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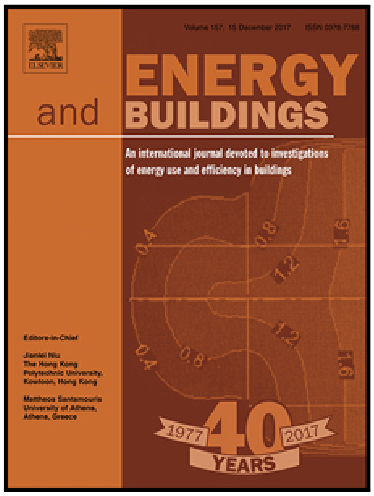

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QUB: a fast dynamic method for in-situ measurement of the whole building heat loss

Florent Alzetto[a,*], Guillaume Pandraud[b], Richard Fitton[c], Ingo Heusler[d], Herbert Sinnesbichler[d]

[a]*Saint-Gobain Recherche, 39 quai Lucien Lefranc, 93303 Aubervilliers Cedex, France*
[b]*Saint-Gobain Isover CRIR, B.P. 10019, 60291 Rantigny Cedex, France*
[c]*Energy House Test Facility, School of the Built Environment, G16a, Cockcroft Building, University of Salford, Salford, M5 4WT, UK*
[d]*Fraunhofer Institute for Building Physics (IBP), Holzkirchen Branch, Fraunhoferstr. 10, 83626 Valley, Germany*

Abstract

QUB is an innovative method enabling the experimental measurement of the total heat loss coefficient (HLC) of a building envelope in one night only. It is based on a simple theory, yet can be demonstrated to be accurate even in a short time and in real buildings, as long as certain experimental conditions are fulfilled.

This study combines analytical and numerical approaches to exactly solve the temperature response of an equivalent building submitted to a QUB test. This allows understanding that even with a short time experiment (less than a night), a reasonable accuracy on the estimated HLC can be obtained. The experiment has to be designed following a simple heating power criterion.

Calculation is then tested experimentally in various cases whether in climate chamber or in real field, and whether on light weight/not insulated

*Corresponding Author
Email address: florent.alzetto@saint-gobain.com (Florent Alzetto)

building or a heavy weighte/highly insulated building. Results show that the QUB method performed by fulfilling this criterion is a promising method to estimate the HLC of a real building in the field with a reasonable accuracy in one night.

Keywords:
Building thermal performance, Coheating, Dwellings, Energy, In-situ measurement, Heat Loss Coefficient (HLC), Performance Gap, RC network, Thermal quadrupole

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1. Introduction

Reducing energy consumption in buildings is one of the main ways of addressing issues around global energy consumption. Technical solutions to reduce consumption generally fall into two main groups; those aimed at optimising the services in the building, such as lighting and HVAC, whilst others are centred on improving the fabric efficiency of the building, such as improved insulation, glazing and airtightness improvements. This paper will focus on the latter category of fabric improvements to a building.

When buildings are constructed or renovated with energy performance as a central design outcome, their performance is generally modelled/predicted at design stage, often as part of a regulatory requirement.

This paper presents a method of validating this prediction using an indicator, representing not the energy consumption, but the intrinsic thermal loss of a building envelope. This indicator is the Heat Loss Coefficient (HLC), expressed in Watts per Kelvin, which represents the thermal power loss due to the thermal difference between interior and exterior temperatures (independently of the solar radiation), divided by this temperature difference. The HLC is the sum of two effects: transmission losses and air infiltration losses. It is a global parameter, related to the overall building envelope, and, therefore, is the sum of the losses through all envelope components.

The main advantage of the HLC is that it is a parameter simple enough so that it can actually be measured in several ways, for example by a co-heating test [1], by the PStar method [2], or by identification methods [3], among others [4, 5]. Additionally, its theoretical value can be easily calculated for a building. It is thus possible to compare design and measured value of this parameter and estimate a performance gap related to the building envelope.

These measurement methods of the HLC have the same drawback: the measurements require several days or even weeks. This might be reasonable for a research project, but it cannot be applied at a larger scale, which means that these methods have a limited application. It is, therefore, important to find a method that would be both much faster, and as reliable as the other ones. The method presented in this paper, called QUB [4, 6, 7, 8, 9, 10], aims at reaching that goal.

QUB has been shown to be able to measure the HLC of a dwelling in at most 48 h [7, 10] and even less [11]. This method can, in principle, be adapted to any kind of building. This may seem deeply counter-intuitive as buildings have time constants that can be much longer than this value. For this reason, this paper presents theoretical, numerical and experimental evidences for the validity of the QUB method in three stages. In the first, RC models are used to explain QUB in a simple way that enables a good understanding of the theoretical bases and the proper experimental conditions for the test. For that purpose, experimental investigations have been carried out in the Energy House of the University of Salford. In a second stage, another approach is shown using a quadrupole model. This method is more complex than the first but is also more detailed, and yields important results for the optimization of the QUB test. Finally the theoretical model is validated using several cases. Experimental investigations have been carried out at a small scale in real field in a bungalow located at Saint-Gobain Recherche, at a full scale dwelling inside a climatic chamber in the Energy House at the University of Salford and at a full scale in the field in one of the Twin houses at Fraunhofer IBP. While limited in number, these examples are different enough to indicate a possible use in a large number of configurations.

2. RC models

2.1. Description of the QUB method

The QUB method is a dynamic analysis method in which the HLC, denoted H_{tot}, is calculated by using the interior air temperature response to two consecutive internal thermal loads. The simplest model one can use to represent a body submitted to transient heat transfer is probably the lumped capacitance analysis with internal energy generation. It supposes that the interior of the body is at homogeneous temperature, that all exchanges happen with a medium of homogeneous temperature through an infinitely thin interface, and that the exterior temperature is constant. Thus, it is an RC model with only one resistance and one capacity. The result is the well-known equation:

$$C_{\mathrm{QUB}}\frac{\mathrm{d}T^*}{\mathrm{d}t} = \varPhi - H_{\mathrm{QUB}}T^* \tag{1}$$

where C_{QUB} is the apparent internal heat capacity [6, 12] of the body in J/K. It corresponds to the total energy stored in the body, going from one steady-state to an another, when its interior temperature increases by 1 K. $\varPhi$ is the internal power in W brought by all internal heating sources , H_{QUB} is the HLC identified with this method and T^* is the difference between the interior and exterior temperatures in K. If two separate experiments (1) and (2) are done, with two different powers, and if we assume H_{QUB} and C_{QUB} to be constant during these two experiments, then:

$$H_{\mathrm{QUB}} = \frac{T'_{(1)}\varPhi_{(2)} - T'_{(2)}\varPhi_{(1)}}{T'_{(1)}T^*_{(2)} - T'_{(2)}T^*_{(1)}} \tag{2}$$

$$C_{\mathrm{QUB}} = \frac{\varPhi_{(1)}T^*_{(2)} - \varPhi_{(2)}T^*_{(1)}}{T'_{(1)}T^*_{(2)} - T'_{(2)}T^*_{(1)}} \tag{3}$$

where $T' = \mathrm{d}T^*/\mathrm{d}t$. Thus it is quite easy in this simple case to calculate H_{QUB} from only two experiments with two different interior heat loads. Of course, such a model is too crude to represent the real behavior of a building; more nodes are needed for that. The model we then use is a larger RC network with an indefinite number of nodes n (but with a unique internal ambient temperature, hence homogeneous inside the building). The problem takes the form of a system of n differential equations with n unknown temperatures. It is well-known that the temperatures evolution in time is a summation of n time exponential decays. If we focus on the interior node, the long-term temperature value is given by $\lim_{t\to\infty} T^*(t) = \varPhi/H_{\mathrm{tot}}$ in the case of heating of constant power $\varPhi$. The general solution takes therefore the form:

$$T^*(t) = \frac{\Phi}{H_{\text{tot}}} + \left[T^*(0) - \frac{\Phi}{H_{\text{tot}}}\right] \sum_{i=1}^{n} a_i e^{-t/\tau_i} \tag{4}$$

where τ_i are time constants (it will be assumed here that they have an increasing value from τ_1, the smallest time constant, to τ_n, the largest) and a_i are constants depending on model resistances and capacitors and on initial conditions.

By injecting Eq. (4) in Eq. (2) it is easy to reach the conclusion that $H_{\text{QUB}} = H_{\text{tot}}$ if:

$$\frac{\sum_{i=1}^{n} \left[a_{i,(1)}/\tau_i\right] e^{-t_{(1)}/\tau_i}}{\sum_{i=1}^{n} a_{i,(1)} e^{-t_{(1)}/\tau_i}} = \frac{\sum_{i=1}^{n} \left[a_{i,(2)}/\tau_i\right] e^{-t_{(2)}/\tau_i}}{\sum_{i=1}^{n} a_{i,(2)} e^{-t_{(2)}/\tau_i}} \tag{5}$$

Equation (5) is obviously true if $n = 1$, but it must be noted that it also becomes true when $t_{(1)}$ and $t_{(2)}$ increase enough so that all values of $\exp(-t/\tau_i)$ become negligible except $\exp(-t/\tau_n)$. This means that after a sufficient time t_{L} such as $t_{\text{L}} \gg \tau_{n-1}$, the problem with multiple nodes and time constants can be treated as if only one time constant existed. If this sufficient time is shorter than a night, then the QUB method can be applied experimentally when solar radiation is nil. This verification, mostly done experimentally, is presented in 2.2.

2.2. Experimental setup

The QUB method is based on simple equations and considerations. Some of them have an important influence on the way the tests have to be done. For instance, in Part 2.1, the thermal power is considered constant and known with accuracy. This means that it is important to eliminate or reduce all sources of uncertainty. The most important step in that direction is to do the test during the night without occupancy. Without solar or internal loads, the heat source used for the test can be measured with accuracy, especially if it is an electrical heater, and in particular a simple Joule effect heater with low inertia. Other heating systems either require conversion coefficients (gas boiler, wood burner, heat pumps...) or decrease the accuracy of the test by reducing the knowledge of the instantaneous power dissipated (inertial heating). However, even with an electrical heater, it is essential to measure the real power consumed. Indeed, the voltage cannot be assumed to be equal to its theoretical value. For instance, a deviation of 5 % of the network voltage leads to a larger deviation of Φ and thus of H_{QUB} of about 10 %. This means that it is difficult to guarantee a good accuracy of the test results unless the heating source is not the one already installed in the house, but is a specific test material that is brought into the building.

Besides, the interior ambiance is considered to be a single node. The internal temperature is thus implicitly considered homogeneous, even if there are several rooms or even floors. But heating a building in a way that the temperatures in all rooms are identical, or at least close to each other, is a difficult task in a dynamic test. It requires the power to be adapted to each room. There are two ways to do this. The first is to regulate the power of each heating element, depending on the temperature of the room in which it is placed. It is thus possible to ensure a perfectly homogeneous heating, but the system required to do this is rather complex. The second way is a heating source that can be easily adapted to each room's surface. This solution has been used and consists in a large number of small power heat sources (approximately 100 W), placed in a way designed to maximize the convective heating, and reduce direct heating of the walls by radiation, or of the ground by conduction (see Fig. 1). An alternative would be to use usual fan heaters with various powers to be selected. In all cases, it is important to ensure a significant temperature difference between the internal and external environments to insure a relatively homogenous temperature in the entire building. The total installed power shall be calculated by optimizing the value of the parameter α presented in section 3 (using the design heat loss figure for the building, or when this is not available, a simple steady state heat loss calculation, for example RdSAP in the UK [13]). This installation has been shown [10] to improve reliability and reproducibility.

The QUB method requires two different powers to be applied. For practical rather than theoretical reasons (mainly related to the possibilities of the equipment used), most of our tests are done with 100 % of the installed power in the first stage and 0 % in the second. But these two stages have to be done during the night. There are two ways to do this. The first is to have each stage during an entire night, which leads to a test duration of about 36 h (less than 48 h with preparation and clean-up); the second is to have both stages in the same night, for a test duration of 8 to 12 h (less than 24 h in total).

Eq. (5) shows that test duration must be as long as possible, but also that if we assume that there is no strong influence of the initial conditions on the values, that is to say if $\forall i, a_{i,(1)} \approx a_{i,(2)}$, then $H_{\mathrm{QUB}} = H_{\mathrm{tot}}$ if $t_{(1)} = t_{(2)}$. For this reason, and because it has been shown to lead to more accurate and more reproducible experimental results [10], this condition is used in all tests in this paper. Furthermore, each variable of Eqs. (2) and (3) is

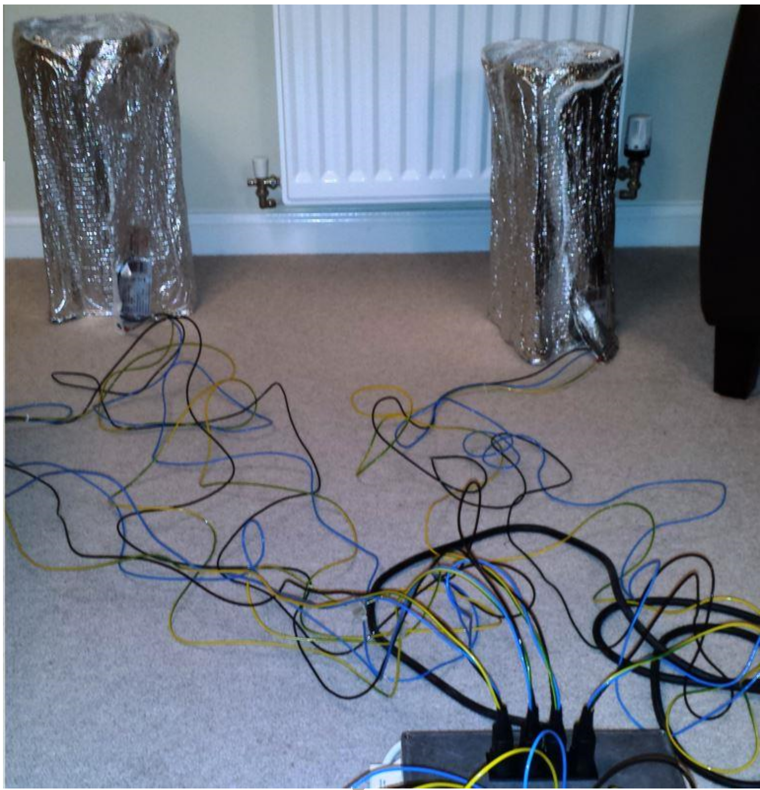


Figure 1: 110W aluminum heat mats connected to boxes allowing to switch simulteously on or off at requested times.

calculated at the end of each stage. This data analysis period must be long enough to reduce the measurement noise, but short enough to ensure that the calculated data are representative. There is no absolute optimal value for this duration; it must be evaluated on a case-by-case basis depending on the measurement noise and the duration of each stage.

2.3. Validation of the QUB method

Although several validation cases exist, either on numerical [9] or on real [8, 9] buildings, the one presented here is probably the most conclusive. It was carried out at the Energy House at the University of Salford [14]. The Energy House shown in Fig. 2 is constructed to meet the specification of a typical 1910 terraced property from the UK that has been through reasonable modifications. The house is located inside a well-insulated concrete chamber which has a solid concrete floor. It consists of a test house, connected via a party wall to a smaller neighboring building. The heating system is a gas condensing combination boiler fed via a wet system to radiators in each room in the test house and electric panel heaters in the neighboring house. The chamber itself is cooled by an air handling unit that is supplied with cooling by 4 condenser units, with a total of 60 kW of cooling (15 kW per unit). This is supplied to the chamber via a ducted HVAC system. This system reacts to the heat load of the house in the chamber and maintains the temperature in a range of $\pm$ 0.5 K around the setpoint. Tests have been done under different configurations, two are presented here: with and without ceiling level insulation. Additional technical information on the Energy House can be found in [14, 7].

The Energy House is therefore a real building which can be submitted to either a variable or a constant external temperature without solar radiation nor wind [15]. It can therefore be used to measure the value of H_{tot} in steady-state conditions, and compare the value obtained with the QUB method with a reference having a low uncertainty—something that is complicated to have in a building in external conditions. An example of a steady-state measurement is presented in Fig. 3. In this case, the HLC is calculated by simply dividing the power by the temperature difference between inside and outside, both values averaged over the considered 12-hour long period

Figure 2: Energy House of the University of Salford. It is a full size typical Victorian House build inside a climate chamber.

(between the two vertical solid black lines).

Such a calculation can be considered to give reference values H_{ref} of the HLC H_{tot}. An example of inside temperature evolution during a QUB test is presented on Fig. 4. It shows the average (weighted by room volume) temperature measurements in the house, and curves derived from an RC model with two time constants. One is the best fit found called Fit, and one is the same model with only the largest time constant identified call Trend. It thus shows the exponential trend towards which the model tends. The model used is derived from Eq. (4) by keeping only two time constants. This corresponds to a RC model with two capacitors and a minimum of two resistances.

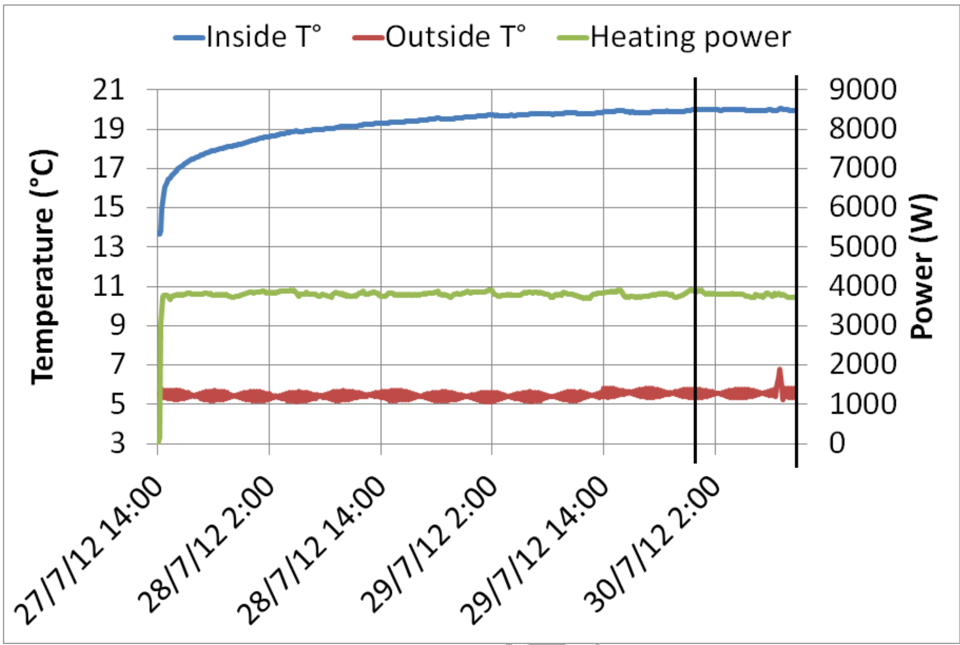


Figure 3: Steady-state measurements at the Energy House. The blue line is the average inside temperature, the red is the outside temperature and the green is the heating power. The period used for average is delimited by the two solid black lines.

Fig. 4 shows that the model fits the data rather well; although three or more time constants would be needed for a perfect fit, two seem sufficient in these specific conditions to describe the behavior of the air temperature. Furthermore, the first time constant is around 23 minutes and has significant effects for only an hour in this specific case. After that, the temperature behaves as a single exponential function. This tends to confirm the logical reasoning presented in Part 2.1 and thus show that the QUB method can indeed be applied.

Yet showing that QUB can be applied does not mean that it actually works. For that, it is necessary to compare the results of the QUB method

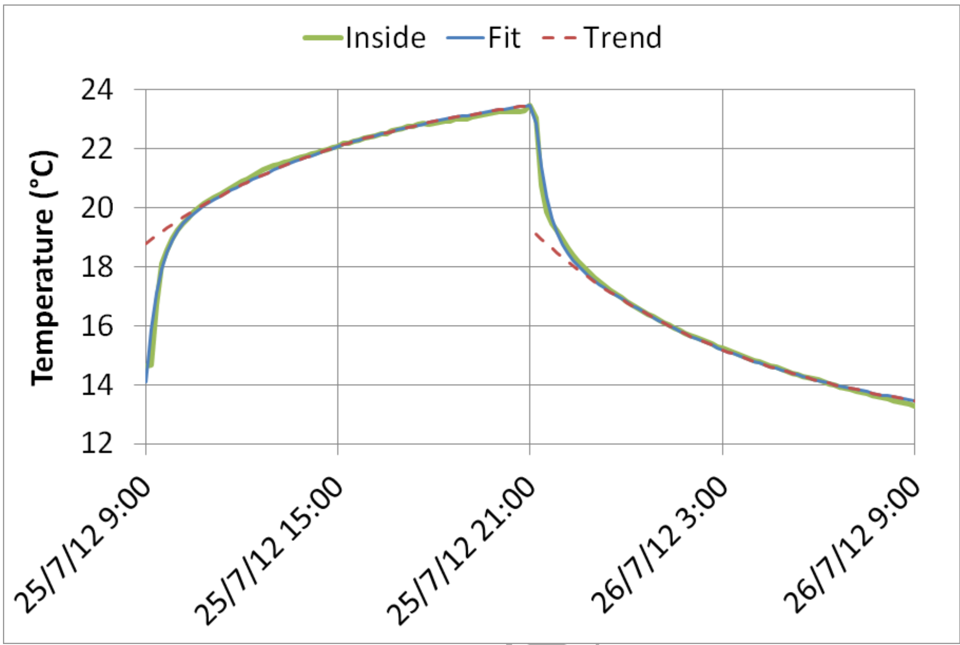


Figure 4: QUB test at the Energy House. The solid green lines represents the average inside temperature, the solid blue is the best fit obtained using a two time constant model and the dashed red is the trend using only the largest time constant of the best fit.

with the reference given by the steady-state measurements. The results of three different QUB tests are presented in Table 1. They show each test's characteristics, the durations of the heating and the cooling phases, the reference values and the results of the QUB test.

Uncertainty for the HLC obtained from the static and QUB test was calculated by error propagation of the uncertainty associated with the measured variables Φ and T in Eq. (2). The differences between H_{ref} and H_{QUB} are low, which is strong evidence of the reliability of the QUB method. The theoretical basis of the QUB method and its experimental feasibility and accuracy are therefore proven.

Case	1	2	3
$t_\mathrm{h} = t_\mathrm{c}$ [h]	12	8	12
Roof insulation?	No		Yes
H_ref [W/K]	263.9 ± 2.7		209.5 ± 2.3
H_QUB [W/K]	255 ± 9	264 ± 8	216 ± 7

Table 1: Results of three measurements at the Energy House in two different building configurations.

Yet some important questions remain, in particular, about the relation between the error, the building characteristics, and the test duration. For instance, H_QUB is theoretically equal to H_ref if $t_\mathrm{h} = t_\mathrm{c} > t_\mathrm{L}$. But the model described in Part 2 does not say how large the error is if $t_\mathrm{h} = t_\mathrm{c} < t_\mathrm{L}$. To have an idea about this, tests with different durations have been done in Salford, with t_h being as low as 0.5 h. The results of several such tests, compared to the reference values, are presented in Fig. 5. The dots are QUB results and the red lines are the reference ± 10 %. They show that results can be good even with the shortest durations. As this effect is not anticipated by the simple RC model, a more complex one has to be developed. This new model and its validation are presented in part 3 and 4.

3. Quadrupole model

In order to understand the behavior of the building for the short times, a different model has been developed. It is based on a quadripolar description of the monodimensional heat transfer through a wall [16]. The principle of this approach is to describe the heat equation in the Laplace frequency space.

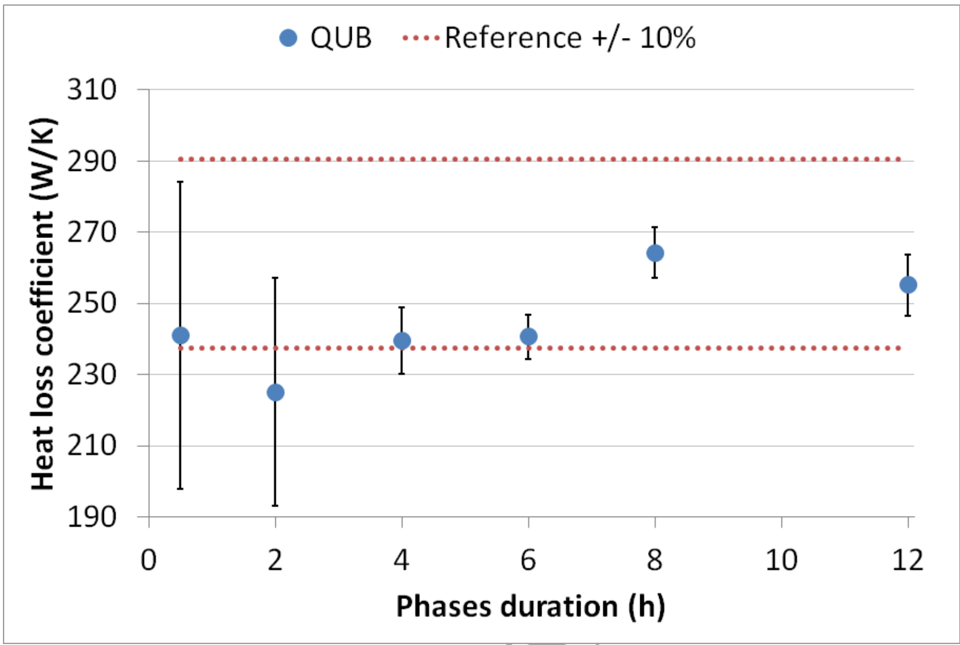


Figure 5: Results of QUB tests of different durations at the Energy House in Salford. The blue dots are the HLC results obtained via the QUB method as a function of the heating and cooling durations and the dashed red lines delimit the reference value ±10 %.

In the frequency space, the equations for the temperatures and heat fluxes can by solved easily, quickly, and exactly. The solution in the time domain is calculated by inverse Laplace transforms of the frequency space solution. For a monodimensional heat transfer, this can be done semi-analytically (it still needs a numerical integration in the complex space). The main advantage of this approach is that there is no differential equation to be solved, so there is no discretization in time and space, which is an approximation of the diffusive process (due to an insufficient number of resistances and capacitors in the nodal network formalism). The main drawback is that analytical expressions are needed in the Laplace domain for all the boundary conditions (temperatures and/or heat fluxes).

In the case of the QUB method, we focus on the understanding of what happens for the shortest durations. This does not require a more detailed spatial analysis of the case, but rather a better description of the dynamic properties of the heat equation than with simpler RC models.

3.1. A quadrupole model of the QUB method

In this section we describe the physical model chosen using the thermal quadrupole formalism and provide the main equation to be solved in the frequency space. We consider the case of a semi-infinite slab of thickness e represented in Fig. 6. The outer face (noted out) is at a constant temperature during the experiment whereas we use a thermal load on the inner face (noted in) as in a QUB test. To prepare the initial state we consider first a constant power P_0 until the time t_0 preceding the QUB test. The QUB test then starts in a first phase with a constant load of power P_{h} on the inner face during a time t_{h}. Then the second phase lets the temperature evolve freely without any power for the same duration t_{h}. A steady state at the beginning of the QUB test is therefore obtained by letting t_0 tend to ∞. A representation of the power evolution is provided in Fig. 7.

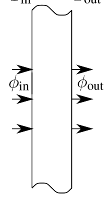


Figure 6: Representation of a semi-infinite slab of a homogeneous material

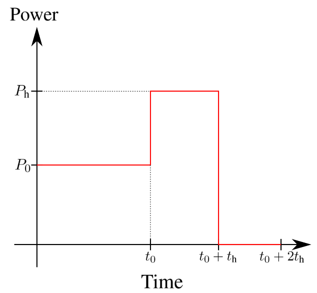


Figure 7: Power on the inner face as a function of the time

The temperature response of the inner face is then fully described by the thermophysical properties of the homogeneous material and the boundary conditions. The properties are the thermal conductivity λ, the specific heat capacity c and the density ρ. These three parameters can be combined with the thickness to obtain the thermal resistance $R = e/\lambda$ and the thermal characteristic time of the slab $\tau = e^2\rho c/\lambda$. The boundary conditions are the power evolution of the inner face and the temperature evolution of the outer face. The interior temperature is $T^*(t) = T_\mathrm{in}(t) - T_\mathrm{out}$, where T_out is supposed to be constant.

Calling $\theta(p)$ and $\phi(p)$ the Laplace transforms of T^* and Φ, these two boundary conditions can be written as:

$$\theta_\mathrm{out}(p) = 0 \tag{6}$$

$$\phi_\mathrm{in}(p) = \frac{P_0}{p} + \frac{P_\mathrm{h} - P_0}{p} e^{-pt_0} + \frac{-P_\mathrm{h}}{p} e^{-p(t_0+t_\mathrm{h})} \tag{7}$$

If the slab is supposed to be constituted of N different layers in series, then standard quadrupole theory [16] gives the following relationship between the interior and exterior temperatures and fluxes:

$$\begin{bmatrix} \theta_{\text{in}}(p) \\ \phi_{\text{in}}(p) \end{bmatrix} = \begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix} \cdot \begin{bmatrix} \theta_{\text{out}}(p) \\ \phi_{\text{out}}(p) \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} A(p) & B(p) \\ C(p) & D(p) \end{bmatrix} = \prod_{i=1}^{N} \begin{bmatrix} \cosh\left(\sqrt{p\tau_i}\right) & \frac{R_i}{\sqrt{p\tau_i}} \sinh\left(\sqrt{p\tau_i}\right) \\ \frac{\sqrt{p\tau_i}}{R_i} \sinh\left(\sqrt{p\tau_i}\right) & \cosh\left(\sqrt{p\tau_i}\right) \end{bmatrix} \tag{9}$$

By injecting Eqs. (6) and (7) in Eq. (8), the temperature of the inner face is given by:

$$\theta_{\text{in}}(p) = \frac{B(p)}{D(p)} \left[\frac{P_0}{p} + \frac{P_{\text{h}} - P_0}{p} e^{-pt_0} + \frac{-P_{\text{h}}}{p} e^{-p(t_0+t_{\text{h}})} \right] \tag{10}$$

Knowing the thermophysical properties of the different materials, Eqs. (9) and (10) describe the exact temperature behavior of the inner face in the frequency space. This model describes thermal conduction through an assembly of homogeneous layers. It does not include air infiltration neither parallel conduction heat transfers. Using this approach we could define such a model which would be more representative of a building but it would be more complex to get a semi-analytical solution in the time domain. In the next section, we show how to calculate the temperature evolution in time during a QUB test and the consequence of the model reduction to a simple RC network.

3.2. Semi-analytical solution and consequence on QUB

Going back to the definition of inverse Laplace transform, the inversion to the time domain rests on the identification of the poles of θ_{in} which are obviously 0 and the poles of $B(p)/D(p)$ and their associated residues.

It has been shown [16] that each individual layer i can be described by an infinity of 2RC circuits in series. Using this description the associated thermal quadrupole is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \prod_{i=1}^{N} \lim_{n_i \to \infty} \left\{ \begin{bmatrix} 1 & \frac{R_i}{2n_i} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ p\frac{C_i}{n_i} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{R_i}{2n_i} \\ 0 & 1 \end{bmatrix} \right\}^{n_i} \tag{11}$$

Note that this is valid by taking the limit where the number n_i of two resistors and one capacitor pairs tend to infinity. In Eq. (11), it is straightforward to show that all the functions entering the matrix are degree n polynomial functions of p with positive and real coefficients, as are the functions entering the matrix in Eq. (9). So $B(p)$ is a holomorphic function of p in the complex plane and the roots of $D(p)$ have real negative parts. Furthermore, the general shape of the solution of this type of problems being known, it can be safely assumed that the temperature response is a sum of exponentially decaying functions [17]. This means that only residues of $D(p)$ will contribute. Each has to be a first order negative pole located at p_i and related to the time constant τ_i by $p_i = -1/\tau_i$.

By using the residue theorem, it is possible to write the following equation:

$$T^*(t) = \sum_{i>0} r_i e^{-t/\tau_i} + r_0 \tag{12}$$

where r_i is the residue of $\theta_{\text{in}}(p)$ for the pole p_i and r_0 the residue for the pole 0. Each τ_i is a time constant of the model which can be calculated numerically by solving:

$$D\left(-1/\tau_i\right) = 0 \tag{13}$$

The associated residue r_i can be calculated using:

$$r_i = \lim_{p \to -1/\tau_i} (p + 1/\tau_i)\, \theta_{\text{in}}(p) = \oint_{\Gamma_i} \theta_{\text{in}}(p)\, dp \tag{14}$$

It consists in calculating the contour integral in Eq. (14) where Γ_i is a contour circling $p_i = -1/\tau_i$ in the positive direction where the only singularity inside the contour is the one of θ_{in} located at $p_i = -1/\tau_i$. The residue of the pole at $p_0 = 0$ is straightforward using Eq. (9). Using Eqs. (14) and (10) we calculate the residues r_i during the two consecutive heat loads (it means for the time periods $t \in [t_0, t_0 + t_\text{h}]$ and $t \in [t_0 + t_\text{h}, \infty[$). The residues are:

$$r_i = \begin{cases} -\frac{B_n(-1/\tau_i)}{D'_n(-1/\tau_i)}\tau_i \left[P_0 + (P_\text{h} - P_0)e^{\frac{t_0}{\tau_i}}\right] & t \in [t_0, t_0 + t_\text{h}] \\ -\frac{B_n(-1/\tau_i)}{D'_n(-1/\tau_i)}\tau_i \left[P_0 + (P_\text{h} - P_0)e^{\frac{t_0}{\tau_i}} - P_\text{h}e^{\frac{t_0+t_\text{h}}{\tau_i}}\right] & t \in [t_0 + t_\text{h}, \infty[\end{cases} \tag{15}$$

$$r_0 = \begin{cases} \sum_{i=1}^{n} R_i P_\text{h} = R_\text{T} P_\text{h} & t \in [t_0, t_0 + t_\text{h}] \\ 0 & t \in [t_0 + t_\text{h}, \infty[\end{cases} \tag{16}$$

where $R_\text{T} = 1/H_\text{tot}$ is the sum of all the resistances in the wall.

In order to calculate the temperature evolution during the QUB test, it is necessary to give an initial condition not only of the interior temperature, but on the entire distribution of temperatures in the envelope. With this aim, a strong assumption, the consequences of which will be examined later, is made: we suppose that $t_0 \to +\infty$. It means that when the heating phase starts, the building is at a steady state with an internal temperature $T_0^* \equiv \lim_{t_0 \to +\infty} T^*_\text{in}(t_0) = P_0/R_\text{T}$. With this assumption it is possible to write the internal temperature evolution during the QUB test:

$$T^*_\text{in}(t) = \begin{cases} R_\text{T} P_\text{h} + (P_0 - P_\text{h}) \sum\limits_{i=1}^{n} s_i \tau_i e^{-\frac{t}{\tau_i}}, & 0 \le t < t_\text{h} \\ \sum\limits_{i=1}^{n} \left[P_\text{h} + (P_0 - P_\text{h})e^{-\frac{t_\text{h}}{\tau_i}}\right] s_i \tau_i e^{-\frac{t - t_\text{h}}{\tau_i}}, & t \ge t_\text{h} \end{cases} \tag{17}$$

where we note $s_i = B_n(-1/\tau_i)/D'_n(-1/\tau_i)$ and we perform the variable change $t + t_0 \to t$. Using Eq. (2) and Eq. (17) with temperatures and temperature derivatives for phase (1) evaluated at $t = t_\mathrm{h}$ and for phase (2) at $t = 2t_\mathrm{h}$, writing $\Phi_{(1)} = P_\mathrm{h}$ and $\Phi_{(2)} = 0$, and simplifying by introducing $\alpha = 1 - T_0^*/R_\mathrm{T}P_\mathrm{h}$ and $\beta_i = e^{-t_\mathrm{h}/\tau_i}$, H_QUB can be written in function of the total heat losses coefficient H_tot:

$$H_\mathrm{QUB} = H_\mathrm{tot}\frac{1}{1 - \alpha^2\frac{\sum\limits_{i>j} s_i s_j \beta_i \beta_j (\tau_i - \tau_j)(\beta_i - \beta_j)}{R_\mathrm{T}\sum\limits_i (1-\alpha\beta_i) s_i \beta_i}} \tag{18}$$

This model leads to the conclusion that there are two main ways to ensure that $H_\mathrm{QUB} = H_\mathrm{tot}$. The first, already reached with the first model, is to have long test durations. If t_h is larger than the second largest time constant, then all β_i except one tend to 0, and the second term in the denominator of Eq. (18) becomes nil. The second way is to have $\alpha = 0$. Taken directly, this simply means that $P_\mathrm{h} = P_0$, thus that the building stays at steady state during the heating phase, implying that the temperature slope is nil during this phase, which transforms Eq. (2) into the simpler $H_\mathrm{QUB} = \Phi_{(1)}/T^*_{(1)}$, which is an obvious conclusion in steady-state conditions.

Yet the consequences are more interesting that this simple equation. For instance, even though a steady state with $\alpha = 0$ is not physically achievable, it is possible to approximate it with $\alpha \to 0$, which should lead to $H_\mathrm{QUB} \approx H_\mathrm{tot}$ whatever is the test duration. On the other hand when α increases, the corrective factor differs from 1 and the error between H_QUB and H_tot increases, with a difference which is reduced when the test duration increases.

Of course, the shorter the test duration, the higher the importance of the initial conditions. Furthermore, low values of α also correspond to low amplitude excitations compared to initial conditions, which once again reinforce the importance of the initial conditions, in particular the hypothesis that the test starts from a steady state. Thus, it is important to understand the influence this hypothesis has on the QUB tests results, both theoretically and experimentally.

3.3. Numerical analysis of the quadrupole model

In order to illustrate the impact of t_{h} and α on the test result, a numerical application is performed with a semi-infinite multi-layered wall, for which inverse Laplace transform is done numerically using Eqs. (9), (13), (14) and (18). A three-layered wall is composed of a 12.5 mm thick plasterboard, 120 mm of insulation and 200 mm of brick. The internal node represents a simple volume of air of about 34 m^3 with an internal convection coefficient $h_{\mathrm{int}} = 10\,\mathrm{W/(m^2\ K)}$. A convective resistance $h_{\mathrm{ext}} = 25\,\mathrm{W/(m^2\ K)}$ between the outer concrete surface and the exterior node is also considered. With these parameters, the envelope HLC is about 12 W/K so $R_T \sim 0.0824\,\mathrm{K/W}$. All the thermophysical properties of the solid materials are given in Table 2.

	Plasterboard	Insulant	Brick
Thickness [mm]	12.5	120	200
$\lambda\,[\mathrm{W/(m\ K)}]$	0.35	0.035	0.39
$\rho\,[\mathrm{kg/m^3}]$	950	30	1150
$c\,[\mathrm{J/(m^3\ K)}]$	1000	1500	1000

Table 2: Thermophysical properties of the wall components namely the thermal conductivity, the specific heat capacity and the density of the plasterboard, the insulant and the brick.

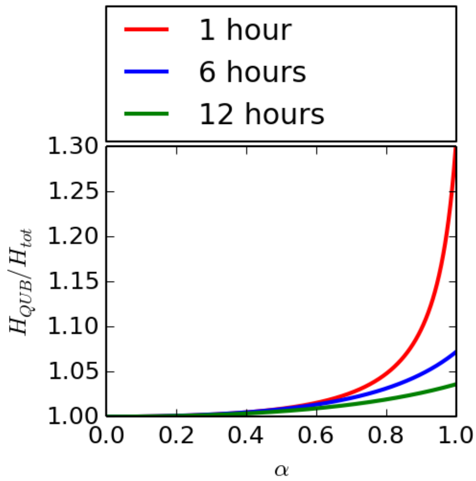
To describe the temperature response in time, we used Eqs. (9), (13) and (14) to compute the time constants longer than 20 minutes and their associated residue. We only keep the ones where the residue is significant. The figures are presented in Table 3 for the previous case (called IWI for internal wall insulation) and another case where the insulation and the brick have been switched (called EWI for external wall insulation).

	IWI case		EWI case	
i	$s_i\tau_i/R_T$	τ_i	$s_i\tau_i/R_T$	τ_i
1	65.97%	21 h 36 min	92.57%	10 d 7 h 37 min
3	30.63%	10 h18 min	2.60%	3 h 38 min
5	0.07%	1 h 42 min	0.70%	57 min
7	0.06%	38 min	0.07%	32 min
9	0.25%	28 min	0.32%	24 min

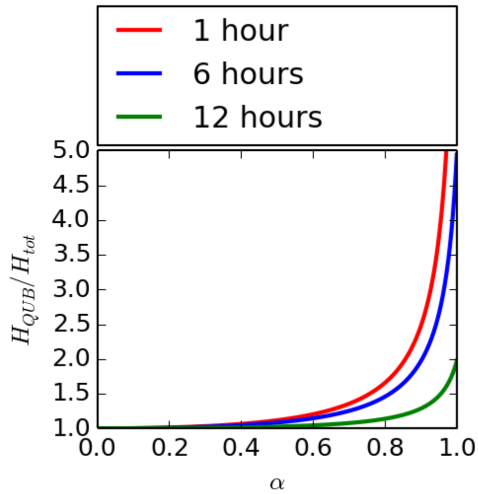
Table 3: Significant time constants and associated weights for the wall component models IWI and EWI. Only the time constants greater than 10 minutes are shown and the ones associated to significant weights.

Using the values in Table 3 and Eq. (18), we can calculate the error on a QUB test at a given heating duration as a function of α. Figure 8 represents the error of a QUB test($H_{\mathrm{QUB}}/H_{\mathrm{tot}}$) for these building envelopes as a function of α for three different durations: $t_{\mathrm{h}} = 1$ h, 6 h and 12 h.

These are extreme cases because in reality there is always a mix between lower and higher inertia systems. As the heat transfer happens in the different parts of the enveloppe in parallel most of the building will behave differently. Figure 8 shows that the HLC measured is overestimated and confirms that



(a) IWI case



(b) EWI case

Figure 8: $H_{\mathrm{QUB}}/H_{\mathrm{tot}} = f(\alpha)$ for IWI wall (8a) and EWI wall (8b), calculated using a numerical resolution of the quadrupole model. The blue lines are the results for 6 hours of heating and cooling, the green ones for 12 hours and the red for 1 hour.

increasing the heating duration will reduce the error during a QUB test. It also shows that the error increases with the inertia of the system.

These results are valid for an initial steady state before the QUB test. The same experiment can be done numerically without the strong hypothesis that the initial condition of the QUB test is a steady-state. In order to assess the effect of a non steady state before the measurement, we modify the power pattern defined in Fig. 7 by adding a zero power phase between the steady regime and the QUB test for a duration t_c. This corresponds to a QUB test after a few hours of free cooling. Using the same approach we can calculate the time evolution of the inside temperature which depends on the same time constants and residues shown in Table 3. Then we can calculate the results of a QUB test as a function of α and for different cooling durations before the QUB test. We show this evolution in Fig. 9 for a QUB test of 4 hours of heating and cooling, for the EWI case and for different duration of t_c. We impose the initial building temperature (before the QUB test) to be 20 $^\circ$C and temperature variations of at least 1 $^\circ$C during heating and cooling phases.

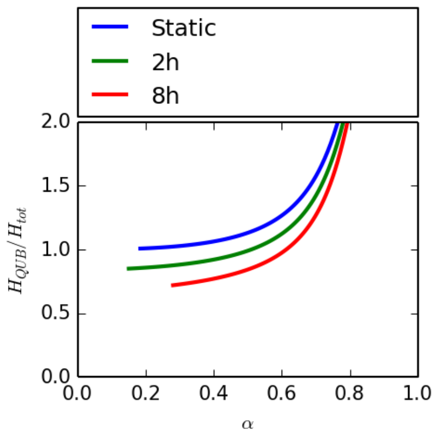


Figure 9: $H_{\rm QUB}/H_{\rm tot} = f(\alpha)$ for a heating and cooling durations of 4 hours in the EWI case, calculated using a numerical resolution of the quadrupole model. The blue line is the result starting from a static initial state, the green for 2 hours of cooling before the QUB and the red corresponds to 8 hours of cooling.

This more realistic model confirms that $H_{\rm QUB}$ presents a strong dependence on α, which is related to the fact that for the shorter measurements, several time constants play a role on the temperature evolution. By preventing large values of α, it is possible to have a correct measurement of the HLC even with a short test duration. It must be noted that the free cooling period before the beginning of the test also creates a underestimation of $H_{\rm QUB}$ for low values of α, although it is much less important than its overestimation at high values of α. These phenomena show that α values around 0.5 should be favorable. In the next section, we investigate experimentally these effects.

4. Experimental validation of the quadrupole model

For all QUB tests presented here, the same protocol has been applied. The temperature difference between the inside and the outside is always positive and the building is heated during the first phase then cooled down with no controlled power (but possibly residual power, like the measurement equipment consumption). The heating is performed using the small heating power sources shown in section 2.2. Temperatures are recorded with Pt100 sensors or aluminum-covered K-type thermocouples. Furthermore, as it has already been explained in section 2.2, heating and cooling phases last for the same duration such as $t_{(1)} = t_{(2)} = t_{\mathrm{h}}$. Several experiments have been presented in a proceeding [10]. The three described here are the ones for which the comparison of H_{QUB} with H_{ref} has been possible. The first is a small bungalow, the second is the Energy House at the University of Salford and the third is one of the Twin Houses at the Fraunhofer Institute of Building Physics.

4.1. Small scale building in real climate

The first test building is a bungalow located in Saint-Gobain Recherche at Aubervilliers, near Paris, France. The bungalow has a floor area of about 13.5 m^2, a volume of about 34 m^3 and a total heat loss area of about 68 m^2. The inertia is low as there is little furniture and the thermal mass mainly comes from plasterboard and glazings. Two kinds of experiments are performed to assess the HLC of this building.

The first one is a quasi-static measurement based on the co-heating methodology proposed by Leeds Metropolitan University [1]. The result of this test is used as a reference. The principle is to maintain the inside air of an unoccupied building at a constant temperature during at least two weeks and to analyze daily averages of energy consumption as a function of external weather conditions. Using a simple model that takes into account total heat losses and solar heat gains we identify the building parameters by performing the following linear regression:

$$\overline{\varPhi_{\mathrm{in}}} + g_{\mathrm{S}}\overline{\phi_{\mathrm{rad}}} = H_{\mathrm{ref}}\overline{T^*} \tag{19}$$

where $\varPhi_{\mathrm{in}}$ is the heat load in the building, g_{S} the solar factor in m^2 and ϕ_{rad} the solar heat gain, measured in $\mathrm{W/m}^2$. All overlined symbols are averaged over 24 h. The reference HLC calculated with this methodology is $H_{\mathrm{ref}} = 33 \pm 2$ W/K

The second experiment is a large number of QUB tests which have been performed during the first semester of 2013. Four different heating durations have been studied (30 min, 1 h, 2 h and 4 h) with different heating powers and initial temperature differences. This allows verifying the correlation of H_{QUB} with α. Results are shown in Figure 10.

Figure 10 confirms the qualitative results obtained from the quadrupole model. It is first possible to observe a strong dependency of H_{QUB} on α, with a low underestimation at low values of α and a high overestimation at high values of α. In both cases, the heating duration increase reduces the error, although it is much clearer for the overestimations (in part because

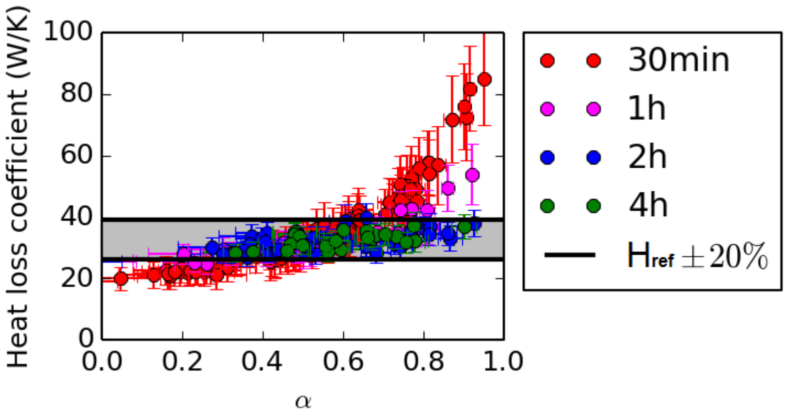


Figure 10: $H_{\text{QUB}} = f(\alpha)$ in SGR bungalows. The red, magenta, blue and green dots are obtained respectively for durations of 30 minutes, 1 hour, 2 hours and 4 hours. The solid black lines delimiting the grey zone corresponds to ± 20 % of the reference value.

low values of α are harder to reach than high values). On the other hand, for $\alpha \approx 0.4 - 0.7$, a good agreement between H_{QUB} and H_{ref} is obtained for all heating durations.

4.2. Real scale building in controlled climate

Additional tests have been done at the Energy House at the University of Salford, already presented in 2.3. The additional short tests have been done later than the longer ones, and the configuration of the house had slightly changed in between (modifications of the window frames and doors), thus the value of H_{ref} had to be measured again. The result is $H_{\text{ref}} = 229.2 \pm 2.4$ W/K.

Short QUB tests were performed with two different heating durations, 1 h and 4 h. As in section 4.1, various settings for the heating power and the initial temperature difference were used in order to have a variation of α. We show in Figure 11 the HLC measured using the QUB test as a function of α for the Energy House, compared to the reference H_{ref}.

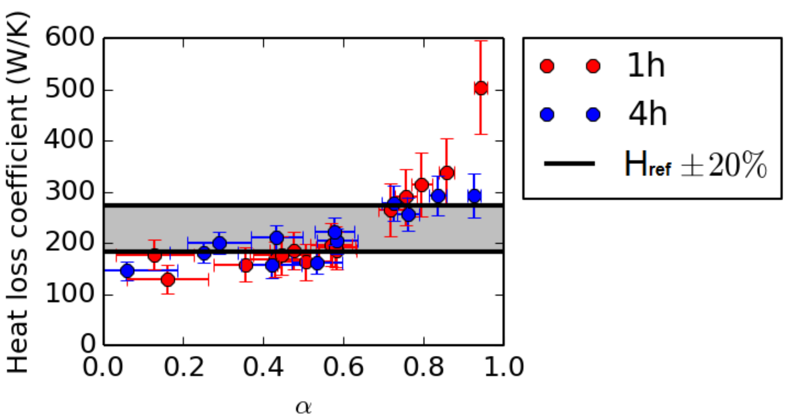


Figure 11: $H_{\mathrm{QUB}} = f(\alpha)$ in the Energy House at the University of Salford. The red and blue dots are obtained respectivelly for durations of 1 hour and 4 hours. The solid black lines delimiting the grey zone corresponds to ±20 % of the reference value.

Figure 11 confirms the previous qualitative conclusions, especially those obtained in the small building in real climate. First, the HLC measured increases with α. Second the error reduces when the heating duration increases. Most of the tests performed at α values between 0.4 and 0.7 are in good agreement with the reference measurement for both values of the heating duration. Finally, for low α values, H_{QUB} is lower than H_{ref}, which also confirms conclusions reached by numerical calculations.

It is important to note that if α has to be chosen between 0.4 and 0.7 during an experimental test, it means that the internal load must be between $1.7\,T_0^*/R_{\mathrm{T}}$ and $3.3\,T_0^*/R_{\mathrm{T}}$, which leaves a rather wide range of acceptable values. This explains why, even though experimental values of α should be controlled, it is often possible to have good results even if α has not been checked, as it was the case in section 2.3.

4.3. Real scale building in real climate

In spring 2014, in-situ tests were performed at one of the twin houses of the Fraunhofer Institute for Building Physics IBP at Holzkirchen near Munich. The house is a solid brickwork construction provided with an External Thermal Insulation Composite System (ETICS) of 8 cm (west/east facade) and 12 cm (south/north facade). It includes a basement, a ground floor and an attic space. The pitched roof is sloped by 30° and insulated with 16 cm of mineral wool. A view and section of the test house can be seen in Figures 12 and 13.

Figure 12: View of the test house

In order to obtain a reference HLC (all roller blinds closed), two methods were applied: A co-heating (baseline) measurement and an assessment

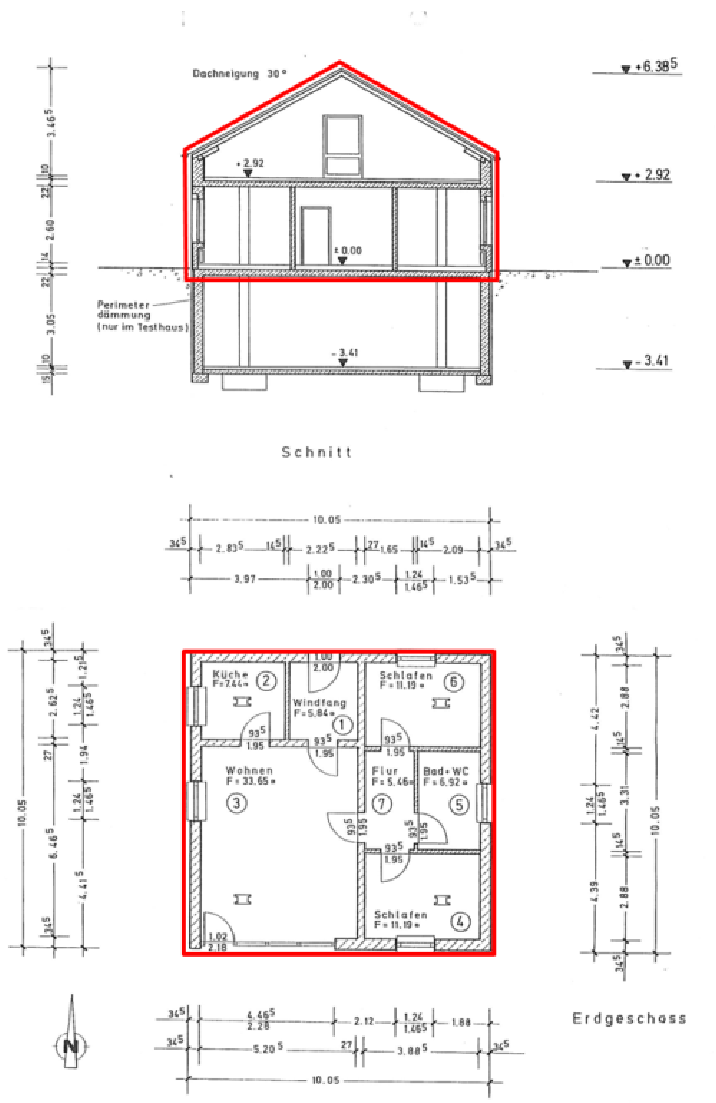


Figure 13: Section and ground floor plan of the test house

according to the German standard for the energy certificate of buildings under public law DIN V 18599-2 [18]. The result of the measurement is

$H_{\mathrm{ref}} = 120.6$ W/K. The calculation according to DIN V 18599, assuming a mean infiltration rate of $n_{\mathrm{inf}} = 0.06$ h^{-1} (according to DIN EN 13829 [19], based on blower-door tests, $n_{50} = 0.9$ h^{-1}) and additional heat losses through thermal bridges of $U_{\mathrm{WB}} = 0.05$ W/(m^2.K), leads to $H_{\mathrm{ref}} = 119.4$ W/K. In the following, the measured value of 120.6 W/K is used as H_{ref}.

During the QUB tests, the basement is heated to a constant temperature of 20 $^\circ$C in order to have an approximately adiabatic system boundary at this point, as the considered zone (ground floor and attic space) is also heated to 20 $^\circ$C before carrying out the QUB-test. The heating mats are activated every evening 15 minutes after sunset and remain active until midnight. This means that the duration of the QUB test is slightly modified in each experiment. Accordingly, the passive cooling phase lasts from midnight until sunrise. In order to mix the air in the zone considered, the existing circulation ventilation system (supply duct in the attic east; extract duct in the bathroom) was activated. The boundary conditions for the QUB tests result from experiences gathered during the experimental optimization of the QUB method [10]. The aim was to realise tests with α-values between 0.4 and 0.7 and to check the reliability and repeatability of the QUB test. In the tests performed, the nominal heating power was 4.4 kW. The effective heating power was measured each time. Figure 14 shows the measured HLC for the QUB tests that were carried out in dependence of the α-parameter.

The average result is 115 ± 10 W/K which is very close to the reference value, and the maximum discrepancy compared to this reference HLC amounts to 16%. The QUB results have therefore an acceptable dispersion and with such a limited number of measurement points it is difficult to con-

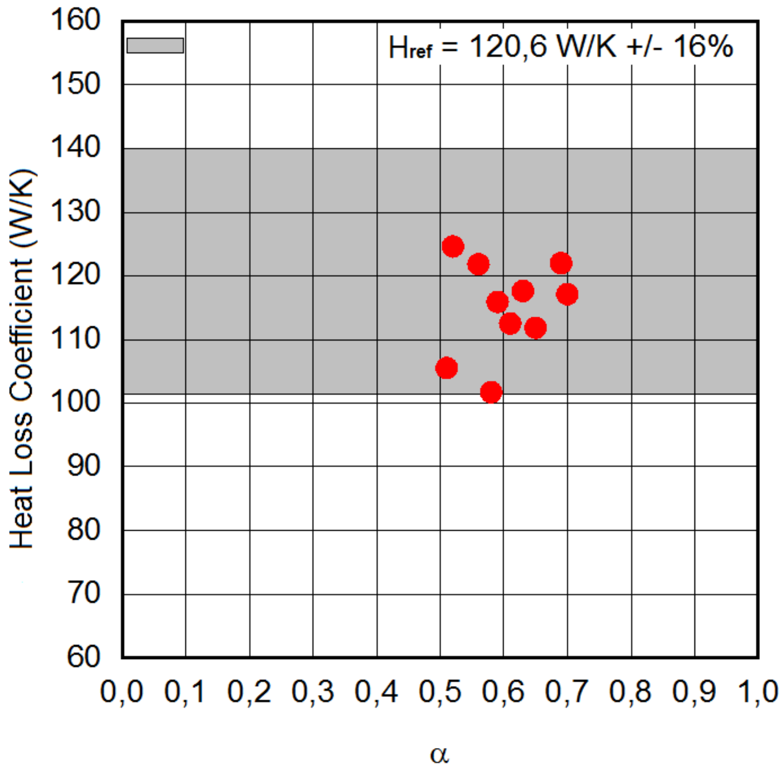


Figure 14: Measured heat loss coefficients in dependence of the α-value

clude whether there is a trend in this case or not. Thus, it can be concluded that by perfoming QUB tests with α-values between 0.4 and 0.7, results are reliable and repeatable. The relevant discrepancy at some measurements may be caused by the following aspects and needs further research studies:

- non-homogeneous temperature in the concerned zone due to convectional effects and stratification (up to $2K$ between ground floor and attic space)
- unavoidable temperature gradient between the zone concerned and the

basement during the heating and cooling phases leads to unwanted heat transfer

- wind pressure conditions vary from a test to another and airtightness of the building in the reference measurement case and the QUB test might have small differences, so infiltration heat losses might be different. Unfortunately wind pressure conditions have not been recorded but as the building airtightness is high this effect should be moderate

- anticipation of the optimal α-value in practice is difficult

When transferring these findings into a measuring method for the practical application of the QUB test in buildings as a method for checking the energy quality of the building envelope, the issues of air tightness and air change due to infiltration must be taken into account. It has been shown here that if the airtightness is good (as can be quickly proven by an air pressurization test), the measured value of HLC should be stable and close to the reference value that could be measured in a co-heating test. For a poor airtighness case, a blower door measurement should not be enough and an appropriate technique (such as tracer-gas method) should be used to quantify the air change rate and its impact on the heat loss coefficient estimated.

5. Discussion

It has been shown in the first section that using an RC model can explain why QUB tests can give good results in only two nights, provided some experimental conditions are respected, in particular homogeneous conductive heating and identical heating and cooling durations. In the second section, a model developed using the quadrupole method has been used to show that it is possible to measure the HLC of a building in one night only, with test durations being, in extreme cases, as short as one hour. In order to achieve such results, experimental requirements are more strict than those required for whole night tests. In particular, it has been shown that the thermal load must be included in a range that depends on the value of the internal and external temperatures. This condition is expressed through the use of an adimensional parameter called α, which should be included between approximately 0.4 and 0.7 (although these values depend on the experimental conditions before the test starts: free cooling or temperature regulation, for instance).

This method has been validated in different ways: by theoretical considerations, by numerical applications, and also by experimental validations in buildings where a good estimation of the heat loss estimation could be found with a second method. The buildings are a bungalow for which extensive co-heating measurements have been done, the Energy House at the University of Salford, which is a Victorian house located in a climatic chamber and can therefore be put in steady-state conditions, and one of the Twin Houses at the Fraunhofer Institute of Building Physics. All these validation cases lead to the same conclusions: low values of α can lead to slight under-evaluations of H_{ref}, high values can lead to high overestimations of H_{ref}, and the error, which depends on the building structure, can be reduced by increasing the measurement duration. Finally it has been shown that even in a real building with a high inertia and a good insulation, and submitted to real climate conditions, if the described experimental conditions are fulfilled, the QUB test can provide a good measurement of the HLC of a building in one night.

Even if this can be considered a very worthy objective, the developed model and associated experimental setup have other advantages, in particular for building scientists. Current HLC measurements take two to three weeks. In that time, it is possible to run as many as 20 QUB tests, and hence study the influence of exterior conditions, like the weather, on the results. For instance, it could be possible to study the impact of wind velocity on the resistance, which is a way of estimating the thermal impact of infiltrations. It is also possible to use them, not for studying the building envelope resistance, but the second parameter of the simplified model—its heat capacity—and in particular the influence of time, as has been presented in [10]. It can therefore be used to complete the understanding scientists have of the buildings behavior in many different conditions.

6. Conclusion

This paper proposes a new and efficient way of measuring the total heat losses of a building envelope. The main problem of existing methods is their duration, which makes them unsuited for use at a large scale. The QUB method aims at solving this issue by using dynamic measurements done only at night, in preferably empty buildings. Furthermore, only two power steps are used, usually a constant heating followed by free cooling, which simplifies the temperature responses. These experimental conditions make it possible to use a simple model to identify the envelope resistance in a short time. The two problems that arise, and that this article tries to solve, are the justification of the thermal model and the validation of the experimental results.

Although we believe that our findings are collectively quite conclusive, we also know that there is much to be done to improve our understanding of the QUB method. In particular, it is important to study its uncertainty, how large it is and how it varies with parameters such as test duration or wall configuration. This would validate this methodology, and so prescribe when the method is suitable and in which cases it could be inaccurate. Besides, it can also be argued that while useful, the information given by a QUB test is insufficient to do a complete energy diagnosis of a building, and should ideally be completed by values of thermal losses for infiltrations, thermal bridges, or specific elements (such as windows, ceiling, etc.). If different methods could be developed to measure these losses without increasing a QUB test duration, a complete and accurate diagnosis of a building would be possible, even if a more complex equipment would probably be needed. Thus it is necessary to study how the QUB method can be improved or completed with other methods to become a more complete assessment of a building's energy performance.

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