb{P}}\left[\frac{\Delta J_\omega}{J_\omega}\right]}$$

where J_ω is the marginal utility of dollar wealth of the world average representative investor, $\frac{\Delta J_\omega}{J_\omega}$ is random percentage jump conditional on a jump occurring and $\frac{dJ_\omega}{J_\omega}$ is percentage shock in the absence of jump.

In the case of vanilla European call option, we have

$$C(t, S(t), v(t), J, K, T) = e^{-rt} E^* max(S(t) - K, 0) \qquad (45)$$

where $J = 1{,}2{,}3$, K is the strike price, and E^* is the expectation with respect to the risk neutral probability measure.

2.7. Option Pricing with the Characteristic Function

There are several methods to price options. Numerical implementation of partial differential equations (PDE) are difficult. The Monte-Carlo method easy to implement, however, this method is computationally heavy as needed a lot of paths to ensure a good approximation.
Hence, we will follow another pricing technique based on the characteristic function, the characteristic function of the asset prices distribution is simply the Fourier transform of its probability distribution function. The probability distribution function can be recovered from the characteristic function through Fourier inversion.

The Fast Fourier Transform (FFT) pricing method is very helpful to efficiently price derivatives under any model with a known characteristic function, some of which are only expressible in this form (see, Carr and Madan, 1999; Duffie et al., 2000; Bakshi and Madan ,2000; Lewis, 2000; Schoutens, 2003; Chourdakis, 2005; Fang and Oosterlee, 2008 and Hirsa and Neftci, 2013).

European call option price C_0 can be expressed by the equation (Bakshi and Madan, 2000 and Schoutens, 2003):

$$C_0 = e^{-q\tau} S_0 \Pi_1 - e^{-r\tau} X \Pi_2 \qquad (46)$$

where S_0 is the spot price of the underlying asset, X is the strike price, r and q are the risk-free rate and dividend yield and τ is time to expiration. The Π_j can be calculated as:

$$\Pi_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{i\omega log(X)}\phi(\omega - i)}{i\omega\phi(-i)}\right) d\omega \qquad (47)$$

$$\Pi_2 = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left(\frac{e^{i\omega log(X)}\phi(\omega)}{i\omega}\right) d\omega \tag{48}$$

where ϕ represent the characteristic function of the log stock price, $Re(.)$ stand in for the real part of a complex number. For call option price, we can compute Π_1 and $\Pi_2$by numerical integration for known characteristic function ϕ of Black-Scholes(BS), Merton Jump-diffusion(MJD), Kou Jump-Diffusion(Kou), the Heston model (HS), the Bates model(BATs) and Variance Gamma Model (VG) from equation (1).

2.7.1. Risk-Neutral Characteristic Functions

The characteristic function of a random variable is the Fourier transform of its distribution. Many probabilistic properties of random variables correspond to analytical properties of their characteristic functions, making this concept very useful for studying random variables.

2.7.1.1 The Black–Scholes Model

For given dynamic of S, the log price $s_\tau = log(S_\tau)$ follows a Gaussian distribution with $s_\tau \sim N\left(s_0 + \tau\left(r - q - \frac{1}{2}v\right), \tau v\right)$, where s_0 represent the natural logarithm of the current spot price. The characteristic function of s_τ can be defined as:

$$\phi_{BS}(\omega) = E\left(e^{i\omega s_\tau}\right) \tag{49}$$

$$= exp\left(\left(i\omega s_0 + i\omega\tau\left(r - q - \frac{1}{2}v\right) + \frac{1}{2}i^2\omega^2\tau v\right)\right)$$

$$= exp\left(\left(i\omega s_0 + i\omega\tau(r - q) - \frac{1}{2}(i\omega - \omega^2)\tau v\right)\right)$$

By inserting equation (4) into equation (1), we will get expression for the Black-Scholes option price.

2.7.1.2. Merton's Jump–Diffusion Model

The characteristic function of Merton's model is given by:

$$\phi_{MJD}(\omega) = e^{A+B} \tag{50}$$

where

$$\mathrm{A} = i\omega s_0 + i\omega\tau\left(r - q - \frac{1}{2}v - \lambda\mu_J\right) + \frac{1}{2}i^2\omega^2\tau v$$

$$\mathrm{B} = \lambda\tau\left(exp\left(i\omega log\left(\mu_J\right) - \frac{1}{2}i\omega v_J - \frac{1}{2}\omega^2 v_J\right) - 1\right)$$

We can then split the characteristic exponent into two parts, A represent a drift part and B-part adds the jump component.

2.7.1.3. Kou Jump-Diffusion Model

The characteristic function of Kou's model is given by:

$$\phi_{Kou}(\omega) = e^{A+B} \tag{51}$$

where

$$\mathrm{A} = i\omega s_0 + i\omega\tau\left(r - q - \frac{1}{2}v - \lambda\left(\frac{p\eta_1}{\eta_1 + 1}\right)\right) + \frac{1}{2}i^2\omega^2\tau v$$

$$\mathrm{B} = \lambda\tau\left(\left(\left(\frac{p\eta_1}{\eta_1 + i\omega}\right) + \left(\frac{(1-p)\eta_2}{\eta_2 + i\omega}\right)\right) - 1\right)$$

We can then split the characteristic exponent into two parts, A represent a drift part and B-part adds the jump component.

2.7.1.4. The Heston Model

The characteristic function of the log price in the Heston model looks as follows (see, Albrecher et al., 2007).

$$\phi_{HS}(\omega) = e^{A+B+C} \tag{52}$$

$$A = i\omega s_0 + i\omega(r - q)\tau$$

$$B = \frac{\theta k}{\sigma^2}\left((k - \rho\sigma i\omega - d)\tau - g\log\left(\frac{1 - ge^{-d\tau}}{1 - g}\right)\right)$$

$$C = \frac{\frac{v_0}{\sigma^2}(k - \rho\sigma i\omega - d)\left(1 - e^{-d\tau}\right)}{1 - ge^{-d\tau}}$$

$$d = \sqrt{(\rho\sigma i\omega - k)^2 + \sigma^2(i\omega + \omega^2)}$$

$$g = \frac{k - \rho\sigma i\omega - d}{k - \rho\sigma i\omega + d}$$

2.7.1.5. The Bates Model

The characteristic function becomes (Schoutens et al., 2004):

$$\phi_{BAT}(\omega) = e^{A+B+C+D} \tag{53}$$

$$A = i\omega s_0 + i\omega(r - q)\tau$$

$$B = \frac{\theta k}{\sigma^2}\left((k - \rho\sigma i\omega - d)\tau - g\log\left(\frac{1 - ge^{-d\tau}}{1 - g}\right)\right)$$

$$C = \frac{\frac{v_0}{\sigma^2}(k - \rho\sigma i\omega - d)(1 - e^{-d\tau})}{1 - ge^{-d\tau}}$$

$$D = \lambda u_J i\omega\tau + \lambda\tau\left(\left(1 + u_J\right)e^{\frac{1}{2}v_J i\omega(i\omega - 1)} - 1\right)$$

$$d = \sqrt{(\rho\sigma i\omega - k)^2 + \sigma^2(i\omega + \omega^2)}$$

$$g = \frac{k - \rho\sigma i\omega - d}{k - \rho\sigma i\omega + d}$$

2.7.1.6. Variance Gamma Model

The characteristic function of the Variance Gamma model can be written as:

$$\phi_{VG}(\omega) = e^{A+B} \tag{54}$$

where

$$A = i\omega dt$$

$$d = r + \frac{ln\left(1 - \theta v - \frac{1}{2}\sigma^2 v\right)}{v}$$

$$\mathrm{B} = \ln\left(1 - i\omega\theta v + \frac{1}{2}\sigma^2\omega^2 v\right)^{-t/v}$$

2.8. Greeks

Option Greek support to compute variation in the worth of option agreement due to elements affecting the underlying stock price. These essential features can be listed as volatility, interest rate, change in the underlying stock price and breaking down of time. The generally known Greeks are the first order derivatives: Delta, Vega, Theta and Rho as well as Gamma, a second-order derivative of the value function.

2.8.1. Delta

Delta is the amount by which the price of an option changes as compared to a $1 rise of the price of an asset indicated as a decimal or percentage. Delta can also be described as a slope of the tangent line fit to the option price function at the underlying stock price.

Delta of call option is the first derivative of the value of call option with respect to the stock price,

$$\Delta_{call} = \frac{\partial C}{\partial S} = N(d_1) \tag{55}$$

Delta of put option is the first derivative of the value of put option with respect to the stock price,

$$\Delta_{put} = \frac{\partial C}{\partial S} = N(d_1) - 1$$

where

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

2.8.2. Vega

Vega measure sensitivity to volatility. It is defined as the measure of volatility of underlying asset that shows the amount of changes in option price due to changes in volatility. Vega is calculated by taking the derivative of the option value with respect to the volatility of the underlying asset,

$$\frac{\partial C}{\partial \sigma} = \frac{\partial P}{\partial \sigma} = v = S\sqrt{T-t}N'(d_1) \tag{56}$$

2.8.3. Theta

Theta indicates the sensitivity of the value of the derivative with respect to the time. Theta is defined as the changes in the option prices as compared to the passage of time which is a negative number because the value of the option decreases with time. Theta can be calculated by taking derivative of function of option price with respect to time,

$$\Theta_{call} = \frac{\partial C}{\partial T} = \frac{SN'(d_1)S}{2\sqrt{T-t}} - Kre^{-r(T-t)}N(d_2) \tag{57}$$

$$\Theta_{put} = \frac{\partial P}{\partial T} = \frac{SN'(d_1)S}{2\sqrt{T-t}} + Kre^{-r(T-t)}N(d_2) \tag{58}$$

where

$$d_2 = \frac{ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

2.8.4. Rho

Rho measures sensitivity to the interest rate. Rho is changes in the option price as compared to changes in the risk-free rate. It is calculated by taking derivative of function of option price with respect to risk free interest rate,

$$\rho_{call} = \frac{\partial C}{\partial r} = KTe^{rT}\ N(d_2) \tag{59}$$

$$\rho_{put} = \frac{\partial P}{\partial r} = -KTe^{rT}\ N(-d_2) \tag{60}$$

where

$$d_2 = \frac{ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

2.8.5. Gamma

Gamma measures the rate of change in the delta relative to changes in the underlying price. Gamma is calculated by taking second derivative of the value function with respect to the underlying price,

$$\Gamma_{call} = \frac{\partial \Delta_{call}}{\partial S} = \frac{\partial^2 C}{\partial S^2} \tag{61}$$

$$\Gamma_{put} = \frac{\partial \Delta_{put}}{\partial S} = \frac{\partial^2 P}{\partial S^2} \tag{62}$$

3. Value at Risk and Expected Shortfall for Options

3.1. The Option Delta based VaR

Consider a portfolio consisting of just one (long) call option on a stock. The change in the dollar value (or the dollar return) of the option portfolio, $DV_{PF,t+1}$is then just the change in the value of the option:

$$DV_{PF,t+1} \equiv c_{t+1} - c_t \tag{63}$$

Using the delta of the option, we have that for small changes in the underlying asset price:

$$\delta \approx \frac{c_{t+1} - c_t}{S_{t+1} - S_t}$$

Defining geometric returns on the underlying stock as:

$$r_{t+1} = \frac{S_{t+1} - S_t}{S_t} \approx ln(S_{t+1}/S_t) = R_{t+1}$$

and combining the previous three equations, we get the change in the option portfolio value to be:

$$DV_{PF,t+1} \approx \delta(S_{t+1} - S_t) \approx \delta S_t R_{t+1}$$

The variance of the portfolio in the delta-based model is:

$$\sigma^2_{DV,t+1} \approx \delta^2 S_t^2 \sigma^2_{t+1}$$

where σ^2_{t+1} is the conditional variance of the return on the underlying stock.

Assuming conditional normality, delta based Value-at-Risk (VaR) in is:

$$VaR^p_{t+1} = \sigma_{DV,t+1} \Phi_p^{-1}$$

When volatility is assumed to be constant and returns are assumed to be normally distributed, we can calculate the dollar VaR at horizon K by:

$$VaR^p_{t+1,t+k} = \sigma_{DV,t+1}\sqrt{k \Phi_p^{-1}} \tag{64}$$

3.2. The Option Delta Gamma based VaR

Gamma based methods present more flexible functional form for capturing nonlinearity than delta method.

As we know the option value for delta base approximation is:

$$DV_{PF,t+1} \approx \delta S_t R_{t+1}$$

When incorporating the second derivative, gamma, we instead rely on the quadratic approximation:

$$DV_{PF,t+1} \approx \boldsymbol{\delta} S_t R_{t+1} + \frac{1}{2}\gamma S_t^2 R_{t+1}^2$$

The variance of the portfolio in the delta gamma-based model is:

$$\sigma_{DV,t+1}^2 \approx \boldsymbol{\delta^2} S_t^2 \sigma_{t+1}^2 + \frac{1}{2}\gamma^2 S_t^4 \sigma_{t+1}^4$$

Gamma based VaR is defined as:

$$VaR_{t+1}^p = \sigma_{DV,t+1}\Phi_p^{-1} \tag{65}$$

3.3. The Simulation-based Delta and Delta Gamma Approximation

Consider the simple case where the portfolio consists of options on only one asset, then the change in the option value for delta base approximation:

$$DV_{PF,t+1} \approx \delta S_t R_{t+1}$$

and for gamma base approximation:

$$DV_{PF,t+1} \approx \boldsymbol{\delta} S_t R_{t+1} + \frac{1}{2}\gamma S_t^2 R_{t+1}^2$$

Using the assumed model for the physical distribution of the underlying asset return, we can simulate MC pseud return on underlying asset:

$$\left\{\hat{R}_h\right\}_{h=1}^{MC}$$

We calculate the hypothetical changes in the portfolio value as:

$$\widehat{DV}_{PF,h} \approx \delta S_t \hat{R}_h$$

$$\widehat{DV}_{PF,h} \approx \boldsymbol{\delta} S_t \hat{R}_h + \frac{1}{2}\gamma S_t^2 \hat{R}_h^2$$

Then value at risk for simulation based delta gamma approach can be calculated as:

$$VaR_{t+1,t+k}^p = -Percentile\left\{\left\{\widehat{DV}_{PF,h}\right\}_i^{MC}, 100p\right\} \tag{66}$$

Expected Shortfall can be calculated as:

$$ES_{t+1,t+k}^{p} = -\frac{1}{p.MC}\sum_{i=1}^{MC.1}\widehat{DV}_{PF,h}\,.\,1\left(\widehat{DV}_{PF,h} < -VaR_{t+1,t+k}^{p}\right) \tag{67}$$

3.4. Portfolio Risk Using Full Valuation

Linear and quadratic approximations to the nonlinearity arising from options can in some cases give a highly misleading picture of the risk from options. Especially, for option portfolio with different strike prices, then issue is likely to occur. In this situation, the full valuation approaches have the potential to improve on the delta and delta-gamma methods to risk because they can allow for alternative methods of approximating changes in portfolio value, (Christoffersen, 2012).

The Returns for a single option in short position:

$$DV_{PF,t+1} = -1.\left(c\left(S_{t+k}, r_f, X, \tilde{T}-\tau; \sigma\right) - c^{mrk}\right) \tag{68}$$

We can simulate future hypothetical returns on the underlying asset by Monte Carlo, as:
$\left\{\hat{R}_h\right\}_{h=1}^{MC}$

The future hypothetical asset prices can be calculated:
$\left\{\hat{S}_h = S_t exp\left(\hat{R}_h\right)\right\}_{h=1}^{MC}$

The hypothetical changes in the portfolio value can be calculated as:

$$\widehat{DV}_{PF,h} = -1.\left(c\left(\hat{S}_h, r_f, X, \tilde{T}-\tau; \sigma\right) - c^{mrk}\right)$$

The full valuation VaR can be calculated as:

$$VaR_{t+1,t+k}^{p} = -Percentile\left\{\left\{\widehat{DV}_{PF,h}\right\}_{i}^{MC}, 100p\right\} \tag{69}$$

Expected Shortfall can be calculated as:

$$ES_{t+1,t+k}^{p} = -\frac{1}{p.MC}\sum_{i=1}^{MC.1}\widehat{DV}_{PF,h}\,.\,1\left(\widehat{DV}_{PF,h} < -VaR_{t+1,t+k}^{p}\right) \tag{70}$$

4. Data and Calibration

All pricing models need a parameter set to completely describe the dynamics of every model. To make a model consistent to real markets and applicable for pricing, risk management, or trading, it is required to carry out calibration (Hirsa and Neftci, 2014).

Usually, it is a simple task to determine the parameters estimation of a selective model. However, for complex models like option pricing, we need to use analytical approaches to estimate parameter. Methods of estimating parameter include, maximum likelihood estimation, method of moments.

To perform calibration for option pricing an objective function need to set. The objective function is usually set as the square root mean error. An optimization technique to minimize the objective function then apply. In general form, optimization function can be written as (Hirsa and Neftci (2014)).

$$\min \sum_{i=1}^{N} (C^{market} - C^{model})^2 \tag{75}$$

For our research, we find the best parameter values by means of minimizing the sum of quadratic deviations between the model's prices and observed prices. For our model calibration, we minimize MAE defined as:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |C^{market} - C^{model}| \tag{76}$$

where C^{market} and C^{model} are the market and model prices, respectively, of the *ith* option used in the calibration.

In this section, we use historical option prices to estimate our models' parameters. By using this technique, we prevent many issues that are related to parameters estimation based on an arbitrary basis. We consider options on S&P500 index traded on every Wednesday from January2005- December 2014. These are daily traded options of weekly record.

There are four different types of the option prices, namely the close price, the bid, the ask and the mean of the bid and ask. For our study, we use the mean of the bid and ask as our option market data to calibrate models. We cannot examine option pricing without considering risk free interest rates and dividends. These two elements are input in our option pricing models and should be correctly chosen. For our analysis of options pricing models, the risk free one-year US Treasury rates are used.

It is critical to consider that changes in interest rates are infrequent and insignificant. However, other determinants of the option price, such as underlying asset price, time to expiry, volatility, and dividend yield alter more commonly and significantly. These other factors have a relatively considerable effect on option prices than interest rates changes.

A dividend is a cash payment made to the holder of stock. A dividend can be paid annually or more regularly. For option pricing, we require measuring the amount of dividend that a stockholder can anticipate gaining till the maturity of the stock. However, dividend is not directly available as an expected value. We used dividend yield per annum for Standard and Poor index.

5. Empirical Results

In this section, we discuss the estimated parameters for option pricing and select the best model based on minimum MAE. We also discuss the evaluation of ES for both one-day and ten-days risk analysis.

5.1. Calibration

The main purpose of our study to compare the most popular and classical Black-Scholes models of options pricing with stochastic volatility models, jump diffusion models, infinite activity models and combined stochastic and jump diffusion models. We include Heston as stochastic volatility model, Merton and Kou as jump diffusion models, Variance Gamma as Infinite Activity model and Bates as combined stochastic and jump diffusion model.

For all the models, we talked about in chapter 2, we tried to calibrate the parameters of these option pricing models on weekly S&P500 traded options data at Chicago Board Options Exchange (CBOE). Over the sample period January 2005 to December 2014. We carry out year-by-year calibrations, considering first six months as in-sample period and the second six months as out-of-sample. On the same data set, we calculate and evaluate Value at risk (VaR) and Expected shortfall (ES).

Tables 63 to 82 show the calibrated model parameters together with the corresponding values of mean absolute error(MAE). The best model is selected by the minimization of error. For 'in the sample' and 'out of sample' result for year 2005 in tables 63 and 64, the results indicate that MJD, HS and BAT have very similar and minimum absolute error. Kou model in jump diffusion category has highest MAE for this sample. It is important to note that all three best models belong to different categories of option pricing models. Moreover, MJD has minimum MAE and Kou has highest MAE are both jump diffusion models.

For January-June 2006 in table 65 BS, BAT and MJD have minimum MAE, while for June-December 2006 in table 66 HS, BS and MJD have minimum MAE. Again, for both sample of 2006 Kou model has highest MAE.

BAT performs better than all other models for in sample 2007, in sample 2008 and out of sample 2008 indicated in table 67 table 69 and table 70. For all three samples HS, VG and MJD have very similar MEA. For out of sample 2007 BS is the best model as shown in table 68.

MJD has lowest MAE for both in sample and out of sample for 2009 in table 71 and 72. For both samples BAT has lowest MAE than HS and VG, HS and VG have similar MAE value. The results in table 73, 74, 76, 77, 78, 79, 80, 81, 82 shows that MJD and BAT are best performing models based on minimum MAE for both in samples and out of samples.

Our results indicate that Bates(BAT) and Merton -Jump diffusion model(MJD) outperforms the Black- Scholes model for both 'in sample' and 'out of sample' analysis. For all data samples, Kou has highest MAE, while HS and VG perform very similar in almost all cases. BS performs better than other models only for in sample 2011 and out of sample 2007.

It is evident from the result that other models perform better than classical model of Black and Scholes model. BAT as a combined stochastic volatility model and jump diffusion model is the best model. However, we cannot draw a clear conclusion about the performance of stochastic volatility models and jump diffusion models. On one hand, MJD is the best model in almost all cases but on the other hand, Kou has the highest MAE. We summarized our results as follows:

1. There is no significant difference in the MAE values of all models except Kou models.
2. For in sample and out of sample Kou has highest MAE.
3. It is the BAT models that has smallest MAE for almost all datasets. As we know that Bat is hybrid model of stochastic model and jump diffusion models.
4. The selection of Bat model give us clear indication of superiority of hybrid models over stochastics volatility models and jump diffusion models.

5.2. Expected Shortfall Evaluation

Risk modelling is an integral part of most, if not all, financial institutions. For risk management, we can calculate the VaR and ES for 5%, 2.5%,1% and 0.5% confidence level. As known from literature, there are many problems associated with VaR. Therefore, we have only measured different ES models as a better measure of risk

We establish partial Monte Carlo and full Monte Carlo approaches to estimated expected shortfall for option risk management. For partial Monte Carlo, we calculated Delta based and

Gamma based. The preceding deltas and gammas were derived from the Black Scholes model(BSM), Variance Gamma model(VG), Heston model(HS), Bates models(Bat), Merton Jump diffusion model(MJD) and Double Exponential Jump diffusion model(Kou). We implied other models beyond BSM to calculate delta and gamma, as know from the previous studies that the BSM model sometimes misprices traded options quite severely.

The purpose of the delta based method is to linearize the option return and thereby make it fit into the risk models. We use gamma of an option is to construct a quadratic model of the portfolio return distribution, as implementations of the quadratic model relies on the Monte Carlo simulation technique. We also measure the expected shortfall for options using the full valuation method, which depends on a detailed version of the Monte Carlo simulation technique.

We have evaluated expected shortfall estimates for European options for all combinations of the following cases: 1-day and 10 days, at a range of confidence levels, and delta and gamma derived from various option models. For longer horizon, ES relies on the typical shortcut to estimate the risk over different time horizons, is to scale by the square root of the ratio of the time horizons.

We compare ES by a loss function that calculated the difference between the actual and the expected losses when a violation occurred. Model ranking by MAE provided in the tables 83 to 102 for in sample and out of sample analysis. The mean absolute error appears small enough to suggest that the best fitting models are reasonable. ES evolution results for 10- days horizon for options also presented for in sample and out of sample analysis. We have used Monte Carlo Delta based, Monte Carlo Gamma based and full Monte Carlo (Full valuation) to calculate one-day risk and long-term risk (10- day risk) for options.

For in sample for Jan-2005 to June 2005, the table 83 shows that MC-MJD-Delta models have minimum MAE for 1-day ahead ES for all significance levels. MC-MJD-Gamma and MC-HS-Delta are second and third best models respectively for one day-ahead ES for all confidence level. Full valuation is the fourth best model at 1 % significance level while MC-BAT-Delta is the third best model at 2.5%, 5% and 10% significance level. For the same sample, we observed that for 10-day ES MC-MJD-Delta, MC-HS-Delta, MC-Bat-Delta and MC-MJD-Gamma are first, second, third and fourth best models respectively. From these results, we observed that as compare to 1-day ES 10-day MAE values are significantly different across models and significance levels. Moreover, ranking the of the models across horizon not the same. MC-Kou-Delta, MC-Kou-Gamma, MC-VG-Delta and MC-VG-Gamma have highest MAE.

MC-HS-Delta, MC-MJD-Delta, and MC-MJD-Gamma are first, second, and third models based on the smallest value of MAE for out of sample data for the period July2005-December 2005 in table 84. At 1% and 2.5% confidence levels MC-Bat-Delta is the fourth best model and at 5% and 10% confidence levels MC-HS-Gamma is the fourth confidence level.

For in sample Jan 2006-June 2006 in table 85 again MC-MJD-Delta has smallest MAE value for both 1-day and 10-days ES. MC-Bates-Delta, MC-HS-Delta and full valuation are second, third and fourth best model for 1-day ES. For 10-days ES MC-MJD-Delta MC-Bates-Delta, and MC-HS-Delta are the first three best models but MC-BS-Delta came fourth instead of full valuation.

MC-HS-Delta, MC-MJD-Delta, MC-HS-Gamma and MC-MJD-Gamma are four top models for out of sample July 2006-Dec 2006 respectively in table 86 for 1-day ES. For 10-days ES full valuation came third after MC-HS-Delta and MC-MJD-Delta. For 10-days sample full valuation performs better than of MC-HS-Gamma and MC-MJD-Gamma.

Table 87 shows that for in sample Jan2007-June 2007 full valuation is the second-best model for 1-day ES, and the third best model for 10-days ES. While the MC-Bat-Delta has the smallest MAE value for both 1-day and 10-day ES. For the out of sample July 2007-Dec 2007 in table 88 full valuation is the fourth best model for 1-day ES, and the third best model for day ES for 10-days ES.

For the out of sample July 2007- Dec2007 in table 88 MC-HS-Delta, MC-MJD-Delta, MC-HS-Gamma and full valuation are the four top models respectively for 1-day ES. For 10- days ES MC-HS-Delta, MC-MJD-Delta, full valuation and MC-BS-Delta are the four top models.

Table 89 represents that MC-BS-Delta, MC-Bat-Delta, MC-Bat-Delta and MC-HS-Delta have smallest MAE respectively for 1-day ES. For 10-days ES Full Valuation, MC-BS-Delta, MC-Bat-Delta and MC-HS-Delta are top four models respectively based on minimum value of MAE.

If we observe at tables 90, table 91, table 92, table 93, table 94, table 95, table 96, table 97, table 98, table 99, table 100, table 101 and table 102 we can summarize our results as follows:

1. MC-HS-Delta, MC-MJD-Delta, MC-HS-Gamma, full valuation and MC-MJD-Gamma are the top models for 1-day ES.
2. Although MC-BS-Delta in many cases remain in top four models based on the minimum value of MAE, but only in few cases is the first best model for 1-day ES. While, for 10-day ES MC-BS-Delta is the best model for many data sets.
3. MC-BS-Gamma models never appear in the four top models.
4. We observed that as compare to 1-day ES 10-day MAE values are significantly different across models and significance levels. Moreover, ranking the of the models across horizon not the same.
5. It is not necessary that top four model for 1-day ES and 10-day ES are same models.
6. In many cases, full valuation is in the four top models. In those cases, where full valuation is not in best four models, the MAE value of full valuation is not substantially different from other models.
7. In the view of Christoffersen (2012) for risk management for longer horizons the full valuation approach may be the only reliable choice.
8. In almost for all the data sets MC-Kou-Delta, MC-Kou-Gamma, MC-VG-Delta and MC-VG-Gamma have the highest MAE for both 1-day and 10- days ES.

6.Concluding Remarks

In this chapter, we calibrate different option pricing models and compare them with traditional Black –Scholes model. The results indicate that non-normal option pricing models are more suitable than the Black Scholes Model. However, we cannot draw any conclusions out of stochastics volatility model, jump diffusion models and infinite actively models, which model performs better. Bates models as combines stochastic and jump diffusion model is the only category of option pricing model that performs better for all samples of options. From jump diffusion category MJD model is one of the best model but Kou has the highest MAE for all data samples. Both Heston model as stochastic volatility model and Variance Gamma model as infinity model, have very similar MAE value.

In the second part of the chapter, we evaluated various ES models based on partial Monte Carlo and full Monte Carlo method. For partial Monte Carlo, we have calculated Delta based and Delta Gamma based. The preceding deltas and gammas were derived from the Black Scholes model(BSM), Heston model, Merton Jump diffusion model and Bates model. We evaluate 1-

day and 10-days Expected Shortfall (ES) for options based on minimum mean absolute error(MAE).

Results for ES evaluation indicates that Delta based Monte Carlo models are the dominate models in the top models. In most of the cases MC-HS-Delta, MC-MJD-Delta MC-HS-Gamma, full valuation and MC-MJD-Gamma are the best performing models. Kou models have highest MAE for both option pricing and ES evaluation. It is also evident from the results that full valuation is one of the top models for 1-day ES and 10-days ES for many datasets. This has clear implication for longer horizon risk analysis, as per Christoffersen (2012) for risk management for longer horizons the full valuation approach may be the only reliable choice.

Table 62: In Sample Model Calibration for Option Traded over the Period Jan 2005-June 2005.

Model	MAE	Parameters							
BS	0.0138	(σ) 0.00300							
VG	0.0736	(σ) 0.0543	(θ) 0.2004	(v) 0.0306					
HS	**0.0136**	(v_0) 0.0694	(v_T) 0.01008	(ρ) 0.9280	(k) 0.1151	(σ) 0.2491			
BAT	**0.0136**	(v_0) 0.02	(v_T) 0.01	(ρ) -0.0009	(k) 1.0000	(σ) 0.0021	(λ) 0.0200	(u_J) -0.000	(v_J) 0.0030
MJD	**0.0130**	(v) 0.202	(λ) 0.9011	(u_J) -0.2000	(v_J) 0.6031				
Kou	0.0841	(v) 0.1311	(λ) 0.1304	(p) 0.0590	(η_1) 0.5413	(η_2) 0.8991			

Note: Calibration with Options traded over the period January 2005-June 2005. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 63: Out of Sample Model Calibration for Option Traded over the Period July 2005-December 2005.

Model	MAE	Parameters							
BS	0.0185	(σ) 0.00843							
VG	0.0147	(σ) 0.0168	(θ) 0.0607	(v) 0.2707					
HS	**0.0141**	(v_0) 0.0832	(v_T) 0.1238	(ρ) 0.5870	(k) 0.3149	(σ) 0.7145			
BAT	**0.0135**	(v_0) 0.0115	(v_T) 0.6544	(ρ) -0.0124	(k) 0.1516	(σ) 0.0147	(λ) 0.3400	(u_J) -0.0322	(v_J) 0.0646
MJD	**0.0139**	(v) 0.0456	(λ) 0.1936	(u_J) -0.0230	(v_J) 0.1470				
Kou	0.0997	(v) 0.1451	(λ) 0.1240	(p) 0.0101	(η_1) 0.4988	(η_2) 0.8000			

Note: Calibration with Options traded over the period July 2005-December 2005. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 64: In Sample Model Calibration for Option Traded over the Period January 2006-June2006.

Model	MAE	Parameters							
BS	**0.0112**	(σ) 0.0020							
VG	0.0132	(σ) 0.0101	(θ) 0.7102	(v) 0.0430					
HS	0.0133	(v_0) 0.900	(v_T) 0.899	(ρ) 0.9000	(k) 0.0100	(σ) 0.4065			
BAT	**0.0113**	(v_0) 0.2001	(v_T) 0.1000	(ρ) -0.0099	(k) 0.0999	(σ) 0.0200	(λ) 0.1000	(u_J) -0.0003	(v_J) 0.0010
MJD	**0.0125**	(v) 0.4020	(λ) 0.7010	(u_J) -0.4000	(v_J) 0.5030				
Kou	0.1148	(v) 0.0240	(λ) 0.0100	(p) 0.0100	(η_1) 0.603	(η_2) 1.000			

Note: Calibration with Options traded over the period January 2006-June 2006. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 65: Out of Sample Model Calibration for Option Traded over the Period July 2006-December 2006.

Model	MAE	Parameters							
BS	**0.0129**	(σ) 0.0019							
VG	0.0132	(σ) 0.0025	(θ) 0.0100	(v) 0.0324					
HS	**0.0125**	(v_0) 0.07450	(v_T) 0.3547	(ρ) 0.4130	(k) 0.7790	(σ) 0.4146			
BAT	0.0161	(v_0) 0.9230	(v_T) 0.7580	(ρ) 0.0001	(k) 0.1307	(σ) 0.02450	(λ) 0.4500	(u_J) -0.0009	(v_J) 0.6230
MJD	**0.0131**	(v) 0.0013	(λ) 0.0103	(u_J) -0.123	(v_J) 0.247				
Kou	0.1001	(v) 0.1451	(λ) 0.1240	(p) 0.0101	(η_1) 0.4414	(η_2) 0.9000			

Note: Calibration with Options traded over the period July 2006-December 2006. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 66: In Sample Model Calibration for Option Traded over the Period January 2007-June 2007.

Model	MAE	Parameters							
BS	0.0174	(σ) 0.00460							
VG	0.0175	(σ) 0.0108	(θ) 0.1124	(v) 0.0792					
HS	0.0172	(v_0) 0.0324	(v_T) 0.0221	(ρ) 1.0132	(k) 0.0549	(σ) 0.2032			
BAT	**0.0138**	(v_0) 0.0923	(v_T) 0.0758	(ρ) -0.0001	(k) 0.0523	(σ) 0.0101	(λ) 0.450	(u_J) -0.0009	(v_J) 0.0623
MJD	0.0172	(v) 0.3210	(λ) 0.8790	(u_J) -0.0651	(v_J) 0.7123				
Kou	0.1180	(v) 0.0114	(λ) 0.0100	(p) 0.5610	(η_1) 0.9530	(η_2) 0.999			

Note: Calibration with Options traded over the period January 2007-June 2007. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 67: Out of Sample Model Calibration for Option Traded over the Period July 2007-December 2007.

Model	MAE	Parameters							
BS	**0.0136**	(σ) 0.0065							
VG	0.0172	(σ) 0.0045	(θ) 0.0126	(v) 0.0432					
HS	0.0161	(v_0) 0.6528	(v_T) 0.7973	(ρ) 0.7332	(k) 0.7790	(σ) 0.4646			
BAT	0.0224	(v_0) 0.5734	(v_T) 0.6408	(ρ) -0.0124	(k) 0.1559	(σ) 0.0214	(λ) 0.4055	(u_J) -0.0214	(v_J) 0.4053
MJD	0.0172	(v) 0.0072	(λ) 0.0420	(u_J) 0.0810	(v_J) 0.7100				
Kou	0.1358	(v) 0.0748	(λ) 0.1441	(p) 0.7087	(η_1) 0.4145	(η_2) 0.2137			

Note: Calibration with Options traded over the period July 2007-December 2007. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 68: In sample Model Calibration for Option Traded over the Period January 2008-June 2008

Model	MAE	Parameters							
BS	0.0251	(σ) 0.00534							
VG	0.0251	(σ) 0.2191	(θ) 0.4418	(v) 0.4698					
HS	0.0255	(v_0) 0.0524	(v_T) 0.0611	(ρ) 0.6848	(k) 0.2890	(σ) 0.5032			
BAT	**0.0211**	(v_0) 0.4495	(v_T) 0.3698	(ρ) -0.0121	(k) 0.1559	(σ) 0.0101	(λ) 0.4499	(u_J) -0.0009	(v_J) 0.1627
MJD	0.0274	(v) 0.983	(λ) 0.0263	(u_J) -0.0005	(v_J) 0.0126				
Kou	0.1347	(v) 0.0608	(λ) 0.0212	(p) 0.0061	(η_1) 0.5208	(η_2) 0.800			

Note: Calibration with Options traded over the period January 2008-June 2008. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 69: Out of Sample Model Calibration for Option Traded over the Period July 2008-December 2008.

Model	MAE	Parameters							
BS	0.0183	(σ) 0.0032							
VG	0.0185	(σ) 0.0025	(θ) 0.0100	(v) 0.0324					
HS	0.0178	(v_0) 0.0480	(v_T) 0.7756	(ρ) 0.7330	(k) 0.7786	(σ) 0.4646			
BAT	**0.0153**	(v_0) 0.9230	(v_T) 0.7580	(ρ) 0.0001	(k) 0.0523	(σ) 0.0214	(λ) 0.4500	(u_J) -0.0009	(v_J) 0.6230
MJD	0.0185	(v) 0.0014	(λ) 0.0001	(u_J) -0.0737	(v_J) 0.6447				
Kou	0.1034	(v) 0.01468	(λ) 0.1440	(p) 0.0101	(η_1) 0.4168	(η_2) 0.9000			

Note: Calibration with Options traded over the period July 2008-December 2008. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 70: In Sample Model Calibration for Option Traded over the Period January 2009-June 2009.

Model	MAE	Parameters							
BS	0.0189	(σ) 0.0004							
VG	0.0188	(σ) 0.0254	(θ) 0.202	(v) 0.0742					
HS	0.0187	(v_0) 0.0361	(v_T) 0.0201	(ρ) 0.9000	(k) 0.1199	(σ) 0.1032			
BAT	0.0148	(v_0) 0.2000	(v_T) 0.1507	(ρ) -0.0011	(k) 0.0500	(σ) 0.0200	(λ) 0.1010	(u_J) 0.1000	(v_J) 0.1000
MJD	**0.0107**	(v) 0.6009	(λ) 0.1635	(u_J) -0.0053	(v_J) 0.6141				
Kou	0.0345	(v) 0.0209	(λ) 0.6859	(p) 0.1813	(η_1) 0.7038	(η_2) 0.2077			

Note: Calibration with Options traded over the period January 2009-July 2009. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 71: Out of Sample Model Calibration for Option Traded over the Period July 2009-December 2009.

Model	MAE	Parameters							
BS	0.0143	(σ) 0.0005							
VG	0.0142	(σ) 0.0478	(θ) 0.0085	(v) 0.1198					
HS	0.0141	(v_0) 0.2733	(v_T) 0.8265	(ρ) 0.1485	(k) 0.6011	(σ) 0.4521			
BAT	0.0086	(v_0) 0.2479	(v_T) 0.1684	(ρ) 0.4982	(k) 0.0740	(σ) 0.0040	(λ) 0.1462	(u_J) -0.0029	(v_J) 0.3294
MJD	**0.0054**	(v) 0.3111	(λ) 0.0501	(u_J) -0.0567	(v_J) 0.2941				
Kou	0.0469	(v) 0.0189	(λ) 0.0810	(p) 0.5041	(η_1) 0.0476	(η_2) 0.1400			

Note: Calibration with Options traded over the period July 2009-December 2009. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 72: In Sample Model Calibration for Option Traded over the Period January 2010-June2010.

Model	MAE	Parameters							
BS	0.0130	(σ) 0.00128							
VG	0.0120	(σ) 0.0560	(θ) 0.0911	(v) 0.0052					
HS	0.0111	(v_0) 0.04791	(v_T) 0.0453	(ρ) 0.8130	(k) 0.2500	(σ) 0.521			
BAT	**0.0094**	(v_0) 0.3632	(v_T) 0.5025	(ρ) -0.0095	(k) 0.9990	(σ) 0.002	(λ) 0.2024	(u_J) 0.0005	(v_J) 0.0629
MJD	**0.0068**	(v) 0.3667	(λ) 0.0497	(u_J) -0.0335	(v_J) 0.5926				
Kou	0.0501	(v) 0.0120	(λ) 0.0403	(p) 0.0601	(η_1) 0.7034	(η_2) 0.700			

Note: Calibration with Options traded over the period January 2010-June 2010. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 73: Out of Sample Model Calibration for Option Traded over the Period July 2010-December 2010.

Model	MAE	Parameters							
BS	0.0145	(σ) 0.0011							
VG	0.0145	(σ) 0.0597	(θ) 0.0266	(v) 0.0980					
HS	0.0141	(v_0) 0.3309	(v_T) 0.0567	(ρ) 0.8990	(k) 0.2998	(σ) 0.6100			
BAT	**0.00845**	(v_0) 0.3155	(v_T) 0.1553	(ρ) 0.4983	(k) 0.2227	(σ) 0.0040	(λ) 0.3902	(u_J) -0.0029	(v_J) 0.0210
MJD	**0.00842**	(v) 0.3079	(λ) 0.0172	(u_J) -0.0049	(v_J) 0.4704				
Kou	0.03805	(v) 0.0086	(λ) 0.0009	(p) 0.0710	(η_1) 0.0878	(η_2) 0.9498			

Note: Calibration with Options traded over the period July 2010-December 2010. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 74: In Sample Model Calibration for Option Traded over the Period January 2011-June 2011.

Model	MAE	Parameters							
BS	**0.0011**	(σ)							
		0.0003							
VG	0.0101	(σ)	(θ)	(v)					
		0.0394	0.0105	0.0183					
HS	0.0111	(v_0)	(v_T)	(ρ)	(k)	(σ)			
		0.3986	0.3659	0.6405	0.6242	0.4965			
BAT	**0.0066**	(v_0)	(v_T)	(ρ)	(k)	(σ)	(λ)	(u_J)	(v_J)
		0.3990	0.1640	-0.0099	0.0999	0.0020	0.1962	-0.0030	0.0100
MJD	**0.0073**	(v)	(λ)	(u_J)	(v_J)				
		0.3753	0.0719	-0.0516	0.3009				
Kou	0.0410	(v)	(λ)	(p)	(η_1)	(η_2)			
		0.0255	0.0403	0.0040	0.4499	0.6990			

Note: Calibration with Options traded over the period January 2011-June 2011. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 75: Out of Sample Model Calibration for Option Traded over the Period July 2011-December 2011.

Model	MAE	Parameters							
BS	0.0102	(σ) 0.0006							
VG	**0.0079**	(σ) 0.0751	(θ) 0.2348	(v) 0.4942					
HS	**0.0098**	(v_0) 0.2607	(v_T) 0.8032	(ρ) 0.4328	(k) 0.7789	(σ) 0.1646			
BAT	**0.0055**	(v_0) 0.2239	(v_T) 0.6287	(ρ) -0.0014	(k) 0.1553	(σ) 0.0710	(λ) 0.2308	(u_J) -0.0251	(v_J) 0.2625
MJD	**0.0070**	(v) 0.2962	(λ) 0.0431	(u_J) -0.0520	(v_J) 0.4952				
Kou	0.0490	(v) 0.0663	(λ) 0.0212	(p) 0.1878	(η_1) 0.5397	(η_2) 0.0058			

Note: Calibration with Options traded over the period January 201-June 2011. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 76: In Sample Model Calibration for Option Traded over the Period January 2012-June 2012.

Model	MAE	Parameters							
BS	0.0059	(σ) 0.0004							
VG	0.0048	(σ) 0.0734	(θ) 0.0097	(v) 0.0688					
HS	0.0060	(v_0) 0.0126	(v_T) 0.0325	(ρ) 0.7461	(k) 0.4701	(σ) 0.6142			
BAT	**0.0036**	(v_0) 0.2341	(v_T) 0.1711	(ρ) 0.2721	(k) 0.0807	(σ) 0.432	(λ) 0.0621	(u_J) 0.2000	(v_J) 0.2433
MJD	**0.0036**	(v) 0.2803	(λ) 0.0386	(u_J) -0.0603	(v_J) 0.2899				
Kou	0.0160	(v) 0.0799	(λ) 0.173	(p) 0.1780	(η_1) 0.3580	(η_2) 0.4500			

Note: Calibration with Options traded over the period January 2012-June 2012. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 77: Out Sample Model Calibration for Option Traded over the Period July 2012-December 2012.

Model	MAE	Parameters							
BS	0.0080	(σ) 0.0003							
VG	0.0080	(σ) 0.03318	(θ) 0.0805	(v) 0.0171					
HS	**0.0077**	(v_0) 0.0328	(v_T) 0.6552	(ρ) 0.3272	(k) 0.0971	(σ) 0.0714			
BAT	**0.0059**	(v_0) 0.0115	(v_T) 0.6229	(ρ) -0.0007	(k) 0.1519	(σ) 0.0701	(λ) 0.1222	(u_J) -0.0323	(v_J) 0.2686
MJD	**0.0048**	(v) 0.1301	(λ) 0.0579	(u_J) -0.1023	(v_J) 0.4510				
Kou	0.0465	(v) 0.0497	(λ) 0.3540	(p) 0.0861	(η_1) 0.4128	(η_2) 0.8141			

Note: Calibration with Options traded over the period July 2012-December 2012. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 78: In Sample Model Calibration for Option Traded over the Period January 2013-June 2013.

Model	MAE	Parameters							
BS	0.0066	(σ) 0.0011							
VG	0.0067	(σ) 0.0722	(θ) 0.0128	(v) 0.0716					
HS	0.0065	(v_0) 0.0689	(v_T) 0.0444	(ρ) 0.5290	(k) 0.5230	(σ) 0.4742			
BAT	**0.0042**	(v_0) 0.1900	(v_T) 0.1598	(ρ) -0.0099	(k) 0.0990	(σ) 0.0040	(λ) 0.3941	(u_J) -0.0029	(v_J) 0.0100
MJD	**0.0040**	(v) 0.1716	(λ) 0.0302	(u_J) -0.0660	(v_J) 0.2824				
Kou	0.0174	(v) 0.0731	(λ) 0.0141	(p) 0.0218	(η_1) 0.0544	(η_2) 0.0950			

Note: Calibration with Options traded over the period January 2013-June 2013. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 79: Out of Sample Model Calibration for Option Traded over the period July 2013-December 2013.

Model	MAE	Parameters							
BS	0.00621	(σ) 0.0001							
VG	0.00622	(σ) 0.0482	(θ) 0.0937	(v) 0.0225					
HS	0.00589	(v_0) 0.1636	(v_T) 0.4238	(ρ) 0.3552	(k) 0.0511	(σ) 0.0619			
BAT	**0.00470**	(v_0) 0.0115	(v_T) 0.6186	(ρ) -0.0012	(k) 0.1518	(σ) 0.0682	(λ) 0.0588	(u_J) -0.0321	(v_J) 0.3084
MJD	**0.00369**	(v) 0.0891	(λ) 0.0416	(u_J) -0.1078	(v_J) 0.4441				
Kou	0.00183	(v) 0.0941	(λ) 0.3552	(p) 0.0918	(η_1) 0.9165	(η_2) 0.9991			

Note: Calibration with Options traded over the period July 2013-December 2013. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 80: In Sample Model Calibration for Option Traded over the Period January 2014-June 2014.

Model	MAE	Parameters							
BS	0.0054	(σ) 0.0039							
VG	0.0055	(σ) 0.0967	(θ) 0.0010	(v) 0.0657					
HS	0.0054	(v_0) 0.1219	(v_T) 0.0295	(ρ) 0.4280	(k) 0.3139	(σ) 0.5470			
BAT	**0.0035**	(v_0) 0.0350	(v_T) 0.0861	(ρ) 0.1028	(k) 0.0531	(σ) 0.5099	(λ) 0.0199	(u_J) 0.1821	(v_J) 0.4020
MJD	**0.0033**	(v) 0.0072	(λ) 0.0486	(u_J) -0.0391	(v_J) 0.2996				
Kou	0.0132	(v) 0.0272	(λ) 0.0008	(p) 0.0010	(η_1) 0.0373	(η_2) 0.9490			

Note: Calibration with Options traded over the period January 2014-June 2014. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives. BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 81: Out of Sample Model Calibration for Option Traded over the Period July 2014-December 2014.

Model	MAE	Parameters							
BS	0.00304	(σ)							
		0.0003							
VG	0.00305	(σ)	(θ)	(v)					
		0.0443	0.0320	0.0092					
HS	**0.00292**	(v_0)	(v_T)	(ρ)	(k)	(σ)			
		0.0594	0.6056	0.3270	0.0980	0.0714			
BAT	**0.00238**	(v_0)	(v_T)	(ρ)	(k)	(σ)	(λ)	(u_J)	(v_J)
		0.9230	0.6563	0.0005	0.1369	0.0624	0.4500	-0.0009	0.6231
MJD	0.00300	(v)	(λ)	(u_J)	(v_J)				
		0.6048	0.9891	-0.9896	0.9945				
Kou	0.00911	(v)	(λ)	(p)	(η_1)	(η_2)			
		0.0351	0.3540	0.0861	0.4127	0.0800			

Note: Calibration with Options traded over the period July 2014-December 2014. We consider Options traded on every Wednesday. We applied FFT approach to price options which is very helpful to efficiently price derivatives.BS stands for Black-Scholes model, VG stands for Variance Gamma infinite activity model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou as Double Exponential Jump diffusion model, and MAE stands for Mean absolute error. The best fitted model selected based on minimum error(MAE) and represent in bold.

Table 82: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2005-June 2005.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	39.38	35.53	32.52	29.22	140.05	121.4	105.7	87.29
MC-BS-Gamma	64.17	53.53	45.24	36.96	461.85	374.4	307.0	235.7
MC-HS-Delta	**27.08**	**25.72**	**24.77**	**23.87**	**73.850**	**63.47**	**56.37**	**47.39**
MC-VG-Delta	80.10	69.52	60.89	50.95	298.87	261.1	229.01	190.19
MC-VG-Gamma	110.4	92.72	78.00	61.74	625.8	518.9	434.9	341.8
MC-HS-Gamma	31.77	28.91	26.80	24.85	165.50	134.6	110.7	85.23
MC-Bat-Delta	27.81	**26.27**	**25.17**	**24.12**	**78.610**	**68.25**	**59.81**	**50.09**
MC-Bat-Gamma	31.50	28.80	26.78	24.89	149.21	122.5	101.6	79.26
MC-MJD-Delta	**24.86**	**24.12**	**23.64**	**23.24**	**57.197**	**50.13**	**44.50**	**38.29**
MC-MJD-Gamma	**26.78**	**25.32**	**24.34**	**23.52**	**106.13**	**87.10**	**72.42**	**57.09**
MC-Kou-Delta	50.238	44.400	39.772	34.701	187.28	162.59	141.86	117.18
MC-Kou-Gamma	79.347	66.003	55.415	44.256	531.77	434.12	357.80	276.32
Full Valuation	**27.24**	27.13	27.08	27.05	164.86	142.9	116.5	87.84

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 83: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2005-Dec 2005.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	32.92	30.62	28.80	26.86	108.2	94.29	82.07	67.012
MC-BS-Gamma	49.03	42.19	37.01	31.68	360.9	294.8	241.1	181.1
MC-VG-Delta	27.663	26.564	25.801	25.136	73.410	63.655	55.531	46.420
MC-VG-Gamma	30.534	28.482	26.991	25.665	138.86	113.73	93.726	72.054
MC-HS-Delta	**25.54**	**24.65**	**24.01**	**23.42**	**62.66**	**54.79**	**48.24**	**40.86**
MC-HS-Gamma-	28.89	26.88	**25.44**	**24.11**	137.0	111.5	91.42	81.35
MC-Bates-Delta	**27.66**	**26.56**	25.80	**25.13**	**73.41**	**63.65**	**55.53**	**46.42**
MC-Bates-Gamma	30.53	28.48	26.99	25.66	138.8	113.7	93.72	72.05
MC-MJD-Delta	**25.65**	**25.20**	**24.96**	**24.85**	**54.45**	**47.90**	**42.62**	**36.75**
MC-MJD-Gamma	**26.91**	**25.93**	**25.32**	**24.91**	97.51	80.01	66.33	51.93
MC-Kou-Delta	48.274	43.122	38.870	33.951	183.13	158.80	137.81	112.38
MC-Kou-Gamma	64.644	55.160	47.701	39.605	388.78	320.86	266.01	204.78
Full Valuation	30.22	29.96	29.88	29.85	**79.41**	**67.23**	**56.44**	**45.49**

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 84: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2006-June 2006.

Models	**1 Day MAE for ES**				**10 Days MAE for ES**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	31.07	29.38	28.12	26.87	**91.31**	**79.68**	**69.69**	**57.89**
MC-BS-Gamma	43.05	37.76	33.85	30.08	300.9	244.5	199.4	150.1
MC-VG-Delta	30.129	28.668	27.589	26.564	84.564	73.343	64.050	53.437
MC-VG-Gamma	36.995	33.376	30.749	28.248	217.00	175.41	142.91	108.00
MC-HS-Delta	**30.58**	**29.02**	**27.84**	**26.71**	**87.69**	**76.02**	**66.31**	**55.20**
MC-HS-Gamma-	37.89	34.05	31.25	28.56	224.8	181.8	148.2	112.0
MC-Bates-Delta	**30.12**	**28.66**	**27.58**	**26.56**	**84.56**	**73.34**	**64.05**	**53.43**
MC-Bates-Gamma	36.99	33.37	30.74	28.24	217.0	175.4	142.9	108.0
MC-MJD-Delta	**29.92**	**28.51**	**27.47**	**26.49**	**83.13**	**72.10**	**63.00**	**52.61**
MC-MJD-Gamma	35.35	32.22	29.94	27.79	191.5	155.5	127.1	96.85
MC-Kou-Delta	39.048	35.688	32.998	30.108	135.53	117.14	101.64	83.496
MC-Kou-Gamma	61.126	51.763	44.619	37.296	438.46	355.05	290.14	220.34
Full Valuation	**30.66**	**29.17**	**28.03**	**26.10**	273.1	236.5	205.83	170.5

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 85: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2006-Dec 2006.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	35.53	33.06	30.95	28.72	117.7	103.6	90.53	74.62
MC-BS Gamma	53.99	46.71	40.72	34.62	392.2	325.3	267.3	202.8
MC-VG-Delta	47.496	42.949	38.909	34.363	175.75	155.14	135.50	111.18
MC-VG-Gamma	66.121	57.123	49.392	41.082	415.26	349.16	290.71	224.19
MC-HS-Delta	**26.23**	**25.97**	**24.81**	**23.77**	**49.47**	**44.60**	**40.26**	**35.36**
MC-HS-Gamma-	**28.07**	**27.04**	31.25	**28.56**	**118.3**	**97.85**	**80.51**	**61.85**
MC-Bates-Delta	42.17	38.53	35.31	31.76	151.5	133.5	116.4	95.68
MC-Bates-Gamma	57.49	50.02	43.70	37.04	361.9	303.7	252.2	193.7
MC-MJD-Delta	**26.61**	**26.21**	**25.94**	**25.78**	**54.87**	**49.16**	**44.01**	**38.16**
MC-MJD-Gamma	**29.52**	**28.04**	**26.97**	**26.14**	143.5	118.2	96.73	73.51
MC-Kou-Delta	49.698	44.790	40.409	35.483	185.32	163.64	143.02	117.40
MC-Kou-Gamma	91.252	76.770	64.347	51.057	681.88	567.34	468.96	360.59
Full Valuation	30.26	30.17	**30.13**	30.10	**107.1**	**89.69**	**77.03**	**61.64**

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 86: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2007-June 2007.

Models	**1 Day MAE for ES**				**10 Days MAE for ES**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	**36.05**	**34.03**	**32.46**	**30.92**	**105.6**	**91.05**	**78.27**	**63.53**
MC-BS-Gamma	52.79	45.67	40.46	35.35	367.8	293.9	234.1	170.1
MC-VG-Delta	51.884	46.828	42.515	37.718	184.92	159.19	136.22	108.80
MC-VG-Gamma	99.782	82.184	68.790	54.572	744.84	602.08	485.23	361.46
MC-HS-Delta	54.86	49.32	44.57	39.17	197.9	170.5	145.9	116.4
MC-HS-Gamma-	78.50	66.60	57.47	47.43	483.2	393.4	318.5	236.1
MC-Bat-Delta	**34.08**	**32.53**	**31.38**	**30.30**	**92.87**	**80.27**	**69.29**	**56.79**
MC-Bat-Gamma	**41.93**	**37.80**	**34.86**	**32.05**	236.8	189.8	151.4	110.9
MC-MJD-Delta	44.86	41.02	37.90	34.45	**152.5**	**131.3**	**112.4**	**89.94**
MC-MJD-Gamma	61.76	53.21	46.69	39.77	380.2	308.2	248.2	182.5
MC-Kou-Delta	53.609	48.279	43.699	38.549	192.55	165.79	141.86	113.27
MC-Kou-Gamma	76.432	64.955	56.123	46.454	471.06	383.33	310.17	229.792
Full Valuation	**34.99**	**34.86**	**34.81**	**30.78**	**118.2**	**97.96**	**80.42**	**62.54**

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 87: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2007-Dec 2007.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	34.11	32.05	30.40	28.87	**108.1**	**94.44**	**81.77**	**67.29**
MC-BS-Gamma	52.23	44.75	38.89	33.54	391.2	319.0	257.9	194.3
MC-VG-Delta	54.436	48.357	43.055	37.457	204.41	179.00	155.25	127.31
MC-VG-Gamma	109.11	90.101	73.998	57.529	832.73	684.65	560.05	430.01
MC-HS-Delta	**27.88**	**25.54**	**24.36**	**22.33**	**54.68**	**48.56**	**43.40**	**37.57**
MC-HS-Gamma	**31.10**	**29.47**	**28.38**	**27.57**	166.6	135.1	108.4	81.28
MC-Bat-Delta	69.72	61.25	53.62	45.24	263.8	231.5	201.0	165.2
MC-Bat-Gamma	113.2	94.85	78.94	62.22	748.1	619.1	507.4	389.7
MC-MJD-Delta	**30.36**	**29.25**	**28.43**	**27.70**	**81.41**	**71.29**	**62.05**	**51.69**
MC-MJD-Gamma	38.74	34.79	31.89	29.43	255.1	207.5	167.0	125.4
MC-Kou-Delta	51.538	45.963	41.128	36.078	192.47	168.49	146.09	119.73
MC-Kou-Gamma	70.014	59.827	51.149	42.204	422.34	351.89	290.16	223.35
Full Valuation	**32.53**	**31.38**	**30.32**	**27.28**	**98.15**	**81.96**	**67.14**	**52.94**

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 88: Back-testing Shortfall for Short S & P 500 Call over the Period Jan 2008-June 2008.

Models	**1 Day MAE for ES**				**10 Days MAE for ES**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	**36.076**	**35.091**	**34.480**	**34.046**	**88.676**	**76.955**	**67.192**	**55.657**
MC-BS-Gamma	45.122	**40.742**	**37.803**	**35.305**	313.73	250.43	199.56	143.99
MC-VG-Delta	49.590	45.056	41.651	38.008	180.79	155.46	133.63	106.58
MC-VG-Gamma	108.01	87.201	71.373	55.065	934.01	760.63	622.79	469.71
MC-HS-Delta	**44.357**	42.110	38.646	36.166	**151.19**	**129.93**	**111.83**	**89.618**
MC-HS-Gamma	65.028	55.304	48.197	41.232	477.52	386.11	312.47	229.49
MC-Bat-Delta	**39.182**	**37.254**	**35.879**	**34.647**	**116.06**	**100.05**	**86.570**	**70.123**
MC-Bat-Gamma	52.645	46.077	41.550	37.321	376.73	303.08	243.76	177.05
MC-MJD-Delta	46.360	**42.616**	39.777	36.848	163.04	140.06	120.52	96.330
MC-MJD-Gamma	69.424	58.664	50.729	42.754	510.40	413.19	334.89	246.64
MC-Kou-Delta	51.171	46.284	42.570	38.589	189.03	162.62	139.78	111.38
MC-Kou-Gamma	57.159	50.535	45.433	40.192	282.90	236.30	197.24	150.57
Full Valuation	**41.984**	**38.284**	**36.214**	**33.184**	**77.549**	62.611	50.965	41.973

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 89: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2008-Dec 2008.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**29.398**	**28.878**	**28.701**	**28.109**	**70.617**	**59.234**	**50.205**	**41.801**
MC-BS-Gamma	35.762	32.120	30.154	29.939	269.49	206.95	161.07	115.00
MC-VG-Gamma	52.607	45.082	39.743	34.675	217.45	183.49	154.36	123.38
MC-VG-Gamma	105.41	81.840	65.075	49.071	819.97	643.39	514.28	383.63
MC-HS-Delta	**31.799**	**30.187**	**28.735**	**26.253**	**60.249**	**49.321**	**38.847**	**28.701**
MC-HS-Gamma	33.657	31.881	29.376	28.928	**76.729**	**57.915**	**45.854**	**36.292**
MC-Bat-Delta	**33.640**	**31.436**	**30.077**	**29.054**	115.51	96.157	80.124	63.935
MC-Bat-Gamma	50.579	41.636	36.128	31.733	412.47	320.72	253.30	184.44
MC-MJD-Delta	**32.413**	**30.635**	**29.587**	**28.857**	105.31	87.618	73.112	58.565
MC-MJD-Gamma	35.453	32.374	30.536	29.194	180.83	143.84	115.67	86.127
MC-Kou-Delta	60.340	51.018	44.211	37.569	248.86	210.68	177.81	142.66
MC-Kou-Gamma	73.462	60.178	50.442	41.088	405.24	329.46	270.89	207.54
Full Valuation	36.983	35.693	34.583	33.533	**86.357**	**62.066**	**46.707**	**35.552**

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 90: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2009-June 2009.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**24.082**	**23.230**	**22.694**	**21.742**	78.030	64.633	53.269	40.804
MC-BS-Gamma	28.545	25.238	23.602	23.097	303.24	243.25	196.56	145.85
MC-VG-Delta	34.923	30.206	27.028	24.346	185.62	160.09	137.95	110.73
MC-VG-Gamma	62.857	48.913	38.803	30.129	509.70	417.17	343.59	262.74
MC-HS-Delta	35.730	29.821	25.106	18.309	**67.101**	**55.679**	**44.766**	**33.967**
MC-HS-Gamma	34.121	32.774	31.653	30.421	**73.164**	**57.306**	**44.367**	**33.0879**
MC-Bat-Delta	**25.760**	**24.489**	**23.765**	**23.258**	**63.581**	**52.131**	**43.106**	**33.975**
MC-Bat-Gamma	29.973	28.947	28.166	26.158	194.96	156.00	124.91	90.460
MC-MJD-Delta	**28.341**	**24.882**	**23.874**	**21.917**	**49.382**	**43.388**	**39.224**	**35.501**
MC-MJD-Gamma	**25.616**	**24.139**	**23.368**	**23.079**	163.98	128.90	101.15	70.903
MC-Kou-Delta	30.438	27.119	25.005	23.438	161.49	138.68	118.90	94.570
MC-Kou-Gamma	57.164	44.277	35.394	28.104	503.95	410.34	336.54	256.15
Full Valuation	47.919	45.139	42.569	38.005	145.130	114.04	88.610	63.283

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 91: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2009-Dec 2009.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	27.189	26.122	25.954	24.904	**64.793**	**53.316**	**44.521**	**43.089**
MC-BS-Gamma	29.188	27.179	26.180	26.077	234.33	185.15	145.45	138.70
MC-VG-Delta	38.179	33.752	30.556	30.056	180.30	153.50	129.48	125.12
MC-VG-Gamma	59.916	47.702	39.225	37.915	465.06	377.04	304.78	292.28
MC-HS-Delta	27.077	26.287	25.932	25.905	**92.799**	**76.738**	**62.578**	**60.127**
MC-HS-Gamma	32.578	29.298	27.292	27.024	263.41	210.679	167.38	159.89
MC-Bates-Delta	**26.845**	**25.737**	**24.275**	**23.975**	**69.333**	**51.740**	**46.092**	**45.183**
MC-Bates-Gamma	**26.503**	**24.986**	**23.952**	**22.939**	158.29	123.94	95.758	90.878
MC-MJD-Delta	**26.055**	**25.888**	**24.015**	**23.071**	**69.720**	**57.154**	**47.358**	**45.752**
MC-MJD-Gamma	**27.116**	**26.222**	**25.898**	**25.888**	144.64	115.49	91.068	86.768
MC-Kou-Delta	55.922	47.473	40.399	39.231	259.20	222.70	190.01	184.08
MC-Kou-Gamma	103.54	81.988	63.890	60.824	734.93	595.92	483.42	464.10
Full Valuation	38.691	38.751	38.141	38.031	288.15	183.23	89.218	74.331

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 92: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2010-June 2010.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	32.181	31.085	30.690	30.481	**65.602**	**53.418**	**45.410**	**44.071**
MC-BS-Gamma	32.524	32.524	30.504	30.479	248.97	192.52	150.71	143.50
MC-VG-Delta	39.331	35.448	33.066	32.705	183.01	153.67	129.94	125.37
MC-VG-Gamma	58.18	46.364	39.404	38.361	463.62	369.52	298.57	286.17
MC-HS-Delta	**30.479**	**27.579**	**25.907**	**23.993**	**71.187**	**57.490**	**48.309**	**46.760**
MC-HS-Gamma	31.989	30.816	29.479	27.474	212.24	164.92	129.25	123.01
MC-Bat-Delta	**31.119**	**31.025**	**30.652**	**30.475**	**67.316**	**54.647**	**46.286**	**44.881**
MC-Bat-Gamma	**31.329**	**30.588**	**30.524**	**29.493**	183.40	142.37	111.29	105.84
MC-MJD-Delta	**31.118**	**31.025**	**30.652**	**30.475**	**67.325**	**54.655**	**46.293**	**44.887**
MC-MJD-Gamma	31.658	30.691	30.474	30.490	203.38	157.71	123.28	117.26
MC-Kou-Delta	55.472	46.423	40.487	39.536	261.49	221.92	189.92	183.74
MC-Kou-Gamma	79.902	62.029	50.125	48.257	520.91	421.66	345.94	332.59
Full Valuation	42.589	42.519	42.289	42.179	134.74	105.84	82.190	78.204

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 93: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2010-Dec 2010.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**31.688**	**30.599**	**29.236**	**29.088**	**62.079**	**51.531**	**43.962**	**42.731**
MC-BS-Gamma	33.286	32.755	31.148	30.108	232.90	182.34	142.08	135.27
MC-VG-Delta	35.184	32.507	30.740	30.482	152.91	128.41	107.34	103.46
MC-VG-Gamma	59.717	47.421	39.478	38.315	538.52	430.35	344.93	330.68
MC-HS-Delta	35.572	34.457	33.847	32.213	79.780	58.351	42.116	39.882
MC-HS-Gamma-	35.550	34.274	33.467	32.618	**77.277**	**52.724**	**40.315**	**34.029**
MC-Bat-Delta	**30.307**	**30.190**	**29.165**	**28.281**	**50.453**	**43.080**	**37.918**	**37.083**
MC-Bat-Gamma	32.499	31.407	30.194	29.116	130.52	101.22	77.312	73.251
MC-MJD-Delta	**30.530**	**29.448**	**29.157**	**28.101**	**66.534**	**54.821**	**46.356**	**44.965**
MC-MJD-Gamma	**31.741**	**30.173**	**29.119**	**29.051**	143.69	113.40	88.450	84.130
MC-Kou-Delta	42.973	37.933	34.333	33.771	202.07	171.37	144.97	140.11
MC-Gamma	59.452	48.318	40.731	39.576	429.09	349.12	283.82	272.57
Full Valuation	43.509	43.409	42.969	42.739	92.660	70.726	55.170	53.003

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 94: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2011-June 2011.

Models	**1 Day MAE for ES**				**10 Days MAE for ES**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	**32.420**	**32.340**	**32.043**	**31.998**	**70.134**	**58.056**	**49.579**	**48.168**
MC-BS-Gamma	34.898	32.974	32.141	32.068	260.60	203.43	158.73	151.13
MC-VG-Delta	48.138	42.328	38.245	37.601	224.01	189.84	161.14	155.77
MC-VG-Gamma	65.576	53.295	45.034	43.761	462.06	375.15	305.31	293.23
MC-HS-Delta	**32.533**	**32.067**	**32.010**	**31.990**	**92.519**	**75.277**	**62.302**	**60.114**
MC-HS-Gamma	36.241	33.776	32.509	32.363	255.69	201.96	159.19	151.84
MC-Bates-Delta	**32.077**	**31.583**	**31.185**	**30.987**	**63.114**	**52.944**	**45.843**	**44.667**
MC-Bates-Gamma	**32.656**	**32.056**	**31.016**	**30.060**	169.319	131.89	101.96	96.809
MC-MJD-Delta	33.410	32.470	32.047	32.011	**109.92**	**90.289**	**74.036**	**71.175**
MC-MJD-Gamma	39.061	35.429	33.368	33.094	293.86	233.29	185.07	176.79
MC-Kou-Delta	46.865	41.422	37.616	37.021	217.26	183.95	155.97	150.74
MC-Kou-Gamma	70.198	56.030	46.635	45.199	529.26	426.52	344.74	330.68
Full Valuation	45.900	45.830	45.540	45.380	296.35	249.33	208.55	201.32

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 95: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2011-Dec 2011.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**33.285**	**33.135**	**32.505**	**32.224**	**81.920**	**65.886**	**54.353**	**52.477**
MC-BS-Gamma	35.703	34.157	32.276	32.233	308.97	237.94	183.98	175.10
MC-VG-Delta	41.590	37.193	34.485	34.115	200.61	168.10	140.55	135.51
MC-VG-Gamma	58.524	47.020	39.955	38.960	467.59	371.60	296.95	284.55
MC-HS-Delta	33.592	32.430	31.703	30.273	**75.581**	**61.108**	**50.912**	**49.266**
MC-HS-Gamma	**34.543**	**33.686**	**32.222**	**31.237**	281.44	216.23	166.55	158.33
MC-Bates-Delta	**33.254**	**33.106**	**32.487**	**32.223**	**82.590**	**66.409**	**54.729**	**52.831**
MC-Bates-Gamma	35.492	33.075	32.262	32.227	300.50	231.55	179.01	170.32
MC-MJD-Delta	36.027	35.811	34.741	33.746	**87.697**	**71.300**	**67.135**	**56.499**
MC-MJD-Gamma	**33.563**	**33.362**	**32.514**	**32.231**	188.73	141.75	105.96	100.04
MC-Kou-Delta	41.522	37.150	34.460	34.093	200.17	167.71	140.22	135.18
MC-Kou-Gamma	70.763	54.238	44.123	42.692	612.40	482.07	382.79	366.37
Full Valuation	52.596	52.436	51.806	51.476	239.07	195.06	158.80	152.99

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 96: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2012-June 2012.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	36.880	35.792	34.472	33.418	76.026	62.727	53.151	51.547
MC-BS-Gamma	36.456	34.448	33.563	33.486	276.01	217.12	169.73	161.68
MC-VG-Delta	36.488	35.233	34.585	33.516	127.00	105.33	86.711	83.232
MC-VG-Gamma	55.108	45.430	39.454	38.606	546.51	436.56	348.99	334.68
MC-HS-Delta	35.363	35.223	34.556	34.002	**49.908**	**43.797**	**39.606**	**38.932**
MC-HS-Gamma	34.990	34.848	34.195	33.097	**69.699**	**56.497**	**47.439**	**46.039**
MC-Bat-Delta	**34.354**	**33.238**	**32.749**	**32.458**	**64.028**	**53.939**	**46.781**	**45.591**
MC-Bat-Gamma	**34.346**	**33.939**	**33.425**	**32.461**	208.32	162.98	126.12	119.75
MC-MJD-Delta	**33.951**	**33.857**	**33.506**	**32.406**	**73.781**	**61.077**	**51.944**	**50.418**
MC-MJD-Gamma	**33.797**	**33.560**	**32.502**	**31.418**	141.72	112.15	87.448	83.123
MC-Kou-Delta	46.376	41.460	38.003	37.465	215.05	182.39	154.31	149.02
MC-Kou-Gamma	55.301	47.049	41.373	40.498	358.96	295.63	242.63	233.27
Full Valuation	48.196	48.136	47.866	47.726	92.123	73.404	59.601	57.629

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 97: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2012-Dec 2012.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**34.870**	**34.105**	**33.897**	**32.949**	**74.883**	**62.201**	**53.239**	**51.702**
MC-BS-Gamma	37.462	35.253	34.198	34.085	279.49	218.49	171.59	163.50
MC-VG-Delta	38.066	36.044	34.795	34.624	145.51	121.32	100.97	97.075
MC-VG-Gamma	48.828	42.373	38.340	37.747	389.23	311.27	249.38	238.58
MC-HS-Delta	35.388	34.424	33.971	**33.928**	112.23	92.251	75.889	72.985
MC-HS-Gamma	41.572	37.748	35.530	35.228	313.57	249.17	198.04	189.12
MC-Bat-Delta	**33.216**	**33.145**	**32.924**	**31.911**	**73.089**	**60.891**	**52.271**	**50.799**
MC-Bat-Gamma	35.898	34.502	33.944	33.905	222.85	174.70	136.48	129.81
MC-MJD-Delta	**34.617**	**33.520**	**33.121**	**32.901**	**62.049**	**52.812**	**46.389**	**45.298**
MC-MJD-Gamma	**34.176**	**34.073**	**34.012**	**32.884**	142.30	110.80	85.447	81.066
MC-VG-Delta	47.061	42.212	38.837	38.301	212.43	179.78	36.019	147.055
MC-VG-Gamma	73.160	58.555	48.947	47.458	578.39	464.91	375.09	359.48
Full Valuation	47.414	47.364	47.124	46.984	**105.73**	**82.483**	**67.016**	**64.655**

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 98: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2013-June 2013.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**38.510**	**38.404**	**37.990**	**37.818**	**78.168**	**65.340**	**56.147**	**54.592**
MC-BS-Gamma	40.909	38.821	37.947	37.876	309.23	242.35	189.24	180.30
MC-VG-Delta	43.596	40.896	39.172	38.928	178.20	149.54	124.92	120.23
MC-VG-Gamma	55.366	47.875	43.074	42.370	427.75	344.77	277.08	265.27
MC-HS-Delta	40.638	40.482	39.723	38.035	**49.233**	**44.609**	**41.545**	**41.058**
MC-HS-Gamma	**39.302**	**38.141**	**37.421**	**36.939**	128.03	96.943	73.518	69.965
MC-Bates-Delta	40.134	39.981	39.254	38.631	**53.254**	**47.399**	**43.426**	**42.780**
MC-Bates-Gamma	**38.869**	**37.725**	**36.122**	**35.825**	144.19	110.28	83.165	78.832
MC-MJD-Delta	40.548	40.393	39.638	38.959	**49.876**	**45.054**	**41.841**	**41.329**
MC-MJD-Gamma	**38.961**	**37.808**	**37.162**	**36.830**	154.92	117.99	88.307	83.423
MC-Kou-Delta	41.224	39.390	38.336	38.201	154.17	128.50	106.45	102.25
MC-Kou-Gamma	62.663	51.855	45.128	44.155	593.16	472.64	377.65	361.69
Full Valuation	53.324	53.264	53.024	52.924	133.10	112.67	93.084	89.557

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 99: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2013-Dec 2013.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
MC-BS-Delta	43.975	41.969	41.404	41.356	**143.00**	**118.12**	**96.906**	**93.069**
MC-BS-Gamma	55.794	48.869	44.731	44.162	493.03	391.66	310.70	297.02
MC-VG-Delta	45.203	43.149	41.938	41.783	170.01	141.76	117.46	112.89
MC-VG-Gamma	57.547	50.228	45.676	45.027	467.55	374.49	298.80	285.71
MC-HS-Delta	**42.148**	**42.031**	**40.549**	**40.321**	**82.740**	**69.419**	**59.921**	**58.336**
MC-HS-Gamma	44.456	42.302	41.418	41.355	334.14	261.67	204.14	194.47
MC-Bates-Delta	**41.798**	**41.704**	**41.370**	**41.354**	**93.360**	**77.223**	**65.593**	**63.641**
MC-Bates-Gamma	43.996	42.146	41.394	41.342	289.32	227.92	177.98	169.34
MC-MJD-Delta	**41.959**	**41.851**	**41.442**	**40.319**	**88.000**	**73.275**	**62.719**	**60.950**
MC-MJD-Gamma	**42.130**	**41.910**	**40.407**	**40.259**	200.26	157.93	122.93	116.80
MC-Kou-Delta	44.537	42.748	41.736	41.613	162.14	134.87	111.43	107.02
MC-Kou-Gamma	55.675	49.030	44.956	44.138	449.34	359.52	286.46	273.82
Full Valuation	57.905	57.845	57.585	57.445	310.68	269.56	231.04	224.39

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 100: Back-testing Expected Shortfall for Short S & P 500 Call over the Period Jan 2014-June 2014.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**45.770**	**45.671**	**44.435**	**43.359**	**107.45**	**87.882**	**74.029**	**71.693**
MC-BS-Gamma	50.800	47.374	45.790	45.627	403.31	314.59	245.52	233.77
MC-VG-Delta	49.097	48.448	47.353	46.386	132.74	108.00	88.814	85.551
MC-VG-Gamma	57.163	50.853	47.432	46.997	514.51	403.02	317.48	302.95
MC-HS-Delta	**45.818**	**43.716**	**41.374**	**40.407**	**105.40**	**86.383**	**72.935**	**70.673**
MC-HS-Gamma	79.824	62.995	53.555	52.290	977.19	786.18	634.34	610.75
MC-Bates-Delta	48.056	47.878	47.019	46.263	**63.708**	**56.543**	**51.791**	**51.034**
MC-Bates-Gamma	**46.761**	**46.584**	**45.819**	**45.373**	156.46	118.26	89.426	84.261
MC-MJD-Delta	46.793	46.642	45.960	45.489	**79.944**	**67.954**	**59.705**	**58.343**
MC-MJD-Gamma	**45.915**	**45.791**	**44.791**	**44.373**	181.61	140.20	106.92	101.32
MC-Kou-Delta	47.405	46.037	45.428	45.377	158.86	130.75	106.99	102.64
MC-Kou-Gamma	60.586	53.017	48.685	48.100	529.14	416.98	329.27	314.37
Full Valuation	61.532	60.516	59.505	58.504	167.56	132.64	100.79	98.823

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Table 101: Back-testing Expected Shortfall for Short S & P 500 Call over the Period July 2014-Dec 2014.

Models	1 Day MAE for ES				10 Days MAE for ES			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
MC-BS-Delta	**49.059**	**48.929**	**48.452**	**48.350**	**105.61**	**87.720**	**75.008**	**72.827**
MC-BS-Gamma	52.993	49.933	48.602	48.479	403.51	314.82	246.01	234.17
MC-VG-Delta	52.926	51.383	50.399	49.455	133.40	108.94	90.769	87.622
MC-VG-Gamma	61.460	54.537	50.744	50.249	568.32	446.86	353.57	337.92
MC-HS-Delta	**48.929**	**47.808**	**45.397**	**43.388**	**109.79**	**90.802**	**77.281**	**74.960**
MC-HS-Gamma	**49.045**	**48.385**	**47.424**	**45.494**	196.86	155.19	121.59	115.73
MC-Bates-Delta	49.132	48.446	48.357	48.396	**139.08**	**113.62**	**94.279**	**90.925**
MC-Bates-Gamma	57.381	52.386	49.730	49.395	462.34	362.34	284.82	271.68
MC-MJD-Delta	**48.741**	**48.639**	**47.342**	**46.095**	**116.97**	**96.151**	**81.244**	**78.669**
MC-MJD-Gamma	53.715	50.346	48.774	48.612	401.79	314.79	246.34	234.63
MC-Kou-Delta	50.963	49.309	48.513	48.434	173.13	142.84	117.66	112.98
MC-Kou-Gamma	66.222	57.670	52.642	51.934	578.44	457.349	363.04	346.84
Full Valuation	69.310	69.200	68.820	68.630	165.51	137.88	112.83	108.77

Note: MAE is mean absolute error for ES at 1%, 2.5%, 5% and 10% significance level. MC delta stands for Monte Carlo Delta based ES calculation method, MC Gamma based stands for Monte Carlo Gamma based ES method and Full Valuation stands for Full Monte Carlo based ES calculation method. The preceding deltas and gammas were derived from the Black Scholes model(BS), Variance Gamma model, Heston model, Bates models, Merton Jump diffusion model and Double Exponential Jump diffusion model. BS stands for Black-Scholes model, HS stands for Heston stochastic volatility model, Bat stands for Bates combined stochastic and volatility model, MJD stands for Merton jump diffusion model and Kou stands for Double Exponential Jump diffusion model(Kou). The best fitted models selected based on minimum error(MAE) and represent in bold.

Chapter Five: Conclusion

1. Thesis Summary

This dissertation examines the estimation of risk measures ES for univariate data, multivariate data and options data for both 1-day ahead and multi-day ahead. In Chapter 2, we tested the performance of a new generalized *t* distribution (GAT) by Baker (2014) alongside other *t* distributions and exponential power distribution as an alternative to the *t* distribution. The primary purpose of this chapter is to compare the performances of the asymmetric *t* distribution(AST) by Zhu and Galbraith (2010) and GAT for calculating the ES for a single asset for both one day and multi-day.

For the one-day ahead ES, our results indicate that EG-GAT significantly outperforms EG-AST for all data sets. EG-AEPD model as an alternative to asymmetric distributions performs better than AST as shown by Zhu and Galbraith (2011). However, our EG-GAT model also performs better than EG-AEPD.

Computing of ES for longer horizons without knowing the detail for the distribution of returns can be found using simulation based methods. We consider Monte Carlo simulation (MCS) with GAT, AEPD, SEPD, AST, SSTD, ST and TTD as standardized distributions of returns and filtered historical simulation (FHS) for 5-days and 10-days. Results indicate that 5-days FHS is the best model for 1% and 2.5% confidence levels and MC-GAT is the best model at 5% and 10% confidence levels. However, for 10-days MC-GAT not only performs better than FHS but also MCS-AST and MCS-AEPD.

There is a critical concern of models particularly those handling the dependence among different assets because increased volatility at international financial markets after the financial crisis of 2007-2009 mean that active risk management became important for any financial organization. The copula has become a popular multivariate modelling tool mainly due to easy implementation and estimation of marginal distribution and copula separately. However, the set of higher-dimensional copulas is rather limited. The pair-copula construction can be a simple and powerful tool for model building and extending bivariate copulas to higher dimensions.

The aim of Chapter 3 is to present the usefulness of copulas and vine copulas in financial risk management. Moreover, we extend our study to examine term structure of risk for bivariate and multivariate data.

For computation of portfolio ES from both copula models and vine copulas as risk management measures, we need to rely on Monte Carlo simulation. We calculate VaR and ES for different GARCH-Copula models with various marginals. So, they are implemented and tested on both bivariate and multivariate data and compared to DCC-*norm* and DDC-*t* models. For multivariate analysis, all developed models and methods are used to analyse the five, seven and fifteen companies from DAX 25 index, a major market indicator for the Eurozone. This study is also the first to explore multivariate term structure of risk with both static and DCC correlation.

The results indicate that copula models for two-dimensional data and vine copula models for five, seven and fifteen-dimensional data provide a good fit and accurately and efficiently forecast the expected shortfall as compared to DCC-*norm* and DCC-*t*. Moreover, for the term structure of risk Monte Carlo simulation with DCC-*t* outperforms Monte Carlo simulation with static correlation for bivariate data but multivariate data Monte Carlo simulation with static correlation perform better than Monte Carlo simulation DCC-*t*.

Options play a major role in the financial markets as they can be used by the investors for hedging, speculative, spreading and synthetic positions. The accurate valuation of an option is critical for financial market analysts. The purpose of Chapter 4 is to compare option pricing models, which are based on a stochastic volatility model, jump diffusion models, an infinite activity model a combined stochastic volatility and jump diffusion model.

For risk analysis purposes, we evaluated various ES models based on partial Monte Carlo and full Monte Carlo methods. For partial Monte Carlo, we calculated Delta based and Gamma based models. The preceding deltas and gammas were derived from the Black Scholes model (BSM), variance gamma model (VG), Heston model (HS), Bates model (Bat), Merton jump diffusion model and double exponential jump diffusion model (Kou). For longer horizons, ES relies on the typical shortcut to estimating the risk over various time horizons, which is to scale by the square root of the ratio of the time horizons.

For option pricing, the Bates model that combines stochastic and jump diffusion models is the only category of option pricing model that apparently perform better for all samples of options. For risk management purposes, the delta based Monte Carlo models are dominant in the top performing models. However, it is also evident that full valuation is one of the top models for 1-day ES and 10-days ES for many datasets.

2. Future Research

This thesis proposes several directions for future research. To date there is no comparison of two asymmetric generalized *t* distributions for calculation of 1– day ahead ES. Our research suggests the asymmetric generalized *t* distributions that allow a separate parameter to control skewness and thickness of the left and right tails are important for financial risk implications. More research is needed to work on these asymmetric generalized *t* distributions for risk management calculations to draw a clear conclusion. In addition, we develop multiday ahead ES with Monte Carlo simulation with asymmetric generalized *t* distributions and other *t* distributions as standardized distributions of returns and filtered historical simulation. However, Monte Carlo simulation and filtered historical simulation are too lengthy for calculating ES for more than 10 days. Therefore, future research is required to extend ES beyond 10-days.

In Chapter 3 we apply copula models and vine copulas for bivariate and multivariate ES for both 1-day ahead and multi-day ahead forecasts. Our experience suggests that parameters estimation becomes very complicated for vine copula models when too many assets are involved in a portfolio. Therefore, work needs to be done to develop more efficient estimation methods for vine copula models. Again, like univariate data for multi-day ES we apply Monte Carlo simulation that is too lengthy. There is more work required on multiday portfolio ES calculations with higher dimensions.

In Chapter 4 we calculate option pricing with various non-normal option pricing models. Estimation of option pricing models is very complicated and initial values turn out to be very important. More work is needed on efficient estimation procedures for option pricing parameter estimation.

We apply mean absolute error (MAE) and mean square error (MSE) to test the performance of various expected shortfall models for univariate data, multivariate data and option data for both 1-day ahead and multi-day ahead risk forecast. It is becoming necessary to develop a more advanced back-testing ES method, with an ability to choose the most appropriate ES model for the assets under study.

Appendix A

Table 102: GARCH Type Models and Test Results for Standard and Poor's 500 for the Periods 1995-2013.

Models	Model Parameters (standard errors)				Residual Diagnostics Statistic (p-values)			Model Comparison			
					Q (5)	$Q^2(5)$	LM (10)	AIC	BIC	SH	HQ
GARCH	α_0	α_1	β_1								
	0.000 (0.000)	0.072 (0.006)	0.920 (0.006)		68.35 (0.071)	5.282 (0.870)	8.182 (0.371)	−6.678	−6.676	−6.678	−6.677
EGARCH	α_0	α_1	β_1	γ							
	-0.128 (0.001)	-0.074 (0.004)	0.986 (0.000)	0.123 (0.005)	55.93 (0.062)	3.054 (0.691)	6.501 (0.772)	**-6.6982**	**−6.6952**	**−6.6982**	**−6.6972**
TGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.019 (0.005)	0.925 (0.010)	0.091 (0.007)	65.21 (0.060)	1.653 (0.894)	2.837 (0.985)	**-6.6980**	**-6.6950**	**-6.6980**	**-6.6970**
NGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.064 (0.003)	0.911 (0.006)	2.556 (0.013)	70.91 (0.053)	2.329 (0.532)	2.923 (0.983)	−6.676	−6.676	−6.676	−6.675

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion*, SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals. $Q^2(5)$: is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals.*

Table 103: GARCH Type Models and Test Results for Standard and Poor's 500 (2) for the Periods 1999-2013.

Models	Models Parameters (standard errors)				Residuals Diagnostic Statistic (p-values)			Model Comparison			
					Q (5)	Q^2**(5)**	**LM (10)**	**AIC**	**BIC**	**SH**	**HQ**
GARCH	α_0	α_1	β_1								
	0.000 (0.000)	0.070 (0.013)	0.922 (0.012)		11.19 (0.041)	12.34 (0.127)	19.32 (0.04)	−6.5296	−6.5243	−6.5296	−6.5271
EGARCH	α_0	α_1	β_1	γ							
	-0.156 (0.0014)	-0.104 (0.003)	0.983 (0.000)	0.114 (0.007)	9.540 (0.089)	11.82 (0.0992)	20.33 (0.026)	**−6.5614**	**−6.556**	**−6.5614**	**−6.560**
TGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.000 (0.005)	0.925 (0.009)	0.123 (0.018)	9.223 (0.101)	19.47 (0.140)	16.15 (0.095)	6.5610	-6.5556	-6.5610	-6.5592
NGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.059 (0.005)	0.917 (0.008)	2.523 (0.019)	11.50 (0.062)	11.79 (0.099)	17.60 (0.062)	−6.5269	−6.5215	−6.5269	−6.5250

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals.* Q^2*(10): is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals.*

Table 104: GARCH Type Models and Test Results Adobe for the Periods 1986-2013.

Models	Models Parameters (standard errors)				Residuals Diagnostic Statistic (p-values)			Model Comparison			
					Q (5)	**Q^2(5)**	**LM (10)**	**AIC**	**BIC**	**SH**	**HQ**
GARCH	α_0	α_1	β_1								
	0.000	0.041	0.958		1.027	1.038	4.213	-4.2082	-4.2043	-4.2082	-4.2078
	(0.000)	(0.002)	(0.001)		(0.960)	(0.960)	(0.937)				
EGARCH	α_0	α_1	β_1	γ							
	-0.156	-0.104	0.983	0.114	9.540	3.621	20.33	$\mathbf{-6.5614}$	$\mathbf{-6.5560}$	$\mathbf{-6.5614}$	$\mathbf{-6.5595}$
	(0.001)	(0.004)	(0.000)	(0.007)	(0.089)	(0.059)	(0.062)				
TGARCH	α_0	α_1	β_1	γ							
	0.000	0.035	0.958	0.013	1.109	11.36	4.530	-4.2088	-4.2039	-4.2088	-4.2071
	(0.000)	(0.003)	(0.001)	(0.005)	(0.953)	(0.146)	(0.920)				
NGARCH	α_0	α_1	β_1	γ							
	0.000	0.051	0.958	1.099	0.867	12.40	9.905	-5.2299	-5.2265	-5.2299	-5.2288
	(0.000)	(0.001)	(0.000)	(0.044)	(0.973)	(0.091)	(0.449)				

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals. Q^2(5): is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals.*

Table 105: GARCH Type Models and Test Results for Bank of America for the Periods 1973-2013.

Models	Models Parameters (standard errors)				Residuals Diagnostic Statistic (p-values)			Model Comparison			
					Q (5)	$Q^2(5)$	LM (10)	AIC	BIC	SH	HQ
GARCH	α_0	α_1	β_1								
	0.000 (0.000)	0.065 (0.009)	0.929 (0.010)		70.60 (0.080)	60.00 (0.330)	11.325 (0.333)	−5.2257	−5.2230	−5.2257	−5.2248
EGARCH	α_0	α_1	β_1	γ							
	-0.057 (0.002)	0.036 (0.004)	0.992 (0.000)	0.108 (0.006)	71.17 (0.101)	61.23 (0.210)	27.87 (0.000)	**−5.2353**	**−5.2319**	**−5.2353**	**−5.2341**
TGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.036 (0.005)	0.937 (0.007)	0.041 (0.004)	13.32 (0.091)	19.36 (0.124)	0.47 (0.199)	**-5.2320**	**-5.2285**	**-5.2320**	**-5.2308**
NGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.073 (0.007)	0.934 (0.008)	1.372 (0.001)	71.22 (0.320)	34.03 (0.530)	17.84 (0.058)	−5.1807	−5.1773	−5.1807	−5.1795

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals. $Q^2(5)$: is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals.*

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Table 106: GARCH Type Models and Test Results J P Morgan for the Periods 1973-2013.

Models	Models Parameters (standard errors)				Residuals Diagnostic Statistic (p-values)			Model Comparison			
					Q (5)	Q^2(5)	LM (10)	AIC	BIC	SH	HQ
GARCH	α_0	α_1	β_1								
	0.000 (0.000)	0.069 (0.009)	0.927 (0.009)		17.50 (0.088)	19.50 (0.103)	9.504 (0.485)	-5.1773	-5.1746	-5.1773	-5.1764
EGARCH	α_0	α_1	β_1	γ							
	-0.058 (0.003)	-0.047 (0.004)	0.992 (0.001)	0.116 (0.013)	14.10 (0.085)	21.58 (0.138)	12.412 (0.258)	-5.1908	-5.1874	-5.1908	-5.1896
TGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.033 (0.005)	0.930 (0.009)	0.065 (0.003)	17.10 (0.100)	31.10 (0.314)	10.498 (0.398)	-5.1877	-5.1843	-5.1877	-5.1866
NGARCH	α_0	α_1	β_1	γ							
	0.000 (0.000)	0.077 (0.005)	0.933 (0.007)	1.333 (0.038)	16.28 (0.090)	14.87 (0.241)	10.52 (0.396)	$\mathbf{-5.3830}$	$\mathbf{-5.3796}$	$\mathbf{-5.3830}$	$\mathbf{-5.3819}$

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals. Q^2(5): is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals*.

Table 107: GARCH Type Models and Test Results for Pfizer for the Periods 1973-2013.

Models	Models Parameters (standard errors)				Residuals Diagnostic Statistic (p-values)			Model Comparison			
					Q (5)	Q^2(5)	LM (10)	AIC	BIC	SH	HQ
GARCH	α_0 0.000 (0.000)	α_1 0.053 (0.009)	β_1 0.934 (0.013)		36.01 (0.090)	51.01 (0.182)	27.22 (0.080)	-5.3817	-5.3790	-5.3817	-5.3808
EGARCH	α_0 -0.117 (0.005)	α_1 -0.037 (0.005)	β_1 0.985 (0.001)	γ 0.110 (0.009)	37.20 (0.070)	38.32 (0.138)	37.20 (0.110)	$\mathbf{-5.3878}$	$\mathbf{-5.3844}$	$\mathbf{-5.3878}$	$\mathbf{-5.3866}$
TGARCH	α_0 0.000 (0.000)	α_1 0.033 (0.007)	β_1 0.939 (0.012)	γ 0.033 (0.007)	36.88 (0.110)	29.32 (0.203)	18.28 (0.060)	-5.3848	-5.3814	-5.3848	-5.3836
NGARCH	α_0 0.000 (0.000)	α_1 0.063 (0.004)	β_1 0.933 (0.008)	γ 1.512 (0.065)	36.24 (0.090)	32.12 (0.182)	35.56 (0.093)	-4.6784	-4.6725	-4.6784	-4.6763

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals. Q^2(5): is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals.*

Table 108: GARCH Type Models and Test Results for Starbucks for the Periods 1993-2013.

Models	Models Parameters (standard errors)				Residuals Diagnostic Statistic (p-values)			Model Comparison			
					Q (5)	$Q^2(5)$	LM (10)	AIC	BIC	SH	HQ
GARCH	α_0	α_1	β_1								
	0.000	0.036	0.961		12.129	22.329	3.285	−4.6692	−4.6644	−4.6692	−4.6675
	(0.000)	(0.001)	(0.001)		(0.053)	(0.063)	(0.974)				
EGARCH	α_0	α_1	β_1	γ	14.53	19.52	4.066				
	-0.03	-0.029	0.995	0.088	(0.083)	(0.351)	(0.944)	**−4.6847**	**−4.6788**	**−4.6847**	**−4.6827**
	(0.001)	(0.005)	(0.000)	(0.002)							
TGARCH	α_0	α_1	β_1	γ	14.66	15.87	2.549				
	0.000	0.018	0.960	0.039	(0.092)	(0.369)	(0.990)	-4.6758	-4.6699	−4.6758	-4.6738
	(0.000)	(0.003)	(0.001)	(0.006)							
NGARCH	α_0	α_1	β_1	γ	12.106	9.30	4.751				
	0.000	0.048	0.961	1.060	(0.073)	(0.249)	(0.907)	−4.6784	−4.6725	−4.6784	−4.6763
	(0.000)	0.001	(0.000)	(0.042)							

Note: AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH is Shibata information criterion, HQ is Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals. $Q^2(5)$ is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10): is ARCH LM test of lag10 of standardized residuals.*

Table 109: EGARCH Model Estimation and Test Results for the Periods 1995-2013.

Parameters	Conditional Variance Equation with Normal Innovation			
	FTSE	**NASDAQ**	**Nikkie**	**DAX**
α_0	-0.1294 (0.0016)	-0.0820 (0.0116)	-0.2282 (0.0089)	-0.1842 (0.0066)
α_1	-0.1001 (0.0057)	-0.0702 (0.0231)	-0.0758 (0.0070)	-0.0930 (0.0060)
β_1	0.9857 (0.0000)	0.9897 (0.0015)	0.9726 (0.0010)	0.9783 (0.0007)
γ	0.1103 (0.0043)	0.1276 (0.0219)	0.1551 (0.0114)	0.1585 (0.0088)
Q (5)	5.7361 (0.1037)	9.195 (0.0947)	2.325 (0.5438)	1.8303 (0.659)
Q^2**(5)**	7.3504 (0.1716)	18.741 (0.1804)	6.397 (0.2551)	13.452 (0.1083)
AIC	**-6.4184**	-5.5320	-5.7913	-5.9052

Note: Standard error and p value presented in parenthesis. AIC is Akaike information criterion, BIC is Bayesian information criterion, *SH Shibata information criterion, HQ: Hannan-Quinn. Q (5): is the Ljung-Box-statistic of lag 5 of standardized residuals.* Q^2*(5): is the Ljung-Box-statistic of lag 5 of standardized squared residuals. LM (10); ARCH LM test of lag 10 of standardized residuals.*

Appendix B

Jargue-Bera Test

The Jarque–Bera test is goodness of test of normality, and test whether sample data have the skewness and kurtosis satisfying a normal distribution. The test is named after Carlos Jarque and Anil K. Bera.

Test the null hypothesis:

H_0: distribution is normal,

skewness is zero and excess kurtosis is zero;

against the alternative hypothesis:

H_1: distribution is non-normal.

The test statistic JB is defined as:

$$JB = \frac{N}{6}\left((SKW)^2 + 1\frac{1}{4}(KUR - 3)^2\right)$$

where *N* is the number of total observations ; *SKW* is the sample skewness, and *KUR* is the sample kurtosis and defined as:

$$SKW = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^3}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^{3/2}}$$

$$KUR = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^4}{\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^{2}}$$

Test statistic can be compared with a chi-square distribution with 2 degrees of freedom. The null hypothesis of normality is rejected if the calculated test statistic exceeds a critical value from the chi-square distribution.

Augmented Dickey–Fuller Unit Root test

An augmented Dickey–Fuller test (ADF) is a unit root test in a time series sample

The null hypothesis of the Augmented Dickey-Fuller test is:

Null Hypothesis: data is not stationary

Alternative Hypothesis: data is stationary.

The ADF is based on fitting the regressing model:

$$\Delta y_t = \alpha_0 + \beta t + \theta y_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \cdots \alpha_p \Delta y_{t-p} + \alpha_t$$

The unit root test is carried out under the null hypothesis $\theta = 0$ against the alternative hypothesis of $\theta < 0$.

The ADF test is:

Regress Δy_t on $y_{t-1}, \Delta y_{t-1} \dots . \Delta y_{t-p}$ and compute the t statistic:

$$\tau = \frac{\hat{\theta}}{se(\hat{\theta})}$$

Phillips-Perron Unit Root Test(PP)

PP test is proposed transformations of the τ statistics from the original Dicey Fuller regressions. The test is robust with respect to unspecified autocorrelation and heteroscedasticity in the disturbance process of the test equation.

The Phillips–Perron test involves fitting the regression:

$$\Delta y_t = \alpha_0 + \beta t + \theta y_{t-1} + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \cdots \alpha_p \Delta y_{t-p} + \alpha_t$$

There are two statistics:

$$Z_t = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{1/2} . t_{\theta=0} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right) . \left(\frac{n. se(\hat{\theta})}{\hat{\sigma}^2}\right)$$

$$Z_\theta = n\hat{\theta} - \frac{1}{2}\frac{n^2. se(\hat{\theta})}{\hat{\sigma}^2}\left(\hat{\lambda}^2 - \hat{\sigma}^2\right)$$

Where $\hat{\sigma}^2$ and $\hat{\lambda}^2$ are standard error of $\hat{\theta}$:

$$\sigma^2 = \lim_{n\to\infty} n^{-1} \sum_{t=1}^{n} E(\mu_t)^2$$

$$\lambda^n = \lim_{n\to\infty} \sum_{t=1}^{n} E(n^{-1} S_n^2)$$

$$S_n = \sum_{t=1}^{n} \mu_t$$

Under the null hypothesis $\theta = 0$. Z_t and Z_θ have the same distribution as the Dickey – Fuller statistic.

Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests

The null hypothesis of the KPPS test is:

Null Hypothesis: data is stationary

Alternative Hypothesis: data is not stationary.

$$y_t = \mu_t + e_t$$

$$\mu_t = \mu_{t-1} + \varepsilon_t \quad \varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$$

he KPSS test is score statistic for testing $\sigma_\varepsilon^2 = 0$ against $\sigma_\varepsilon^2 > 0$. KPSS Statistics is:

$$KPSS = \frac{1}{2} . \frac{n^{-1} \sum_{t=1}^{n} S_t^2}{\lambda^2}$$

Where

$$S_t = \sum_{j=1}^{n} e_j$$

and λ^2 is a consistent estimate of the long-run variance of e_j.

Ljung–Box test

The sample autocorrelation function (ACF) and partial autocorrelation function (PACF) are helpful qualitative tools to test the presence of autocorrelation at individual lags. The Ljung-Box Q-test is a more convenient way to test for autocorrelation at multiple lags jointly (Ljung and Box, 1978).

The Ljung–Box test can be defined as follows.

Null Hypothesis: data is independently distributed.

Alternative hypothesis: data is not independently distributed

Ljung and Box (1978) defined test statistics as:

$$Q = N(N+2) \sum_{k=1}^{h} \frac{\rho_k^2}{N-k}$$

where N is the sample size,ρ_k is the sample autocorrelation at lag k, and h is number of laga being tested.

ARCH LM Test

We want to model a time series with ARCH process effect and return residual are:

$$\varepsilon_t = z_t \sigma_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots \alpha_p \varepsilon_{t-p}^2$$

The null hypothesis is α_i=0 mean no ARCH effect.

Akaike information criterion (AIC)

The **Akaike information criterion** (**AIC**) is a measure of selection of models from a set of models, for a given set of data. Given a set of models for the data, the favored model is the one with the minimum AIC value. AIC is define as:

$$AIC = 2\big(k - ln(L)\big)$$

where *k* is the number of free parameters of the data, and *L* is the maximized value of the likelihood function for the model.

Bayesian information criterion (BIC)

BIC feature the same goodness-of-fit term as AIC and defined as:

$$BIC = kln(n) - 2ln(L)$$

where k is the number of free parameters of the data, and L is the maximized value of the likelihood function for the model. Given a set of models for the data, the favored model is the one with the minimum BIC value.

Hannan–Quinn information criterion(HQ)

HQ is alternative to AIC and BIC is define as:

$$HQ = 2k.\,ln\big(ln(n)\big) - 2ln(L)$$

The aim is to find the model with the lowest value of the selected information criterion.

Maximum Likelihood Method

Maximum-likelihood estimation (MLE) is the parameters estimation procedure of a statistical model. Given a statistical model and data set, maximum-likelihood estimation gives estimates for the model's parameter.

Suppose a vector of independent and identically distributed observations $x_0, x_1 \ldots \ldots \ldots x_N$ coming from a distribution with an unknown density function $f(x|\theta)$.θ is unknown and is the true value of the parameter. It is desirable to find some estimator $\widehat{\theta}$ which would be as close to the true value θ as possible.

The likelihood function is the density function regarded as a function of θ is:

$$L(x|\theta) = f(x|\theta\ )$$

The maximum likelihood estimator (MLE) is:

$$\hat{\theta}(x) = argmax_{\theta}\ L(x|\theta)$$

In practice, it is often more convenient to work with the logarithm of the likelihood function, called the log-likelihood:

$$lnL(x|\theta) = ln\,f(x|\theta\ )$$

Determination of Threshold

There are several plots available in the literature to determine the threshold level θ. The choice of threshold is to some extent subjective because in practice the explanation of threshold level determined plots is very difficult. A choice of the threshold is an important issue, as sufficiently high threshold θ result in too few exceedance and result in high variance estimator. While, too high threshold θ provides biased estimators.

Quantile-Quantile Plot(QQ-Plot)

Let $x_1, x_2, \dots x_n$ be a sequence of randome variables with *i.i.d* condition,$x_1 n < \cdots < x_m$.The empirical distribution is :

$$F_m(x_k, n) = \frac{(n-k+1)}{n}$$

F is the estimated parametric distribution of data.
The Q-Q plot is defined as:

$$x_{k,n} F^{-1}\left(\frac{n-k+1}{n}\right), \qquad k = 1,2, \dots n \qquad 9$$

Q-Q plot can be used to differentiate between distribution functions. For the success of the model Q-Q plot should me linear. No linear plot indicates model failure.

Mean Excess Function

Another method that enables us to provide a graphical tool to choose the threshold θ is mean excess function. If x hs a GPD distribution function $G_{\xi,\sigma}$, the mean excess function is:

$$e(\theta) = E(x - \theta | x > \theta) = \frac{\sigma + \xi\theta}{1 - \xi} \qquad 10$$

If the plot is a straight line, the model is good fit, if plot is flat line then data may follow exponential, and if it is in curved form then data may follow Weiball or gamma.

Hill Plot

Let $x_1 > \cdots > x_n$ be the order statistic random variable i.i.d. The Hill estimator of the tail index ξ using k+1 ordered statistic is defined as:

$$H_{k.n} = \frac{1}{k} \sum_{i=1}^{k} ln x_{j.n} - ln x_{k,n} = \widehat{\xi} \qquad 11$$

$$\hat{\xi} = \frac{1}{k} \sum_{i=1}^{k} ln\left(\frac{x_i}{x_{k+1}}\right)$$

The Hill plot is defined by the set of points (Dress de Haan, and Resnick, 1999)):

$$\left(k, x_{k,n}^{-1}\right), \qquad 1 \leq k \leq n-1$$

Hill plot is considered good instrument to find the optimal threshold θ for the GPD distribution (Dress De Hannn,Rcsnick, 1999 and 1998 and Pilipihas, 2010).

Appendix C
Table 110: Bivariate Copula Estimation.

				BMW/SEI				
Copula Model	**Marginal**	θ_1	θ_2	τ	λ_L	λ_U	**AIC**	**BIC**
Normal	Skew t	0.1756 (0.0065)		0.0878			-681.58	-675.24
Student t	Skew t	0.1391 (0.009)	5.303 (0.002)	**0.0889**	0.0694	0.0694	2335.5	2348.2
Clayton	Skew t	0.0755 (0.0054)		0.0363	0.0001		-267.44 (2)	-261.11
Gumbel	Skew t	1.0031 (0.003)		0.0029		0.0041	0.6438	6.9771
Frank	Skew t	2.0197 (0.072)		**0.2158**			-820.93 (1)	-814.60
Joe	Skew t	1.0001 (0.0031)		0.0053		0.0001	2.3012 (3)	8.6345

				BMW/VOL				
Model	**Marginal**	θ_1	θ_2	τ	λ_L	λ_U	**AIC**	**BIC**
Normal	Skew t	0.3218 (0.0047)		0.2085			75.207	81.540
Student t	Skew t	0.2818 (0.0073)	9.0443 (0.2780)	**0.1819**	0.0390	0.0390	1107.6	1120.3
Clayton	Skew t	0.2140 (0.0059)		0.0966	0.0392		176.99 (2)	183.32
Gumbel	Skew t	1.1574 (0.0050)		0.1359		0.1799	811.44	817.78
Frank	Skew t	2.7366 (0.0684)		**0.2838**			-491.23 (1)	-484.90
Joe	Skew t	1.1084 (0.0045)		0.0935		0.2010	616.84 (3)	623.17

				BMW/THY				
Model	**Marginal**	θ_1	θ_2	τ	λ_L	λ_U	**AIC**	**BIC**
Normal	Skew t	0.1874 (0.0063)		0.1200			-746.62	-740.28
Student t	Skew t	0.1788 (0.0072)	6.625 (0.0124)	**0.0974**	0.0468	0.0468	728.8	741.5
Clayton	Skew t	0.0950 (0.0056)		0.0453	0.0006		-405.31 (2)	-398.98
Gumbel	Skew t	1.0317 (0.0041)		0.0307		0.0421	-127.66 (3)	-121.33
Frank	Skew t	1.8878 (0.0701)		**0.2027**			-718.33 (1)	-711.99
Joe	Skew t	1.0099 (0.0029)		0.0057		0.01354	-37.684	-31.351

BMW/LIN

Model	Marginal	θ_1	θ_2	τ	λ_L	λ_U	AIC	BIC
Normal	Skew t	0.1810 (0.0063)		0.1158			-549.06	-542.72
Student t	Skew t	0.4403 (0.0090)	2.2314 (0.0126)	**0.2902**	0.3388	0.3388	-764.61 (2)	-751.94
Clayton	Skew t	0.0895 (0.0055)		0.0428	0.0004		-264.40 (3)	-258.07
Gumbel	Skew t	1.0338 (0.0039)		0.0326		0.0448	-187.95	-181.62
Frank	Skew t	1.7814 (0.0694)		**0.1919**			-783.57 (1)	-777.24
Joe	Skew t	1.0142 (0.0031)		0.0082		0.0194	-51.392	-45.058

SEI/VOL

Model	Marginal	θ_1	θ_2	τ	λ_L	λ_U	AIC	BIC
Normal	Skew t	0.1611 (0.0067)		0.1030			-2756.8	-2750.5
Student t	Skew t	0.1254 (0.0098)	4.3261 (0.0145)	**0.0800**	0.09408	0.09408	-5648.4 (1)	-5635.8
Clayton	Skew t	0.0742 (0.0058)		0.0357	0.0023		-2258.2 (2)	-2251.8
Gumbel	Skew t	1.0042 (0.0027)		0.0041		0.0057	-1647.3 (3)	-1640.9
Frank	Skew t	1.7739 (0.0702)		**0.1912**			-1411.3	-1404.9
Joe	Skew t	1.0041 (0.5321)		0.0023		0.0056	-1532.6	-1526.3

SEI/THY

Model	Marginal	θ_1	θ_2	τ	λ_L	λ_U	AIC	BIC
Normal	Skew t	0.5336 (0.0106)		0.3583			-360.92	-354.59
Student t	Skew t	0.5572 (0.0115)	6.8512 (0.8382)	**0.3762**	0.1741	0.1741	-1011.3 (1)	-998.71
Clayton	Skew t	0.8435 (0.0318)		0.2966	0.4396		-506.08 (3)	-499.75
Gumbel	Skew t	1.5339 (0.0203)		0.3480		0.4287	-327.82	-321.49
Frank	Skew t	3.8739 (0.1116)		**0.3786**			-598.39 (2)	-592.06
Joe	Skew t	1.6741 (0.0293)		0.2730		0.4870	-235.59	-229.25

SEI/LIN

Model	Marginal	θ_1	θ_2	τ	λ_L	λ_U	AIC	BIC
Normal	Skew t	0.4895 (0.0116)		0.3256			3254.9 (2)	3261.3
Student t	Skew t	0.5071 (0.0124)	8.2720 (1.2317)	**0.3386**	0.1146	0.1146	3637.5	3650.2

Clayton	Skew t	0.7648 (0.0307)		0.2766	0.4040		4444.0	4450.4
Gumbel	Skew t	1.4427 (0.0189)		0.2877		0.3616	3664.9 (3)	3670.7
Frank	Skew t	3.3648 (0.1086)		**0.3382**			691.99 (1)	698.32
Joe	Skew t	1.5447 (0.0273)		0.2339		0.4337	4159.0	4165.4
				VOL/THY				
Model	**Marginal**	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	**τ**	$\boldsymbol{\lambda_L}$	$\boldsymbol{\lambda_U}$	**AIC**	**BIC**
Normal	Skew t	0.1820 (0.0065)					-96.723	-90.390
Student t	Skew t	0.1291 (0.0102)	3.3151 (0.1284)	**0.0824**	0.1226	0.1226	3715.0	3727.6
Clayton	Skew t	0.0986 (0.0060)		0.0470	0.0009		-141.82 (1)	-135.48
Gumbel	Skew t	1.0359 (0.0041)		0.0346		0.0474	-108.23 (3)	-101.89
Frank	Skew t	1.7936 (0.0687)		**0.1932**			44.719	51.052
Joe	Skew t	1.0149 (0.0032)		0.0085		0.0202	-99.959 (2)	-93.626
				VOL/LIN				
Model	Marginal	θ_1	θ_2	τ	λ_L	λ_U	AIC	BIC
Normal	Skew t	0.1714 (0.0065)		0.1096			-614.16	-607.83
Student t	Skew t	0.1710 (0.0065)	3.5188 (0.8371)	**0.1093**	0.1399	0.1399	-40.853	-28.187
Clayton	Skew t	0.09327 (0.0059)		0.0445	0.0006		-116.95 (3)	-110.62
Gumbel	Skew t	1.0350 (0.0039)		0.0339		0.0464	-187.91 (2)	-181.58
Frank	Skew t	1.6134 (0.0675)		**0.1747**			-384.38 (1)	-378.04
Joe	Skew t	1.0174 (0.0032)		0.0100		0.02364	-40.366	-34.033
				Thy/LIN				
Model	**Marginal**	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	**τ**	$\boldsymbol{\lambda_L}$	$\boldsymbol{\lambda_U}$	**AIC**	**BIC**
Normal	Skew t	0.4458 (0.0113)		0.2941			7428.5	7434.8
Student t	Skew t	0.4616 (0.0124)	7.5897 (0.9899)	**0.3054**	0.1105	0.1105	4529.6 (2)	4542.2
Clayton	Skew t	0.6891 (0.0274)		0.2562	0.9998		5626.0	5632.4
Gumbel	Skew t	1.3778 (0.0164)		0.2742		0.3461	5524.8	5531.2
Frank	Skew t	2.9991		**0.3071**			1852.4	1858.8

		(0.0994)			(1)	
Joe	Skew t	1.4550	0.2039	0.3898	4593.3	4599.6
		(0.0234)			(3)	

*Note***:***Siemens:SIE,BMW:BMW,Linde:LIN,Thyssenkrup:THY,Volkswagen:VOL. Standard errors are presented in parenthesis. All models are ranked on the basis of smallest AIC. AIC is Akaike information criterion, and BIC is* Bayesian information criterion. *τ is Kendell's Tau measure of dependence.*

Table 111: Bivariate DCC Model Parameter Estimation.

DCC-mvnorm

	BMW/SEI	BMW/VOL	BMW/THY	BMW/LIN	SEI/VOL	SEI/THY	SEI/LIN	VOL/THY	SEI/LIN	THY/LIN
$\boldsymbol{\alpha}$	0.0288	0.0464	0.0431	0.0718	0.0480	0.0300	0.0155	0.0241	0.0294	0.0206
$\boldsymbol{\beta}$	0.9658	0.9371	0.9469	0.6822	0.9404	0.9679	0.9821	0.9714	0.9539	0.9701
AIC	7.3272	10.047	7.8530	7.9142	7.2414	4.8512	4.9517	7.7565	7.8058	5.4410

DCC-mvt

	BMW/SEI	BMW/VOL	BMW/THY	BMW/LIN	SEI/VOL	SEI/THY	SEI/LIN	VOL/THY	SEI/LIN	THY/LIN
$\boldsymbol{\alpha}$	0.0464	0.0428	0.0367	0.0486	0.0450	0.0302	0.0172	0.0291	0.0299	0.0192
$\boldsymbol{\beta}$	0.9356	0.9476	0.9540	0.8266	0.9421	0.9674	0.9793	0.9626	0.9547	0.9743
df	6.5275	5.0671	5.6556	5.3163	5.9866	6.0869	5.9913	6.5342	6.4307	6.5275
AIC	**7.0623**	**9.8216**	**7.6728**	**7.7229**	**5.9866**	**4.6741**	**4.7719**	**7.6262**	**7.6682**	**5.3126**

Note**:**Siemens:SIE,BMW:BMW,Linde:LIN,Thyssenkrup:THY,Volkswagen:VOL. Standard errors are presented in parenthesis. All models are ranked on the basis of smallest AIC. AIC is Akaike information criterion, and BIC is Bayesian information criterion. τ is Kendell's Tau measure of dependence.

Table 112: Back-testing Value at Risk (VaR) and Expected Shortfall (ES) for Bivariate Copulas.

BMW/SEI								
Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.1140	0.2924	0.5944	1.2038	0.1125	0.1143	0.1157	0.1174
Student t	0.1138	0.2909	0.5919	1.2058	**0.1095** **(1)**	0.1129 **(2)**	0.1151 **(2)**	0.1173 **(3)**
Clayton	0.1148	0.2922	0.5924	1.2048	0.1127	0.1144	0.1161	0.1177
Gumbel	0.1143	0.2919	0.5934	1.2048	0.1127	0.1144	0.1162	0.1179
Frank	0.1123	0.2889	0.5889	1.1948	**0.1100** **(2)**	0.1124 **(1)**	0.1145 **(1)**	0.1165 **(1)**
Joe	0.1138	0.2884	0.5864	1.2028	0.1121 (3)	0.1136 **(3)**	0.1151 **(2)**	0.1170 **(2)**
DCC-norm	3.061	2.345	1.725	1.0225	0.5418	0.5233	0.5008	0.4745
DCC-t	57.56	32.81	24.25	15.89	0.9475	0.8060	0.7051	0.6067

BMW/VOL								
Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.1160	0.2949	0.5964	1.2148	0.1130	0.1154	0.1169	0.1187
Student t	0.1161	0.2952	0.6014	1.2178	0.1142 **(2)**	0.1159 **(2)**	0.1176 **(2)**	0.1193 **(2)**
Clayton	0.1169	0.2962	0.6004	1.2168	0.1144 **(3)**	0.1162 **(3)**	0.1178 (3)	0.1193 (2)
Gumbel	0.1168	0.2972	0.6009	1.2198	0.1155	0.1169	0.1182	0.1197
Frank	0.1162	0.2939	0.5974	1.2158	0.1132 **(1)**	0.1154 **(1)**	0.1170 **(1)**	0.1189 **(1)**
Joe	0.1169	0.2972	0.6039	1.2218	0.1148	0.1166	0.1182	0.1198
DCC-norm	1.908	1.206	0.611	0.041	0.4237	0.4048	0.3823	0.3557
DCC-t	4.594	2.126	1.248	0.447	0.7796	0.6271	0.4922	0.4228

BMW/THY								
Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.1107	0.2855	0.5835	1.1900	0.1083	0.1110	0.1132	0.1156
Student t	0.1115	0.2837	0.5810	1.1940	0.1070 **(1)**	0.1103 **(2)**	0.1126 **(1)**	0.1154 **(3)**
Clayton	0.1105	0.2835	0.5790	1.1890	0.1081	0.1106 **(3)**	0.1126 **(1)**	0.1152 **(1)**
Gumbel	0.1122	0.2875	0.5830	1.1970	0.1080 (3)	0.1113	0.1135	0.1159

Frank	0.1112	0.2852	0.5815	1.1860	0.1081	0.1109	0.1130	0.1152 **(1)**
Joe	0.1106	0.2850	0.5845	1.1910	0.1072 **(2)**	0.1102 **(1)**	0.1129 **(2)**	0.1155
DCC-norm	3.0729	2.3555	1.7337	1.0286	0.5424	0.5236	0.5011	0.4747
DCC-t	5.8300	3.4232	2.4870	1.6001	0.9490	0.7906	0.6815	0.5781

BMW/LIN

Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.1205	0.3078	0.6267	1.2735	0.1191	0.1207	0.1225	0.1244
Student t	0.1192	0.3048	0.6232	1.2755	0.1151 **(1)**	0.1186 **(1)**	0.1211 **(1)**	0.1237 **(2)**
Clayton	0.1213	0.3088	0.6282	1.2825	0.1190	0.1211	0.1228	0.1248
Gumbel	0.1193	0.3066	0.6267	1.2775	0.1183	0.1201 **(3)**	0.1221 **(2)**	0.1244 **(3)**
Frank	0.1196	0.3041	0.6232	1.2715	0.1169 **(2)**	0.1191 **(2)**	0.1211 **(1)**	0.1235 **(1)**
Joe	0.1223	0.3098	0.6297	1.2815	0.1179 (3)	0.1210	0.1229	0.1250
DCC-norm	2.8278	2.1395	1.5441	0.8716	0.5281	0.5100	0.4883	0.4627
DCC-t	5.6454	3.0882	2.2072	1.3756	0.8850	0.7356	0.6328	0.5355

SEI/VOL

Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.1319	0.3369	0.6838	1.3846	0.1300	0.1320	0.1338	0.1358
Student t	0.1334	0.3399	0.6878	1.3936	0.1304 (1)	0.1331 (1)	0.1349 (1)	0.1367 (3)
Clayton	0.1332	0.3386	0.6878	1.3896	0.1314 (2)	0.1332 (2)	0.1349 (1)	0.1366 (2)
Gumbel	0.1343	0.3396	0.6863	1.3916	0.1329	0.1342	0.1354	0.1368
Frank	0.1339	0.3404	0.6858	1.3866	0.1316	0.1335 (3)	0.1350 (2)	0.1365 (1)
Joe	0.1333	0.3404	0.6858	1.3876	0.1312 (1)	0.1336	0.1351	0.1366
DCC-norm	0.9966	0.4358	0.1911	0.0638	0.3884	0.3881	0.3841	0.3791
DCC-t	0.8726	0.3583	0.1376	0.0360	0.3343	0.3038	0.2830	0.2635

SEI/THY

Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.0069	0.0232	0.0640	0.1541	0.0029	0.0060	0.0085	0.0114

Student t	0.0081	0.0265	0.0675	0.1591	0.0011 (1)	0.0061 (2)	0.0091 (2)	0.0119 (3)
Clayton	0.0066	0.0222	0.0625	0.1551	0.0024 (2)	0.0058 (1)	0.0083 (1)	0.0112 (1)
Gumbel	0.0077	0.0242	0.0645	0.1621	0.0044 (3)	0.0071 (3)	0.0093	0.0120
Frank	0.0082	0.0247	0.0615	0.1511	0.0051	0.0075	0.0092 (3)	0.0115 (2)
Joe	0.0085	0.0290	0.0690	0.1651	0.0044	0.0078	0.0103	0.0128
DCC-norm	6.0279	4.6398	0.4173	0.9676	0.5096	0.5096	0.5050	0.4996
DCC-t	8.6118	6.3266	6.3266	6.3266	0.4548	0.4269	0.4073	0.3882

SEI/LIN

Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.0156	0.0496	0.1128	0.2506	0.0132	0.0157	0.0184	0.0211
Student t	0.0167	0.0476	0.1068	0.2466	0.0109 (2)	0.0151 (1)	0.0176 (1)	0.0203 (1)
Clayton	0.0153	0.0464	0.1073	0.2446	0.0138	0.0157 (2)	0.0178 (2)	0.0204 (2)
Gumbel	0.0179	0.0521	0.1133	0.2516	0.0108 (1)	0.0160 (3)	0.0189	0.0215
Frank	0.0161	0.0499	0.1078	0.2426	0.0136 (3)	0.0165	0.0186 (3)	0.0208 (3)
Joe	0.0187	0.0516	0.1143	0.2496	0.0160	0.0183	0.0200	0.0219
DCC-norm	2.2012	1.3294	0.5828	0.0246	0.4909	0.4891	0.4879	0.4845
DCC-t	4.4584	2.7626	1.5993	0.5521	0.4013	0.3803	0.3659	0.3525

VOL/THY

Model	**VaR-Loss Function**				**ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Normal	0.1298	0.3297	0.6749	1.3718	0.1317	0.3349	0.6849	1.3918
Student t	0.1315	0.3339	0.6789	1.3801	0.1249 (1)	0.1276 (1)	0.1304 (1)	0.1340 (2)
Clayton	0.1313	0.3349	0.6779	1.3818	0.1298	0.1316	0.1333	0.1352
Gumbel	0.1299	0.3322	0.6729	1.3688	0.1289	0.1312	0.1330	0.1350
Frank	0.1308	0.3344	0.6754	1.3748	0.1278 (2)	0.1301 (2)	0.1318 (2)	0.1338 (1)
Joe	0.1317	0.3349	0.6849	1.3918	0.1287 (3)	0.1308 (3)	0.1326 (3)	0.1344 (3)
DCC-norm	0.9638	0.4149	0.1784	0.0577	0.3906	0.3905	0.3859	0.3805

DCC-t	0.8388	0.3419	0.1309	0.0297	0.3512	0.3224	0.3022	0.2825

VOL/LIN

Model	VaR-Loss Function				ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Normal	0.1367	0.3485	0.7101	1.4503	0.1339	0.1364	0.1386	0.1411
Student t	0.1383	0.3510	0.7146	1.4573	0.1357	0.1378	0.1399	0.1422
Clayton	0.1391	0.3523	0.7156	1.4523	0.1368	0.1385	0.1403	0.1423
Gumbel	0.1366	0.3485	0.7091	1.4403	0.1338 (2)	0.1363 (2)	0.1383 (2)	0.1406 (1)
Frank	0.1360	0.3480	0.7066	1.4433	0.1329 (1)	0.1361 (1)	0.1382 (1)	0.1407 (2)
Joe	0.1381	0.3510	0.7151	1.4493	0.1349 (3)	0.1376 (3)	0.1396 (3)	0.1419 (3)
DCC-norm	1.1983	0.5531	0.2568	0.0934	0.3712	0.3728	0.3696	0.3659
DCC-t	1.1143	0.5046	0.2250	0.07453	0.2940	0.2745	0.2607	0.2473

THY/LIN

Model	VaR-Loss Function				ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Normal	0.0149	0.0464	0.1074	0.2428	0.0063	0.0127	0.0164	0.0196
Student t	0.0096	0.0384	0.0954	0.2228	0.0060 (2)	0.0099 (2)	0.0135 (2)	0.0172 (2)
Clayton	0.0081	0.0357	0.0884	0.2178	0.0047 (1)	0.0086 (1)	0.0123 (1)	0.0160 (1)
Gumbel	0.0169	0.0464	0.1074	0.2438	0.0112	0.0151	0.0175	0.0202
Frank	0.0146	0.0452	0.1009	0.2308	0.0107 (3)	0.0141 (3)	0.0166 (3)	0.0191 (3)
Joe	0.0176	0.0482	0.1079	0.2418	0.0155	0.0171	0.0187	0.0208
DCC-norm	3.0806	2.0492	1.1797	0.2176	0.4927	0.4940	0.4905	0.4865
DCC-t	5.1497	3.4240	2.2057	1.0546	0.4249	0.4036	0.3885	0.3739

Note: Siemens: SIE, BMW: BMW, Linde: LIN, ThyssenKrupp: THY, Volkswagen:VOL. All models are ranked based on the minimum of MAE for ES on 1%, 2.5%, 5% and 10% significance level. The best models are highlighted by bold.

Table 113: Back-testing Term structure of Risk for Bivariate Data.

BMW/SEI

Model	5- Day VaR-Loss Fuction				10- Days VaR-Loss Function			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	147.40	100.23	69.712	69.712	286.52	196.00	130.47	77.734
Monte-Carlo(DCC)	144.97	100.12	68.249	39.439	279.82	192.07	132.67	81.66
	5-Day ES-MAE				10-Days ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	2.5397	1.9812	1.5846	1.2161	5.8295	4.1613	3.2499	2.4343
Monte-Carlo(DCC)	**2.0602**	**1.5428**	**1.1931**	**0.8637**	**4.1767**	**2.9914**	**2.3385**	**1.7267**

BMW/VOL

Model	5- Day VaR-Loss Fuction				10 -Days VaR-Loss Fuction			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	133.41	87.51	56.07	27.53	257.87	167.69	108.94	55.245
Monte-Carlo(DCC)	130.25	85.66	55.22	27.09	249.03	163.24	101.60	47.856
	5-Day ES-MAE				10-Days ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	2.3756	1.8391	1.4525	1.0907	5.7101	4.1102	3.1305	2.3149
Monte-Carlo(DCC)	**1.8858**	**1.3876**	**1.0462**	**0.7231**	**4.0191**	**2.8163**	**2.0577**	**1.3817**

BMW/THY

Model	5- Day VaR-Loss Fuction				10 -Days VaR-Loss Fuction			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	145.27	99.303	67.639	38.483	269.72	181.89	121.18	67.067
Monte-Carlo(DCC)	144.48	100.04	67.687	39.239	259.94	175.90	113.06	59.876
	5-Day ES-MAE				10-Days ES-MAE			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	2.4773	1.9410	1.5504	1.1915	5.6712	4.0545	3.0818	2.2887
Monte-Carlo(DCC)	**2.0225**	**1.5259**	**1.1820**	**0.8578**	**4.1577**	**2.9503**	**2.1927**	**1.5166**

BMW/LIN

Model	5- Day VaR-Loss Fuction				10 -Days VaR-Loss Fuction			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	170.57	120.72	85.618	50.386	292.68	194.06	128.54	73.44
Monte-Carlo(DCC)	185.00	133.75	133.75	60.331	307.48	204.84	136.69	75.613
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	2.8213	2.2183	1.7964	1.4115	6.0938	4.3549	3.3148	2.4566
Monte-Carlo(DCC)	**2.5842**	**2.0013**	**1.6053**	**1.2252**	**4.8904**	**3.4998**	**2.6382**	**1.8854**

SEI/VOL

Model	5- Day VaR-Loss Fuction				10 -Days VaR-Loss Fuction			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	105.07	62.032	34.734	9.299	190.24	118.202	78.168	37.937
Monte-Carlo(DCC)	1.2486	0.7159	0.5753	0.2651	114.19	88.377	65.980	43.166
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	2.0201	1.4978	1.1351	0.8101	4.4975	2.9756	2.2394	1.6163
Monte-Carlo(DCC)	**0.0811**	**0.0924**	**0.1018**	**0.1116**	**1.4760**	**1.1698**	**0.9850**	**0.7738**

SEI/THY

Model	5- Day VaR-Loss Fuction				10- Days VaR-Loss Fuction			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	116.86	73.67	46.14	21.229	248.59	165.62	108.01	61.405
Monte-Carlo(DCC)	3.3054	2.0463	0.9869	0.0121	11.502	5.1273	3.4474	1.5919
	5Day ES-MAE				**10Days ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	2.0837	1.5911	1.2402	0.9227	4.6169	3.1727	2.3588	1.7357
Monte-Carlo(DCC)	**0.0425**	**0.0324**	**0.0236**	**0.0132**	**0.1820**	**0.1098**	**0.0764**	**0.0503**

SEI/LIN

Model	5- Day VaR-Loss Fuction				10 -Days VaR-Loss Fuction			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	180.66	130.67	95.307	59.565	302.78	204.01	138.23	82.623
Monte-Carlo(DCC)	121.84	83.219	56.347	34.068	151.03	105.52	80.373	48.108
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	2.9233	2.3203	1.8984	1.5135	6.1958	4.4569	3.4168	2.5586
Monte-Carlo(DCC)	**2.0903**	**1.6138**	**1.2886**	**0.9919**	**2.9231**	**2.1367**	**1.6872**	**1.3173**

VOL/THY

Model	5- Day VaR-Loss Fuction				10- Days VaR-Loss Fuction			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	105.08	62.040	34.809	10.486	202.54	139.94	96.378	43.822
Monte-Carlo(DCC)	**1.2098**	**0.5469**	**0.2477**	**0.0867**	**1.0218**	**0.4214**	**0.1719**	**0.0485**
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	2.0202	1.4979	1.1352	0.8105	4.5084	3.0351	2.3687	1.7809
Monte-Carlo(DCC)	**0.1063**	**0.0959**	**0.0868**	**0.0760**	**0.3991**	**0.1922**	**0.9177**	**0.0849**

VOL/LIN

Model	5 -Day VaR-Loss Fuction				10- Days VaR-Loss Fuction			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	116.95	76.202	51.438	28.229	235.93	153.14	95.860	49.892
Monte-Carlo(DCC)	69.513	46.601	30.367	15.890	71.592	46.961	32.286	15.863
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	1.9558	1.4632	1.1123	0.7951	4.4998	3.1037	2.3601	1.7723
Monte-Carlo(DCC)	0.9038	0.6925	0.5381	0.3798	0.9524	0.7226	0.5575	0.3937

THY/LIN

Model	5- Day VaR-Loss Function	10- Days VaR-Loss Function

	1%	2.5%	5%	10%	1%	2.5%	5%	10%
Monte-Carlo(Static)	128.79	87.839	62.863	38.953	254.15	167.99	109.44	61.939
Monte-Carlo(DCC)	120.81	83.003	56.564	34.129	220.01	141.72	94.080	48.034
	5-Day ES-MAE				**10-Days ES-MAE**			
	1%	**2.5%**	**5%**	**10%**	**1%**	**2.5%**	**5%**	**10%**
Monte-Carlo(Static)	2.1867	1.6961	1.3545	1.0542	5.3775	3.8114	2.9068	2.1481
Monte-Carlo(DCC)	2.0897	1.6130	1.2884	0.9924	4.9913	3.4633	2.5864	1.8851

Note: Siemens: SIE, BMW: BMW, Linde: LIN, ThyssenKrupp: THY, Volkswagen:VOL. All models are ranked based on the minimum of MAE for ES on 1%, 2.5%, 5% and 10% significance level. The best models are highlighted by bold.

Table 114: Estimates from the Univariate GARCH-Normal Models.

Parameters	Conditional Variance Equation with normal Innovation				
	BMW	**SEI**	**VOL**	**THY**	**LIN**
α_0	0.0007 (0.0002)	0.0001 (0.0000)	0.0001 (0.0001)	0.0000 (0.0000)	0.0000 (0.0000)
α_1	0.0849 (0.0086)	0.0826 (0.0097)	0.0958 (0.0083)	0.0926 (0.0176)	0.0696 (0.0080)
β_1	0.9255 (0.0081)	0.9270 (0.0089)	0.9044 (0.0092)	0.9119 (0.0180)	0.9346 (0.0090)
γ	1.2910 (0.1564)	1.0421 (0.1188)	1.3995 (0.1463)	1.4875 (0.2267)	1.3071 (0.1935)
Q(5)	15.19 (0.0003)	4.665 (0.1819)	9.331 (0.0136)	15.150 (0.0004)	11.933 (0.0029)
Q2(5)	5.336 (0.1283)	0.1945 (0.9930)	3.005 (0.4061)	4.8518 (0.1653)	2.979 (0.4109)
AIC	-5.1120	-5.1722	-4.9122	-4.9429	-5.4676

Siemens: SIE, BMW: BMW, Linde: LIN, ThyssenKrupp: THY, Volkswagen: VOL. Standard errors are presented in parenthesis. All models are ranked based on smallest AIC. AIC is Akaike information criterion, and BIC is Bayesian information criterion.

Table 115: Estimates from the Univariate GARCH-t Models.

Parameters	Conditional Variance Equation with t distribution Innovation				
	BMW	**SEI**	**VOL**	**THY**	**LIN**
α_0	0.0000 (0.0000)	0.0001 (0.0001)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)
α_1	0.0758 (0.0110)	0.0750 (0.0141)	0.1072 (0.0113)	0.0889 (0.0132)	0.0719 (0.0125)
β_1	0.9368 (0.0100)	0.9391 (0.0120)	0.9016 (0.0109)	0.9218 (0.0131)	0.9351 (0.0139)
γ	1.2922 (0.2023)	0.9937 (0.1828)	1.2787 (0.1662)	1.3248 (0.1922)	1.4408 (0.2434)
df	6.406 (0.5926)	6.4361 (0.5605)	5.9080 (0.5008)	6.0429 (0.5291)	5.2149 (0.4189)
Q(5)	14.60 (0.0005)	4.242 (0.2254)	9.268 (0.0141)	15.0100 (0.0004)	11.497 (0.0037)
Q2(5)	12.551 (0.0135)	0.3602 (0.9770)	2.2037 (0.5711)	7.2432 (0.0452)	2.423 (0.5225)
AIC	-5.1555	-5.2428	-4.9686	-4.9964	-5.5300

Siemens: SIE, BMW: BMW, Linde: LIN, ThyssenKrupp: THY, Volkswagen: VOL. Standard errors are presented in parenthesis. All models are ranked based on smallest AIC. AIC is Akaike information criterion, and BIC is Bayesian information criterion.

Table 116: Estimates from the Univariate GARCH-Skewed *t* Models.

Parameters	Conditional Variance Equation with skewed t distribution Innovation				
	BMW	**SEI**	**VOL**	**THY**	**LIN**
α_0	0.0000 (0.0000)	0.0001 (0.0001)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0000)
α_1	0.0758 (0.0218)	0.0746 (0.0138)	0.0109 (0.0114)	0.0894 (0.0133)	0.0728 (0.0125)
β_1	0.9368 (0.0205)	0.9394 (0.0117)	0.9009 (0.0109)	0.9215 (0.0131)	0.9345 (0.0138)
γ	1.2923 (0.2957)	0.9950 (0.1803)	1.2703 (0.1669)	1.3206 (0.1921)	1.4420 (0.2420)
Skew	1.000 (0.0173)	0.9799 (0.0195)	1.0344 (0.0202)	1.0115 (0.0195)	1.0474 (0.0197)
Shape	6.4059 (0.6079)	6.4578 (0.5634)	5.8526 (0.4932)	6.0204 (0.5268)	5.2133 (0.4182)
Q(5)	14.60 (0.0005)	4.238 (0.2258)	9.2130 (0.0146)	14.98 (0.0004)	11.480 (0.0038)
Q2(5)	12.551 (0.4135)	0.3762 (0.9750)	2.1361 (0.5866)	7.1428 (0.4478)	2.2390 (0.5632)
AIC	-5.1550	-5.2426	-4.9684	-4.9961	-5.5302

Siemens: SIE, BMW: BMW, Linde: LIN, ThyssenKrupp: THY, Volkswagen: VOL. Standard errors are presented in parenthesis. All models are ranked based on smallest AIC. AIC is Akaike information criterion, and BIC is Bayesian information criterion.

Table 117: Estimates from the Univariate GARCH-GED Models.

Parameters	Conditional Variance Equation with Generalized Error Distribution Innovation				
	BMW	**SEI**	**VOL**	**THY**	**LIN**
α_0	0.0000 (0.0000)	0.0001 (0.0000)	0.0001 (0.0001)	0.0000 (0.0000)	0.0000 (0.0000)
α_1	0.0803 (0.0122)	0.0792 (0.0176)	0.1030 (0.0109)	0.0923 0.0115	0.0717 (0.0157)
β_1	0.9315 (0.0116)	0.9337 (0.0155)	0.9010 (0.0113)	0.9155 (0.0115)	0.9338 (0.0187)
γ	1.2810 (0.2025)	1.0288 (0.1930)	1.3226 (0.1704)	1.3921 (0.1828)	1.3768 (0.3037)
Shape	1.3187 (0.0386)	1.2970 (0.0355)	1.3002 (0.0367)	1.2907 (0.0371)	1.2041 (0.0367)
Q(5)	14.92 (0.0004)	4.430 (0.2051)	9.407 (0.0130)	13.72 (0.0001)	11.569 (0.0036)
Q2(5)	6.997 (0.5190)	0.2589 (0.9877)	2.3910 (0.5293)	5.4468 (0.1210)	2.631 (0.4786)
AIC	-5.1571	-5.2335	-4.9655	-4.9965	-5.5349

Siemens: SIE, BMW: BMW, Linde: LIN, ThyssenKrupp: THY, Volkswagen: VOL. Standard errors are presented in parenthesis. All models are ranked based on smallest AIC. AIC is Akaike information criterion, and BIC is Bayesian information criterion.

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