STATISTICAL MODELLING OF TRAINING AND PERFORMANCE USING POWER OUTPUT AND HEART RATE DATA COLLECTED IN THE FIELD

PhD Thesis

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Abstract

This thesis develops statistical models of performance and training that make use of power output and heart rate data. These data were collected during training and competition, and were recorded every five seconds using a power meter and heart rate monitor. Using these data, we estimate the parameters of the Banister model of training and performance. In principle, knowledge of these parameters allows one to provide quantitative decision support for the scheduling of training in advance of a major competition.

The methodology proceeds in a number of steps. In the first, measures of both training and performance must be specified. The training experienced by an athlete in a single session, the training load, can be measured in a number of ways. We use the TRIMP measure. This measure in its simplest form is essentially the total number of heart beats in a training session. Then the training loads of successive sessions are accumulated into a single measure of training up to time *t*. This we term the accumulated training effect (at time *t*). Performance during a session at time *t* is defined as a function of the power output observed during the session. We consider various performance measures and describe these in detail in the thesis. Then in the second step, we relate the performance at time *t* to the training load up to time *t* using a regression model, estimating the parameters of the performance training relationship. The final step is the training optimisation step, whereby the known training-performance model parameters can be used to specify training loads up to time *T* that will maximise (in expectation) the performance at time *T*.

We demonstrate the methodology using the training data histories of ten competitive male cyclists. As each athlete has his own specific characteristics, we should focus on optimising training and performance individually. We compare and contrast the different performance measures that we propose.

Our principal findings are that: Banister model parameters can be estimated; that the different performance measures yield different Banister model parameter estimates and therefore that the performance measure specification is a matter for athlete/coach choice; and that finally the Banister model has a serious shortcoming for the optimisation of training. The articulation of this shortcoming is an important contribution of this thesis.

CHAPTER ONE

INTRODUCTION

1.1 Background

The fundamental aim of this thesis is to develop a model that relates performance to training in cycling. The purpose of this model is to allow training to be quantified and planned systematically in order to improve the capability of an athlete in advance of a particular competition.

Training in sport, in particular, is the approach through which an athlete can improve his or her individual performance. It builds specific abilities and attributes that would optimise his or her overall performance required in specific competitions (Fister et al., 2015). The process of training essentially involves carrying out the same exercises numerous times to develop the skills, strength and endurance of the athlete, which lead to increased physical performance. Cycling training mainly aims to increase the ability of a rider to produce a power output or speed over a specified time or distance. By monitoring training sessions and performances during races with the help of a power meter and heart rate monitor, one can attempt to understand and model the relationship between training and performance (Passfield et al., 2016). Banister et al. (1975) suggested that a systematic theory can be adopted to model the response of an athlete to training. This paper suggested that there are two opposing responses to a training load: the positive fitness response and the negative fatigue response. This idea was reinforced later by Calvert et al. (1976), Morton (1997), and Busso (2003) who expressed the process of training as an impulse oriented mathematical model. The basic characteristic of their model was the mathematical link between preparedness and the training impulse (Busso & Thomas, 2006).

Hellard et al. (2006a) observed that useful information can be obtained from a modelling oriented approach and that this will be helpful in shaping individual training programs. However, Taha and Thomas (2003) observed that models so far developed did not relate to strictly physiological mechanisms. These models are also not able to differentiate between the particular impacts of various impulses of training. Moreover, inter-subject and inter-study variance limit the potential for developing and applying a general model, so that Jobson et al. (2009) observed that the prediction of performance output using training input was still an unsolved issue. Having regard to this, we evaluate whether the individual parameter values of the performance-training relationship can be deduced from the link between heart rate and power output data.

Maintaining a good balance in training is vital for a rider to develop his or her individual capabilities. Smith (2003) illustrated that the right amount of training would allow a rider to achieve the required skills and prevent him or her from acquiring an injury or illness due to over-training. On the other hand, under-training can prevent a rider from gaining peak potential. The systematic model of Banister et al. (1975) in a sense trades-off under/over training, aiming to strike the optimum balance. However, for the model to be specific rather than just indicative, both performance and training must be quantified, and the parameters of the Banister model must be estimated. A number of studies have estimated Banister model parameters in different types of sports (Busso et al., 2002; Calvert et al., 1976; Hayes & Quinn, 2009; Wood et al., 2005), but these studies do not report the precision of estimates of parameters.

In the PhD study of Shrahili (2014), a quantitative model was established to relate training to performance based on the Banister model (Banister et al., 1975). We extend that work to consider other alternative performance measures and to consider the effect of cardiovascular drift on performance measurement (Wingo et al., 2005). Cardiovascular drift is the gradual increase in heart rate during exercise at a fixed workload (Hamilton et al., 1991; Morales-Palomo et al., 2017). The performance measures are compared in terms of their statistical and practical significance. The selection of a performance measure is then down to athlete choice, albeit with the support of the analysis that this thesis provides. We also consider the usefulness of the Banister model for optimising training in advance of a particular competition.

Thus in our study, we also use the Banister model to specify the accumulated training effect at a time t. The model has a number of parameters that must be estimated for an individual rider. These parameters are necessary for using the model to plan training for each rider. We use power output and heart rate data collected in the field to make this estimation possible.

Our method includes some stages that must be achieved. Firstly we require two measures, those of training and performance. The training measure that we consider is associated with the training impulse TRIMP measure. In its most basic form, this measure is the sum of the total heart beats of the athlete during the session. The training loads of consecutive sessions are combined to determine a measurement of training up to time t. This is termed the accumulated training effect at time t. The performance in a session at time t is determined as a function of the power output achieved throughout the session. Various performance measures are considered and explained in detail in the thesis. Importantly, we suppose performance is measured with error. We quantify this error in a statistical model. In this way, we distinguish between the notion of preparedness of Busso and Thomas (2006), which is the expectation of performance, and performance itself which is a random variable with this expectation. Secondly, the performance calculated at time t is related to the training load up to time t with the use of a regression model. In this method, the parameters of the relationship between performance and training are estimated. Lastly, the parameters estimated are utilised to specify the training loads required preceding time t to maximise the athlete's performance at time T.

We note that while elite athletes and coaches can gain benefit from understanding the relationship between training and performance when they devise the training programs, such a relationship is not universal in nature but is highly individual. The reason behind the individual nature of such relationships is the individuals capacity, background of the trainee (Avalos et al., 2003; Mujika et al., 1996), genetic reasons (Wolfarth et al., 2000), technical factors (Wakayoshi et al., 1995), speciality (Stewart & Hopkins, 2000) and psychological factors (Saw et al., 2015).

Finally, here, we make a brief statement of the methodology in this thesis. Primarily this PhD is concerned with the field of statistics, and in particular, we use statistical modelling to quantify the uncertainty in estimates of parameters that arises because of the limited information that data provide about the real, underlying relationships. This point about uncertainty in the training-performance relationship has been not considered by the sports science literature to date. A statistical model is a set of assumptions about the generation of observed data and, in principle, we test the veracity of these assumptions given the data available. Statistical modelling then proceeds by accepting the model or modifying it according to the evidence for the model, and finally using the model to make deductive statements. In our case, these deductive statements concern the nature of the training-performance relationship.

1.2 Research Motivation

The quantification of the relationship between training and performance is an unsolved problem. This is the motivation of this study. In particular, if a coach and an athlete know that one additional unit of training on day t prior to competition on day T produces β units of improvement in performance on day T, this would provide very useful information for planning training. This is based on the presumption that better performance is desirable because better performance implies a higher chance of winning. This is the axiom of training.

Cycling lends itself to the statistical methodology we develop because power output is directly measurable and power output data can be and is routinely collected by riders using power meters and cycle computers. In other sports, the measurement of power output (and data collection) is more difficult.

1.3 Aim and Objectives

The aim of this study is to develop a model that can be used to optimise a training programme for an individual cyclist. To do so, we use power output and heart rate data collected every five seconds in training and competition. To achieve this aim, we have the following objectives:

- To develop statistical models that link power output to heart rate. Through these models, a performance measure can be specified and calculated for each session for each rider.
- To develop a statistical model that relates performance to training.
- To apply these models to the power output and heart rate data of a number of athletes.
- To compare different performance measures in terms of their statistical and practical significance.

Further, we discuss the limitations of the Banister model of training. Then we present some points of interest that could be undertaken to develop our methodology in the future.

1.4 Research Contributions

The main contributions of this thesis are as follows:

- To introduce various measures of performance, calculated using power output and heart rate data, where these data are recorded using a power meter and heart-rate monitor, and where each one of these performance measures depends on a specific performance concept.
- To relate these performance measures to a training measure, which is defined using the Banister model, and through this relationship, to estimate the Banister model parameters for each performance measure.
- To compare the different performance measures in terms of the statistical and practical significance of the models pertaining to them, in order to suggest a best measure of performance that a cyclist should use to optimise performance at a future competition.
- To demonstrate more realistically that while different performance measures can be specified, a methodology used to estimate the Banister model parameters that are appropriate for them is common to the various different performance measures.
- To present the idea that performance is a random variable and therefore that the performances of an athlete in a session (training or competition) is different from the readiness to perform (preparedness) of the athlete at the time of this session.

1.5 Research Question

We formulate the research question as:

- Can a practical method be established that quantitatively relates performance to training in cycling using power output and heart rate data collected in the field?

1.6 Thesis Structure

The structure of the thesis is as follows. In next chapter, chapter two, we describe the power output and heart rate data that we use in this thesis. In chapter three, we present the training measure we use, which is based on the Banister model. Then, we propose performance measures that consider the relationship between power output and heart rate. In chapter four, we link the training measure to the performance measures to determine the Banister model parameters and discuss the results statistically and practically. In chapter five, we relate the training measure to a modified performance measure that is defined as a function of power output alone, and repeat the estimation of the Banister model parameters and compare the results with those of chapter four. In chapter six, we consider other simpler performance measures and again relate each one in turn to the training input. In chapter seven, we discuss and compare the results from each of the different performance measures. In final chapter, chapter eight, we summarise our work and discuss further potential developments of the Banister model that might be studied by others in future.

CHAPTER TWO

DATA DESCRIPTION

2.1 Introduction

In this chapter, we describe the data that we use in this thesis. These data are power output and heart rate collected every five seconds during training and competitions. A summary for each athlete is presented. Moreover, how data such as these are collected is described, and the instrument (SRM power meter) that is used by riders to collect data is illustrated. Furthermore, some examples of heart rate and power output from two training sessions for one rider are presented to show the format of the data.

2.2 Training Data

Our methodology is illustrated using data from ten competitive, male road cyclists. These riders collected data on power-output and heart rate for nominally all their sessions (training, testing and competition) over a period in 2006-2008. Missing data on a particular day might be due to either a lack of recording or no ride that day. At the time the data were collected the ages, masses, and heights of the riders were as in Table 2.1. Measurements of power output were recorded every 5 seconds using power-meter cranks (SRM, Julich, Germany). The riders gave written, informed consent for their data to be used in this study, and the data collection received ethics-committee approval at the University of Kent and was carried out according to the principles of the Declaration of Helsinki (World Medical Association, 2013).

Table 2-1 Summary data for each rider

Rider	Start date	End date	Age (years)	Height (cm)	Weight (kg)	Training period (days)	Training Sessions recorded (number)
1	04/03/2007	11/08/2007	52	175.0	74.5	160	112
2	21/11/2006	28/07/2007	34	185.5	88.2	249	88
3	19/04/2007	31/01/2008	42	178.5	78.2	287	108
4	10/11/2006	23/09/2007	29	174.5	71.5	317	112
5	02/11/2006	02/08/2007	27	183.7	71.8	273	101
6	27/10/2006	30/09/2007	35	181.0	71.0	338	146
7	06/12/2006	04/09/2007	34	182.0	77.0	272	152
8	27/10/2006	07/10/2007	40	177.5	75.5	348	162
9	01/11/2006	30/09/2007	21	171.4	60.9	333	197
10	28/10/2006	12/12/2007	45	183.0	74.3	410	251

The data were not collected specifically for the study in this thesis. The data were collected by sports scientists at the University of Kent as part of an extended study of training and performance that received EPSRC support through grant number EP/F006136/1. Collaboration on this grant led to the opportunity to use these data for the study in this thesis. We are satisfied that the data are robust.

The data then are secondary data. A consequence of this is that, for our study, it would not have been possible to extend the data with contextual information, relating to, for example, qualitative reporting of: the nature of sessions; descriptions of any activity between sessions; periods of illness and injury if applicable; etc. To collect data specifically for this thesis would have been very difficult, and beyond its scope. This difficulty arises principally because athletes (and coaches) are protective of data about their performance. The riders whose data were used in this study were developing riders and had the trust of the scientists at the University of Kent. The riders are anonymised throughout this thesis.

2.3 Power Meters

A power meter measures power output in units of Watts and is considered to be a validated system for the measurement of exercise workload and energy exhaustion (Bertucci et al., 2005; Gardner et al., 2004; Maier et al., 2014). The data that are generated by such power meters can thus be utilized to evaluate and monitor the performance of the cyclist. It can also be used to develop a cyclist's training program. Another advantage in this regard is that this meter can be used in different environments such as on road, off-road, and on the track. Power meters were developed in the 1980s by SRM (Schoberer Rad Messtecnik, Jülich, Welldorf, Germany). Such power meters have been commercially available since 1986, and SRM is generally regarded as a pioneer manufacturer of these meters. SRM is further regarded as the best meter available based on its reproducibility and validity rate (Gardner et al., 2004; Lawton et al., 1999). The SRM meter comes with a monitor that displays the power output and heart rate on the cycle's handlebars (Figure 2.1). With the SRM computer, other factors such as cadence that is defined as the number of pedal-crank revolutions per minute, speed, temperature, distance and altitude are also displayed (Haakonssen et al., 2013). After the completion of a training session, athletes upload the power output and heart rate measurements to a spreadsheet through the software provided by the SRM (Hurst et al., 2015). Power meters are subject to continued marginal improvements (Lu et al., 2015), but we confine our discussion to the SRM meter. The efficacy of power meters has been investigated by many (Allen & Coggan, 2012; Craig & Norton, 2001; Ebert et al., 2005; Stapelfeldt et al., 2004; Vogt et al., 2006). Power meters are becoming increasingly popular as a training tool (Allen & Coggan, 2012).

Figure 2-1 An example of SRM monitor and SRM power meter

2.4 Heart Rate Monitoring

Heart rate monitors (HRMs) have been used as popular training tools among coaches and athletes for a long time. Their cost-effectiveness and easy application have made HRMs a very common tool in measuring the extent of exercise and training load (Achten & Jeukendrup, 2003; Jeukendrup & Diemen, 1998; Mazzoleni et al., 2016). Furthermore, HRMs are also useful to identify overtraining (Achten & Jeukendrup, 2003). The use of HRMs for estimating exercise intensity, energy exhaustion, and exercise load in cycling competitions has been researched for many years (Andez-Garcia et al., 2000; Impellizzeri et al., 2005; Mujika & Padilla, 2001). At the same time, the use of HRMs has some barriers. Within and between sessions, variations in the heart rate occur due to multiple factors such as hydration status, ambient temperature, cardiovascular drift (Rowell et al., 1996), and altitude (Achten & Jeukendrup, 2003). Understanding these factors is essential for analysing appropriately the heart rate data accumulated throughout training.

Heart rate is typically measured using a chest strap monitor that interfaces with a recording device. In our data, heart rate was recorded every 5 seconds. Examples of heart rate recorded every 5 seconds from two training sessions for rider (1) are shown in Figure 2.2. Maximum heart rate and resting heart rate for each rider in our study are shown in Table 2-2. The duration of each training session for each rider is shown in Figure 2.3. Furthermore, the average heart rate for each training session with maximum and minimum average heart rate are for each rider shown in Figure 2.4. Figure 2.5 presents the histogram of the entire heart rate measurements for each rider.

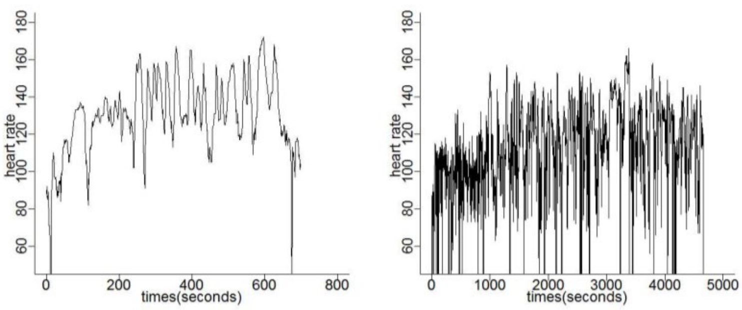

Figure 2-2 Examples of heart rate (bpm) from two training sessions for rider (1)

Table 2-2 Maximum heart rate and resting heart rate for each rider

Rider	$H_{maximum}$	$H_{resting}$	Rider	$H_{maximum}$	$H_{resting}$
1	180	45	6	187	39
2	203	48	7	187	49
3	182	45	8	173	42
4	192	42	9	192	53
5	184	42	10	176	42

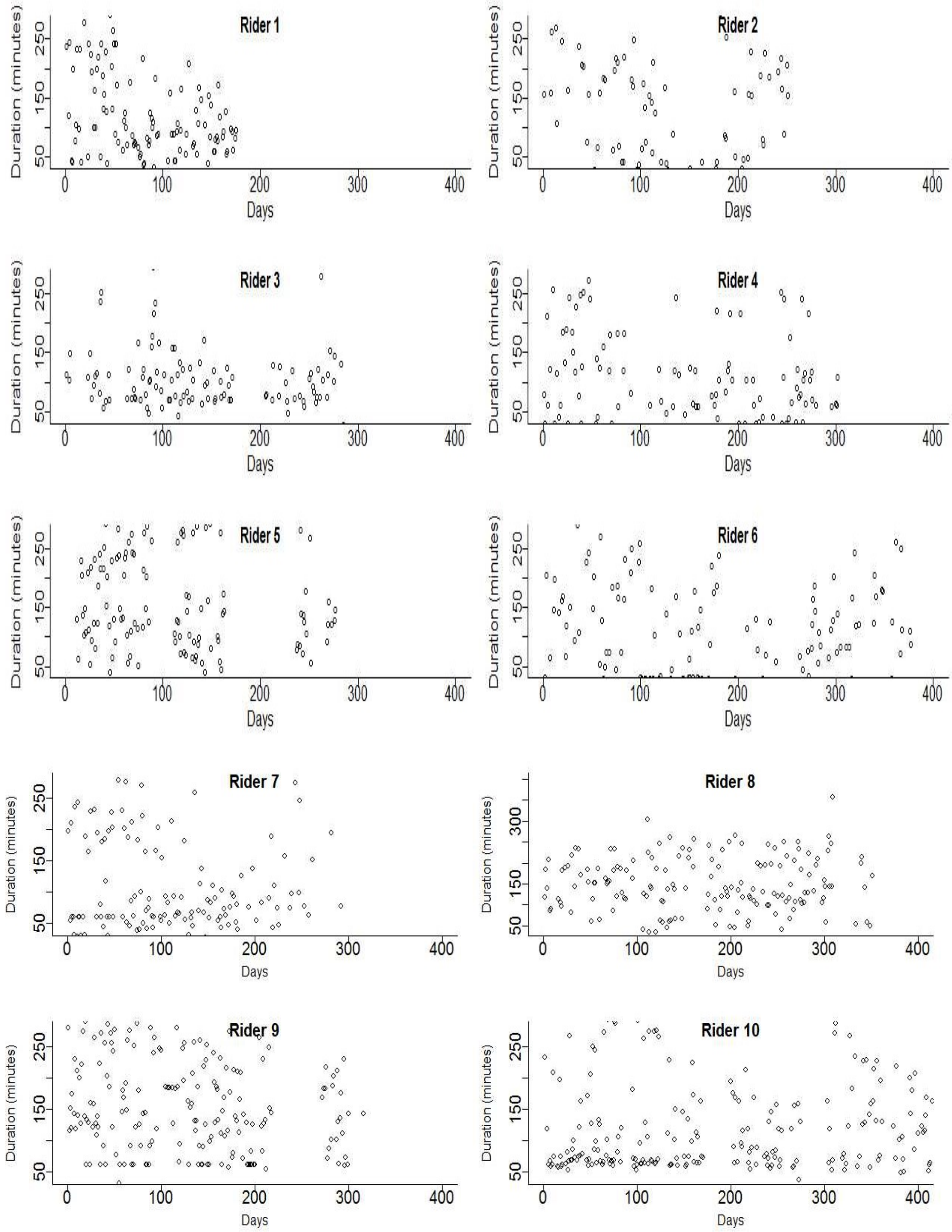

Figure 2-3 The duration of each training session for each rider

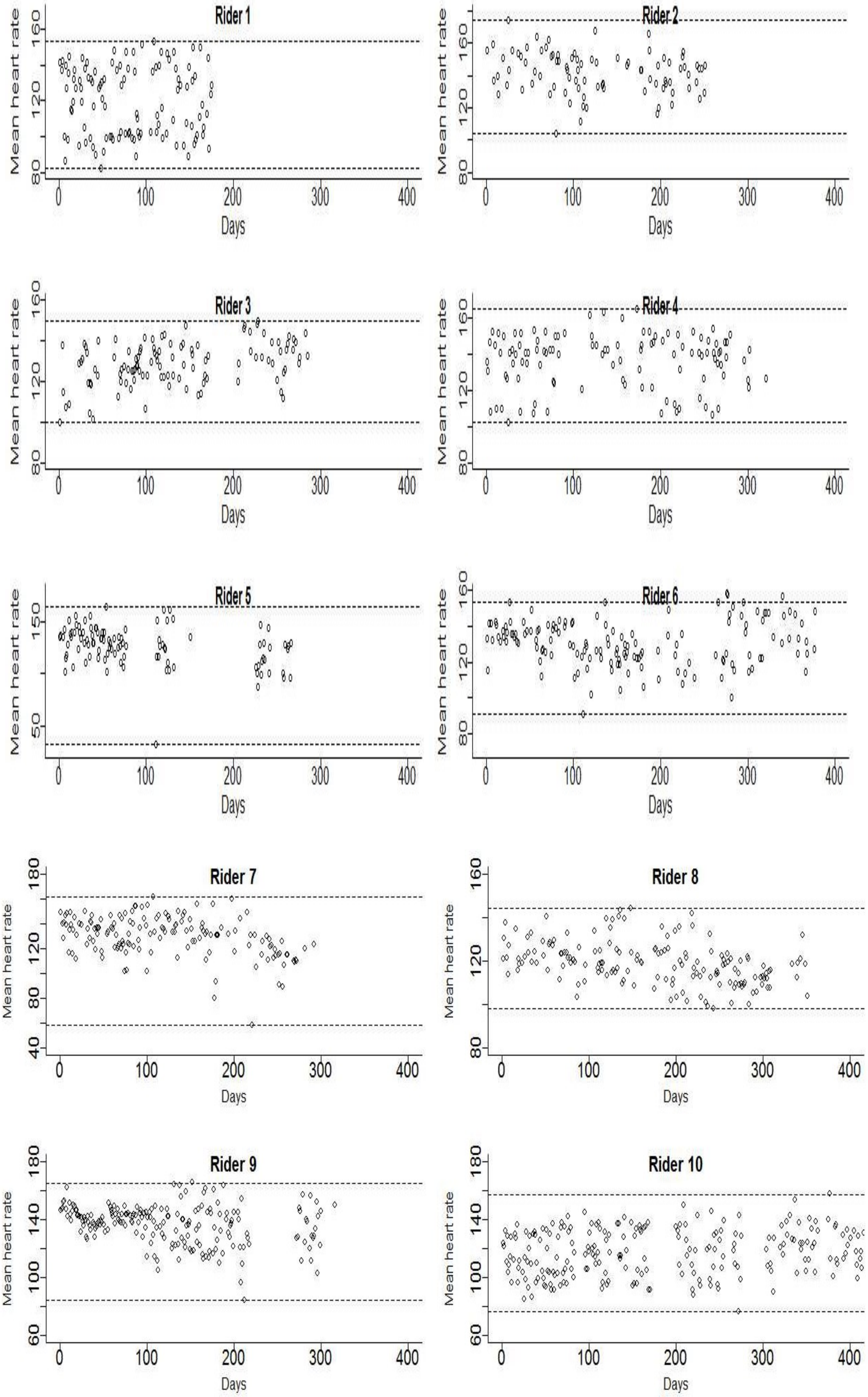

Figure 2-4 The average heart rate for each training session for each rider, with maximum and minimum average heart rate

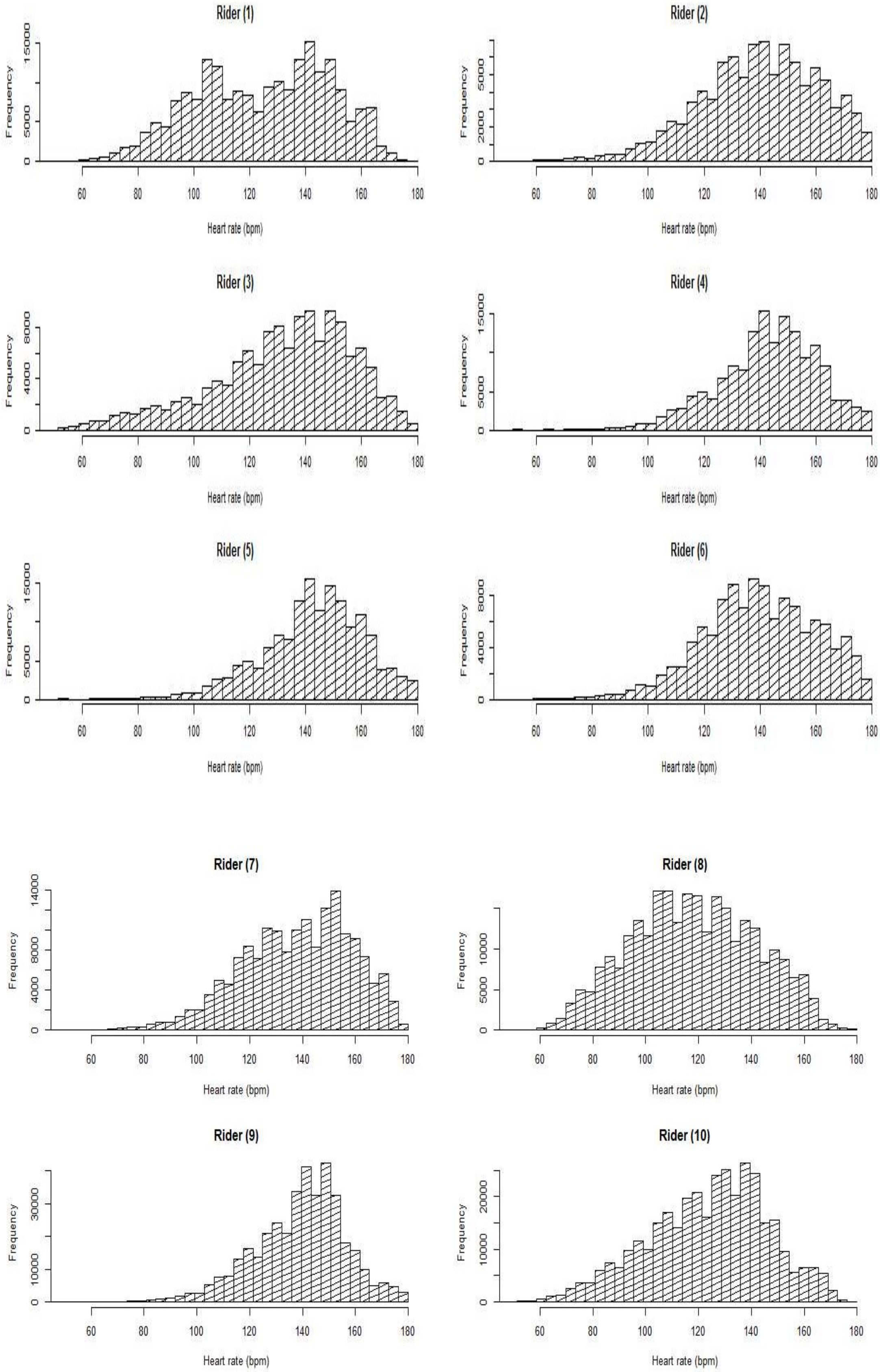

Figure 2-5 The histogram of the entire heart rate measurements for each rider

2.5 Power Output Monitoring

Power output has been considered as the most direct measure for describing performance in cycling (Stannard et al., 2015; Vogt et al., 2006). This is because it gives a measurement or feedback instantly. Many sport scientists and coaches now use power output instead of heart rate to specify training intensity in cycling (Duc et al., 2007). Power output can be estimated by using mathematical models, or measured directly on the cyclist's bicycle using mobile power meters (Martin et al., 1998; Olds, 2001). We described the SRM power meter in section 2.3. Generating and sustaining the power output is considered a vital factor for athletes (Soriano et al., 2015).Various factors contribute to power output such as nutrition fitness, bike design, and riding position. Other factors can be seen in Figure 2.6 (Atkinson et al., 2007).

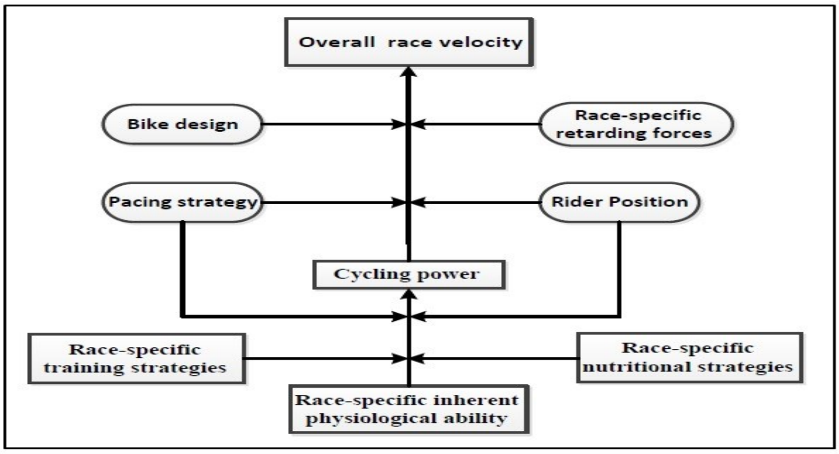

Figure 2-6 Factors influencing cycling power output and consequential velocity

(Atkinson et al., 2007)

In our data, power output was measured using SRM cranks. Examples of power output recorded every five seconds from two training sessions for rider (1) are shown in Figure 2.7. The average power output for each training session for each rider with maximum and minimum average is shown in Figure 2.8. Figure 2.9 presents the histogram of the entire power output measurements for each rider.

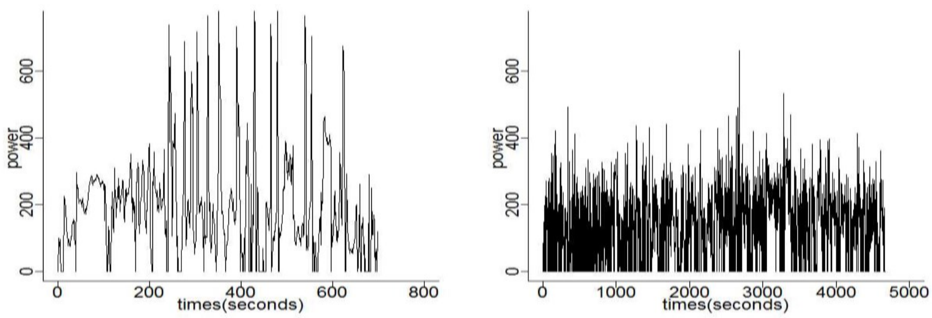

Figure 2-7 Examples of power output (watts) from two training sessions for rider 1

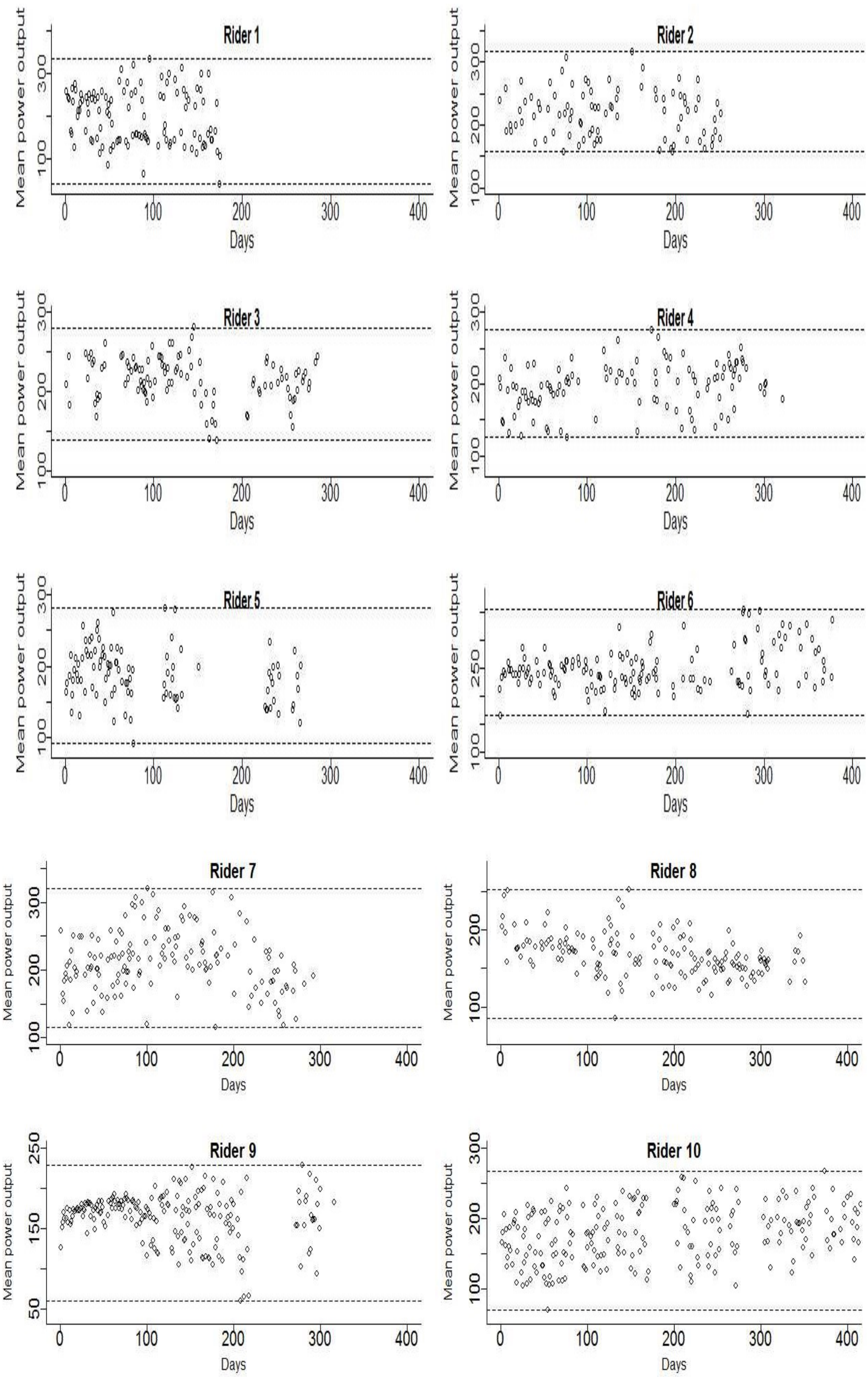

Figure 2-8 The average power output for each training session for each rider, with maximum and minimum average power output

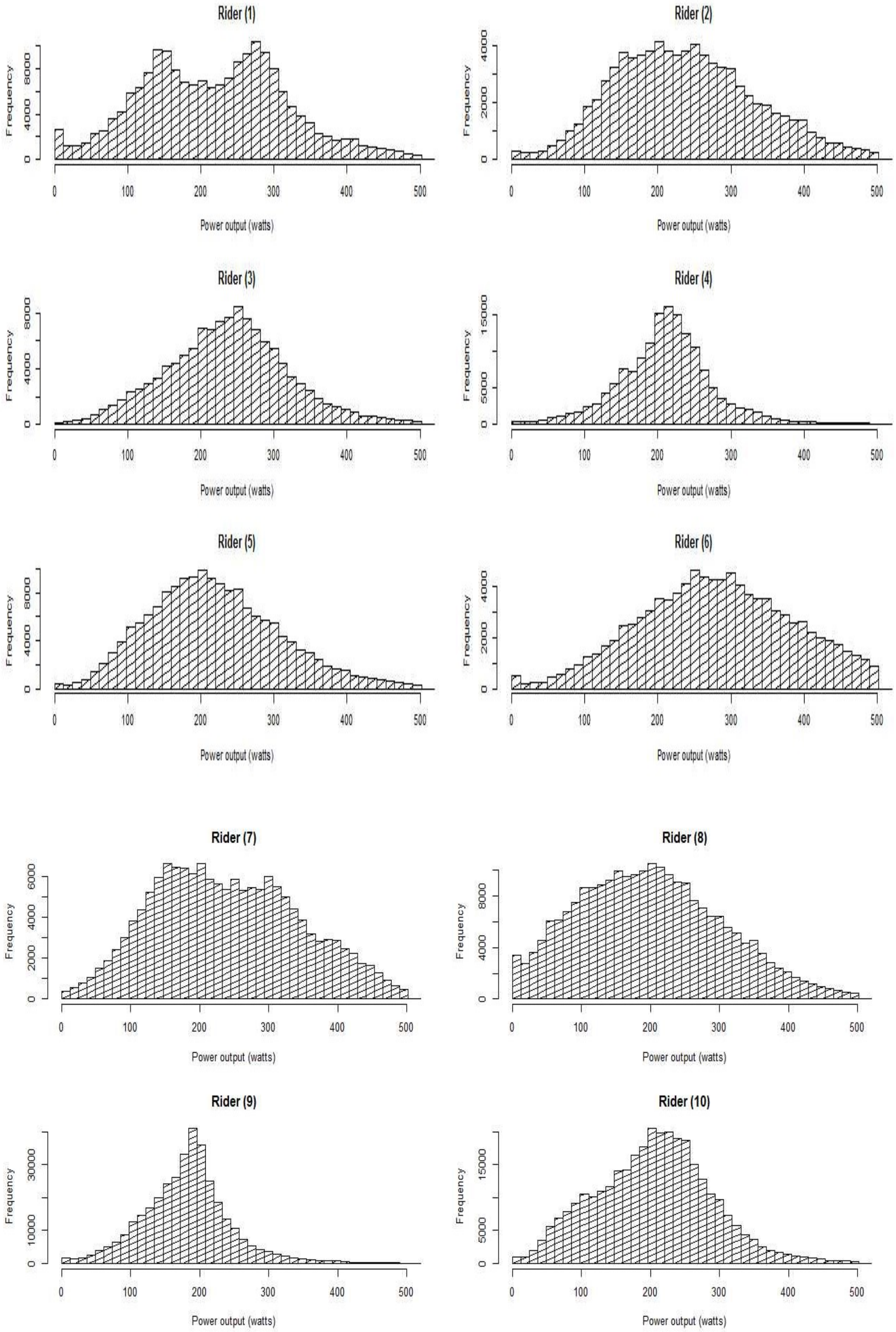

Figure 2-9 The histogram of the entire power output measurements for each rider

2.6 Summary

In this chapter, we described our data on power output and heart rate. The data we used are secondary data collected by the athletes in collaboration with sports scientists. Further contextual information about the nature of sessions was not available. The power output and heart rate were recorded every 5 seconds during training sessions. We briefly described the instruments that are used to record power output and heart rate. A summary of the data for each rider is presented and explained. Some examples from two training sessions for one rider are provided to describe the format of the data. Furthermore, examples of the average power output and heart rate for each session for each rider are presented. Finally, the histograms of the entire power output and heart rate data for each rider are shown. Next, we discuss measures of training and performance.

CHAPTER THREE

MEASURING PERFORMANCE AND TRAINING

3.1 Introduction

In this chapter, we describe measures of training and performance in general and in particular. We review the literature on training and performance measures. We describe the particular measures that we use initially in our study. For training, this is the accumulated training effect at time *t*, which is denoted (ATE). This quantity quantifies the training load accumulation over time. For performance measurement, we describe a measure that is a function of power output and heart rate. These latter quantities are measured in the field, that is, in training and competition using a power meter and a heart rate monitor. These measures will be used in chapter 4 to estimate the relationship between training and performance.

3.2 The Relationship between Training and Performance

Knowledge of the relationship between training and performance is important to athletes and coaches for determining the optimum amount and period of training. This knowledge can enhance the performance of the athlete (Avalos et al., 2003; Foster et al., 1996; Gabbett et al., 2014). In a fundamental contribution, a mathematical model was proposed by Banister et al. (1975) that aims to describe the response of the athlete to particular training stimuli. The model proposes that readiness to perform or preparedness is the result of a positive response component (fitness) and a negative response component (fatigue). Nonetheless, these studies are qualitative rather than quantitative.

Studies have investigated the relationship between training and performance as analogous to the dose–response relationship (Morton, 1997). Moreover, some studies indicate that the primary aim of investigating such a relationship is the prescription of training stimuli that enhances the potential of an athlete to perform better by maximising the positive effects of training such as fitness, improvement in body composition, burning fat and increasing muscle mass and minimising the negative effects of training such as fatigue, stress, and injury (Borresen & Lambert, 2009; Morton, 1997).

Additionally, several studies have highlighted the results of investigations conducted on the effects of volume, frequency, and intensity of training on performance of the athlete. The general observation was that the performance of the athlete increases with increasing load of training (Foster et al., 1996; Krebs et al., 1986; Stewart & Hopkins, 2000), but the direct link of training to performance is not quantified. Several early studies that investigated the relationship between training and performance found a positive relationship between volume of training and performance (Foster et al., 1977; Stewart & Hopkins, 2000) and a positive relationship between intensity of training and performance

(Mujika et al., 1995; Scrimgeour et al., 1986). However, that a positive relationship is reported is not surprising.

Foster et al. (1996) studied the relationship between training and performance among 56 cyclists, runners and speed skaters during 12 weeks of training. They observed that a ten-fold increase in load of training resulted in a nearly 10% increase in performance. Again, the precision of this finding is not given. However, it has also been noted that increasing the dosage of training can also sometimes lead to negative effects on performance. Additionally, it can also result in injury, fatigue, stress, and illness when the dosage of training is at its highest level (Foster, 1998; Gabbett, 2004). Qualitative approaches have been used by many researchers in order to find a relation between training and performance (Grazzi, et al. 1999; Stewart & Hopkins, 2000).

However, Banister and his colleagues were the first to attempt to model the relationship between training and performance. Banister et al. (1975) suggested a model which the benefit and detriment of training is described. Moreover, a system model with the ability to relate athletic performance profiles to training profiles was proposed by these authors. This model will be explained in more detail. We aim to utilise the Banister model to find the relationship between performance and training over time with the use of data accumulated over an extended period of training. To use the Banister model, a measure of training load and performance must be known. To optimise athletes' training, and in doing so maximising their future performance, the parameters of Banister model have to be available. Few studies have been able to relate training to performance quantitatively. Even though this is the case, there are a few such studies that were conducted prior to the present study.

For example, Hellard et al. (2006) conducted a study for swimming. Nine leading swimmers, of whom 5 were females and 4 were males, took part in the research, which was carried out over a one year period. Actual performances during competitions were measured during the study period. The parameters of the Banister model were estimated for every swimmer with the use of the nonlinear least squares method among actual and modelled performances. The values of the parameters were reported as τ_a=38 days and τ_f=19 days. The Banister model was applied to different sports by Morton et al. (1990), particularly for running. The values of the parameters τ_a and τ_f were reported as 45 and 15 days respectively. Precision of estimates was not reported.

Busso et al. (1997) reported the Banister model parameter estimates for cycling. Two subjects took part in a 16-week study. To determine the model parameters, they utilised the least squares method between actual and modelled performances. The values of the parameter τ_a were reported as 60 days for both of the cyclists, and the values of the parameter τ_f were reported as 4 days for subject one and 6 days for subject two. Again, the precision of estimates was not reported.

Nonetheless, Hellard et al. (2006) indicated three probable limitations involved in the modelling approach suggested by Banister. They also suggested some modifications in this regard. The main limitations include the inaccuracy of the model for predicting the performance in future, variation between estimated performance and actual performance and poor correspondence to physiological elements by the model. All modifications which have been explained focus primarily on the processes of the model without having regard to the standard of the input data.

Thus, in this thesis, one of our purposes is to estimate the parameter values of the Banister model for cycling, and to provide the precision of the estimates. Unlike previous studies, using a different approach, we develop new models to estimate these parameters using power output and heart rate data collected in the field. This will be explained in chapter 4.

3.3 Measuring Training Load

A number of methods have been utilised to quantify training load, such as diaries and questionnaires (Lambert et al., 2002; Shephard, 2003), direct observation (Foster et al., 2001; Hopkins, 1991) and physiological monitoring in terms of heart rate (Achten & Jeukendrup, 2003; Robinson et al., 1991). It has also been proposed to use indices of training stress, such as training impulse that uses heart rate measurements and training load (Morton et al., 1990). Despite the fact that physiological adaptation is documented adequately with respect to training in the literature, its influence on performance is not yet accurate (Borresen & Lambert, 2009; Jobson et al., 2009). Despite these developments, focus on training impulse (TRIMP) as the most suitable measure of training remains. Therefore, we use TRIMP to quantify training load. In the next section, this measure is discussed in more detail.

3.3.1 Training Impulse (TRIMP)

The training impulse (TRIMP) measure has been established to evaluate the volume or amount of training undertaken in any one given bout (Morton, 1997). Banister et al (1975) and Banister and Calvert (1980) presented the training impulse measure (TRIMP) as follows

$$TRIMP = D \times \overline{H}, \tag{3.1}$$

where D is the duration of the training session in minutes and $\overline{H}$ is the average heart rate during the session in beats per minute. Thus this simple measure is the total number of heart beats in a session. It can be interpreted as an athlete's heart rate response to training over the duration of the training session (Borresen & Lambert, 2009). This equation was further modified by Morton et al. (1990) to

$$TRIMP = D \times F \times Y, \tag{3.2}$$

where D is as above (duration), F is the fraction of heart rate reserve, and Y is the factor ae^{bF} that gives higher weight to high heart rates during a session. The heart rate reserve is given by

$$F = (HR_{ex} - HR_{rest})/(HR_{max} - HR_{rest}),$$

where HR_{ex} is the average heart rate in a training session during exercise. HR_{max} and HR_{rest} are the maximal heart rate and the average heart rate during rest respectively.

A number of studies have proposed values for the TRIMP parameters (Akubat & Abt, 2011; Morton et al., 1990; Stagno et al., 2007). The values of a and b were reported as 0.1225 and 3.9434 respectively in the study of Stagno et al. (2007). They estimated these values by fitting an exponential line for the blood lactate concentration against the fractional elevation in heart rate for eight participants. In our study, we use the modified TRIMP, equation (3.2), with the values $a = 0.64$ and $b = 1.92$ reported by Borresen and Lambert (2009). We use these values in order to maintain continuity with the work of Shrahili (2014). The modified training impulse (TRIMP) for each session for each rider is shown in Figure 3.1.

Edwards (1994) adopted TRIMP measurements, considering the time duration as well. He basically divided the data into five parts, considering the maximal heart rate into percentage groups such as 50-60%, 60-70%, 70-80%, 80-90% and 90-100%. The time duration of each part or segment intensity was then multiplied by the segment or zone numbers. Further modification of these zones was done by Lucia et al. (1999) in which the heart rates were based on the aerobic and anaerobic thresholds. According to this modification, their model contained three zones below the aerobic threshold, similarly for below thresholds and between thresholds. In this, the time intervals of sessions were also multiplied by the zone numbers. Depending on measurements of heart rate is the main problem with these methods and this has many implications regarding possible changes such as temperature, hydration, cardiovascular drift or the position on the bicycle (Achten & Jeukendrup, 2003; Leweke et al., 1995).

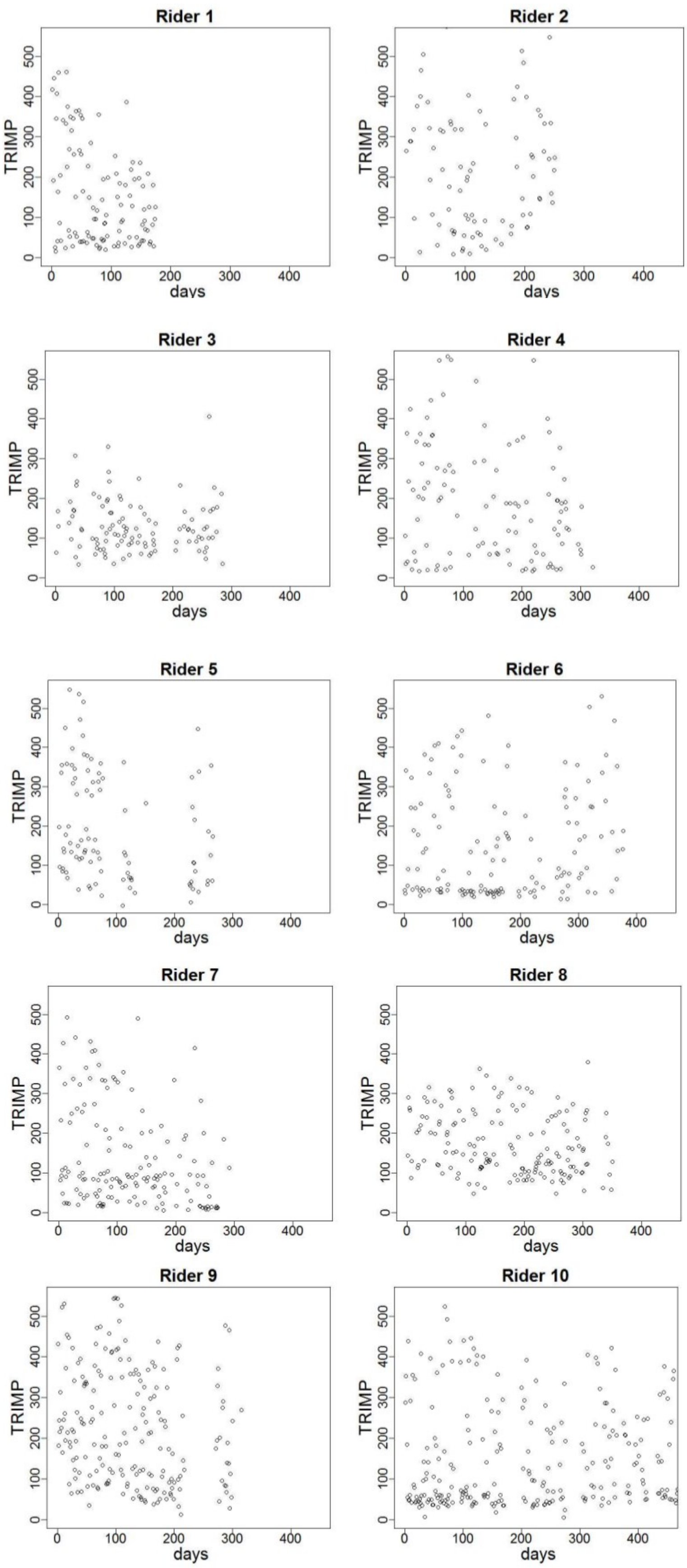

Figure 3-1 The modified training impulse (TRIMP), equation (3.2) for each rider for each session

3.3.2 Training Stress Score (TSS)

The training stress score (TSS) system was modelled on Banister's model for training impulse. It relies on the concept of Normalized Power (NP). TSS was used to quantify the training load in running (McGregor et al., 2009) and cycling (MacLeod & Sunderland, 2009). This measure is defined as follows:

$$TSS = (S \times NP^2 \times IF)/(FTP^2 \times 3600), \tag{3.3}$$

where S is the duration of the activity in seconds and NP is calculated for a session. The functional threshold power (FTP) is defined as the maximal power that can be continued by the individual for one hour. This number is individual for each athlete. The training stress score (TSS) for each rider for each session in our study is shown in Figure 3.2. The training stress scores (TSS) and training impulses (TRIMP) for each session for each rider are shown in Figure 3.3. The correlation coefficient between training stress score (TSS) and training impulse (TRIMP) for each rider is presented in Table 3-1. Table 3-1 and Figure 3.3 show very strong positive correlation between TRIMP and TSS.

Table 3-1 The correlation coefficient between training stress score (TSS) and modified training impulse (TRIMP) for each rider

Rider	Corr.(TRIMP,TSS)
1	0.91
2	0.89
3	0.76
4	0.95
5	0.71
6	0.92
7	0.93
8	0.92
9	0.93
10	0.93

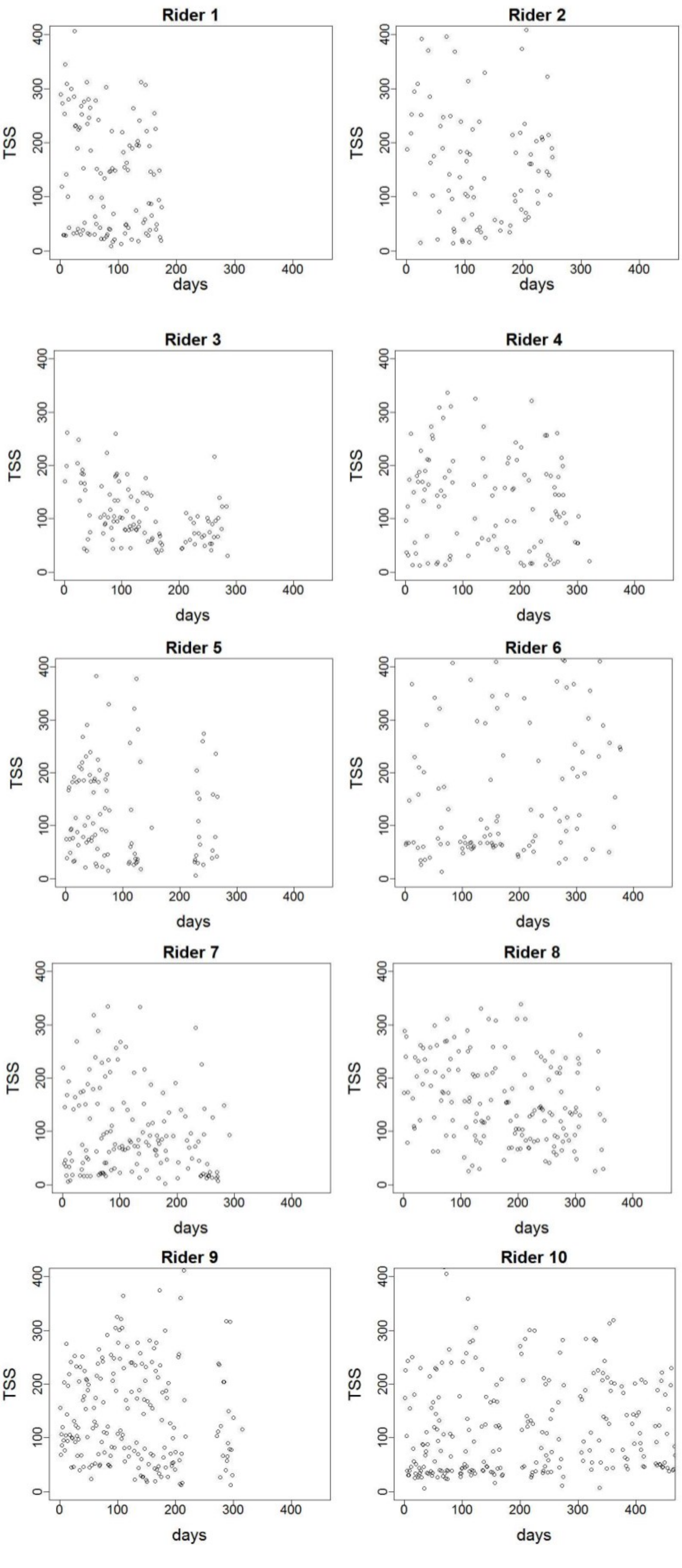

Figure 3-2 Training stress score (TSS) for each session for each rider

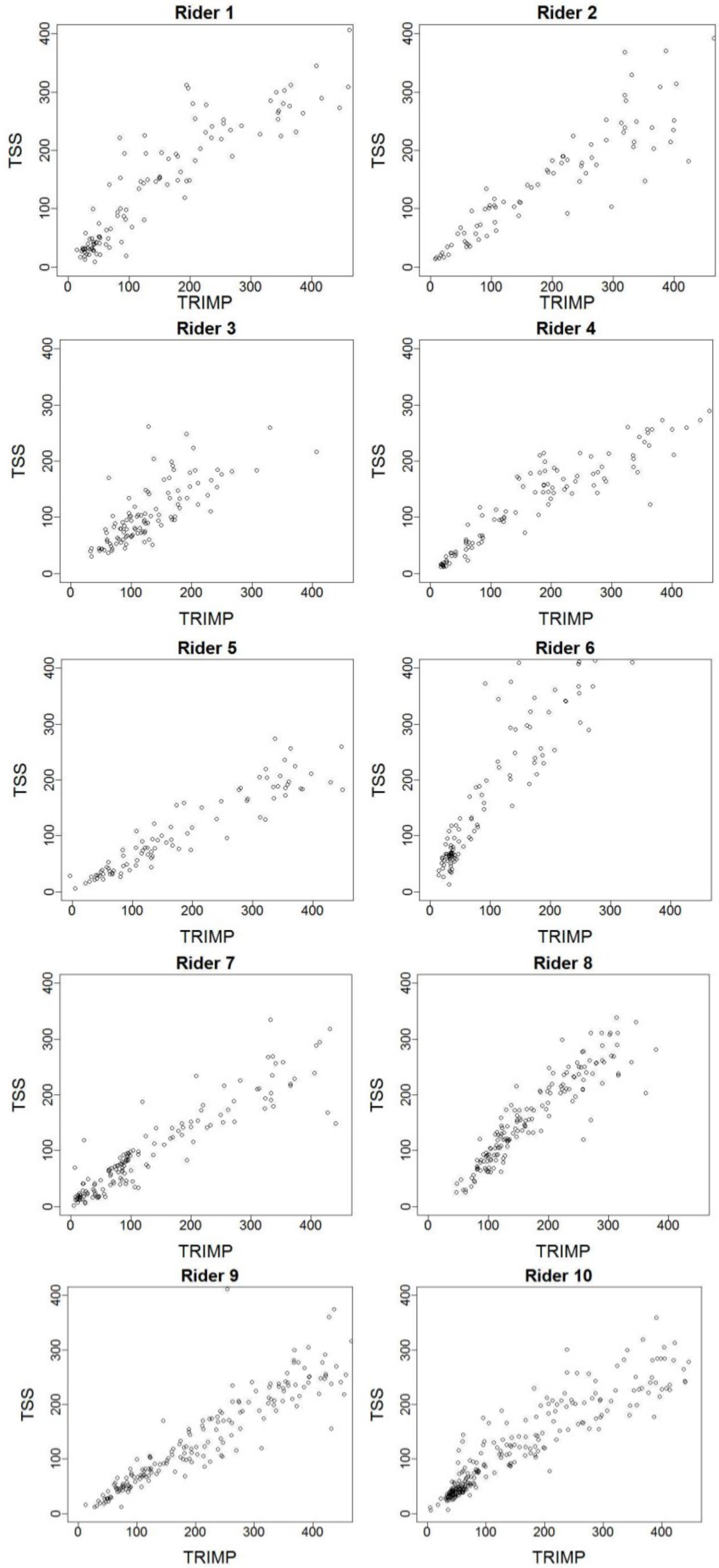

Figure 3-3 Training stress score (TSS) against modified training impulse (TRIMP) for each rider

3.4 Banister Model

Banister et al. (1975) proposed a model for the accumulation of training. This model specifies the training effect at time t. The Banister model was simplified by Calvert et al. (1976) to include two components, which are fitness (positive impact) and fatigue (negative impact). These components include parameters that must be estimated for an individual athlete in order to optimise training.

The Banister model is proposed to measure the accumulated training effect over a number of sessions as:

$$W_i = W_0 + k_a \sum_{j=1}^{i-1} w_j e^{-(i-j)/\tau_a} - k_f \sum_{j=1}^{i-1} w_j e^{-(i-j)/\tau_f} \qquad (3.4)$$

where W_i is the accumulated training effect on day i, in arbitrary units, and w_j is the known training load of the session on day j, in arbitrary units. In our study, training impulse (TRIMP) is used to measure the training load. k_a and k_f are the scale constants that determine the size of the immediate training benefit with respect to the immediate training detriment or fatigue. In this study, we set $k_a = 1$. τ_a and τ_f are the fitness and fatigue decay time constants, respectively and W_0 is the initial training effect. $w_j e^{-(i-j)/\tau_a}$ and $w_j e^{-(i-j)/\tau_f}$ are training benefit and training detriment of session $j < i$ respectively.

We show how this function looks in Figures 3.4 and 3.5. In Figure 3.4, the response to a single session according to the Banister model is shown. In Figure 3.5, shows the response to a series of sessions. In this latter example, the accumulation of decaying responses at different lags shows as an increasing "saw tooth" curve during the training phase, a final peak at the trained stage, and then a gradual decay to the initial state once training has ceased.

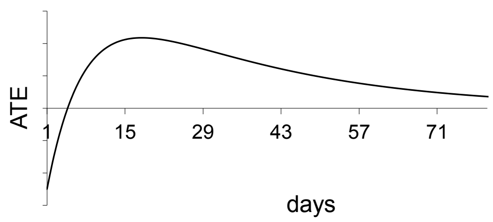

Figure 3-4 ATE given one unit of training load on day 1 with parameters τ_a=30, τ_f=7, k_f=1.5 and k_a=1

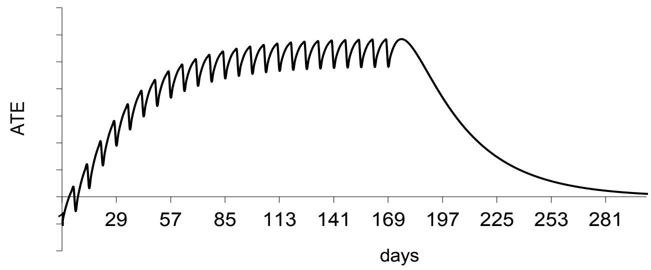

Figure 3-5 ATE given one unit of training load once per week for 25 weeks with parameters τ_a=30, τ_f=7, k_f=1.5 and k_a=1

Banister and Calvert (1980) also stated that an athlete must avoid over/under-training as this will affect his performance in the future. The concept of over-training is discussed in more detail in the next section.

3.5 Over-Training

In its general sense, over-training is regarded as an imbalance between recovery and training (Halson & Jeukendrup, 2004; Lehmann et al., 1993). Various terms have been utilized to describe over-training (Smith, 2003). It has also been described as excessive training. The basic characteristics of excessive training include long term fatigue and a falling level of performance. Overtraining has also been described as overwork, chronic fatigue, and burnout (Gleeson, 2002). Matos et al. (2011) defined over-training as a reduction in the athlete's potential or ability to continue to perform at a particular level. This reduction can range from weeks to even months. When an athlete carries out over-training, he subjects himself to intense pressure (Fister Jr et al., 2014). Sometimes, athletes fail to perform not because of lack of preparation but because of over-training or infection.

Kuipers (1998) observed that diagnosing over-training is a gradual process. There are various symptoms which indicate that the athlete has subjected himself to over-training. However, symptoms may vary from one athlete to another (Hartmann & Mester, 2000). The easiest and most common manner of detecting over-training includes changes in the behaviour of the athlete and falling performance (Hooper & Mackinnon, 1995). However, some other symptoms may also point towards the fact that the athlete has over-trained himself/herself. These include loss of appetite, sleep disorders, hormonal changes, and emotional instability. It may also happen that one symptom can lead to another symptom (Lehmann et al., 1998).

There are various elements which can contribute towards over-training. The fact that the phenomenon can take place in almost any sporting activity indicates that there may be some common elements giving rise to over-training (MacKinnon, 2000). Hooper and Mackinnon (1995) highlighted the elements which can lead to over-training. These include increases in the volume of training, increases in the intensity of training, short schedules, overdoing exercises, and lack of programmed coordination between different exercises.

Some studies have summarized the strategies to avoid indulging in over-training (Foster et al., 1999; Fry et al., 1992). Common strategies in this regard include low-intensity training, simple training, conducting hard sessions only twice or thrice a week, and resting before competitions (Daniels, 2013; Noakes, 1992; Wenger & Bell, 1986).

While the Banister model of training takes account of the negative impacts of training, it assumes the contributions to fatigue of individual sessions are additive and not persistent. Therefore in a sense, it does not model long-term over-training episodes, illness, or chronic fatigue. We will return to this issue of additivity of the model later in the thesis.

3.6 Measuring Performance

3.6.1 Introduction

The primary aim of any sports coach, as well as any athlete, is to produce a winning performance, or a performance which is at least his/her personal best at a particular time (Borresen & Lambert, 2009; Röthlin et al., 2016). The nature of prescription for accomplishing these goals is largely instinctive and develops from experience gained over years. The potential for achieving the pinnacle of performance corresponding to the date of the competitive event, such as achieving excellence in performance on the day of competition, is variably successful. The general belief is that if training is increased then performance would automatically increase. However, this approach is vague in nature and is also regarded as fragile because an excessive increase in training may also lead to injury due to over-exercise (Budgett et al., 2000; Williams & Eston, 1989). Therefore, the importance of scientific research in this field is also gaining popularity.

Optimal performance strategy revolves around the issue of designing the training programme which serves to enhance performance at a future date and minimises the risk of overtraining and fatigue (Calvert et al., 1976; Morton, 1997). It is widely acknowledged that the training must be continued periodically to gain improvement in performance (Matveyev, 1981). The positive/importance difference in performance can be achieved through variance in intensity and volume of training.

There are various factors which the athlete has to integrate to perform better. These factors can be trainable, such as certain psychological, physiological and biomechanical aspects, or teachable. There can also be some factors which are outside the control of the athlete, such as those related to age and genetics. Other elements which influence performance include the technical and material constraints, the condition of the environment in which the competition is taking place, coordination, and mindset of the athlete as well. It has also been argued by academics and coaches that genetic endowment is the vital element in determining the potential of an athlete to excel in his/her sport. This not only includes inherited traits of cardiovascular drift and anthropometric characteristics, but also fibre proportions of muscles (Bouchard, 1986).

In this section, we explain a new measure of performance based on the relationship between power output and heart rate. Firstly, we will review some previous studies that discussed this relationship. Then, we present our performance measure based on the relationship between power output and heart rate data under the effect of cardiovascular drift.

There have been several studies that have attempted to illustrate the relationship between heart rate and power output. Grazzi et al. (1999) found that there was a high correlation (approximately 0.98) between heart rate and power output data collected from 290 athletes. Furthermore, this study also concluded that there was a common time lag between the heart rate response and the power output. Stirling et al. (2008) found that the change of heart rate generally arose between 30 and 60 seconds. Different time lags of 10, 15, 20, and 30 seconds between heart rate and power output were investigated by Shrahili (2014). However, he concluded that a time lag of 15 seconds often portrayed the strongest relationship between heart rate and power output. Consequently, as there is no strong consensus for a single value for this lag, we investigate different time lags of 5 and 25 seconds. We find that the best time lag is still one of 15 seconds, as shown in Appendix 1. An example of the relationship between power output and heart rate for all sessions for rider 10 is presented in Appendix 2. Schniepp et al. (2002) illustrated that many factors during races and competition could influence cycling performance. For example, cold conditions are considered to be the most effective. Changes in metabolism and muscle blood flow can be found stemming from this factor. Figures 3.6, 3.7, and 3.8 show examples of recorded power output and heart rate from a single session on different timescales. Next, a new performance measure is presented.

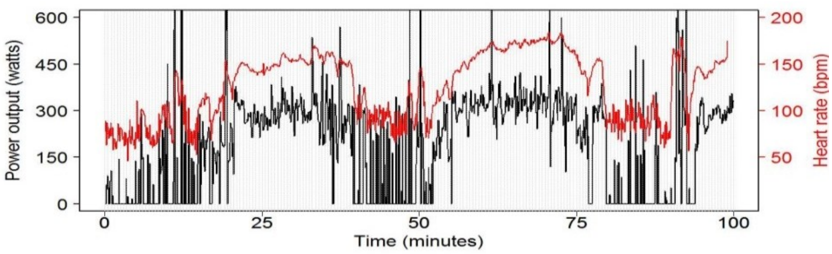

Figure 3-6 Power output and heart rate vs time for rider 3 in session 13, from minute 0 to minute 100

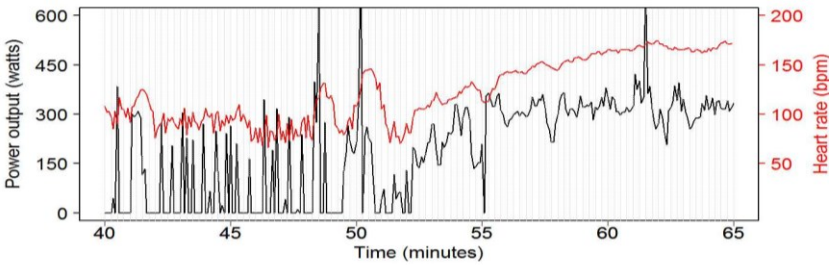

Figure 3-7 Power output and heart rate vs time for rider 3 in session 13, from minute 40 to minute 65

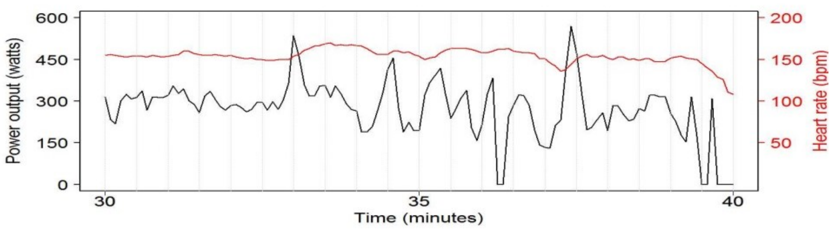

Figure 3-8 Power output and heart rate vs time for rider 3 in session 13, from minute 30 to minute 40

3.6.2 A Performance Measure based on the Relationship between Power output and Heart Rate

We describe the performance measure that we relate to training. Our performance measure depends on a linear relationship between power output and heart rate. We assume that expected power for each rider at time t on session i (P_{it}) is related to the heart rate ($H_{i,t+l}$) at time $t + l$ as follows:

$$P_{it} = a_i + b_i H_{i,t+l} + ctT_i, \qquad (3.5)$$

where T_i is the ambient outside temperature in°C for a specific session i, a_i and b_i are constants for a given rider in a particular session, l is the heart rate lag (l =15 seconds) and c is a global rider constant for each rider that models cardiac drift. We expect $c < 0$, so that for a given expected power output, the heart rate will drift upwards at rate $-ctT_i/b_i$. To improve the relationship between power output and heart rate compared to the work of Shrahili (2014) , the term that includes c is needed to model the drift in heart rate as the session proceeds. This is because at a fixed power output, heart rate has been observed to increase with time (Lafrenz et al., 2008). In this way, better estimates of a_i and b_i can be found, and a better performance measure obtained. The model, equation (3.5), is fitted to data by the method of least squares, the estimates and variances of the estimates are determined. These estimates are presented in Appendix 4. Secondly, we take into account a percentile of power output for each rider using his entire data history. It is denoted by P_q. For a specific rider, we determine some percentiles (e.g. the 75th , the 90th) of power output data that divide the ordered data with $q\%$ below it and $(100 - q)\%$ above it. Some Percentile values of power output for each rider are recorded. These percentile values are shown in Table 3-2.The suitable percentile relies on the nature of each competition.

Now, our proposed performance measure for a session is defined as the heart rate when the expected power is equal to this power output percentile, the ambient temperature is T_R °C on session i and t_R is time units into the session. This performance measure denoted h_{Pq} .The performance measure for session i is as follows:

$$h_{Pq,i} = (P_q - a_i - ct_R T_R)/b_i \qquad (3.6)$$

To calculate the performance measure h_{Pq} for each session i, a reference time t_R and a reference temperature T_R must be fixed. In our study, t_R=1 hour and T_R= 20 °C. Other times and ambient temperatures such as t_R=2 hours and T_R= 30 °C could be chosen to calculate and determine performance measure for each rider for each session.

Figure 3.9 shows an example for power output versus heart rate at lag time of 15 seconds with fitted line. The performance measures at P_{50} and P_{75} for each session for each rider are presented in Figures 3.10 and 3.11.

Table 3-2 Various percentiles of power output data for each rider

Rider	1	2	3	4	5	6	7	8	9	10
P_{50}	225	235	239	213	213	293	238	197	184	208
P_{75}	291	307	291	246	280	384	323	274	214	260
P_{90}	360	387	347	289	350	488	405	350	257	312
P_{99}	615	573	508	451	536	776	595	514	407	469

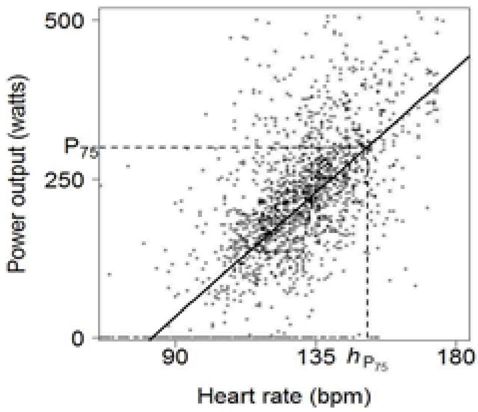

Figure 3-9 Power output vs heart rate with fitted line for rider 3 in a single session

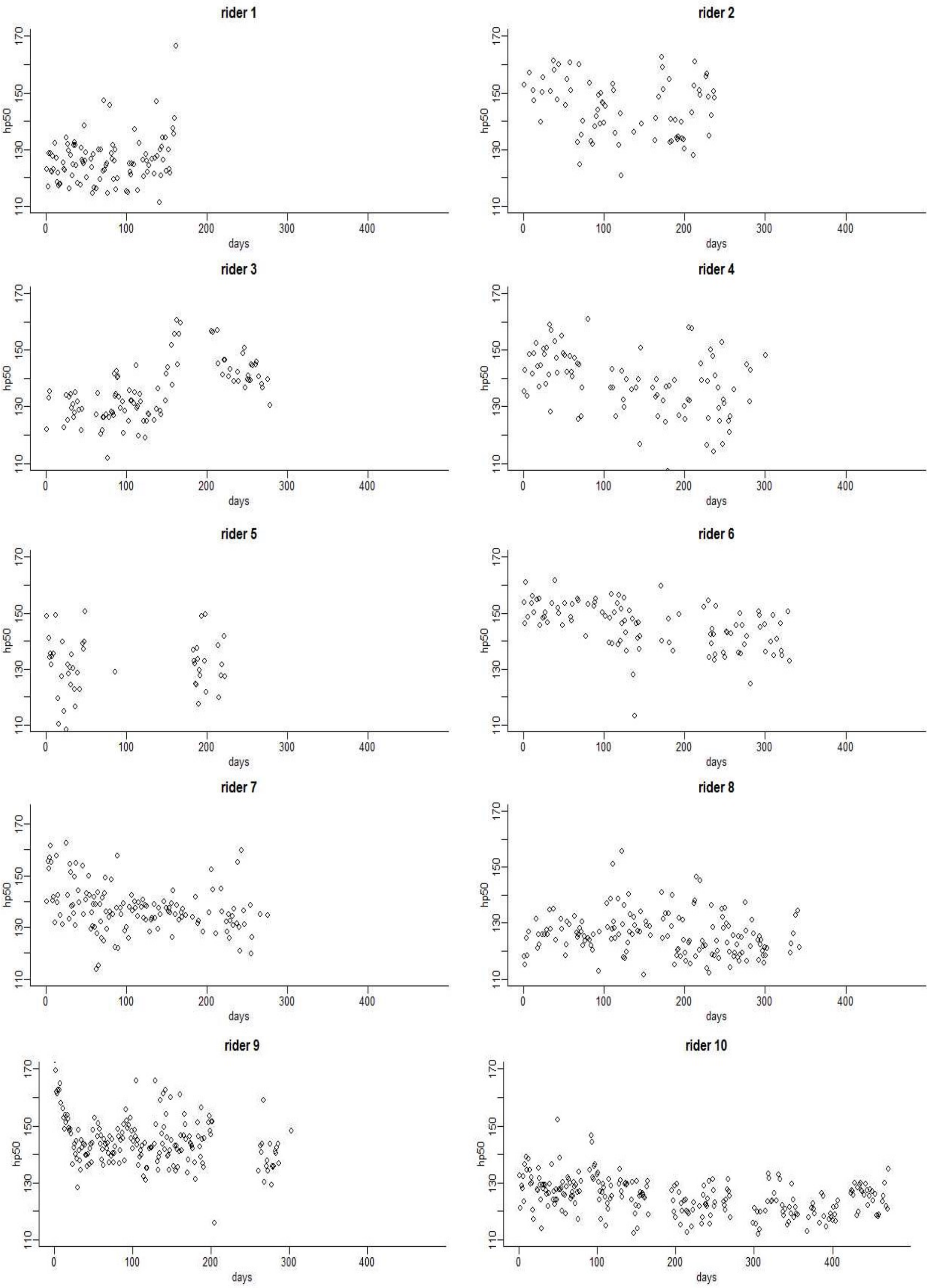

Figure 3-10 Performance measure h_{P50} for each session for each rider

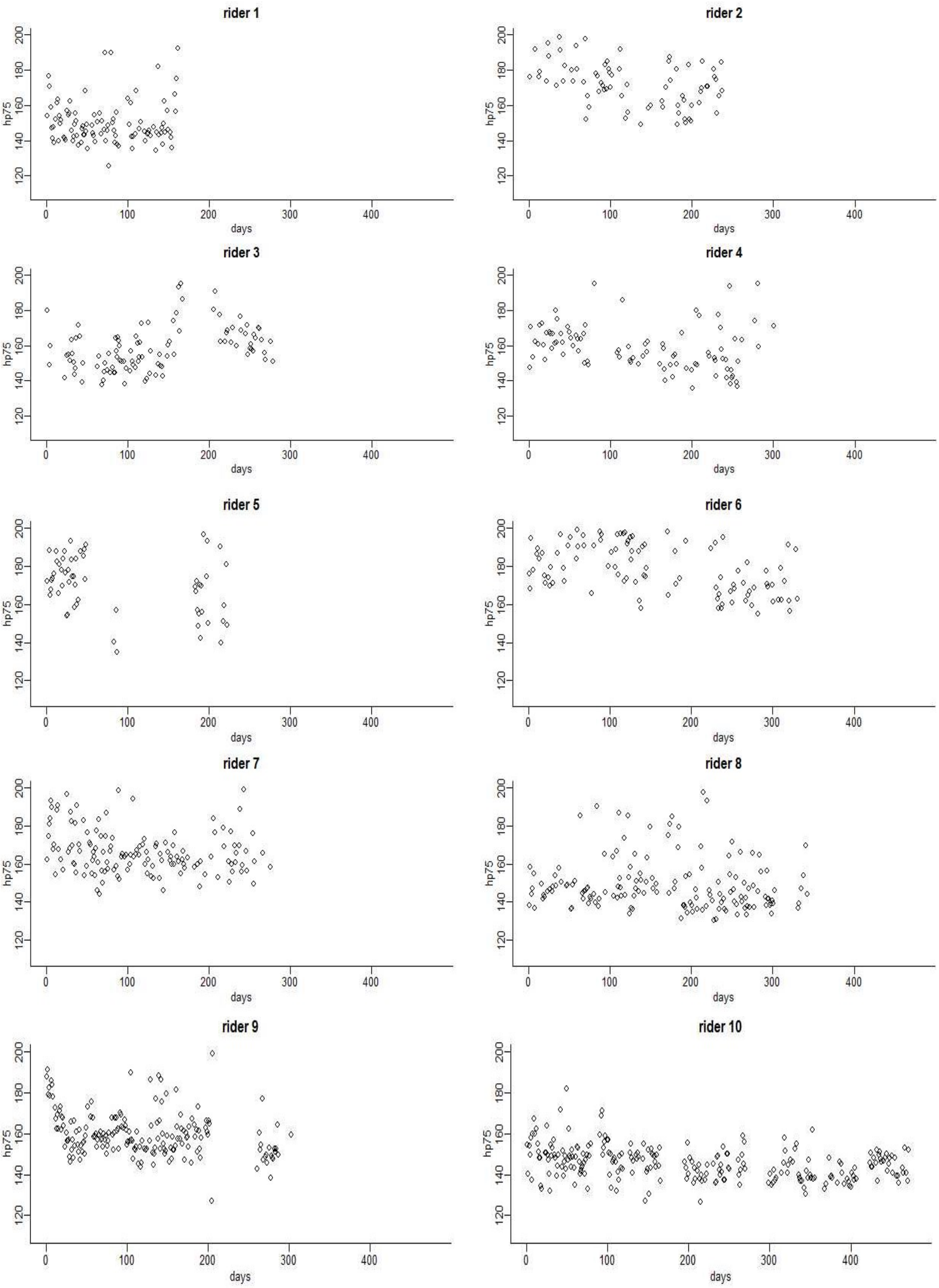

Figure 3-11 Performance measure h_{P75} for each session for each rider

3.6.3 A Modified Performance Measure based on the Relationship between Power output and Heart Rate

In this section, another new performance measure is presented. This performance is slightly different to the one we described in subsection 3.6.2. This measure will be related to training later to estimate the values of the Banister model parameters. It depends on the linear relationship between heart rate and power output. An example of the relationship between heart rate and power output for a single session for rider 10 is presented in Appendix 3. To calculate this measure, we suppose that the expected heart rate $H_{i,t+l}$developed by individual rider on session i at time $t+l$ is related to the power output P_{it} at time t on session i as follows:

$$H_{i,t+l} = a_i + b_i P_{it} + ctT_i \tag{3.7}$$

It should be noted that a_i, b_i and c here in equation (3.7) are different from those defined in equation (3.5). Nonetheless, we retain the notation for consistency of presentation. In equation (3.7), T_i is the ambient outside temperature for a session i, a_i and b_i are rider-session constants. c is the a global rider constant that models cardiac drift and we expect that $c > 0$. The coefficients of the model in equation (3.7) are determined for each rider and for each session using the method of least squares with a time lag of 15 seconds. These estimates are shown in Appendix 5. Then we specify a particular percentile of heart rate for each rider using his data history. Percentile values of heart rate for each rider are shown in Table 3-3. Now, our performance measure for a session is denoted by P_{hq} and is defined as follows:

$$P_{hq,i} = (hq - a_i - ct_i T_i)/b_i \tag{3.8}$$

where T is the ambient temperature in°C in session i and t_R is the time units into the session. To calculate the performance measure for each rider and for each session, a reference time t_R and a reference temperature T_R must be fixed. In our study, t_R=1 hour and T_R= 20 °C. Figure 3.12 shows an example of heart rate at lag 15 seconds versus power output with fitted line for rider 1 at a single session. The performance measures at h_{50} , h_{75} and h_{90} for all sessions for each rider are presented in Figures 3.13, 3.14 and 3.15.

3.7 Summary

In this chapter, two measures that quantify training load are introduced. These two measures are training impulse (TRIMP) and training stress score (TSS) respectively. Then, we present the Banister model. Our proposed performance measures using power output and heart rate data are presented. These performance measures will be related to the accumulated training effect (ATE) to determine the Banister model parameters later on. Examples of these performance measures for all sessions for each rider are shown. Furthermore, another performance measure that can be calculated from the relationship between heart rate and power output is proposed. Through this measure, the parameters of the Banister model with respect to the measures will also be estimated. Finally, examples of these performance measures for each session for each rider are presented.

Table 3-3 Various percentiles of heart rate data for each rider

Rider	1	2	3	4	5	6	7	8	9	10
h_{50}	127	142	135	145	129	140	140	118	142	126
h_{75}	144	158	150	156	147	155	154	136	151	140
h_{90}	155	170	161	167	163	168	164	151	160	151

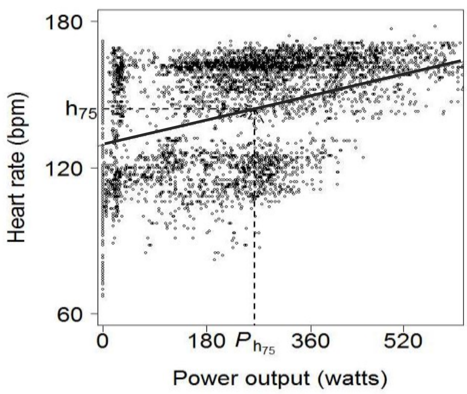

Figure 3-12 Heart rate vs power output with fitted line for rider 1 in a single session

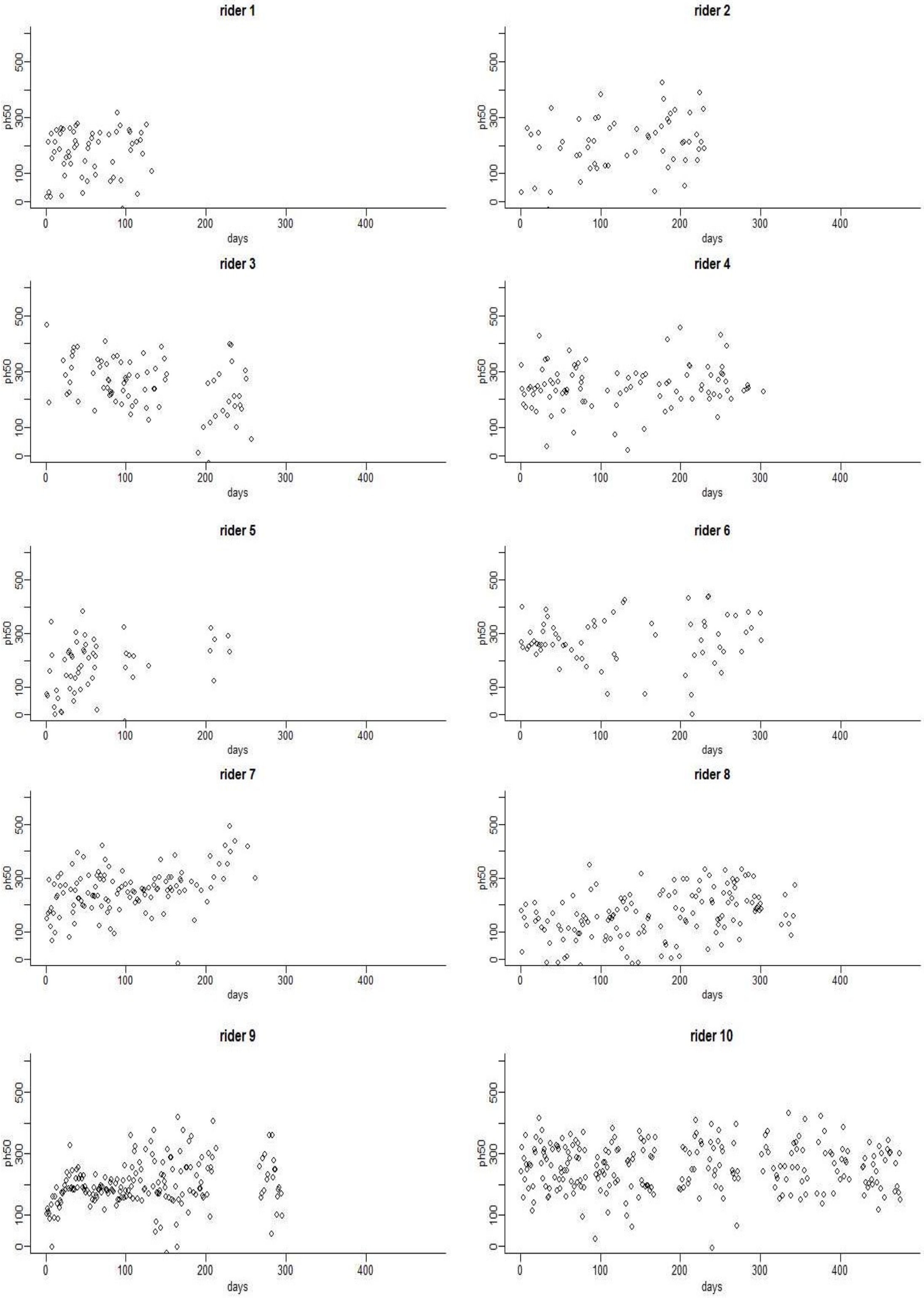

Figure 3-13 Performance measure P_{h50} for each session for each rider

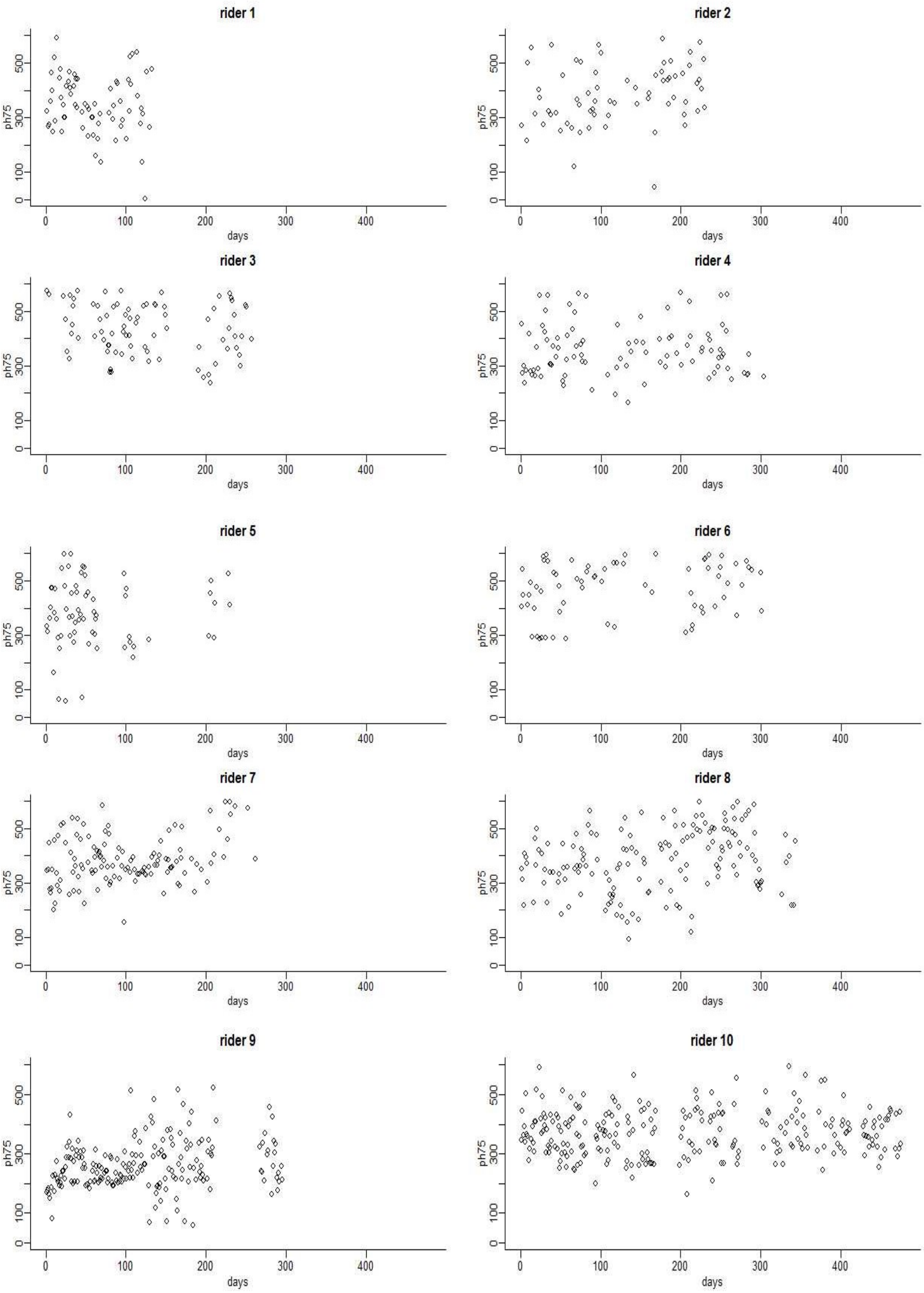

Figure 3-14 Performance measure P_{h75} for each session for each rider

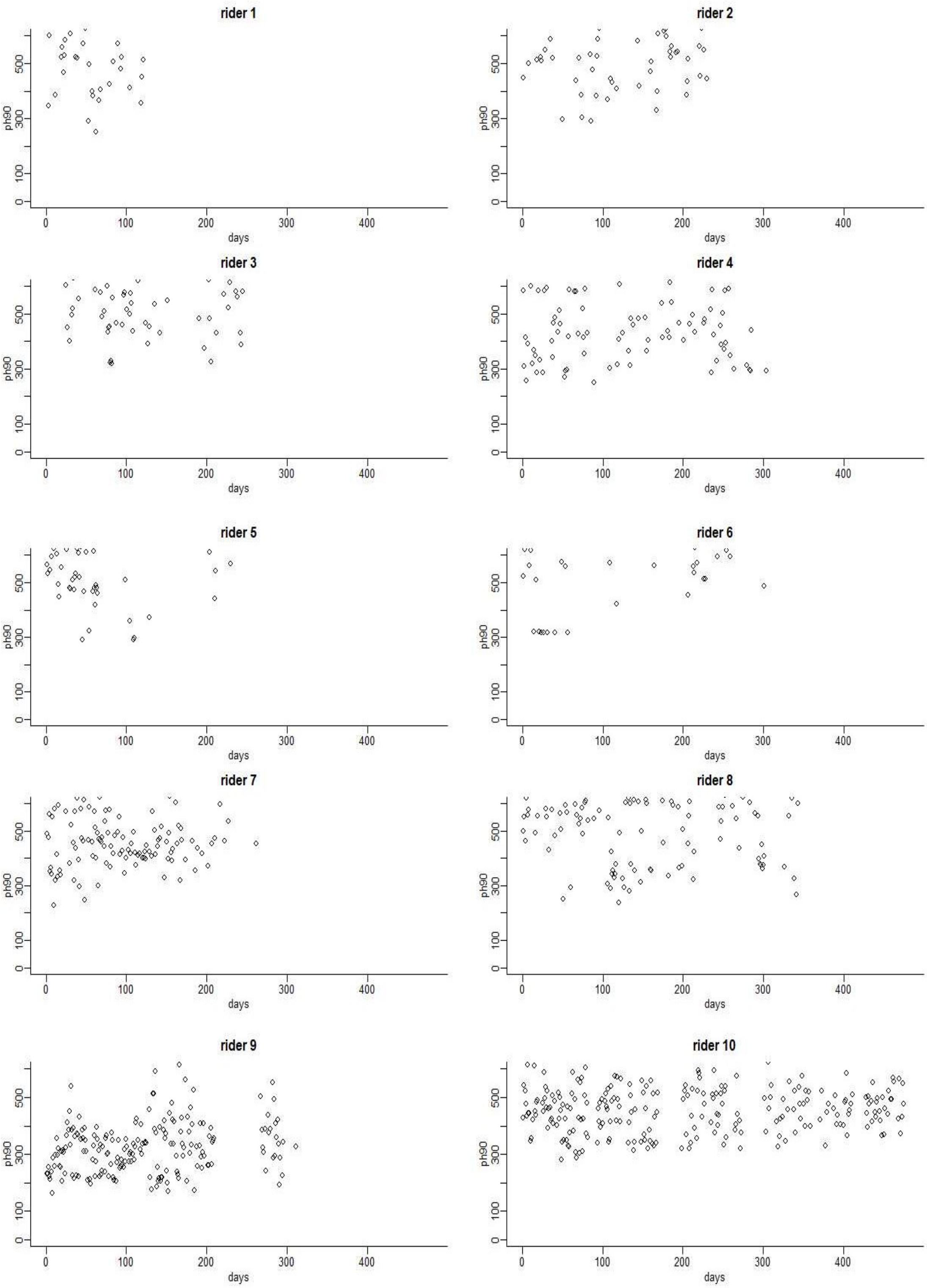

Figure 3-15 Performance measure P_{h90} for each session for each rider

CHAPTER FOUR

RELATING TRAINING TO PERFORMANCE

4.1 Introduction

In this chapter, the accumulation of training is related to the first of the performance measures described in the previous chapter, in order to determine the Banister model parameters. First, we describe the statistical distribution of our proposed performance measure. The performance measure itself is defined in section 3.6.2. This measure depends on the approximately linear relationship between power output and heart rate. Then we use the Banister model as a measure of training. These measures of training and performance are related to present a statistical model of training and performance. Through this, the Banister model parameters are estimated. The results of this model are discussed statistically and practically for each rider. Our methodology shows that the Banister model parameters can be estimated using data acquired in the field.

4.2 The Parameters of the Performance – Training Model

We relate the accumulated training effect (ATE) to our performance measure as follows: Firstly, we suppose that the relationship between performance and training is negatively linear. So the performance in session i, $h_{Pq,i}$, is related to the accumulated training effect in session i, ATE_i, as

$$h_{Pq,i} \sim N(\alpha + \beta ATE_i, \sigma^2). \tag{4.1}$$

with parameters α, β, and σ, the latter measuring the variability in the performance-training relationship. The ATE was previously defined in chapter 3 by equation (3.4). Then, we obtain the estimated performance for each session i for each rider which is defined from (3.6) in chapter 3 as

$$\hat{h}_{Pq,i} = (P_q - \hat{a}_i - \hat{c} t_R T_R)/\hat{b}.$$

This estimated performance is assumed to be distributed as

$$\hat{h}_{Pq,i} \sim N\left(h_{Pq,i}, \lambda_i\right). \tag{4.2}$$

The variances λ_i (i=1,…,n) are the variability in the relationship between power output and heart rate; these variances must be estimated, n is the training session for a rider . To accomplish this, we use the delta method (Casella & Berger, 2002, p.240) as follows:
Through the relationship between power output and heart rate given in equation (3.5), we can define power output for a session as:

$$P_q = a + b.h_q + ctT$$

Therefore

$$h_{Pq} = \frac{P_q - a - ctT}{b}$$

In the general form the delta method provides the variance of a function of parameter estimates:

$$var[y(\underline{\hat{\theta}})] = \sum_i \sum_j \frac{\partial y}{\partial \theta_i} \frac{\partial y}{\partial \theta_j} . cov(\hat{\theta}_i, \hat{\theta}_j).$$

In our case here, we have $\theta = (\theta_1, \theta_2, \theta_3) = (a, b, c)$ and

$$y(\theta) = h_{Pq} = \frac{P_q - a - ctT}{b},$$

so that

$$\frac{\partial y}{\partial a} = -\frac{1}{b}, \quad \frac{\partial y}{\partial b} = -\frac{P_q - a - ctT}{b^2}, \quad \frac{\partial y}{\partial c} = \frac{-tT}{b}.$$

Hence, the variances λ_i (i=1,…,n) for each session can be obtained and $\hat{\lambda}_i \geq 0$ for all i as follows

$$\hat{\lambda}_i = \frac{1}{\hat{b}_i^{\,2}} \Big\{ \hat{\text{var}}(\hat{a}_i) + \hat{h}_{Pq,i}^{\,2} \hat{\text{var}}(\hat{b}_i) + (tT)^2 \hat{\text{var}}(\hat{c}) + 2\hat{h}_{Pq,i} \hat{\text{cov}}(\hat{a}_i, \hat{b}_i) + 2tT \hat{\text{cov}}(\hat{a}_i, \hat{c}) + 2\hat{h}_{Pq,i} tT \hat{\text{cov}}(\hat{b}_i, \hat{c}) \Big\},$$

where $t = t_R$ and $T = T_R$.

The final step in our method is, through (4.1) and (4.2), to write the model of training–performance as

$$\hat{h}_{Pq,i} \sim N(\alpha + \beta ATE_i, \sigma^2 + \lambda_i).$$

The parameters of this model α, β, σ, k_f, τ_a and τ_f are estimated using the method of the maximum likelihood. Maximum likelihood estimation is considered as a preferred method of parameter estimation and is a fundamental tool for many statistical modelling techniques (Stuart et al., 1999). In this study, the estimation of the parameters is carried out in R (R Development Core Team, 2005).

This procedure is done to determine the values that maximise the log-likelihood function:

$$\log L = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{n} \log(\sigma^2 + \lambda_i) - \frac{1}{2}\sum_{i=1}^{n} \frac{\left(\hat{h}_{Pq,i} - \alpha - \beta ATE_i\right)^2}{(\sigma^2 + \lambda_i)}$$

The estimates for performance measures h_{P50} and h_{P75} when k_f=2 are shown in Tables 4-1 and 4-2. Moreover, Table 4-3 and Table 4-4 present the estimates for performance measures h_{P50} and h_{P75} when $k_f \neq 2$.

Table 4-1 Estimated parameters with standard errors of the model for performance measure h_{p50} and k_f=2 for each rider, with the *t* statistic and *p* value for the test of *β*=0.

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	12.2	3.5	78	61.1	9.8	1.9	139	6.4	-0.0027	0.0009	-3.00	0.00
2	3.1	1.9	5	2.1	0.7	0.2	160	4.1	-0.0406	0.0198	-2.05	0.02
3	12.3	6.1	78	87.3	9.3	6.6	140	5.6	-0.0015	0.0005	-3.00	0.00
4	2.4	1.6	137	28.4	44.2	10.8	139	2.8	-0.0034	0.0006	-5.00	0.00
5	23.4	3.5	184	103.2	0.2	1.5	156	6.2	-0.0026	0.0012	-2.17	0.02
6	0.2	1.6	182	88.1	17.0	2.3	156	4.7	-0.0024	0.0014	-1.71	0.04
7	7.1	1.3	175	118.6	0.5	2.1	146	3.9	-0.0016	0.0008	-2.00	0.02
8	2.1	1.2	62	25.4	32.1	11.7	122	2.3	-0.0030	0.0011	-2.73	0.00
9	3.2	0.8	7	2.1	3.3	0.9	145	2.2	-0.0128	0.0069	-1.86	0.03
10	2.4	0.7	166	40.7	33.1	11.1	130	2.1	-0.0013	0.0003	-4.30	0.00

Table 4-2 Estimated parameters with standard errors of the model for performance measure h_{P75} and k_f=2 for each rider, with the *t* statistic and *p* value for the test of *β*=0.

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	6.9	1.2	33	16.3	1.3	2.5	158	4.4	-0.0030	0.0014	-2.14	0.02
2	6.4	1.7	7	2.7	0.3	0.1	186	5.5	-0.0288	0.0126	-2.29	0.01
3	3.9	1.4	9	4.2	3.2	1.1	159	2.8	-0.0213	0.0127	-1.68	0.05
4	1.2	2.1	194	62.1	24.2	10.6	162	2.7	-0.0019	0.0005	-3.80	0.00
5	2.4	3.6	118	143.4	62.1	30.8	174	12.1	-0.0037	0.0021	-1.76	0.04
6	3.2	2.9	139	31.8	0.1	13.6	197	4.5	-0.0042	0.0009	-4.60	0.00
7	4.9	1.1	82	31.9	0.5	2.6	172	3.6	-0.0020	0.0008	-2.50	0.01
8	7.9	0.9	5	3.1	0.1	0.1	150	2.90	-0.0130	0.0088	-1.50	0.07
9	4.7	0.9	7	2.2	3.6	0.9	158	2.40	-0.0129	0.0071	-1.82	0.04
10	5.4	0.7	144	26.5	35.3	10.4	151	2.50	-0.0017	0.0004	-4.20	0.00

Table 4-3 Estimated parameters with standard errors of the model for performance measure h_{p50} when $k_f \neq 2$ for each rider, with the *t* statistic and *p* value for the test of *β*=0.

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	6.2	1.1	6	5.9	6.3	1.3	0.2	0.1	129	2.5	-0.0050	0.0050	-1.00	0.16
2	3.1	2.2	16	2.1	2.4	4.4	0.7	0.8	160	4.1	-0.0400	0.0200	-2.00	0.02
3	2.9	1.6	12	8.4	4.2	3.5	1.7	1.2	135	2.6	-0.0062	0.0047	-1.32	0.09
4	1.6	2.2	106	24.1	1.1	0.1	76.1	22.1	149	3.6	-0.0129	0.0078	-1.65	0.05
5	20	2.7	12	2.3	8.0	6.7	1.1	1.4	135	5.8	-0.0044	0.0070	-0.63	0.27
6	0.3	2.2	48	9.1	2.0	2.1	12.2	7.6	148	5.6	-0.0035	0.0018	-1.95	0.03
7	7.1	1.3	149	75.3	8.0	6.9	0.5	1.1	147	3.7	-0.0017	0.0007	-2.43	0.01
8	1.1	1.4	110	26.1	1.4	0.2	72	24.6	123	2.0	-0.0039	0.0023	-1.69	0.05
9	3.8	0.8	106	15.6	5.0	3.3	2.3	3.2	147	5.1	-0.0003	0.0001	-3.00	0.00
10	3.1	0.7	105	17.3	1.2	0.3	58.1	9.2	131	2.8	-0.0035	0.0012	-2.92	0.00

Table 4-4 Estimated parameters with standard errors of the model for performance measure h_{p75} when $k_f \neq 2$ for each rider, with the *t* statistic and p value for the test of β=0.

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	6.9	1.2	32	16.7	1.7	4.5	2.0	3.2	158	4.5	-0.0031	0.0015	-2.10	0.02
2	9.0	1.8	86	65.4	4.0	2.1	0.1	2.6	181	8.1	-0.0032	0.0017	-1.88	0.03
3	4.3	1.5	16	6.3	4.7	4.3	1.4	1.1	162	2.9	-0.0091	0.0047	-1.94	0.03
4	2.1	2.3	98	27.4	1.1	0.3	33.1	6.2	171	4.1	-0.0054	0.0029	-1.86	0.03
5	25	3.6	181	10.6	1.4	0.4	25.6	77.1	175	9.5	-0.0085	0.0073	-1.17	0.12
6	0.8	3.1	164	51.2	2.0	0.9	27.3	16.6	187	5.6	-0.0040	0.0018	-2.22	0.01
7	4.9	1.1	90	62.3	7.1	4.8	0.1	4.3	172	3.5	-0.0015	0.0008	-1.87	0.03
8	7.1	0.7	201	79.1	1.1	0.1	61.4	80.6	142	4.3	-0.0103	0.0041	-2.51	0.01
9	4.4	0.9	155	78.4	3.2	2.9	8.2	3.4	162	5.4	-0.0004	0.0002	-2.00	0.02
10	6.1	0.8	75	21.6	2.0	0.5	34.3	9.7	149	3.7	-0.0038	0.0018	-2.11	0.02

4.3 Statistical Discussion of the Training Effect

In this section, the relationship between the accumulated training effect and our proposed performance are discussed from a statistical perspective. This relationship is expected to be linearly negative. Therefore, we would like to reject the hypothesis $H_0: \beta = 0$ in favour of $H_1: \beta < 0$. To test this hypothesis, we use T test at a significance level of 0.05 using $t_{\hat{\beta}} = \hat{\beta} \,/\, (s.e.(\hat{\beta}))$, where $s.e.(\hat{\beta})$ is the standard error for $\widehat{\beta}$. From Tables 4-1, 4-2, 4-3 and 4-4, we conclude that the relationship between training and performance is statistically significant at level 5% when $t_{\hat{\beta}} \leq -1.64$. This relationship is negative for all riders (β < 0). However, in some riders, it can be observed that the value of the $t_{\hat{\beta}}$ is greater than -1.64. For example, rider 8 for performance measure h_{p75} at k_f=2, and rider 5 for performance measure h_{p75} when k_f is free.

4.4 Practical Discussion of the Training Effect

The practical interpretation of the training effect for each rider is discussed in this section. To accomplish this, the changes in power output from the beginning of training until the point at which a rider has completed the optimal amount of training must be calculated. This is expressed as follows:

$$\Delta_{P_q} = \hat{b} \times \left(-\hat{\beta}\right) \times \Delta_{ATE},$$

where b is the coefficient of the model relating power output and heart rate. The coefficients of this model (a, b, c) and their standard errors for each rider are shown in Table 4-5. The values of $\hat{\beta}$ are given in Tables 4-1, 4-2, 4-3 and 4-4. Δ_{ATE} is the difference between the maximum ATE and the initial value of the ATE. If Δ_{P_q}/P_q > 0.05, the accumulated training has a practical positive effect on performance. The changes in performance are shown in Tables 4-6, 4-7, 4-8 and 4-9 with free and fixed values of k_f

with percentiles of power output data $P_{50}$and P_{75} . These changes are greater or equal to 5% for all riders, excluding rider 5 at ($h_{p75}, k_f \neq 2$), which his results may have been influenced by the multiple gaps in his data.

Table 4-5 The coefficients of the model between power output and heart rate from the last 60 days

Rider	$\hat{a}$ $(s.e.)$	$\hat{b}$ $(s.e.)$	$\hat{c}$ $(s.e.)$
1	-102 (2.1)	2.66 (0.02)	-0.00040 (0.000004)
2	-195 (2.5)	2.95 (0.02)	-0.00008 (0.000040)
3	-35 (2.4)	1.94 (0.02)	-0.00030 (0.000010)
4	59 (2.5)	1.51 (0.02)	-0.00001 (0.000001)
5	-58 (2.1)	2.08 (0.02)	-0.00010 (0.000008)
6	-248 (4.3)	3.89 (0.03)	-0.00015 (0.000010)
7	-36 (3.1)	1.90 (0.03)	-0.00019 (0.000010)
8	-220 (1.8)	3.35 (0.02)	-0.00002 (0.000005)
9	-167 (2.9)	2.54 (0.02)	-0.00008 (0.000007)
10	-142 (2.1)	2.79 (0.02)	-0.00030 (0.000009)

Table 4-6 Performance gain and the ATE change for each rider at performance measure h_{P50} and k_f=2

Rider	$\hat{\beta}$	Δ_{ATE}	Δ_{p50}	P_{50}	Δ_{p50}/P_{50}
1	-0.0027	4166	30	225	0.13
2	-0.0406	801	96	235	0.41
3	-0.0015	3340	10	239	0.05
4	-0.0034	3455	14	213	0.07
5	-0.0026	13700	74	213	0.35
6	-0.0024	7541	70	293	0.24
7	-0.0016	10024	31	238	0.13
8	-0.0030	892	9	197	0.05
9	-0.0128	486	16	184	0.09
10	-0.0013	6992	25	208	0.12

Table 4-7 Performance gain and the ATE change for each rider at performance measure h_{P75} and k_f=2

Rider	$\hat{\beta}$	Δ_{ATE}	Δ_{p75}	P_{75}	Δ_{p75}/P_{75}
1	-0.0030	3541	28	291	0.10
2	-0.0288	1252	106	307	0.35
3	-0.0213	501	21	291	0.07
4	-0.0019	7919	17	246	0.07
5	-0.0037	1794	12	280	0.05
6	-0.0042	8483	139	384	0.36
7	-0.0020	6808	26	323	0.08
8	-0.0130	807	35	274	0.13
9	-0.0129	478	16	214	0.08
10	-0.0017	5636	26	260	0.10

Table 4-8 Performance gain and the ATE change for each rider at performance measure h_{P50} and k_f is free with percentile of power output P_{50}

Rider	$\hat{\beta}$	Δ_{ATE}	Δ_{p50}	P_{50}	Δ_{p50}/P_{50}
1	-0.0050	1579	21	225	0.09
2	-0.0400	799	94	235	0.40
3	-0.0062	882	11	239	0.04
4	-0.0129	1534	23	213	0.10
5	-0.0044	2244	21	213	0.09
6	-0.0035	2153	29	293	0.09
7	-0.0017	9321	30	238	0.13
8	-0.0039	980	13	197	0.07
9	-0.0003	15242	12	184	0.07
10	-0.0035	2614	26	208	0.13

Table 4-9 Performance gain and the ATE change for each rider at performance measure h_{P75} and k_f is free with percentile of power output P_{75}

Rider	$\hat{\beta}$	Δ_{ATE}	Δ_{p75}	P_{75}	Δ_{p75}/P_{75}
1	-0.0031	3433	28	291	0.10
2	-0.0032	6774	64	307	0.20
3	-0.0091	1315	23	291	0.08
4	-0.0054	3826	24	246	0.10
5	-0.0085	399	7	280	0.03
6	-0.0040	5738	89	384	0.23
7	-0.0015	7224	21	323	0.06
8	-0.0103	896	31	274	0.11
9	-0.0004	17829	18	214	0.08
10	-0.0038	2155	22	260	0.09

4.5 Discussion of Results

In this section, due to the individuality of the capacities of each of the riders, the findings are discussed statistically and practically for each rider from the results obtained.

For rider (1), significant relationships between the performance measures and the ATE are observed, as presented in Tables 4-1, 4-2, 4-3, and 4-4. Moreover, the training effect for this rider is practically significant in the practical sense for all cases and performance for this rider is increasing by about 13%, 10%, 9% and 10%, as this can be seen in Tables 4-6, 4-7, 4-8 and 4-9. Furthermore, the ATE decreases slightly after 50 days, as shown in Figures 4.1, 4.2 and 4.4.

The training effect for rider (2) is statistically significant for both performance measures h_{P50} and h_{P75} with either free to vary or fixed values of the fatigue decay time constant k_f. This is indicated in Tables 4-1, 4-2, 4-3, and 4-4. Furthermore, a slight improvement in the performance measures for this rider with free or fixed values of k_f is illustrated in Figures 4.1, 4.2, 4.3, and 4.4. There is a similarity between the fitness and the fatigue decay time constants, as presented in Tables 4-1, 4-2 and 4-3, the exception being the case when the performance measure is h_{P75} and k_f is free. For this rider performance is increasing by about 41%, 35%, 40% and 20%, as shown in Tables 4-6, 4-7, 4-8 and 4-9.

Although the results are statistically and practically significant for rider (3), the training effect is not statistically significant with the performance measure h_{P50} and the free to vary value of k_f. The parameter values of the fitness and fatigue decay constants are similar, as seen in Tables 4-2, 4-3, and 4-4. The performance for this rider is increasing by 5%, 7%, 4% and 8%, as presented in Tables 4-6, 4-7, 4-8 and 4-9.

Rider (4) shows a clear improvement in his performance measures h_{P50} and $h_{P75}$with fixed or free k_f, as seen in Figures 4.1, 4.2, 4.3, and 4.4. Moreover, the relationship between the ATE and the performance measures (h_{P50} and h_{P75}) is significant for all cases, as shown in Tables 4-1, 4-2, 4-3, and 4-4. Additionally, as presented in Tables 4-6, 4-7, 4-8, and 4-9, this rider demonstrates a significant practical training effect for all cases by about 7%, 7%, 10% and 10%, as this clearly explained in Tables 4-6, 4-7, 4-8 and 4-9 .

For rider (5), there is a significant statistical relationship between the ATE and the performance measures (h_{P50} and h_{P75}) for fixed k_f, as shown in Tables 4-1 and 4-2. However, the statistical effect of training for this rider is not significant with the performance measures h_{P50} and h_{P75} and free to vary k_f, as configured in Tables 4-3 and 4-4. Furthermore, this rider shows just 3% in practical training effect with the performance measure h_{P75} and free k_f, as observed in Table 4-9. This poor effect occurred because the data states multiple gaps over time for this rider, which may have influenced this rider's results.

The relationship between the ATE and the performance measures h_{P50} and h_{P75} with fixed or free to vary k_f is practically and statistically significant for rider (6) in all cases. . the performance is increasing by 24%, 36%, 9% and 23%, as this illustrated in Tables 4-6, 4-7, 4-8, and 4-9. This rider's performance measures improve slightly with h_{P50} and free to vary or fixed k_f, as seen in Figures 4.1 and 4.3.

Although rider (7) demonstrates both a significant practical and statistical relationship between the ATE and the performance measures h_{P50} and $h_{P75}$when k_f is fixed or free to vary, as presented in Tables 4-1, 4-2, 4-3, and 4-4, he also shows a huge variation in the parameters values in the Banister model between the different cases . This can be observed in Tables 4-1, 4-2, 4-3, and 4-4. Furthermore, the performance measure for this rider for all cases indicates clear improvement, as seen in Figures 4.1, 4.2, 4.3, and 4.4 and is increasing by 13%, 8%, 13% and 6%, as can be seen in Tables in 4-6, 4-7, 4-8, and 4-9.

For rider (8), there is a significant practical effect of training in all cases and performance is increasing by 5%, 13%, 7% and 11%, as shown in Tables in 4-6, 4-7, 4-8, and 4-9. Although there is a statistical significant relationship between the ATE and the performance measures in most cases, the training effect for this rider is not statistically significant with the performance measure h_{P75} and fixed value of k_f, as presented in Table 4-2.

The results for rider (9) are statistically significant for all performance measures when k_f is fixed or free to vary. Moreover, this rider offers significant practical effects of training for the both performance measures by 8%, whether k_f is fixed or free to vary, as explained in Tables 4-6, 4-7, 4-8, and 4-9. Furthermore, a slight improvement over time is observed for this rider with the performance measure h_{P75} and fixed k_f, as seen in Figure 4.2.

Rider (10) expresses similar fitness and fatigue decay time constants, as can be observed in Tables 4-1, 4-2, 4-3, and 4-4. Furthermore, the training effect for this rider is practically and statistically significant in all cases. The performance for this rider is increasing by 11%, as this presented in 4-6, 4-7, 4-8, and 4-9. Although rider 10 has had many training sessions, his performance measure has improved only slightly in just one case, with h_{P75} and fixed k_f, Figure 4.2 clearly shows this.

In conclusion, taking into account the fact that cardiovascular drift could strengthen the relationship between performance and training, according to the $t_{\hat{\beta}} = \hat{\beta} \,/\, (s.e.(\hat{\beta}))$ and Δ_{P_q}/P_q values shown in Tables 4-1, 4-2, 4-3, 4-4, 4-6, 4-7, 4-8, and 4-9 and compare them with the results of Shrahili (2014). Our findings are statistically and practically acceptable for the majority of the riders while half of them were acceptable in Shrahili's study. In our findings, training has a significant positive effect for all riders, with the exception of rider 5. There are a lot of gaps in rider 5's training data, which could have affected his results.

4.6 Summary

In this chapter, the Banister model parameters for each rider are determined by relating the ATE to our performance measure, which is defined in equation (3.6). Then, we discuss our results practically and statistically, with plots of the ATE and the performance measures for each rider. Finally, we present a brief discussion of our results, comparing these with the previous study of Shrahili (2014). Next, we describe how to estimate the Banister model parameters using the second of the two performance measures that were described in chapter 3.

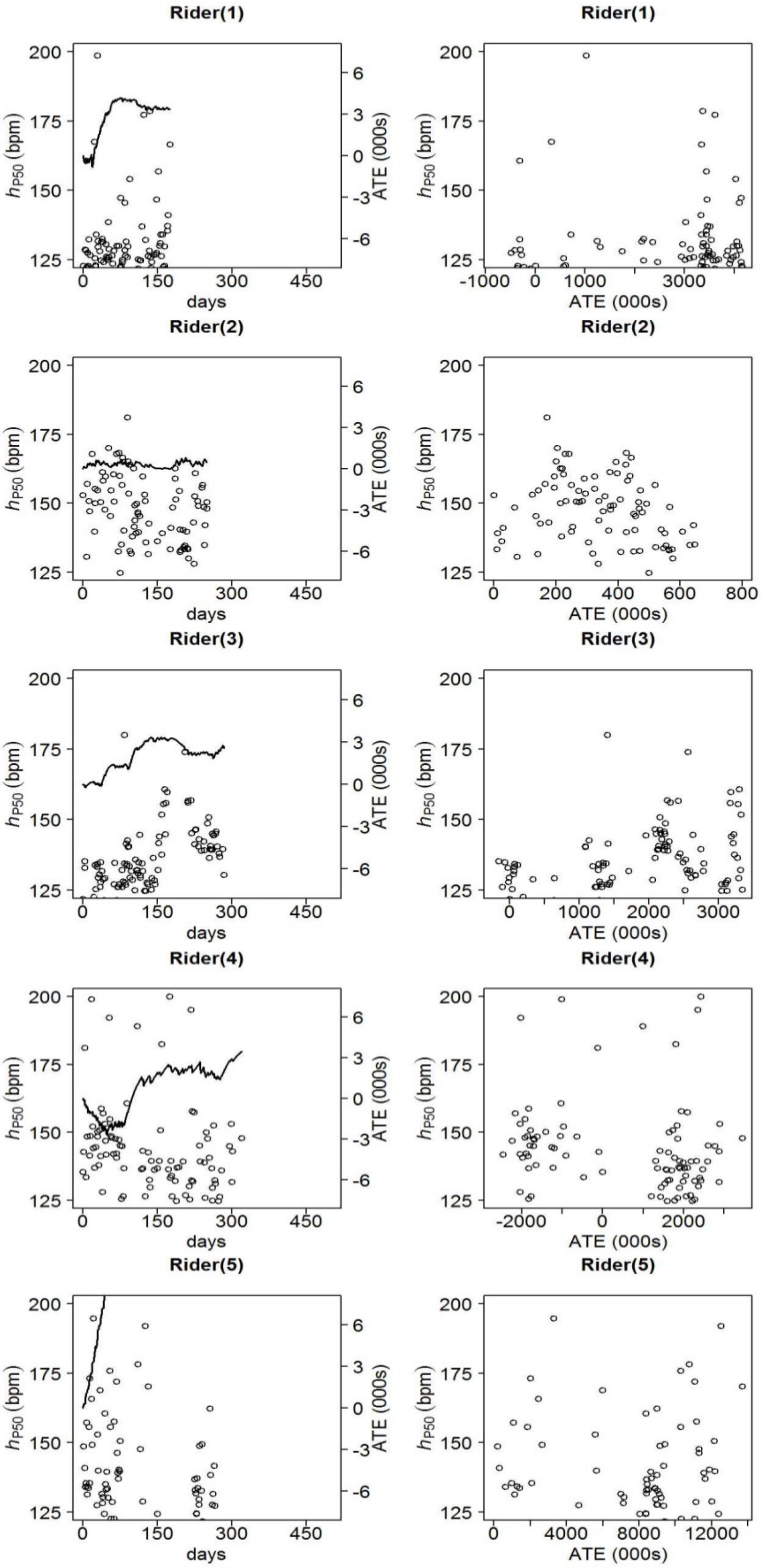

Figure 4-1 Two plots for each rider: left h_{P50} (symbols) vs time in days and ATE (line) when k_f=2 vs time in days; right h_{P50} vs ATE (all sessions)

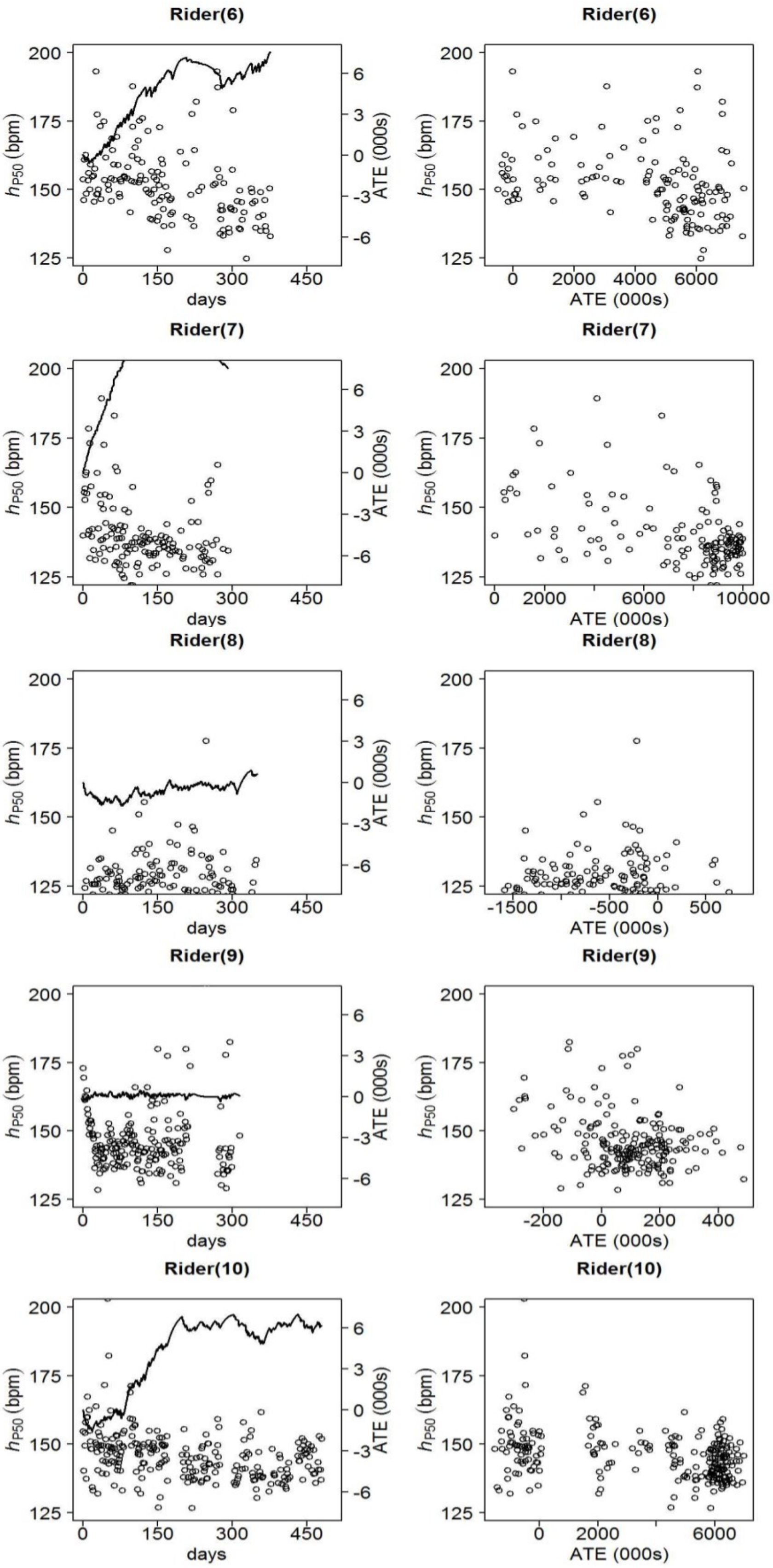

Figure 4.1 Continued

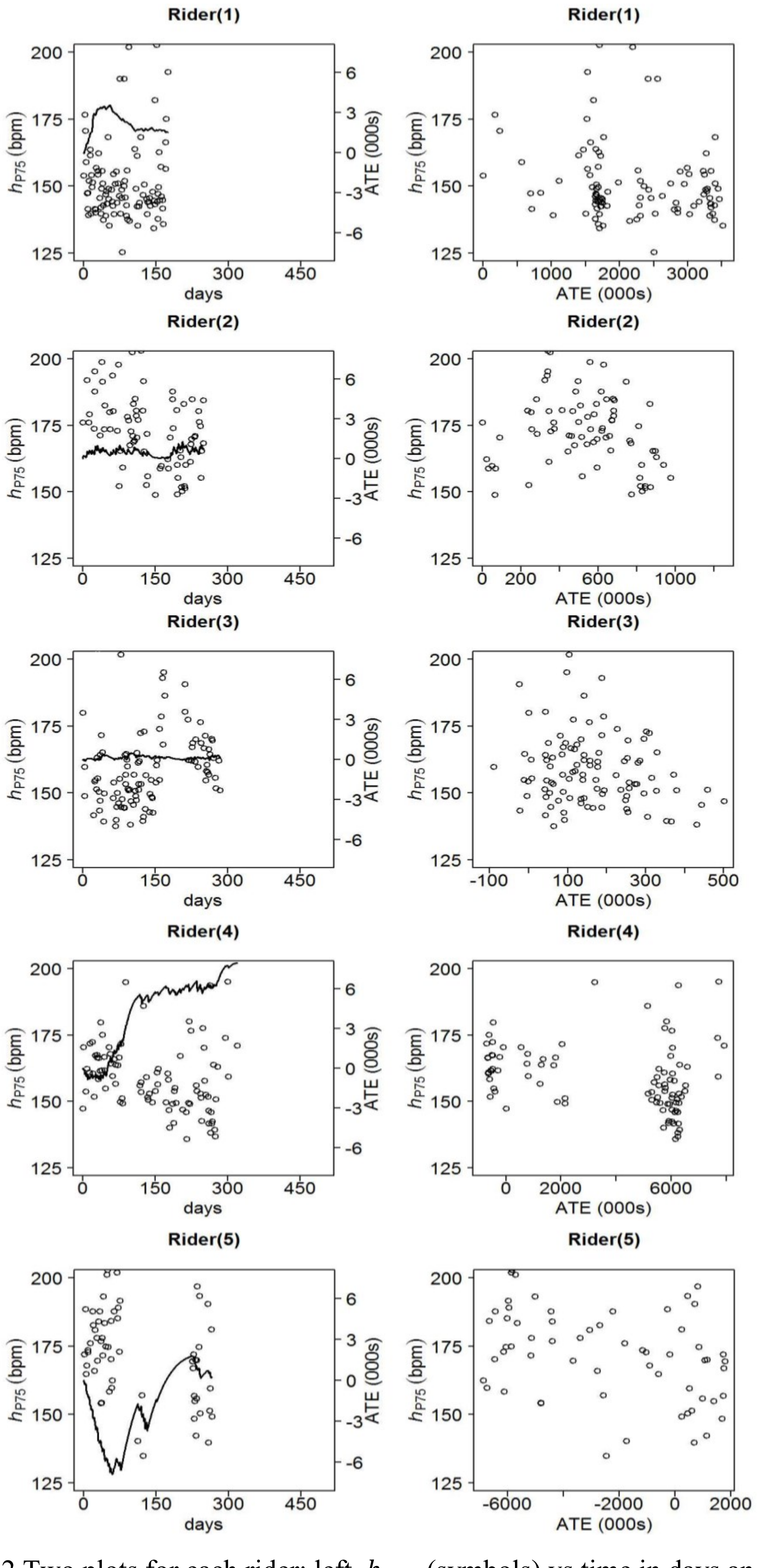

Figure 4-2 Two plots for each rider: left h_{P75} (symbols) vs time in days and ATE (line) when k_f = 2 vs time in days; right h_{P75} vs ATE (all sessions)

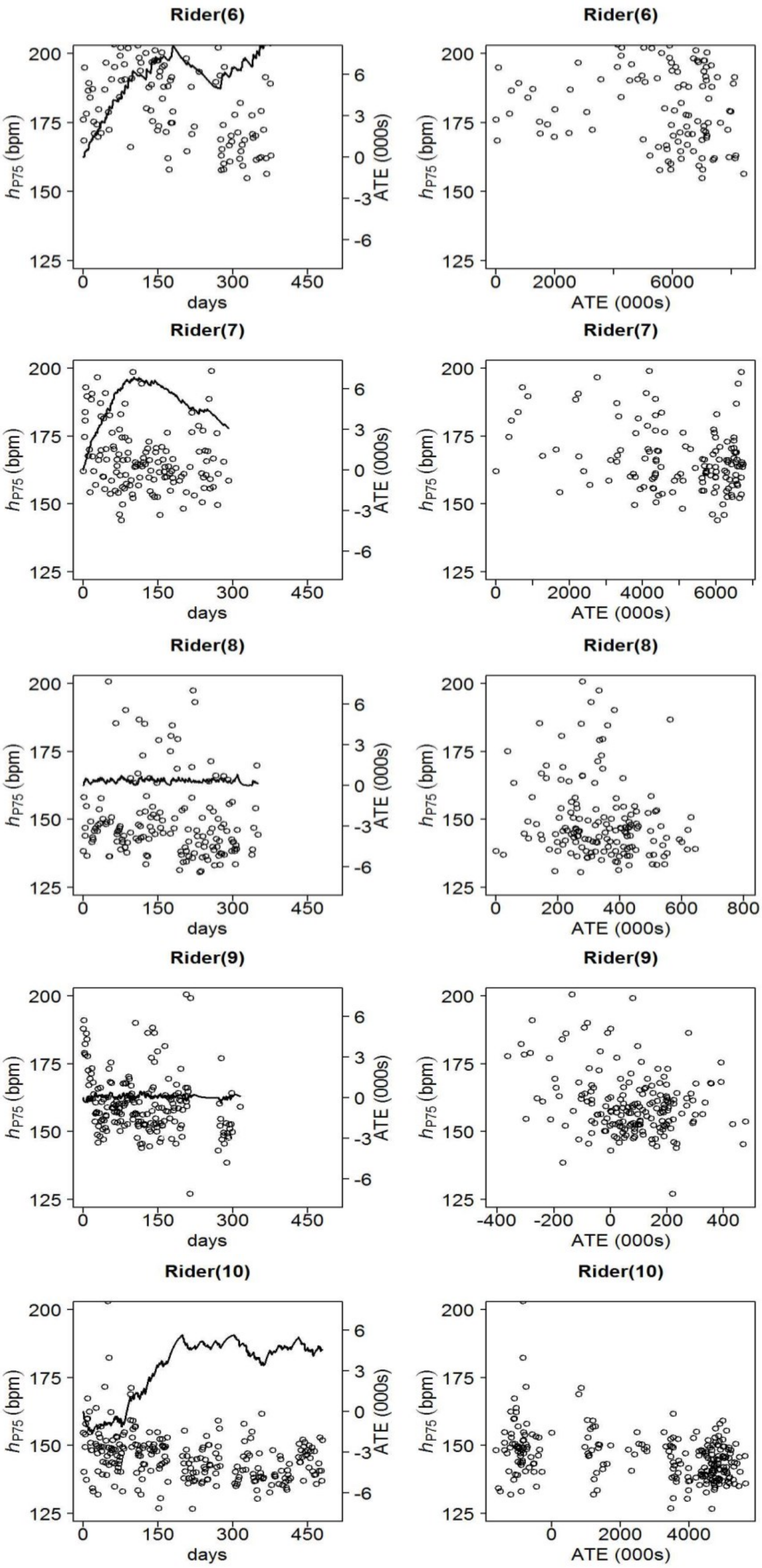

Figure 4.2 Continued

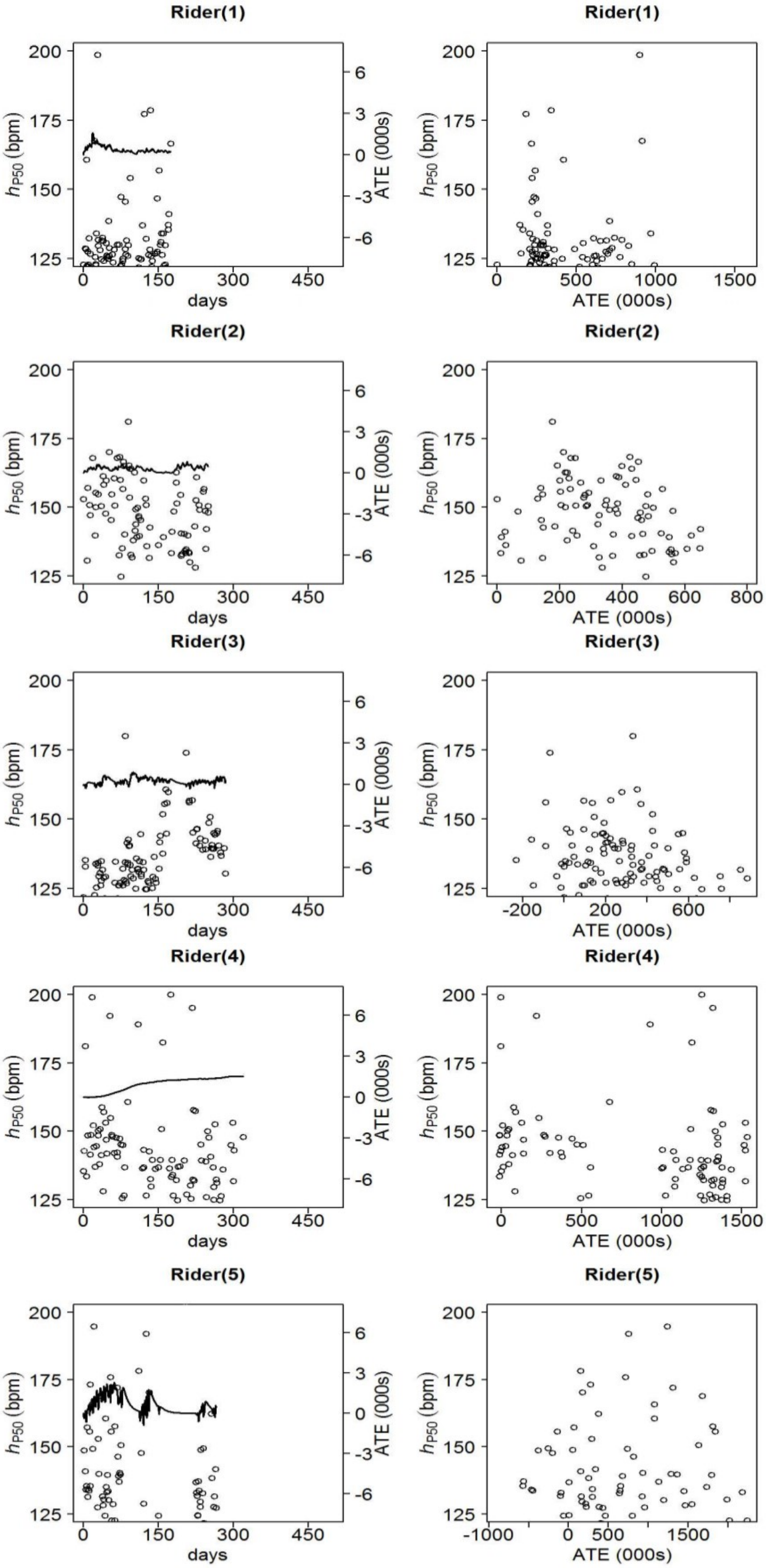

Figure 4-3 Two plots for each rider: left h_{P50} (symbols) vs time in days and ATE (line) when $k_f \neq 2$ vs time in days; right h_{P50} vs ATE (all sessions)

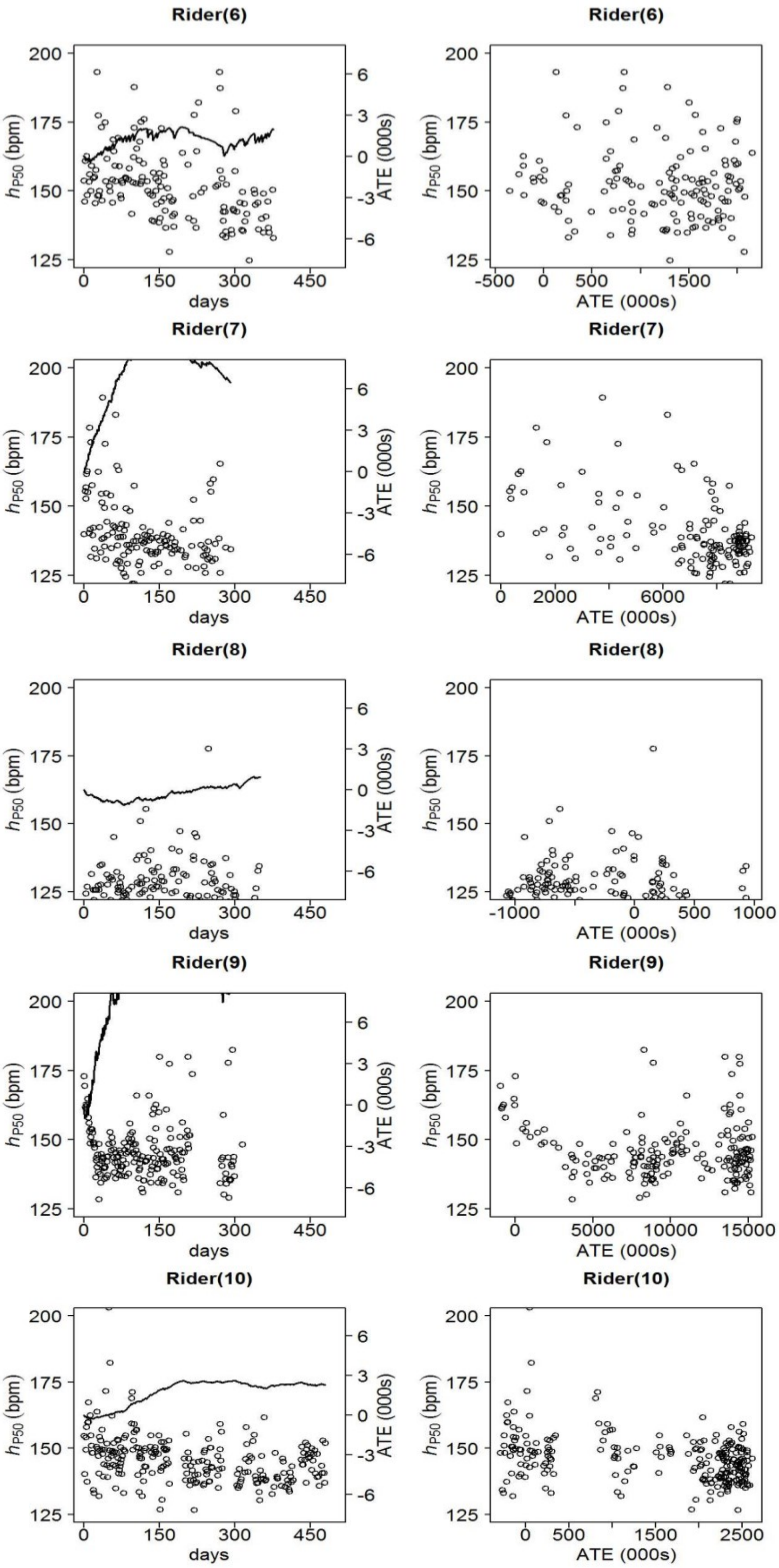

Figure 4.3 Continued

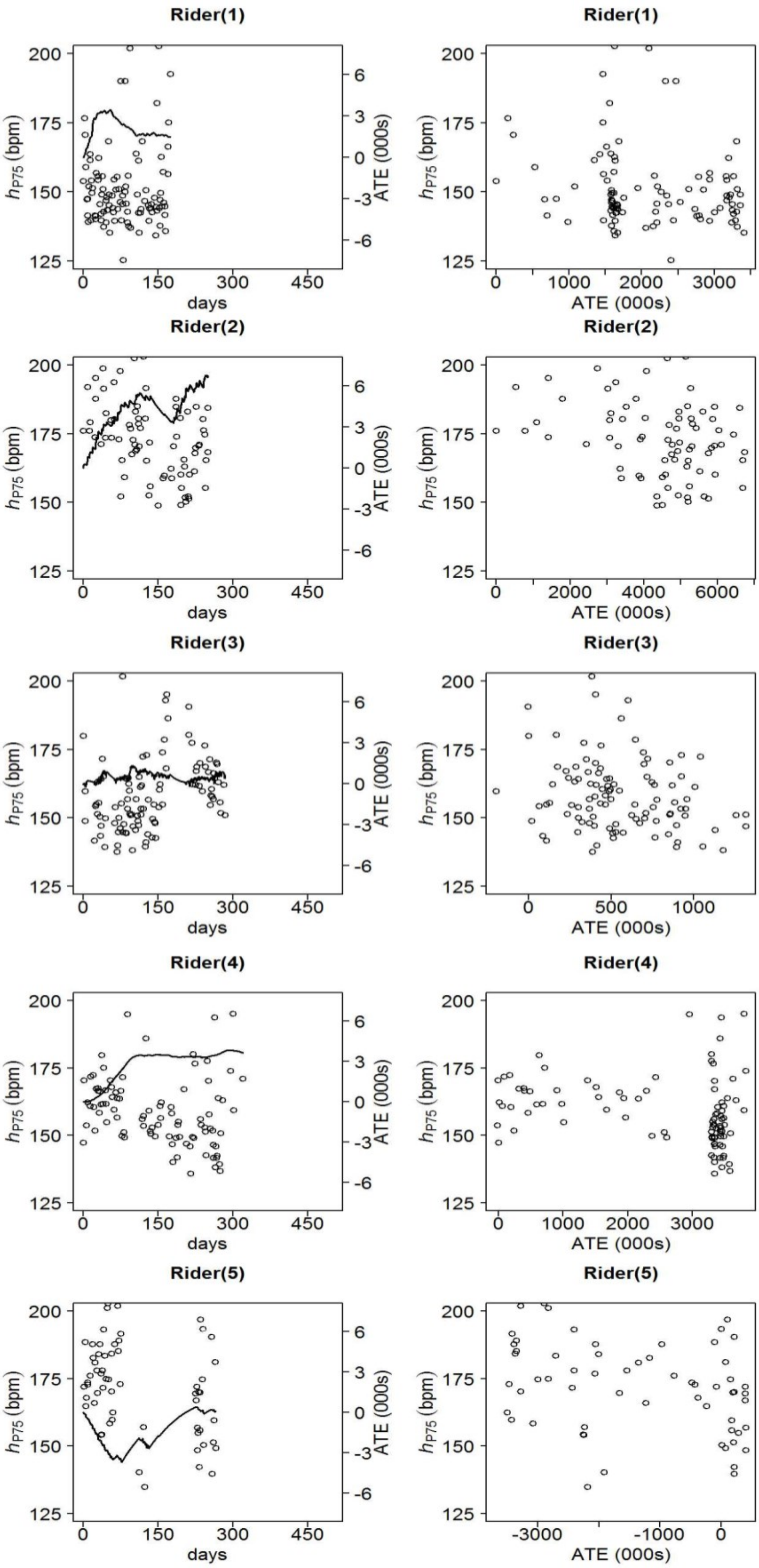

Figure 4-4 Two plots for each rider: left h_{P75} (symbols) vs time in days and ATE (line) when $k_f \neq 2$ vs time in days; right h_{P75} vs ATE (all sessions)

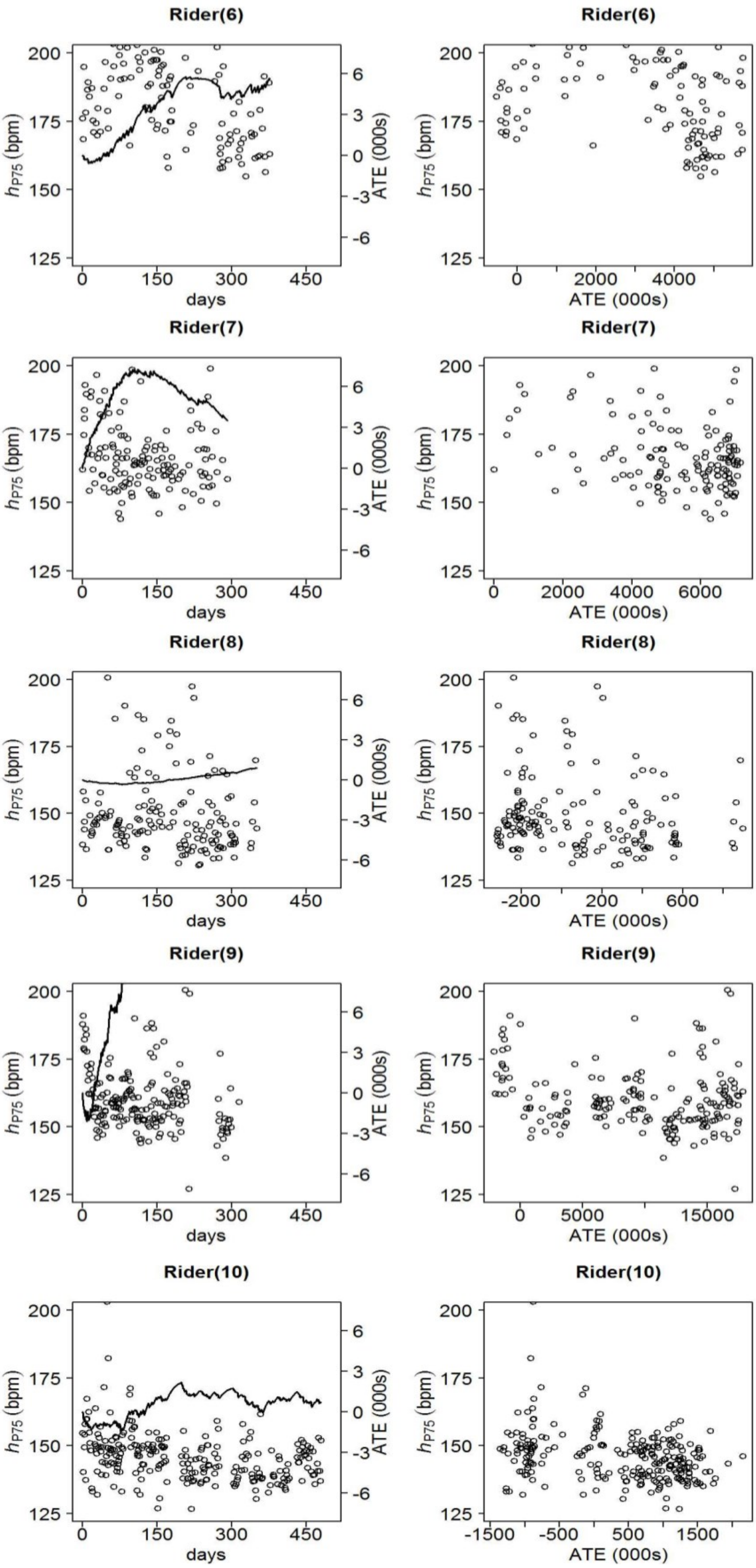

Figure 4.4 Continued

CHAPTER FIVE

A MODIFIED TRAINING - PERFORMANCE MODEL

5.1 Introduction

In this chapter, the training–performance model parameters are estimated using an alternative performance measure, the second performance measure described in chapter 3. This measure still depends on the relationship between heart rate and power output. However, in this case, we regard the power output as the performance measure rather than heart rate. So the relationship for this performance measure is one of positive related to the ATE rather than the negative relationship of the previous chapter. The measure itself is defined in section 3.6.3. The measure of training that is presented in chapter 3 is used. Then, we present a statistical model that links the training measure to our new performance measure, and estimate the Banister model parameters as before. Our methodology is carried out individually for each athlete. The results of this model are again discussed statistically and practically.

5.2 Relating Training to Performance

In this section we consider $P_{h_{q,i}}$as performance measure that is defined in equation (3.8). Again we have:

$$P_{h_{q,i}} \sim N(\alpha + \beta ATE_i, \sigma^2), \tag{5.1}$$

and

$$\hat{P}_{h_{q,i}} \sim N\left(P_{h_{q,i}}, \lambda_i\right), \tag{5.2}$$

where $\lambda_i (i = 1, \dots, n)$ is the variance in the estimate $\hat{P}_{hq,i}$, which must be calculated and $\hat{\lambda}_i \geq 0$ for all i. To do so, we use the delta method (Casella & Berger, 2002, p.240) as follows:

$$\hat{\lambda}_i = \frac{1}{\hat{b}_i^{\,2}}\Big\{\text{var}(\hat{a}_i) + (\hat{P}_{h_{q,i}})^2 \text{var}\big(\hat{b}_i\big) + (tT)^2 \,\text{var}(\hat{c}) + 2\hat{P}_{h_{q,i}} \text{cov}\big(\hat{a}_i, \hat{b}_i\big) + 2tT\text{cov}(\hat{a}_i, \hat{c}) + 2\hat{P}_{h_{q,i}} tT\text{cov}\big(\hat{b}_i, \hat{c}\big)\Big\}$$

Finally, through (5.1) and (5.2), we conclude the modified model of training–performance as

$$\hat{P}_{h_{q,i}} \sim N(\alpha + \beta ATE_i, \sigma^2 + \lambda_i)$$

So α, β, σ, λ are different from those used in equations (4.1) and (4.2). The parameters can again be estimated using the method of maximum likelihood. This is done using R programming. Tables 5-1 and 5-2 show the parameters for each rider when the performance measures are $\hat{P}_{h75}$ and $\hat{P}_{h90}$.

Table 5-1 Estimated parameters with standard errors of the model of training–performance when the performance measure is P_{h75} for each rider with the *t* statistic and p value for the test of β=0

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	25.4	1.2	35	2.3	2.9	0.5	2.8	0.9	195	17.9	0.085	0.024	3.54	0.00
2	12.4	0.9	31	7.1	1.8	0.5	4.1	0.8	193	30.2	0.120	0.032	3.75	0.00
3	30.6	3.3	16	1.4	3.2	0.4	1.0	0.1	145	25.9	0.318	0.032	9.93	0.00
4	27.2	8.4	95	13.8	2.5	0.2	13.3	0.9	174	10.5	0.043	0.009	4.78	0.00
5	57.5	3.2	139	10.5	0.2	0.1	69.3	3.5	157	36.5	0.023	0.003	7.67	0.00
6	29.9	1.8	161	7.2	3.5	0.1	23.2	0.4	139	10.5	0.094	0.015	6.26	0.00
7	22.9	4.8	132	13.1	1.1	0.2	1.1	0.5	143	61.8	0.033	0.009	3.66	0.00
8	10.8	4.4	162	10.4	1.4	0.2	40.1	2.3	134	57.7	0.044	0.013	3.38	0.00
9	82.9	5.7	168	49.3	2.6	1.8	1.6	0.2	187	19.1	0.004	0.001	4.00	0.00
10	17.4	1.3	126	15.7	1.2	0.1	20.4	1.5	147	10.9	0.031	0.007	4.23	0.00

Table 5-2 Estimated parameters with standard errors of the model of training–performance when the performance measure is P_{h90} for each rider with the *t* statistic and p value for the test of β=0

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	22.5	2.2	59	3.1	1.2	0.1	1.9	0.2	111	50.5	0.103	0.018	5.75	0.00
2	18.8	5.2	100	14.5	2.1	0.6	8.4	1.1	143	49.1	0.095	0.030	3.17	0.00
3	42.7	9.3	31	6.2	1.2	0.1	4.9	0.8	163	14.6	0.266	0.042	6.33	0.00
4	13.2	2.3	95	9.4	2.6	0.4	4.8	0.6	158	23.4	0.056	0.018	3.11	0.00
5	47.2	6.6	145	35.3	0.4	0.1	81.4	2.2	159	53.8	0.058	0.014	4.14	0.00
6	16.8	1.8	91	7.1	2.6	0.3	9.3	0.8	155	24.9	0.146	0.015	9.73	0.00
7	72.9	8.7	59	3.4	2.3	0.4	5.7	0.3	127	96.8	0.113	0.013	8.69	0.00
8	18.1	2.6	93	20.2	2.2	0.1	17.9	0.4	146	36.0	0.076	0.020	3.80	0.00
9	44.1	2.7	114	7.8	1.5	0.2	3.9	0.5	145	25.5	0.012	0.002	6.00	0.00
10	31.5	1.7	164	6.1	2.1	0.2	11.8	0.1	195	18.9	0.024	0.010	2.40	0.00

5.3 Statistical Discussion of the Training Effect

In this section, we study statistically the training effect on performance. The expected relationship between the accumulated training effect and our proposed performance is linearly positive. Therefore, we would like to reject the hypothesis $H_0: \beta = 0$ in favour of $H_1: \beta > 0$. Through Tables 5-1 and 5-2, we conclude that there is a statistically significant linear relationship between the accumulated training effect and our proposed performance measure when the performance measures are P_{h75} or P_{h90} for all riders.

5.4 Practical Discussion of the Training Effect

In this section, the training effect is discussed practically. As we conclude that there is a statistically significant relationship between the accumulated training effect and our proposed performance measures, it is useful to determine the practical significance of the increase. This can be calculated using the change in heart rate from the beginning of training until the point at which a rider has completed the optimal training, as follows:

$$\Delta_{h_q} = b \times (\hat{\beta}) \times \Delta_{ATE},$$

where $q \in \{75,90\}$ and b is the coefficient of the model giving heart rate as a linear function of power output. The coefficients of this model are shown in table 5-3. The values of $\hat{\beta}$ are shown in Tables 5-1 and 5-2. Δ_{ATE} is defined as the change between the maximum and initial accumulated training effect ($\Delta_{ATE} = ATE_{max} - ATE_1$). The changes in performance are presented in Tables 5-4 and 5-5 for all riders with different performance measures. These changes range between 4% and 20% for different riders when the performance measure is P_{h75} and between 5% and 35% when the performance measure is P_{h90}.

Table 5-3 The coefficients of the model between heart rate and power output from the last sixty days

Rider	$\hat{a}$ $(s.e.)$	$\hat{b}$ $(s.e.)$	$\hat{c}$ $(s.e.)$
1	97.02 (0.20)	0.108 (0.0006)	0.000062 (0.000001)
2	114.31 (0.21)	0.038 (0.0006)	0.000008 (0.000001)
3	99.30 (0.37)	0.015 (0.0001)	0.000056 (0.000003)
4	104.21 (0.60)	0.077 (0.0026)	0.000023 (0.000004)
5	85.73 (0.32)	0.078 (0.0012)	0.000090 (0.000002)
6	112.50 (0.26)	0.063 (0.0011)	0.000055 (0.000002)
7	92.93 (0.40)	0.043 (0.0014)	0.000056 (0.000003)
8	85.99 (0.19)	0.016 (0.0008)	0.000011 (0.000001)
9	105.12 (0.24)	0.153 (0.0010)	0.000064 (0.000002)
10	93.54 (0.19)	0.065 (0.0008)	0.000130 (0.000002)

Table 5-4 The change in heart rate and the ATE for each rider when the performance measure is P_{h75}

Rider	$\hat{\beta}$	Δ_{ATE}	Δ_{h75}	h_{75}	Δ_{h75}/h_{75}
1	0.085	3251	30	144	0.20
2	0.120	2394	11	158	0.07
3	0.318	1526	07	150	0.05
4	0.043	4688	16	156	0.10
5	0.023	10616	19	147	0.13
6	0.094	5177	31	155	0.20
7	0.033	8761	12	154	0.08
8	0.044	7540	05	136	0.04
9	0.004	20881	13	151	0.08
10	0.031	7613	15	140	0.11

Table 5-5 The change in heart rate and the ATE for each rider when the performance measure is P_{h90}

Rider	$\hat{\beta}$	Δ_{ATE}	Δ_{h90}	h_{90}	Δ_{h90}/h_{90}
1	0.103	4938	55	155	0.35
2	0.095	5736	21	170	0.12
3	0.266	1926	8	161	0.05
4	0.056	5993	26	167	0.16
5	0.058	7937	36	163	0.22
6	0.146	4831	44	168	0.26
7	0.113	4459	22	164	0.13
8	0.076	5087	7	151	0.05
9	0.012	16221	30	160	0.18
10	0.024	11045	17	151	0.11

5.5 Discussion of Results

As it is well known that personal capacities vary from one individual to another, the results obtained are discussed statistically and practically for each rider.

The accumulated training effect and the performance measures P_{h75} and P_{h90} for rider (1) have significantly improved both practically and statistically. However, the practical significance of the training–performance effect is high for this rider when the performance measure is P_{h90} by about 35%, as seen in Table 5-5.

Rider (2) shows practically and statistically significant effects of training in both cases, as well as presenting a clear improvement in his performance in terms of both P_{h75} and P_{h90}, as seen in Figures 5.1 and 5.2. The performance for this rider is increasing by 7% and 12%, as explained in Tables 5-4 and 5-5. Furthermore, the parameters values of the Banister model for this rider are similar. Tables 5-1 and 5-2 clearly show this.

The training effect for rider (3) is both practically and statistically significant for both performance measures P_{h75} and P_{h90} by about 5%. This is shown in Tables 5-1, 5-2, 5-4, and 5-5. Furthermore, there is a slight improvement in the performance measures of this rider, as highlighted in Figures 5.1 and 5.2.

Rider (4) has statistically significant linear relationship between his performance measures and the accumulated training effect in both cases, as presented in Tables 5-1 and 5-2. The training effect for this rider is practically significant by 10% and 16%, this can be seen in Tables 5-4 and 5-5.

The results for rider (5) are statistically and practically significant as regards to the training effect and performance for this rider is increasing by 13% and 22%, as explained in Tables 5-4 and 5-5. However, he has little improvement in his performance in both cases, as seen in Figures 5.1 and 5.2, and the estimate of the parameter k_f is less than 1, whereas it should be greater than 1.

Rider (6) expresses a statistically significant relationship between the accumulated training effect and the performance measures P_{h75} and P_{h90}, as seen in Tables 5-1 and 5-2.

Furthermore, the training effect is practically significant when the performance measures are P_{h75} and P_{h90} by 20% and 26%, as this is presented in Tables 5-4 and 5-5.

The performance for rider (7) for both cases is clearly improved, Figures 5.1 and 5.2 demonstrate this. Moreover, the training effect of this rider is statistically and practically significant for both performance measures, and performance for this rider is increasing by 8% and 13%, as can be seen in Tables 5-1, 5-2, 5-4, and 5-5.

Rider (8) reveals a clear improvement in his performance when the performance measures are P_{h75} and P_{h90}, as presented in Figures 5.1 and 5.2. Furthermore, the results for this rider are statistically significant and practically significant. The performance for this rider is improving by about 5%. This is shown in Tables 5-4 and 5-5.

Rider (9) illustrates statistically significant linear relationship in his performance measures. Moreover, this rider shows practically significant effects of training for both performance measures by 8% and 18%, as presented in Tables 5-4 and 5-5. This is clear in Figures 5.1 and 5.2, the performance measures P_{h75} and P_{h90} has clearly improved.

Although rider (10) has had many training sessions, his performance measure improves only slightly, as presented in Figures 5.1 and 5.2. Moreover, the training effects for this rider are both practically and statistically significant in all cases. For this rider, performance is improving by 11%, as shown in Tables 5-4 and 5-5.

In conclusion, according to the $t = \hat{\beta} \,/\, (s.e.(\hat{\beta}))$ and (Δ_{hq}/hq) values and in comparison to our results in chapter 4, the results in this chapter are statistically and practically acceptable for all riders. The parameters of the Banister model have been estimated for all riders using the proposed performance measure. Furthermore, training has a significant effect for all riders, except for rider 5. This rider has many gaps in his training data over time, which could have an influence on his results.

5.6 Summary

In this chapter, we have used a measure of performance that is related to power output. We relate this measure to the training measure (ATE) to estimate Banister model parameters. This performance measure depends on power output obtained by a rider in each session, and this differs from the measure of performance used in chapter 4, which is related to heart rate. Additionally, the relationship between the new performance measure and the ATE is positive. The methodology is the same and the Banister model parameter estimates are similar between the approaches of chapter 4 and 5. Thus, we show that our methodology can be applied to different performance measures. The choice of performance measure is not a statistical one, but a matter for the coach and athlete.

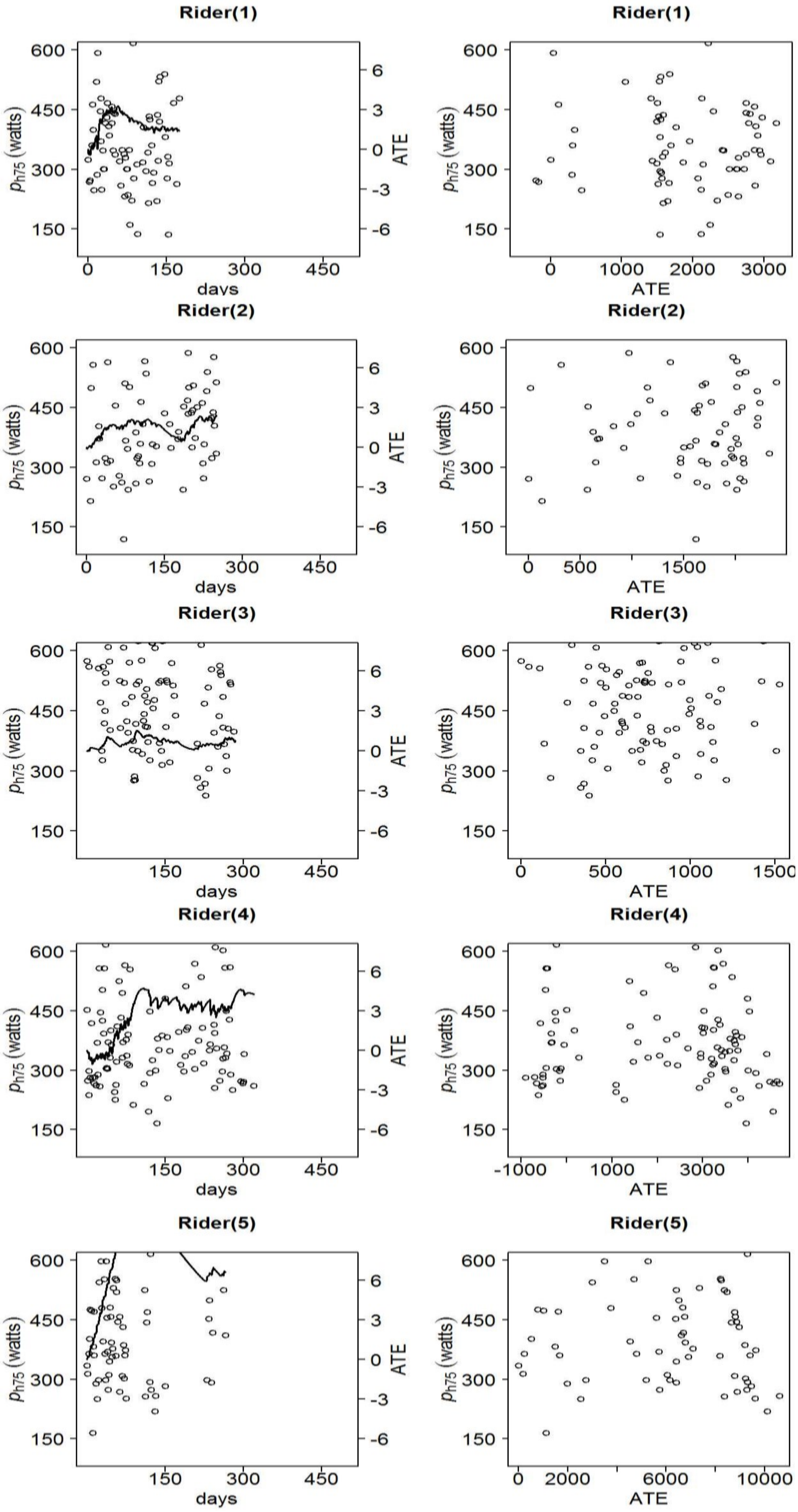

Figure 5-1 Two plots for each rider: left P_{h75} (symbols) vs time in days and ATE (line) vs time in days; right P_{h75} vs ATE (all sessions)

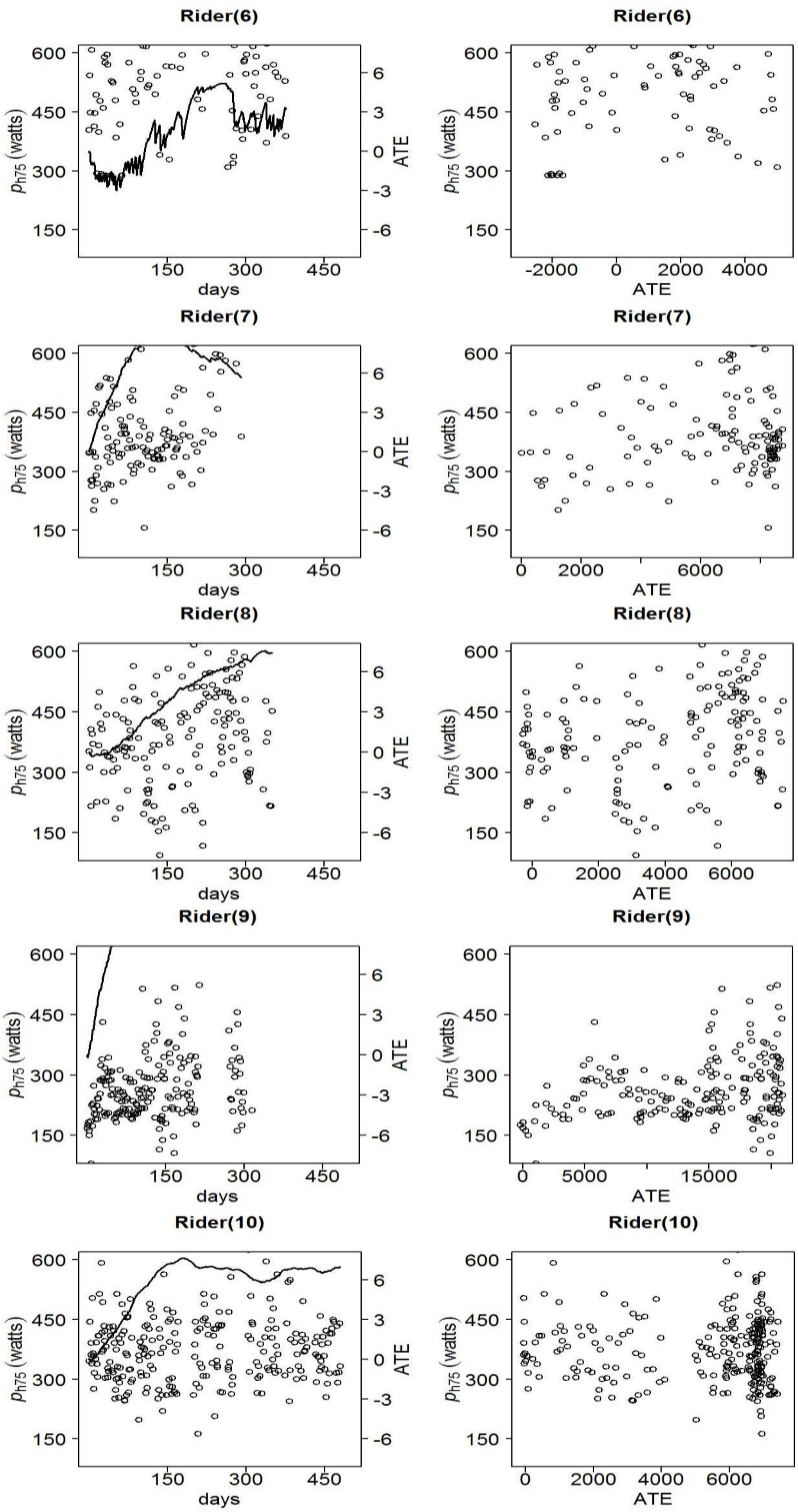

Figure 5.1 Continued

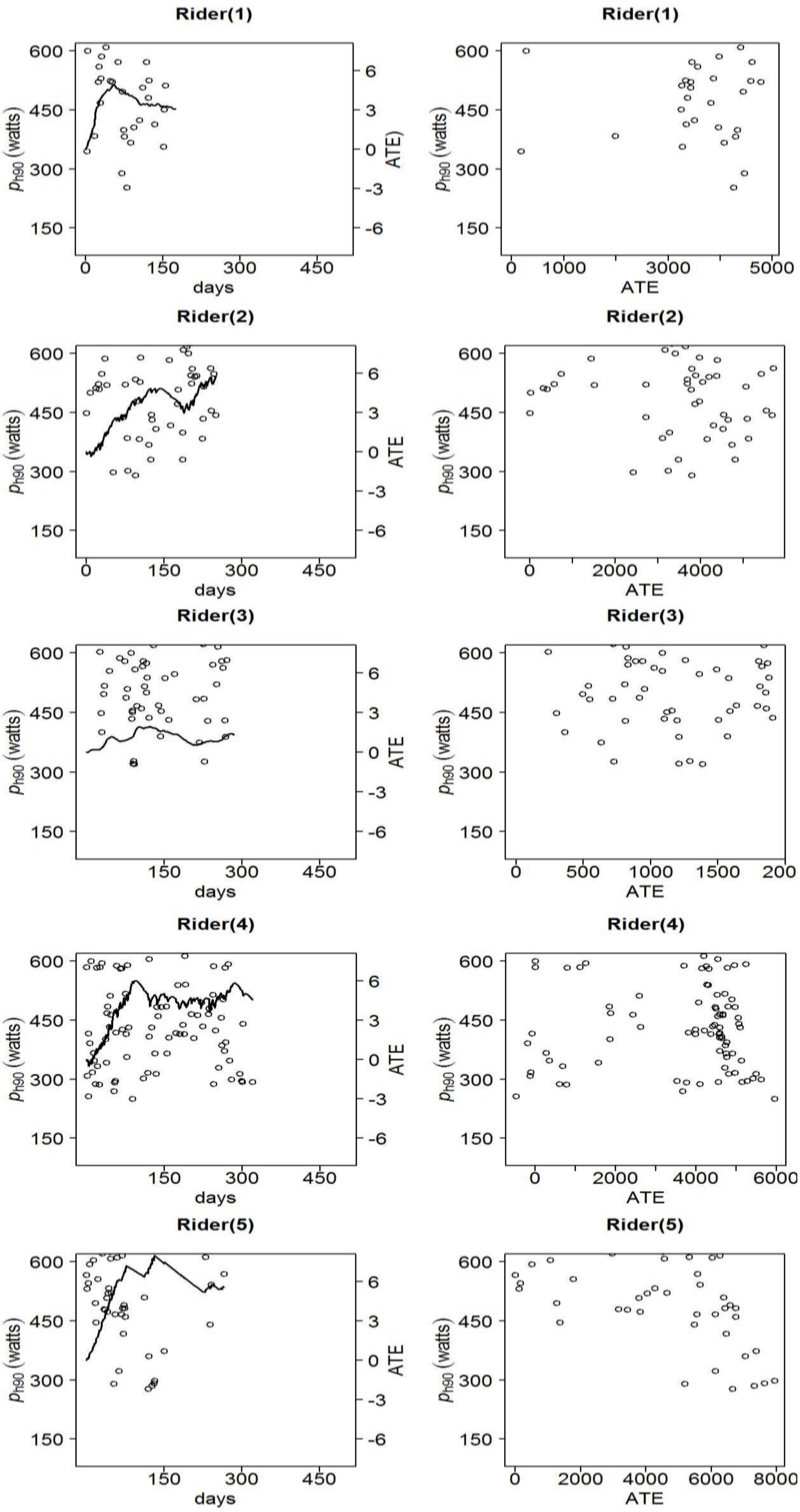

Figure 5-2 Two plots for each rider: left P_{h90}, (symbols) vs time in days and ATE (line) vs time in days; right P_{h90}, vs ATE (all sessions)

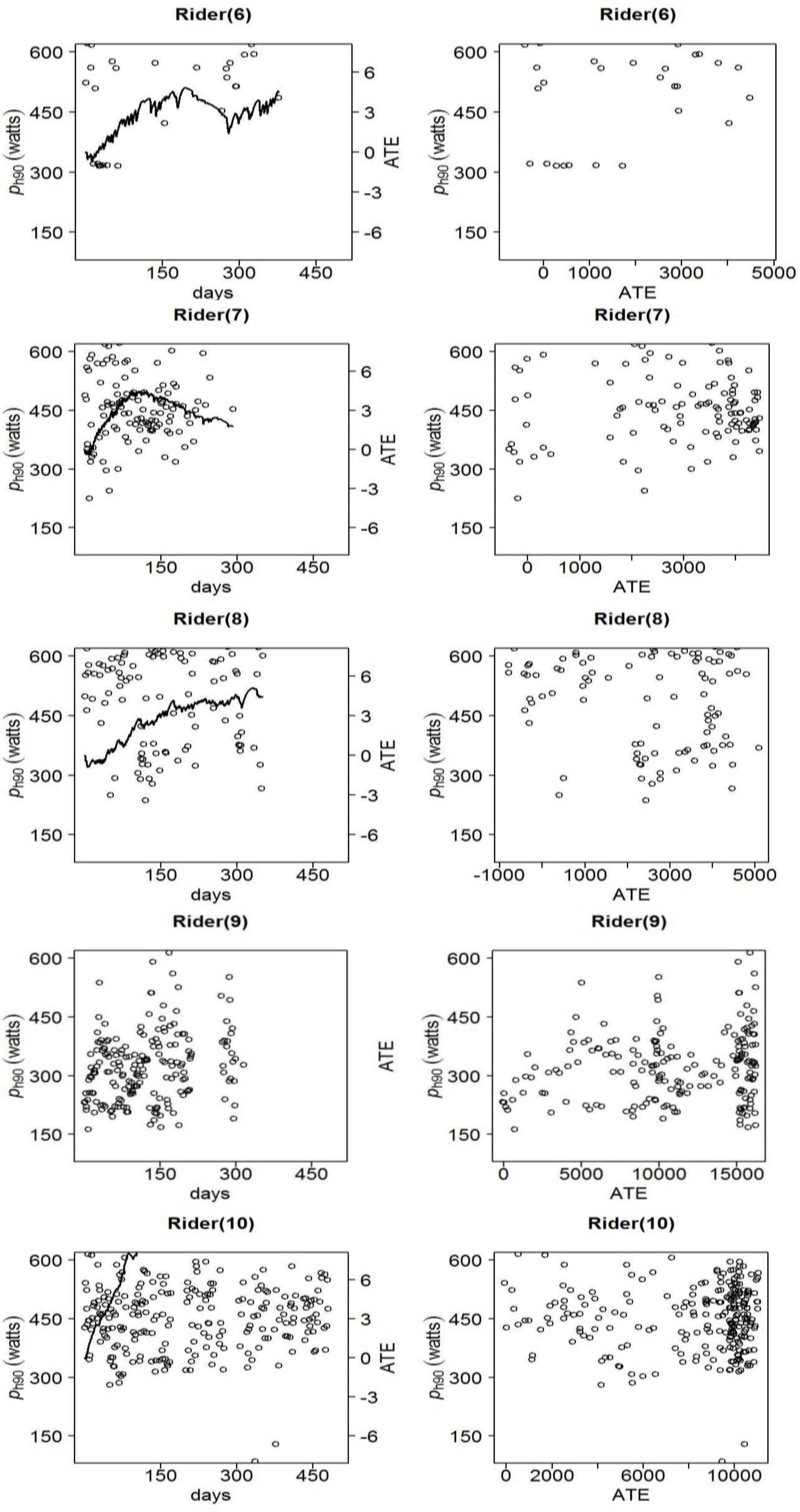

Figure 5.2 Continued

CHAPTER SIX

OTHER PERFORMANCE MEASURES

6.1 Introduction

In this chapter, three further, new performance measures are proposed. These performance measures depend on power output data only, and are therefore different to those considered in the previous two chapters. They have been inspired by feedback from Sports Scientists who reviewed the earlier measures. These new measures that are related only to power output are simpler. The first performance measure is the 75th percentile of the distribution power output for each training session. The second measure proposed is the maximum power that is sustained by a rider for at least d seconds during a session. The third performance measure is the peak power output for each training session for each rider. The peak power concept is defined using the critical power concept, which is outlined in this chapter. In each case, again, the Banister model parameters are estimated.

6.2 Performance Measure using the 75th Percentile of the Power Output

To study the characteristics of a random variable such as variance, the bootstrap method can be used to provide more information about its variance. The bootstrap is a powerful statistical method, which may be used to estimate the parameter θ of an unknown distribution, e,g, mean, median, variance. The goal here is to estimate the 75th percentile of power output for every single session. We apply the nonparametric bootstrap proposed in Efron, (1979). The idea of the bootstrap is to perform a large number of resamples of the original data to allow the calculation of the desired parameter θ from each of the resamples. The full bootstrap procedure used in this study is performed as follows:

1. Resample m observations from the sample with replacement.
2. Calculate the 75th percentile $\hat{\theta}$ from the bootstrap sample.

The steps are repeated k times to obtain $\hat{\theta}_1,\ldots, \hat{\theta}_k$, and the bootstrap estimator of 75th percentile is calculated as the mean of $(\hat{\theta}_1,\ldots, \hat{\theta}_k)$. The variance of the estimated parameter is obtained as var$(\hat{\theta}_1,\ldots, \hat{\theta}_k)$. As the result of repeated bootstrap procedure, we obtain estimators of the 75th percentile of the random variable considered and the corresponding standard error, for each one of n sessions. The estimates of P_{75} with their standard errors for each session for each rider are shown in Appendix 6. In the next section, these values of P_{75} are taken as the performance measure and then related to the training measure that is defined using the Banister model. Figure 6.1 shows the value of P_{75} for each session for each rider. The confidence interval of P_{75} for each session for each rider is presented in Figure 6.2.

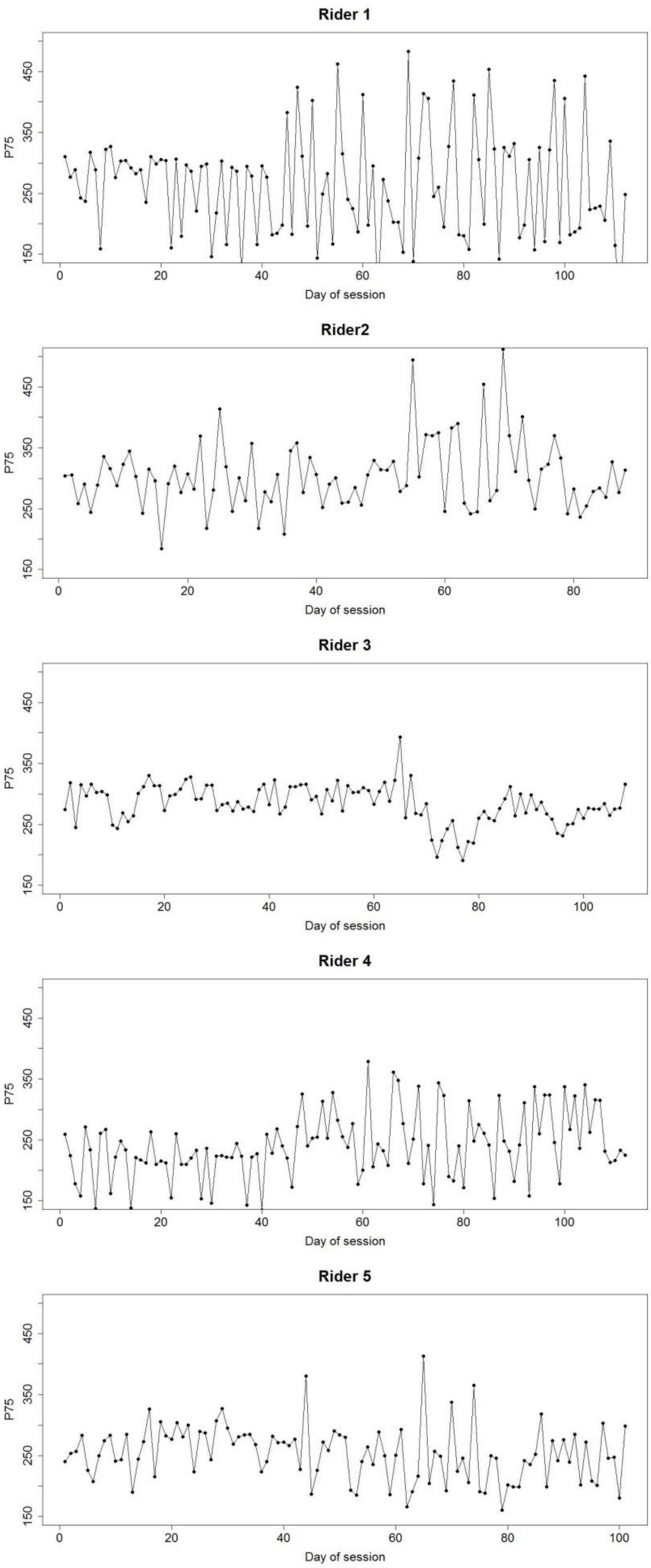

Figure 6-1 P_{75} (in watts) varied from session to session for each rider

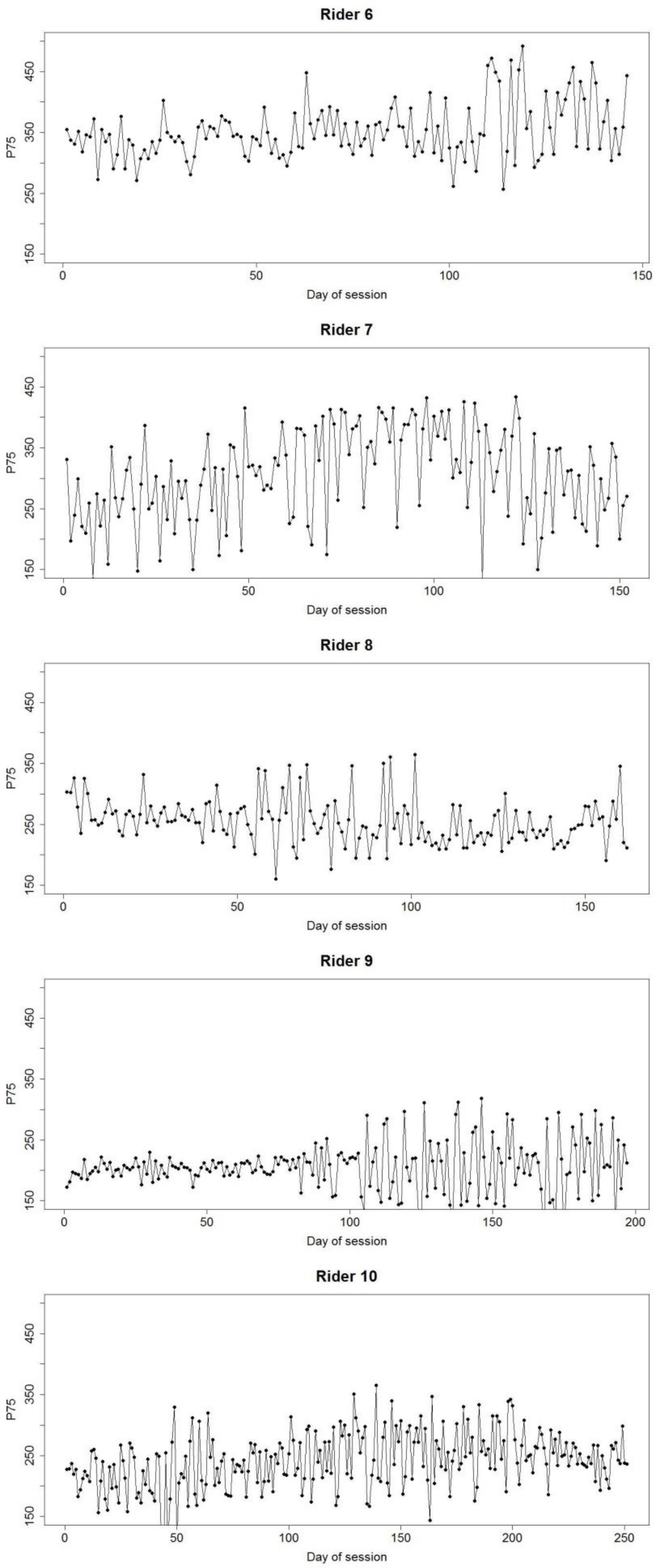

Figure 6.1 Continued

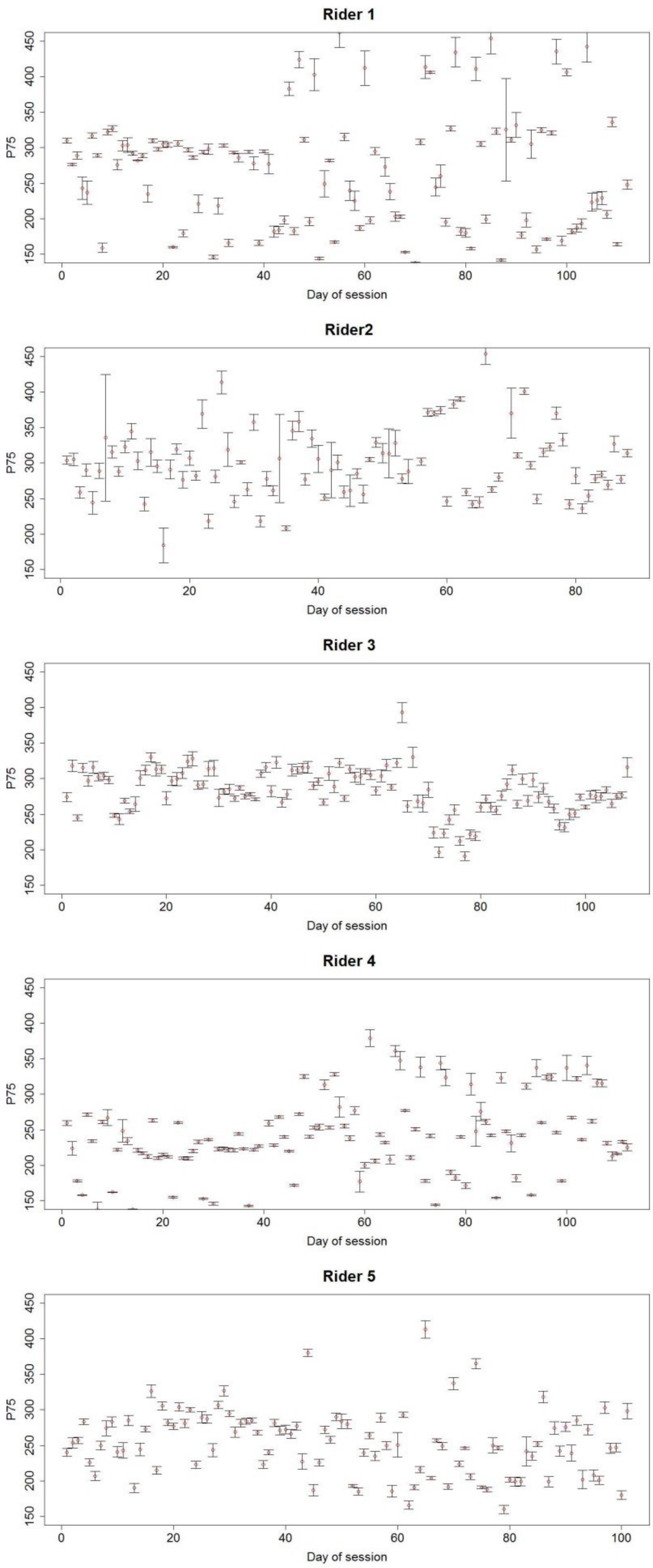

Figure 6-2 The confidence interval of the P_{75} (in watts) for all sessions for each rider

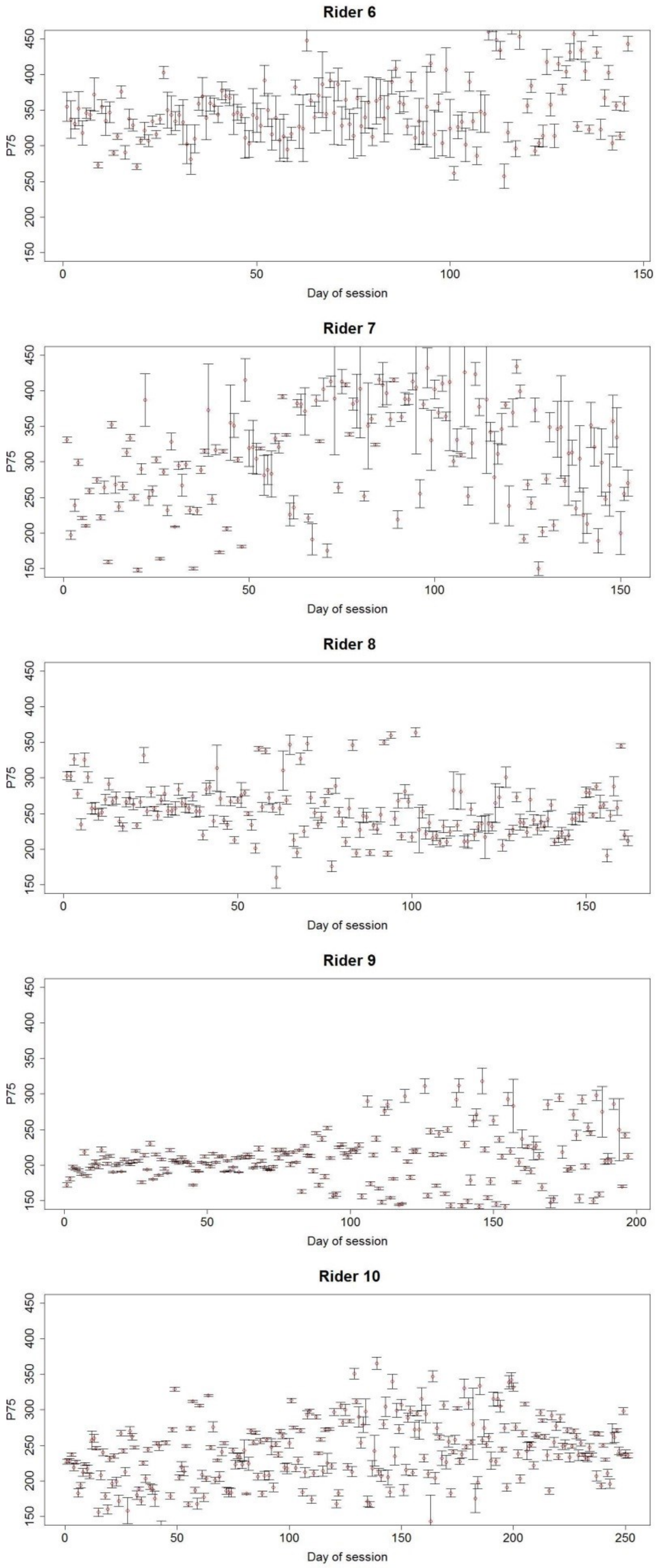

Figure 6.2 Continued

6.3 Relating Training to the Performance Measure P_{75}

In this section, we consider $P_{75,i}$, as a performance measure, which was defined in the previous section. So through this we have:

$$P_{75,i} \sim N(\alpha + \beta ATE_i, \sigma^2), \tag{6.1}$$

and

$$\widehat{P}_{75,i} \sim N\left(P_{75,i}, \lambda_i\right), \tag{6.2}$$

where $\lambda_i (i = 1, \ldots, n)$ is the variance in the estimate $\hat{P}_{75,i}$, which must be calculated. The estimate $\widehat{P}_{75,i}$ and its variance are obtained as described in section 6.2. Finally, through (6.1) and (6.2), we conclude the model of training–performance to be

$$\hat{P}_{75,i} \sim N(\alpha + \beta ATE_i, \sigma^2 + \lambda_i)$$

The parameters can again be estimated using the method of maximum likelihood. This is done using R programming. The parametersα, β, σ are fixed for each individual rider, but λ_i varies session by session. Table 6-1 shows the estimates for each rider.

Table 6-1 Estimated parameters with standard errors of the training and performance model for each rider with the *t* statistic and *p* value for the test of β=0

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	88.2	6.1	31.2	15.8	1.9	0.8	2.2	5.8	215	22.1	0.0081	0.0046	1.76	0.04
2	51.5	4.3	84.5	26.5	1.4	0.7	12.7	1.7	264	31.6	0.0098	0.0052	1.88	0.03
3	36.6	4.3	36.6	17.6	6.5	0.8	1.3	0.2	233	17.9	0.0319	0.0190	1.67	0.05
4	61.5	3.7	60.6	28.4	3.3	1.4	5.2	1.5	207	13.1	0.0113	0.0018	6.28	0.00
5	60.6	8.7	57.6	13.3	0.8	0.7	5.1	3.4	250	14.7	0.0062	0.0012	5.17	0.00
6	53.2	5.4	85.4	10.1	1.8	0.9	3.7	1.1	295	89.7	0.0093	0.0019	4.89	0.00
7	61.8	3.5	74.1	19.5	3.9	3.4	2.8	2.6	197	26.8	0.0264	0.0050	5.28	0.00
8	51.3	1.5	46.7	13.3	1.1	0.8	1.8	0.9	214	21.8	0.0095	0.0053	1.79	0.04
9	41.9	2.2	58.2	18.3	3.4	5.8	12.5	8.5	213	11.7	0.0020	0.0006	3.33	0.00
10	44.8	3.2	93.8	24.6	4.8	3.5	5.1	0.6	203	21.1	0.0090	0.0020	4.50	0.00

6.3.1 Statistical Discussion of the Training Effect

In this section, we statistically study the effect of training on performance. The relationship between the accumulated training effect and our proposed performance is linearly positive. Therefore, we would like to reject the hypothesis $H_0: \beta = 0$ in favour of $H_1: \beta > 0$. Through the Table 6-1, we see that there is a statistically significant relationship at 5% level between the accumulated training effect and the performance measure P_{75} for all riders. Since all estimates of β are positive and $t = \widehat{\beta}/se(\hat{\beta})$ >1.64 in all cases.

6.3.2 Practical Discussion of the Training Effect

Since there is a statistically significant increase in performance as a result of the training process, is important to determine the practical significance of that increase. This can be calculated using the change in power output from the beginning of training until the point at which a rider has completed the optimal training, as follows:

$$\Delta_{P75} = \hat{\beta} \times \Delta_{ATE}$$

The values of $\hat{\beta}$ are shown in Table 6-1. Δ_{ATE} is defined as the change between the maximum and initial accumulated training effect ($\Delta_{ATE} = ATE_{max} - ATE_1$). The change in power output is presented in Table 6-2 for each rider. This change ranges between 4% and 35%.

Table 6-2 Performance gain and the ATE change when the performance measure is P_{75} with percentile of power output P_{75} for each rider

Rider	Δ_{ATE}	Δ_{P75}	P_{75}	Δ_{p75}/P_{75}
1	3227	26	291	0.09
2	4734	43	307	0.13
3	2457	74	291	0.25
4	4472	45	246	0.18
5	7795	47	280	0.16
6	5448	49	384	0.12
7	5794	116	323	0.35
8	4242	38	274	0.13
9	3873	08	214	0.04
10	7068	64	260	0.24

6.3.3 Discussion of Results

In this section, we discuss the results for each rider in terms of the impact of training on performance. The reason for this is that each rider's training program differs from those of the others as well as his individual capacities.

For rider (1), there is a statistically significant relationship that training has an impact on his performance measure P_{75}, as seen in Table 6-1. Also, as noted in Table 6-2, the practical impact of the training for this rider is significant with the P_{75} measure increasing by about 9%. In addition, a slight improvement in the performance measure of this rider is seen in Figure 6.3.

Rider (2) has a statistically significant relationship between the accumulated training effect and the performance measure, as highlighted in Table 6-1. In Figure 6.3, there is a clear improvement in performance over time for this rider. Practically, for this rider, the effect of training is significant with the P_{75} measure increasing by about 13%, as presented in Table 6-2. Furthermore, the accumulated training effect has clearly improved, Figure 6.3 illustrates this.

The effect of training for rider (3) is practically significant with the P_{75} measure increasing by about 25%, as seen in Table 6-2. The relationship between the accumulated training effect and the performance measure for this rider is also statistically significant, as seen in Table 6-1. Moreover, the training effect over time is stable, as presented in Figure 6.3.

Rider (4) shows a statistically and practically significant relationship between the accumulated training effect and the performance measure, as presented in Tables 6-1 and 6-2. The impact of training on this rider's performance over time is increasing, as seen in Figure 6.3.

The results for Rider (5) appear statistically and practically significant in the relationship between the accumulated training effect and the performance measure, as seen in Tables 6-1 and 6-2. However, the parameter k_f is less than 1 in this case. It is supposed to be greater than one. It seems probable that the reason for this issue is that this rider has a lot of gaps in his training programme.

Rider (6) has statistically and practically significant results in terms of training effect on performance, this is demonstrated in Tables 6-1 and 6-2. The performance measure of this rider has obviously improved by 12%, as shown in Figure 6.3. Also, the accumulated effect of training is improved.

Rider (7) reveals a significant practical improvement of 35% performance after training, this is explained in Table 6-2. Furthermore, the relationship between the accumulated training effects and the performance measure for this rider is highly statistically significant, Table 6-1 proves this. After initially rising, the performance measure declines slightly after 100 days. This is noted in Figure 6.3.

For rider (8), a statistically and practically significant training effect is illustrated in Tables 6-1 and 6-2. Although the effect of the training on performance for this rider is clearly improved, the performance is fairly stable, as shown in Figure 6.3.

Rider (9) presents a statistically significant relationship between the accumulated training effect and his performance, this is presented in Table 6-1. However, this rider expresses only a very small practical effect from training on his performance by about 4%, as can be visualised in Table 6-2. Furthermore, the accumulation of training effect for this rider is fluctuates rather a lot, this is evident in Figure 6.3.

Finally, rider (10) indicates a statistically and practically significant training effect increased by about 24%, as can be seen in Tables 6-1 and 6-2. The performance measure for this rider initially increases and after 150 days fluctuates, this is noted in Figure 6.3.

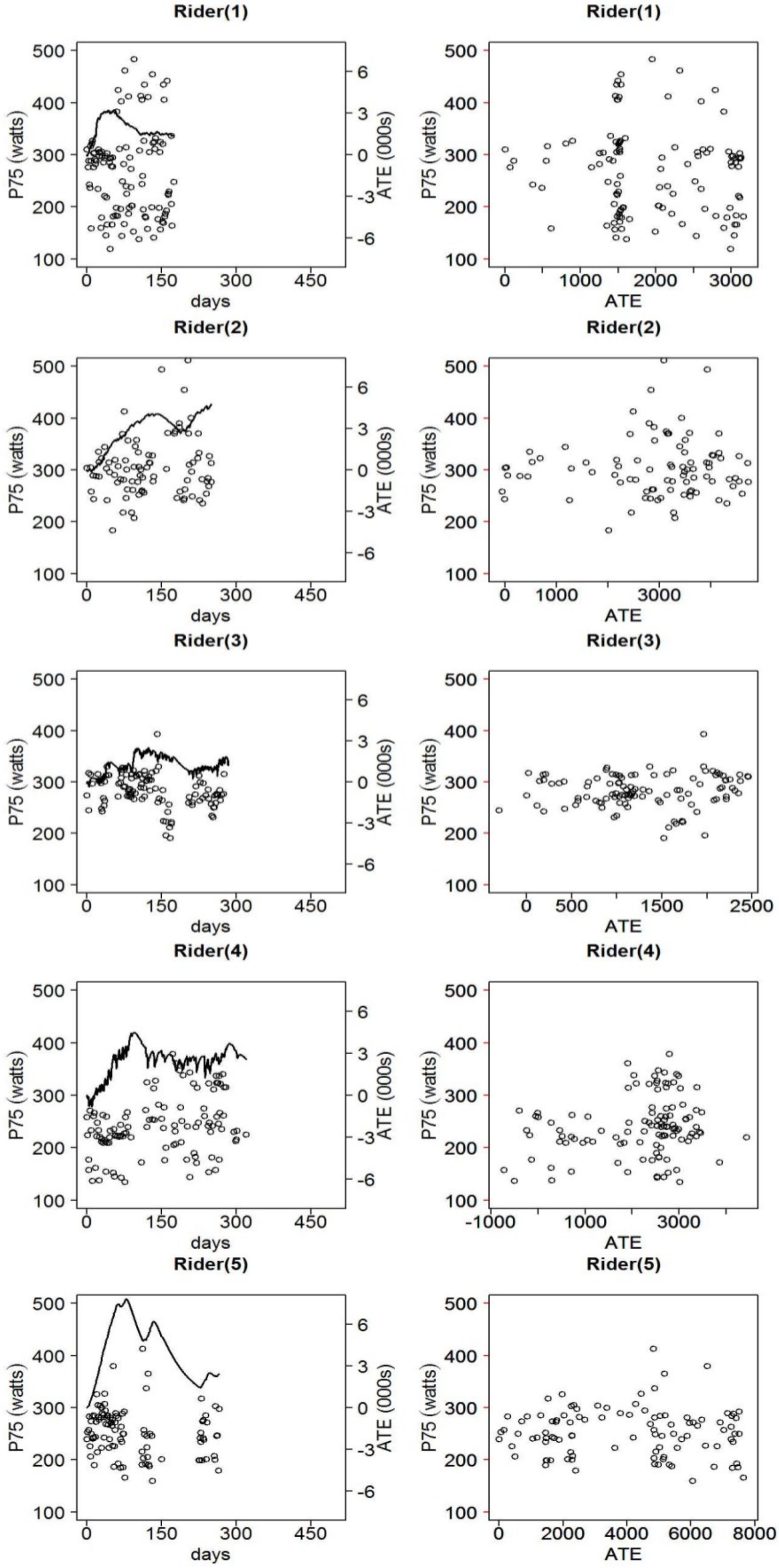

Figure 6-3 Two plots for each rider: left P_{75} , (symbols) vs time in days and ATE (line) vs time in days; right P_{75} , vs ATE (all sessions)

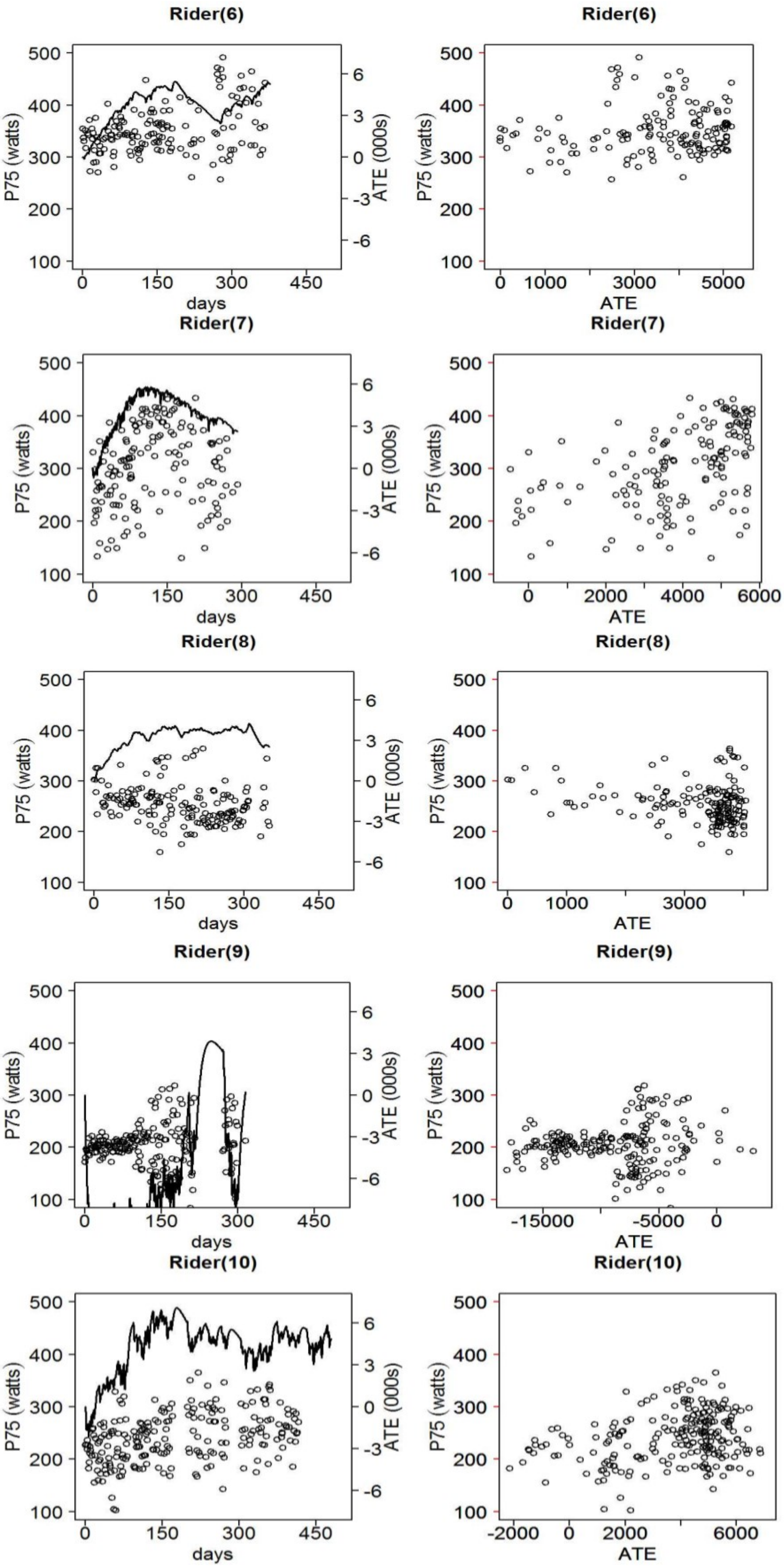

Figure 6.3 Continued

6.4 Performance Measure using Maximum Power

Now we consider the maximum power that is maintained by a rider for d seconds. To do so, we follow some steps that are required to calculate this measure using field data. Firstly, a set of increasing power outputs $p_1, p_2, \ldots$ is determined. In the second step, for the specified power output level p_k, the longest duration d_k for which this power output level is sustained by the rider for each session i is determined. Then a parametric model is fitted to the pairs (p_k, d_k) $(k = 1,2, \ldots, m)$. In this step we assume that $\log_e p_k$ is linearly linked to $\log_e d_k$ so that

$$\log_e p_k = u_i + v_i \log_e d_k$$

So

$$p_k = e^u {d_k}^v,$$

where u_i and v_i are rider-session constants, which are estimated using a simple linear regression. Also, it is necessary to specify the reference duration *d.* This is a choice for the rider or coach. The appropriate reference duration depends on the nature of the competition for which the rider is training. Multiple values of d might be considered for an individual rider. However, in this case, we use d = 10 seconds as the reference duration. Table 6-3 shows an example of the estimates of the model parameters for each session for rider 1. The estimate of the model parameters for each session for some riders with its fitted line is presented in Appendices 9 and 10. Figure 6.4 shows the relationship between power p and duration d for a session for rider 1 with the best fitting of the form $p_k = e^u {d_k}^v$. In our case, the delta method (Casella & Berger, 2002, p.240) is used to calculate, which is the variance of $\hat{p}$ varies from session to session for each rider and $\lambda > 0$ as

$$\lambda_i = e^{\widehat{2u}} d^{\widehat{2v}}[var(\hat{u}) + (\ln(d))^2 var(\hat{v}) + 2\ln(d)\, cov(\hat{u}, \hat{v})]$$

We use this formula to evaluate λ_i at $d = 10\ s$ and at $d = 30\ s$ for each session for each rider with estimates of u and v.

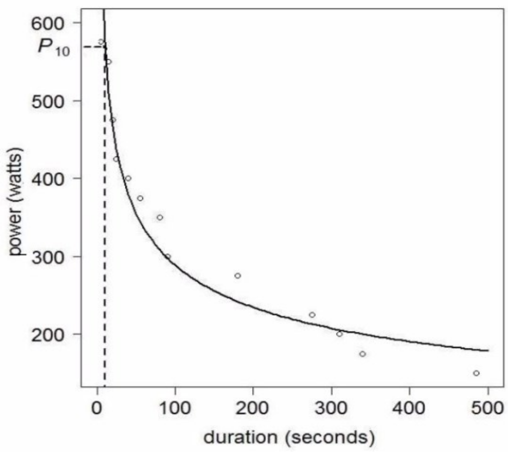

Figure 6-4 Observed power output against duration (points) and fitted power-duration curve for a single session for rider 1

Table 6-3 Estimated values of the parameters u, v, with their standard error (s.e.), the R^2 value of the regression model and the predicted value and respective standard error for each session for rider 1

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\hat{P}_{d=10}$	$s.e.$	$\hat{P}_{d=30}$	$s.e.$
1	6.880	0.093	-0.2450	0.0180	0.94	553.0	30.1	422.4	16.5
2	6.065	0.176	-0.0990	0.0270	0.73	342.8	41.1	307.5	29.4
3	6.822	0.042	-0.2450	0.0090	0.98	522.6	12.4	399.5	7.0
4	6.894	0.172	-0.3390	0.0410	0.91	451.5	36.7	311.1	14.1
5	6.997	0.140	-0.3550	0.0370	0.94	483.4	28.9	327.4	9.7
6	6.769	0.045	-0.2290	0.0100	0.98	513.9	12.8	399.6	6.9
7	6.781	0.066	-0.2150	0.0140	0.96	536.7	20.6	423.8	11.8
8	6.362	0.107	-0.2550	0.0290	0.93	321.9	15.0	243.1	7.4
9	6.881	0.082	-0.2580	0.0180	0.95	538.2	24.2	405.6	13.0
10	6.812	0.100	-0.2150	0.0200	0.94	554.2	31.8	437.6	17.5
11	7.041	0.107	-0.2990	0.0240	0.94	573.5	32.6	412.7	15.8
12	6.991	0.209	-0.2250	0.0370	0.79	646.5	82.9	504.7	46.6
13	6.818	0.131	-0.2910	0.0320	0.89	468.3	30.1	340.3	13.7
14	7.010	0.102	-0.2350	0.0200	0.91	644.4	39.2	497.7	22.0
15	5.981	0.053	-0.0760	0.0080	0.93	332.7	12.4	306.2	9.4
16	6.664	0.037	-0.1950	0.0080	0.98	500.4	10.7	403.9	6.3
17	6.619	0.152	-0.2730	0.0390	0.86	399.8	27.9	296.3	12.8
18	6.749	0.088	-0.2270	0.0180	0.94	505.8	25.7	394.1	14.4
19	6.787	0.051	-0.2150	0.0100	0.98	540.2	16.3	426.5	9.5
20	6.654	0.046	-0.1850	0.0100	0.98	506.8	13.0	413.7	7.6
21	6.880	0.211	-0.2510	0.0410	0.82	546.4	66.8	414.9	35.2
22	5.359	0.060	-0.0570	0.0150	0.94	186.4	6.5	175.1	5.2
23	6.992	0.066	-0.2640	0.0140	0.97	592.7	21.9	443.6	11.3
24	6.705	0.073	-0.3320	0.0190	0.99	380.4	12.7	264.2	5.4
25	6.568	0.093	-0.1970	0.0200	0.91	451.8	24.0	363.6	14.4
26	6.568	0.102	-0.1950	0.0210	0.92	454.3	27.4	366.8	16.3
27	7.042	0.115	-0.3780	0.0280	0.96	479.5	25.1	316.6	8.6
28	6.673	0.078	-0.2120	0.0160	0.94	484.9	22.5	384.1	13.3
29	7.570	0.140	-0.3750	0.0290	0.92	817.3	62.3	541.1	26.6
30	6.583	0.046	-0.3280	0.0130	0.99	339.5	6.5	236.7	3.2
31	7.160	0.208	-0.3140	0.0490	0.84	623.8	64.8	441.6	28.7
32	6.768	0.124	-0.2080	0.0240	0.89	538.1	39.7	428.0	23.0
33	6.812	0.466	-0.3010	0.1090	0.56	455.0	101.9	327.0	39.8
34	6.482	0.079	-0.1660	0.0160	0.93	445.9	20.9	371.7	12.7
35	7.564	0.138	-0.3800	0.0290	0.94	803.2	60.3	529.1	25.6

Table 6-3 continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\hat{P}_{d=10}$	$s.e.$	$\hat{P}_{d=30}$	$s.e.$
36	6.315	0.274	-0.2500	0.0750	0.74	310.8	36.0	236.2	16.9
37	6.672	0.067	-0.2180	0.0140	0.96	478.0	18.6	376.1	10.8
38	6.907	0.089	-0.2250	0.0170	0.95	595.4	31.7	465.1	18.1
39	6.815	0.145	-0.3690	0.0400	0.95	389.4	24.9	259.5	11.4
40	6.701	0.043	-0.2100	0.0090	0.99	501.6	12.5	398.3	7.2
41	6.864	0.104	-0.2160	0.0210	0.90	582.5	35.5	459.6	20.3
42	6.323	0.071	-0.2380	0.0190	0.96	321.9	10.7	247.8	5.7
43	6.470	0.208	-0.2760	0.0530	0.85	342.2	32.0	252.8	13.1
44	6.732	0.084	-0.3230	0.0240	0.97	398.8	13.6	279.7	7.0
45	7.574	0.122	-0.3820	0.0260	0.96	807.6	52.2	530.7	21.3
46	6.423	0.029	-0.2730	0.0080	1.00	328.1	4.4	243.0	2.3
47	7.199	0.063	-0.3470	0.0140	0.98	601.7	19.6	411.1	8.7
48	6.920	0.074	-0.2610	0.0150	0.96	555.2	23.3	417.0	12.2
49	6.625	0.114	-0.3110	0.0310	0.94	368.5	18.9	261.9	9.2
50	7.973	0.197	-0.5750	0.0490	0.95	772.8	69.2	411.1	18.9
51	5.636	0.053	-0.1510	0.0160	0.99	197.8	3.7	167.5	2.2
52	7.117	0.223	-0.2960	0.0470	0.79	623.6	75.9	450.4	35.8
53	6.231	0.087	-0.1280	0.0150	0.90	378.2	21.3	328.5	14.4
54	5.602	0.083	-0.0910	0.0160	0.97	219.8	10.4	198.9	6.4
55	7.971	0.234	-0.5550	0.0600	0.95	807.0	81.8	438.7	21.8
56	6.884	0.167	-0.2360	0.0330	0.81	567.0	55.5	437.4	30.6
57	7.150	0.099	-0.4140	0.0260	0.97	491.4	21.4	311.8	7.9
58	6.957	0.159	-0.3550	0.0420	0.86	463.9	31.9	314.0	12.8
59	6.563	0.060	-0.2920	0.0160	0.98	362.2	10.1	262.9	5.1
60	8.453	0.229	-0.6470	0.0530	0.94	1055.9	116.2	518.5	30.1
61	6.726	0.039	-0.3480	0.0110	0.99	374.2	5.9	255.3	2.7
62	6.883	0.070	-0.2650	0.0150	0.97	529.4	20.5	395.6	10.7
63	7.531	0.143	-0.3740	0.0300	0.94	789.2	61.4	523.5	26.2
64	7.709	0.431	-0.4130	0.0880	0.71	861.3	202.2	547.3	80.3
65	7.047	0.173	-0.3060	0.0390	0.86	568.4	52.1	406.2	25.2
66	6.402	0.104	-0.1600	0.0220	0.89	417.6	24.1	350.5	14.3
67	7.012	0.079	-0.2630	0.0170	0.97	606.6	26.2	454.6	12.9
68	5.484	0.060	-0.0740	0.0150	0.93	203.3	7.0	187.5	5.5
69	6.837	0.049	-0.2470	0.0100	0.98	527.0	14.8	401.6	8.1
70	5.809	0.115	-0.2320	0.0440	0.93	195.5	5.8	151.6	7.2
71	7.287	0.078	-0.3350	0.0160	0.98	674.7	29.0	466.7	13.2
72	7.841	0.168	-0.5030	0.0400	0.95	798.2	64.3	459.2	20.2
73	6.762	0.402	-0.2160	0.0760	0.47	526.4	122.9	415.4	65.8
74	6.701	0.186	-0.2920	0.0450	0.86	414.9	37.3	300.9	16.2
75	7.638	0.111	-0.4880	0.0270	0.97	674.6	34.9	394.7	11.1

Table 6-3 continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\hat{P}_{d=10}$	$s.e.$	$\hat{P}_{d=30}$	$s.e.$
76	6.669	0.102	-0.3060	0.0280	0.94	389.2	17.5	278.1	8.3
77	6.969	0.174	-0.2670	0.0350	0.87	575.3	56.5	429.1	28.4
78	6.670	0.176	-0.2950	0.0460	0.84	399.6	31.6	288.9	14.3
79	6.400	0.096	-0.2580	0.0260	0.93	332.1	13.9	250.1	6.7
80	7.886	0.299	-0.4610	0.0630	0.86	921.1	147.3	555.3	54.9
81	5.472	0.053	-0.0830	0.0130	0.98	196.6	5.3	179.5	3.4
82	7.025	0.270	-0.3070	0.0590	0.79	554.6	77.9	395.8	34.3
83	6.791	0.050	-0.2420	0.0110	0.98	509.3	14.4	390.2	7.9
84	7.356	0.264	-0.4710	0.0680	0.87	529.9	61.4	316.0	18.9
85	8.496	0.182	-0.6890	0.0440	0.96	1002.5	83.2	470.4	18.7
86	6.965	0.066	-0.2710	0.0150	0.97	567.1	20.0	421.1	10.0
87	5.562	0.073	-0.1570	0.0270	0.97	181.3	3.9	152.6	4.6
88	7.389	0.262	-0.3360	0.0530	0.80	746.7	108.1	516.4	48.0
89	6.808	0.052	-0.2490	0.0110	0.98	510.1	14.8	387.9	8.0
90	7.866	0.152	-0.5180	0.0360	0.96	791.3	56.4	447.9	17.0
91	6.572	0.068	-0.2830	0.0170	0.97	372.1	11.8	272.6	5.7
92	7.864	0.298	-0.6020	0.0760	0.94	650.0	84.4	335.3	22.2
93	8.051	0.174	-0.5510	0.0420	0.95	882.7	71.5	481.9	20.1
94	6.545	0.092	-0.3010	0.0250	0.96	347.5	13.8	249.5	6.0
95	6.615	0.048	-0.1900	0.0100	0.98	482.2	12.8	391.5	7.3
96	5.493	0.041	-0.0690	0.0080	0.99	207.4	5.1	192.3	3.4
97	6.175	0.245	-0.1120	0.0400	0.61	371.6	59.5	328.7	40.6
98	8.537	0.168	-0.7060	0.0420	0.97	1002.9	75.0	461.6	16.4
99	6.556	0.155	-0.2870	0.0410	0.89	362.7	25.0	264.5	11.4
100	6.366	0.256	-0.1070	0.0500	0.54	454.6	69.1	404.2	44.4
101	6.715	0.066	-0.3010	0.0160	0.98	412.8	13.1	296.6	6.1
102	6.804	0.120	-0.3260	0.0330	0.93	425.3	22.7	297.1	10.7
103	7.766	0.364	-0.5540	0.0900	0.86	658.6	108.5	358.3	29.8
104	6.318	0.157	-0.2090	0.0410	0.87	342.3	25.1	271.9	13.3
105	6.503	0.247	-0.1880	0.0580	0.68	433.1	56.9	352.4	33.2
106	6.383	0.107	-0.2230	0.0260	0.91	354.1	19.0	277.2	9.9
107	6.816	0.102	-0.3070	0.0250	0.95	450.6	22.6	321.8	10.7
108	7.439	0.362	-0.4320	0.0840	0.82	629.9	113.4	392.1	42.4
109	8.435	0.699	-0.5670	0.1330	0.90	1249.0	493.6	670.0	168.8
110	6.833	0.128	-0.2950	0.0290	0.93	470.3	31.3	340.1	15.0
111	6.664	0.053	-0.2670	0.0110	0.99	424.2	12.5	316.4	6.4
112	6.869	0.102	-0.2100	0.0190	0.90	593.6	36.5	471.5	21.1

6.5 Relating Training to the Performance Measure P_{10}

Again we have

$$P_{10,i} \sim N(\alpha + \beta \mathrm{ATE}_i, \sigma^2), \tag{6.3}$$

and

$$\widehat{P}_{10,i} \sim N\left(P_{10,i}, \lambda_i\right), \tag{6.4}$$

where λ_i is the variance of the estimate $\widehat{P}_{10,i}$. These variances λ_i $(i = 1, \dots n)$ are estimated using the parametric model fitted to the (p_k, d_k) $(k = 1,2, \dots, m)$ data discussed in the previous section. Finally, through (6.3) and (6.4), we have

$$\widehat{P}_{10,i} \sim N(\alpha + \beta ATE_i, \sigma^2 + \lambda_i).$$

The parameters can again be estimated using the method of maximum likelihood. The estimates are presented in Table 6-4.

Table 6-4 Estimated parameters with standard errors of the training and performance model for each rider with the *t* statistic and *p* value for the test of *β*=0

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	40.9	1.4	33.6	4.2	4.8	1.3	3.8	2.1	452	23.5	0.0180	0.0096	1.88	0.03
2	10.8	7.8	105.8	27.2	1.7	0.3	22.5	1.7	533	33.7	0.0319	0.0083	3.84	0.00
3	59.4	5.1	67.9	19.1	3.1	0.6	3.4	2.3	519	15.5	0.0105	0.0020	5.25	0.00
4	11.8	7.8	57.9	20.7	2.6	2.5	2.7	2.4	422	37.6	0.0073	0.0028	2.61	0.01
5	22.2	6.9	83.9	18.2	0.2	0.1	5.1	1.6	514	19.3	0.0063	0.0021	3.00	0.00
6	53.3	2.1	87.9	30.8	1.8	0.3	8.5	1.9	498	13.9	0.0343	0.0080	4.29	0.00
7	35.8	9.8	76.3	24.6	1.7	0.5	9.1	5.9	507	50.4	0.0224	0.0092	2.43	0.01
8	66.9	5.1	84.7	26.6	2.1	0.4	2.4	0.6	503	30.8	0.0113	0.0053	2.13	0.02
9	49.4	6.7	100.5	23.3	2.3	1.2	24.6	11.5	325	34.9	0.0151	0.0057	2.65	0.00
10	76.8	3.6	90.8	26.1	2.6	0.4	10.7	1.5	400	15.2	0.0146	0.0036	4.06	0.00

6.5.1 Statistical Discussion of the Training Effect

Table 6-4 shows the estimates of the training and performance model parameters obtained for each rider using our proposed performance measure, with their standard errors. Through the estimates of β and its standard error we see that there is a statistically significant relationship between performance and training for every rider. This is done using the rejection of the hypothesis $H_0 : \beta = 0$ in favour of $H_1 : \beta > 0$. For all riders, we see that the training has effect on performance in regarding statistical significance, as shown in Table 6-4.

6.5.2 Practical Discussion of the Training Effect

In this section, the training effect on performance is discussed in practical terms. As we discussed that there is a statistically significant relationship between the accumulated training effect and the performance measure P_{10}, it is useful to determine the practical significance of that increase. This can be calculated using the change in power output from the beginning of training until the point at which a rider has completed the optimal training, as follows:

$$\Delta_{P_{10}} = \hat{\beta} \times \Delta_{ATE}$$

The values of $\hat{\beta}$ are shown in Table 6-4 and $\bar{P}_{10}$ is the mean maximum power output sustained for at least 10 seconds, averaged across sessions. The change in power output ranges between 7% and 34%, as seen in Table 6-5.

Table 6-5 Performance gain and the ATE change when the performance measure is P_{10} for each rider

Rider	Δ_{ATE}	$\hat{\beta}$	$\bar{P}_{10}$	$\widehat{\beta}\text{ATE}_{\text{max}}$	$\Delta_{P10}/\bar{P}_{10}$
1	2631	0.0180	518	47	0.09
2	3898	0.0319	584	124	0.21
3	3469	0.0105	534	37	0.07
4	4978	0.0073	433	36	0.08
5	10128	0.0063	535	64	0.12
6	4886	0.0343	660	168	0.25
7	5248	0.0224	603	118	0.19
8	6903	0.0113	593	78	0.13
9	8580	0.0151	376	130	0.34
10	6083	0.0146	459	89	0.19

6.5.3 Discussion of Results

Through Figure 6.5 and Tables 6-4 and 6-5, we argue the results obtained rider by rider, as it is known that each rider has a different training program. Therefore the capacity of rider for training is individualised from one to other.

For rider (1), a statistically and practically significant training effect by 9% is highlighted in Tables 6-4 and 6-5. The performance of this rider has somewhat improved over training, as can be seen in Figure 6.5. Furthermore, there is a simple increase in the accumulation of training, Figure 6.5 illustrates this.

Rider (2) shows a noticeable increase in the accumulation of training, as demonstrated in Figure 6.5. A statistically and practically significant training effect by 21% as explained in Tables 6-4 and 6-5. The performance of this rider has clearly improved, as noted in Figure 6.5.

Rider (3) has a statistically and practically significant relationship between the accumulated training effect by 7% and his performance measure, this can be seen in Tables 6-4 and 6-5. The training effect for this rider slightly increased over time, as shown in

Figure 6.5. Also, there is a noticeable improvement in his performance, as presented in Figure 6.5.

For rider (4), a statistically and practically significant of training effect by 8% is clarified in Tables 6-4 and 6-5. The performance of this rider was slightly enhanced, as can be visualised in Figure 6.5. Furthermore, the accumulation of training fluctuated over time, Figure 6.5 clearly demonstrates this.

Rider (5) presents a statistically and practically significant relationship between training effect increased by 12% and performance measure, as shown in Tables 6-4 and 6-5. However, the parameter k_f for this rider is still less than 1, as explained in Table 6-4. The performance of this rider was clearly stable, Figure 6.5 shows this.

Rider (6) displays a statistically significant training effect, this can be seen in Table 6-4. The performance of this rider has clearly improved, as presented in Figure 6.5. Furthermore, the accumulation of training is enhanced by 25%, as can be noted in Figure 6.5.

Rider (7) has a statistically significant relationship between the accumulated training effect and his performance, this is shown in Table 6-4. Furthermore, for this rider, the training effect was practically significant by19%, as presented in Table 6-5. Moreover, there was a slight enhancement in the performance of this rider, as can be seen in Figure 6.5.

Rider (8) demonstrates a statistically and practically significant of training effect by 13%, as shown in Tables 6-4 and 6-5. The performance of this rider is obviously improved, this is evident in Figure 6.5. Furthermore, the accumulation of training has slightly improved, as seen in Figure 6.5.

The results for rider (9) highlight a statistically and practically significant relationship between his performance measure and the accumulated training effect by 34%, as explained in Tables 6-4 and 6-5. The performance of this rider has improved, as can be seen in Figure 6.5. Furthermore, there was a slight enhancement in the accumulation of training effect, as shown in Figure 6.5.

Rider 10 has the greatest number of training sessions over the longest time period amongst all the riders (251 sessions over 481 days), and the statistical significance of the performance and training effect was high for this rider, this is clearly explained in Table 6-4. The training effect of this rider using his performance measure was increasing by 19%, as presented in Table 6-5. Additionally, the performance of this rider was improved, as configured in Figure 6.5.

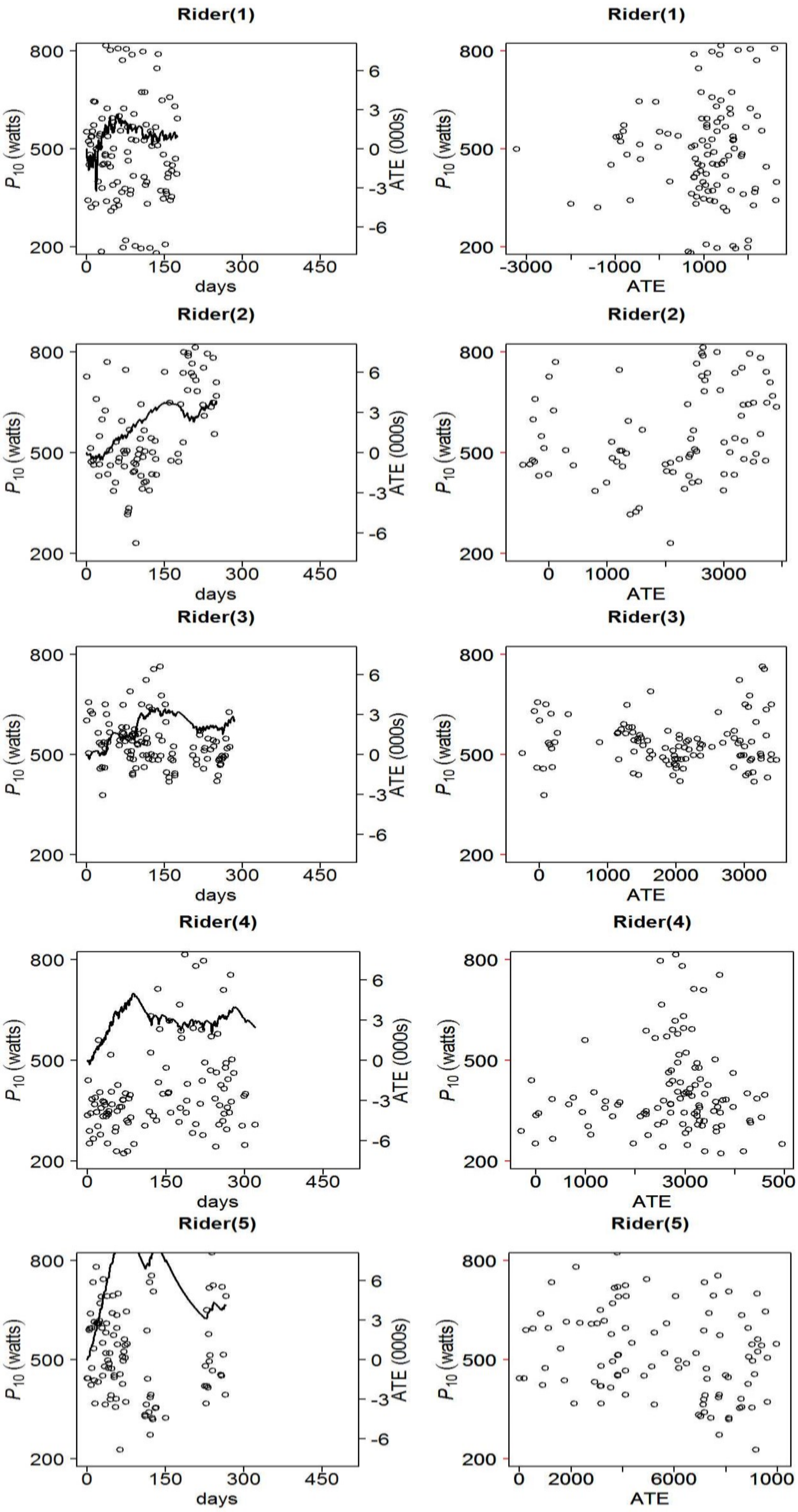

Figure 6-5 Two plots for each rider: left P_{10} , (symbols) vs time in days and ATE (line) vs time in days; right P_{10} , vs ATE (all sessions)

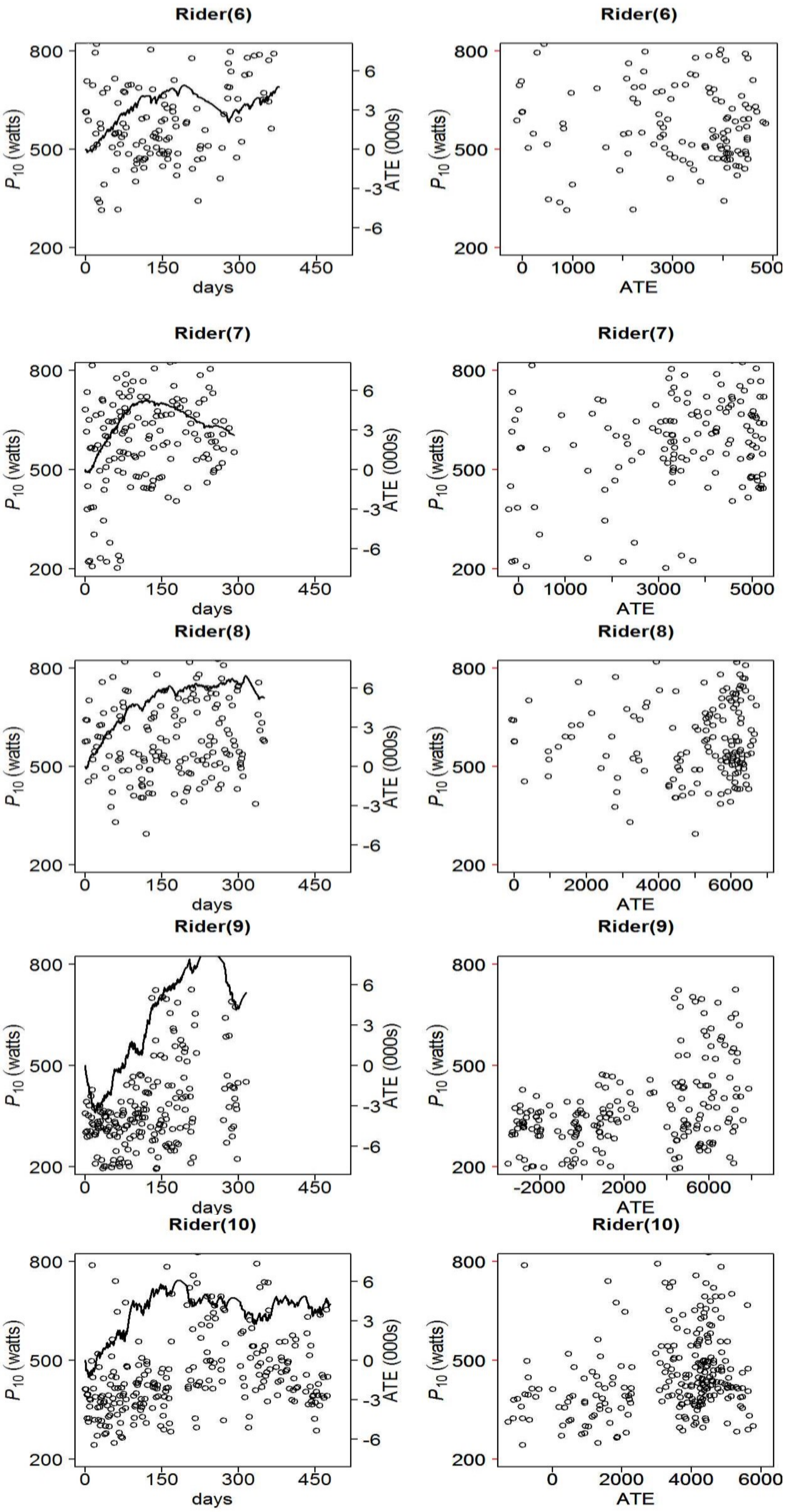

Figure 6.5 Continued

6.6 Critical Power Concept

Critical power (CP) is considered to be a measure of performance. Monod and Scherrer (1965) defined Critical Power as the maximum power that can be sustained for a long period of time without exhaustion. CP can be defined as an estimate of the maximum power output that can be maintained at a physiological steady state without fatigue and exhaustion (Borresen & Lambert, 2009). The basis for the concept of critical power is that there exists a hyperbolic relationship between power output and the for which time that power can be sustained (Shepley et al., 1992; Vanhatalo & Jones, 2009). Walsh (2000) has stated that the fundamental aim of the critical power concept is to describe fatigue and exhaustion. Moreover, the maximum time duration for which the Critical Power is able to be held is probably 60 minutes (Burnley, 2009). In cycling, CP can be continued for 20.3 minutes (McLellan & Cheung, 1992). In addition to cycling, the CP model has been applied in various sports such as swimming (Wakayoshi et al., 1992), running (Hughson et al., 1984) and rowing (Kennedy & Bell, 2000).

A number of studies have developed the CP concept to improve its validity. Linear and nonlinear mathematical models were used to determine the model that gives the estimate of the CP. Gaesser et al. (1995) estimated CP using five mathematical models during cycle ergometry. These mathematical models were discussed by Bull et al. (2000) , of which two are linear, and three are nonlinear.

The first linear model was the linear-TW model based on a regression model of total work done (TW) versus time to exhaustion (t). It was proposed by Moritani et al. (1981) and was formulated as follows:

$$TW = AWC + CP.t, \tag{6.5}$$

where CP is the critical power and AWC is the anaerobic work capacity. The results showed strong linear relationships between TW and t.

The linear-P model is the second linear model. It plots power output p against $1/t$, that is, the inverse of time, to describe the relationship between time to exhaustion and power output. The formula for this model is as follows:

$$P = AWC/t + CP. \tag{6.6}$$

The third model, which is nonlinear, uses the linear-P model to solve for t like so;

$$t = AWC/(P - CP) + k, \tag{6.7}$$

where k is determined by setting $P = P_{Max}$ at $t = 0$

So,

$$k = -AWC/(P_{Max} - CP).$$

The fourth model, also nonlinear, known as the nonlinear-2 model, is as follows:

$$t = AWC/(P - CP) - AWC/(P_{Max} - CP). \tag{6.8}$$

The fifth regression model is an exponential model (EXP), defined as follows:

$$P = CP + (P_{Max} - CP).e^{-(t/\tau)}, \quad (6.9)$$

where τ represents a time constant that is unspecified. An example of the relationship between power output and duration with CP value is shown in Figure 6-6.

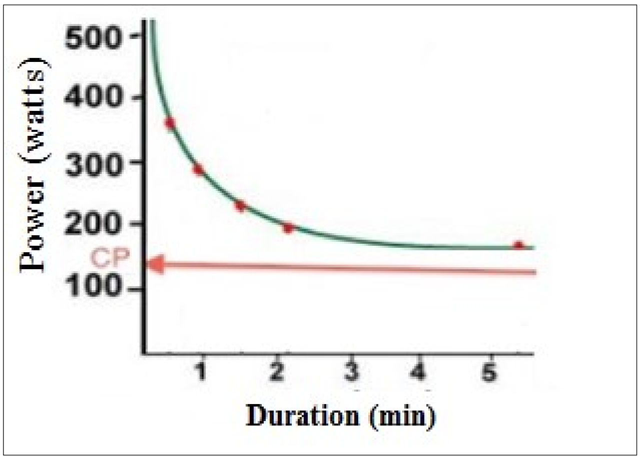

Figure 6-6 An example of the power output and duration with CP

In this section, two models are used to model critical power. These models are determined as follows:

Model 1 $\quad p_i = (p_0 - CP)e^{-\theta d_i} + cp + \epsilon_i \quad , \quad p_0, CP, \theta \geq 0\ , \ \epsilon_i \sim N(0, \sigma^2)$

Model 2 $\quad p_i = p_0 e^{-\theta d_i} + \epsilon_i \quad , \quad p_0, \theta \geq 0\ , \ \epsilon_i \sim N(0, \sigma^2)$

where p is the power output that can be sustained for duration d, p_0 and CP are parameters, the former the "peak power" and the latter the critical power (the power output that can be notionally sustained indefinitely), and $\theta > 0$ so that the shorter the duration the greater the power output that can be sustained. Model 2 is considered as a special case of Model 1 when $CP = 0$. To estimate the parameters of these models, we use the log likelihood functions for the suggested models that are defined as:

$$\log\ L1 = -\frac{n}{2} Log(2\pi) - n\ Log(\sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \left(p_i - ((p_0 - CP)e^{-\theta d_i} - CP)\right)^2$$

$$\log\ L2 = -\frac{n}{2} log(2\pi) - n\ log(\sigma) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \left(p_i - p_0 e^{-\theta d_i}\right)^2$$

In our study, the following procedure is performed to determine the critical power models for each rider. Firstly, the ten levels of power, 100, 150, 200,..., 600 watts, are specified, and the duration for which each power output can be maintained during each session, is determined for each rider. Table 6-6 shows the estimates of the parameters of the Model 1 with their standard errors. The estimates of the parameters of the Model 2 with their standard errors are presented in Table 6-7. Figure 6.7 shows the fitted critical power curve for the Model 1 every session for each rider. The fitted critical power curve for the Model 2 for all riders for all sessions is shown in Figure 6.8.

Furthermore, we let p_0 vary from session to session for the suggested critical power models. The estimates of p_0 with their standard errors are presented in Appendices 7 and 8. These estimates are called peak power and used as performance measure for each rider for each session as explained in section 6.7. Figures 6.9 and 6.10 present p_0 for each session for each rider and the confidence intervals of p_0 for critical power Model 1, respectively. Similarly, for critical power Model 2, Figures 6.11 and 6.12 present p_0 for each session for each rider and the confidence intervals of p_0, respectively. Also these figures clearly present the behaviour and changes of the estimate p_0 from session to session for each rider.

Table 6-6 Estimates of the parameters of critical power Model (1) with standard errors for each rider

Rider	$\widehat{p_0}\,(s.e.)$	$\hat{\theta}\,(s.e.)$	$\widehat{CP}$ (s.e.)	$\hat{\sigma}\,(s.e.)$	*AIC*
1	492 (4.68)	0.010 (0.0006)	160 (6.18)	95(1.90)	4881
2	529 (4.80)	0.011 (0.0005)	139 (6.51)	75(1.70)	3638
3	554 (3.87)	0.013 (0.0005)	157 (3.35)	53 (1.07)	4105
4	468 (3.76)	0.011 (0.0008)	149 (5.61)	88 (1.78)	4799
5	519 (4.37)	0.012 (0.0005)	148 (5.22)	71 (1.43)	4121
6	545 (4.26)	0.010 (0.0004)	147 (5.51)	91 (1.82)	6300
7	519 (4.36)	0.009 (0.0004)	163 (5.03)	102 (2.06)	6725
8	537 (3.01)	0.008 (0.0002)	138 (3.97)	77(1.54)	6731
9	472 (2.83)	0.019 (0.0009)	140 (4.37)	114 (2.29)	8923
10	508 (2.70)	0.016 (0.0006)	159 (3.06)	112 (2.26)	11324

Table 6-7 Estimates of the parameters of critical power Model (2) with standard errors for each rider

Rider	$\widehat{p_0}\,(s.e.)$	$\hat{\theta}\,(s.e.)$	$\hat{\sigma}\,(s.e.)$	*AIC*
1	457 (4.43)	0.003 (0.0001)	108(2.18)	5016
2	472 (3.89)	0.002 (0.0001)	85(1.71)	3741
3	493 (3.68)	0.004 (0.0002)	77(1.56)	4488
4	441 (3.63)	0.003 (0.0001)	101(2.03)	4945
5	480 (3.90)	0.004 (0.0001)	85(1.70)	4293
6	512 (4.37)	0.005 (0.0001)	100(2.01)	6430
7	474 (4.33)	0.003 (0.0008)	117(2.39)	6922
8	509 (3.43)	0.004 (0.0007)	89(1.79)	6954
9	451 (4.05)	0.006 (0.0003)	132(2.60)	9197
10	468 (3.74)	0.004 (0.0001)	138(2.78)	11822

We use the Akaike information criterion (AIC) to select the best model from Model 1 and Model 2 .The Akaike information criterion (AIC) method was first proposed by

Akaike in 1974, and broadly speaking it measures the information in a model for predictive purposes (Burnham & Anderson, 2003). It is defined as

$$AIC = 2k - 2\ln(L),$$

where k is the number of the parameters in the model and L is the value of the likelihood function.

6.6.1 Discussion of Results

In this section, we discuss the proposed critical power models. The critical power estimate CP has been estimated in many studies. In cycling, it was reported as 195W by Gaesser et al., (1995), as 160 and 176W by Bull et al. (2000), and as 185W by Clarke and Skiba (2013). Our estimates of CP (Table 6-6) are similar.

According to Tables 6-6 and 6-7, there are differences between p_0 and CP for each rider. This is because each rider has different training program and this confirms that the values of p_0 and CP differ from rider to another. The values of p_0 are consistent with previous studies that reported these values ranging between 337 and 562 watt for the first model (Bull et al., 2000).

Furthermore, according to the Figures 6.7 and 6.8 and the values of the AIC that are shown in Tables 6-6 and 6-7, we see that the Model 1 is the better model. Thus, the results we obtain are in support of the non-zero critical power.

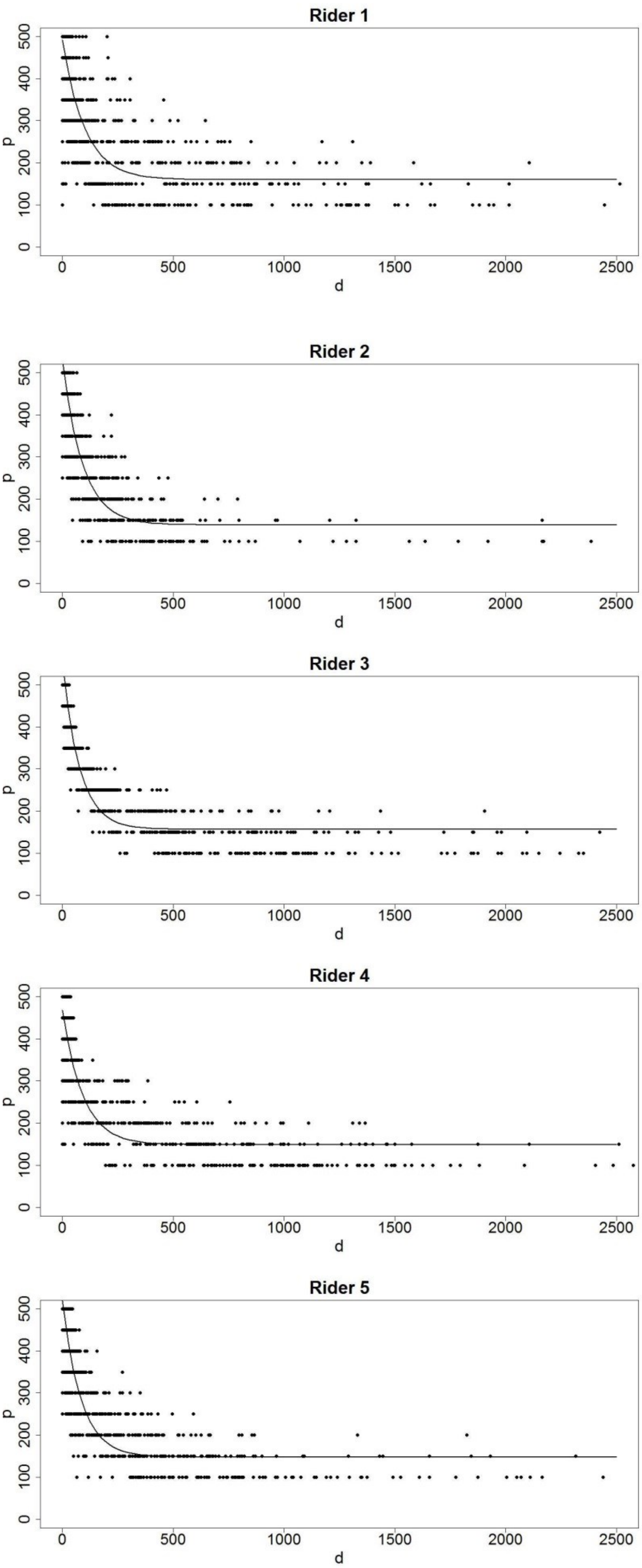

Figure 6-7 Observed power output and duration, and fitted critical power curve for Model 1 for each rider for all sessions

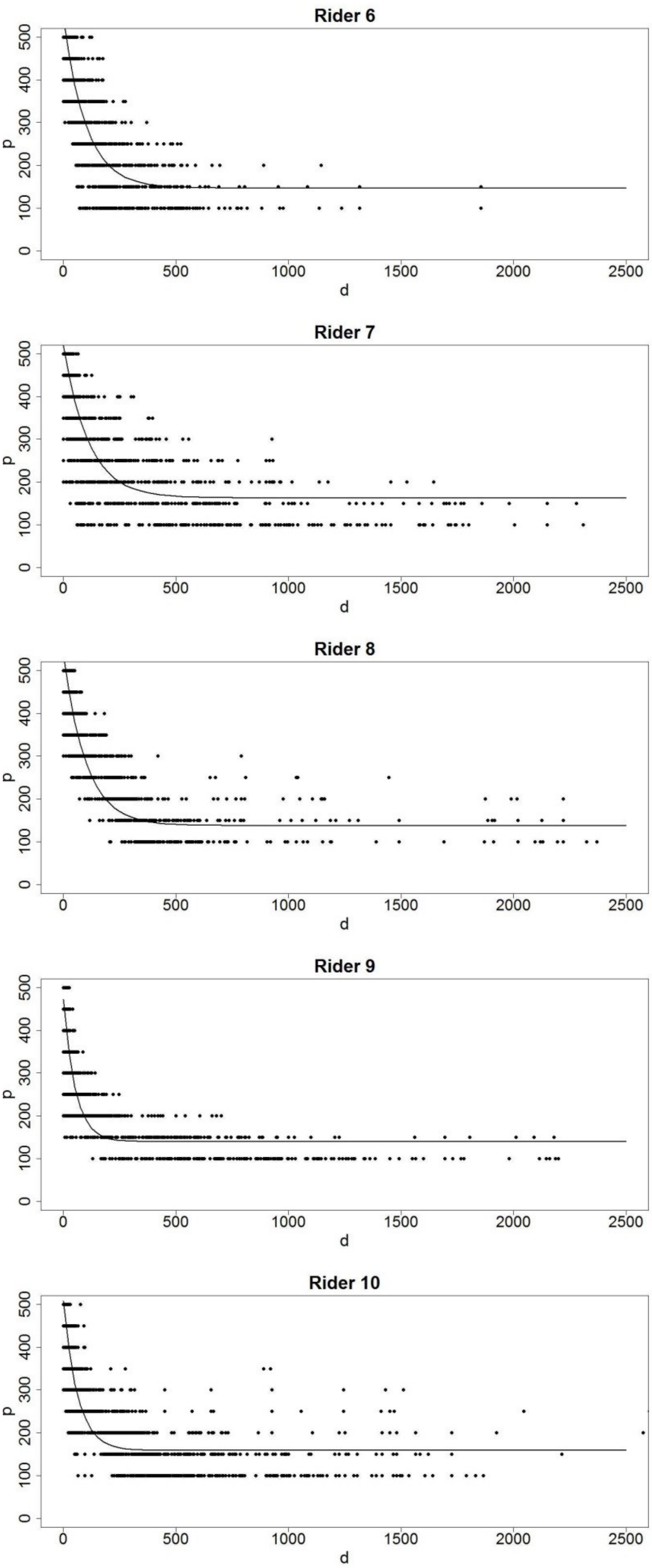

Figure 6.7 Continued

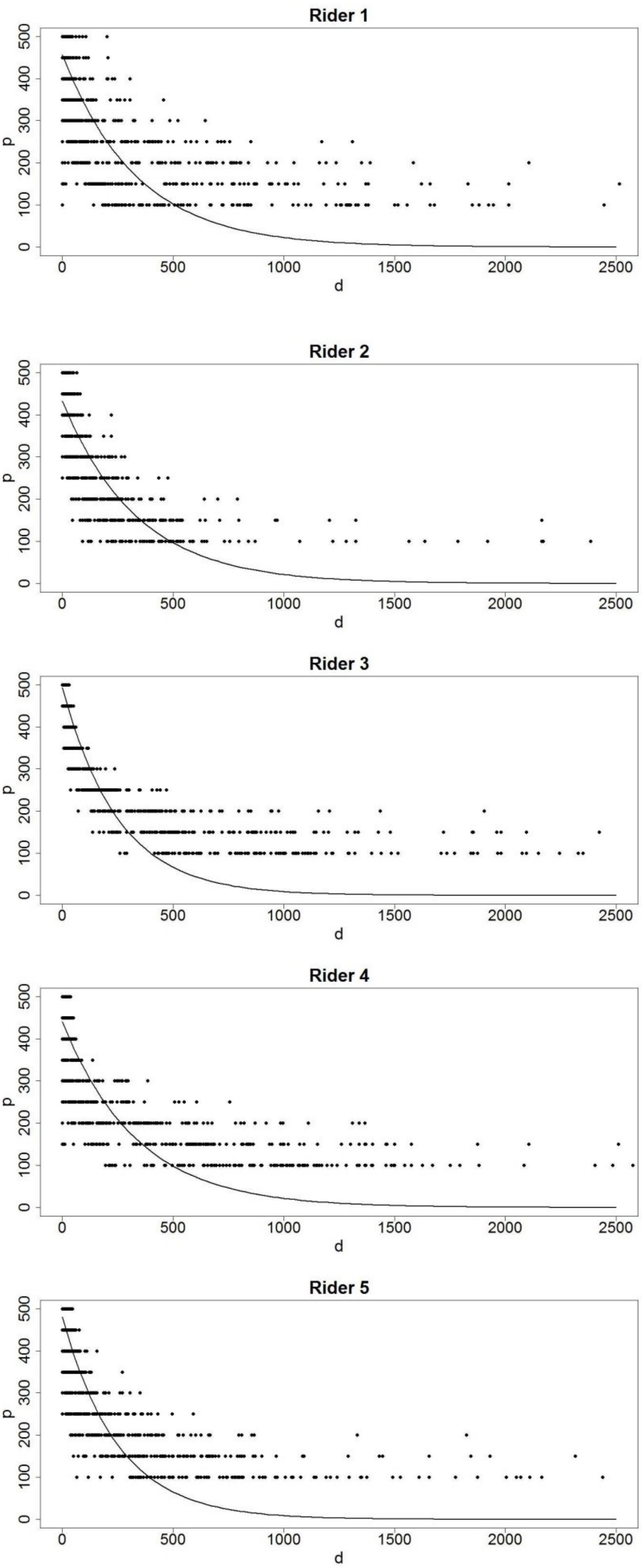

Figure 6-8 Observed power output and duration, and fitted critical power curve for Model 2 for each rider for all sessions

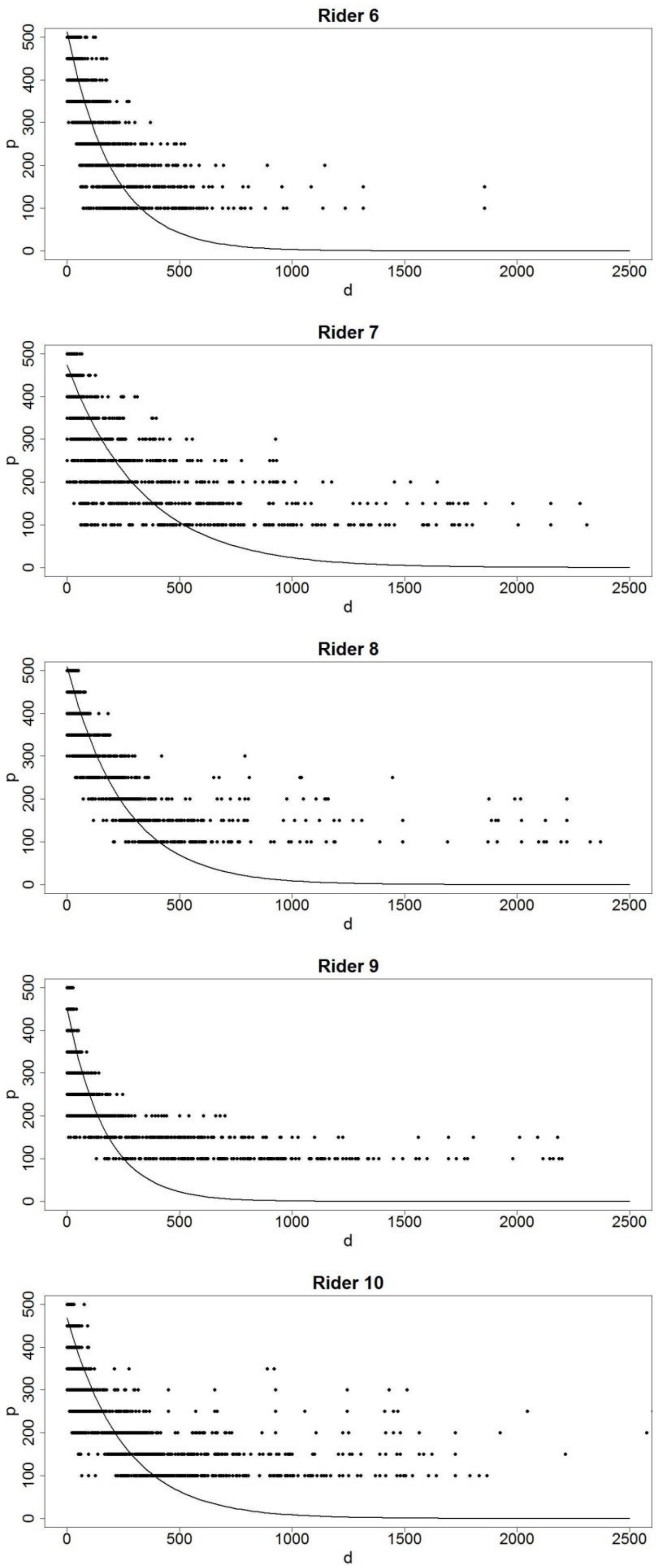

Figure 6.8 Continued

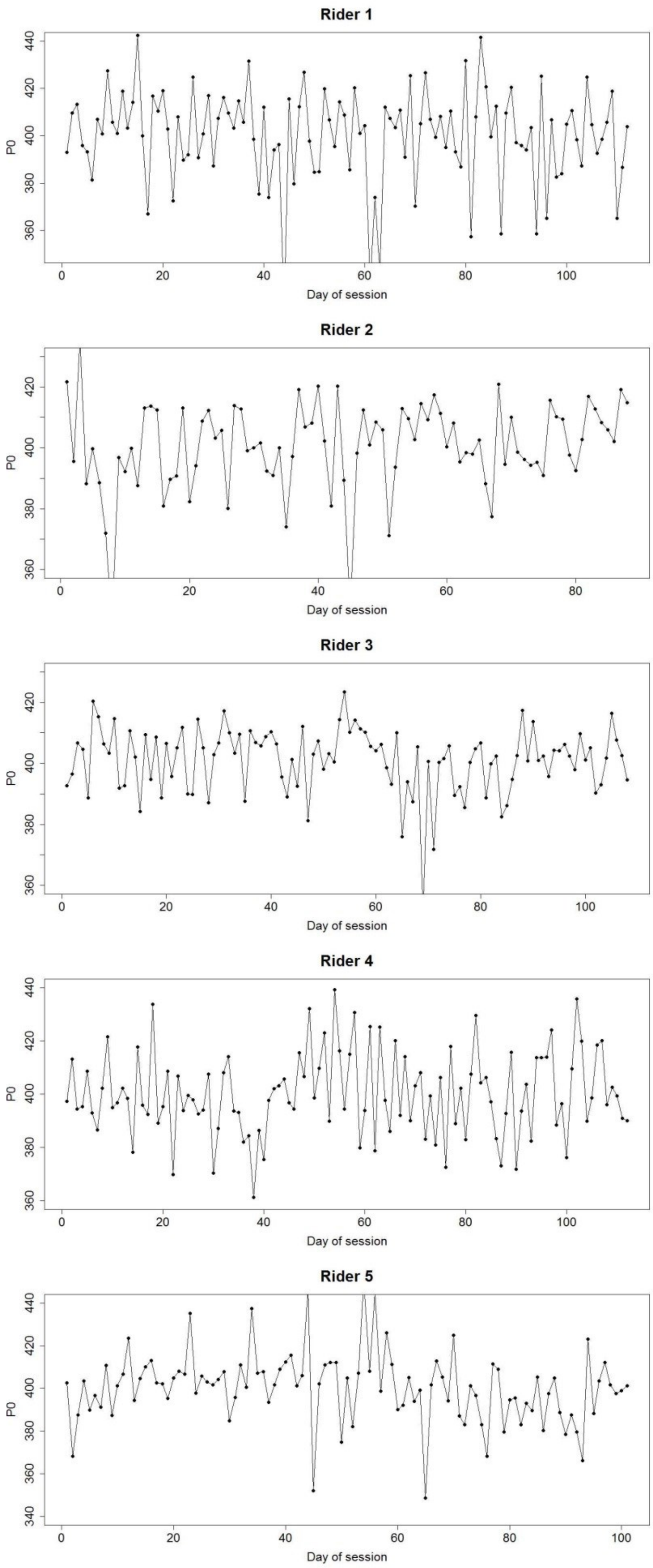

Figure 6-9 Estimate of p_0 for each session for each rider for critical power Model 1

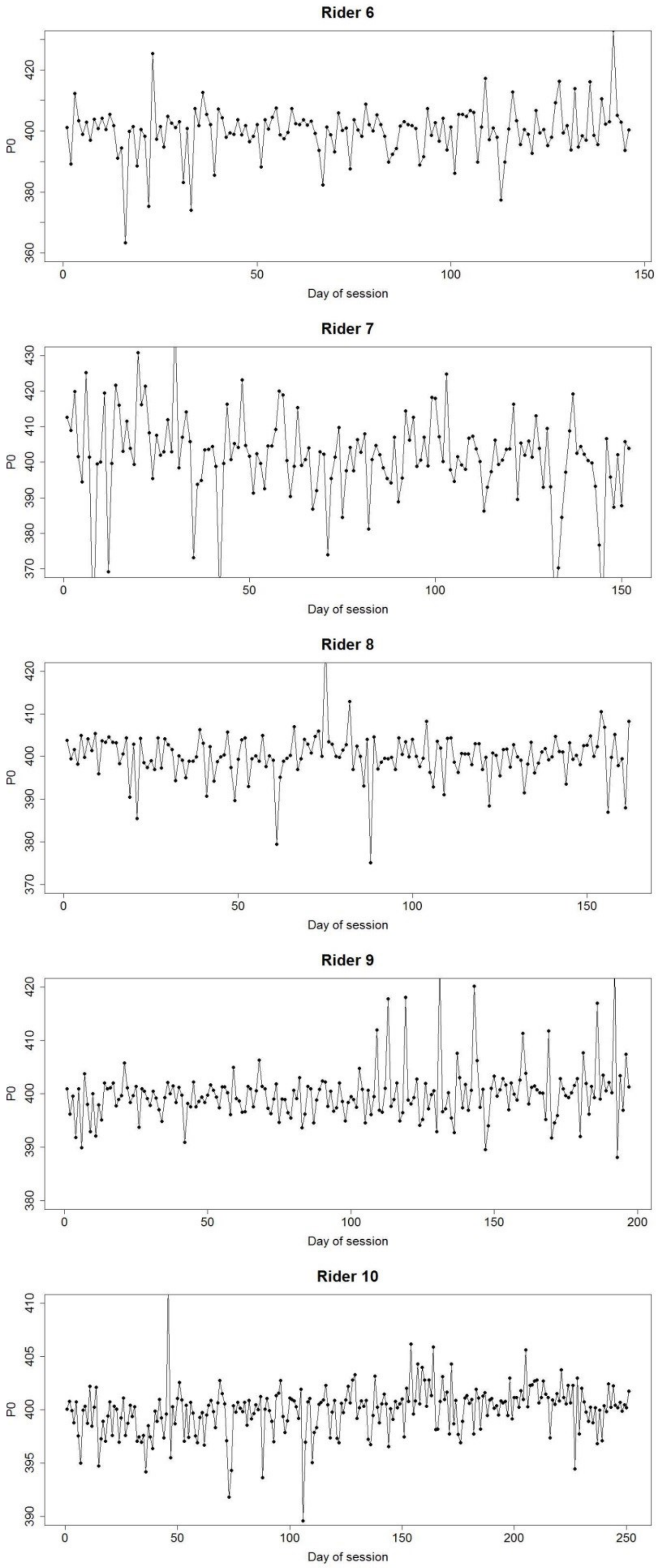

Figure 6.9 Continued

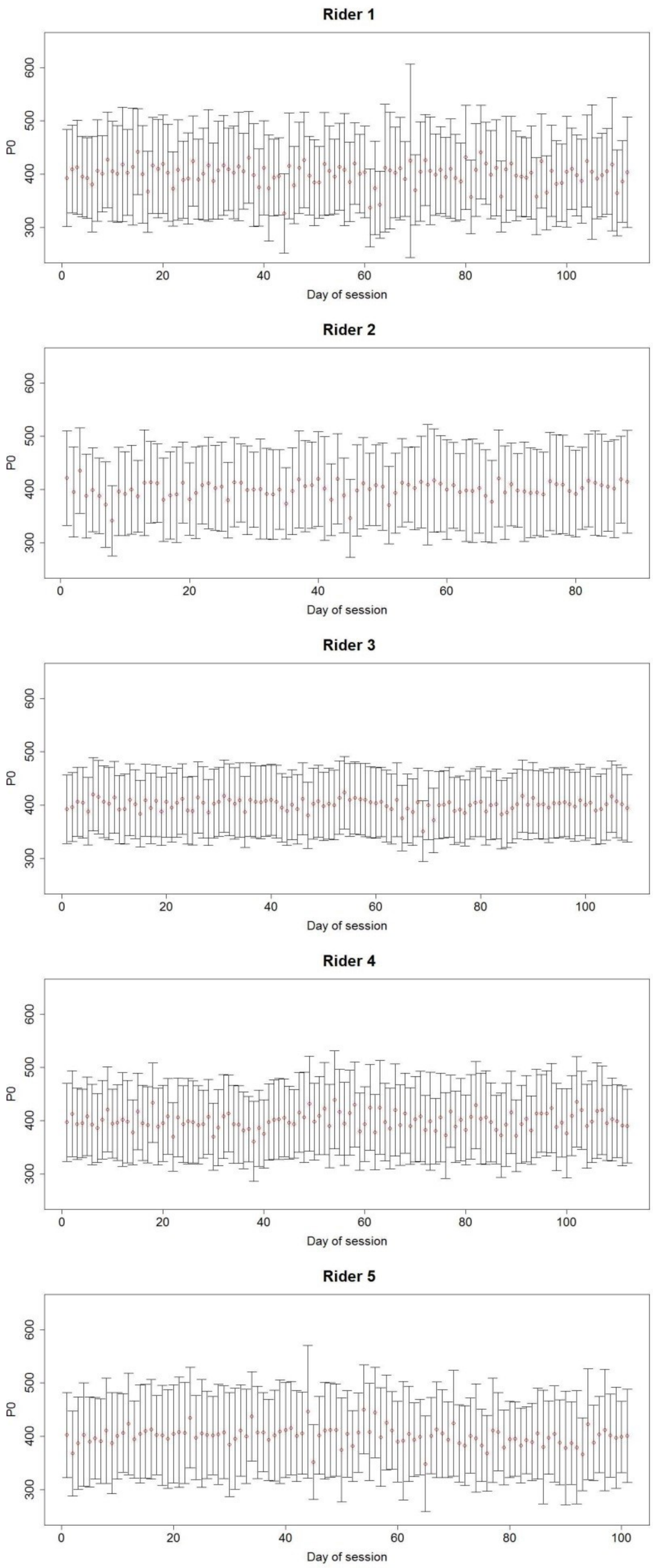

Figure 6-10 The confidence intervals of the parameter p_0 for each session for each rider for critical power Model 1

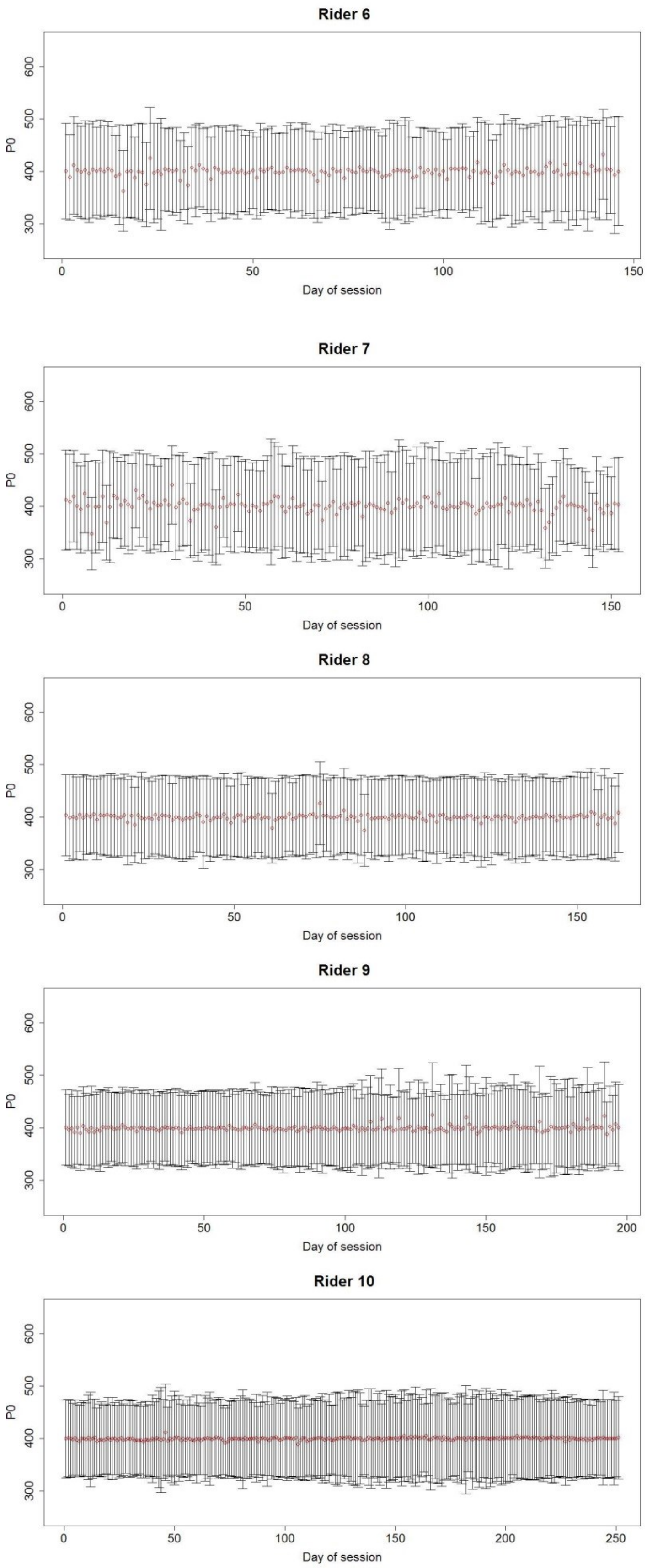

Figure 6.10 Continued

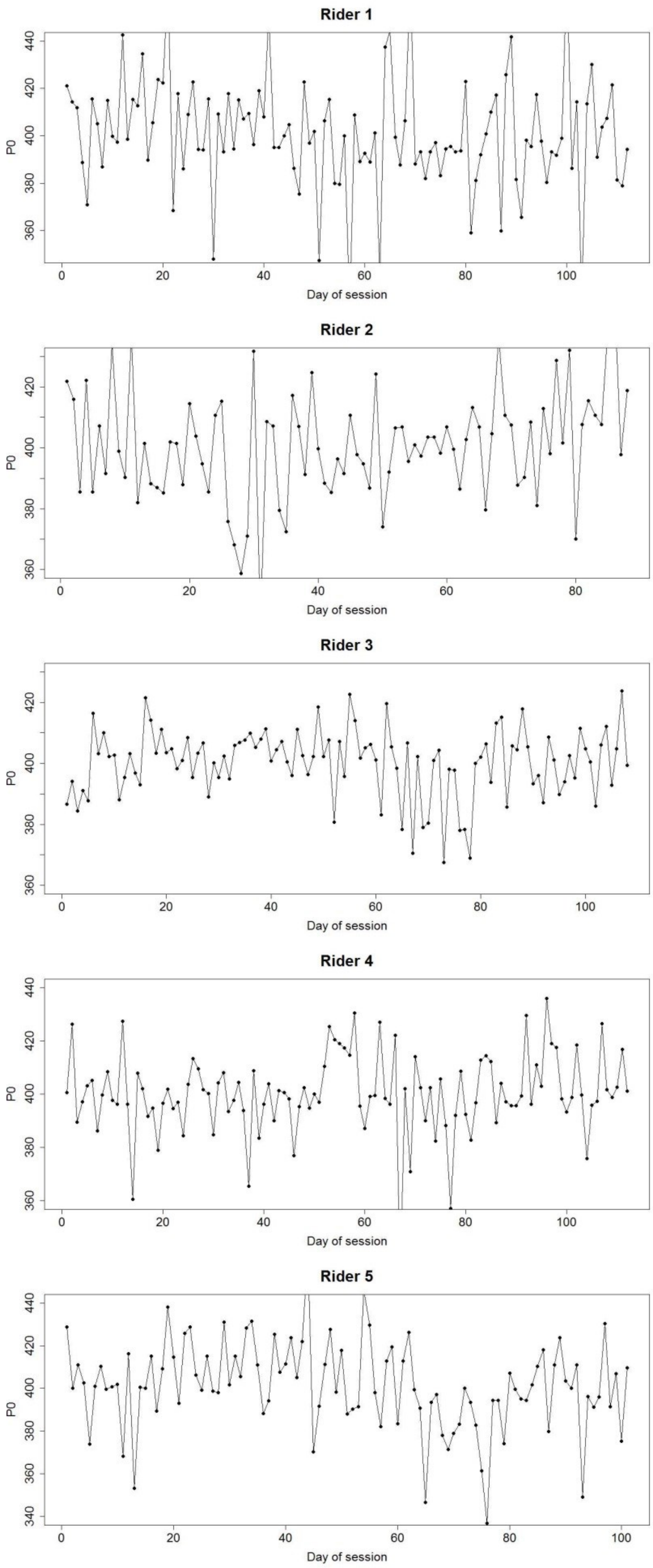

Figure 6-11 Estimate of p_0 for each session for each rider for critical power Model 2

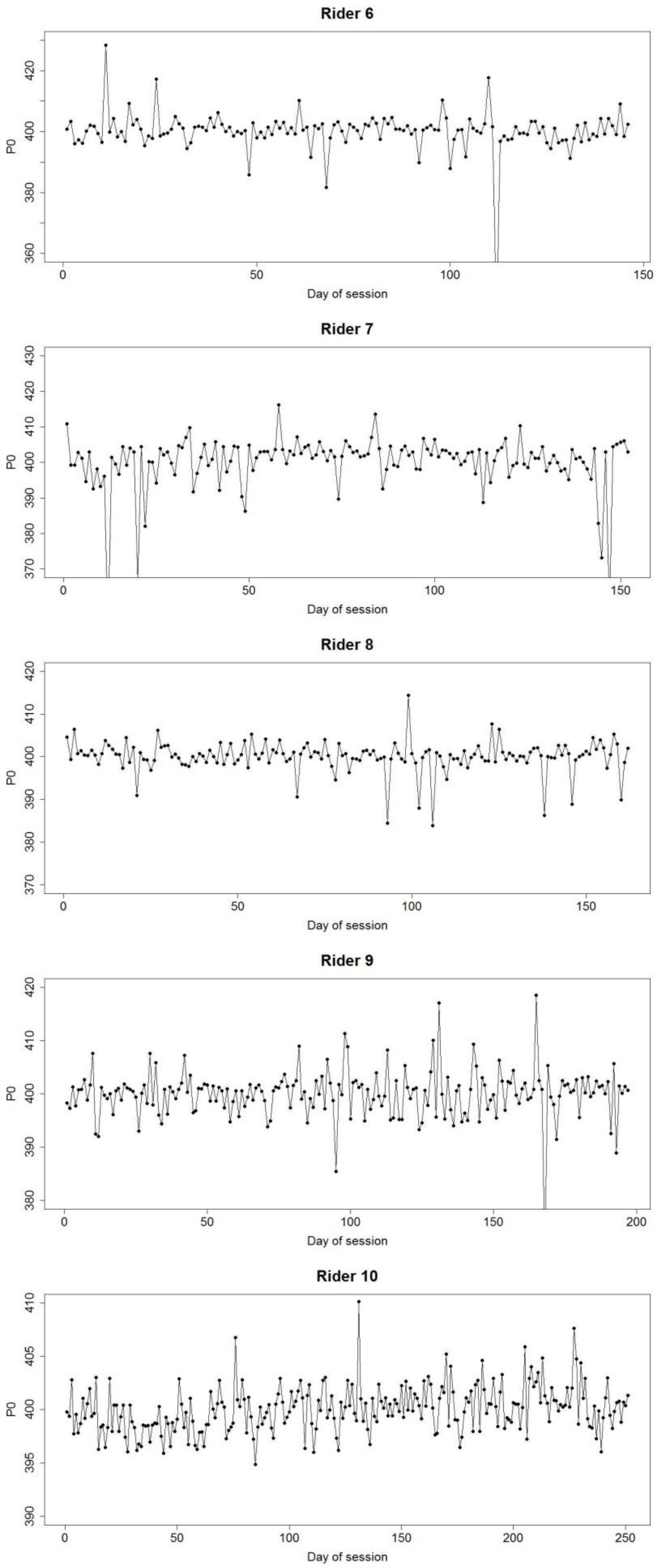

Figure 6.11 Continued

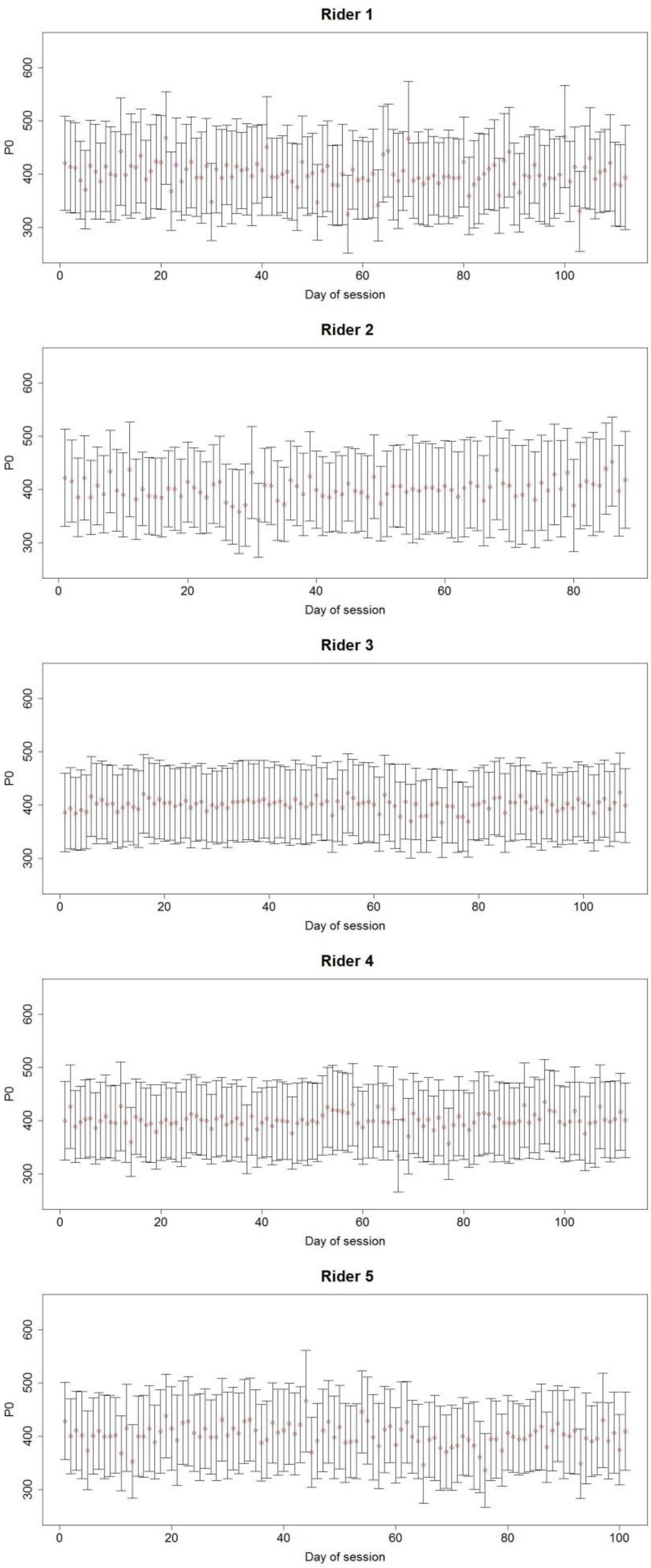

Figure 6-12 The confidence intervals of parameter p_0 for each session for each rider for critical power Model 2

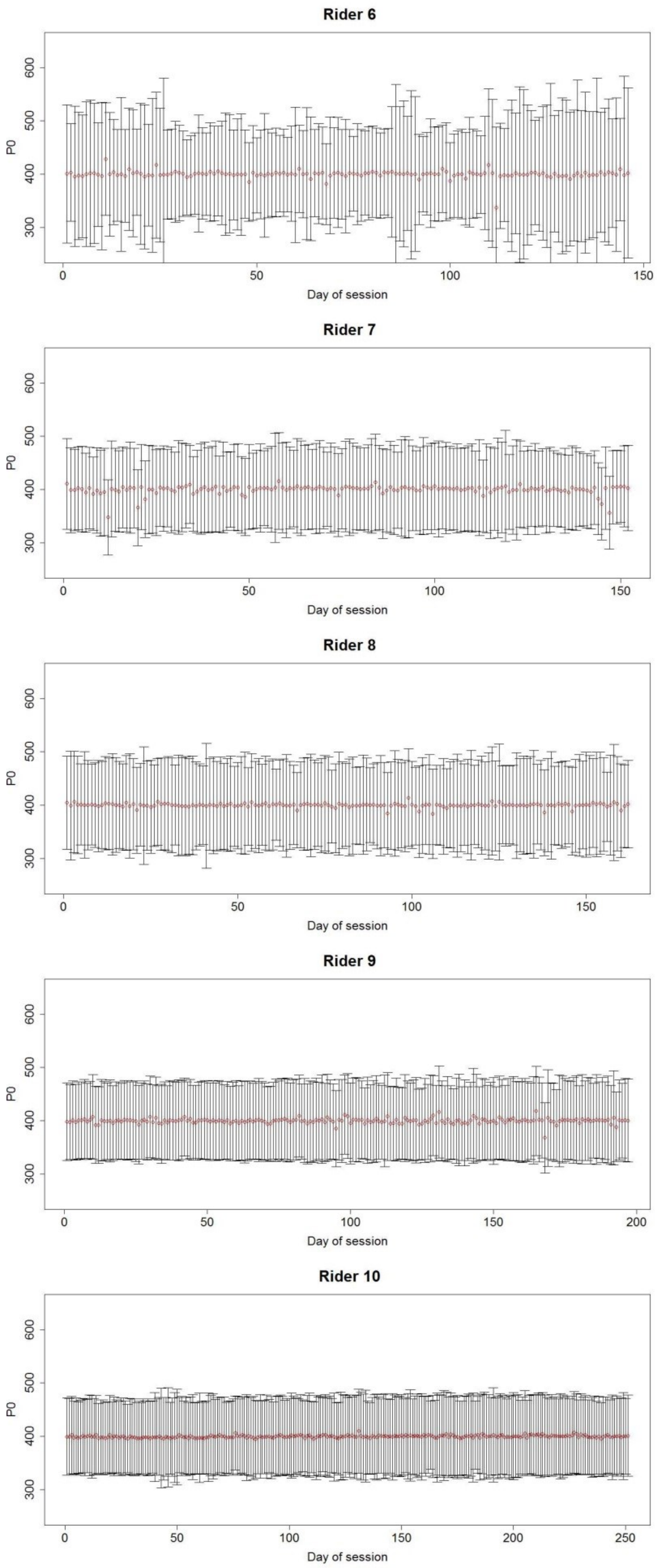

Figure 6.12 Continued

6.7 Relating Training to the Performance Measure p_0

In this section, we consider $p_{0,i}$ for session i for a given rider, as the performance measure that is defined in section 6.6. So we have

$$p_{0,i} \sim N(\alpha + \beta ATE_i, \sigma^2), \tag{6.10}$$

and

$$\widehat{p}_{0,i} \sim N(\, p_{0,i}\,, \lambda_i), \tag{6.11}$$

where $\lambda_i (i = 1, \ldots, n)$ is the variance in the estimate $\widehat{p}_{0,i}$, which must be calculated. The estimates of $\widehat{p}_{0,i}$ and their variances were obtained in section 6.6. Finally, through (6.10) and (6.11), we conclude our model of training–performance to be

$$\hat{p}_{0,i} \sim N(\alpha + \beta ATE_i, \sigma^2 + \lambda_i).$$

The parameters can again be estimated using the method of maximum likelihood. Table 6-8 shows the estimates obtained for each rider.

Table 6-8 Estimated parameters with standard errors of the training and performance model for each rider with the *t* statistic and *p* value for the test of β=0

Rider	$\hat{\sigma}$	$se(\hat{\sigma})$	$\widehat{\tau_a}$	$se(\widehat{\tau_a})$	$\widehat{k_f}$	$se(\widehat{k_f})$	$\widehat{\tau_f}$	$se(\widehat{\tau_f})$	$\hat{\alpha}$	$se(\hat{\alpha})$	$\hat{\beta}$	$se(\hat{\beta})$	t	p
1	23.6	4.8	68.1	12.6	1.1	0.1	5.2	4.3	389	16.2	0.0020	0.0012	1.67	0.05
2	8.2	5.4	96.2	69.7	1.3	0.5	11.6	2.3	380	12.4	0.0048	0.0026	1.85	0.03
3	24.9	3.5	86.8	8.5	1.8	0.1	2.2	1.9	390	11.5	0.0026	0.0008	3.25	0.00
4	10.9	3.9	91.5	29.9	4.7	2.9	12.2	9.8	401	12.7	0.0030	0.0007	4.29	0.00
5	21.4	6.9	80.5	44.8	0.7	0.4	4.8	0.3	376	15.6	0.0038	0.0019	2.00	0.02
6	4.1	0.1	89.8	25.8	1.2	0.2	3.4	1.3	391	16.7	0.0030	0.0011	2.73	0.00
7	5.9	3.9	86.2	25.6	3.1	2.9	11.1	1.3	405	12.4	0.0008	0.0003	2.67	0.00
8	16.8	4.2	76.5	34.9	3.2	2.5	2.2	0.9	397	15.9	0.0010	0.0003	3.33	0.00
9	26.9	2.6	123.8	23.9	2.4	0.7	19.8	8.1	395	15.7	0.0005	0.0003	1.67	0.05
10	12.3	2.1	66.2	58.4	1.1	0.2	4.9	1.1	399	23.6	0.0002	0.0001	2.00	0.02

6.7.1 Statistical Discussion of the Training Effect

In this section, the effect of training on performance is studied statistically. As the relationship between the accumulated training effect and performance is assumed to be linear and positive, we would like to test this relationship using the hypothesis $H_0\colon \beta = 0$ in favour of $H_1\colon \beta > 0$. We conclude that there is a statistically significant relationship between the accumulated training effect and the performance measure p_0 for each rider.

6.7.2 Practical Discussion of the Training Effect

In this section, we discuss the training effect on performance in practical term. As we concluded that there is a statistically significant relationship between the accumulated training effect and the performance measure p_0, it is important to determine the practical significance of that increase. This can be calculated using the change in average power output from the beginning of training until the point at which a rider has completed the optimal training, as follows

$$\Delta_{p_0} = \hat{\beta} \times \Delta_{ATE}$$

The values of $\hat{\beta}$ are shown in Table 6-8 and $\overline{p_0}$ is the mean peak power output across sessions for each rider. Δ_{ATE} is defined as the change between the maximum and initial accumulated training effect ($\Delta_{ATE} = ATE_{max} - ATE_1$). The change in power ranges between 1% and 9%, as seen in Table 6-9.

Table 6-9 Performance gain and the ATE change when the performance measure is p_0 for each rider

Rider	Δ_{ATE}	$\hat{\beta}$	$\overline{p_0}$	$\hat{\beta}\Delta_{ATE}$	$\Delta_{p0}/\overline{p_0}$
1	4879	0.0020	400	10	0.03
2	5221	0.0048	402	25	0.06
3	4375	0.0026	401	11	0.03
4	4210	0.0030	400	13	0.03
5	9440	0.0038	401	36	0.09
6	5825	0.0030	399	17	0.04
7	4644	0.0008	402	4	0.01
8	6382	0.0010	400	6	0.02
9	12163	0.0005	399	6	0.02
10	7808	0.0002	400	2	0.01

6.7.3 Discussion of Results

In this section, as each rider has a different training programme and different individual capacities, we discuss the results obtained for each rider in terms of the impact of training on his performance.

Rider (1) has a statistically significant relationship between the accumulated training effect and his performance measure, as explained in Table 6-8. However for this rider, practically, the training effect is not significant just 3% increasing with performance, as presented in Table 6-9. Also there is a slight enhancement in the performance of this rider, this is seen in Figure 6.13.

Rider (2) presents a statistically and practically significant relationship between the accumulated training effect and performance measure and practical significance is increasing by 6% with this performance measure, as presented in Tables 6-8 and 6-9. There is a clear improvement in the performance measure of this rider, as noted in Figure 6.13.

The results for rider (3) show a statistically significant relationship between the performance measure and the training effect, as demonstrated in Table 6-8. However, this relationship is not significant practically only 3% with this performance measure, this is highlighted in Table 6-9. Furthermore, there is a slight enhancement in the accumulation of training effect, as shown in Figure 6.13. The performance measure of this rider is stable, this can be shown in Figure 6.13.

Rider (4) has a statistically significant relationship between the accumulated training effect and his performance, as explained in Table 6-8. Furthermore, this relationship is not significant practically only 3% with this performance. Also, there is a noticeable enhancement in his performance, as presented in Figure 6.13.

Rider (5) displays statistically and practically significant relationship between training effect and performance, also this relationship is significant practically by about 9% increasing with this performance, as illustrated in Tables 6-8 and 6-9. The parameter k_f for this rider is still less than 1, where it should be greater than one, as this rider had lots of gaps in his training programme.

For rider (6), a statistically and practically significant training effect, this relationship is significant practically by 4% increasing with this performance, this is evident in Tables 6-8 and 6-9. The performance measure of this rider is slightly improved, as can be seen in Figure 6.13. Furthermore, there is an apparent increase in the accumulation of training, Figure 6-13 clarifies this.

Rider (7) has a statistically significant relationship between the accumulated training effect and his performance, as presented in Table 6-8. However, this relationship is not practically significant only 1% increasing with this performance, as identified in Table 6-9. Moreover, there is not significant change in his performance and the accumulation of training of this rider, as can be seen in Figure 6.13.

Rider (8) reveals a statistically significant training effect, as shown in Table 6-8. The performance measure of this rider is stable, this is presented in Figure 6.13. However, no practical improvement effect from training for this rider just 2%, as noted in Table 6-9.

For rider (9), a statistically significant relationship between training effect and his performance is demonstrated in Table 1. However, this relationship is not practically significant about 2%, as presented in Table 6-9. For this rider, the accumulation of training increases immensely, Figure 6.13 highlights this.

Rider (10) expresses a statistically significant relationship between the accumulated training effect and his performance measure, this is explained in Table 6-8. However, this rider reveals almost no practical improvement effect from training just 1%, as shown in Table 6-9. Furthermore, the training effects increases somewhat, this is evident in Figure 6.13.

6.8 Summary

In this chapter, other performance measures that relate directly to power output alone are considered. These are the 75th percentile of power output P_{75} for each training session, the maximum power output sustained in a session for a minimum duration *d,* with d=10 seconds, and the peak power p_0 that depends on the critical power concept. Again, these performance measures are related to the training measure ATE, and the Banister model parameters are estimated. We present these measures as alternative choices, and demonstrate that Banister model parameters can be estimated in each case using data obtained in the field. In the next chapter, we present a critical comparison of the five performance measures, how they can be statistically related to training, and the results in each case.

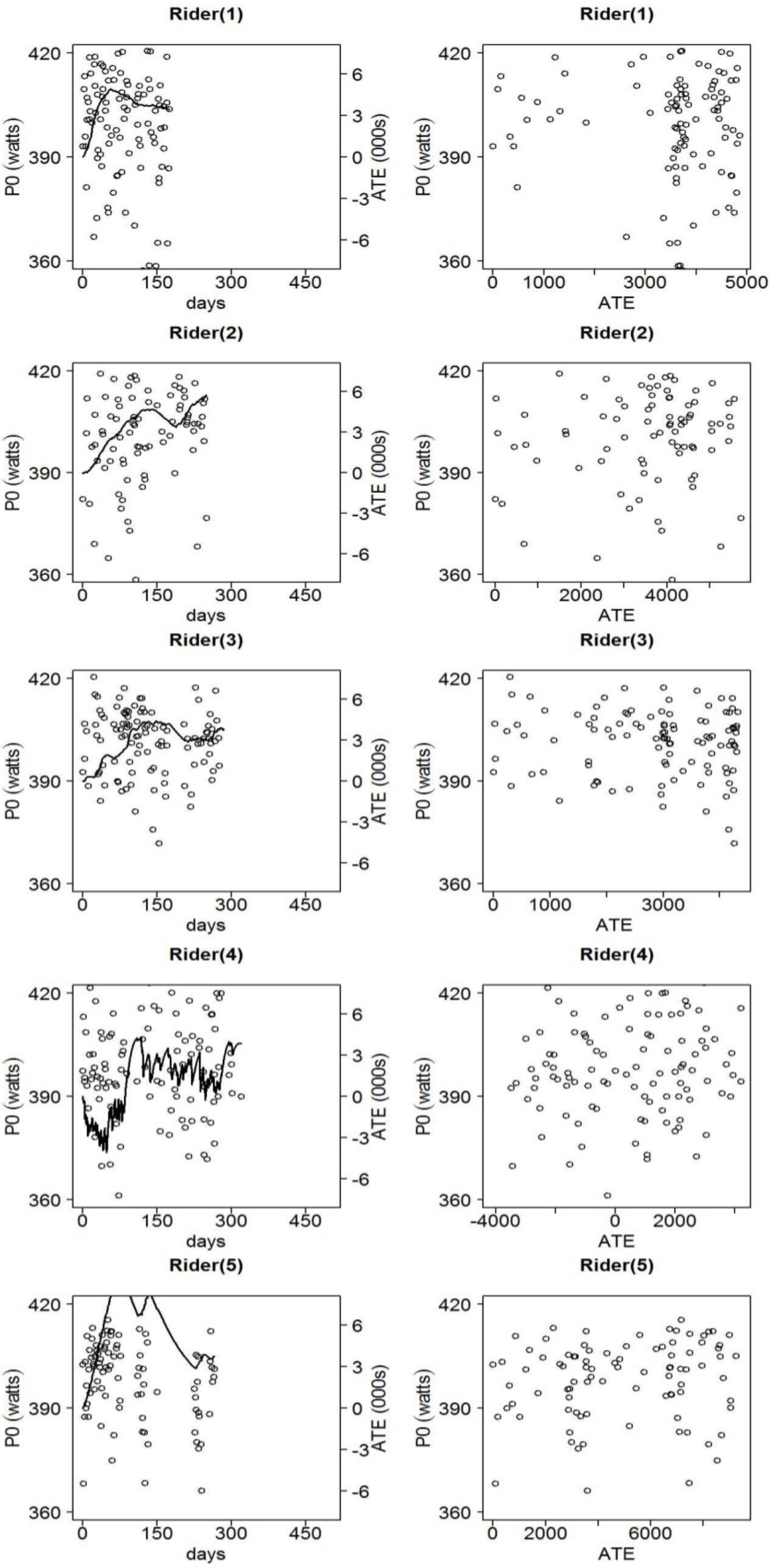

Figure 6-13 Two plots for each rider: left p_0, (symbols) vs time in days and ATE (line) vs time in days; right p_0, vs ATE (all sessions)

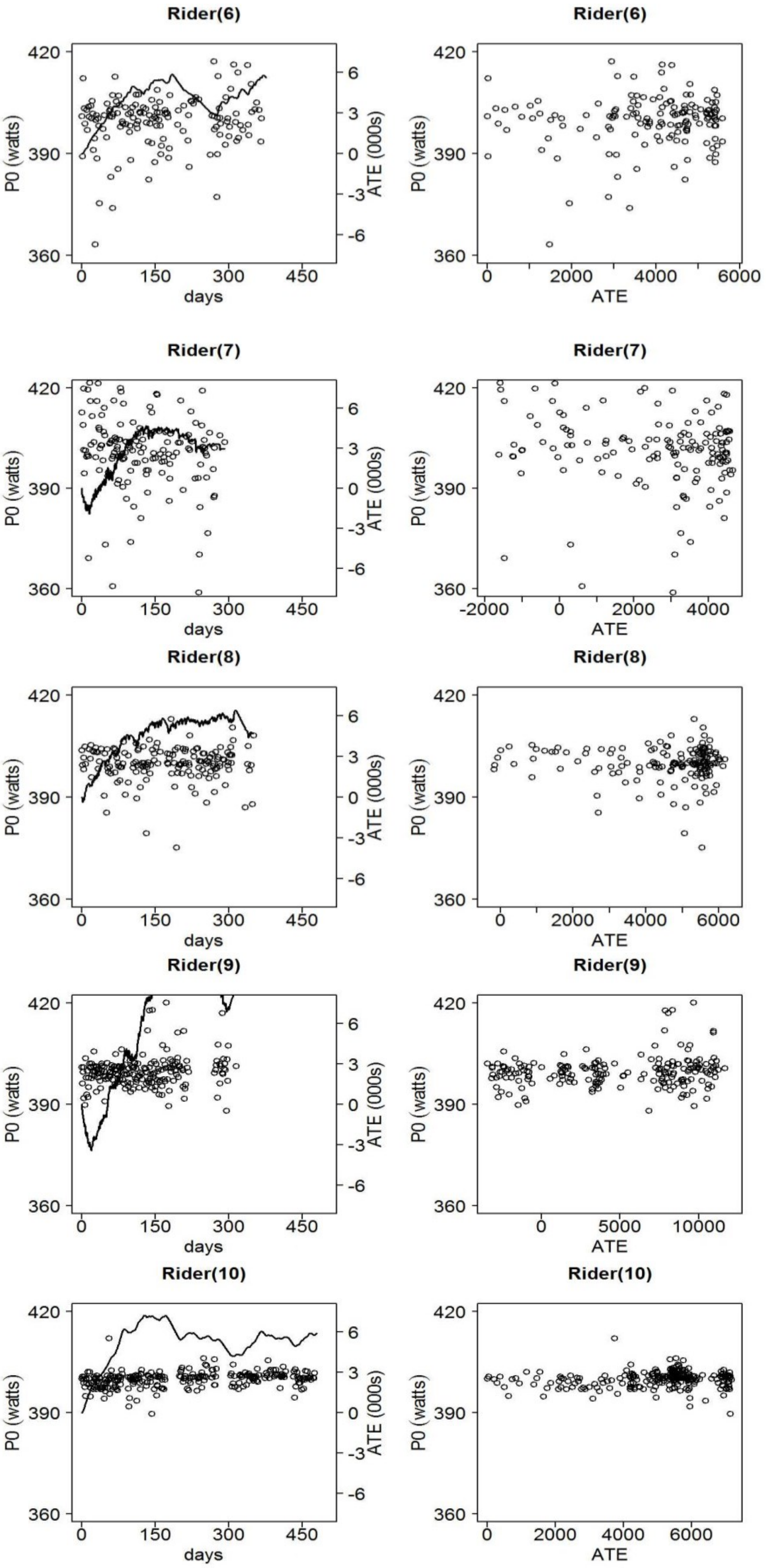

Figure 6.13 Continued

CHAPTER SEVEN

COMPARISON OF PERFORMANCE MEASURES

7.1 Introduction

In this chapter, we compare the performance measures proposed in this thesis. These measures are related to the training measure that is defined using the Banister model, with the purpose of estimating the Banister model parameters. The fundamental aim of the discussion here is to critically examine and contrast these measures in terms of methods, results, and ease of use, and furthermore to make a comparison with previous studies that discuss such results. The best measure is suggested and some justifications are given.

7.2 Discussion of the Proposed Performance Measures

In this thesis, we have proposed five performance measures that use power output and heart rate data collected in the field. These measures depend on specific performance concepts that are different from one to another. These measures are related to the training measure. In each case, the parameters of the Banister model are estimated. We aim to suggest a best performance measure among the proposed measures, taking account of the estimated parameters.

The basic purpose of selecting the performance measure is the estimation of the parameters of the Banister model; therefore, the training-performance model that is presented in sections 4.2, 5.2, 6.5 and 6.7 is useful in practice. Parameter estimates for the Banister model have been reported previously by other authors. Mujika et al. (1996) reported values of the fitness and detriment decay time constants τ_a and τ_f that were 41.4 and 12.4 days respectively and values of k_a and k_f that were 0.062 and 0.128, in arbitrary units respectively, for swimming. Hellard et al. (2006) reported values of τ_a and τ_f as 38 and 19 days and values of k_a and k_f as 0.036 and 0.050, again for swimming. Notice here that both k_a and k_f are estimated in spite of the fact that it is the relative sizes of these parameters that is important. This indicates that these authors do not appreciate this point. Further analysis has been done for swimming by Gouba et al. (2013), the values of τ_a and τ_f reported as 42.25 and 15.29 days. For running, Morton et al. (1990) reported values of τ_a and τ_f as 45 and 15 days respectively and the values of k_a and k_f as 1 and 2, in arbitrary units, respectively; their justification for this is that the value of k_f should be twice of k_a, as argued by Banister et al. (1975). Again for running, the values of τ_a and τ_f were given as 20 and 10 days respectively by Millet et al. (2002). For cycling, Busso et al. (1997) reported values of τ_a and τ_f as 60 and 4 days respectively, and values of k_a and k_f as 0.0021 and 0.0078 for participant A. For participant B, the values of τ_a and τ_f were 60 and 6 days and the values of k_a and k_f were 0.0019 and 0.0073. Again, the authors here do not appreciate that the model is over-parameterised. Another study was carried out in cycling by Busso et al. (2002). They found that the values of τ_a range from 30 to 60 days, and τ_f from 1 to 20 days. Shrahili (2014), for cycling, reported values of τ_a ranges from 5.7 to 228 days, and τ_f from 0.4 to 57 days, and the value of k_f from 0.93 to 7.2, supposing fixed k_a=1. The study of this author uses a performance measure related to that in chapter 3, but without the addition of the cardiovascular drift factor.

We can observe in these results that the parameter estimates vary depending on the type of sport. Also, we note the values of the parameter τ_a in swimming are close to each other at about 40 days (Hellard et al., 2006b; Mujika et al., 1996). Additionally, the values of this parameter in cycling are close to each other in only two studies at about 60 days (Busso et al., 2002; Busso et al., 1997), but they differ from the values mentioned by Shrahili (2014). In addition, the value of k_f is very approximately twice the value of k_a in each sport, corresponding to what was argued by Banister et al. (1975). An important point that should be taken into account here is that these authors, apart from Shrahili (2014), do not report the standard errors for the estimated parameters. Therefore, we do not know if the parameters are well estimated. Nonetheless, they provide a useful comparator for our estimates.

Next, we discuss our results. Tables 7-1, 7-2 and 7-3 show the estimates obtained for each rider using our proposed performance measures, along with their standard errors. Looking at the rider estimates, a number of points are important. Firstly, all values of $k_f > 1$, implying that the immediate training detriment is larger than the immediate benefit, as required. This can be seen in Table 7-3. Secondly, there is noticeable variation in the fitness and detriment decay constants between riders; this can be seen in Tables 7-1 and 7-2. This may be due to variation in the personal characteristics of the riders. We can conclude from this that parameter estimation should be individualised. Thirdly, the very large values of the benefit decay constant imply that the training benefit is extremely persistent. Additionally, the values of τ_a and τ_f using performance measures h_{P75} and P_{h75} are quite similar to the values obtained by Shrahili (2014) and those using the performance measure P_{75} are close to the estimates obtained by Busso et al.(2002). However, using performance measures P_{10} and p_0, we observe that these values are somewhat different from those mentioned previously. Finally, we note that the values of τ_a and τ_f obtained in this thesis using performance measures h_{P75} and P_{h75} vary, according to Tables 7-1 and 7-2. Also, using P_{75}, P_{10}, and p_0, we can observe that these values are similar, as seen in Tables 7-1 and 7-2. It is also observed, in Table 7-3, that the values of k_f are similar to each other and are greater than unity in most cases, and that when using P_{10} and p_0 they are more similar, as shown in Table 7-3.

Next, we discuss each performance measure and its characteristics. The first of our proposed measures, h_{P75}, is based on the relationship between power output and heart rate, taking into account the influence of cardiovascular drift. This measure is the estimated heart rate required by the rider to produce power output at high level. The relationship between heart rate and power output can be affected significantly by the cardiovascular drift phenomenon (Jeukendrup & Diemen, 1998). Heart rate drifts up as exercise progresses even when power output is stable. This drift therefore needs to be allowed for in the calculation of performance. Thus, we consider the effect of this phenomenon in this measure. For this measure, the estimation methodology is similar in principle to that of

Shrahili (2014); however, we account for cardiovascular drift. The estimates we obtain using h_{P75} are themselves somewhat different from those obtained by Shrahili (2014), thus reinforcing the importance of accounting for cardiovascular drift.

Table 7-1 The estimate with standard error of fitness decay constant τ_a for all riders using our performance measures

Rider	h_{P75}		P_{h75}		P_{75}		P_{10}		p_0	
	$\hat{\tau}_a$	*s.e.*	$\hat{\tau}_a$	*s.e.*	$\hat{\tau}_a$	*s.e.*	$\hat{\tau}_a$	*s.e.*	$\hat{\tau}_a$	*s.e.*
1	32.1	16.3	35.3	2.3	31.2	15.8	33.6	4.2	68.1	12.6
2	86.2	65.4	31.2	7.1	84.5	26.5	105.8	27.2	96.2	69.7
3	16.8	6.3	16.3	1.4	36.6	17.6	67.9	19.1	86.8	8.5
4	98.4	27.3	95.1	13.8	60.6	28.4	57.9	20.7	91.5	29.9
5	181.2	10.2	139.6	10.2	57.6	13.3	83.9	18.2	80.5	44.8
6	164.7	51.2	161.7	7.2	85.4	10.1	87.9	30.8	89.8	25.8
7	90.1	62.3	132.4	13.1	74.1	19.5	76.3	24.6	86.2	25.6
8	201.3	79.1	162.3	10.4	46.7	13.3	84.7	26.6	76.5	34.9
9	155.9	78.4	168.7	49.3	58.2	18.3	100.5	23.3	123.8	23.9
10	75.4	21.6	126.2	15.7	93.8	24.6	90.8	26.1	66.2	58.4

Table 7-2 The estimate with standard error of detriment decay constant τ_f for all riders using our performance measures

Rider	h_{P75}		P_{h75}		P_{75}		P_{10}		p_0	
	$\hat{\tau}_f$	*s.e.*	$\hat{\tau}_f$	*s.e.*	$\hat{\tau}_f$	*s.e.*	$\hat{\tau}_f$	*s.e.*	$\hat{\tau}_f$	*s.e.*
1	2.0	3.2	2.8	0.9	2.2	5.8	3.8	2.1	5.2	4.3
2	0.1	2.6	4.1	0.8	12.7	1.7	22.5	1.7	11.6	2.3
3	1.4	1.1	1.0	0.1	1.3	0.2	3.4	2.3	2.2	1.9
4	33.1	6.2	13.3	0.9	5.2	1.5	2.7	2.4	12.2	9.8
5	25.6	77.1	69.3	3.5	5.1	3.7	5.1	1.6	4.8	0.3
6	27.3	16.6	23.2	0.4	3.7	1.1	8.5	1.9	3.4	1.3
7	0.1	4.3	1.1	0.5	2.8	2.6	9.1	5.9	11.1	1.3
8	61.4	80.6	40.1	2.3	1.8	0.9	2.4	0.6	2.2	0.9
9	8.2	3.4	1.6	0.2	12.5	8.5	24.6	11.5	19.8	8.1
10	34.3	9.7	20.4	1.5	5.1	0.6	10.7	1.5	4.9	1.1

Table 7-3 The estimate with standard error of immediate training detriment k_f for all riders using our performance measures

Rider	h_{P75}		P_{h75}		P_{75}		P_{10}		p_0	
	$\hat{k}_f$	*s.e.*	$\hat{k}_f$	*s.e.*	$\hat{k}_f$	*s.e.*	$\hat{k}_f$	*s.e.*	$\hat{k}_f$	*s.e.*
1	1.7	4.5	2.9	0.5	1.9	0.8	4.8	1.3	1.1	0.1
2	4.0	5.1	1.8	0.5	1.4	0.7	1.7	0.3	1.3	0.5
3	4.7	4.3	3.2	0.4	6.5	0.8	3.1	0.6	1.8	0.1
4	1.1	0.3	2.5	0.2	3.3	1.4	2.6	2.5	4.7	2.9
5	1.4	0.4	0.2	0.1	0.8	0.7	0.2	0.1	0.7	0.4
6	2.0	0.9	3.5	0.1	1.8	0.9	1.8	0.3	1.2	0.2
7	7.1	4.8	1.1	0.2	3.9	3.4	1.7	0.5	3.1	2.9
8	1.1	0.1	1.4	0.2	1.1	0.8	2.1	0.4	3.2	2.5
9	3.2	2.9	2.6	1.8	3.4	5.8	2.3	1.2	2.4	0.7
10	2.0	0.5	1.2	0.1	4.8	3.5	2.6	0.4	1.1	0.2

So, for the performance measure h_{P75}, over the training period for which data were collected, the difference in heart rate between when the rider was most trained (maximum ATE) and least trained (minimum ATE) is between 6% and 23% (excluding rider 5 because of the lack of sufficient data on him), as shown in Table 4-9. These are similar to values reported by others (Foster et al., 1996; Gabbett & Domrow, 2007), where a ten-fold increase in training load is associated with 10% improvement in performance. Furthermore, most riders show a progression in training. Although the training effects are statistically and practically significant, there are some weaknesses in using this measure. One of these weaknesses is that it is not clear whether the best percentile of power to choose is 50, 75, or 90. In fact, determination of this percentile should be based on the type of competition, with a high value appropriate for sprint events and a lower value for endurance. The choice should thus vary between coaches and between athletes. In our opinion, despite its statistical and practical significance, we do not prefer this measure as a best measure of performance. The reason is that this measure is rather counter-intuitive because, as fitness increases, the performance measure decreases. It therefore appears contrary to the notion that as a rider becomes fitter he or she is able to sustain a higher power output, although this is implied indirectly because if the heart rate required to produce a fixed power output decreases then if a rider increases heart-rate (in competition), power output will be increased.

Therefore, we consider a second performance measure, P_{h75}, that considers the power output at a high percentile of heart rate. This measure again accounts for the influence of cardiovascular drift. For this measure, as fitness increases the performance measure increases. Furthermore, over the training period for which data were collected, the difference in power output between when the rider was most trained and least trained is between 5% and 20% for all riders, as shown in Table 5-4. The estimates we have obtained for these parameters are a little different to the estimates obtained using the first performance measure h_{P75}. This may be because that the relationship between power output and heart rate is very noisy. Thus, it is important to account for this noise in the modelling, and this is what we try to do in this thesis. Furthermore, while a power output related measure is a sensible performance measure, there are the issues with this measure because power output is, in the modelling methodology, regarded as a response to heart rate. However, the actual physiology is the other way round, and so the model (equation 3.7) in which power output at some lag l is linearly related to heart rate may be contrary to the standard model of sports scientists in which heart rate responds to the demand for power output. For this reason, it may be sensible and more straightforward to focus directly on power output as a performance measure.

Thus, our third proposed measure P_{75}, uses power output only and considers the 75th percentile of power output for each session. This measure is thus different from the two previous measures in that it does not consider heart rate measurement at all. One of the advantages of using this method is that this measure is easy to calculate. However, to proceed with this measure, we require a measure of its sampling variability. We measure the sampling variability of this measure for each session using the bootstrap method, as described in section 6.2. Using P_{75}, we obtained similar estimates to those reported by Busso et al. (2002) in terms of the values of τ_a and τ_f, as seen in Tables 7-1 and 7-2. Furthermore, using this measure, the difference in power output between when the rider was most trained (maximum ATE) and least trained (minimum ATE) ranges between 4% and 25% for all riders, except rider 7 who shows a 35% improvement, as shown in Table 6-2. This performance measure may be made more general by specifying it for other percentiles, e.g. 50 or 90. The analysis proceeds in the same way, and the choice of percentile is to be based on the type of competition of interest (sprint or endurance), as with the other measures.

Two further measures that are related only to power output are proposed in this thesis. The first one of these, and the fourth measure overall is P_{10}. This measure does not use heart rate data in its calculation. This measure is based on determining a level of power output that a rider is able to sustain for a particular length of time. In the case of P_{10}, it is the level of power output that a rider can sustain for 10 seconds. The advantage of this measure is that it shows the ability of an athlete to maintain a high power output for a very short time. Again we find that the Banister model parameter estimates obtained are similar to those using performance measure P_{75} (see Tables 7-1, 7-2, and 7-3). Moreover, the difference in power output between when the rider was most trained and least trained ranges between 7% and 34%, as seen in Table 6-5. In fact these values are close to the values reported by others (Foster et al., 1996; Gabbett & Domrow, 2007).

The fifth and final measure of performance is denoted p_0, which is based on the concept of critical power. Calculation initially proceeds in a similar way to P_{10}. Five mathematical models of critical power are presented by Bull et al. (2000). In their study, which considered nine male cyclists riding cycle ergometers in the laboratory, they re-examined the findings of previous studies to estimate critical power and to determine the time to exhaustion. Accordingly, we use a nonlinear model mentioned in that study, which relates power output sustained to the corresponding duration. This model provides a theoretical peak power value for each session for each rider. To fit these models, we fix some levels of power output and then determine the maximum time that the rider successfully sustains each power level. Through this procedure, we obtain the values of peak power p_0 for all sessions. The values of p_0 obtained are consistent with previous studies that reported these values as ranging between 337 and 562 watts for our preferred model (Bull et al., 2000). Using p_0 as the performance measure, our estimates of the Banister model parameters are similar to those mentioned by Busso et al. (2002), as seen in Tables 7-1, 7-2, and 7-3. However, using this measure, we can see that the changes in peak power output between when the rider was most trained and least trained are less than 5% for most riders, as shown in Table 6-9. This may be because the training data relate principally to endurance riding when this measure itself is peak-power and so is much more related to sprinting.

To summarise then, according to the results from using the five performance measures, we can see a variation among these estimates. Some of them give estimated parameters that are similar to those stated by Shrahili (2014) and others, while other measures support the estimates reported by Busso et al. (2002). Overall, we do not recommend a single measure for coaches and athletes to use, but instead offer analysis of the characteristics of a range of performance measures, and recommend that coaches and athletes make an informed choice.

CHAPTER EIGHT

DISCUSSION AND CONCLUSIONS

8.1 Introduction

The primary objective of training in elite sport is to increase performance for competition. Training should develop athletes' ability to achieve performance at the highest level at a particular time in the future. In training, the primary objective of monitoring is to find out whether the training is suitable and beneficial for the athletes and hence whether the training needs to be modified. Generally, the aim is to control the training programmes of athletes in order to ensure that the performance of a given athlete attains its maximum level at a given competition at a known future time. Furthermore, balanced training that can support athletes in developing their capabilities is essential for avoiding over and under-training.

In this thesis, we discuss models of training and performance in cycling with this aim in mind, namely to determine quantitative models that athletes and coaches might use to optimise their training before a major competition. In particular, we consider these models in the context of data that are collected in the field during actual training sessions, rather than in the testing laboratory. We develop statistical models that relate performance to accumulated training. For this purpose, both performance and training should be measured. We use the Banister model of accumulated training as the basis of our training measure. This measure has a number of parameters that have to be estimated. We propose a number of new performance measures, and we compare and contrast these measures with respect to their statistical and practical properties. In particular, we relate parameter variability in the Banister model to sampling variability in the performance measures. In this way, we demonstrate that for a particular athlete the training-performance relationship contains uncertainly, and therefore that the use in practice of these models needs to take account of this uncertainty. Thus, we distinguish between preparedness or readiness to perform at the point in time of a session and performance during the session. So preparedness is unknown and must be estimated, and performance is a random variable with expectation that is the preparedness.

In our analysis, we use heart rate and power output data for ten male cycling competitors from sessions recorded over a number of months. The characteristics of the riders are described. These data are secondary data, collected by the athletes during a period of collaboration with sports scientists at the University of Kent. The power output and heart rate data for each rider are presented in an exploratory analysis in sections 2.4 and 2.5. We then describe a measure of the training load for each training session, known as the "training impulse" or TRIMP in section 3.3. Next, in section 3.4, we explain how the Banister model accounts for the accumulation of TRIMP into a single training load measure for training sessions up to time t. The performance measures we propose are related, using a statistical model, to the accumulated training load and the parameters of the model are estimated. Standard errors of all parameters estimates are also reported. These analyses are presented in sections 4.2, 5.2, 6.3, 6.5, and 6.7.

The performance measures themselves depend on the linear relationship between heart rate and power output, the concept of critical power, and the percentile of the power distribution. We consider five performance measures, and in each case estimate the Banister model parameters. We compare and contrast the results of these different measures in section 7.2. Some of these performance measures use power output alone; others use both power and heart rate in their definition. Finally, and importantly, we consider the practical usefulness of these performance measures and the Banister model of training.

8.2 Conclusions

A significant finding to emerge from this thesis is that the parameters of the Banister model can be estimated using field data, and that this estimation is practical for a number of different performance measures. Broadly speaking, in the context of estimation, estimates between riders are dissimilar, while estimates across the performance measures but within each single rider are similar. On this basis, we conclude that, firstly, performance-training models should be individualised. Therefore it would be a mistake to seek estimates of a performance-training model that could be used in general, regardless of the specific athlete who uses the model and its estimates, to plan or even optimise training. So in summary:

- Estimation of parameters in the performance-training model should be individualised.

Secondly, we conclude that the choice of the general specification of the performance measure is less critical than the athlete-specific individualisation, but that the particular specification of the performance measure should be training-specific. That is, for a general performance measure, there is some choice regarding its exact specification that corresponds to the choice of a threshold. For example, for the performance measure P_{10}, the threshold is 10 seconds duration. Other choices are possible but we do not regard the choice of this threshold as a statistical problem, although choosing a very high threshold will lead to some difficulties with estimation as the training data available at high thresholds may be sparse. Nonetheless, the choice of threshold is a matter for coach and athlete, and must be determined for the most part by the nature of the training (such as sprinting or endurance) and performance. So in summary:

- A performance measure that is calculated from power outputs (and heart rates) must be carefully specified.

With regard to the different measures of performance, each of which is based on a different concept, given the importance and sensitivity of the relationship between training and performance, performance measurement must account for the variability in measurement. In each of the cases we discuss, we present a model to specify this variability and a method to estimate the variability. In the literature published to date, there has been a failure to acknowledge this variability and so estimates are presented in a way that overestimates their precision. Thus, we distinguish between preparedness (expectation) and performance (random variable) in a way that has not been done in the sport science literature. This thesis attempts to address this deficiency. Thus:

- A performance measure should not be considered as a deterministic quantity; performance is measured with error.

For each performance measure, parameter estimates are obtained and we explain some of the advantages and disadvantages of each performance measure and our methods in the light of these. We also compare those parameters with the parameters that have been found in previous studies and suggest the best measure for athletes and coaches to follow. Thus:

- Banister model parameters are estimable, but their values are imprecise (some have large standard errors).

There is scope to use the methodology in other sports where power-output (or another measure from which performance can be derived) can be routinely collected. However, cycling lends itself to such study because of the availability of power meters. The methodology might also be used outside elite sport, for example, in studies of the effect of exercise on well-being or on rehabilitation following illness or injury. Thus:

- Our modelling could be used in studies of exercise and well-being and rehabilitation.

The study in this thesis sets out to obtain a methodology for planning training. The development of performance measures and the relating of performance to training in a quantitative model had this aim in mind. The notion existed that parameters in the training model, the Banister model, which describes quantitatively the accumulation of the benefits and detriments of training sessions, once estimated, could be used to plan training. However, the Banister model is lacking in this respect. We end our conclusions by noting, importantly, that in fact the Banister model alone cannot be used to plan training. The evidence to support this claim is set out in the following paragraphs. Nonetheless, it should also be noted that the Banister model may be used to broadly, quantitatively describe the persistence of the effects of training inputs.

To show how the Banister model is inadequate for planning training, we determine here, if n training sessions are available for an athlete to prepare for a competition at time t, the optimal times to carry out these training sessions. By optimal here we mean to find the times of the training sessions such that the accumulated training effect (preparedness) under the Banister model is maximised. Then the athlete is “most trained”. We suppose that the training parameters are known. Consider n identical sessions each with a standard unit of TRIMP conducted in advance of t. Suppose that the first session is carried out at time s, and the other sessions are carried out at times $s+d_2, s+d_3, \dots, s+d_n$. Thus we set $d_1 = 0$. The accumulated training effect (ATE) at time t is then given by

$$ATE = W_0 + \sum_{j=1}^{n} e^{-(t-s-d_j)/\tau_a} - k_f e^{-(t-s-d_j)/\tau_f}.$$

Now we find the values of s, and d_2, …, d_n such that ATE is maximised. Differentiating with respect to s, and d_2, …, d_n in turn we have

$$\frac{\partial ATE}{\partial s} = \frac{1}{\tau_a}\sum_{j=1}^{n} e^{-(t-s-d_j)/\tau_a} - \frac{k_f}{\tau_f}\sum_{j=1}^{n} e^{-(t-s-d_j)/\tau_{f,}}$$

and

$$\frac{\partial ATE}{\partial d_j} = \frac{1}{\tau_a} e^{-(t-s-d_j)/\tau_a} - \frac{k_f}{\tau_f} e^{-(t-s-d_j)/\tau_f}.$$

Setting $\frac{\partial ATE}{\partial s} = 0$ and $\frac{\partial ATE_t}{\partial d_j} = 0$, we obtain the equations

$$\frac{1}{\tau_a}\sum_{j=1}^{n} e^{-(t-s-d_j)/\tau_a} - \frac{k_f}{\tau_f}\sum_{j=1}^{n} e^{-(t-s-d_j)/\tau_f} = 0, \tag{8.1}$$

and

$$\frac{1}{\tau_a} e^{-(t-s-d_j)/\tau_a} - \frac{k_f}{\tau_f} e^{-(t-s-d_j)/\tau_f} = 0. \tag{8.2}$$

Substituting equation (8.2) into equation (8.1), we then obtain

$$\frac{1}{\tau_a} e^{-(t-s)/\tau_a} - \frac{k_f}{\tau_f} e^{-(t-s)/\tau_f} = 0.$$

Therefore

$$\frac{1}{\tau_a} e^{-(t-s)/\tau_a} = \frac{k_f}{\tau_f} e^{-(t-s)/\tau_f},$$

so that

$$(t-s)\left(\frac{1}{\tau_f} - \frac{1}{\tau_a}\right) = log\left(\frac{\tau_a k_f}{\tau_f}\right).$$

Thus

$$t - s^* = log\left(\frac{\tau_a\, k_f}{\tau_f}\right) \Big/ \left(\frac{1}{\tau_f} - \frac{1}{\tau_a}\right). \tag{8.3}$$

We now re-express equation (8.3) as

$$s^* = t - c^*,$$

where

$$c^* = log\left(\frac{\tau_a\, k_f}{\tau_f}\right) \Big/ \left(\frac{1}{\tau_f} - \frac{1}{\tau_a}\right).$$

This c^* is how far in advance of t to carry out the first training session in order to maximise the ATE at time t. Notice the requirement that $\tau_a > \tau_f$ for $s*$ and hence for c^* to exist. Further, from (8.2), we get that

$$t - s^* - d_i^* = log\left(\frac{\tau_a\, k_f}{\tau_f}\right) \Big/ \left(\frac{1}{\tau_f} - \frac{1}{\tau_a}\right) = t - s^*, \quad i = 2,3, \dots \dots n. \qquad (8.4)$$

Therefore $d_i^* = 0$ for $i = 2,3, \dots \dots n$. Thus it follows that, under the Banister model, to maximise the ATE at time t, it is optimal to do all n sessions concurrently. From a practical training point of view this is clearly neither possible nor sensible. This property of the Banister model is a consequence of its linearity; training loads add, albeit after some period of decay, to produce the accumulated training effect (ATE). This additive property is therefore a shortcoming of the Banister model, and the application of the Banister model for planning training is inappropriate. Thus:

- The Banister model is inadequate for planning training.

This inadequacy has been acknowledged in the sports science literature. However, as far as we are aware, we are the first to demonstrate this result mathematically. To compensate for this inadequacy, some authors introduce the notion of a work capacity, W'. W' is defined as the maximum amount of work that can be done above the critical power threshold, CP (Bergstrom et al., 2014; Hill, 1993; Noordhof et al., 2013; Skiba et al., 2012). These studies argue that the relationship between power output and duration in cycling depends on two quantities. These are W' and CP, both of which vary as a function of age, health, and training. Furthermore, it is suggested that these quantities should be the focus of modelling for optimising athletic training programs and performance. Work in this area is developing (see for example (Poole et al., 2016)), and the estimation of the parameters of the work capacity and critical power models using field data is an important issue for future research. We will return briefly to this point below.

8.3 Limitations of our Study

The findings in this thesis are subject to some limitations: Firstly, the relationship between heart rate and power output relies on the heart rate lag. According to Stirling et al. (2008), heart rate lags behind the changes in power output by approximately 30s or less. In this context, we have studied the range of heart rate lags and suggest that 15s is a reasonable time lag between changes in power output and heart rate response for competitive cycling training sessions. However, parameter estimates are sensitive to this choice and so care is needed with its specification.

Secondly, TRIMP parameters that are used to calculate the training load of every session must be carefully selected and determined, as discussed in section 3.3.1. We use values published in the literature reported by Borresen and Lambert (2009). However, other values may be appropriate, particularly given that we conclude above that Banister model parameters themselves should be individually estimated. There are also other ways to formulate TRIMP, and we might expect Banister model parameter estimates to be sensitive to this. Some alternative formulations are discussed in section 3.3.1.

While we use TRIMP, and this may be specified in different ways, other training metrics might be used such as training stress score (TSS), which has been used to quantify the training load in running (McGregor et al., 2009) and in cycling (MacLeod & Sunderland, 2009), as discussed in section 3.3.2. Also, it is over simplistic to consider scalar measures of training load and performance when a vector training load and performance metrics might be specified. In this way, an analysis might accommodate multiple aspects of training and performance simultaneously, in a multivariate analysis, for example corresponding to volume, intensity and frequency (Stewart & Hopkins, 2000; Avalos et al., 2003; Nimmerichter et al., 2011). Related to this is the problem of determining the appropriate threshold or reference power output. This requires investigation. Modelling developments may help athletes and coaches to choose these parameters appropriately. Also, multivariate models of training and performance may assist in this respect. Also, the type of sport may play a role in selecting the appropriate threshold or reference of power output.

There are also some data-related limitations such as the registration of some values of heart rate as zero. Moreover, the data also lack particular information about the training programme that explains and describes the applicable performances by each rider. Thus, the data provide limited information about the type of sessions. Additionally, there is the fact that the data themselves are limited. These data were recorded more than 10 years ago. Another point must be mentioned here is that some of the riders are non-elite athletes and they have modest power outputs. The performance literature in sports science tends to focus on elite performance, so the analysis of performance non-elite athletes may be less interesting. Nonetheless, we claim that our methodology is appropriate regardless of the performance level of the individual athletes. Finally, collection of new data for further study would be very worthwhile. We would recommend that complete data over an extended period be collected in cooperation with a small number of athletes. Heart-rate and power output data should form the basis of these new data, but other experiential and contextual data should also be collected, so that a complete picture of the athletes' inputs and outputs can be established.

8.4 Future Work

It may be possible to improve the work in this thesis by focusing on its limitations and provide some solutions to the problems that these limitations raise.

For the question of the time lag of the heart rate response to the power output, a study might consider optimising the lag, choosing that lag which minimises the variance of the estimated parameters in the performance-training model. It is likely, however, that without a very large volume of data, such an analysis will be inconclusive. Thus:

- Performance measurement requires further development.

A similar approach could be considered for estimation of parameters in the TRIMP measure, and for choosing between different measures of training load that are themselves alternatives to TRIMP, such as the training stress score or Edwards' TRIMP (Edwards, 1994) and Lucia's TRIMP model (Lucia et al., 1999). Thus:

- The measurement of training load requires further development.

In this work, we focus on five different performance measures. Other choices are possible and could be the subject of future work. One suggestion is to model critical power using a different model to those described in Bull et al. (2000) and to compare results with the results presented in this thesis. However, as the Banister model has a fundamental weakness, as we establish in section 8.2 above, researchers should in our view modify the Banister model to avoid this limitation, by perhaps focusing on models for work capacity and critical power. Thus:

- Future developments should focus on a nonlinear model of training and performance.

The Banister model parameters have been estimated for other sports, and we discuss these findings. However, there, standard errors of parameter estimates are not provided, and we suspect that these would be very large. Cycling lends itself to the analysis we develop because power output is directly measurable. For other sports, this is more difficult. However, in principle, our methodology can be applied to any sport in which heart-rate and power output can be either measured or calculated routinely. Thus:

- Parameter estimates in training-performance models are generally not well estimated, and athletes and coaches should be sceptical about the precision of estimates.

Finally, given the developments in data recording and the spread of the routine use of power meters and heart rate monitors, it would be interesting to conduct a similar study, or a new study, of work capacity and critical power, on a large, detailed dataset that considers a small number of elite athletes who have data recorded for a long period of time. This would require very close cooperation with the athletes. Thus:

- A new study should collect new data that provides complete information about training and performance and context for a small number of athletes over an extended period of time.

Appendix 1: Correlation of power output and heart rate at different lags

Table A1.1 The correlation coefficient between power output and heart rate at different lags (5,15, 25 seconds) for each session for rider 3

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
1	0.59	0.70	0.70	55	0.55	0.63	0.64
2	0.57	0.68	0.69	56	0.67	0.72	0.70
3	0.54	0.60	0.57	57	0.58	0.63	0.58
4	0.56	0.64	0.62	58	0.28	0.36	0.36
5	0.49	0.57	0.53	59	0.63	0.71	0.71
6	0.44	0.49	0.52	60	0.43	0.49	0.45
7	0.66	0.72	0.73	61	0.68	0.72	0.65
8	0.49	0.54	0.55	62	0.65	0.69	0.68
9	0.41	0.48	0.49	63	0.48	0.56	0.56
10	0.70	0.74	0.71	64	0.41	0.40	0.35
11	0.84	0.87	0.83	65	0.45	0.49	0.44
12	0.70	0.76	0.71	66	0.66	0.72	0.68
13	0.51	0.53	0.50	67	0.51	0.56	0.53
14	0.31	0.39	0.37	68	0.55	0.62	0.62
15	0.35	0.42	0.37	69	0.62	0.69	0.68
16	0.63	0.70	0.70	70	0.59	0.65	0.63
17	0.55	0.60	0.59	71	0.68	0.70	0.69
18	0.50	0.55	0.57	72	0.63	0.67	0.64
19	0.55	0.66	0.67	73	0.61	0.70	0.68
20	0.65	0.69	0.66	74	0.55	0.56	0.52
21	0.56	0.62	0.61	75	0.34	0.41	0.45
22	0.64	0.69	0.69	76	0.54	0.59	0.55
23	0.59	0.64	0.61	77	0.22	0.19	0.19
24	0.60	0.66	0.63	78	0.11	0.11	0.12
25	0.62	0.68	0.69	79	0.51	0.47	0.46
26	0.44	0.50	0.47	80	0.44	0.54	0.57
27	0.20	0.26	0.25	81	0.25	0.37	0.38
28	0.64	0.71	0.74	82	0.63	0.70	0.71
29	0.24	0.24	0.20	83	0.50	0.61	0.62
30	0.12	0.14	0.14	84	0.54	0.61	0.64
31	0.58	0.62	0.60	85	0.48	0.55	0.53
32	0.65	0.69	0.68	86	0.64	0.68	0.69
33	0.66	0.68	0.67	87	0.50	0.58	0.58
34	0.79	0.81	0.81	88	0.54	0.57	0.57
35	0.75	0.76	0.73	89	0.56	0.62	0.62
36	0.85	0.85	0.85	90	0.44	0.44	0.41
37	0.84	0.85	0.84	91	0.46	0.49	0.53
38	0.88	0.87	0.86	92	-0.05	-0.02	-0.07
39	0.61	0.70	0.70	93	0.60	0.70	0.71
40	0.54	0.59	0.55	94	0.55	0.65	0.65
41	0.64	0.68	0.67	95	0.70	0.74	0.72
42	0.73	0.74	0.69	96	0.65	0.67	0.67
43	0.64	0.73	0.70	97	0.62	0.67	0.66
44	0.53	0.59	0.57	98	0.63	0.71	0.67
45	0.56	0.64	0.64	99	0.58	0.66	0.64
46	0.58	0.67	0.68	100	0.57	0.65	0.64
47	0.60	0.66	0.62	101	0.57	0.67	0.67
48	0.65	0.73	0.69	102	0.61	0.64	0.64
49	0.41	0.46	0.48	103	0.60	0.66	0.67
50	0.60	0.72	0.70	104	0.56	0.64	0.60
51	0.65	0.67	0.68	105	0.67	0.73	0.70
52	0.56	0.57	0.54	106	0.65	0.71	0.66
53	0.32	0.37	0.33	107	0.44	0.53	0.52
54	0.34	0.40	0.37	108	0.56	0.59	0.58

Table A1.2 The correlation coefficient between power output and heart rate with different lags (5, 15, 25 seconds) for each session for rider 4

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
1	0.75	0.73	0.67	57	0.48	0.47	0.45
2	0.58	0.65	0.66	58	0.24	0.26	0.23
3	0.54	0.60	0.54	59	0.56	0.61	0.59
4	0.29	0.25	0.22	60	0.24	0.23	0.19
5	0.45	0.44	0.38	61	0.32	0.37	0.30
6	0.55	0.60	0.56	62	0.24	0.19	0.12
7	0.28	0.31	0.34	63	0.31	0.35	0.34
8	0.63	0.64	0.63	64	0.41	0.43	0.40
9	0.58	0.64	0.65	65	0.55	0.64	0.61
10	0.28	0.22	0.18	66	0.54	0.57	0.50
11	0.59	0.61	0.57	67	0.32	0.40	0.37
12	0.81	0.81	0.81	68	0.61	0.62	0.56
13	0.76	0.76	0.66	69	0.62	0.58	0.52
14	0.35	0.35	0.29	70	0.27	0.23	0.21
15	0.44	0.48	0.41	71	0.45	0.50	0.43
16	0.54	0.56	0.52	72	0.50	0.43	0.40
17	0.61	0.64	0.57	73	0.35	0.36	0.32
18	0.45	0.48	0.42	74	0.28	0.28	0.27
19	0.55	0.56	0.50	75	0.29	0.35	0.35
20	0.56	0.55	0.48	76	0.51	0.50	0.44
21	0.49	0.53	0.47	77	0.62	0.51	0.37
22	0.16	0.13	0.13	78	0.23	0.12	-0.03
23	0.34	0.33	0.29	79	0.34	0.37	0.32
24	0.62	0.61	0.52	80	0.69	0.56	0.47
25	0.64	0.65	0.60	81	0.57	0.57	0.48
26	0.66	0.67	0.61	82	0.48	0.55	0.56
27	0.67	0.68	0.63	83	0.69	0.76	0.76
28	0.57	0.57	0.55	84	0.51	0.55	0.52
29	0.73	0.73	0.70	85	0.48	0.53	0.49
30	0.51	0.50	0.48	86	-0.07	-0.10	-0.11
31	0.63	0.57	0.56	87	0.49	0.51	0.43
32	0.50	0.56	0.51	88	0.49	0.53	0.47
33	0.63	0.66	0.58	89	0.67	0.78	0.82
34	0.45	0.50	0.46	90	0.36	0.35	0.25
35	0.36	0.37	0.33	91	0.32	0.32	0.31
36	0.59	0.58	0.53	92	0.83	0.84	0.84
37	0.18	0.07	0.05	93	0.03	-0.01	0.02
38	0.44	0.49	0.45	94	0.39	0.41	0.38
39	0.57	0.56	0.48	95	0.59	0.59	0.54
40	0.29	0.31	0.34	96	0.41	0.44	0.43
41	0.59	0.55	0.46	97	0.28	0.32	0.35
42	0.36	0.39	0.34	98	0.40	0.40	0.35
43	0.50	0.50	0.46	99	0.11	0.09	0.08
44	0.66	0.64	0.56	100	0.43	0.46	0.35
45	0.51	0.54	0.53	101	0.68	0.69	0.66
46	0.29	0.28	0.28	102	0.30	0.36	0.34
47	0.41	0.39	0.38	103	0.11	0.10	0.09
48	0.60	0.68	0.72	104	0.43	0.44	0.37
49	0.50	0.53	0.51	105	0.56	0.54	0.48
50	0.33	0.29	0.27	106	0.45	0.44	0.42
51	0.77	0.78	0.74	107	0.46	0.44	0.39
52	0.39	0.41	0.39	108	0.32	0.32	0.33
53	0.50	0.49	0.47	109	0.67	0.72	0.69
54	0.63	0.64	0.63	110	0.37	0.30	0.28
55	0.69	0.73	0.69	111	0.62	0.63	0.60
56	0.45	0.47	0.46	112	0.62	0.50	0.49

Table A1.3 The correlation coefficient between power output and heart rate at different lags (5,15, 25 seconds) for each session for rider 5

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
1	0.09	0.10	0.08	52	0.66	0.66	0.65
2	0.53	0.61	0.57	53	0.13	0.09	0.10
3	0.29	0.36	0.31	54	0.27	0.27	0.25
4	0.38	0.45	0.43	55	0.37	0.37	0.34
5	0.60	0.59	0.54	56	0.60	0.58	0.59
6	0.31	0.33	0.32	57	0.40	0.39	0.37
7	0.36	0.42	0.34	58	0.38	0.38	0.37
8	0.42	0.45	0.45	59	0.22	0.21	0.19
9	0.15	0.20	0.19	60	0.58	0.58	0.54
10	0.23	0.27	0.25	61	0.42	0.43	0.42
11	0.23	0.23	0.21	62	-0.09	-0.10	-0.11
12	0.38	0.41	0.41	63	-0.14	-0.10	-0.10
13	0.17	0.12	0.10	64	0.18	0.20	0.11
14	0.20	0.27	0.27	65	0.26	0.30	0.24
15	0.28	0.36	0.38	66	0.18	0.17	0.15
16	0.21	0.16	0.09	67	0.14	0.15	0.12
17	0.08	0.17	0.21	68	0.28	0.27	0.28
18	0.30	0.34	0.32	69	0.11	0.10	0.10
19	0.21	0.27	0.23	70	0.18	0.20	0.21
20	0.10	0.11	0.12	71	0.08	0.10	0.07
21	0.21	0.22	0.20	72	0.87	0.88	0.88
22	0.32	0.38	0.35	73	0.03	0.03	0.02
23	0.11	0.17	0.17	74	0.24	0.27	0.27
24	0.34	0.30	0.28	75	0.12	0.03	0.03
25	0.35	0.39	0.38	76	0.23	0.20	0.18
26	0.20	0.25	0.24	77	0.15	0.17	0.17
27	0.41	0.37	0.29	78	0.33	0.27	0.25
28	0.19	0.22	0.24	79	0.70	0.56	0.47
29	0.33	0.36	0.37	80	0.31	0.21	0.17
30	0.19	0.25	0.27	81	0.62	0.62	0.52
31	0.34	0.37	0.37	82	0.63	0.63	0.52
32	0.42	0.46	0.39	83	0.49	0.51	0.31
33	0.19	0.25	0.26	84	0.52	0.51	0.41
34	0.12	0.07	0.07	85	0.52	0.60	0.56
35	0.30	0.30	0.28	86	0.57	0.62	0.60
36	0.31	0.33	0.33	87	0.60	0.58	0.50
37	0.45	0.47	0.47	88	0.72	0.70	0.63
38	0.19	0.17	0.13	89	0.43	0.47	0.45
39	0.30	0.34	0.32	90	0.56	0.64	0.60
40	0.27	0.31	0.30	91	0.47	0.50	0.46
41	0.31	0.36	0.34	92	0.42	0.42	0.41
42	0.12	0.11	0.09	93	0.44	0.47	0.37
43	0.05	0.10	0.04	94	0.67	0.68	0.66
44	0.27	0.25	0.24	95	0.22	0.23	0.15
45	0.37	0.34	0.24	96	0.56	0.57	0.49
46	0.33	0.35	0.36	97	0.65	0.68	0.63
47	0.28	0.33	0.31	98	0.43	0.52	0.46
48	0.26	0.27	0.25	99	0.56	0.61	0.58
49	0.27	0.31	0.26	100	0.67	0.64	0.53
50	0.46	0.49	0.46	101	0.66	0.66	0.64
51	0.27	0.28	0.27				

Table A1.4 The correlation coefficient between power output and heart rate at different lags (5,15, 25 seconds) for each session for rider 8

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
1	0.78	0.79	0.73	56	0.75	0.81	0.84
2	0.43	0.43	0.39	57	0.78	0.81	0.75
3	0.67	0.71	0.65	58	0.77	0.82	0.85
4	0.60	0.59	0.54	59	0.79	0.81	0.75
5	0.75	0.76	0.70	60	0.76	0.79	0.71
6	0.58	0.58	0.52	61	-0.01	-0.01	-0.06
7	0.72	0.76	0.71	62	0.74	0.78	0.73
8	0.77	0.78	0.70	63	0.71	0.76	0.80
9	0.80	0.82	0.74	64	0.68	0.69	0.65
10	0.71	0.71	0.64	65	0.64	0.68	0.66
11	0.76	0.77	0.70	66	0.78	0.80	0.73
12	0.74	0.76	0.68	67	0.76	0.77	0.70
13	0.70	0.72	0.67	68	0.68	0.71	0.69
14	0.74	0.75	0.68	69	0.77	0.79	0.72
15	0.70	0.74	0.71	70	0.71	0.73	0.69
16	0.70	0.73	0.68	71	0.56	0.58	0.55
17	0.66	0.69	0.65	72	0.73	0.76	0.69
18	0.75	0.76	0.69	73	0.68	0.70	0.64
19	0.70	0.74	0.70	74	0.73	0.74	0.68
20	0.77	0.78	0.72	75	0.86	0.89	0.88
21	0.78	0.81	0.82	76	0.87	0.89	0.88
22	0.79	0.81	0.73	77	0.55	0.56	0.52
23	0.74	0.76	0.72	78	0.71	0.73	0.71
24	0.68	0.70	0.66	79	0.36	0.37	0.35
25	0.62	0.64	0.60	80	0.52	0.56	0.52
26	0.74	0.75	0.74	81	0.13	0.14	0.13
27	0.64	0.66	0.60	82	0.74	0.76	0.71
28	0.77	0.79	0.72	83	0.79	0.85	0.89
29	0.69	0.70	0.65	84	0.07	0.08	0.09
30	0.78	0.79	0.73	85	0.48	0.51	0.47
31	0.77	0.78	0.72	86	0.36	0.38	0.39
32	0.74	0.75	0.68	87	0.32	0.34	0.32
33	0.72	0.74	0.69	88	0.76	0.78	0.69
34	0.74	0.74	0.67	89	0.72	0.75	0.68
35	0.73	0.75	0.71	90	0.75	0.79	0.73
36	0.78	0.81	0.73	91	0.79	0.82	0.79
37	0.76	0.78	0.72	92	0.73	0.80	0.85
38	0.73	0.74	0.66	93	0.77	0.78	0.69
39	0.69	0.72	0.64	94	0.70	0.78	0.83
40	0.79	0.81	0.73	95	0.80	0.82	0.80
41	0.74	0.77	0.72	96	0.72	0.76	0.71
42	0.74	0.76	0.69	97	0.81	0.82	0.74
43	0.78	0.80	0.73	98	0.68	0.68	0.66
44	0.75	0.79	0.82	99	0.70	0.73	0.71
45	0.80	0.83	0.77	100	0.82	0.82	0.74
46	0.87	0.88	0.88	101	0.47	0.53	0.55
47	0.84	0.86	0.83	102	0.63	0.72	0.78
48	0.88	0.90	0.89	103	0.77	0.78	0.72
49	0.72	0.73	0.73	104	0.36	0.38	0.32
50	0.86	0.87	0.86	105	0.69	0.74	0.71
51	0.90	0.91	0.90	106	0.34	0.41	0.40
52	0.87	0.88	0.87	107	0.59	0.62	0.53
53	0.86	0.88	0.90	108	0.72	0.73	0.67
54	0.81	0.84	0.80	109	0.67	0.70	0.65
55	0.45	0.49	0.45	110	0.81	0.81	0.72

Table A1.4 Continued

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
111	0.80	0.80	0.74	137	0.75	0.80	0.78
112	0.59	0.68	0.68	138	0.74	0.76	0.69
113	0.71	0.75	0.68	139	0.45	0.49	0.49
114	0.68	0.74	0.72	140	0.65	0.67	0.63
115	0.82	0.83	0.76	141	0.77	0.78	0.71
116	0.72	0.76	0.73	142	0.63	0.65	0.60
117	0.79	0.82	0.75	143	0.55	0.56	0.52
118	0.20	0.21	0.17	144	0.68	0.70	0.66
119	0.81	0.80	0.72	145	0.82	0.83	0.76
120	0.51	0.55	0.51	146	0.70	0.71	0.67
121	0.72	0.76	0.76	147	0.56	0.57	0.54
122	0.68	0.72	0.68	148	0.75	0.76	0.71
123	0.48	0.49	0.48	149	0.86	0.88	0.87
124	0.64	0.72	0.72	150	0.90	0.91	0.91
125	0.71	0.74	0.71	151	0.88	0.89	0.89
126	0.64	0.64	0.55	152	0.89	0.92	0.91
127	0.66	0.72	0.66	153	0.90	0.91	0.90
128	0.77	0.80	0.74	154	0.89	0.90	0.89
129	0.50	0.49	0.42	155	0.73	0.74	0.72
130	0.59	0.62	0.59	156	0.13	0.21	0.22
131	0.76	0.75	0.70	157	0.76	0.80	0.73
132	0.57	0.58	0.52	158	0.78	0.81	0.78
133	0.71	0.75	0.71	159	0.65	0.71	0.64
134	0.76	0.81	0.78	160	0.79	0.84	0.87
135	0.77	0.79	0.73	161	0.92	0.93	0.93
136	0.80	0.80	0.72	162	0.74	0.76	0.72

Table A1.5 The correlation coefficient between power output and heart rate at different lags (5,15, 25 seconds) for each session for rider 9

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
1	0.67	0.69	0.63	56	0.85	0.82	0.81
2	0.58	0.53	0.45	57	0.63	0.65	0.59
3	0.58	0.56	0.50	58	0.90	0.92	0.92
4	0.48	0.43	0.39	59	0.91	0.89	0.86
5	0.61	0.64	0.60	60	0.65	0.67	0.59
6	0.47	0.41	0.38	61	0.83	0.84	0.82
7	0.52	0.58	0.55	62	0.53	0.57	0.49
8	0.60	0.58	0.46	63	0.75	0.76	0.69
9	0.51	0.51	0.48	64	0.59	0.56	0.48
10	0.64	0.67	0.62	65	0.63	0.63	0.54
11	0.64	0.66	0.59	66	0.86	0.86	0.86
12	0.59	0.61	0.55	67	0.64	0.68	0.61
13	0.59	0.64	0.59	68	0.67	0.66	0.56
14	0.58	0.63	0.59	69	0.63	0.62	0.53
15	0.58	0.60	0.54	70	0.90	0.91	0.91
16	0.65	0.70	0.64	71	0.90	0.92	0.93
17	0.94	0.94	0.94	72	0.90	0.91	0.91
18	0.51	0.48	0.42	73	0.80	0.81	0.75
19	0.59	0.61	0.52	74	0.68	0.72	0.67
20	0.95	0.95	0.95	75	0.61	0.57	0.50
21	0.55	0.61	0.55	76	0.76	0.78	0.76
22	0.65	0.72	0.66	77	0.69	0.67	0.61
23	0.52	0.50	0.43	78	0.73	0.73	0.68
24	0.59	0.61	0.54	79	0.87	0.88	0.86
25	0.63	0.66	0.56	80	0.75	0.78	0.75
26	0.58	0.55	0.47	81	0.54	0.53	0.48
27	0.50	0.51	0.43	82	0.72	0.74	0.69
28	0.63	0.67	0.59	83	0.82	0.81	0.72
29	0.93	0.93	0.92	84	0.68	0.69	0.65
30	0.64	0.65	0.57	85	0.76	0.78	0.74
31	0.9	0.90	0.90	86	0.74	0.76	0.72
32	0.64	0.67	0.60	87	0.49	0.48	0.45
33	0.89	0.90	0.90	88	0.71	0.73	0.70
34	0.700	0.74	0.65	89	0.05	0.04	0.02
35	0.77	0.77	0.69	90	0.75	0.77	0.73
36	0.85	0.83	0.82	91	0.84	0.82	0.76
37	0.65	0.65	0.56	92	0.62	0.65	0.63
38	0.68	0.68	0.57	93	0.77	0.79	0.74
39	0.57	0.57	0.46	94	0.79	0.79	0.70
40	0.65	0.64	0.57	95	0.71	0.66	0.53
41	0.73	0.74	0.66	96	0.70	0.73	0.65
42	0.58	0.52	0.42	97	0.68	0.74	0.70
43	0.58	0.55	0.45	98	0.67	0.66	0.56
44	0.70	0.70	0.63	99	0.80	0.83	0.82
45	0.90	0.91	0.91	100	0.61	0.62	0.55
46	0.83	0.84	0.84	101	0.60	0.63	0.60
47	0.75	0.74	0.75	102	0.70	0.73	0.67
48	0.48	0.50	0.45	103	0.68	0.72	0.67
49	0.55	0.58	0.50	104	0.70	0.67	0.56
50	0.63	0.63	0.52	105	0.75	0.75	0.75
51	0.84	0.85	0.84	106	0.50	0.55	0.52
52	0.52	0.47	0.42	107	0.59	0.56	0.45
53	0.55	0.50	0.41	108	0.63	0.64	0.55
54	0.64	0.64	0.56	109	0.61	0.63	0.57
55	0.56	0.59	0.53	110	0.61	0.64	0.53

Table A1.5 Continued

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
111	0.77	0.72	0.62	155	0.24	0.22	0.17
112	0.44	0.44	0.42	156	0.62	0.66	0.62
113	0.54	0.56	0.51	157	0.78	0.83	0.85
114	0.65	0.65	0.61	158	0.63	0.67	0.64
115	0.88	0.89	0.88	159	0.69	0.69	0.59
116	0.64	0.68	0.62	160	0.63	0.64	0.62
117	0.70	0.70	0.68	161	0.78	0.81	0.82
118	0.78	0.78	0.77	162	0.66	0.68	0.63
119	0.55	0.58	0.55	163	0.72	0.72	0.63
120	0.61	0.66	0.58	164	0.68	0.72	0.69
121	0.66	0.70	0.62	165	0.73	0.76	0.74
122	0.61	0.64	0.56	166	0.76	0.79	0.75
123	0.68	0.70	0.64	167	0.77	0.76	0.67
124	0.71	0.72	0.71	168	0.72	0.67	0.57
125	0.70	0.70	0.68	169	0.56	0.57	0.56
126	0.48	0.52	0.47	170	0.79	0.79	0.69
127	0.72	0.71	0.61	171	0.77	0.79	0.69
128	0.67	0.71	0.66	172	0.78	0.75	0.62
129	0.59	0.61	0.54	173	0.48	0.48	0.45
130	0.63	0.62	0.52	174	0.15	0.16	0.14
131	0.61	0.65	0.61	175	0.12	0.14	0.14
132	0.50	0.53	0.47	176	0.69	0.72	0.63
133	0.04	0.02	0.02	177	0.59	0.62	0.57
134	0.61	0.66	0.64	178	0.62	0.65	0.59
135	0.17	0.17	0.13	179	0.66	0.69	0.66
136	0.73	0.72	0.71	180	0.69	0.70	0.61
137	0.46	0.48	0.46	181	0.68	0.70	0.70
138	0.39	0.39	0.38	182	0.69	0.73	0.66
139	0.26	0.26	0.16	183	0.66	0.66	0.60
140	0.54	0.59	0.54	184	0.64	0.67	0.63
141	0.35	0.35	0.26	185	0.33	0.35	0.33
142	0.78	0.78	0.72	186	0.56	0.60	0.57
143	0.63	0.65	0.61	187	0.78	0.77	0.67
144	0.68	0.69	0.65	188	0.82	0.86	0.87
145	0.12	0.13	0.16	189	0.68	0.72	0.63
146	0.19	0.26	0.26	190	0.77	0.78	0.74
147	0.69	0.74	0.67	191	0.66	0.73	0.65
148	0.14	0.14	0.09	192	0.56	0.58	0.53
149	0.69	0.76	0.71	193	0.79	0.76	0.75
150	0.50	0.52	0.51	194	0.78	0.84	0.85
151	0.60	0.59	0.50	195	0.83	0.85	0.86
152	0.66	0.66	0.62	196	0.58	0.62	0.56
153	0.42	0.44	0.42	197	0.57	0.64	0.60
154	0.69	0.65	0.55				

Table A1.6 The correlation coefficient between power output and heart rate at different lags (5,15, 25 seconds) for each session for rider 10

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
1	0.64	0.71	0.66	56	0.70	0.71	0.69
2	0.59	0.63	0.59	57	0.91	0.91	0.91
3	0.41	0.45	0.42	58	0.71	0.76	0.69
4	0.63	0.69	0.64	59	0.83	0.84	0.76
5	0.50	0.56	0.54	60	0.90	0.90	0.90
6	0.69	0.69	0.60	61	0.69	0.72	0.62
7	0.73	0.69	0.63	62	0.83	0.84	0.75
8	0.61	0.65	0.56	63	0.66	0.73	0.62
9	0.58	0.59	0.56	64	0.87	0.88	0.88
10	0.60	0.62	0.57	65	0.65	0.70	0.65
11	0.69	0.71	0.64	66	0.47	0.55	0.53
12	0.48	0.58	0.54	67	0.69	0.72	0.62
13	0.57	0.66	0.66	68	0.67	0.69	0.63
14	0.54	0.56	0.54	69	0.73	0.77	0.65
15	0.79	0.82	0.72	70	0.70	0.72	0.67
16	0.79	0.73	0.64	71	0.43	0.46	0.44
17	0.66	0.71	0.63	72	0.66	0.73	0.68
18	0.23	0.21	0.23	73	0.75	0.75	0.71
19	0.76	0.78	0.68	74	0.75	0.79	0.70
20	0.64	0.65	0.57	75	0.67	0.67	0.61
21	0.64	0.77	0.75	76	0.64	0.73	0.66
22	0.57	0.65	0.62	77	0.54	0.60	0.58
23	0.67	0.73	0.64	78	0.55	0.64	0.61
24	0.82	0.82	0.73	79	0.62	0.66	0.63
25	0.61	0.63	0.58	80	0.62	0.70	0.66
26	0.53	0.53	0.49	81	0.32	0.34	0.35
27	0.69	0.75	0.66	82	0.74	0.76	0.73
28	0.53	0.51	0.47	83	0.56	0.57	0.55
29	0.43	0.50	0.50	84	0.92	0.92	0.93
30	0.56	0.59	0.55	85	0.50	0.52	0.49
31	0.53	0.52	0.50	86	0.68	0.74	0.67
32	0.76	0.81	0.71	87	0.52	0.57	0.55
33	0.78	0.79	0.75	88	0.78	0.79	0.68
34	0.75	0.81	0.71	89	0.74	0.79	0.68
35	0.55	0.63	0.61	90	0.65	0.69	0.67
36	0.66	0.72	0.65	91	0.71	0.75	0.64
37	0.52	0.53	0.46	92	0.60	0.68	0.64
38	0.77	0.75	0.69	93	0.77	0.81	0.71
39	0.73	0.77	0.66	94	0.60	0.64	0.59
40	0.80	0.80	0.80	95	0.64	0.67	0.59
41	0.64	0.66	0.64	96	0.59	0.61	0.58
42	0.61	0.66	0.62	97	0.51	0.56	0.53
43	0.67	0.67	0.68	98	0.65	0.70	0.61
44	0.48	0.54	0.56	99	0.75	0.77	0.71
45	0.62	0.66	0.64	100	0.69	0.74	0.67
46	0.77	0.78	0.79	101	0.79	0.81	0.80
47	0.81	0.81	0.75	102	0.56	0.59	0.57
48	0.71	0.73	0.69	103	0.71	0.75	0.70
49	0.87	0.88	0.89	104	0.45	0.52	0.47
50	0.78	0.79	0.79	105	0.53	0.54	0.53
51	0.75	0.76	0.66	106	0.61	0.65	0.59
52	0.69	0.75	0.69	107	0.81	0.80	0.73
53	0.72	0.78	0.66	108	0.62	0.66	0.67
54	0.68	0.71	0.66	109	0.67	0.69	0.69
55	0.79	0.81	0.73	110	0.74	0.79	0.72

Table A1.6 Continued

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
111	0.72	0.74	0.66	166	0.59	0.63	0.61
112	0.70	0.71	0.70	167	0.64	0.62	0.57
113	0.48	0.50	0.49	168	0.57	0.61	0.50
114	0.63	0.66	0.66	169	0.73	0.74	0.71
115	0.58	0.65	0.62	170	0.64	0.68	0.62
116	0.60	0.63	0.61	171	0.65	0.74	0.70
117	0.63	0.71	0.68	172	0.70	0.70	0.68
118	0.71	0.72	0.72	173	0.54	0.59	0.57
119	0.63	0.63	0.59	174	0.70	0.71	0.65
120	0.72	0.76	0.76	175	0.66	0.67	0.67
121	0.79	0.81	0.68	176	0.69	0.69	0.64
122	0.79	0.80	0.71	177	0.63	0.66	0.61
123	0.74	0.76	0.75	178	0.55	0.59	0.55
124	0.52	0.58	0.56	179	0.59	0.63	0.60
125	0.54	0.58	0.55	180	0.62	0.65	0.63
126	0.50	0.55	0.52	181	0.56	0.59	0.57
127	0.56	0.58	0.56	182	0.55	0.62	0.61
128	0.74	0.75	0.71	183	-0.03	0.07	0.10
129	0.49	0.54	0.51	184	0.57	0.62	0.51
130	0.67	0.69	0.68	185	0.54	0.57	0.54
131	0.59	0.65	0.63	186	0.62	0.66	0.63
132	0.67	0.71	0.66	187	0.83	0.83	0.80
133	0.74	0.76	0.74	188	0.61	0.67	0.62
134	0.81	0.82	0.80	189	0.69	0.72	0.67
135	0.79	0.78	0.70	190	0.82	0.83	0.76
136	0.78	0.81	0.72	191	0.67	0.70	0.69
137	0.69	0.70	0.66	192	0.66	0.71	0.63
138	0.60	0.66	0.64	193	0.54	0.58	0.56
139	0.48	0.53	0.48	194	0.69	0.72	0.69
140	0.72	0.70	0.62	195	0.63	0.68	0.64
141	0.74	0.74	0.68	196	0.30	0.33	0.30
142	0.67	0.69	0.66	197	0.22	0.24	0.19
143	0.59	0.62	0.58	198	0.51	0.54	0.49
144	0.80	0.77	0.69	199	0.64	0.66	0.64
145	0.60	0.62	0.55	200	0.53	0.57	0.56
146	0.46	0.51	0.50	201	-0.01	-0.05	-0.06
147	0.68	0.67	0.59	202	0.71	0.73	0.67
148	0.62	0.64	0.62	203	0.48	0.51	0.45
149	0.67	0.68	0.66	204	0.16	0.12	0.10
150	0.62	0.68	0.67	205	0.58	0.60	0.59
151	0.69	0.68	0.62	206	0.61	0.66	0.60
152	0.70	0.74	0.65	207	0.73	0.74	0.67
153	0.62	0.66	0.63	208	0.69	0.69	0.65
154	0.75	0.77	0.75	209	0.68	0.71	0.64
155	0.65	0.67	0.60	210	0.61	0.59	0.54
156	0.59	0.64	0.60	211	0.63	0.63	0.57
157	0.60	0.63	0.62	212	0.63	0.66	0.64
158	0.66	0.67	0.65	213	0.61	0.61	0.56
159	0.73	0.77	0.77	214	0.70	0.71	0.64
160	0.69	0.69	0.59	215	0.75	0.78	0.72
161	0.61	0.62	0.59	216	0.72	0.75	0.62
162	0.58	0.59	0.51	217	0.72	0.73	0.71
163	0.48	0.49	0.32	218	0.69	0.68	0.61
164	0.51	0.55	0.51	219	0.62	0.64	0.59
165	0.63	0.70	0.63	220	0.73	0.73	0.65

Table A1.6 Continued

Session	Lag time			Session	Lag time		
	5 sec	15 sec	25 sec		5 sec	15 sec	25 sec
221	0.64	0.67	0.63	237	0.71	0.77	0.68
222	0.59	0.62	0.59	238	0.64	0.63	0.61
223	0.63	0.65	0.60	239	0.76	0.79	0.68
224	0.64	0.64	0.61	240	0.53	0.54	0.48
225	0.67	0.72	0.67	241	0.63	0.66	0.62
226	0.60	0.63	0.58	242	0.69	0.71	0.62
227	0.49	0.58	0.58	243	0.74	0.77	0.67
228	0.61	0.59	0.55	244	0.55	0.64	0.60
229	0.68	0.70	0.63	245	0.63	0.64	0.54
230	0.60	0.66	0.62	246	0.54	0.56	0.53
231	0.58	0.62	0.58	247	0.63	0.65	0.59
232	0.67	0.66	0.56	248	0.61	0.62	0.58
233	0.68	0.70	0.60	249	0.59	0.63	0.59
234	0.59	0.63	0.58	250	0.67	0.69	0.60
235	0.67	0.69	0.66	251	0.56	0.62	0.59
236	0.52	0.55	0.52				

Appendix 2: Power output versus heart rate for all training sessions

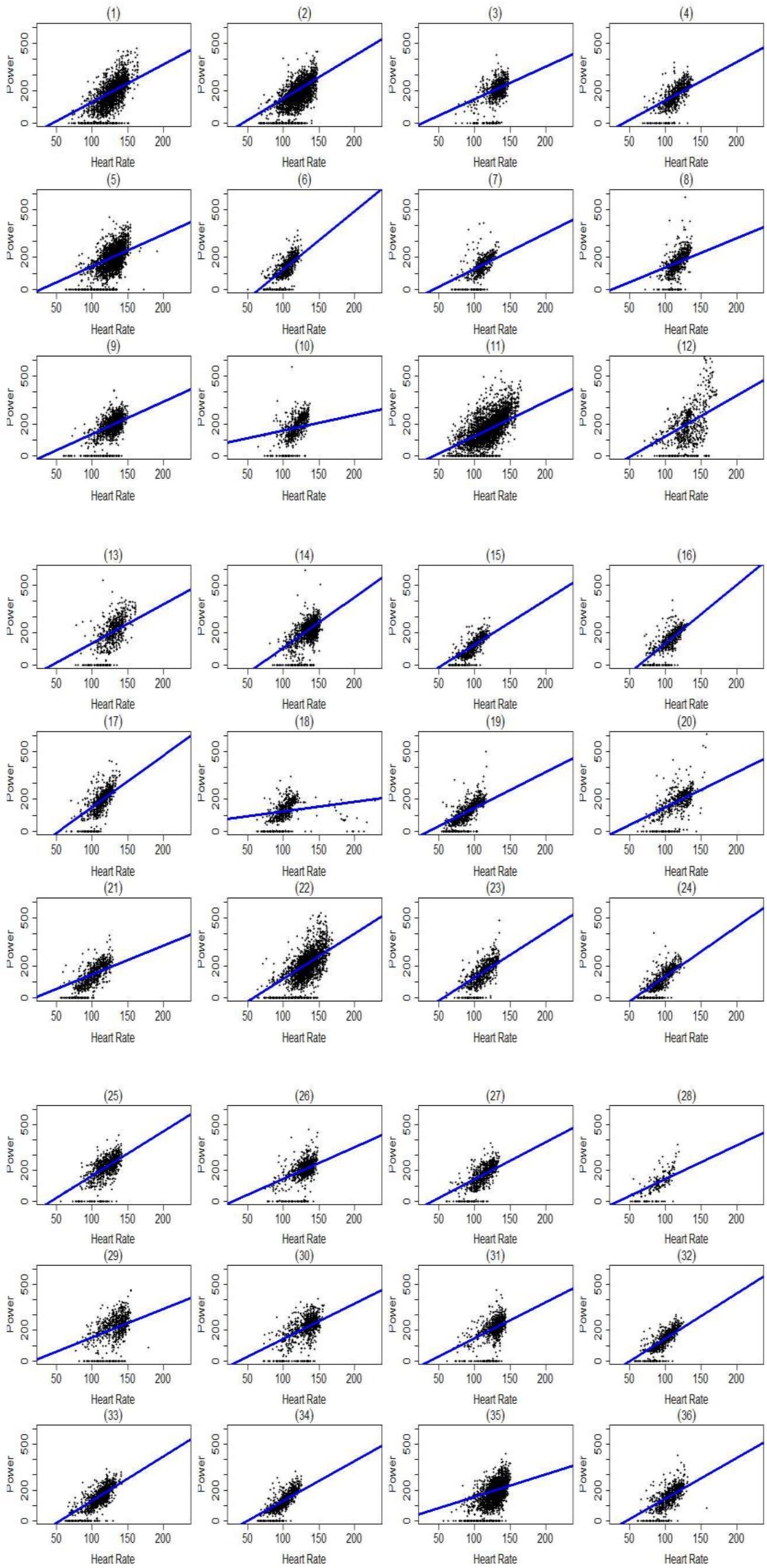

Figure A2.1 Power output and heart rate for rider 10 for all training sessions

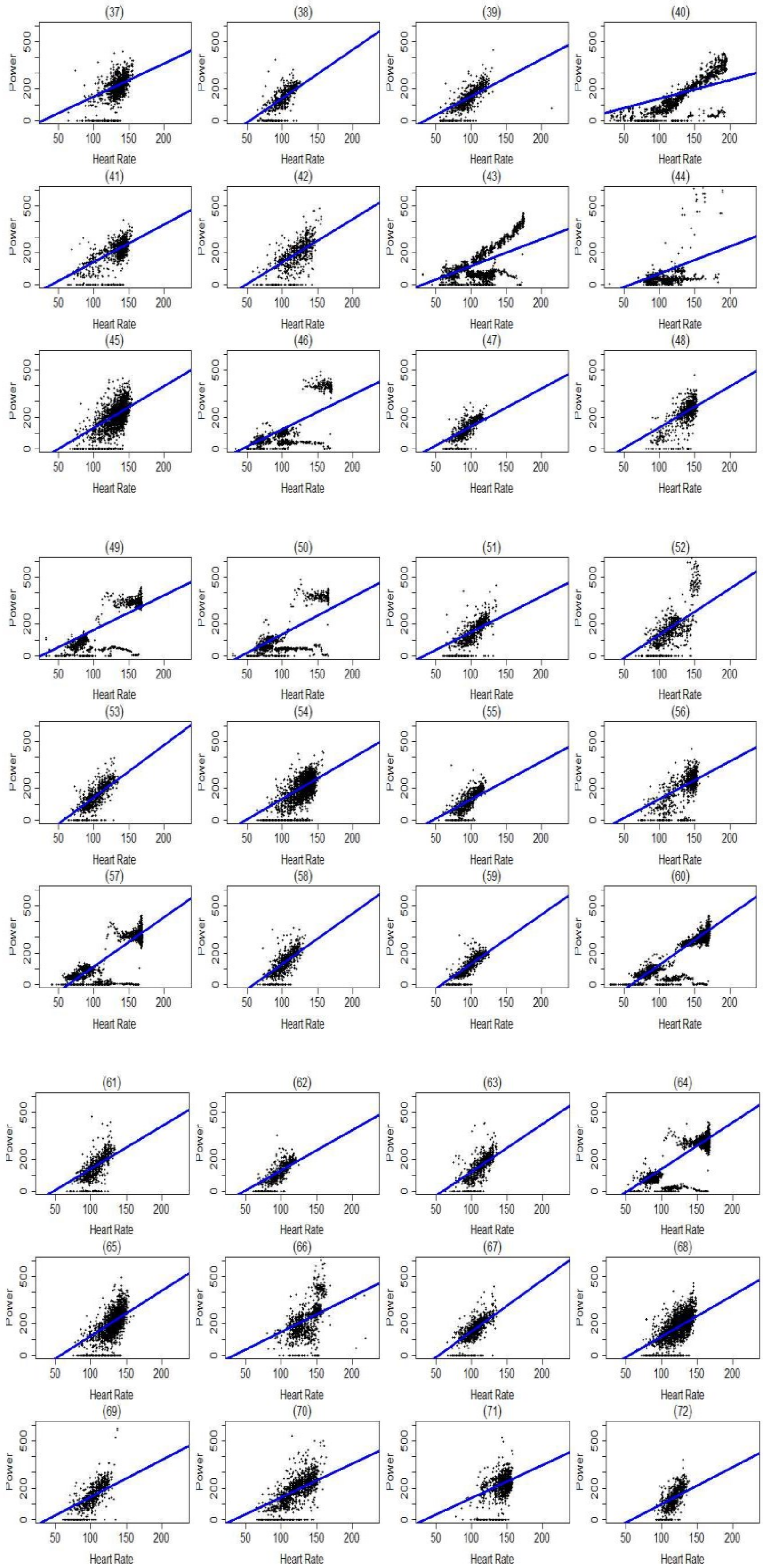

Figure A2.1 Continued

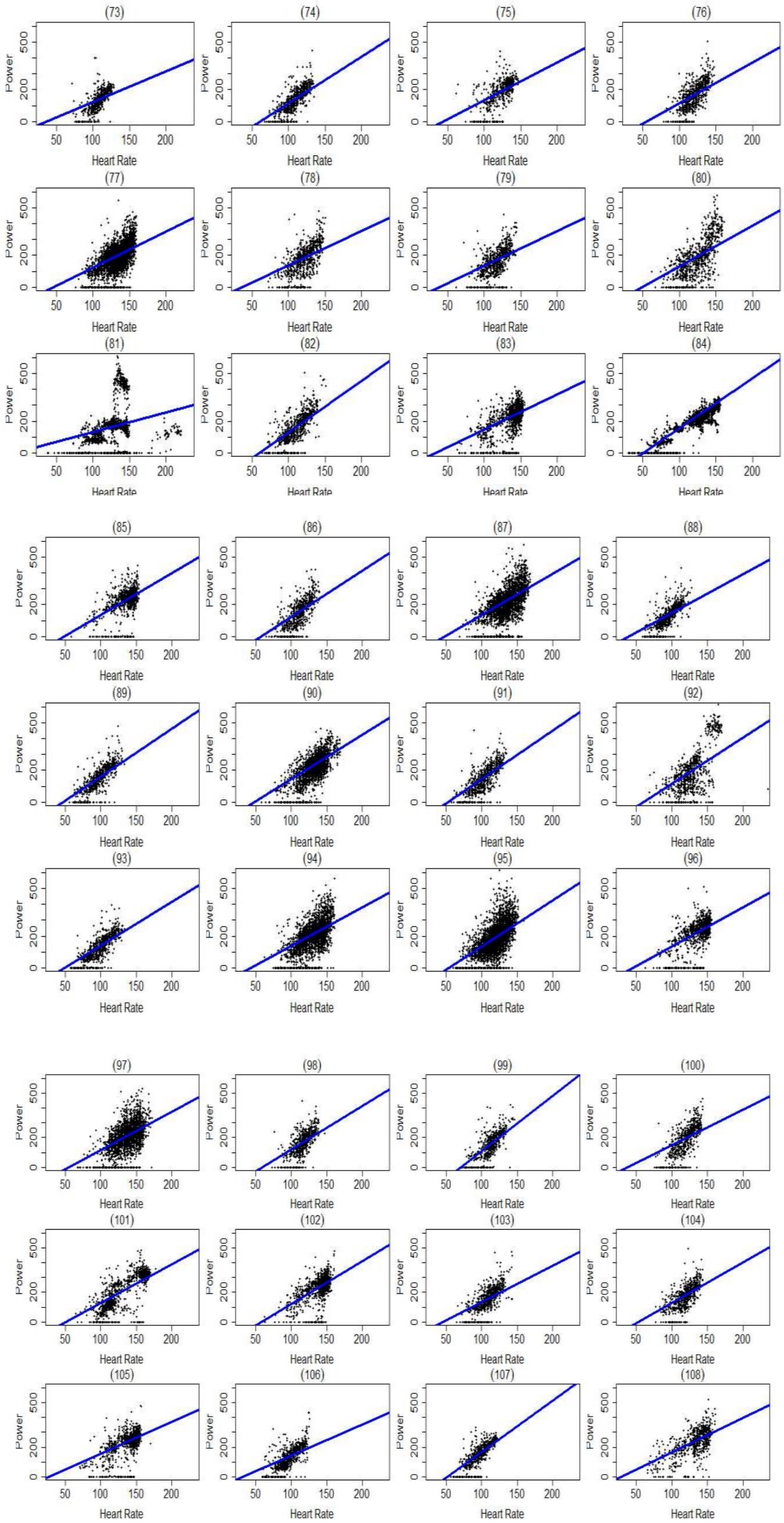

Figure A2.1 Continued

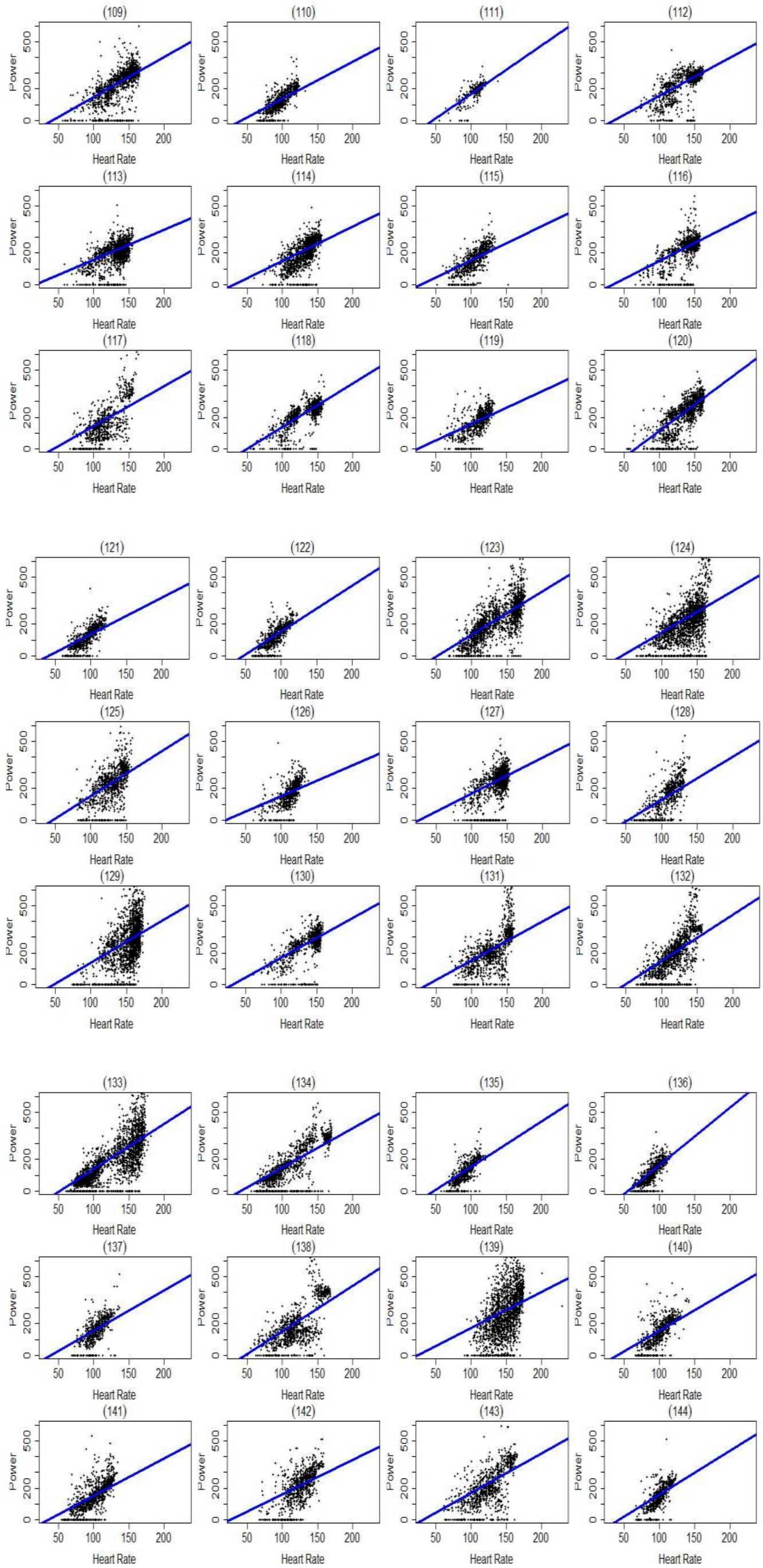

Figure A2.1 Continued

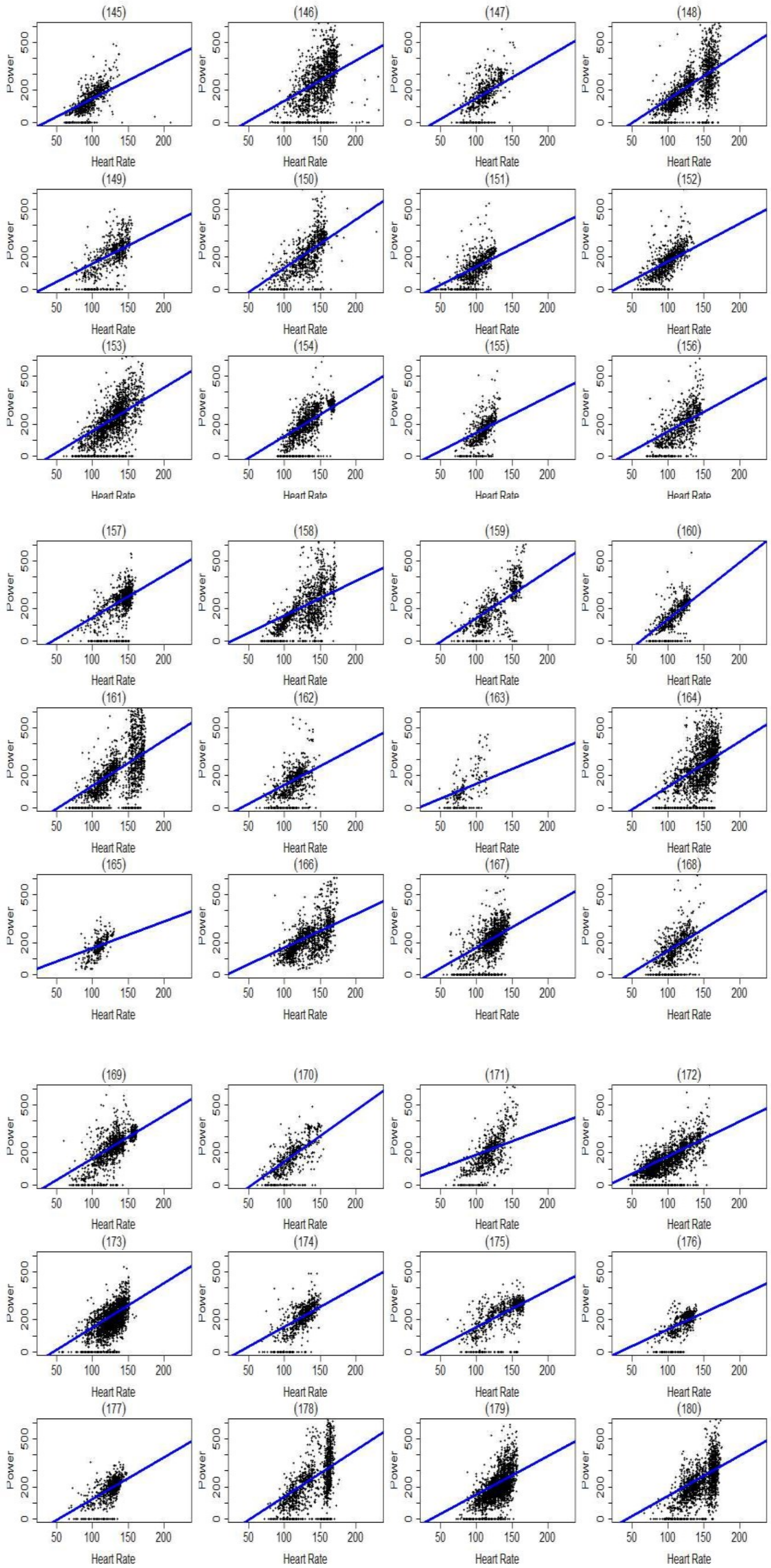

Figure A2.1 Continued

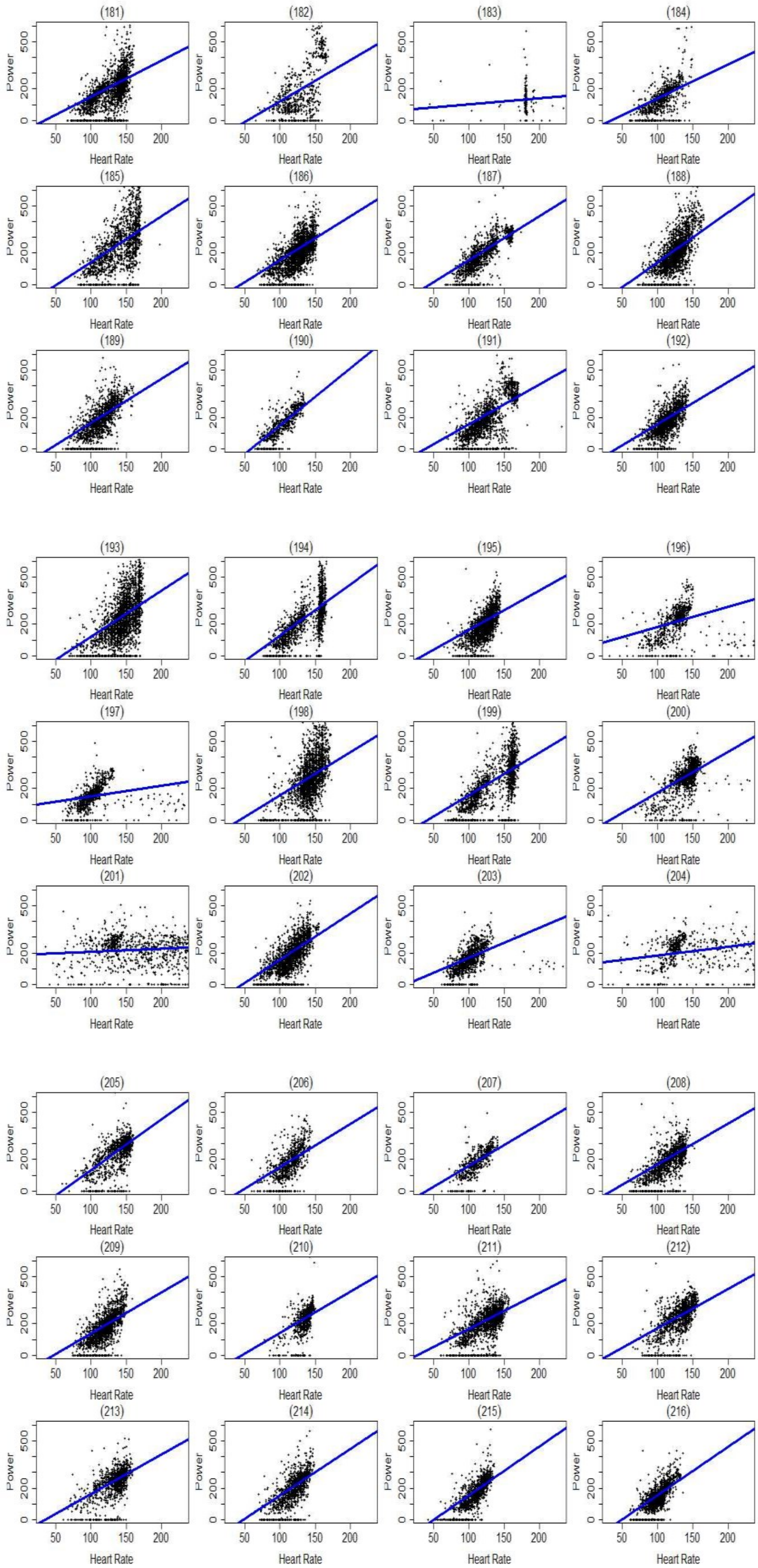

Figure A2.1 Continued

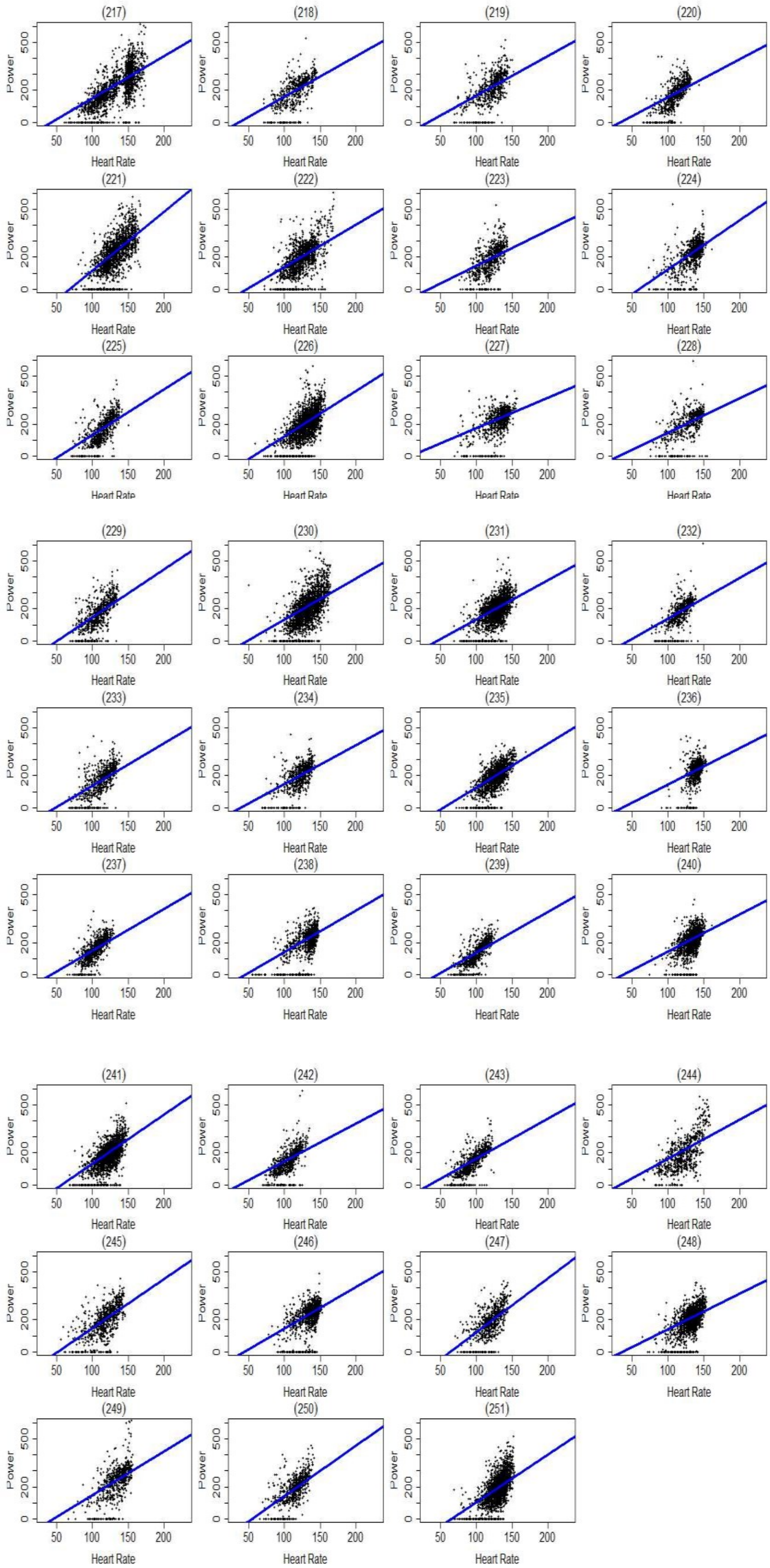

Figure A2.1 Continued

Appendix 3: Heart rate versus power output for all training sessions

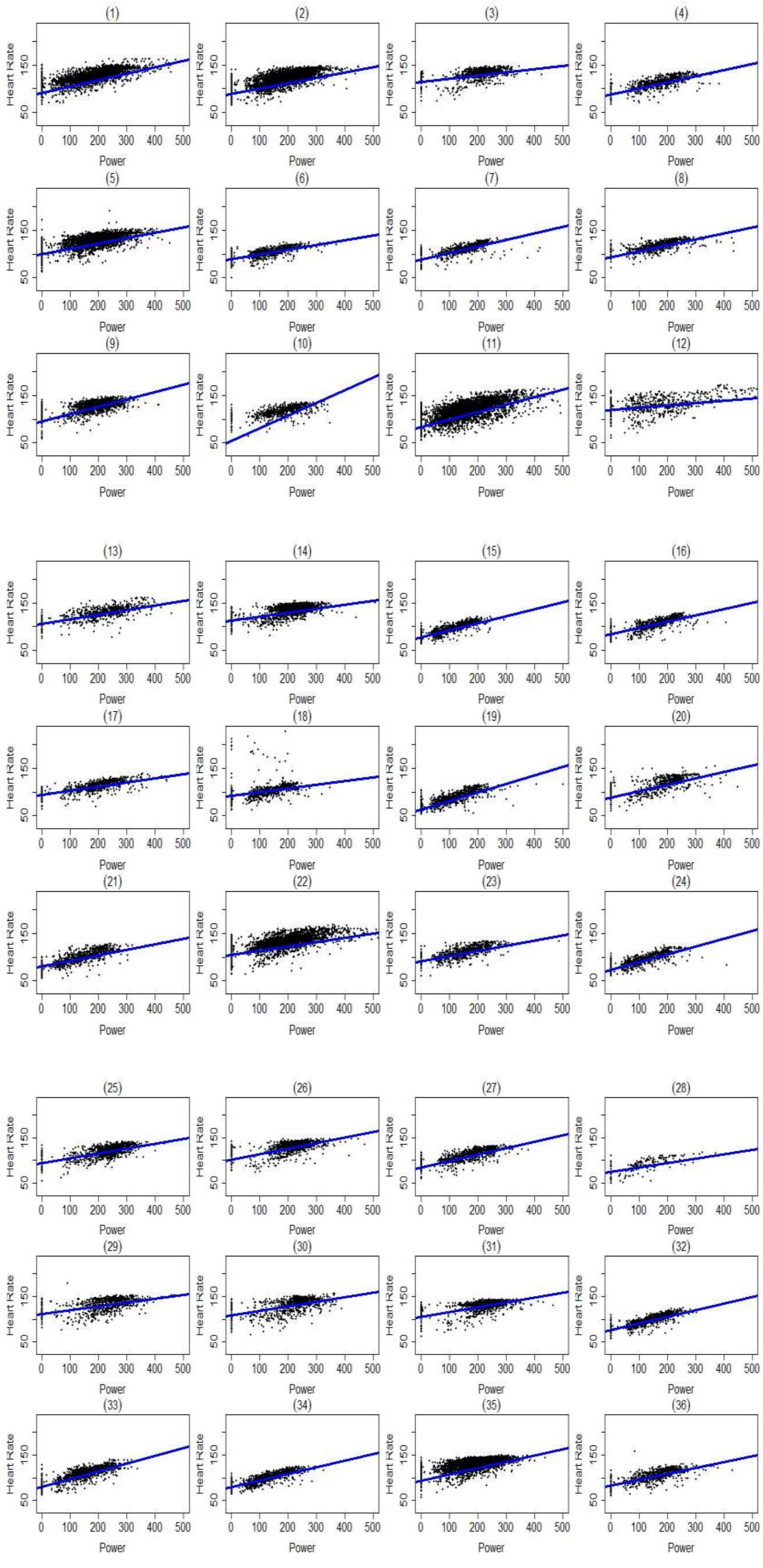

Figure A3.1 Heart rate and power output for rider 10 for all training sessions

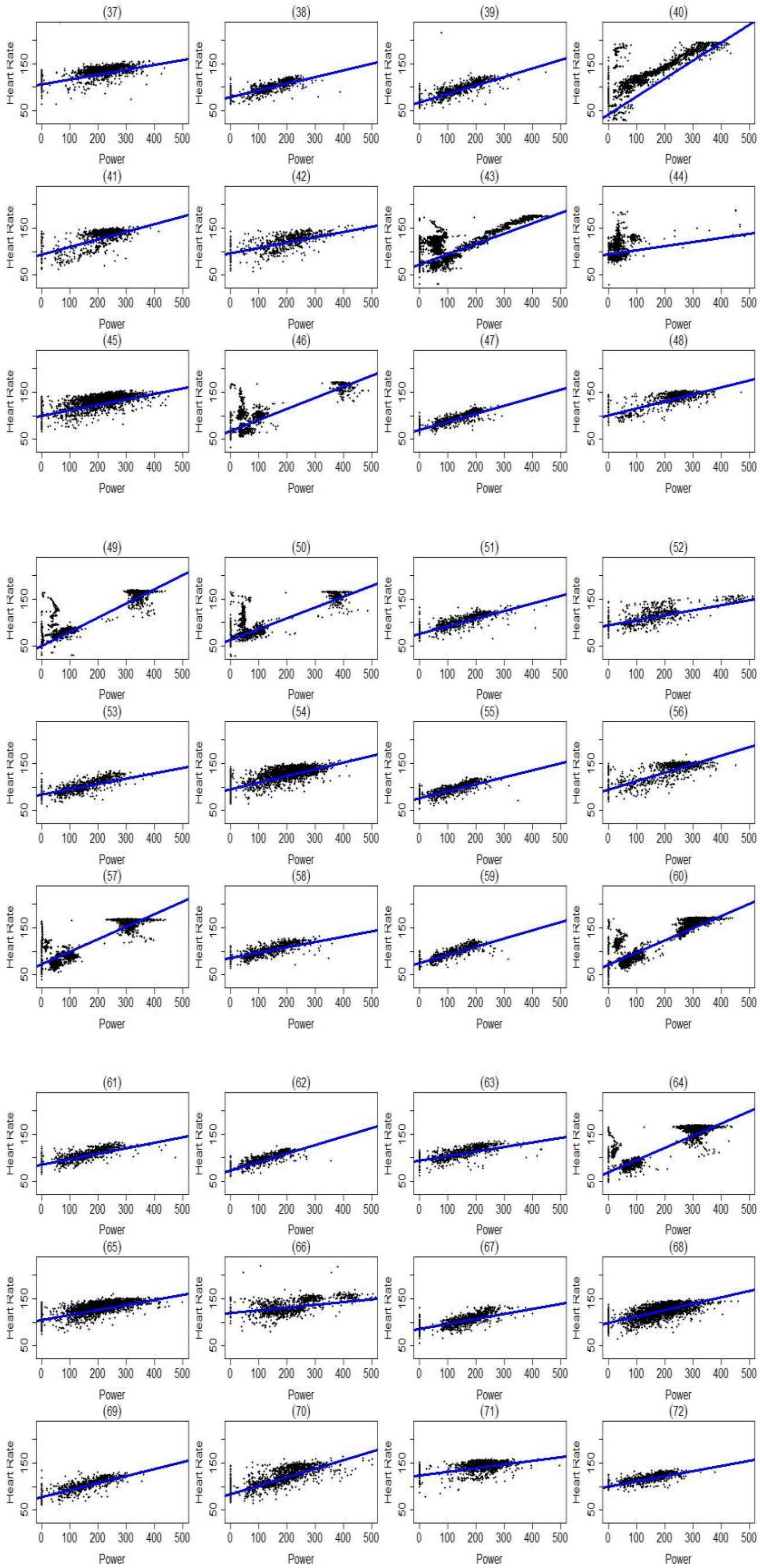

Figure A3.1 Continued

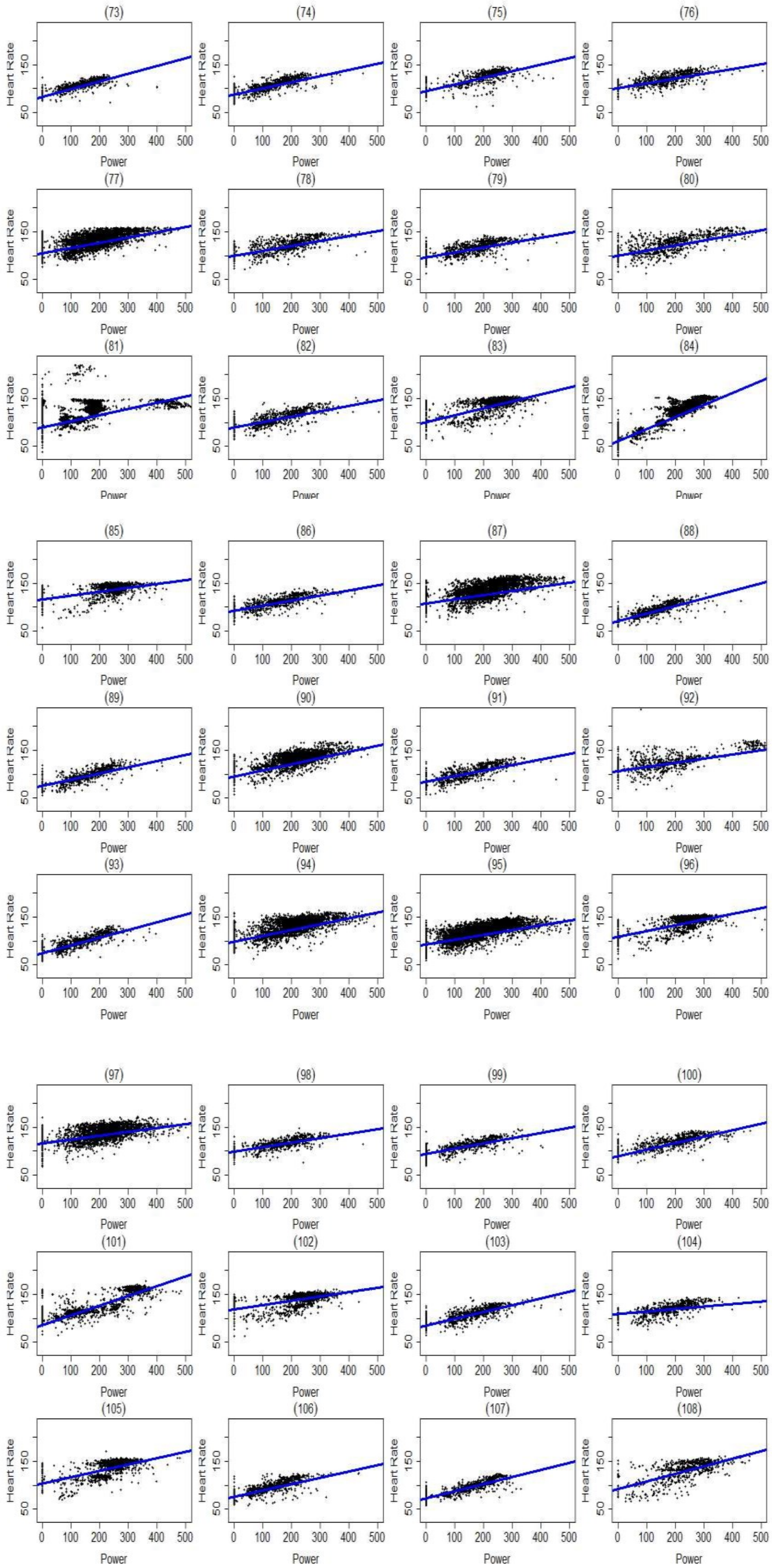

Figure A3.1 Continued

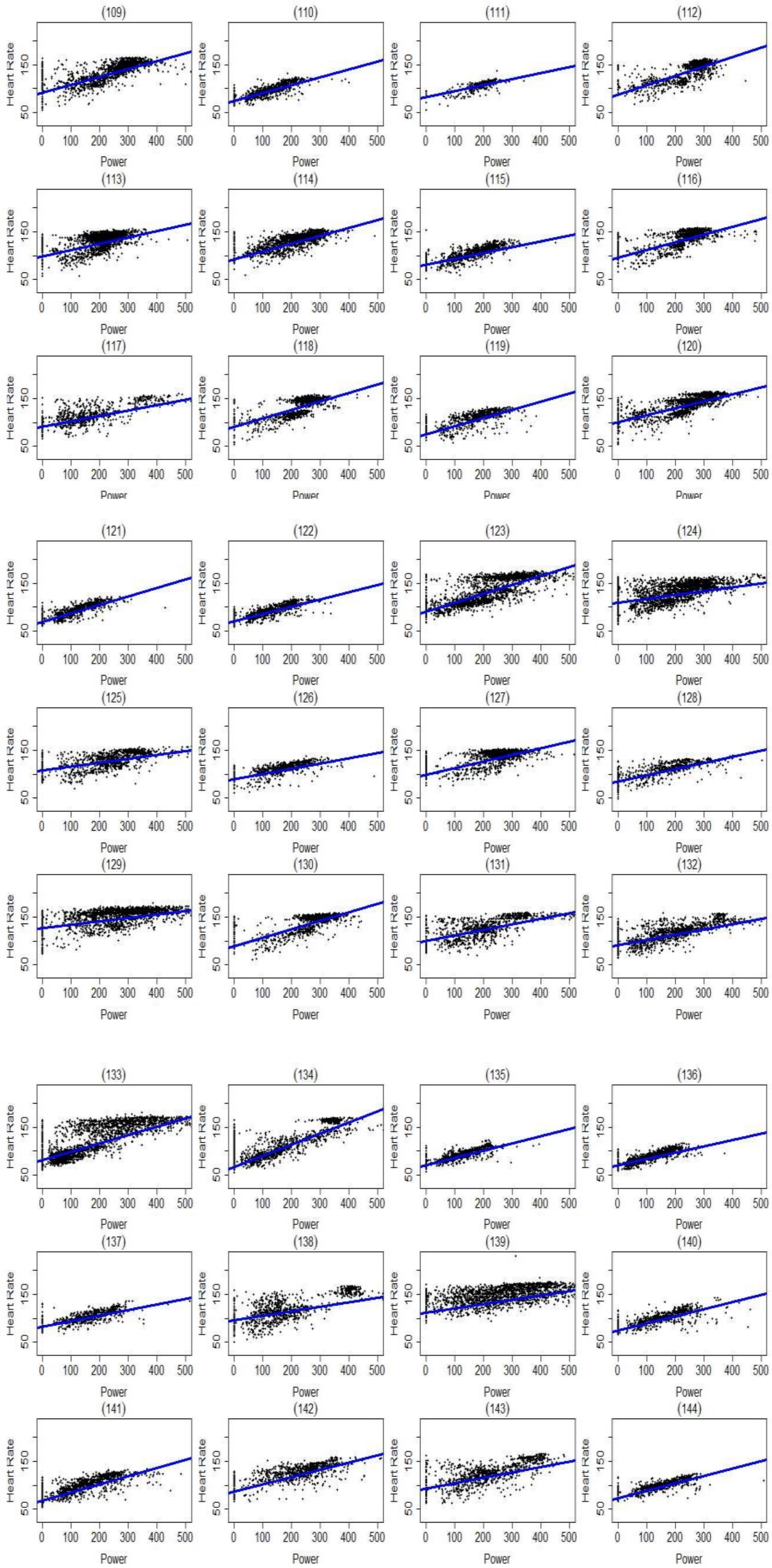

Figure A3.1 Continued

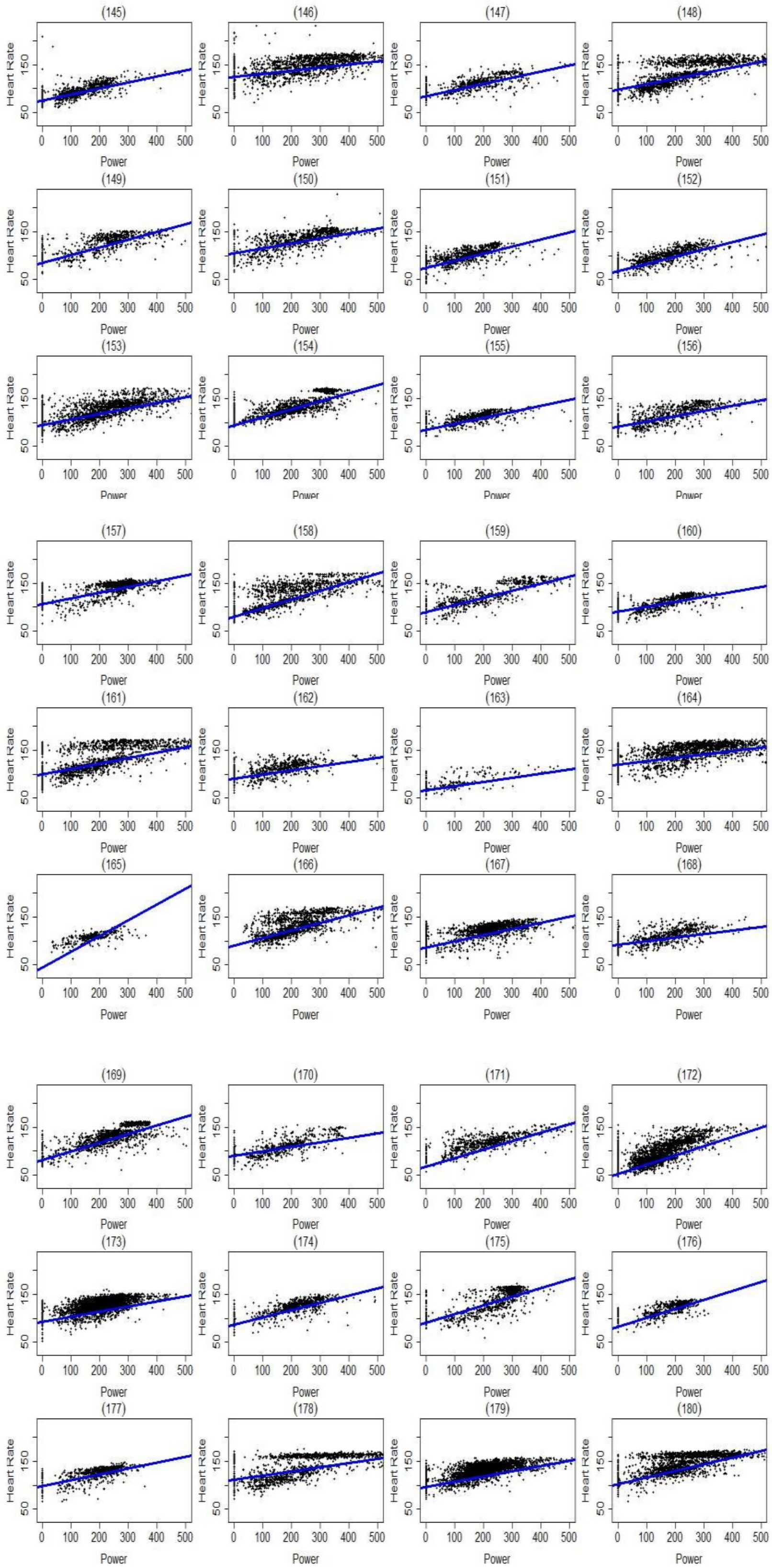

Figure A3.1 Continued

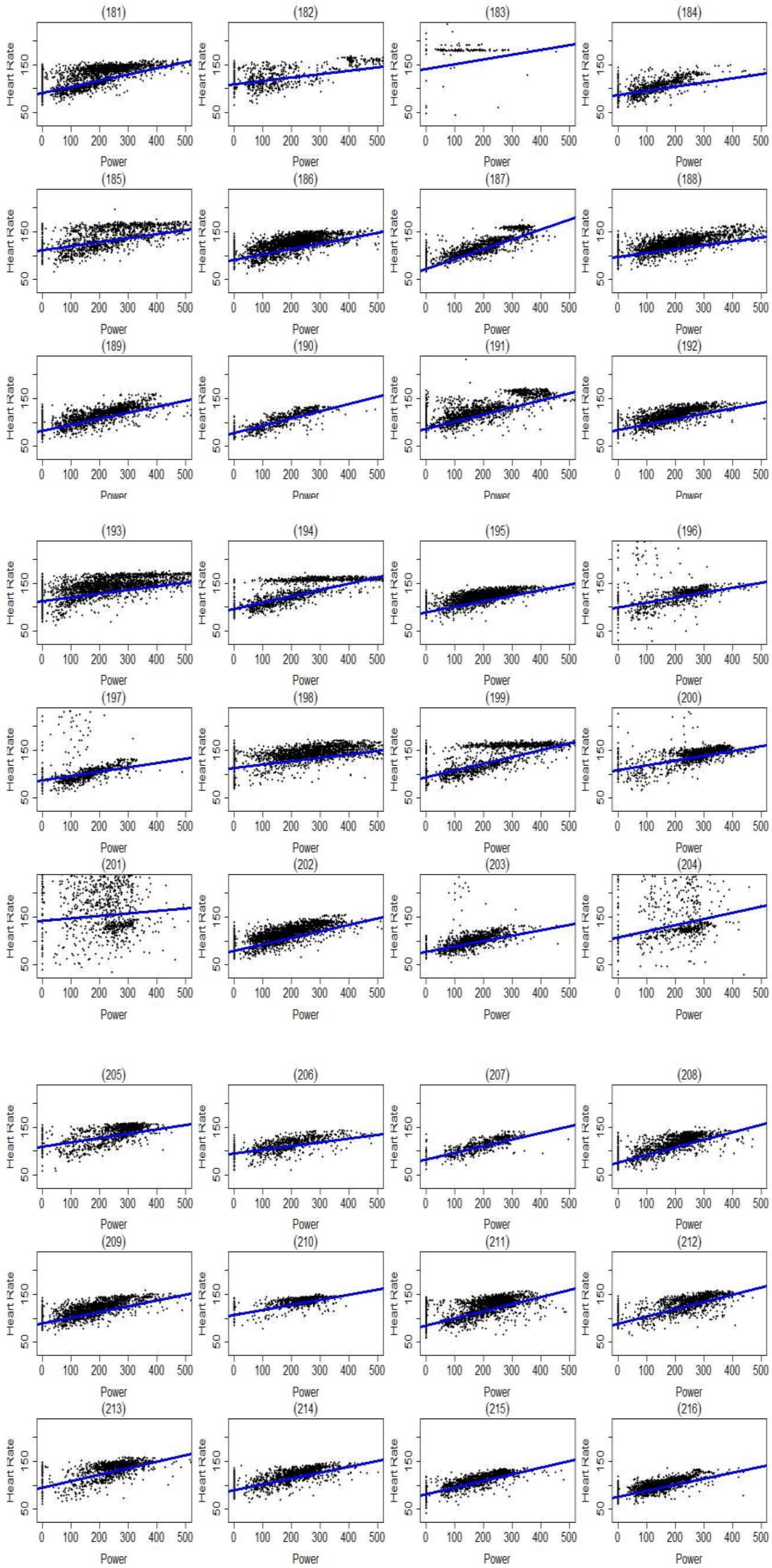

Figure A3.1 Continued

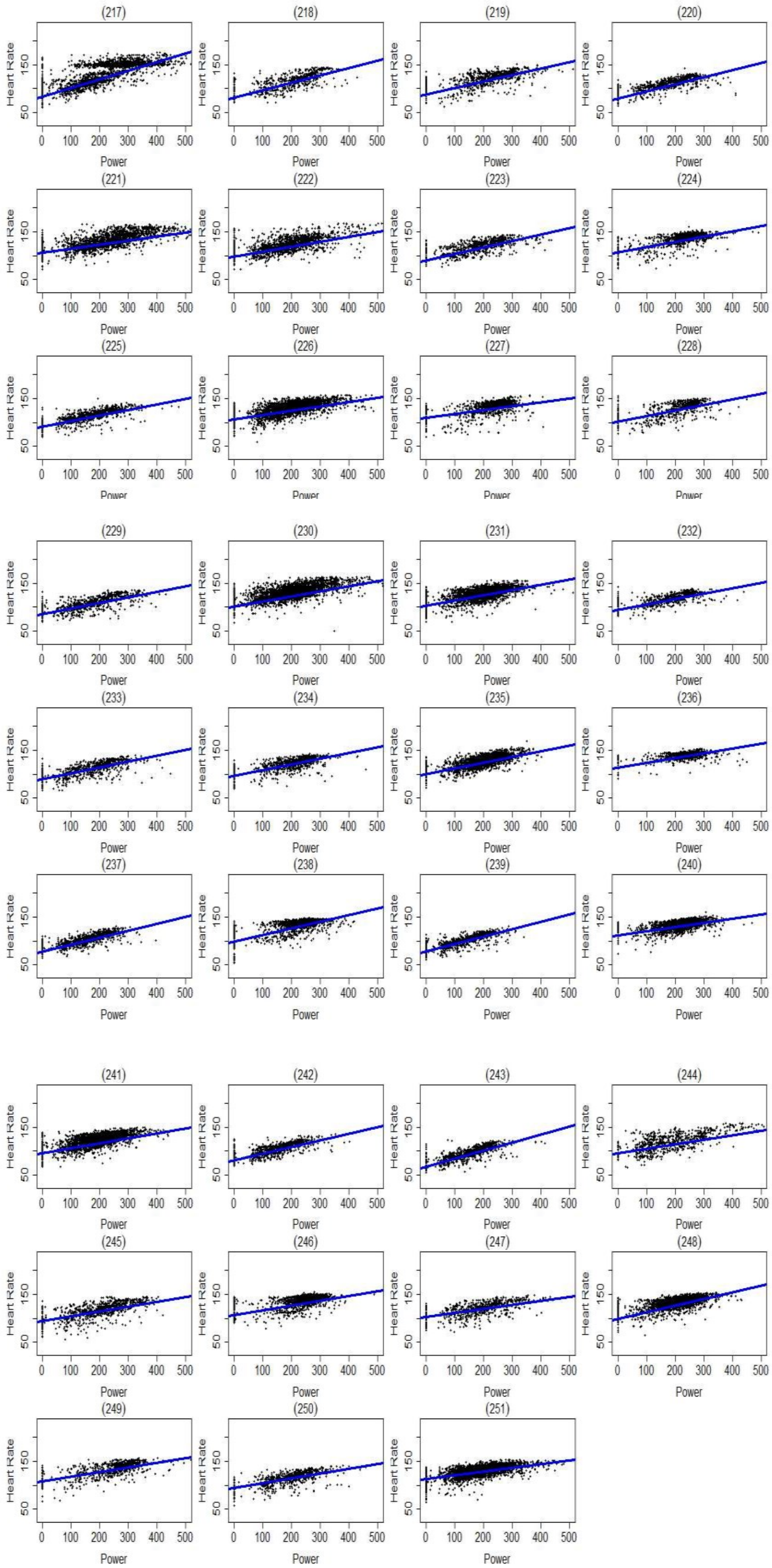

Figure A3.1 Continued

Appendix 4: The estimated parameters of the model between power output and heart rate

Table A4.1 The estimated parameters of the model between power output and heart rate for each rider

	Rider																			
	1		2		3		4		5		6		7		8		9		10	
c	-9.16E-05		-0.00011		-0.0002		-0.00024		-0.00022		-9.86E-05		-0.00021		-0.0001		-6.61E-05		-0.00022	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
1	-35.9	2.12	-237	3.09	130	0.894	-162	2.77	108	0.472	-334	4.08	-299	3.84	-250	3.79	-162	2	-103	2.35
2	95.9	1.1	121	0.87	-194	3.26	42.5	1.19	-215	2.88	-299	4.05	-452	4.44	-7.12	1.78	-51.4	1.39	-118	2.69
3	22.1	1.58	-87.4	2.05	-47.3	2.12	-32.8	1.36	14.2	1.41	-136	2.67	-226	3.03	-293	3.93	-99.4	1.75	-59.1	2.07
4	-52.6	2.16	-196	2.86	-94.8	2.72	116	0.341	-80.7	2.19	-48.2	2.1	-258	3.16	-122	2.7	-45.9	1.43	-103	2.43
5	77.8	0.915	-95.9	2.25	-102	2.54	-6.52	1.64	-66.8	2.07	-162	3.06	-200	2.71	-150	2.74	-128	1.92	-56.9	2
6	-99.4	2.65	-86.3	1.91	18.5	1.76	-142	2.39	50.3	1.03	-126	2.73	-84.2	1.98	5.36	1.97	-21.6	1.27	-246	3.68
7	-192	3.42	-60.2	2.11	-143	2.86	111	0.222	-0.334	1.62	-135	2.74	-142	2.45	-372	4.33	-74.6	1.57	-97.5	2.24
8	-197	3.31	-3.86	1.59	-1.36	1.79	-28.3	1.7	-16.8	1.71	-109	2.68	-14	1.15	-253	3.72	-55.1	1.51	-48.4	1.84
9	-275	4.07	-110	2.22	-19.2	1.99	0.0201	1.43	115	0.734	24.5	1.69	-198	3.11	-350	4.35	-99.7	1.82	-66.6	2.04
10	-217	3.34	20.1	1.39	-103	2.62	94.7	0.594	81.6	0.843	-142	2.81	-60.8	1.67	-261	3.75	-137	2.1	64.3	0.958
11	-16.1	1.9	-288	3.48	-140	3	-36.6	1.64	85.5	0.736	-404	4.49	-186	2.99	-288	3.86	-147	2.23	-93	2.17
12	51	1.47	-74.4	1.92	-107	2.7	-78.6	2.02	-10.9	1.65	-224	3.55	-64	1.74	-266	3.68	-114	1.94	-137	2.56
13	-225	3.7	-107	2.16	-0.489	1.77	-94.9	2.25	63.5	0.682	7.52	1.81	-259	3.77	-336	4.19	-225	2.71	-109	2.44
14	15.9	1.79	-177	2.79	67.3	1.3	83.9	0.46	55.4	0.95	273	0.104	-195	2.75	-238	3.46	-75.7	1.69	-210	3.17
15	-20.4	2.08	-271	3.17	54.9	1.43	2.23	1.46	-46.3	1.74	-226	3.5	6.25	1.66	-346	4.04	-35.2	1.44	-158	2.82
16	-6.4	1.96	-42.1	1.63	-114	2.91	-75.3	1.92	185	0.52	109	1.03	-242	3.38	-278	3.72	-162	2.32	-234	3.68
17	88.1	0.817	-68.8	2.09	-88.2	2.53	-57.8	1.82	105	0.552	-339	4.26	-179	3.1	-261	3.39	-39	1.5	-172	3.24
18	-289	4.1	-356	3.82	-76.7	2.48	-10.8	1.62	40.5	1.44	-167	3.06	-193	3.28	-164	2.91	-49.9	1.57	64.7	0.61
19	-219	3.61	-112	2.16	-125	2.7	-106	2.11	108	0.951	174	0.565	-166	2.48	-401	4.55	-80.4	1.8	-83.1	2.28
20	-236	3.76	-130	2.42	-121	2.99	-18.1	1.64	122	0.767	47	1.42	7.01	1.11	-263	3.6	-51.7	1.73	-72.8	2.21
21	-166	2.91	-8.47	1.45	-102	2.81	-36.8	1.57	127	0.858	-248	3.69	-275	3.6	55.5	1.09	-49.3	1.64	-35.5	1.8
22	61.1	0.825	-1.16	1.78	-112	2.78	120	0.276	10.8	1.59	138	0.887	-89.2	2.45	-308	4.26	-144	2.29	-168	2.86
23	-158	2.91	-75.1	1.84	-34.2	2.17	86.6	0.986	147	0.63	-125	2.72	-162	2.59	-455	5.34	-68	1.8	-166	2.89
24	-103	2.53	-68.4	1.9	-111	2.75	-71.3	1.81	62	0.987	-128	2.6	-178	2.75	-345	4.15	-160	2.38	-178	3.12
25	59.2	1.43	-93.6	2.63	105	1.19	-157	2.41	1.04	1.52	-217	3.36	-135	2.69	-268	3.6	-211	2.86	-122	2.9
26	-256	3.76	-85.3	2.37	-111	2.78	-116	2.24	107	0.921	-313	4.05	-13.3	1.33	19.3	1.22	-34.8	1.7	-61.5	2.08
27	-192	3.45	86.7	0.947	140	0.746	-148	2.54	48.6	0.974	65.1	1.48	-305	3.92	4.74	1.45	-71.9	1.72	-104	2.44
28	-251	3.81	8.13	1.36	-101	2.65	52.6	0.835	126	0.872	-1.83	2.02	-230	3.46	-349	4.31	-216	2.83	-75.2	2.19
29	-275	3.8	63.9	1.04	150	0.665	-114	2.11	55.5	1.46	-3.99	1.76	-158	2.65	-369	4.54	-31.3	1.56	-34.3	1.87
30	-153	2.85	-293	3.77	172	0.374	70	0.612	83.8	1.27	-2.16	1.79	96.5	0.82	-293	3.89	-112	2.2	-86.7	2.32
31	-214	3.34	9.2	1.25	-158	3.11	-43.6	1.72	23.5	1.44	-35.7	2.06	-212	3.44	-357	4.32	-20.3	1.41	-89.2	2.36
32	-215	3.54	-4.75	1.45	-135	2.95	-91.6	2.06	15.2	1.54	-43.3	2.19	-127	2.36	-294	3.87	-180	2.56	-151	2.95
33	-181	3.43	-212	2.91	-83.1	2.28	-54.9	1.88	73.6	1.07	179	0.441	-235	3.28	-394	4.53	-64.7	1.73	-160	2.92
34	-136	3.07	18.1	1.63	-61.3	2.25	-53.5	1.8	204	0.282	17.7	1.63	-203	3.16	-350	4.49	-194	2.64	-131	2.62

Table A4.1. continued

	Rider																			
	1		2		3		4		5		6		7		8		9		10	
c	-9.16E-05		-0.00011		-0.0002		-0.00024		-0.00022		-9.86E-05		-0.00021		-0.0001		-6.61E-05		-0.00022	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
35	-305	4.05	10.9	1.37	-126	2.72	-5.77	1.54	72	1.13	-24.7	2	48.7	0.802	-359	4.44	-194	2.7	6.14	1.48
36	-158	3.03	-29.7	2.01	-98.6	2.37	-78.9	2.07	30.2	1.14	-84.8	2.54	-216	2.95	-327	4.24	-4.78	1.35	-122	2.66
37	-238	3.69	-51.7	2.08	-88.8	2.34	94.8	0.394	-14.5	1.68	35	1.75	-367	4.49	-294	3.95	-179	2.67	-60.3	2.11
38	-225	3.6	-58.9	1.81	-138	2.68	-85.2	2.02	131	0.619	3.5	1.88	-422	4.69	-412	4.91	-116	2.14	-171	3.09
39	-82.6	2.22	-109	2.43	-147	2.89	-11.7	1.55	61.1	1.09	-81	2.44	-127	2.55	59.5	1.13	-87.7	1.99	-85.2	2.36
40	-211	3.46	-184	2.91	-71.9	2.4	89.4	0.306	84.4	0.964	-49.5	2.21	-355	3.96	-334	4.41	-127	2.19	20.1	1.18
41	-196	3.27	-83.9	2.14	-115	2.69	42.8	1.36	17.8	1.5	-96.8	2.52	-197	3.05	-412	4.85	-164	2.42	-98.3	2.4
42	-294	4.33	-97.1	2.39	-119	2.97	34	1.24	200	0.241	53.8	1.55	15.7	1.21	31.7	1.47	-129	2.28	-133	2.75
43	-153	2.98	-71.1	2.04	-126	2.83	43.8	1.34	139	0.221	-35.8	2.08	-190	3.31	-334	4.18	-106	2.01	-54.1	1.72
44	-170	3.19	-77.1	2.13	-74.5	2.51	-151	2.66	79.7	1.26	-236	3.73	40.1	1.2	-199	2.89	-130	2.12	-102	1.72
45	-45.2	2.36	-88.5	2.21	-97.6	2.48	58.1	0.964	8.19	1.16	-74.1	2.4	-209	3.29	-477	5.25	-40.1	1.47	-130	2.65
46	-97.7	2.52	-89.2	2.32	-126	2.77	54.4	0.839	52.8	1.03	-55.2	2.14	-69.8	2.36	-178	2.71	-13.1	1.35	-95.4	2.19
47	-109	2.87	-95.4	2.27	-170	3.1	-18.6	1.7	40.3	1.41	-15.1	1.78	-230	3.37	-336	4.29	0.504	1.22	-99.9	2.41
48	2.55	1.91	-30	1.66	-178	3.18	-128	2.38	97.9	0.989	57.9	1.25	-47.4	1.75	-290	3.81	-52	1.58	-126	2.61
49	-112	2.59	-167	2.62	5.98	1.73	-55.8	1.96	87.7	1.2	47.7	1.61	-748	6.95	-128	2.15	-102	1.99	-55.1	2.19
50	-98.9	2.71	-31	1.76	-207	3.08	143	0.555	-42.7	1.99	70.8	1.34	-50.8	2.22	-257	3.48	-171	2.42	-103	2.38
51	39.1	0.9	-98.2	2.45	-71.2	2.4	-73.9	2.01	86.9	1.03	-59.1	2.18	-67.9	2.2	-312	3.98	-19.6	1.47	-79.4	2.28
52	-182	3.13	-217	3.44	-54.3	2.25	-16.2	1.73	77.9	0.857	240	0.315	-60.8	2.62	-319	4.13	-46.5	1.63	-162	2.94
53	-109	2.73	-115	2.46	92.1	1.23	5.23	1.6	95.5	0.343	-9.8	1.99	-91.1	2.58	-109	2.21	-50.1	1.72	-189	3.33
54	-2.23	1.54	-10.8	2.04	54.6	1.37	-131	2.46	89.4	0.718	81.7	1.21	-70.5	2.15	-346	4.31	-187	2.55	-128	2.61
55	-253	3.89	-534	5.64	-80	2.42	-121	2.45	36.4	1.21	-55.2	2.27	-103	2.96	-206	2.59	-86	1.88	-113	2.43
56	-157	3.08	-276	3.67	-199	3.51	-50	1.92	10.8	1.45	-44.9	2.18	-94.9	2.39	-228	3.28	-37.5	1.56	-108	2.41
57	-126	2.8	130	1.11	-41	2.35	-1.48	1.54	37	1.28	-28.2	2.01	-231	3.55	-369	4.81	-147	2.3	-202	3.15
58	-467	6.04	-95.4	2.48	104	1.08	116	0.832	71.7	1.01	16.2	1.57	-207	3.54	-277	3.47	-57.3	1.72	-189	3.2
59	9.21	1.48	-340	4.07	-167	3.18	-221	2.88	85.9	0.424	-6.99	1.95	-244	3.42	-273	4	-70.5	1.85	-186	3.14
60	-169	3.07	-252	3.28	16.3	1.75	118	0.522	-67.5	2.01	-163	3.03	-103	2.73	-357	4.63	-128	2.14	-193	3.16
61	-104	2.6	-296	3.26	-127	2.93	-131	2.52	-33.5	1.64	9.98	1.65	-149	2.7	94.4	-0.055	-49.9	1.68	-130	2.7
62	-391	4.68	-164	2.51	-91.1	2.55	133	0.401	99.3	0.0306	-56.6	2.35	-98.1	2.25	-279	3.87	-135	2.2	-128	2.58
63	26.6	0.483	-246	3.18	-144	2.81	51.1	1.21	159	0.095	-24.4	2.28	-228	3.43	-238	3.1	-245	3.06	-183	3.04
64	-177	3.36	-190	2.75	189	0.689	-34.3	1.77	118	0.534	-40.3	2.25	-153	3.02	-276	3.74	-59.2	1.77	-155	2.95
65	-269	3.92	-341	4.35	-98.4	2.63	-142	2.64	116	1.17	-60.5	2.26	-110	2.6	-376	4.31	-76.7	1.86	-165	2.89
66	-104	2.54	-282	3.67	-182	3.31	-93.9	2.43	120	0.441	-53.8	2.49	-113	2.58	-291	3.87	-57.4	1.69	-78.2	2.26
67	-124	3.01	-191	3.20	-76.6	2.39	-44.2	1.95	159	0.503	4.69	1.92	-265	3.39	-253	3.41	-52.7	1.59	-175	3.27
68	11.7	1.38	-166	2.86	-158	2.8	-26	1.92	110	0.699	38.6	1.65	-287	3.89	-245	3.42	-147	2.26	-142	2.61

Table A4.1 continued

	Rider																			
	1		2		3		4		5		6		7		8		9		10	
c	-9.16E-05		-0.00011		-0.0002		-0.00024		-0.00022		-9.86E-05		-0.00021		-0.0001		-6.61E-05		-0.00022	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
69	-228	3.79	-98.8	2.49	-167	2.82	-52.8	1.94	145	0.15	56.8	1.63	-44.5	2.31	-357	4.36	-81.6	1.94	-91.6	2.36
70	50.6	0.773	-284	3.89	-119	2.36	139	0.688	135	0.689	-25.6	2.07	-493	5.31	-215	3.25	-39.5	1.58	-73.5	2.15
71	68.9	1.36	-386	4.61	-177	3.02	-159	2.71	151	0.286	-83.3	2.71	-88	2.07	-155	2.63	-5.17	1.3	-74.5	2.1
72	5.49	1.91	2.51	1.66	-110	2.25	143	0.226	-92.6	2.37	-52.7	2.21	-105	2.81	70	1.14	-34.4	1.48	-133	2.33
73	-3.08	1.82	-287	3.91	-19.7	1.61	48.5	1.18	107	0.508	-84.2	2.69	-206	3.23	-258	3.49	-90.5	1.89	-68.2	1.92
74	-167	3.21	-300	4.01	-83.4	2.23	66	0.671	126	1.14	-51.6	2.27	-143	2.82	-355	4.28	-192	2.5	-183	2.96
75	-339	4.65	-76.3	2.39	33.5	1.32	4.62	1.64	130	0.274	71.3	1.28	-244	3.47	-280	3.7	-93	2.01	-108	2.4
76	70.8	0.739	-328	3.94	-71.4	1.94	-59.9	2.1	105	0.561	11.9	1.92	-64.7	2.35	-299	3.95	-140	2.08	-140	2.56
77	-242	3.74	4.08	1.8	69.5	0.584	-181	3.14	56.2	0.65	-80.9	2.32	-173	3.17	-121	2.25	-76	1.71	-104	2.28
78	-204	3.44	-403	4.18	145	0.266	85.6	0.653	175	0.458	-65	2.3	-78.3	2.51	-276	3.8	-128	2.09	-81.2	2.18
79	97.5	0.322	-247	3	-7.12	1.41	-40.6	1.92	78.2	0.791	-75.6	2.5	-195	3.15	-8.56	1.56	-163	2.31	-84.8	2.2
80	-63.3	2.1	-317	3.66	-107	2.21	-21.7	1.49	178	0.284	55.5	1.49	4.26	1.64	-2.68	1.5	-136	2.09	-125	2.56
81	37.2	1.06	-257	3.3	3.71	1.51	-40.8	1.92	-67.2	2.05	4.33	2.02	-187	3.11	96.7	0.415	-49.7	1.61	11.5	1.21
82	-19.8	2.12	-220	2.92	-158	2.53	-56.3	1.71	-48.6	1.97	83.3	1.54	-226	3.32	-250	3.57	-149	2.25	-198	3.27
83	-248	3.58	-342	3.68	-206	3.06	-224	3.01	-6.52	1.67	-369	4.38	-156	2.87	-404	4.5	-153	2.37	-74.7	2.23
84	-221	3.53	-174	2.75	-115	2.51	-102	2.26	-46.9	2.08	-40.8	2.26	-136	2.79	96	0.295	-192	2.6	-161	3.16
85	-187	3.43	-241	3.53	-128	2.51	108	0.904	-131	2.77	-46.1	2.65	-161	2.86	-52	1.93	-180	2.45	-126	2.64
86	-314	4.2	-201	3.07	-105	2.34	112	0.279	-307	3.79	64.2	2.02	-120	2.6	-74	1.94	-247	2.95	-172	2.94
87	18.3	1.16	-83.1	2.11	-107	2.46	-157	2.66	-33	1.84	-8.01	2.06	-135	2.65	-1.37	1.35	-23.6	1.25	-127	2.61
88	-230	3.61	-293	3.56	-36.1	1.92	-35.5	1.97	-105	2.71	-83.5	2.67	-196	3.25	-351	4.75	-130	2.19	-99.6	2.46
89	-118	2.81			-107	2.49	33	1.2	-3.52	1.67	-4.8	2.03	-191	3.09	-267	3.91	119	0.146	-140	3.03
90	-223	3.6			21.9	1.53	-2.93	1.46	-89.8	2.37	-37.6	2.41	-278	3.88	-324	4.33	-191	2.59	-134	2.79
91	-169	3.11			-6.06	1.76	127	0.756	6.07	1.39	-52.6	2.44	-251	3.53	-365	4.67	-170	2.6	-165	3.09
92	-385	5.03			227	0.0057	-181	2.8	1.18	1.59	-8.52	1.84	-198	3.4	-268	3.53	-117	2.16	-173	2.9
93	-249	3.73			-190	2.88	144	0.113	-14.7	1.52	111	0.848	-192	3.24	-355	4.68	-219	2.85	-136	2.76
94	-49	1.87			-139	2.51	-85.8	2.18	-70.7	2.33	-82.7	2.35	-360	4.48	-237	3.31	-113	2.13	-109	2.46
95	-312	4.21			-151	2.85	-139	2.72	49	1.01	-227	3.71	-151	2.8	-398	4.81	-54.8	1.67	-159	2.93
96	-1.44	1.44			-152	2.8	22.3	1.53	35.1	1.29	-44.8	2.28	-67.8	2.29	-282	4.02	-121	2.31	-111	2.46
97	1.26	2.01			-130	2.62	90.7	0.802	-189	3.36	-108	2.88	-230	3.62	-296	4.23	-254	3.04	-141	2.59
98	-274	3.84			-155	2.83	35.9	1.52	-149	2.84	127	0.936	-183	3.06	-204	3.27	-119	2.31	-176	2.94
99	-238	3.84			-173	2.95	136	0.302	-103	2.4	-41.9	2.45	-193	3.19	-320	4.18	-206	2.88	-266	3.74
100	-237	3.53			-115	2.44	-110	2.44	-27.2	1.7	72.8	1.34	53.7	1.84	-229	3.68	-8.6	1.42	-91.8	2.4
101	-84.7	2.31			-138	2.61	-200	3.14	-175	3.05	82.5	1.16	-145	2.74	-129	2.38	-216	2.82	-129	2.6
102	-227	3.57			-59.8	2.06	125	0.849			-17.5	2.08	-166	2.94	-340	3.88	-245	3.01	-168	2.89

Table A4.1 continued

	Rider															
	1		3		4		6		7		8		9		10	
c	-9.16E-05		-0.0002		-0.00024		-9.86E-05		-0.00021		-0.0001		-6.61E-05		-0.00022	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
103	-157	2.85	-77.6	2.17	229	0.106	47.3	1.63	-212	3.31	-316	4.34	-233	2.93	-111	2.47
104	-118	2.8	-85.9	2.31	-71.4	2.28	36.1	1.69	-210	3.26	-24.8	1.51	-151	2.33	-147	2.74
105	-341	4.36	-169	2.95	-40.2	2.09	-77.8	2.43	-242	3.53	-342	4.48	-60.5	1.47	-60.8	2.17
106	-209	3.53	-236	3.47	42.6	1.35	148	0.752	-156	2.83	-36.6	1.61	-145	2.19	-65.4	2.1
107	-335	4.6	-81.5	2.3	46.9	1.22	108	0.988	-78.7	2.51	-227	3.43	-125	2.25	-184	3.5
108	-86.5	2.27	-89.2	2.52	47.3	1.14	-27.8	2.04	-142	2.63	-224	3.46	-218	2.99	-67.4	2.31
109	-199	3.13			115	0.639	-80.6	2.42	-160	2.95	-301	4.09	-197	2.74	-102	2.54
110	-48.7	1.94			144	0.52	-55.7	2.6	-279	3.73	-327	4.6	-143	2.33	-101	2.37
111	27.4	0.255			-75.3	2.02	-332	4.39	-278	3.88	-257	4.05	-77.5	1.64	-145	3.1
112	-198	2.54			0.507	1.44	-383	4.86	204	0.736	-534	5.36	-60.2	1.66	-85.7	2.41
113							-326	4.3	59	0.631	-331	4.45	-122	2.13	-30.9	1.91
114							-114	2.86	-134	2.74	-440	4.96	4.88	1.11	-70.7	2.2
115							-195	3.66	-208	3.32	-226	3.57	-84	1.79	-64.2	2.17
116							-200	3.64	-316	4.05	-326	4.24	-208	2.77	-78.1	2.27
117							-30.8	2.13	-160	2.96	-372	4.73	-21.6	1.26	-111	2.54
118							-107	2.95	-228	3.48	61.9	0.761	-31.5	1.4	-139	2.78
119							-166	3.42	-414	4.61	-308	4.31	-132	2.16	-51.7	2.08
120							-192	3.38	-429	5.08	-160	2.64	-81.8	1.91	-220	3.33
121							-222	3.6	-153	2.96	-257	3.44	-136	2.29	-93	2.31
122							106	1.19	-178	3.25	-379	4.5	-73.7	1.92	-138	2.91
123							160	0.74	-173	3.03	-91	2.13	-231	2.93	-145	2.77
124							-73.9	2.57	-176	2.72	-394	4.55	-61.3	1.53	-116	2.62
125							-223	3.54	-150	2.68	-363	4.55	-25.1	1.16	-133	2.85
126							-183	3.5	164	0.0942	-216	3.21	-128	2.15	-45.3	1.96
127							-128	2.81	-189	3.35	-284	4.02	-213	2.92	-61.5	2.29
128							-129	3.11	137	0.388	-259	4	-244	2.99	-146	2.74
129							-325	4.25	-124	2.5	-42	1.9	-118	2.25	-128	2.66
130							-327	4.46	-110	2.56	-169	3.11	-90.2	1.92	-79.9	2.5
131							-185	3.37	-149	2.93	-249	3.74	-215	2.82	-100	2.51
132							-83.3	3.01	-250	3.8	-141	2.77	-58.6	1.79	-142	2.91
133							-207	3.33	-34.7	2.02	-279	4.05	124	0.134	-147	2.87
134							-332	4.19	-78.7	2.52	-377	4.58	-146	2.34	-102	2.5
135							-245	3.71	-243	3.63	-337	4.58	58.9	0.503	-140	2.9
136							-250	3.73	-82.2	2.39	-281	4.03	-51	1.46	-203	3.65

Table A4.1 continued

	Rider									
	6		7		8		9		10	
c	-9.86E-05		-0.00021		-0.0001		-6.61E-05		-0.00022	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
137	-194	3.58	-136	2.86	-235	3.46	-76.2	1.77	-101	2.58
138	-277	4.07	-170	2.98	-309	4.24	-78.3	1.85	-139	2.91
139	-156	3.01	-54.5	2.23	-174	2.7	39	0.694	-55.1	2.29
140	-153	3.3	14	1.42	-262	3.61	-226	2.8	-108	2.62
141	-112	2.88	-156	2.54	-238	3.74	-2.11	1.05	-83.6	2.37
142	-1.28	2.01	-196	3.34	-189	3.17	-115	1.94	-59.1	2.19
143	-195	3.57	-28.7	2.21	-104	2.29	-194	2.51	-84.3	2.53
144	-278	4.24	-108	2.17	-132	2.61	-273	3.14	-120	2.78
145	-60.8	2.35	-144	2.8	-309	4.09	50.4	0.491	-74.9	2.25
146	-107	3.01	-201	3.35	-221	3.58	41.2	1.07	-120	2.55
147			-74.9	2.26	-112	2.47	-268	3.27	-112	2.61
148			-99.5	2.82	-287	3.96	86.7	0.365	-152	2.94
149			-66.5	2.42	-339	4.32	-128	2.13	-68.5	2.27
150			-84.2	1.95	-302	4.16	-132	2.19	-171	3.05
151			-138	2.79	-244	3.72	-99.5	1.98	-87	2.26
152			-239	3.55	-299	4.29	-219	2.81	-66	2.37
153					-292	4.03	-13.6	1.39	-115	2.72
154					-242	3.71	-294	3.49	-148	2.72
155					-171	3.04	75.6	0.827	-80.7	2.27
156					87	0.407	-160	2.28	-89.5	2.44
157					-320	4.34	-171	2.43	-116	2.64
158					-385	4.74	-40.4	1.57	-61.1	2.18
159					-276	3.75	-162	2.49	-154	2.96
160					-280	3.59	-93.8	1.78	-221	3.56
161					-95.2	2.17	-91.1	1.89	-146	2.85
162					-207	3.33	-76.9	1.91	-94.1	2.36
163							-139	2.38	-35.1	1.85
164							-164	2.39	-152	2.83
165							-195	2.51	-2.87	1.68
166							-178	2.36	-39	2.1
167							-162	2.34	-90.2	2.57
168							-77	1.45	-122	2.73
169							-169	2.4	-101	2.67
170							-124	2.03	-177	3.22

Table A4.1 continued

		Rider								
	9		10			10			10	
c	-6.61E-05		-0.00022		c	-0.00022		c	-0.00022	
session	a_i	b_i	a_i	b_i	session	a_i	b_i	session	a_i	b_i
171	-160	2.27	16.7	1.71	198	-122	2.77	226	-162	2.85
172	-9.49	0.895	-38.9	2.18	199	-112	2.69	227	-18.5	1.91
173	-119	2.62	-125	2.78	200	-85.7	2.6	228	-69.1	2.16
174	-20.9	1.18	-90.4	2.49	201	189	0.187	229	-151	3
175	-11.2	0.673	-75.9	2.3	202	-141	2.96	230	-124	2.57
176	-279	3.45	-69.8	2.09	203	-22.3	1.9	231	-112	2.46
177	-60.7	1.71	-137	2.62	204	130	0.546	232	-114	2.53
178	-178	2.58	-153	2.93	205	-187	3.22	233	-129	2.66
179	-202	2.69	-91.9	2.43	206	-126	2.77	234	-99.4	2.45
180	-80.4	1.66	-110	2.52	207	-108	2.66	235	-149	2.76
181	-45.3	1.76	-76.2	2.3	208	-91.6	2.59	236	-79.9	2.26
182	-129	2.31	-142	2.63	209	-121	2.6	237	-110	2.6
183	-169	2.56	65	0.387	210	-124	2.65	238	-118	2.6
184	-169	2.63	-75.9	2.16	211	-62.3	2.29	239	-117	2.55
185	-6.39	1.07	-149	2.93	213	-84.4	2.5	240	-86.4	2.32
186	-271	3.16	-127	2.8	214	-146	2.97	241	-172	3.07
187	-223	3.16	-125	2.81	215	-159	3.11	242	-92.6	2.38
188	-112	2.18	-175	3.17	216	-153	3.09	243	-88.8	2.51
189	-129	2.31	-117	2.83	217	-113	2.64	244	-79.4	2.44
190	-156	2.51	-215	3.64	218	-94.8	2.55	245	-156	3.06
191	-186	2.62	-99.6	2.54	219	-82.4	2.49	246	-115	2.62
192	-210	2.81	-111	2.69	220	-82.3	2.39	247	-218	3.39
193	-19.3	1.11	-173	2.94	221	-247	3.68	248	-83.3	2.24
194	-225	2.87	-198	3.27	222	-130	2.68	249	-124	2.73
195	-28	1.47	-88.7	2.52	223	-77	2.24	250	-175	3.18
196	-128	2.28	51.2	1.29	224	-182	3.07	251	-196	3
197	-219	2.72	82.9	0.689	225	-151	2.86			

Appendix 5: The estimated parameters of the model between heart rate and power output for each rider

Table A5.1 The estimated parameters of the model between heart rate and power output for each rider

	Rider															
	1		2		3		4		5		6		7		8	
c	8.67E-06		9.18E-06		2.48E-05		8.28E-05		5.41E-05		4.06E-05		2.33E-05		6.87E-06	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
1	126.0881	0.055149	139.6921	0.067226	69.67453	0.139152	117.0032	0.083584	123.1029	0.069065	110.4436	0.108719	129.3053	0.070245	99.7089	0.10268
2	63.76	0.299246	148.8424	0.042022	127.0033	0.040417	70.11602	0.310343	123.1868	0.073436	97.82127	0.104218	126.7497	0.077294	116.357	0.062626
3	124.8512	0.070198	124.3367	0.067225	99.10718	0.078546	126.949	0.093527	114.1948	0.088001	120.8492	0.074814	113.6434	0.089288	122.7283	0.060795
4	96.83573	0.023503	130.0286	0.049979	111.194	0.06919	27.78566	0.536615	129.7422	0.041049	133.9982	0.0336	116.2036	0.135113	107.1658	0.070609
5	81.5612	0.036763	126.1802	0.015299	117.6595	0.049731	126.365	0.100944	81.79901	0.135521	122.7757	0.038495	127.8852	0.098071	99.49456	0.092104
6	126.2747	0.049184	139.0673	0.060117	110.8963	0.082485	129.2979	0.060924	103.4137	0.060882	118.3303	0.087384	110.7595	0.154015	108.9502	0.072577
7	108.23	0.077225	116.9714	0.101385	110.3687	0.112055	70.84388	0.2987	104.2425	0.064154	120.429	0.075974	136.2828	0.049647	123.0555	0.05651
8	90.65958	0.059918	170.3767	0.014337	100.9897	0.149122	125.2892	0.110387	112.8282	0.070586	115.7146	0.078295	67.44944	0.425322	103.2492	0.070615
9	116.3041	0.069272	125.0262	0.088142	121.5879	0.050184	107.4534	0.167819	140.5121	0.034694	16.16107	0.470159	117.6942	0.079004	107.0009	0.078908
10	132.5106	0.046426	145.9903	0.043543	90.32911	0.141765	40.98025	0.434128	126.8785	0.050589	107.3947	0.117859	129.3915	0.107522	108.5413	0.054786
11	105.5395	0.044536	143.2192	0.045405	76.99686	0.16166	135.8542	0.051411	128.2927	0.038078	126.4046	0.0586	110.2945	0.128917	107.9042	0.06656
12	108.7419	0.024768	140.0163	0.057458	97.17062	0.100996	81.71514	0.2525	122.5399	0.065703	118.0795	0.053147	83.6984	0.240591	110.8395	0.061564
13	113.9323	0.025988	118.9188	0.06901	98.29199	0.094254	107.9392	0.084185	99.6426	0.050901	18.32832	0.463915	114.5063	0.082985	119.8566	0.053462
14	118.3181	0.04942	150.3196	0.001644	84.29876	0.085889	49.99569	0.403779	123.6377	0.078021	18.32514	0.470936	121.9573	0.117458	109.9702	0.074212
15	76.88026	0.234659	155.3184	0.008075	111.0127	0.06346	119.4722	0.079235	143.5611	0.041804	123.2589	0.067635	46.19725	0.347368	118.9704	0.074042
16	114.0757	0.050506	92.46653	0.259976	103.1742	0.08105	127.3915	0.06461	142.243	0.015894	18.56748	0.469931	117.0459	0.071415	109.6844	0.059207
17	91.1287	0.026575	127.8296	0.065938	120.7428	0.07203	120.2246	0.068727	113.5002	0.044616	123.1199	0.053113	127.236	0.051005	114.3557	0.063409
18	114.6968	0.065767	159.4021	0.016449	115.5777	0.064897	142.8841	0.030376	127.6698	0.062161	118.6227	0.062187	117.7302	0.080572	100.3894	0.104825
19	109.4381	0.072231	139.0383	0.007169	124.9242	0.060842	125.7904	0.052129	128.0231	0.033429	18.85677	0.468135	133.147	0.08021	118.7307	0.056893
20	87.28034	0.152858	150.0344	0.028122	97.78256	0.066697	114.1107	0.066022	129.1062	0.011987	111.036	0.072823	114.9079	0.061452	107.454	0.085703
21	125.6119	0.073837	150.9043	0.026625	111.2842	0.049553	120.1619	0.113696	130.2352	0.026779	113.5256	0.071642	116.3791	0.090829	93.04004	0.231653
22	77.209	0.191961	153.3689	0.037623	103.0596	0.076689	64.90885	0.294419	115.2088	0.064729	18.61312	0.469256	112.9006	0.075789	101.7855	0.076936
23	113.6617	0.100716	121.2622	0.044854	105.6739	0.084687	134.5735	0.066832	144.6233	0.02728	116.1562	0.072365	125.3546	0.083669	113.4957	0.063054
24	87.29601	0.06821	148.4111	0.018542	103.1804	0.099238	119.2669	0.095949	98.22065	0.075855	119.8229	0.065938	100.5084	0.197658	117.7606	0.058213
25	119.7815	0.080544	129.1441	0.078198	77.59274	0.170269	118.9368	0.108171	118.0205	0.071235	118.4793	0.074853	132.5548	0.054624	117.4923	0.05144
26	116.7171	0.065595	124.8609	0.042085	116.0625	0.039384	115.0956	0.099115	115.4102	0.055764	128.167	0.068093	106.7174	0.130214	97.45605	0.182455
27	93.83448	0.035973	50.88391	0.309273	111.5566	0.096268	114.5939	0.110465	95.30252	0.139604	129.0348	0.015156	119.9911	0.07054	96.45911	0.091235
28	115.1562	0.066955	108.3076	0.203728	97.39858	0.091668	44.38094	0.450135	148.0425	0.016849	120.7742	0.036435	100.889	0.09856	110.2439	0.072727
29	118.0334	0.05561	154.4117	0.00445	122.1949	0.014781	116.9676	0.167554	119.9121	0.088358	130.0947	0.029025	108.1908	0.141021	114.9752	0.04377
30	86.63626	0.066209	139.3227	0.036955	115.858	0.04431	66.05763	0.337981	122.6652	0.039442	115.8888	0.091916	65.46114	0.331446	102.5649	0.091975
31	97.21067	0.114578	126.2747	0.018945	106.693	0.062782	118.4674	0.113251	111.555	0.076118	111.6218	0.055727	114.7529	0.084378	111.0592	0.073023
32	118.0961	0.067275	140.6329	0.02394	103.6788	0.094914	129.4008	0.061929	103.9982	0.114487	107	0.033455	119.8888	0.092758	111.6369	0.06733
33	81.16232	0.071092	139.2722	0.020715	102.3301	0.134132	116.4841	0.073009	124.2905	0.08026	18.48184	0.470854	122.0399	0.089589	119.5681	0.063892
34	101.498	0.102108	125.957	0.082315	95.68405	0.143882	122.5967	0.074496	122.2688	0.076862	119.4053	0.033144	100.8118	0.102391	109.2118	0.062922

Table A5.1 continued

	Rider															
	1		2		3		4		5		6		7		8	
c	8.67E-06		9.18E-06		2.48E-05		8.28E-05		5.41E-05		4.06E-05		2.33E-05		6.87E-06	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
35	114.656	0.064195	54.35286	0.398262	97.81352	0.138259	140.2501	0.043931	116.8253	0.085201	132.9915	0.031203	48.5972	0.468533	109.3107	0.068963
36	75.69596	0.067789	132.8395	0.077462	81.85706	0.24547	123.5425	0.063202	97.49682	0.101411	130.7577	0.018532	113.2926	0.107948	106.137	0.073499
37	98.81868	0.130186	145.7984	0.036741	74.24209	0.263698	82.01678	0.196698	102.6084	0.095351	111.0665	0.059522	113.2257	0.085872	105.3398	0.084673
38	100.1769	0.099277	106.0825	0.166924	79.10177	0.254595	128.3629	0.04682	116.9087	0.074576	128.8235	0.035715	123.0895	0.08842	111.3334	0.048011
39	81.56162	0.090061	132.2038	0.07147	117.4248	0.077307	125.1917	0.07853	111.6688	0.09709	128.8879	0.044386	106.8958	0.139094	88.98233	0.083272
40	101.8035	0.125044	137.8173	0.026655	102.0579	0.092268	107.071	0.141552	122.6084	0.062637	127.8983	0.033878	122.8815	0.071344	97.26511	0.080088
41	98.01238	0.104539	113.3627	0.095845	116.5278	0.094648	67.44527	0.275995	118.9376	0.051492	129.0516	0.049942	120.2946	0.084429	112.2042	0.07099
42	91.07723	0.05635	115.2166	0.001644	87.23325	0.100214	133.2432	0.055293	133.2804	0.017571	122.1431	0.065014	92.56846	0.176973	93.28469	0.089282
43	95.18647	0.031575	135.3943	0.055013	102.8439	0.089296	126.1428	0.091935	112.9792	0.000419	127.9694	0.055776	98.73311	0.132094	105.7412	0.078425
44	95.43299	0.02699	109.8376	0.062753	114.2362	0.06155	126.2936	0.051471	141.0349	0.07309	126.34	0.013179	52.906	0.368493	108.4627	0.138917
45	120.8553	0.072212	125.2268	0.016973	117.6778	0.093406	93.09793	0.291085	88.12346	0.105002	134.1187	0.027044	111.8637	0.05206	112.0886	0.07093
46	89.11398	0.059637	123.954	0.059855	116.8032	0.077336	72.72278	0.308568	92.30599	0.149909	132.0313	0.042022	87.38449	0.162224	86.71609	0.221153
47	124.857	0.073697	102.3207	0.103655	113.9435	0.08068	137.1103	0.090633	115.2439	0.056342	118.0263	0.065916	133.7172	0.048158	94.46861	0.133294
48	115.0261	0.082977	127.2721	0.115615	114.9679	0.071259	126.8233	0.095372	104.7886	0.079719	112.2721	0.010672	97.662	0.141493	91.01111	0.175221
49	87.15457	0.082761	161.8152	0.024339	106.3503	0.105248	123.409	0.070005	120.069	0.023969	110.0576	0.071726	139.2588	0.019681	109.4648	0.116833
50	122.4533	0.063609	130.6186	0.088313	123.962	0.050933	120.9852	0.103802	103.433	0.096373	107.2466	0.042363	89.75766	0.161051	90.32305	0.184508
51	52.31302	0.395803	99.08403	0.164219	99.64273	0.122098	104.794	0.167198	122.4396	0.051914	112.1471	0.068482	111.1475	0.067117	89.30943	0.18067
52	98.96337	0.13692	130.2206	0.019311	98.73139	0.107802	142.377	0.074841	67.17307	0.294246	85.91115	0.018825	84.31147	0.088964	93.34686	0.152056
53	74.62278	0.231126	125.1446	0.046996	124.9644	0.066401	114.524	0.106037	117.8044	0.042284	107.8902	0.090919	123.1726	0.084869	87.73284	0.266244
54	59.30149	0.281325	82.31566	0.214096	116.0301	0.023844	119.2558	0.101067	114.792	0.10174	129.2436	0.007587	103.9063	0.085331	98.59661	0.105859
55	134.4371	0.040579	132.2458	0.058879	117.1893	0.099562	110.3078	0.11478	108.6375	0.087041	113.7338	0.078506	85.12832	0.089966	123.6431	0.0333
56	117.6385	0.075626	129.6936	0.068948	106.785	0.069155	130.7654	0.050008	81.82532	0.166879	116.4009	0.001644	91.95919	0.162291	106.4493	0.135839
57	97.76405	0.029905	96.69998	0.175549	98.54203	0.08258	112.9078	0.109068	104.0536	0.139348	115.4752	0.044691	107.0214	0.106096	102.8769	0.066805
58	103.4516	0.251649	113.7834	0.118943	123.8223	0.05663	136.173	0.081177	101.6155	0.123615	86.29637	0.10494	96.00811	0.118108	113.043	0.129801
59	85.39018	0.089863	118.9236	0.100295	112.5903	0.077618	117.1822	0.049555	100.9005	0.030754	111.6278	0.063289	107.2245	0.147426	100.089	0.084215
60	133.2327	0.04864	120.0652	0.05452	105.3411	0.073072	87.81895	0.192317	91.41404	0.146884	132.3969	0.044447	126.6465	0.075103	104.911	0.057546
61	89.79278	0.057889	155.9335	0.042087	99.36486	0.096711	157.241	0.020565	126.9798	0.076514	112.486	0.077414	129.2989	0.048027	127.6989	-0.02912
62	129.7241	0.02312	139.0767	0.077209	108.5878	0.111277	125.9128	0.027267	129.2847	-0.02459	109.6694	0.063287	105.224	0.101009	103.8602	0.076027
63	72.06019	0.258701	123.0358	0.076923	127.4449	0.04236	124.288	0.07709	19.38432	0.071231	135.3189	0.055971	114.4016	0.133537	117.2519	0.120606
64	108.0962	0.024946	120.6697	0.079554	53.30533	0.274943	130.0849	0.03835	99.72087	0.088524	131.2288	0.03214	131.812	0.071804	111.9533	0.065012
65	106.2089	0.023354	100.2421	0.098095	124.6942	0.079317	122.5357	-0.00716	130.0093	0.063297	127.4184	0.039438	87.13015	0.183573	130.94	0.052593
66	89.03071	0.087105	130.6721	0.062685	100.2761	0.079151	120.8272	0.108431	109.0185	0.032257	120.6289	0.032359	104.9917	0.077977	99.75945	0.076597
67	84.76292	0.085087	97.72741	0.120284	114.044	0.086875	131.8219	0.077554	116.6017	0.066869	117.0364	0.025006	130.9683	0.034039	101.4028	0.080844
68	66.79858	0.246819	109.1665	0.111563	122.2771	0.051988	108.4058	0.138403	111.5263	0.073967	118.8784	-0.00115	133.8793	0.061606	119.5248	0.088487

Table A5.1 continued

	Rider															
	1		2		3		4		5		6		7		8	
c	8.67E-06		9.18E-06		2.48E-05		8.28E-05		5.41E-05		4.06E-05		2.33E-05		6.87E-06	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
69	139.6555	0.031576	133.633	0.06919	112.9258	0.07046	98.51268	0.109846	124.9209	0.019095	125.8034	0.038789	116.9559	0.094087	104.4706	0.076539
70	75.56687	0.215408	112.9246	0.101362	117.257	0.100598	122.9031	0.079326	149.9516	0.044175	109.0456	0.080035	118.9968	0.080913	119.3179	0.101192
71	123.4196	0.050651	115.4773	0.083811	101.141	0.071655	136.0818	0.045822	122.2527	0.009331	115.6317	0.049868	89.66384	0.104949	110.301	0.08149
72	146.0569	0.022479	131.2125	0.071513	102.1456	0.08355	94.9573	0.075882	75.89097	0.239569	108.3256	0.061677	121.2295	0.102355	94.5116	0.074231
73	111.8199	0.108958	125.3228	0.044575	104.1945	0.088467	122.8451	0.092253	87.59063	0.095289	102.0611	0.082238	99.75932	0.1493	109.8451	0.066925
74	99.8094	0.02358	106.9849	0.080561	112.9994	0.050145	99.63966	0.096943	136.2358	0.036432	117.3568	0.020148	87.56051	0.159823	111.215	0.066236
75	110.4742	0.00615	99.05988	0.130407	116.1983	0.068627	121.8286	0.108895	96.55187	0.078594	108.38	0.139604	145.3703	0.052903	94.86243	0.155804
76	101.0069	0.038954	128.6823	0.063183	105.216	0.101397	108.0189	0.124771	95.15833	0.044953	118.9664	0.054218	85.93778	0.194128	91.62151	0.166462
77	121.3165	0.066249	107.5878	0.162082	127.5679	0.03097	107.2242	0.015353	153.0743	0.032024	120.797	0.034258	107.4111	0.130854	96.98719	0.088266
78	134.6387	0.043491	137.6748	0.074116	109.0435	0.05901	109.9338	0.001644	98.36682	0.218825	130.811	0.041804	110.9661	0.126283	106.0376	0.098368
79	104.1997	0.092019	130.6652	0.076067	121.3898	0.039132	127.3492	0.051296	41.72088	0.404427	113.3092	0.055244	99.6317	0.141701	117.5681	0.043232
80	77.1118	0.157232	129.73	0.057237	134.2351	0.054591	103.5212	0.040542	96.34584	0.176503	119.6951	0.024333	115.4101	0.097175	101.9327	0.063197
81	75.45168	0.19038	118.9432	0.07218	142.1299	0.020416	103.5981	0.125686	97.46903	0.053177	124.812	0.028867	127.0718	0.040812	120.4757	0.0174
82	120.2507	0.089132	121.0542	0.086807	125.2053	0.094769	125.0358	0.093933	93.36234	0.038724	99.28907	0.069412	104.7989	0.140378	115.0608	0.046657
83	128.4671	0.053222	128.6234	0.090682	126.4158	0.0378	121.3729	0.095407	88.6016	0.001644	133.0017	0.011887	109.2052	0.145948	112.0236	0.115511
84	108.6728	0.003855	130.0568	0.063576	116.206	0.071465	119.5108	0.096679	99.8351	0.031587	121.5984	0.019675	112.2313	0.124952	123.5091	0.005954
85	135.4685	0.038666	108.4952	0.085703	136.0485	0.050798	108.7191	0.110985	113.4178	0.035216	112.4263	0.062569	114.0099	0.119091	97.42936	0.088019
86	129.5839	0.032958	124.0447	0.083505	120.214	0.123598	68.01754	0.340195	130.9648	0.051083	103.2355	0.059344	95.55774	0.170452	123.3803	0.046314
87	62.99595	0.251649	112.7701	0.087782	117.8609	0.062594	132.3072	0.057133	93.63592	0.031943	96.41092	0.103268	98.69664	0.16167	117.881	0.046427
88	111.7288	0.061867	120.9314	0.110278	122.2197	0.089543	116.1205	0.063441	94.17036	0.08011	103.6236	0.080435	102.938	0.142821	94.74532	0.052866
89	113.8994	0.071615			118.0234	0.057109	97.89557	0.160132	101.4185	0.06103	104.9011	0.052299	104.0667	0.14979	98.58386	0.065911
90	116.2838	0.052038			124.7073	0.063092	107.6526	0.009256	108.8012	0.082658	101.4158	0.089161	100.3304	0.082927	100.7491	0.069179
91	86.72893	0.069104			127.3827	0.005596	123.5113	0.04683	95.69545	0.101212	93.47449	0.062162	124.9153	0.087051	102.3648	0.082588
92	105.8975	0.012779			118.1797	0.005479	100.8998	0.197268	114.8738	0.107441	121.5383	0.035336	100.2562	0.149565	113.516	0.102555
93	116.0434	0.051903			124.8768	0.068665	106.4647	0.000149	91.25224	0.066439	109.8142	0.017849	109.6778	0.132439	95.04164	0.0662
94	78.44734	0.08857			122.8207	0.061512	134.5168	0.068534	92.87703	0.127955	129.8828	0.038292	131.2478	0.056734	117.2909	0.090094
95	125.7461	0.047949			98.68823	0.090506	124.4078	0.050435	92.64711	0.036359	136.6215	0.036893	110.9821	0.104974	103.7163	0.093475
96	62.2551	0.294345			96.1467	0.09778	92.05169	0.190651	88.83703	0.033253	97.39787	0.124674	79.76179	0.202464	105.6869	0.066635
97	80.03914	0.192602			109.9569	0.073541	127.7798	0.075578	111.4765	0.054964	121.4387	0.033144	105.9723	0.130314	92.9285	0.083973
98	138.6966	0.038725			115.0634	0.050958	107.1135	0.085329	117.9368	0.012723	116.9334	-0.00964	109.5169	0.115828	102.6961	0.106474
99	90.55598	0.040486			123.0154	0.054993	63.82746	0.274129	105.7626	0.077011	124.8627	0.049409	96.85505	0.141652	107.0577	0.078913
100	107.2205	0.116667			123.3295	0.064379	127.3112	0.043735	83.05968	0.075796	98.34105	0.060144	80.22913	0.161995	86.71243	0.105747
101	89.40562	0.061776			129.1221	0.056047	117.8377	0.082279	104.8476	0.100782	98.97895	0.074617	115.719	0.145107	127.2674	0.072951
102	97.10365	0.028718			109.8979	0.117892	83.57871	0.208543			109.4573	0.046438	99.68063	0.139533	125.2306	0.060781

Table A5.1 continued

	Rider											
	1		3		4		6		7		8	
c	8.67E-06		2.48E-05		8.28E-05		4.06E-05		2.33E-05		6.87E-06	
session	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
103	107.1056	0.025792	112.4154	0.123962	97.97343	0.088827	93.93445	0.081817	97.79301	0.16555	100.0542	0.075968
104	143.934	0.021673	124.1984	0.062686	126.1825	0.066931	116.2941	0.035713	118.4238	0.091586	109.2003	0.052073
105	102.9302	0.087934	113.7342	0.068929	118.1354	0.065462	126.6046	0.089888	117.5052	0.073539	104.6469	0.057481
106	96.39052	0.033264	117.7985	0.061712	100.4557	0.188323	82.00736	0.1331	99.6948	0.151692	112.216	0.047863
107	100.5209	0.029146	132.1105	0.044017	99.24907	0.22197	114.1481	0.035374	56.23281	0.274316	104.6041	0.052445
108	109.9044	0.015276	125.0923	0.028758	81.68227	0.268304	108.9002	0.051731	105.9281	0.13271	91.54643	0.090898
109	129.2629	0.055946			51.32936	0.387344	98.32905	0.123615	97.27874	0.110171	106.4918	0.053807
110	79.69685	0.100989			19.81195	0.49733	135.1445	0.060191	104.2533	0.131041	91.35518	0.071182
111	122.2139	0.003597			117.2197	0.110232	139.364	0.044624	140.4428	0.04482	85.34664	0.097662
112	122.125	0.045756			67.25508	0.335875	127.6608	0.039876	32.43284	0.184195	126.6305	0.031407
113							134.3002	0.032479	55.65296	0.338774	103.8207	0.050218
114							82.80307	0.097774	106.3731	0.112238	116.3842	0.045749
115							95.75481	0.093258	100.945	0.134359	86.01981	0.10256
116							121.9895	0.079325	115.8954	0.074528	104.3243	0.062664
117							102.3233	0.056387	96.52278	0.169625	102.4669	0.074243
118							107.3091	0.116927	100.5316	0.136769	109.5538	0.034025
119							117.237	0.097492	123.6884	0.112334	96.87661	0.078348
120							117.6409	0.063674	104.9009	0.078224	111.3802	0.067364
121							120.3626	0.058488	99.56414	0.147008	103.2434	0.10168
122							81.70251	0.13284	101.928	0.148035	107.5959	0.080782
123							98.12946	0.094423	107.993	0.150361	106.6603	0.075439
124							96.19163	0.084755	110.785	0.075991	115.53	0.047251
125							126.3705	0.069069	106.4696	0.126632	106.8378	0.069912
126							98.7304	0.090219	60.59026	-0.01451	100.6426	0.070714
127							106.775	0.075436	96.33871	0.142388	113.1395	0.040922
128							119.2334	0.068136	102.9907	0.009583	87.80428	0.091071
129							127.1452	0.049657	101.981	0.078338	94.4896	0.083294
130							134.2287	0.03417	104.7933	0.098807	102.2312	0.075212
131							122.6427	0.072279	96.96074	0.143746	94.84421	0.094722
132							93.4774	0.124288	100.3839	0.059626	102.5439	0.068841
133							111.343	0.076461	122.5855	0.035064	98.95965	0.063932
134							141.4789	0.034733	117.6856	0.01798	106.0704	0.082297
135							132.1593	0.035674	106.4104	0.078971	97.92685	0.070922
136							115.3243	0.059439	103.2255	0.063925	95.37505	0.085032

Table A5.1 continued

	Rider					
	6		7		8	
c	4.06E-05		2.33E-05		6.87E-06	
session	a_i	b_i	a_i	b_i	a_i	b_i
137	125.6624	0.059581	92.72321	0.132948	89.66582	0.139975
138	122.3397	0.056177	100.5936	0.079436	100.7416	0.058886
139	113.4873	0.055245	102.5739	0.077724	114.5377	0.047731
140	106.2128	0.087569	73.61588	0.134147	109.3767	0.066563
141	117.5647	0.068289	103.3863	0.090958	88.50065	0.088909
142	90.93482	0.101306	125.9487	0.016027	94.65675	0.07546
143	109.2395	0.066037	73.11727	0.107104	99.59822	0.084822
144	113.464	0.03784	102.1984	0.04056	95.91946	0.070638
145	103.0193	0.097247	110.7136	0.041418	98.81516	0.092504
146	102.7681	0.132965	97.28612	0.09679	98.19352	0.064236
147			99.31509	0.073868	101.3791	0.071708
148			92.97135	0.077087	102.6367	0.086889
149			113.5497	0.00301	89.46853	0.154136
150			108.3882	0.032566	85.33253	0.17401
151			102.3404	0.089414	82.0967	0.182815
152			93.32764	0.155045	84.58332	0.147244
153					86.14662	0.179286
154					78.74131	0.191806
155					90.6821	0.147332
156					100.8698	0.135482
157					99.89055	0.075554
158					104.2723	0.084077
159					109.2263	0.067026
160					105.7421	0.138218
161					70.60357	0.300814
162					90.30229	0.100765

Table A5.1 continued

	Rider																
	9		9			9			9			9			9		
c	1.79E-05		1.79E-05			1.79E-05			1.79E-05			1.79E-05			1.79E-05		
session	a_i	b_i	session	a_i	b_i	session	a_i	b_i	session	a_i	b_i	session	a_i	b_i		a_i	b_i
1	1.27E+02	1.41E-01	35	1.05E+02	1.48E-01	69	1.12E+02	1.37E-01	103	1.20E+02	1.17E-01	137	1.34E+02	1.13E-01	171	1.02E+02	1.65E-01
2	1.23E+02	1.58E-01	36	3.56E+01	5.60E-01	70	4.52E+01	5.24E-01	104	1.03E+02	1.24E-01	138	1.42E+02	8.17E-02	172	6.69E+01	2.61E-01
3	1.28E+02	1.24E-01	37	1.16E+02	1.01E-01	71	3.05E+01	6.22E-01	105	7.79E+01	3.75E-01	139	1.03E+02	9.24E-02	173	1.11E+02	7.58E-02
4	1.26E+02	1.53E-01	38	1.12E+02	1.35E-01	72	4.38E+01	5.60E-01	106	1.45E+02	8.44E-02	140	1.30E+02	7.85E-02	174	1.23E+02	1.82E-02
5	1.29E+02	1.44E-01	39	1.16E+02	1.13E-01	73	9.14E+01	2.51E-01	107	1.06E+02	1.05E-01	141	9.56E+01	1.50E-01	175	1.20E+02	2.28E-02
6	1.19E+02	1.71E-01	40	1.13E+02	1.31E-01	74	1.27E+02	1.14E-01	108	1.17E+02	8.35E-02	142	9.67E+01	1.87E-01	176	1.11E+02	9.67E-02
7	1.42E+02	1.09E-01	41	1.13E+02	1.51E-01	75	1.13E+02	1.34E-01	109	1.24E+02	9.18E-02	143	1.26E+02	1.15E-01	177	1.05E+02	1.42E-01
8	1.19E+02	1.41E-01	42	1.17E+02	1.09E-01	76	1.15E+02	1.77E-01	110	1.10E+02	8.41E-02	144	1.23E+02	1.13E-01	178	1.25E+02	1.07E-01
9	1.32E+02	1.08E-01	43	1.22E+02	1.08E-01	77	1.16E+02	1.70E-01	111	9.30E+01	1.78E-01	145	1.05E+02	9.75E-02	179	1.20E+02	1.29E-01
10	1.21E+02	1.30E-01	44	1.12E+02	1.54E-01	78	1.12E+02	1.69E-01	112	1.36E+02	1.27E-01	146	1.46E+02	6.71E-02	180	9.31E+01	1.71E-01
11	1.21E+02	1.09E-01	45	5.48E+01	5.01E-01	79	9.50E+01	2.48E-01	113	1.34E+02	1.01E-01	147	1.16E+02	1.01E-01	181	8.82E+01	2.98E-01
12	1.26E+02	1.14E-01	46	3.36E+01	5.98E-01	80	1.14E+02	1.79E-01	114	4.81E+01	5.42E-01	148	1.11E+02	5.21E-02	182	1.04E+02	1.27E-01
13	1.35E+02	7.71E-02	47	1.68E+01	7.30E-01	81	1.06E+02	1.69E-01	115	7.00E+01	4.20E-01	149	1.05E+02	1.44E-01	183	1.17E+02	1.14E-01
14	1.25E+02	1.35E-01	48	1.27E+02	1.16E-01	82	1.18E+02	1.53E-01	116	1.22E+02	9.59E-02	150	1.32E+02	9.15E-02	184	1.15E+02	1.15E-01
15	1.17E+02	1.67E-01	49	1.27E+02	9.88E-02	83	9.11E+01	1.94E-01	117	5.48E+01	5.15E-01	151	9.28E+01	1.44E-01	185	1.08E+02	9.38E-02
16	1.27E+02	1.06E-01	50	1.27E+02	8.95E-02	84	1.20E+02	1.21E-01	118	5.53E+01	4.79E-01	152	1.21E+02	1.16E-01	186	1.39E+02	7.23E-02
17	4.54E+01	5.55E-01	51	4.23E+01	5.28E-01	85	1.16E+02	1.60E-01	119	1.35E+02	1.13E-01	153	1.11E+02	1.20E-01	187	9.36E+01	1.34E-01
18	1.21E+02	1.23E-01	52	1.21E+02	1.42E-01	86	1.15E+02	1.48E-01	120	1.14E+02	1.19E-01	154	1.04E+02	1.06E-01	188	8.61E+01	2.51E-01
19	1.18E+02	1.36E-01	53	1.16E+02	1.11E-01	87	9.64E+01	2.07E-01	121	1.09E+02	1.21E-01	155	1.46E+02	7.92E-02	189	1.03E+02	1.39E-01
20	4.63E+01	4.87E-01	54	1.26E+02	9.59E-02	88	1.15E+02	1.64E-01	122	1.09E+02	1.43E-01	156	1.21E+02	1.19E-01	190	1.01E+02	1.64E-01
21	1.18E+02	1.29E-01	55	1.24E+02	1.14E-01	89	1.21E+02	5.79E-02	123	1.19E+02	1.09E-01	157	1.10E+02	1.92E-01	191	1.16E+02	1.04E-01
22	1.17E+02	1.17E-01	56	5.22E+01	4.87E-01	90	1.10E+02	1.66E-01	124	7.89E+01	3.33E-01	158	1.02E+02	1.75E-01	192	1.34E+02	7.67E-02
23	1.17E+02	1.04E-01	57	1.20E+02	1.19E-01	91	8.68E+01	2.16E-01	125	6.39E+01	4.82E-01	159	1.08E+02	1.27E-01	193	4.72E+01	5.90E-01
24	1.25E+02	9.00E-02	58	6.13E+01	4.28E-01	92	1.20E+02	1.41E-01	126	1.44E+02	9.35E-02	160	1.10E+02	1.71E-01	194	1.10E+02	1.74E-01
25	1.23E+02	8.17E-02	59	6.45E+01	3.96E-01	93	1.09E+02	1.55E-01	127	1.00E+02	1.34E-01	161	9.09E+01	2.75E-01	195	4.45E+01	5.14E-01
26	8.85E+01	2.13E-01	60	1.21E+02	1.19E-01	94	8.98E+01	1.70E-01	128	1.26E+02	9.98E-02	162	1.05E+02	1.78E-01	196	1.24E+02	1.04E-01
27	1.14E+02	8.51E-02	61	6.66E+01	3.93E-01	95	8.32E+01	1.80E-01	129	1.15E+02	9.33E-02	163	9.87E+01	1.50E-01	197	1.34E+02	7.85E-02
28	1.25E+02	9.03E-02	62	1.33E+02	7.52E-02	96	1.05E+02	1.56E-01	130	1.04E+02	1.32E-01	164	1.19E+02	1.39E-01			
29	3.95E+01	5.37E-01	63	1.14E+02	1.25E-01	97	1.25E+02	1.10E-01	131	1.27E+02	9.76E-02	165	1.15E+02	1.72E-01			
30	1.12E+02	1.22E-01	64	1.13E+02	1.56E-01	98	1.05E+02	1.35E-01	132	1.15E+02	1.08E-01	166	1.11E+02	1.86E-01			
31	3.90E+01	5.64E-01	65	1.14E+02	1.51E-01	99	9.85E+01	2.05E-01	133	1.19E+02	1.40E-02	167	1.01E+02	1.62E-01			
32	1.23E+02	1.01E-01	66	6.28E+01	4.38E-01	100	9.31E+01	1.97E-01	134	1.24E+02	1.21E-01	168	8.43E+01	1.92E-01			
33	6.34E+01	4.29E-01	67	1.11E+02	1.83E-01	101	1.28E+02	9.31E-02	135	1.09E+02	5.37E-02	169	1.32E+02	1.05E-01			
34	1.16E+02	1.18E-01	68	1.20E+02	1.06E-01	102	1.21E+02	1.13E-01	136	7.06E+01	3.73E-01	170	8.90E+01	2.06E-01			

Table A5.1 continued

	Rider																
	10		10			10			10			10			10		
c	9.49E-05		9.49E-05			9.49E-05			9.49E-05			9.49E-05			9.49E-05		
session	a_i	b_i	session	a_i	b_i	session	a_i	b_i	session	a_i	b_i	session	a_i	b_i		a_i	b_i
1	9.19E+01	1.35E-01	35	9.34E+01	1.40E-01	69	7.83E+01	1.49E-01	103	8.50E+01	1.41E-01	137	8.25E+01	1.15E-01	171	6.83E+01	1.76E-01
2	8.84E+01	1.13E-01	36	8.23E+01	1.30E-01	70	8.44E+01	1.81E-01	104	1.09E+02	5.25E-02	138	9.59E+01	9.42E-02	172	5.31E+01	1.93E-01
3	1.14E+02	6.81E-02	37	1.06E+02	1.03E-01	71	1.23E+02	7.86E-02	105	1.04E+02	1.33E-01	139	1.11E+02	8.98E-02	173	9.26E+01	1.05E-01
4	8.77E+01	1.30E-01	38	7.90E+01	1.41E-01	72	9.94E+01	1.09E-01	106	7.53E+01	1.33E-01	140	7.48E+01	1.46E-01	174	8.64E+01	1.52E-01
5	1.00E+02	1.14E-01	39	6.85E+01	1.79E-01	73	8.30E+01	1.60E-01	107	7.24E+01	1.49E-01	141	6.86E+01	1.70E-01	175	8.99E+01	1.83E-01
6	8.91E+01	9.83E-02	40	4.30E+01	3.79E-01	74	8.78E+01	1.29E-01	108	9.13E+01	1.58E-01	142	8.61E+01	1.54E-01	176	8.23E+01	1.86E-01
7	8.81E+01	1.38E-01	41	9.45E+01	1.61E-01	75	9.41E+01	1.40E-01	109	9.18E+01	1.66E-01	143	9.36E+01	1.10E-01	177	9.83E+01	1.21E-01
8	9.30E+01	1.27E-01	42	9.66E+01	1.12E-01	76	1.01E+02	9.89E-02	110	7.40E+01	1.64E-01	144	7.30E+01	1.54E-01	178	1.11E+02	8.93E-02
9	9.56E+01	1.56E-01	43	7.28E+01	2.19E-01	77	1.05E+02	1.09E-01	111	8.22E+01	1.25E-01	145	7.51E+01	1.25E-01	179	9.70E+01	1.07E-01
10	5.43E+01	2.67E-01	44	9.47E+01	8.55E-02	78	9.99E+01	1.02E-01	112	8.68E+01	1.97E-01	146	1.25E+02	6.54E-02	180	1.02E+02	1.38E-01
11	8.38E+01	1.56E-01	45	9.94E+01	1.18E-01	79	9.64E+01	1.04E-01	113	9.87E+01	1.32E-01	147	8.40E+01	1.28E-01	181	9.17E+01	1.27E-01
12	1.19E+02	5.00E-02	46	6.74E+01	2.34E-01	80	9.98E+01	1.07E-01	114	9.20E+01	1.66E-01	148	9.77E+01	1.18E-01	182	1.09E+02	7.29E-02
13	1.06E+02	9.65E-02	47	6.97E+01	1.71E-01	81	8.90E+01	1.32E-01	115	8.15E+01	1.22E-01	149	8.56E+01	1.59E-01	183	1.41E+02	1.01E-01
14	1.13E+02	8.37E-02	48	1.01E+02	1.48E-01	82	8.97E+01	1.13E-01	116	9.55E+01	1.61E-01	150	1.05E+02	1.02E-01	184	8.76E+01	8.56E-02
15	7.81E+01	1.48E-01	49	5.03E+01	3.03E-01	83	1.00E+02	1.45E-01	117	9.16E+01	1.13E-01	151	7.49E+01	1.46E-01	185	1.11E+02	8.41E-02
16	8.43E+01	1.33E-01	50	6.33E+01	2.29E-01	84	6.07E+01	2.51E-01	118	9.15E+01	1.76E-01	152	6.74E+01	1.52E-01	186	9.05E+01	1.13E-01
17	9.39E+01	8.69E-02	51	7.55E+01	1.61E-01	85	1.16E+02	8.11E-02	119	7.51E+01	1.72E-01	153	9.47E+01	1.15E-01	187	7.15E+01	2.08E-01
18	9.16E+01	7.95E-02	52	9.37E+01	1.09E-01	86	9.23E+01	1.06E-01	120	1.01E+02	1.43E-01	154	9.34E+01	1.69E-01	188	9.69E+01	8.30E-02
19	6.34E+01	1.80E-01	53	8.48E+01	1.10E-01	87	1.08E+02	8.55E-02	121	6.89E+01	1.80E-01	155	8.48E+01	1.24E-01	189	8.22E+01	1.26E-01
20	8.83E+01	1.36E-01	54	9.48E+01	1.43E-01	88	7.12E+01	1.58E-01	122	7.07E+01	1.53E-01	156	9.19E+01	1.07E-01	190	7.75E+01	1.51E-01
21	8.03E+01	1.18E-01	55	7.55E+01	1.50E-01	89	7.54E+01	1.29E-01	123	9.00E+01	1.87E-01	157	1.06E+02	1.22E-01	191	8.65E+01	1.49E-01
22	1.05E+02	8.98E-02	56	9.36E+01	1.82E-01	90	9.51E+01	1.30E-01	124	1.10E+02	8.04E-02	158	8.06E+01	1.79E-01	192	8.41E+01	1.14E-01
23	9.18E+01	1.08E-01	57	7.38E+01	2.64E-01	91	8.39E+01	1.15E-01	125	1.07E+02	8.17E-02	159	9.04E+01	1.46E-01	193	1.13E+02	7.70E-02
24	7.35E+01	1.64E-01	58	8.56E+01	1.14E-01	92	1.07E+02	8.65E-02	126	8.88E+01	1.12E-01	160	8.99E+01	1.04E-01	194	9.72E+01	1.31E-01
25	9.36E+01	1.07E-01	59	7.49E+01	1.74E-01	93	7.42E+01	1.62E-01	127	9.84E+01	1.39E-01	161	9.95E+01	1.14E-01	195	8.85E+01	1.17E-01
26	1.01E+02	1.24E-01	60	7.15E+01	2.57E-01	94	9.86E+01	1.23E-01	128	8.39E+01	1.30E-01	162	9.03E+01	8.67E-02	196	9.95E+01	1.02E-01
27	8.52E+01	1.40E-01	61	8.63E+01	1.15E-01	95	9.34E+01	9.93E-02	129	1.27E+02	7.10E-02	163	6.64E+01	8.76E-02	197	8.69E+01	9.17E-02
28	7.54E+01	9.41E-02	62	7.31E+01	1.82E-01	96	1.08E+02	1.22E-01	130	8.89E+01	1.78E-01	164	1.20E+02	7.08E-02	198	1.13E+02	7.06E-02
29	1.12E+02	8.12E-02	63	9.40E+01	9.77E-02	97	1.17E+02	7.90E-02	131	9.97E+01	1.15E-01	165	4.48E+01	3.28E-01	199	9.40E+01	1.39E-01
30	1.09E+02	9.72E-02	64	7.01E+01	2.57E-01	98	9.91E+01	9.27E-02	132	9.16E+01	1.10E-01	166	8.96E+01	1.60E-01	200	1.08E+02	9.87E-02
31	1.05E+02	1.05E-01	65	1.04E+02	1.07E-01	99	9.45E+01	1.09E-01	133	8.10E+01	1.75E-01	167	8.71E+01	1.26E-01	201	1.43E+02	5.09E-02
32	7.62E+01	1.45E-01	66	1.19E+02	6.03E-02	100	8.89E+01	1.36E-01	134	6.75E+01	2.31E-01	168	9.30E+01	7.30E-02	202	8.00E+01	1.33E-01
33	8.04E+01	1.70E-01	67	8.60E+01	1.05E-01	101	8.57E+01	2.03E-01	135	7.08E+01	1.51E-01	169	8.12E+01	1.80E-01	203	7.76E+01	1.12E-01
34	7.92E+01	1.45E-01	68	9.91E+01	1.35E-01	102	1.19E+02	8.94E-02	136	6.98E+01	1.34E-01	170	9.06E+01	9.43E-02	204	1.07E+02	1.29E-01

Table A5.1 continued

	Rider				
	10		10		
c	9.49E-05		9.49E-05		
session	a_i	b_i	session	a_i	b_i
205	1.10E+02	8.87E-02	229	8.59E+01	1.15E-01
206	9.48E+01	7.97E-02	230	1.02E+02	1.06E-01
207	8.22E+01	1.41E-01	231	1.03E+02	1.08E-01
208	7.66E+01	1.58E-01	232	9.48E+01	1.13E-01
209	8.93E+01	1.19E-01	233	8.90E+01	1.22E-01
210	1.07E+02	1.05E-01	234	9.63E+01	1.18E-01
211	8.56E+01	1.46E-01	235	1.00E+02	1.19E-01
212	8.83E+01	1.51E-01	236	1.13E+02	1.00E-01
213	9.56E+01	1.34E-01	237	7.83E+01	1.43E-01
214	8.97E+01	1.24E-01	238	9.93E+01	1.36E-01
215	8.06E+01	1.41E-01	239	7.80E+01	1.55E-01
216	7.61E+01	1.26E-01	240	1.11E+02	8.76E-02
217	8.39E+01	1.80E-01	241	9.59E+01	1.03E-01
218	8.15E+01	1.55E-01	242	8.09E+01	1.39E-01
219	8.76E+01	1.37E-01	243	6.76E+01	1.66E-01
220	7.91E+01	1.48E-01	244	9.64E+01	9.38E-02
221	1.06E+02	8.57E-02	245	9.44E+01	9.95E-02
222	9.85E+01	1.02E-01	246	1.07E+02	1.00E-01
223	9.03E+01	1.33E-01	247	1.02E+02	8.45E-02
224	1.07E+02	1.08E-01	248	9.81E+01	1.39E-01
225	9.18E+01	1.15E-01	249	1.08E+02	9.63E-02
226	1.06E+02	9.12E-02	250	9.41E+01	1.01E-01
227	1.09E+02	8.32E-02	251	1.13E+02	7.70E-02
228	1.01E+02	1.17E-01			

Appendix 6: Estimate of $\boldsymbol{P_{75}}$ varied from session to session

Table A6.1 Estimate of P_{75} varied from session to session for rider 6

Session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$	Session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$	Session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$
1	355	10.41002	50	339	10.51598	99	406	15.88491
2	337	13.39036	51	328	11.97989	100	324	20.84447
3	331	3.962350	52	391	11.00175	101	261	4.882689
4	352	12.27743	53	350	11.94490	102	326	8.423851
5	318	8.573719	54	316	11.05721	103	334	6.757686
6	346	3.650689	55	339	15.93954	104	301	12.08057
7	343	3.621892	56	307	14.99371	105	390	6.916619
8	372	11.71607	57	313	15.52710	106	334	6.400745
9	273	2.241523	58	295	8.661528	107	286	6.246048
10	355	5.001092	59	317	4.752792	108	348	14.80713
11	335	10.42871	60	382	5.264503	109	345	13.39200
12	347	7.485250	61	327	15.61183	110	460	5.899502
13	290	1.812566	62	324	23.85318	111	471	5.899558
14	313	2.121743	63	448	7.837197	112	448	7.624634
15	376	4.337871	64	364	4.447928	113	434	6.266813
16	290	4.842705	65	340	11.03745	114	257	8.742518
17	338	7.016416	66	370	12.35195	115	319	7.145996
18	329	4.307448	67	386	23.40781	116	469	6.240422
19	271	2.000231	68	345	11.99802	117	296	5.705383
20	307	2.383162	69	392	5.360795	118	453	9.040678
21	321	6.087152	70	346	22.79400	119	492	8.091564
22	307	4.161582	71	386	11.53286	120	356	5.022445
23	334	3.773358	72	328	13.67470	121	384	4.586548
24	316	2.718794	73	364	13.61960	122	293	3.268606
25	337	3.505776	74	330	11.08709	123	304	3.103142
26	403	4.346306	75	314	16.06551	124	314	7.669029
27	350	12.91889	76	366	6.948257	125	417	9.018291
28	343	13.74127	77	327	21.31731	126	357	8.203618
29	334	6.719513	78	340	11.36216	127	314	8.428024
30	343	6.970868	79	360	18.31018	128	415	4.853552
31	333	16.03056	80	312	6.098354	129	379	3.608784
32	302	13.98299	81	363	16.51182	130	404	4.319860
33	281	10.81445	82	366	13.61468	131	431	6.246871
34	310	10.07823	83	338	16.46133	132	456	17.77375
35	359	4.391698	84	353	19.02634	133	327	3.514917
36	369	13.65608	85	389	8.650038	134	434	6.388145
37	339	15.66927	86	408	6.039488	135	405	6.610391
38	359	5.497476	87	360	7.795452	136	323	3.058090
39	356	5.783763	88	358	5.528996	137	465	5.131619
40	344	5.828989	89	326	5.691916	138	431	4.129580
41	377	6.352847	90	390	6.752671	139	323	7.228653
42	370	4.859154	91	311	8.329972	140	367	5.991444
43	367	5.486066	92	334	16.31556	141	403	4.929512
44	344	14.30567	93	318	8.398536	142	304	4.976719
45	347	5.612856	94	355	28.83102	143	356	2.943779
46	343	5.731751	95	415	6.258702	144	314	2.777567
47	311	14.69535	96	316	17.28652	145	359	5.355087
48	302	9.506884	97	360	6.539706	146	443	5.486599
49	343	18.89016	98	303	9.183595			

Table A6.2 Estimate of P_{75} varied from session to session for rider 7

Session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$
1	331	2.182126	52	304	11.12223	103	364	3.116684
2	197	3.175558	53	319	1.056933	104	412	49.09037
3	239	4.299093	54	281	14.37232	105	301	3.976704
4	299	2.230395	55	289	10.62975	106	330	8.320848
5	221	1.035962	56	283	16.58355	107	309	1.087084
6	210	0.605303	57	333	3.634823	108	426	38.90773
7	259	2.013979	58	321	4.437072	109	252	6.414872
8	134	1.003359	59	392	1.324916	110	326	10.54936
9	274	1.851470	60	338	0.932153	111	423	8.073661
10	222	2.009160	61	226	8.171397	112	377	6.007071
11	264	4.656757	62	236	8.319491	113	131	1.865975
12	159	1.276386	63	382	2.798801	114	387	52.93370
13	352	2.449284	64	380	7.483591	115	342	6.768015
14	268	6.111733	65	371	16.81224	116	278	33.01457
15	237	3.212853	66	221	2.769812	117	311	7.293810
16	266	2.823145	67	191	11.26426	118	346	11.30209
17	313	2.762691	68	386	4.016626	119	380	2.245933
18	334	2.175781	69	329	1.075017	120	238	14.25783
19	250	2.221217	70	401	8.295078	121	369	9.958891
20	148	1.378695	71	175	4.877939	122	434	4.916253
21	290	3.752383	72	413	3.597431	123	399	4.625772
22	387	18.89768	73	389	40.47025	124	192	3.000624
23	250	5.565190	74	263	3.664239	125	268	3.654916
24	259	3.713700	75	413	8.625442	126	242	4.266377
25	303	2.040151	76	408	1.366333	127	373	8.606538
26	164	0.893516	77	339	1.135997	128	150	4.752680
27	286	2.050268	78	381	4.201907	129	202	3.418831
28	231	3.628319	79	385	19.39015	130	276	3.628647
29	328	6.001074	80	402	35.13932	131	348	10.42862
30	209	0.379079	81	252	3.539999	132	211	3.764265
31	295	2.713164	82	350	30.96664	133	345	20.15330
32	267	7.878041	83	360	3.161911	134	349	36.80293
33	296	2.604581	84	324	0.991668	135	273	3.808683
34	232	2.645359	85	416	3.590206	136	312	37.29256
35	150	1.160311	86	408	15.81618	137	313	8.771861
36	231	2.515937	87	396	8.304808	138	235	4.977331
37	289	2.442499	88	359	4.916810	139	304	23.60067
38	315	1.451285	89	415	1.358146	140	225	19.67507
39	372	33.07134	90	219	6.175679	141	213	7.135018
40	247	3.381480	91	363	3.229841	142	351	16.19059
41	317	3.729545	92	388	4.434014	143	321	13.71120
42	173	0.809744	93	388	3.621961	144	189	8.674881
43	315	1.153071	94	413	6.025836	145	299	24.22019
44	206	1.264032	95	404	47.76884	146	248	4.697737
45	355	27.01561	96	255	10.07393	147	267	22.30917
46	351	8.615169	97	381	3.115986	148	357	18.78145
47	303	1.815507	98	432	14.25998	149	334	21.38477
48	181	0.888781	99	330	21.50681	150	200	15.24404
49	415	15.20197	100	402	6.773751	151	255	4.710463
50	319	13.02116	101	369	3.778691	152	270	9.419217
51	321	19.08917	102	410	5.568659			

Table A6.3 Estimate of P_{75} varied from session to session for rider 8

session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$s.e.(\widehat{P_{75}})$
1	303	2.884678	55	201	3.528672	109	232	4.646028
2	302	3.452381	56	341	1.604921	110	210	2.891882
3	326	3.999792	57	259	3.461001	111	224	3.454605
4	278	3.165949	58	338	2.109955	112	282	13.15528
5	235	3.973116	59	271	4.198105	113	233	3.651160
6	325	4.950891	60	258	3.269987	114	280	12.65298
7	301	3.733604	61	160	7.852385	115	211	4.030352
8	257	3.790424	62	257	4.455306	116	211	5.005959
9	257	4.179388	63	310	14.01293	117	256	4.334818
10	249	4.014862	64	269	2.959345	118	220	4.028681
11	252	2.916313	65	346	7.057930	119	231	3.576234
12	269	3.785059	66	213	4.582285	120	236	5.105328
13	291	4.124895	67	195	3.338030	121	217	15.25967
14	267	2.983801	68	327	4.057805	122	235	6.734609
15	272	4.406317	69	225	4.063300	123	232	3.272550
16	239	3.178289	70	348	4.780463	124	265	13.96406
17	231	2.920208	71	272	4.211246	125	273	7.297727
18	266	3.331132	72	251	3.318438	126	205	4.259130
19	271	4.366203	73	235	2.963132	127	301	7.183706
20	263	4.021614	74	244	3.818360	128	220	4.161874
21	233	1.822629	75	266	3.074748	129	227	3.168729
22	266	3.892787	76	281	2.078079	130	273	3.329636
23	332	5.405703	77	176	3.814749	131	238	5.801415
24	253	3.224902	78	289	5.492669	132	237	2.948560
25	280	3.700884	79	252	3.453493	133	224	3.449461
26	257	1.556911	80	238	3.487161	134	269	8.302846
27	247	3.036474	81	210	3.088220	135	241	3.852672
28	269	3.318014	82	257	6.846562	136	228	3.924525
29	278	5.434220	83	346	3.896919	137	239	4.332971
30	254	4.624186	84	194	2.843759	138	232	3.314696
31	254	2.830624	85	227	5.541735	139	242	5.549219
32	257	4.830311	86	247	5.275896	140	262	3.928699
33	284	4.072438	87	245	3.452990	141	210	2.500340
34	265	3.048713	88	195	2.135391	142	217	4.256817
35	262	5.161412	89	233	3.108339	143	223	3.881184
36	257	3.267288	90	228	3.631514	144	212	3.046821
37	274	3.363647	91	248	5.311541	145	220	3.521015
38	253	4.204451	92	350	1.698465	146	242	3.847181
39	253	3.381089	93	194	1.751621	147	243	4.406508
40	219	3.572495	94	360	2.250050	148	249	4.797613
41	284	4.389060	95	243	5.097654	149	250	6.000223
42	287	4.760793	96	268	5.285363	150	279	3.716181
43	239	4.302381	97	218	2.600508	151	279	2.805216
44	314	16.29067	98	281	5.126917	152	248	1.748118
45	271	4.864866	99	267	4.642761	153	288	2.551762
46	241	2.658636	100	217	3.383810	154	259	8.285633
47	234	2.957381	101	364	3.412050	155	262	2.274719
48	267	3.253065	102	227	16.16693	156	191	4.361064
49	213	2.512216	103	253	4.699375	157	247	3.796296
50	269	2.084548	104	222	4.007052	158	288	6.807778
51	276	9.020181	105	237	5.780782	159	258	5.329545
52	279	2.201305	106	215	4.676399	160	345	1.422366
53	250	1.396162	107	219	3.571278	161	220	3.327543
54	234	3.792308	108	209	2.854357	162	211	3.591690

Table A6.4 Estimate of P_{75} varied from session to session for rider 9

session	$\widehat{P_{75}}$	$se\ (\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$se\ (\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$se\ (\widehat{P_{75}})$	session	$\widehat{P_{75}}$	$se(\widehat{P_{75}})$
1	172	1.515701	51	198	0.806334	101	222	1.048906	151	145	1.687999
2	181	1.380019	52	216	0.909613	102	219	1.459185	152	236	2.149678
3	197	1.476330	53	204	0.834041	103	228	1.620292	153	212	1.726660
4	195	1.055037	54	212	1.036589	104	156	1.663726	154	141	1.837549
5	193	1.142816	55	213	0.763056	105	134	1.571991	155	293	4.585306
6	187	0.907388	56	191	0.369722	106	290	4.009845	156	220	2.269281
7	218	1.802514	57	206	0.652448	107	174	1.311646	157	283	19.18416
8	185	1.090537	58	192	0.498189	108	214	1.370127	158	176	1.043773
9	195	1.073967	59	197	0.425312	109	237	1.901779	159	204	2.632489
10	199	1.233047	60	210	1.095248	110	167	1.098434	160	237	6.363978
11	205	1.316762	61	190	0.243506	111	148	1.468816	161	195	1.519507
12	198	1.443772	62	212	0.854224	112	276	2.849110	162	226	1.329245
13	222	1.923372	63	211	1.214681	113	285	3.337020	163	192	2.084285
14	211	1.052161	64	215	0.892089	114	154	0.732932	164	224	8.542008
15	202	1.153423	65	211	0.779246	115	181	0.713217	165	227	2.805567
16	213	1.039601	66	196	0.442635	116	222	1.701600	166	213	2.350710
17	190	0.466050	67	200	1.165806	117	144	0.535290	167	169	2.451857
18	200	1.000335	68	223	1.686597	118	146	0.444353	168	84	2.593400
19	202	1.134671	69	206	0.953070	119	297	4.766240	169	285	3.661448
20	191	0.456682	70	197	0.618359	120	205	1.342792	170	147	3.785097
21	208	0.972618	71	194	0.415247	121	182	1.380239	171	152	2.516531
22	204	1.055835	72	193	0.496131	122	219	1.265180	172	119	5.447189
23	201	1.007899	73	198	0.630344	123	220	1.888785	173	295	2.842075
24	205	1.162896	74	221	1.435901	124	130	0.600493	174	218	4.660815
25	220	0.939249	75	210	1.369024	125	119	0.677402	175	80	10.42551
26	206	1.092116	76	222	0.912181	126	311	5.275645	176	193	1.504256
27	176	1.171056	77	217	0.874985	127	157	1.604810	177	196	1.466519
28	214	1.289809	78	215	1.030405	128	247	2.082583	178	271	3.802632
29	194	0.370220	79	201	1.529000	129	215	1.382780	179	242	3.551635
30	230	1.555752	80	218	0.698096	130	171	0.986545	180	153	2.836846
31	180	0.466929	81	204	0.719545	131	244	2.258398	181	292	3.134151
32	215	0.992962	82	221	0.829372	132	215	0.951030	182	198	1.912952
33	186	0.421708	83	163	1.256226	133	160	1.114796	183	253	3.091057
34	208	0.887059	84	227	0.921911	134	250	1.803025	184	245	1.643423
35	195	0.940823	85	214	0.780435	135	143	1.821254	185	150	2.073745
36	189	0.367641	86	213	1.037859	136	124	0.838729	186	298	3.587127
37	221	1.084760	87	192	1.505102	137	292	5.170742	187	159	1.820241
38	207	0.832776	88	245	0.970194	138	312	4.944567	188	275	18.0165
39	205	0.818087	89	172	1.418666	139	143	1.553466	189	205	1.958801
40	203	0.833062	90	237	1.741325	140	229	2.349770	190	209	3.582890
41	211	0.961911	91	184	1.506152	141	149	1.664462	191	206	2.182782
42	205	0.870804	92	252	1.193880	142	179	3.730427	192	286	3.818326
43	204	0.776909	93	210	0.881700	143	262	4.290120	193	102	0.521743
44	200	1.250251	94	156	1.794121	144	271	4.189132	194	249	22.31476
45	172	0.485970	95	159	1.526217	145	142	1.647062	195	170	0.836563
46	192	0.769077	96	225	0.884998	146	318	9.126044	196	242	1.901814
47	191	0.501573	97	229	1.072481	147	222	2.018060	197	212	1.746341
48	204	0.747301	98	217	1.117386	148	154	1.273428			
49	212	1.031140	99	211	1.140319	149	177	2.445563			
50	202	0.844016	100	220	1.765878	150	263	3.270088			

Appendix 7: Estimate of $\boldsymbol{p_o}$ varied from session to session for critical power Model (2)

Table A7.1 Estimate of p_0 varied from session to session for rider 6 (Model 2)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	400.8532	67.71744	50	397.9464	35.90838	99	404.3964	47.47883
2	403.4243	47.03214	51	399.8216	42.74067	100	387.8756	36.70479
3	396.0947	68.98283	52	397.9086	60.28590	101	397.4239	40.10464
4	397.2751	62.31705	53	401.4212	42.98891	102	400.4919	41.38092
5	396.2581	59.66429	54	399.0996	40.25687	103	400.6292	43.01497
6	400.1303	70.90412	55	403.3307	42.81585	104	391.7694	44.68249
7	402.1538	71.71055	56	401.1422	36.63734	105	404.1726	53.58915
8	401.6938	68.53939	57	403.0224	42.03076	106	401.1315	43.00993
9	399.4263	50.39891	58	399.3915	35.90713	107	400.1386	41.73891
10	396.4711	72.35960	59	401.2166	46.04925	108	399.5546	37.31907
11	428.4326	54.47862	60	399.2143	66.17416	109	402.5009	55.29240
12	399.8265	60.57396	61	410.2049	42.24311	110	417.6265	75.04165
13	404.2554	50.76221	62	400.4860	42.10437	111	401.6535	72.27719
14	398.2806	44.17646	63	401.3652	64.42307	112	337.4417	78.89449
15	399.9532	74.88714	64	391.5122	60.61233	113	396.8498	51.00228
16	396.7621	44.68522	65	401.9419	42.18268	114	398.5960	45.29586
17	409.3263	59.75700	66	401.0324	48.53980	115	397.2718	64.37273
18	402.1624	62.00253	67	402.5887	44.03798	116	397.5516	73.96201
19	403.9477	41.53936	68	381.6086	38.71442	117	401.6685	58.88909
20	400.7955	68.89658	69	397.9690	57.08577	118	399.4426	85.91085
21	395.4088	70.28088	70	402.2714	43.99017	119	399.6050	83.51770
22	398.5672	49.03134	71	403.1360	53.55858	120	399.0714	68.43331
23	397.8018	74.99965	72	400.1367	43.15928	121	403.3338	60.44758
24	417.2657	71.35152	73	396.5403	55.05327	122	403.3237	50.31855
25	398.5962	65.83529	74	402.4252	41.87799	123	399.5686	55.34459
26	399.2573	94.60866	75	401.4126	35.77639	124	401.6766	65.58489
27	399.5811	43.17262	76	400.2857	48.68658	125	396.3871	69.83721
28	400.8538	43.34435	77	397.7679	44.48297	126	394.4441	92.04268
29	404.9711	49.08405	78	402.4077	40.51979	127	401.1238	56.41969
30	402.6314	46.47635	79	401.9012	44.93446	128	396.2646	64.26668
31	401.0893	41.23437	80	404.5117	44.15341	129	397.0572	77.03262
32	394.4852	35.92163	81	402.6597	43.12279	130	397.3281	74.25841
33	396.3501	38.71902	82	397.4641	41.19731	131	391.2542	65.42373
34	401.3705	39.11751	83	404.3101	41.44301	132	397.7804	63.78409
35	401.7601	57.41711	84	402.6258	42.80686	133	402.0684	91.57407
36	401.4640	44.04715	85	404.6605	63.59979	134	396.7060	64.12365
37	400.3355	39.73697	86	400.8630	88.27764	135	402.8808	79.12915
38	404.4374	47.49508	87	400.8435	65.36489	136	397.3615	62.54910
39	401.3928	45.84271	88	400.3727	71.15012	137	399.1522	61.99222
40	406.1594	43.47078	89	401.8627	55.48075	138	398.4107	95.76330
41	402.4133	50.20716	90	399.2086	81.73778	139	404.2862	58.64851
42	400.0543	59.43890	91	400.5999	75.72486	140	399.1394	65.11599
43	401.4095	58.02975	92	389.8737	43.59324	141	404.3281	71.50100
44	398.6451	43.24303	93	400.4281	36.41349	142	401.8726	52.71365
45	399.9279	54.11548	94	401.2224	39.82839	143	399.1235	74.85044
46	399.3481	59.42067	95	402.1532	52.85740	144	409.1356	49.61977
47	400.2807	42.44252	96	400.6295	45.14187	145	398.4579	97.25911
48	385.8248	37.69723	97	400.5460	45.70532	146	402.4583	83.39548
49	402.9125	45.65217	98	410.2874	42.20801			

Table A7.2 Estimate of p_0 varied from session to session for rider 7 (Model 2)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	410.8061	43.59618	52	401.3474	37.04476	103	403.3027	43.63739
2	399.2033	40.69247	53	402.8799	42.44336	104	402.4510	40.35470
3	399.2837	39.06142	54	403.0262	35.66708	105	401.1203	38.83869
4	402.7931	39.63967	55	402.9980	37.90674	106	402.5632	42.67713
5	401.1835	41.06381	56	400.7112	41.12427	107	399.4046	39.96643
6	394.5383	37.94890	57	403.5606	52.18073	108	400.3086	39.86484
7	402.9864	38.99827	58	416.1171	46.37047	109	402.6288	40.72373
8	392.4965	34.98447	59	403.5524	43.09117	110	402.8931	42.27872
9	398.1468	40.86976	60	399.6573	45.79186	111	396.8075	38.33540
10	393.1935	41.15941	61	403.1816	39.32107	112	403.5755	45.95940
11	396.1178	41.81639	62	402.1565	36.53257	113	388.7009	34.35332
12	348.1351	35.93733	63	407.0952	45.37784	114	402.5980	42.46726
13	401.3580	46.10648	64	402.4687	45.19032	115	394.3686	44.06437
14	399.5087	39.65397	65	404.2687	40.47312	116	400.4994	39.36669
15	396.5777	38.88664	66	404.7854	41.06756	117	403.3800	47.75052
16	404.4129	38.62299	67	401.1072	37.92908	118	404.1671	43.36279
17	399.2477	39.92103	68	402.1513	43.16415	119	406.7317	53.24559
18	403.9822	43.82019	69	405.7384	44.06047	120	395.8565	35.83534
19	402.9624	37.96023	70	402.9992	39.22260	121	399.0981	42.26205
20	366.1796	36.53284	71	400.4093	35.56268	122	399.7221	47.96521
21	404.4730	40.88577	72	403.3425	43.08212	123	410.2397	41.28718
22	381.9901	36.60556	73	401.5456	40.46996	124	399.5692	36.88758
23	400.2039	42.44693	74	389.6336	39.66258	125	398.5342	42.15403
24	400.0770	41.76957	75	401.6550	38.13372	126	402.7422	36.48515
25	394.2161	41.11385	76	406.0331	43.22520	127	401.1418	43.63120
26	403.8906	37.14190	77	404.4210	46.22263	128	401.0867	36.11965
27	402.1597	40.91830	78	402.8348	43.26526	129	404.4467	36.80494
28	402.9145	37.20778	79	403.2289	40.06586	130	397.5596	40.45861
29	399.7919	41.25547	80	401.5410	39.51407	131	399.8034	40.51235
30	396.5387	37.78155	81	401.7803	39.32172	132	402.0257	36.16474
31	404.6966	44.80112	82	402.4291	41.49074	133	399.8539	34.92158
32	404.1462	42.47601	83	406.9628	45.22858	134	397.6504	34.99478
33	406.9874	40.29894	84	413.5459	46.17908	135	398.1485	40.80011
34	409.7544	37.40529	85	403.8164	40.40949	136	395.1603	35.41893
35	391.7351	35.80197	86	392.5767	42.81873	137	403.6365	40.08863
36	396.9071	39.45448	87	398.0230	39.35730	138	401.0738	39.55764
37	401.4243	43.20727	88	404.5600	43.96450	139	401.4729	35.69775
38	405.1771	43.25138	89	399.2847	42.90814	140	400.0965	34.55621
39	399.0945	40.57929	90	398.7683	36.49741	141	398.0947	35.45965
40	400.9231	40.96619	91	403.5359	46.27491	142	395.3066	35.16971
41	405.8284	43.81717	92	404.6059	48.63491	143	403.8194	35.27416
42	392.1635	36.15545	93	401.9159	46.75967	144	382.7960	34.40007
43	404.4528	43.34747	94	402.9363	40.63398	145	373.1748	34.57545
44	397.3239	37.38626	95	398.0850	38.41970	146	402.9555	39.31403
45	400.2812	36.14161	96	398.0260	41.48309	147	356.8698	34.92673
46	404.5665	41.12833	97	406.6966	46.55213	148	404.4068	35.27519
47	404.2225	41.73614	98	403.7185	40.16836	149	405.0909	35.23746
48	390.3283	36.59524	99	402.1368	43.16857	150	405.6320	34.29173
49	386.2502	37.04981	100	406.4670	41.64848	151	405.9982	38.68243
50	404.9029	40.66028	101	401.5492	46.02120	152	402.9255	40.91404
51	397.7544	36.23963	102	403.4421	39.11620			

Table A7.3 Estimate of p_0 varied from session to session for rider 8 (Model 2)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	404.6410	44.86247	55	400.6668	41.77793	109	397.6894	45.77485
2	399.3704	51.97024	56	399.4321	45.21575	110	394.6729	37.61571
3	406.4718	48.56459	57	400.8959	38.85319	111	400.4708	41.17581
4	400.7724	46.67850	58	404.1789	44.79318	112	399.4927	44.08850
5	401.4105	38.74051	59	398.5968	43.07626	113	399.6094	49.33258
6	400.3897	50.96373	60	401.6440	43.37715	114	398.2589	43.52837
7	400.3056	42.76959	61	401.0163	33.98926	115	401.4728	39.09827
8	401.5453	38.61671	62	403.9128	41.22274	116	397.4647	44.96451
9	400.4181	40.57393	63	400.7045	44.04697	117	399.8781	52.56351
10	398.2334	43.71947	64	398.9543	49.23463	118	400.6793	37.75187
11	400.6857	41.66918	65	399.4709	42.13371	119	402.5503	42.22455
12	403.8341	44.77200	66	401.1215	37.91798	120	399.9370	47.06342
13	402.7171	43.69892	67	390.6182	36.20191	121	398.9885	42.09648
14	401.7673	48.56697	68	400.6699	41.04790	122	399.0376	51.20927
15	400.6782	45.21050	69	402.0915	43.97085	123	407.7075	51.89233
16	400.5318	41.58013	70	403.2785	44.23835	124	398.7597	43.22046
17	397.3512	41.10853	71	399.9597	48.97796	125	406.4627	55.48886
18	404.5048	45.52756	72	401.2146	42.72548	126	401.0424	37.14722
19	398.7144	49.93747	73	400.9335	40.13071	127	399.3373	38.54901
20	402.2534	43.71676	74	399.5233	45.12358	128	400.8824	37.13575
21	390.9441	41.11913	75	404.0516	45.75385	129	400.2598	37.97762
22	400.9765	39.54441	76	400.2822	46.16704	130	399.0400	50.77094
23	399.3589	56.51617	77	397.7978	34.81037	131	400.1913	44.46357
24	399.2352	42.97786	78	394.5832	42.60056	132	400.0826	47.04528
25	396.9151	44.48462	79	403.0772	49.49523	133	398.5811	45.66162
26	399.1800	39.05264	80	400.2604	41.28302	134	401.0220	46.34373
27	406.2012	41.64128	81	400.7403	39.77707	135	402.0088	51.66579
28	402.1633	44.37718	82	396.2574	46.25065	136	402.0509	38.09784
29	402.6076	43.89757	83	399.6018	46.56133	137	400.3215	42.94708
30	402.6849	45.29375	84	399.4610	36.84696	138	386.2759	41.10333
31	399.9086	45.57293	85	399.1932	37.23768	139	400.0628	44.75935
32	400.5956	38.28076	86	401.3263	47.43634	140	399.8649	51.60810
33	399.7064	48.13863	87	401.5599	44.51604	141	399.7507	37.32765
34	398.1913	46.48727	88	400.4566	35.84779	142	402.7179	39.68384
35	398.1562	47.46953	89	401.4160	38.95407	143	400.3764	41.39995
36	397.8325	45.86075	90	399.2342	41.41821	144	402.6231	41.33486
37	400.0297	47.13102	91	399.5562	47.93508	145	400.7792	36.92016
38	398.8913	41.99894	92	399.9278	40.96144	146	388.8603	42.18082
39	400.7318	39.38016	93	384.4391	39.05306	147	399.2460	44.45239
40	400.2052	37.73516	94	399.4602	41.80484	148	400.0297	48.76413
41	398.6465	60.05194	95	403.2859	50.37886	149	400.4771	42.15446
42	401.5257	43.92667	96	400.8765	43.59361	150	401.2558	50.29447
43	400.0937	46.24999	97	399.4917	40.65540	151	400.6005	47.34783
44	398.5750	42.71186	98	398.7690	43.75373	152	404.4729	42.68310
45	403.3483	44.35744	99	414.3847	46.67805	153	401.7679	50.02406
46	398.2158	41.04452	100	400.7425	41.46386	154	403.9082	48.92640
47	400.4732	44.49487	101	398.5531	37.25436	155	402.0588	44.88098
48	403.1795	45.37597	102	388.0260	43.83431	156	397.3111	33.30018
49	398.3187	38.39315	103	399.7953	46.09904	157	400.5069	47.86289
50	399.3108	41.44874	104	401.2403	38.10047	158	405.2964	55.76752
51	400.5021	46.43752	105	401.5987	47.93829	159	403.0001	45.44668
52	403.8326	48.78772	106	383.8828	42.98430	160	389.9656	44.83222
53	397.4104	37.22619	107	400.9563	37.99013	161	398.6434	39.47673
54	405.3042	46.61098	108	400.1839	38.42552	162	401.9349	42.03253

Table A7.4 Estimate of p_0 varied from session to session for rider 9 (Model 2)

session	$\widehat{p_0}$	$se(\widehat{p_0})$	session	$\widehat{p_0}$	$se(\widehat{p_0})$	session	$\widehat{p_0}$	$se(\widehat{p_0})$	session	$\widehat{p_0}$	$se\ (\widehat{p_0})$
1	398.2998	37.07705	51	398.6908	37.09522	101	402.1633	39.96960	151	395.4757	34.83600
2	397.2877	35.87322	52	401.4980	37.61865	102	402.4567	40.17784	152	406.2988	39.82001
3	401.2800	37.80800	53	398.6934	37.22025	103	401.2060	39.99064	153	402.4067	38.55549
4	397.7281	36.38134	54	401.2601	38.11915	104	401.7756	37.17828	154	396.9460	35.55120
5	400.7723	36.73154	55	400.5535	37.29710	105	394.9618	34.10537	155	402.3031	39.40669
6	400.8852	36.72281	56	397.3653	35.81159	106	400.8292	41.39488	156	402.0719	39.48568
7	402.6958	38.31124	57	400.9619	37.29900	107	397.1406	35.84882	157	404.3811	40.35680
8	398.8092	36.29588	58	394.7833	35.65480	108	398.9259	39.12637	158	399.7868	38.12476
9	401.6760	36.95421	59	398.5418	37.45717	109	403.9859	41.45720	159	398.1904	38.40842
10	407.5871	40.36312	60	400.5445	38.65799	110	399.5669	36.30939	160	400.9666	40.22518
11	392.4472	36.78794	61	395.7864	35.72786	111	397.7476	35.21246	161	401.9971	39.68062
12	392.0359	36.86215	62	400.6029	37.78857	112	399.5405	43.86559	162	398.9507	38.54840
13	401.2198	38.82585	63	397.6277	38.98956	113	408.2039	41.92906	163	399.3235	36.87884
14	399.8072	37.91079	64	399.3961	38.33168	114	395.0735	35.26595	164	400.8976	41.74474
15	399.1482	36.79260	65	401.7725	37.31246	115	395.4688	35.58210	165	418.4979	42.57838
16	400.0376	39.18084	66	398.8833	36.66111	116	402.4535	39.64661	166	402.4908	40.48258
17	396.0979	35.81213	67	401.1270	37.23859	117	395.2294	34.53838	167	400.8154	36.04334
18	400.5542	36.89602	68	401.6683	39.78498	118	395.2482	34.29612	168	368.1533	33.76481
19	401.0777	36.93926	69	400.5254	37.49659	119	405.3145	41.59371	169	405.3627	46.18761
20	398.8681	36.66965	70	398.7189	36.96761	120	401.2392	37.62584	170	399.3546	36.51978
21	401.8888	37.87297	71	393.8330	35.94136	121	399.1212	36.81751	171	398.0522	36.79920
22	401.1649	38.24137	72	394.9082	35.66549	122	400.8647	38.82223	172	391.4706	34.34818
23	400.8523	36.85986	73	400.6062	36.74312	123	401.1059	40.78153	173	399.5626	43.90785
24	400.5166	37.23273	74	401.3055	39.58540	124	393.3088	33.87385	174	402.5292	41.46131
25	399.4238	38.54767	75	401.1427	37.47263	125	394.5431	34.11359	175	401.5985	39.83847
26	392.9941	37.51178	76	402.3484	39.42388	126	400.6791	40.16405	176	401.8334	37.66960
27	400.0893	36.52461	77	403.6467	38.35882	127	397.8204	36.05632	177	400.3195	37.75086
28	401.6777	37.36195	78	401.3789	40.23700	128	404.0891	41.67657	178	400.6624	41.20901
29	398.1953	36.57928	79	397.4263	37.68464	129	410.0442	38.22887	179	402.6881	42.17208
30	407.5696	38.85486	80	401.5801	38.20457	130	395.6455	36.13287	180	395.6037	35.87376
31	397.9415	35.58166	81	402.4911	38.61028	131	417.0255	44.18952	181	403.0259	42.69691
32	405.8904	39.12728	82	408.9326	39.22767	132	399.9479	38.06372	182	400.2179	38.03404
33	396.0485	35.65422	83	399.0230	36.15000	133	395.3124	36.09911	183	403.2419	39.84094
34	394.3926	38.02534	84	400.3979	40.17656	134	403.1175	40.79934	184	399.4514	41.13167
35	400.8712	37.05484	85	394.5664	38.36683	135	396.9859	35.22697	185	400.2481	35.74103
36	396.2509	35.66491	86	399.1216	38.91442	136	394.0344	33.88279	186	402.4167	43.07113
37	401.3008	38.13846	87	397.5125	38.71213	137	400.6080	40.91599	187	401.3468	36.70229
38	400.3765	37.64469	88	402.4571	39.78289	138	401.5752	43.88422	188	401.5472	40.68459
39	399.1576	37.06132	89	399.9439	36.80409	139	394.7729	35.14173	189	400.0420	37.11943
40	400.8138	38.79385	90	403.3333	42.25320	140	396.3617	41.14672	190	402.3223	41.65735
41	402.0137	37.88887	91	397.2231	37.27493	141	395.0514	35.20547	191	392.5595	37.81942
42	407.2082	37.28900	92	406.5250	41.63446	142	400.8498	36.55726	192	405.6793	45.18697
43	400.3172	37.32396	93	402.0229	38.19203	143	409.3517	45.67616	193	388.8895	33.30978
44	403.5350	38.02143	94	398.7182	35.97229	144	405.2431	42.13351	194	401.5178	38.41600
45	396.4727	35.66462	95	385.4946	36.37197	145	394.7878	35.11219	195	400.1321	35.81038
46	396.8433	35.72713	96	401.7397	38.53438	146	403.0815	38.99701	196	401.4397	39.81234
47	401.0748	36.43064	97	399.8625	39.83447	147	401.6568	38.74223	197	400.6815	39.91973
48	400.9484	37.33329	98	411.3431	37.84646	148	397.1470	35.52361			
49	401.8647	37.60045	99	408.8974	40.44000	149	398.8326	37.55441			
50	401.6342	37.64293	100	395.2695	38.61633	150	399.8903	41.78839			

Appendix 8: Estimate of $\boldsymbol{p_0}$ varied from session to session for critical power Model (1)

Table A8.1 Estimate of p_0 varied from session to session for rider 6 (Model 1)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	401.0714	46.54909	50	402.1121	37.36126	99	393.8455	41.78896
2	389.1502	41.66487	51	388.1846	39.63769	100	401.3162	37.46394
3	412.2962	47.26937	52	403.7129	44.99861	101	386.1544	38.32589
4	403.2988	45.35421	53	400.6087	40.11334	102	405.4852	39.67907
5	398.9176	44.78730	54	404.5299	39.51902	103	405.3941	40.21303
6	402.8447	47.93313	55	407.4897	40.38380	104	404.7549	40.10422
7	396.9343	48.25219	56	398.7227	37.72177	105	406.6304	43.29487
8	403.897	47.07843	57	397.4523	39.61887	106	406.089	40.20828
9	400.8795	42.79776	58	399.5984	37.39522	107	389.7301	39.13253
10	404.1625	47.99132	59	407.3651	40.90320	108	401.2180	38.37071
11	400.4037	44.34007	60	402.3513	46.31740	109	417.1762	44.05067
12	405.4806	45.55211	61	402.0306	40.20734	110	397.1378	49.60218
13	401.7478	42.96286	62	403.7376	39.91282	111	400.9933	48.95023
14	391.0327	40.06183	63	401.9182	46.53021	112	397.8759	47.42583
15	394.4922	48.89263	64	403.2241	44.71976	113	377.3043	42.65723
16	363.2878	39.42775	65	399.1640	40.12696	114	389.8405	41.23608
17	399.8773	44.92779	66	393.5920	42.05274	115	400.6162	45.90895
18	401.4203	45.13389	67	382.3660	40.63660	116	412.7815	48.91275
19	388.5997	39.43276	68	401.3546	38.13156	117	403.3968	44.80407
20	400.4164	47.01547	69	398.7953	44.75133	118	395.6154	52.16647
21	398.2212	47.87407	70	393.0787	40.57881	119	400.5260	51.32350
22	375.3248	41.31479	71	405.8706	43.26874	120	398.8636	46.98036
23	425.3445	49.46228	72	400.2280	40.17264	121	392.6408	45.59906
24	397.2825	48.79431	73	400.8859	44.77280	122	406.6525	42.32120
25	401.3848	46.46366	74	387.5197	39.61165	123	399.4420	43.76614
26	394.8112	54.58365	75	403.7273	37.26677	124	400.4340	46.74158
27	404.7622	40.23117	76	400.3725	42.49439	125	395.2026	47.55594
28	402.5914	40.34977	77	398.1976	40.47464	126	397.9845	55.36574
29	401.0772	42.51993	78	408.7785	40.01267	127	409.1894	43.98548
30	403.0704	41.39803	79	402.1223	41.23483	128	416.2296	46.39925
31	383.1003	39.15106	80	400.0277	40.76863	129	399.3390	49.42164
32	400.8619	37.22332	81	405.2299	40.54239	130	401.7615	48.71330
33	374.0556	37.55136	82	402.0955	39.30923	131	393.7409	46.52320
34	407.2589	39.35754	83	398.2822	39.76379	132	413.8895	46.01146
35	401.7027	44.76815	84	389.8405	40.14589	133	394.7015	54.04794
36	412.6508	40.81860	85	392.3591	45.59564	134	398.3567	45.91542
37	405.4197	39.37982	86	394.2787	53.12457	135	396.9524	50.69071
38	402.0482	41.81196	87	401.6143	46.28864	136	416.1014	46.05189
39	385.4410	40.98094	88	402.9601	48.17061	137	398.6216	45.58292
40	407.1642	40.78476	89	402.1548	43.77128	138	395.5037	55.21121
41	404.3335	42.55145	90	401.7482	51.63509	139	410.5887	44.82358
42	397.9597	44.61426	91	400.7597	49.20080	140	402.3026	46.01382
43	399.4319	44.86798	92	388.8167	39.33016	141	403.0821	48.39383
44	398.8384	40.17853	93	391.5520	37.50735	142	432.9397	43.72855
45	403.7161	43.57021	94	407.2802	39.33807	143	405.1483	49.47751
46	398.6605	44.95758	95	398.5670	43.30194	144	402.9141	42.32276
47	401.6838	39.75396	96	402.7530	40.69334	145	393.5775	57.10974
48	396.4424	37.48721	97	396.6678	41.54543	146	400.3915	53.01530
49	398.3155	41.03450	98	404.1448	39.96911			

Table A8.2 Estimate of p_0 varied from session to session for rider 7 (Model 1)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	412.6556	48.56708	52	402.3875	41.09829	103	424.7753	50.50531
2	408.9814	46.05829	53	399.7002	44.93345	104	397.8537	45.72031
3	419.8060	44.19024	54	392.5121	38.40671	105	394.5992	44.98919
4	401.4972	42.29151	55	404.5878	42.60114	106	401.5493	51.36454
5	394.5172	42.33173	56	404.6071	47.27469	107	399.2978	43.15008
6	425.2476	38.11246	57	409.1492	61.04311	108	398.0138	47.07665
7	401.4043	43.27412	58	419.9321	52.47061	109	406.7981	48.31398
8	348.5852	35.25158	59	418.8622	48.70361	110	407.2401	51.02042
9	399.4897	45.00245	60	400.4468	50.60931	111	403.7973	44.81189
10	400.1112	42.17386	61	390.3982	44.43013	112	400.1997	56.06578
11	419.4713	45.22514	62	398.7629	39.80420	113	386.2125	34.51913
12	369.1842	36.28810	63	415.3215	51.26289	114	393.0086	49.75833
13	399.6836	54.43289	64	399.0554	52.70339	115	397.3417	52.88572
14	421.5799	41.39076	65	400.6938	45.79945	116	406.2532	45.35451
15	416.1057	39.73369	66	404.0595	46.99521	117	399.4359	56.06184
16	403.0251	42.38519	67	386.8378	42.13558	118	400.6599	51.67914
17	411.5570	44.45891	68	391.9329	50.27896	119	403.6502	60.31742
18	403.8879	48.01105	69	402.9322	47.33928	120	403.7288	38.52551
19	399.4029	41.71632	70	402.1841	42.49022	121	416.2865	48.97273
20	430.7892	36.64079	71	373.9853	38.01145	122	389.5339	55.21168
21	416.1479	46.60376	72	395.3445	50.75674	123	405.4061	46.33661
22	421.3404	40.52057	73	401.3891	45.57376	124	401.9891	39.98108
23	408.2537	47.23171	74	409.8003	43.88476	125	405.9462	45.04943
24	395.3798	42.86173	75	384.5454	44.11613	126	401.4551	39.66244
25	407.5051	45.70020	76	397.5662	50.65852	127	413.0820	51.40814
26	401.9025	38.07572	77	404.1529	50.49962	128	403.9252	38.85276
27	402.8851	45.64841	78	397.5917	50.72071	129	392.9572	39.99605
28	411.9004	40.03429	79	406.3357	45.33319	130	409.5402	46.22772
29	402.9643	45.20485	80	402.7191	44.47622	131	393.1303	46.51832
30	441.1815	38.02846	81	407.9736	41.89731	132	358.9097	38.89163
31	398.4019	50.82462	82	381.1818	48.00133	133	370.2702	36.90367
32	406.9728	45.95783	83	400.6798	53.74424	134	384.4616	36.91958
33	414.0654	45.54951	84	404.6704	51.91487	135	397.1234	46.41152
34	405.8468	40.72085	85	402.1075	46.00616	136	408.7844	37.95543
35	373.1746	36.03821	86	398.3731	50.06258	137	419.2194	46.46779
36	393.7571	44.32647	87	395.4365	45.19429	138	402.5103	44.60951
37	394.8066	50.06151	88	394.2236	53.27537	139	404.4921	38.38731
38	403.4104	46.32401	89	407.0626	50.29826	140	402.2006	35.95402
39	403.6343	44.04541	90	388.8053	39.14882	141	400.5235	37.54513
40	404.3688	49.24287	91	395.5113	56.21825	142	399.7228	37.34459
41	398.8584	53.38053	92	414.4424	57.58100	143	393.2621	37.52748
42	360.7523	36.39138	93	406.2525	55.29685	144	376.6693	35.69622
43	399.5929	46.16123	94	412.6901	48.27994	145	354.6117	36.24768
44	416.2739	37.68873	95	398.8176	42.58615	146	406.5824	44.92591
45	400.6849	39.65865	96	400.5631	48.54728	147	395.7723	36.73737
46	405.2759	46.42281	97	407.0829	54.24795	148	387.2885	37.57968
47	404.1081	46.86218	98	398.9653	47.19871	149	402.1665	37.48209
48	423.1999	36.57316	99	418.2898	52.36964	150	387.7742	35.36889
49	404.7106	41.55952	100	417.9355	48.98331	151	405.8530	44.23937
50	401.6295	44.74479	101	407.1182	53.17156	152	403.9014	46.23464
51	391.3303	39.89917	102	400.2317	44.97430			

Table A8.3 Estimate of p_0 varied from session to session for rider 8 (Model 1)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	403.8586	39.44322	55	400.1262	37.92955	109	391.1075	40.29661
2	399.4848	41.92973	56	398.9411	39.03808	110	404.2913	35.79456
3	401.6004	40.87471	57	404.9377	36.54834	111	404.4051	37.53682
4	398.2019	40.08406	58	397.6953	38.92524	112	398.7353	39.15278
5	404.9863	36.54432	59	400.1611	38.19872	113	396.2722	41.02577
6	399.8113	41.78925	60	399.1172	38.70985	114	400.7425	39.17039
7	404.1824	38.27669	61	379.4337	33.89089	115	400.6041	36.88246
8	401.4143	36.42874	62	395.2066	37.70021	116	400.5885	39.61706
9	405.3507	37.47737	63	398.9237	38.52251	117	398.1599	42.56631
10	395.9491	39.29747	64	399.6394	41.06163	118	402.9755	36.11383
11	403.6789	37.98788	65	400.2562	38.45323	119	402.9794	38.32411
12	403.3119	39.36063	66	407.0122	36.38794	120	396.9261	40.52067
13	404.6005	38.91990	67	396.9961	35.16564	121	399.8432	37.97604
14	403.3537	40.76354	68	399.4727	38.06791	122	388.4061	42.32599
15	403.1952	39.52647	69	404.0714	38.89847	123	400.8031	42.73852
16	398.3496	38.17403	70	402.9178	39.19214	124	400.2999	39.04071
17	400.6401	37.83203	71	400.8175	40.91107	125	395.5551	43.87708
18	404.3429	39.53512	72	404.7232	38.32305	126	401.6471	35.65238
19	390.4648	41.50062	73	405.9878	37.37404	127	401.7575	36.79551
20	402.8482	38.75325	74	400.0187	39.49899	128	397.5481	35.87525
21	385.5347	36.85622	75	426.7554	40.42365	129	402.7968	36.03897
22	404.2181	36.82642	76	403.4381	40.26605	130	399.8958	41.72014
23	398.5746	44.39162	77	402.8531	34.37741	131	399.1154	39.57176
24	397.4157	38.81822	78	400.1078	38.37676	132	391.5343	40.21671
25	399.0283	39.12041	79	399.7885	41.24312	133	398.1990	39.95666
26	396.9418	36.34129	80	401.5117	37.78091	134	403.3523	40.13645
27	404.4055	37.98343	81	402.8265	37.24329	135	396.2082	42.20374
28	397.3055	38.83822	82	412.9697	40.76472	136	398.4929	36.30693
29	404.1496	39.09817	83	396.9469	39.57409	137	401.1207	38.32676
30	402.8426	39.30344	84	402.4755	35.80082	138	401.8606	37.45133
31	401.6966	39.54098	85	400.0381	35.77211	139	399.2357	39.84404
32	394.4124	36.19486	86	393.0835	40.99653	140	399.9469	42.01515
33	400.1297	40.54047	87	404.0035	39.23889	141	404.7723	36.11737
34	399.1924	40.15424	88	375.1694	35.16921	142	401.1643	37.28215
35	394.9968	40.49881	89	404.6204	36.75685	143	401.0954	37.87298
36	398.8918	39.59376	90	397.0971	38.59044	144	393.6042	37.79434
37	398.9066	40.07865	91	398.7366	40.57283	145	403.1973	35.55236
38	399.9821	38.32313	92	399.6527	37.31041	146	399.3919	38.04267
39	406.3133	36.79868	93	399.4302	36.73416	147	400.2514	39.36833
40	403.0877	35.97711	94	399.8357	37.71917	148	398.0808	40.94748
41	390.7402	45.43949	95	396.9997	41.73937	149	402.5049	38.06759
42	402.2750	38.80639	96	404.3313	39.04734	150	402.6207	42.72865
43	394.3047	40.05804	97	400.5315	37.17074	151	404.8421	41.01630
44	398.7798	37.86405	98	403.4864	39.05207	152	400.0938	38.50327
45	399.889	38.99348	99	399.9441	40.54973	153	402.3828	42.45190
46	400.3576	37.32553	100	404.0743	37.77894	154	410.5386	41.94935
47	405.7982	39.51602	101	400.0410	35.74995	155	406.8676	39.70352
48	397.4005	39.67549	102	397.6919	37.98814	156	386.9748	33.41039
49	389.7251	35.77093	103	399.5614	39.97407	157	399.8704	40.63102
50	399.3979	37.70115	104	408.2214	36.46256	158	405.1568	44.16742
51	403.9138	40.49791	105	396.3483	41.09651	159	397.8786	39.41292
52	404.3431	41.30168	106	392.9285	38.75683	160	399.5410	38.50548
53	393.0051	35.34534	107	403.5489	36.37894	161	388.0398	36.47680
54	399.5355	40.14474	108	401.9743	36.45084	162	408.2542	38.36849

Table A8.4 Estimate of p_0 varied from session to session for rider 9 (Model 1)

session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$	session	$\widehat{p_0}$	$s.e.(\widehat{p_0})$
1	400.9753	36.58291	51	401.6591	34.59986	101	398.9511	40.70705	151	399.5785	33.47579
2	396.1967	34.48211	52	400.6593	36.39084	102	397.4872	41.63632	152	400.7308	42.79517
3	399.5711	37.87446	53	399.4051	35.65667	103	404.7886	40.27872	153	402.8172	39.09671
4	391.8409	35.02778	54	397.3846	37.47539	104	400.8295	36.34581	154	401.6413	34.85157
5	400.9745	35.56890	55	401.2668	35.97284	105	394.5815	31.78025	155	397.0394	43.16689
6	389.9081	36.05848	56	401.3363	33.27031	106	400.6384	46.38029	156	402.0482	42.66374
7	403.7474	38.13999	57	400.2206	35.90352	107	396.1541	34.59952	157	399.9449	40.50598
8	397.9976	35.12001	58	396.1137	32.99477	108	399.4974	39.72869	158	398.8107	38.69442
9	392.8999	35.55734	59	404.9751	34.65583	109	411.9445	44.51242	159	402.5682	40.15584
10	400.0152	40.48741	60	399.1664	38.54082	110	396.9685	35.27540	160	411.3488	45.75502
11	392.1082	36.16461	61	398.6755	33.14381	111	396.5981	33.97652	161	403.9062	39.38410
12	397.9142	36.10954	62	396.5688	37.04009	112	400.9993	48.62282	162	398.0894	38.76833
13	395.1298	39.49811	63	396.6665	38.57746	113	417.7601	48.24977	163	401.1952	36.07607
14	402.0611	36.48989	64	401.4241	37.62607	114	397.6839	32.95391	164	401.5381	43.9713
15	400.9542	35.31171	65	400.9821	35.98383	115	398.9626	32.83451	165	400.7298	42.80635
16	401.1172	38.77606	66	397.5696	34.65763	116	402.0805	40.78412	166	400.2241	40.28782
17	402.0748	33.26861	67	400.5685	35.79933	117	394.9819	32.41876	167	400.1563	34.98837
18	397.7498	35.47161	68	406.3624	40.95475	118	396.5082	32.05467	168	395.2117	32.19979
19	398.9356	35.57942	69	401.3909	36.20396	119	418.0445	48.44780	169	411.8232	54.19066
20	399.6988	34.36951	70	400.9061	34.53795	120	398.8933	36.48057	170	391.7616	36.63773
21	405.7564	36.75322	71	397.2979	33.51346	121	398.1270	35.91427	171	394.6149	36.78686
22	401.1322	37.22244	72	396.2694	32.99433	122	399.3417	38.32348	172	395.9183	32.84068
23	398.4305	35.89683	73	399.0086	35.14650	123	402.7296	43.46712	173	402.8561	48.80433
24	399.6669	36.18348	74	401.8358	39.28563	124	394.1603	31.38756	174	400.9234	46.82241
25	401.4072	38.24328	75	394.6826	36.54372	125	395.2391	31.84033	175	399.6703	43.19744
26	393.7621	36.73901	76	399.0075	40.35721	126	401.9825	45.78387	176	399.3001	36.69858
27	400.9863	36.34773	77	398.8989	37.12837	127	397.2546	34.93091	177	400.1868	38.27933
28	400.4939	36.11724	78	396.5087	41.61917	128	399.8374	44.43851	178	401.6149	46.42369
29	399.1014	34.31631	79	395.4785	35.78392	129	400.6152	37.48708	179	402.8861	46.47031
30	397.8827	37.98448	80	400.6852	36.77902	130	392.8941	34.61026	180	392.0515	34.45432
31	400.4865	32.83346	81	399.1536	37.77515	131	424.4684	50.63320	181	407.6614	44.33126
32	399.2285	38.37456	82	403.0382	38.27614	132	396.6233	37.27512	182	401.9255	38.14034
33	397.0466	32.99373	83	393.6664	34.09417	133	397.1806	34.73916	183	396.2366	43.60966
34	394.8298	37.50191	84	396.2505	41.27867	134	400.1984	42.09128	184	401.4394	41.54024
35	399.3213	35.88509	85	401.4089	37.62547	135	395.4665	33.99567	185	399.3013	34.92204
36	402.1764	32.98904	86	400.9472	38.17310	136	392.7343	31.38605	186	416.9922	50.19631
37	400.0311	37.14615	87	394.5855	37.55512	137	407.6128	47.57162	187	399.0357	35.43642
38	401.4632	36.25251	88	398.8954	38.75117	138	403.0384	50.32975	188	403.4884	40.50812
39	398.3961	35.48963	89	400.8421	35.31195	139	397.3933	33.69106	189	400.6199	36.05977
40	401.1975	38.19405	90	402.4213	43.60824	140	401.7521	42.00064	190	402.1001	42.58515
41	399.7818	36.84298	91	402.2642	35.95164	141	396.8995	33.63863	191	400.1913	38.61383
42	390.9609	36.23482	92	397.6406	42.63280	142	400.6761	36.06307	192	423.4956	52.31315
43	398.1899	35.74686	93	400.5107	36.76492	143	420.1687	50.68451	193	388.1372	31.06880
44	397.5379	37.32458	94	396.7147	34.55816	144	406.2600	46.60442	194	403.4450	38.99017
45	402.2517	32.98943	95	397.3852	35.53733	145	397.4894	34.03033	195	396.9765	33.32094
46	397.5539	33.13731	96	402.0757	37.69547	146	400.8341	43.05879	196	407.4205	40.92748
47	398.5565	34.18922	97	398.6002	39.57921	147	389.5684	39.23060	197	401.2727	41.73151
48	399.4234	35.84556	98	394.9665	37.05636	148	394.0706	34.25893			
49	398.4355	36.53290	99	398.3610	40.73597	149	401.0117	38.10173			
50	399.7894	36.31597	100	399.4804	38.18456	150	403.3175	47.77560			

Appendix 9: Estimates of $\boldsymbol{u}, \boldsymbol{v}$ and standard error of both parameters for all sessions

Table A9.1 Estimated of u,v, standard error of both parameters, the predicted value $\hat{p}$ and respective standard error for each session for rider 3

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
1	7.129	0.113	-0.3160	0.0260	0.93	602.8	426.1	35.7	16.8
2	7.215	0.134	-0.3150	0.0280	0.91	657.5	465.0	48.0	22.4
3	6.918	0.081	-0.3020	0.0180	0.97	504.6	362.2	21.4	9.7
4	7.122	0.107	-0.2930	0.0230	0.94	631.0	457.4	37.2	18.8
5	7.313	0.196	-0.3820	0.0440	0.88	622.7	409.5	61.8	24.3
6	6.937	0.068	-0.2600	0.0150	0.97	565.7	425.0	21.1	11.1
7	6.822	0.065	-0.2340	0.0120	0.97	535.0	413.6	20.6	11.5
8	6.601	0.053	-0.2060	0.0110	0.97	457.8	364.9	14.0	8.1
9	6.849	0.101	-0.2510	0.0200	0.94	529.3	401.7	30.8	16.3
10	6.662	0.060	-0.2280	0.0130	0.97	462.6	360.1	15.6	8.5
11	6.391	0.061	-0.1970	0.0130	0.97	379.4	305.7	12.7	7.3
12	6.875	0.081	-0.2560	0.0190	0.94	537.3	405.7	23.1	12.3
13	6.631	0.074	-0.2160	0.0150	0.95	461.8	364.4	19.9	11.7
14	6.875	0.130	-0.2700	0.0290	0.89	519.3	385.9	36.1	18.1
15	7.304	0.087	-0.3590	0.0190	0.97	650.9	439.0	29.5	12.5
16	7.143	0.111	-0.3100	0.0230	0.94	620.6	441.7	37.9	17.9
17	6.884	0.064	-0.2600	0.0140	0.97	536.9	403.7	18.9	9.9
18	7.039	0.109	-0.2850	0.0240	0.94	591.7	432.8	34.9	17.0
19	6.994	0.086	-0.2840	0.0200	0.95	567.5	415.6	26.2	13.2
20	6.956	0.140	-0.2990	0.0320	0.93	527.3	379.8	38.4	18.3
21	7.003	0.087	-0.3060	0.0200	0.96	544.0	388.8	25.1	12.2
22	7.051	0.187	-0.2970	0.0410	0.84	582.1	420.0	57.6	26.6
23	7.074	0.100	-0.3180	0.0230	0.94	567.3	400.0	29.5	13.7
24	7.342	0.126	-0.3760	0.0300	0.94	649.7	430.0	40.2	16.1
25	6.961	0.081	-0.2720	0.0170	0.95	563.3	417.6	25.1	12.9
26	6.989	0.103	-0.3030	0.0220	0.96	539.7	386.7	30.2	14.1
27	7.080	0.087	-0.3090	0.0190	0.96	582.5	414.7	27.2	13.0
28	6.990	0.186	-0.2940	0.0430	0.83	551.5	399.2	52.1	23.9
29	6.938	0.165	-0.3060	0.0390	0.87	509.4	363.9	42.6	19.9
30	7.065	0.172	-0.3290	0.0400	0.87	548.5	382.0	47.6	20.7

Table A9.1. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
31	6.778	0.072	-0.2540	0.0160	0.95	489.7	370.6	19.0	9.9
32	7.341	0.087	-0.3500	0.0180	0.97	689.0	469.1	32.7	14.6
33	6.752	0.063	-0.2330	0.0130	0.97	499.9	386.8	18.4	10.2
34	6.722	0.114	-0.2120	0.0210	0.92	510.2	404.4	35.8	21.0
35	6.831	0.122	-0.2330	0.0230	0.91	541.2	419.0	39.8	22.4
36	6.462	0.087	-0.1640	0.0160	0.93	438.6	366.2	23.9	15.5
37	6.525	0.070	-0.1870	0.0140	0.95	443.6	361.3	19.0	11.7
38	6.658	0.112	-0.2060	0.0220	0.92	485.4	387.2	32.7	19.4
39	6.984	0.086	-0.2830	0.0200	0.95	563.1	412.7	25.5	12.8
40	7.123	0.061	-0.3330	0.0140	0.98	576.1	399.5	18.3	8.1
41	6.887	0.069	-0.2430	0.0130	0.97	559.1	427.9	23.0	12.7
42	6.751	0.046	-0.2690	0.0110	0.99	459.7	341.9	11.0	5.6
43	6.934	0.082	-0.2850	0.0190	0.95	532.3	389.0	22.6	11.4
44	6.893	0.073	-0.2700	0.0160	0.96	528.9	393.1	20.8	10.8
45	6.969	0.052	-0.2770	0.0110	0.98	562.3	415.0	16.1	8.4
46	6.852	0.107	-0.2440	0.0250	0.92	539.6	412.9	30.7	16.6
47	7.353	0.109	-0.3800	0.0250	0.96	650.6	428.6	35.6	13.8
48	6.769	0.041	-0.2420	0.0090	0.99	497.9	381.5	11.3	6.2
49	6.980	0.062	-0.2740	0.0120	0.98	571.5	422.8	20.3	10.5
50	6.747	0.114	-0.2650	0.0260	0.92	463.0	346.2	28.2	14.5
51	6.914	0.064	-0.2740	0.0140	0.97	536.1	396.9	18.9	9.7
52	7.621	0.135	-0.4510	0.0310	0.96	723.2	440.8	48.5	17.2
53	6.885	0.064	-0.2500	0.0140	0.97	549.9	418.0	19.8	10.7
54	6.794	0.065	-0.2530	0.0150	0.97	498.4	377.6	17.0	8.7
55	6.825	0.074	-0.2360	0.0140	0.96	534.1	411.9	23.1	12.7
56	6.748	0.080	-0.2420	0.0170	0.95	488.1	374.2	21.9	11.9
57	7.210	0.054	-0.3240	0.0120	0.98	642.4	450.2	18.5	8.5
58	6.715	0.053	-0.2170	0.0110	0.97	501.0	394.9	15.7	9.2
59	7.164	0.057	-0.3090	0.0130	0.98	635.1	452.5	19.1	9.0
60	6.761	0.075	-0.2510	0.0170	0.95	484.9	368.2	20.0	10.6
61	7.664	0.133	-0.4500	0.0310	0.95	756.3	461.4	49.4	17.4
62	6.693	0.028	-0.2100	0.0060	0.99	497.3	394.7	7.4	4.1
63	6.720	0.045	-0.2330	0.0100	0.98	484.8	375.3	12.5	7.0
64	6.837	0.055	-0.2230	0.0100	0.97	557.6	436.4	18.6	10.9
65	7.846	0.145	-0.5240	0.0380	0.97	765.0	430.3	47.1	12.9
66	6.858	0.128	-0.2820	0.0290	0.92	496.7	364.2	34.1	16.8
67	7.432	0.097	-0.3970	0.0240	0.97	676.6	437.3	30.6	10.7
68	7.111	0.092	-0.3540	0.0230	0.96	542.6	367.8	23.8	10.5

Table A9.1. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
69	7.394	0.108	-0.3980	0.0270	0.96	650.9	420.5	32.5	12.4
70	7.303	0.211	-0.3950	0.0500	0.86	597.6	387.2	61.2	23.4
71	6.755	0.071	-0.2820	0.0170	0.98	447.8	328.4	16.7	8.6
72	6.800	0.099	-0.3180	0.0250	0.95	431.6	304.3	20.5	9.5
73	6.690	0.069	-0.2820	0.0180	0.96	419.6	307.7	13.8	6.7
74	6.842	0.050	-0.2840	0.0110	0.99	487.2	356.8	13.1	6.8
75	6.914	0.087	-0.3010	0.0210	0.96	503.0	361.3	22.5	10.8
76	6.910	0.125	-0.3170	0.0280	0.94	483.4	341.4	30.3	12.8
77	7.203	0.162	-0.4080	0.0420	0.95	524.9	335.2	40.0	16.9
78	6.792	0.150	-0.3030	0.0350	0.91	443.4	317.9	33.9	15.7
79	6.817	0.092	-0.3200	0.0220	0.97	437.4	307.8	20.2	9.5
80	6.912	0.074	-0.3050	0.0170	0.97	498.1	356.4	19.2	9.1
81	6.968	0.072	-0.3090	0.0170	0.97	521.9	371.8	19.5	9.6
82	6.843	0.072	-0.2860	0.0170	0.97	485.6	354.8	18.1	8.7
83	6.765	0.066	-0.2660	0.0150	0.97	470.2	351.2	16.2	8.6
84	6.961	0.078	-0.2750	0.0170	0.95	559.6	413.7	23.8	11.9
85	6.947	0.091	-0.2600	0.0190	0.94	571.2	429.1	29.3	15.5
86	6.602	0.069	-0.2060	0.0150	0.94	458.5	365.6	17.6	10.0
87	6.912	0.079	-0.2930	0.0180	0.97	511.8	371.0	21.5	10.3
88	6.880	0.051	-0.2720	0.0110	0.98	520.5	386.1	14.6	7.6
89	6.804	0.058	-0.2680	0.0130	0.98	486.8	362.9	15.4	7.8
90	6.989	0.102	-0.2960	0.0220	0.94	547.9	395.6	29.7	14.1
91	6.980	0.076	-0.3190	0.0180	0.97	515.6	363.1	19.3	8.8
92	6.990	0.125	-0.3000	0.0300	0.91	544.1	391.4	33.8	15.9
93	6.908	0.092	-0.2800	0.0200	0.96	524.9	386.0	26.6	13.7
94	7.009	0.094	-0.3130	0.0220	0.96	538.1	381.4	26.7	13.2
95	6.643	0.037	-0.2610	0.0090	0.99	420.5	315.7	8.1	4.3
96	6.712	0.077	-0.2740	0.0180	0.97	437.1	323.4	17.6	8.9
97	6.850	0.078	-0.2910	0.0180	0.97	482.5	350.4	20.1	10.2
98	6.882	0.085	-0.3000	0.0200	0.96	488.1	351.1	21.5	10.8
99	6.753	0.129	-0.2610	0.0280	0.90	469.1	352.0	32.7	16.8
100	6.702	0.040	-0.2370	0.0090	0.98	471.4	363.3	10.3	5.7
101	6.754	0.037	-0.2440	0.0080	0.99	488.3	373.4	10.0	5.5
102	6.800	0.107	-0.2570	0.0240	0.91	496.9	374.7	28.5	14.6
103	6.745	0.062	-0.2370	0.0130	0.97	492.0	379.1	17.5	9.6
104	6.769	0.054	-0.2430	0.0110	0.98	497.1	380.5	15.3	8.5
105	7.027	0.116	-0.3130	0.0250	0.95	548.5	389.0	34.6	16.1
106	6.848	0.130	-0.2580	0.0260	0.91	519.3	390.9	38.6	20.1
107	7.095	0.149	-0.2840	0.0280	0.93	626.9	458.8	55.7	29.1
108	6.850	0.052	-0.2560	0.0120	0.98	523.1	394.6	13.8	6.8

Table A9.2 Estimates of u,v, standard error of both parameters, the predicted value $\hat{p}$ and respective standard error for each session for rider 4

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
1	6.108	0.043	-0.1260	0.0090	0.97	336.2	292.8	8.5	5.6
2	6.498	0.347	-0.1790	0.0720	0.75	440.2	361.8	82.9	43.5
3	5.862	0.036	-0.1430	0.0090	0.98	253.1	216.4	4.6	2.9
4	6.017	0.044	-0.1510	0.0080	0.99	289.6	245.2	7.4	4.2
5	6.184	0.094	-0.1500	0.0190	0.90	343.5	291.4	19.0	11.8
6	6.421	0.074	-0.2030	0.0160	0.95	385.1	308.1	15.6	8.6
7	5.927	0.077	-0.1480	0.0160	0.98	266.8	226.8	11.2	6.1
8	6.339	0.140	-0.1860	0.0280	0.88	369.1	300.9	29.3	16.1
9	6.254	0.125	-0.1260	0.0250	0.81	389.6	339.3	28.6	17.7
10	6.232	0.250	-0.1680	0.0430	0.83	346.1	287.9	52.7	30.9
11	6.011	0.040	-0.1270	0.0090	0.97	304.4	264.7	6.7	4.2
12	6.835	0.123	-0.2200	0.0220	0.91	561.0	440.7	42.4	24.6
13	6.470	0.085	-0.2030	0.0180	0.93	404.9	324.1	19.1	10.8
14	5.928	0.139	-0.1290	0.0270	0.89	278.8	241.9	22.4	13.4
15	6.351	0.053	-0.2040	0.0120	0.98	358.5	286.6	10.3	5.9
16	6.235	0.063	-0.1840	0.0140	0.97	334.0	272.8	11.3	6.2
17	6.509	0.044	-0.2490	0.0110	0.99	378.0	287.4	8.4	4.3
18	6.351	0.107	-0.1830	0.0220	0.91	375.9	307.5	23.0	13.2
19	6.445	0.080	-0.2330	0.0190	0.96	367.9	284.8	15.3	8.4
20	6.217	0.064	-0.1780	0.0140	0.97	333.0	274.0	11.9	7.0
21	6.299	0.068	-0.2000	0.0160	0.97	343.6	275.9	12.6	7.4
22	5.888	0.066	-0.1540	0.0160	0.97	253.1	213.8	8.2	4.4
23	6.163	0.093	-0.1450	0.0180	0.92	340.5	290.4	19.2	12.2
24	6.344	0.042	-0.2110	0.0100	0.98	349.8	277.4	7.5	4.2
25	6.912	0.104	-0.2890	0.0230	0.94	516.6	376.2	28.3	13.5
26	6.509	0.076	-0.2180	0.0170	0.95	406.6	320.2	16.6	9.0
27	6.317	0.085	-0.1770	0.0180	0.92	368.7	303.5	18.0	11.0
28	6.134	0.116	-0.2040	0.0290	0.93	288.3	230.4	15.5	7.3
29	5.988	0.065	-0.1230	0.0130	0.95	300.4	262.4	12.0	7.9
30	5.969	0.145	-0.2320	0.0440	0.93	229.5	178.0	11.3	5.1
31	6.270	0.048	-0.1800	0.0100	0.98	349.2	286.6	9.6	5.8
32	6.584	0.133	-0.2290	0.0280	0.90	427.3	332.4	31.6	16.9
33	6.370	0.062	-0.2090	0.0150	0.95	361.4	287.4	11.5	6.5
34	6.305	0.041	-0.1800	0.0090	0.98	361.3	296.4	8.2	4.9
35	6.410	0.101	-0.1990	0.0210	0.94	383.9	308.4	22.7	13.3

Table A9.2. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
36	6.452	0.143	-0.2210	0.0310	0.89	381.1	298.9	30.4	17.3
37	5.878	0.082	-0.2050	0.0270	0.95	222.6	177.6	6.3	4.8
38	6.505	0.104	-0.2210	0.0230	0.92	401.6	314.9	23.3	13.2
39	6.165	0.063	-0.1700	0.0150	0.95	321.6	266.8	10.8	6.5
40	5.769	0.106	-0.1450	0.0270	0.93	229.1	195.3	11.1	5.7
41	6.099	0.120	-0.1500	0.0240	0.88	315.5	267.6	21.7	13.0
42	6.431	0.057	-0.2060	0.0130	0.97	386.3	308.1	12.1	6.9
43	6.352	0.124	-0.1590	0.0270	0.88	397.8	334.0	28.5	18.0
44	6.142	0.042	-0.1470	0.0090	0.98	331.3	281.7	8.1	5.1
45	5.703	0.118	-0.0770	0.0200	0.83	251.3	231.0	19.4	13.9
46	6.131	0.171	-0.1780	0.0310	0.97	305.2	251.1	30.6	16.9
47	6.113	0.028	-0.1140	0.0060	0.98	347.9	307.1	5.6	3.7
48	6.845	0.290	-0.2540	0.0560	0.81	523.4	395.9	88.4	46.4
49	7.193	0.119	-0.3230	0.0260	0.94	632.2	443.4	40.7	19.0
50	6.195	0.110	-0.1650	0.0230	0.90	335.3	279.7	20.7	11.9
51	6.091	0.124	-0.1380	0.0250	0.84	321.9	276.7	23.4	14.6
52	7.531	0.090	-0.4180	0.0200	0.98	712.9	450.6	33.5	13.0
53	6.522	0.097	-0.1940	0.0190	0.92	434.9	351.4	24.9	14.6
54	6.865	0.149	-0.2080	0.0320	0.88	593.4	472.1	49.1	26.8
55	6.331	0.196	-0.1460	0.0440	0.69	401.0	341.5	42.0	24.8
56	6.822	0.046	-0.2830	0.0110	0.99	478.3	350.4	11.6	5.8
57	6.438	0.178	-0.1930	0.0380	0.78	401.4	324.9	39.7	22.6
58	6.359	0.139	-0.1540	0.0270	0.82	405.1	342.0	33.2	19.9
59	7.737	0.370	-0.5690	0.0950	0.90	617.8	330.6	99.3	27.1
60	6.339	0.110	-0.2150	0.0250	0.94	344.8	272.2	20.4	11.5
61	8.382	0.291	-0.6650	0.0730	0.93	945.5	455.5	121.0	27.5
62	6.211	0.053	-0.1980	0.0130	0.97	316.0	254.4	8.5	4.9
63	7.321	0.082	-0.3560	0.0180	0.97	666.6	451.0	28.7	12.6
64	6.323	0.058	-0.1910	0.0130	0.97	358.8	290.8	11.6	6.9
65	7.382	0.131	-0.4360	0.0330	0.96	588.4	364.3	36.3	13.7
66	6.942	0.109	-0.2610	0.0220	0.93	567.5	426.0	35.7	18.8
67	8.328	0.200	-0.7050	0.0530	0.95	815.2	375.6	66.0	13.4
68	6.149	0.127	-0.1330	0.0250	0.82	344.9	298.1	25.9	16.5
69	6.484	0.113	-0.2360	0.0260	0.93	380.2	293.4	23.4	13.1
70	6.502	0.068	-0.1900	0.0150	0.95	430.4	349.4	16.3	9.5
71	8.419	0.228	-0.6370	0.0530	0.97	1046.2	519.7	115.2	31.3
72	6.039	0.065	-0.1690	0.0150	0.97	284.4	236.3	9.5	5.3
73	7.059	0.143	-0.2900	0.0300	0.89	596.3	433.4	47.2	23.4
74	6.124	0.047	-0.1680	0.0120	0.98	310.0	257.7	7.1	4.4

Table A9.2. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
75	7.829	0.193	-0.5070	0.0490	0.95	781.3	447.5	66.0	19.3
76	8.384	0.244	-0.6650	0.0610	0.94	945.8	455.4	102.1	24.1
77	6.730	0.123	-0.3530	0.0330	0.96	371.5	252.1	20.2	8.8
78	7.599	0.197	-0.5280	0.0490	0.98	591.4	330.9	52.7	15.5
79	6.585	0.063	-0.2170	0.0130	0.97	438.9	345.7	16.0	9.0
80	6.182	0.041	-0.2410	0.0130	0.99	278.0	213.3	4.5	2.9
81	8.076	0.171	-0.6060	0.0430	0.96	797.5	409.9	61.3	15.7
82	6.710	0.210	-0.2200	0.0400	0.78	494.0	387.8	60.9	33.0
83	6.642	0.123	-0.2120	0.0250	0.87	470.5	372.7	33.1	18.5
84	7.017	0.131	-0.2900	0.0280	0.92	572.3	416.2	41.9	20.9
85	6.416	0.035	-0.1980	0.0080	0.99	387.3	311.5	7.4	4.3
86	5.767	0.080	-0.1190	0.0190	0.93	243.2	213.5	10.7	7.1
87	8.661	0.260	-0.7540	0.0670	0.95	1016.5	443.9	112.0	21.4
88	6.753	0.099	-0.2660	0.0240	0.95	464.0	346.2	24.5	13.3
89	6.956	0.243	-0.2590	0.0450	0.89	578.7	435.5	81.7	40.8
90	6.304	0.117	-0.2490	0.0320	0.91	308.6	234.8	15.2	7.4
91	6.337	0.053	-0.1880	0.0110	0.98	366.2	297.8	11.0	6.5
92	6.417	0.080	-0.1560	0.0160	0.93	427.6	360.3	20.7	13.2
93	6.182	0.063	-0.1810	0.0150	0.97	319.1	261.6	10.8	6.5
94	7.609	0.218	-0.4530	0.0550	0.91	710.0	431.4	68.6	21.8
95	6.105	0.070	-0.1170	0.0130	0.94	342.1	300.8	15.0	10.3
96	6.360	0.111	-0.1430	0.0200	0.89	416.0	355.5	29.1	18.9
97	6.695	0.183	-0.2270	0.0400	0.82	478.7	372.9	48.4	26.8
98	6.768	0.084	-0.2460	0.0180	0.95	492.8	375.9	23.3	12.6
99	6.000	0.104	-0.1370	0.0220	0.90	294.3	253.2	17.6	11.4
100	8.072	0.266	-0.5670	0.0640	0.93	868.6	465.9	106.7	29.9
101	6.207	0.073	-0.1400	0.0150	0.92	359.5	308.2	15.0	9.3
102	6.421	0.065	-0.1410	0.0120	0.94	443.8	380.0	17.3	10.9
103	6.188	0.032	-0.1510	0.0070	0.98	344.2	291.7	6.0	3.7
104	7.967	0.232	-0.5820	0.0590	0.93	755.1	398.4	76.3	20.1
105	6.315	0.070	-0.1660	0.0140	0.95	377.4	314.6	15.9	9.9
106	6.615	0.065	-0.1710	0.0130	0.95	502.9	416.6	19.5	11.8
107	6.592	0.113	-0.1980	0.0220	0.90	462.6	372.3	31.2	18.4
108	6.054	0.066	-0.1430	0.0140	0.94	306.0	261.4	11.5	7.2
109	6.360	0.118	-0.1660	0.0220	0.89	394.9	329.2	28.2	17.3
110	5.633	0.087	-0.0520	0.0150	0.86	248.1	234.4	14.4	10.8
111	6.494	0.081	-0.2180	0.0170	0.96	399.8	314.5	18.1	10.0
112	6.099	0.116	-0.1590	0.0240	0.92	309.1	259.6	20.9	12.9

Table A9.3 Estimates of u,v, standard error of both parameters, the R^2 value of the regression model, the predicted value $\hat{p}$ and respective standard error for each session for rider 5

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
1	6.641	0.068	-0.2370	0.0160	0.96	443.7	342.0	15.7	8.5
2	6.767	0.140	-0.2920	0.0350	0.88	443.7	321.9	29.7	13.6
3	7.213	0.062	-0.3620	0.0140	0.98	590.0	396.5	18.0	7.4
4	7.051	0.114	-0.2880	0.0240	0.93	595.3	434.0	37.7	18.8
5	7.363	0.193	-0.3920	0.0420	0.91	640.0	416.2	64.1	25.8
6	6.628	0.070	-0.2520	0.0160	0.97	423.1	320.7	15.6	7.9
7	6.858	0.044	-0.3020	0.0100	0.99	475.1	341.0	10.6	5.2
8	7.008	0.191	-0.2680	0.0460	0.75	595.7	443.6	56.5	27.5
9	7.577	0.194	-0.4250	0.0450	0.90	734.2	460.4	71.2	26.5
10	7.083	0.142	-0.3490	0.0350	0.89	533.8	363.9	36.1	15.1
11	6.809	0.052	-0.3160	0.0130	0.99	437.8	309.5	10.8	5.0
12	7.065	0.060	-0.2800	0.0130	0.98	614.5	451.7	19.6	9.5
13	6.460	0.042	-0.2390	0.0110	0.99	368.2	283.1	7.7	4.2
14	7.693	0.147	-0.4490	0.0330	0.95	780.9	477.0	58.5	21.3
15	7.228	0.101	-0.3520	0.0220	0.96	611.6	415.2	32.0	13.5
16	7.117	0.080	-0.3070	0.0180	0.97	608.4	434.3	25.8	12.2
17	6.761	0.082	-0.3000	0.0200	0.96	433.0	311.5	17.2	8.3
18	7.141	0.086	-0.3160	0.0190	0.95	610.3	431.3	27.6	12.7
19	7.165	0.081	-0.3200	0.0170	0.97	618.8	435.4	27.4	12.8
20	7.300	0.170	-0.3440	0.0350	0.94	670.6	459.6	63.1	28.6
21	7.270	0.073	-0.3180	0.0150	0.97	690.5	486.9	27.9	13.2
22	7.093	0.082	-0.3050	0.0190	0.95	595.7	425.9	25.7	12.5
23	6.830	0.075	-0.2260	0.0150	0.95	550.6	429.7	24.8	14.3
24	6.740	0.068	-0.2720	0.0160	0.97	452.3	335.6	15.3	7.6
25	7.426	0.084	-0.3540	0.0180	0.98	743.3	504.0	33.5	14.5
26	6.767	0.052	-0.2570	0.0120	0.98	480.2	361.9	13.4	7.1
27	6.410	0.112	-0.2220	0.0270	0.92	364.1	285.2	20.8	11.4
28	7.003	0.049	-0.2760	0.0100	0.98	582.9	430.5	15.6	8.0
29	6.823	0.064	-0.2470	0.0130	0.97	520.4	396.8	18.9	10.3
30	7.066	0.064	-0.2830	0.0130	0.98	610.5	447.4	22.8	11.6
31	7.394	0.053	-0.3710	0.0120	0.99	692.1	460.4	18.2	7.1
32	6.786	0.081	-0.2500	0.0190	0.96	498.5	379.0	20.9	10.7
33	6.618	0.062	-0.1980	0.0130	0.95	474.7	381.9	16.4	9.4
34	6.687	0.069	-0.2150	0.0140	0.96	488.6	385.7	19.9	11.5
35	6.888	0.067	-0.2750	0.0150	0.97	520.4	384.7	18.6	9.2

Table A9.3. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
36	6.391	0.091	-0.1960	0.0200	0.92	379.7	306.1	19.2	11.2
37	6.437	0.065	-0.2020	0.0140	0.97	392.2	314.2	14.5	8.3
38	6.691	0.088	-0.2310	0.0200	0.94	473.0	367.1	22.2	11.7
39	7.169	0.108	-0.3060	0.0230	0.93	642.1	459.0	38.2	18.4
40	7.436	0.051	-0.3880	0.0110	0.99	694.5	453.7	18.5	7.5
41	7.083	0.097	-0.3170	0.0230	0.95	574.4	405.5	28.9	13.9
42	6.623	0.100	-0.2210	0.0210	0.92	452.6	355.1	25.4	13.7
43	6.735	0.101	-0.2750	0.0240	0.94	446.5	330.1	23.0	11.5
44	7.463	0.196	-0.3170	0.0390	0.86	840.9	593.9	93.4	44.0
45	6.568	0.167	-0.2990	0.0470	0.89	357.3	257.2	26.5	13.9
46	6.485	0.050	-0.2340	0.0120	0.98	382.0	295.3	9.6	4.8
47	7.256	0.054	-0.3480	0.0120	0.98	635.2	433.2	17.7	7.5
48	7.116	0.040	-0.3140	0.0090	0.99	596.8	422.5	12.6	5.9
49	6.958	0.040	-0.2850	0.0090	0.99	545.9	399.4	11.8	5.9
50	7.389	0.149	-0.3630	0.0310	0.92	700.6	470.0	56.5	24.2
51	6.664	0.074	-0.2340	0.0170	0.95	456.7	353.1	17.6	9.0
52	5.604	0.035	-0.0770	0.0070	0.99	227.2	208.8	4.9	3.4
53	6.688	0.084	-0.2750	0.0220	0.95	426.2	315.2	16.8	8.2
54	6.802	0.080	-0.2470	0.0170	0.95	509.1	388.1	22.7	12.2
55	6.678	0.070	-0.2040	0.0140	0.95	496.2	396.4	20.7	12.3
56	6.773	0.310	-0.2210	0.0580	0.71	525.3	412.1	97.5	54.6
57	6.948	0.131	-0.2680	0.0270	0.90	562.0	418.8	41.7	21.1
58	6.869	0.126	-0.2480	0.0260	0.91	543.9	414.2	38.9	21.0
59	7.166	0.117	-0.4080	0.0300	0.95	505.9	323.2	26.5	9.8
60	6.417	0.207	-0.2160	0.0470	0.78	372.6	294.0	40.0	20.1
61	7.031	0.231	-0.2440	0.0380	0.84	645.6	494.0	95.7	54.6
62	6.973	0.117	-0.2890	0.0260	0.92	548.1	398.8	34.7	17.3
63	6.306	0.063	-0.2220	0.0160	0.97	328.9	257.8	10.2	6.1
64	6.250	0.065	-0.1920	0.0150	0.96	333.2	269.9	11.6	6.9
65	8.510	0.164	-0.6610	0.0390	0.97	1083.1	524.0	82.5	20.4
66	6.385	0.113	-0.2120	0.0260	0.92	364.3	288.7	22.8	13.5
67	7.020	0.120	-0.2790	0.0260	0.95	588.5	433.2	38.5	18.8
68	6.537	0.115	-0.1930	0.0220	0.91	442.6	358.0	31.1	18.8
69	6.340	0.049	-0.2190	0.0120	0.98	342.1	268.9	8.6	4.9
70	7.370	0.187	-0.3340	0.0430	0.88	735.3	509.2	68.2	27.9
71	6.505	0.105	-0.2370	0.0240	0.94	387.2	298.3	22.2	12.6
72	5.761	0.124	-0.0650	0.0210	0.76	273.6	254.8	22.1	16.1
73	6.610	0.157	-0.2750	0.0370	0.92	393.9	291.1	31.6	15.5
74	7.473	0.082	-0.3680	0.0180	0.98	754.8	504.0	32.2	13.7

Table A9.3. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
75	6.405	0.140	-0.2710	0.0390	0.91	323.8	240.3	20.9	11.7
76	6.253	0.054	-0.2100	0.0130	0.98	320.2	254.1	8.6	4.3
77	7.341	0.165	-0.3390	0.0400	0.87	706.6	486.9	54.6	21.2
78	6.210	0.056	-0.1490	0.0110	0.97	353.6	300.3	11.9	7.5
79	6.435	0.025	-0.2450	0.0060	1.00	354.9	271.2	4.0	1.9
80	6.128	0.103	-0.1510	0.0210	0.91	323.7	274.2	19.5	12.2
81	6.935	0.056	-0.3870	0.0160	0.99	421.1	275.2	9.8	4.2
82	7.176	0.127	-0.4340	0.0350	0.96	481.6	299.1	24.2	7.8
83	7.010	0.162	-0.4780	0.0510	0.96	368.3	217.8	20.6	9.4
84	6.808	0.109	-0.3330	0.0280	0.95	420.9	292.0	21.4	9.9
85	7.408	0.075	-0.4040	0.0180	0.98	650.8	417.6	23.9	9.3
86	7.801	0.175	-0.4390	0.0360	0.94	888.3	548.3	83.5	32.2
87	6.880	0.055	-0.3680	0.0160	0.99	416.6	278.0	8.8	4.4
88	7.112	0.150	-0.3270	0.0350	0.88	577.1	402.7	43.8	19.8
89	6.982	0.081	-0.3380	0.0210	0.96	495.1	341.6	18.8	8.2
90	7.401	0.166	-0.3590	0.0360	0.89	717.0	483.6	63.4	27.4
91	7.079	0.138	-0.3630	0.0380	0.90	514.8	345.6	30.3	12.6
92	7.624	0.137	-0.3950	0.0290	0.95	824.8	534.6	59.5	23.6
93	7.139	0.165	-0.4310	0.0500	0.95	467.1	290.8	28.4	12.9
94	7.350	0.125	-0.3320	0.0250	0.95	724.9	503.5	50.9	23.5
95	7.061	0.130	-0.4090	0.0360	0.94	454.9	290.3	25.1	10.1
96	6.946	0.175	-0.3610	0.0450	0.89	452.5	304.4	35.3	13.8
97	7.288	0.121	-0.3080	0.0250	0.91	720.3	513.7	49.1	23.7
98	7.029	0.063	-0.3400	0.0150	0.98	516.2	355.3	16.3	7.3
99	8.071	0.223	-0.5320	0.0510	0.90	939.3	523.3	103.0	31.8
100	6.900	0.065	-0.4010	0.0190	0.99	394.6	254.1	10.5	4.7
101	7.544	0.125	-0.4350	0.0300	0.96	693.4	429.8	42.7	16.1

Table A9.4 Estimates of u,v, standard error of both parameters, the predicted value $\hat{p}$ and their respective standard errors for each session for rider 6

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
1	7.094	0.196	-0.2920	0.0420	0.82	615.8	447.0	65.5	31.4
2	7.272	0.206	-0.3710	0.0510	0.87	612.9	407.7	58.8	22.2
3	7.395	0.207	-0.3610	0.0450	0.86	709.0	476.9	77.7	33.2
4	7.032	0.117	-0.2840	0.0260	0.93	588.6	430.8	36.7	17.7
5	7.921	0.187	-0.4950	0.0420	0.92	879.9	510.6	82.8	27.3
6	7.885	0.153	-0.4450	0.0320	0.95	953.1	584.3	78.0	29.5
7	7.883	0.127	-0.4410	0.0260	0.96	959.7	591.0	67.1	26.2
8	7.303	0.142	-0.3290	0.0290	0.92	696.1	484.9	55.4	25.3
9	6.670	0.085	-0.1940	0.0130	0.97	503.9	407.0	28.1	17.2
10	7.546	0.128	-0.3770	0.0280	0.94	795.4	525.9	53.8	22.2
11	6.922	0.167	-0.2680	0.0350	0.85	547.4	408.0	50.0	24.8
12	7.712	0.255	-0.4340	0.0570	0.83	822.0	510.1	105.1	37.2
13	6.683	0.143	-0.1910	0.0220	0.93	515.0	417.7	49.0	30.4
14	6.170	0.064	-0.1380	0.0130	0.96	347.8	298.7	13.0	7.8
15	8.184	0.158	-0.4950	0.0310	0.96	1146.9	666.0	101.3	37.5
16	6.202	0.090	-0.1640	0.0190	0.91	338.7	282.9	17.1	10.1
17	7.016	0.154	-0.2850	0.0320	0.88	578.5	423.2	49.5	24.3
18	6.951	0.098	-0.2680	0.0230	0.92	563.4	419.8	28.8	14.3
19	6.037	0.085	-0.1240	0.0160	0.93	314.3	274.2	16.6	10.9
20	7.244	0.171	-0.3190	0.0350	0.89	672.1	473.6	63.7	29.8
21	8.387	0.141	-0.5570	0.0300	0.97	1217.1	659.9	89.7	28.9
22	6.372	0.128	-0.1740	0.0240	0.88	392.0	323.7	29.9	17.5
23	7.331	0.144	-0.3480	0.0330	0.91	686.2	468.4	49.9	20.7
24	8.044	0.212	-0.4710	0.0420	0.93	1052.1	626.8	123.8	46.9
25	7.878	0.108	-0.4590	0.0230	0.97	916.3	553.2	51.1	18.1
26	8.247	0.137	-0.4760	0.0270	0.96	1275.3	756.0	96.7	36.2
27	7.050	0.193	-0.3580	0.0470	0.89	505.8	341.4	47.2	20.0
28	7.239	0.168	-0.4060	0.0420	0.93	547.1	350.3	42.5	16.0
29	7.604	0.232	-0.4470	0.0560	0.86	716.7	438.6	78.1	26.6
30	7.129	0.163	-0.3560	0.0400	0.88	549.4	371.5	42.6	17.9
31	6.687	0.161	-0.2650	0.0440	0.86	436.1	326.1	30.6	15.1
32	7.180	0.223	-0.4300	0.0660	0.91	487.4	303.8	41.4	17.7
33	6.248	0.106	-0.2130	0.0270	0.91	316.6	250.5	15.8	8.0
34	7.702	0.247	-0.5380	0.0640	0.92	641.5	355.3	68.6	20.3
35	6.793	0.131	-0.2080	0.0310	0.85	551.9	439.1	37.3	20.2

Table A9.4. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
36	7.032	0.167	-0.3420	0.0420	0.88	514.8	353.5	41.3	18.1
37	7.258	0.130	-0.4210	0.0350	0.94	537.8	338.5	29.4	10.6
38	7.150	0.062	-0.3430	0.0160	0.99	578.7	397.1	16.4	6.6
39	7.569	0.143	-0.4560	0.0350	0.96	677.4	410.3	45.2	14.8
40	7.153	0.156	-0.3700	0.0420	0.88	544.8	362.8	36.9	15.5
41	7.446	0.158	-0.4050	0.0370	0.92	674.6	432.5	52.3	19.2
42	6.901	0.127	-0.2430	0.0340	0.87	567.5	434.6	33.1	17.7
43	7.322	0.379	-0.3380	0.0860	0.72	695.3	479.7	131.9	53.6
44	6.930	0.152	-0.3230	0.0400	0.89	486.4	341.3	34.3	15.8
45	7.464	0.113	-0.3980	0.0260	0.96	698.0	450.8	39.1	15.2
46	7.790	0.336	-0.4580	0.0760	0.84	841.9	509.2	141.2	48.9
47	6.788	0.185	-0.3060	0.0460	0.88	438.1	312.9	40.3	19.8
48	6.809	0.102	-0.3530	0.0300	0.96	401.4	272.3	16.7	8.0
49	7.004	0.175	-0.3310	0.0460	0.90	513.8	357.1	42.6	20.4
50	7.040	0.148	-0.3870	0.0440	0.95	467.9	305.8	27.7	13.1
51	6.818	0.197	-0.3020	0.0470	0.84	456.3	327.5	44.1	19.8
52	7.314	0.082	-0.3390	0.0190	0.97	687.4	473.5	28.0	11.3
53	6.869	0.156	-0.3080	0.0390	0.88	473.1	337.2	35.5	16.0
54	7.529	0.126	-0.4970	0.0340	0.97	592.6	343.3	31.6	10.5
55	7.073	0.090	-0.3520	0.0220	0.97	524.7	356.5	23.5	10.3
56	7.211	0.165	-0.4620	0.0460	0.93	467.1	281.1	31.6	11.7
57	6.733	0.108	-0.2770	0.0270	0.94	444.0	327.6	22.8	11.0
58	7.027	0.142	-0.3780	0.0430	0.93	471.4	311.1	25.0	12.0
59	6.747	0.071	-0.2560	0.0180	0.96	472.3	356.5	16.3	8.3
60	7.114	0.125	-0.3130	0.0310	0.90	598.0	424.0	36.6	16.6
61	7.001	0.112	-0.3380	0.0280	0.95	504.2	347.8	26.7	11.7
62	6.952	0.134	-0.3300	0.0360	0.92	488.2	339.6	29.0	13.6
63	7.636	0.280	-0.4110	0.0620	0.83	804.8	512.6	115.2	43.6
64	7.413	0.159	-0.3850	0.0360	0.91	683.3	447.8	55.6	22.2
65	7.167	0.199	-0.3800	0.0510	0.89	540.6	356.2	48.4	18.0
66	7.019	0.235	-0.3080	0.0590	0.77	549.6	391.7	61.5	28.5
67	7.028	0.209	-0.3230	0.0550	0.83	535.5	375.3	51.4	23.8
68	7.229	0.172	-0.4360	0.0500	0.94	505.8	313.4	34.9	14.9
69	8.365	0.297	-0.5940	0.0680	0.90	1092.6	568.7	156.5	42.9
70	7.047	0.194	-0.3330	0.0500	0.85	533.4	369.9	47.7	20.5
71	6.981	0.157	-0.2860	0.0350	0.89	557.3	407.2	45.6	21.6
72	6.952	0.177	-0.3310	0.0450	0.86	487.1	338.5	40.7	17.8
73	6.778	0.046	-0.1760	0.0120	0.97	586.0	483.0	12.2	5.7
74	6.994	0.208	-0.3420	0.0510	0.90	495.7	340.3	50.5	22.8
75	7.068	0.289	-0.4180	0.0880	0.82	448.5	283.5	48.4	22.1

Table A9.4. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
76	7.402	0.256	-0.3930	0.0650	0.82	662.7	430.1	75.7	27.3
77	6.895	0.164	-0.3080	0.0410	0.88	486.2	346.7	38.4	17.8
78	7.440	0.209	-0.4590	0.0550	0.90	591.5	357.2	52.6	17.3
79	7.449	0.146	-0.4310	0.0350	0.94	636.4	396.2	44.4	15.8
80	7.094	0.201	-0.3580	0.0510	0.84	528.8	356.9	49.3	20.8
81	7.359	0.148	-0.3990	0.0360	0.96	626.4	403.9	44.0	16.0
82	7.080	0.083	-0.3890	0.0220	0.97	485.2	316.6	18.1	8.0
83	6.848	0.154	-0.3020	0.0390	0.91	470.0	337.3	34.1	16.0
84	6.922	0.141	-0.3120	0.0370	0.91	494.3	350.8	31.9	14.9
85	6.791	0.071	-0.2200	0.0150	0.96	535.8	420.7	21.0	11.6
86	7.676	0.371	-0.3670	0.0780	0.73	926.7	619.3	183.0	75.5
87	7.041	0.181	-0.2690	0.0410	0.82	614.9	457.5	56.5	26.4
88	7.063	0.089	-0.2670	0.0190	0.94	631.1	470.5	30.8	15.5
89	7.513	0.244	-0.4110	0.0540	0.89	710.8	452.5	89.7	34.8
90	6.821	0.126	-0.1890	0.0260	0.87	593.0	481.6	41.6	22.7
91	6.865	0.230	-0.2260	0.0540	0.72	569.2	444.0	69.5	38.3
92	6.664	0.153	-0.2710	0.0390	0.89	420.3	312.1	31.6	16.2
93	6.921	0.078	-0.3530	0.0250	0.98	449.8	305.4	12.0	6.0
94	6.951	0.204	-0.3310	0.0570	0.85	487.5	339.0	42.9	20.5
95	7.128	0.226	-0.3330	0.0540	0.79	579.5	402.1	64.9	28.9
96	7.293	0.160	-0.4010	0.0360	0.96	583.6	375.7	47.5	18.6
97	7.865	0.118	-0.5240	0.0290	0.97	778.7	437.8	42.8	13.1
98	6.757	0.139	-0.2900	0.0360	0.91	440.6	320.3	29.5	15.7
99	7.299	0.190	-0.3740	0.0460	0.92	625.1	414.4	55.8	21.0
100	7.077	0.247	-0.4020	0.0740	0.81	469.6	302.0	43.6	19.2
101	6.336	0.060	-0.2160	0.0150	0.97	343.6	271.1	10.7	6.0
102	7.101	0.090	-0.3840	0.0240	0.97	501.6	329.1	20.2	8.9
103	7.026	0.128	-0.3430	0.0330	0.95	511.4	351.0	30.7	14.4
104	6.930	0.102	-0.3360	0.0250	0.96	471.4	325.9	23.8	11.3
105	7.027	0.066	-0.3040	0.0160	0.97	559.6	400.7	18.2	8.5
106	7.026	0.128	-0.3430	0.0330	0.95	511.4	351.0	30.7	14.4
107	6.673	0.081	-0.2840	0.0200	0.96	411.1	300.9	16.1	7.8
108	6.882	0.063	-0.2860	0.0200	0.98	505.2	369.1	11.4	7.0
109	7.170	0.180	-0.3310	0.0430	0.87	606.9	422.1	52.9	22.3
110	7.785	0.113	-0.4100	0.0240	0.97	934.5	595.3	56.3	22.0
111	7.679	0.086	-0.3950	0.0170	0.98	870.9	564.3	41.1	17.1
112	8.374	0.132	-0.5570	0.0280	0.97	1202.8	652.5	82.1	25.5
113	8.106	0.140	-0.5500	0.0340	0.97	934.6	510.7	60.5	17.1
114	7.607	0.178	-0.4640	0.0440	0.90	691.3	415.1	55.6	17.8

Table A9.4. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
115	7.186	0.155	-0.3050	0.0350	0.92	655.1	468.7	52.5	23.9
116	7.437	0.082	-0.3480	0.0180	0.96	762.1	520.1	32.8	13.8
117	7.421	0.101	-0.3840	0.0230	0.96	690.1	452.7	35.7	14.4
118	8.300	0.149	-0.5000	0.0300	0.96	1272.5	734.8	103.8	37.5
119	7.408	0.126	-0.3150	0.0260	0.94	798.3	564.7	54.7	24.5
120	7.401	0.107	-0.3470	0.0230	0.96	737.1	503.7	41.4	17.6
121	8.400	0.252	-0.5830	0.0550	0.94	1161.9	612.6	149.8	45.4
122	6.759	0.054	-0.2580	0.0120	0.98	475.6	358.1	14.1	7.5
123	7.352	0.091	-0.3780	0.0200	0.97	653.5	431.4	30.6	12.4
124	6.952	0.095	-0.2470	0.0200	0.93	591.9	451.3	30.7	15.9
125	8.124	0.164	-0.4970	0.0350	0.96	1074.9	622.7	92.1	31.3
126	7.622	0.521	-0.3310	0.1100	0.70	951.7	661.2	260.9	108.0
127	6.869	0.164	-0.2640	0.0340	0.86	523.7	391.9	47.9	24.2
128	7.379	0.087	-0.3430	0.0200	0.97	727.0	498.7	31.5	12.4
129	8.330	0.159	-0.5270	0.0320	0.96	1233.3	691.4	107.2	37.6
130	7.465	0.177	-0.3500	0.0380	0.89	779.4	530.5	72.4	30.4
131	8.189	0.221	-0.5190	0.0480	0.94	1089.6	616.1	122.7	39.3
132	7.444	0.129	-0.3700	0.0290	0.93	729.9	486.3	48.7	20.1
133	7.487	0.273	-0.3220	0.0590	0.75	850.5	597.3	121.8	52.1
134	7.699	0.112	-0.4090	0.0250	0.97	860.0	548.6	47.9	17.3
135	7.440	0.139	-0.3360	0.0290	0.93	786.1	543.6	60.5	27.3
136	7.635	0.132	-0.4190	0.0300	0.94	788.9	497.9	52.8	19.8
137	7.970	0.084	-0.4870	0.0190	0.98	941.4	551.1	39.7	13.1
138	8.194	0.223	-0.4620	0.0430	0.94	1249.3	752.2	159.1	63.2
139	7.246	0.185	-0.3330	0.0370	0.90	652.0	452.4	67.6	30.9
140	7.247	0.077	-0.3190	0.0170	0.97	672.9	473.8	26.8	11.7
141	8.103	0.202	-0.4960	0.0440	0.94	1053.8	610.8	108.6	36.6
142	7.359	0.192	-0.3860	0.0450	0.88	645.4	422.3	61.7	24.5
143	7.426	0.066	-0.3380	0.0130	0.98	772.1	532.8	28.1	12.8
144	7.096	0.072	-0.3310	0.0170	0.98	563.6	392.0	20.2	8.7
145	8.293	0.310	-0.4620	0.0610	0.88	1379.7	830.8	235.7	88.2
146	7.370	0.058	-0.3020	0.0110	0.98	791.7	567.9	27.2	13.7

Table A9.5 Estimates of u,v, standard error of both parameters, the predicted value $\hat{p}$ and respective standard error for each session for rider 7

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
1	7.202	0.129	-0.2950	0.0230	0.95	681.5	493.1	52.7	26.9
2	7.158	0.108	-0.3200	0.0240	0.94	615.3	433.2	35.3	16.8
3	7.571	0.078	-0.4220	0.0180	0.98	735.1	462.6	28.5	10.3
4	6.370	0.413	-0.1870	0.0780	0.66	379.6	309.1	92.9	53.6
5	6.515	0.106	-0.1750	0.0160	0.96	450.8	371.8	31.4	19.7
6	5.529	0.137	-0.0550	0.0230	0.75	222.0	209.0	20.4	15.3
7	7.411	0.078	-0.4050	0.0180	0.98	651.0	417.3	25.2	9.6
8	5.808	0.032	-0.1720	0.0100	0.99	224.3	185.7	2.8	2.0
9	6.987	0.040	-0.2820	0.0090	0.99	565.2	414.5	11.8	5.6
10	6.286	0.148	-0.1450	0.0240	0.85	384.6	327.9	36.6	23.4
11	6.889	0.137	-0.2380	0.0250	0.92	567.6	437.1	47.8	27.6
12	5.541	0.015	-0.0890	0.0030	1.00	208.0	188.7	1.6	1.0
13	7.465	0.117	-0.3300	0.0220	0.95	816.1	567.8	55.8	26.8
14	6.349	0.083	-0.1710	0.0160	0.93	386.0	320.0	19.0	11.3
15	5.985	0.245	-0.1160	0.0430	0.65	304.2	267.8	46.3	29.7
16	7.116	0.104	-0.3410	0.0260	0.94	561.6	386.1	27.6	12.4
17	7.340	0.068	-0.3640	0.0150	0.98	665.4	445.9	23.6	10.0
18	6.911	0.086	-0.2420	0.0170	0.94	574.6	440.4	29.0	15.9
19	6.899	0.096	-0.3000	0.0220	0.95	497.4	357.9	24.4	11.5
20	5.675	0.078	-0.0990	0.0150	0.98	231.8	207.8	10.6	6.5
21	7.311	0.085	-0.3490	0.0190	0.97	670.2	456.8	29.9	13.2
22	7.533	0.171	-0.4190	0.0460	0.90	712.3	449.6	50.1	15.5
23	7.252	0.170	-0.2990	0.0310	0.93	708.5	510.0	73.0	37.9
24	6.110	0.165	-0.1140	0.0270	0.75	346.4	305.7	37.5	25.6
25	7.161	0.107	-0.3140	0.0230	0.95	625.2	442.9	35.9	16.5
26	6.728	0.117	-0.2780	0.0280	0.95	439.9	324.0	25.7	12.7
27	7.092	0.102	-0.3000	0.0210	0.96	602.6	433.4	33.4	15.8
28	6.877	0.113	-0.3170	0.0270	0.95	467.3	329.8	26.4	12.1
29	6.777	0.281	-0.2380	0.0560	0.75	507.9	391.2	81.7	43.2
30	5.531	0.130	-0.0550	0.0210	0.77	222.1	209.0	19.3	14.4
31	6.844	0.083	-0.2110	0.0180	0.92	577.4	458.0	26.3	14.7
32	7.008	0.066	-0.2650	0.0130	0.97	599.7	448.1	23.1	12.5
33	7.229	0.047	-0.3290	0.0110	0.99	646.3	450.3	15.7	7.0
34	7.074	0.063	-0.3500	0.0160	0.98	527.4	359.0	16.0	7.3
35	6.038	0.052	-0.1770	0.0130	0.98	278.9	229.6	7.1	3.7

Table A9.5. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
36	7.335	0.182	-0.3760	0.0390	0.92	644.9	426.6	61.5	25.1
37	7.243	0.152	-0.3030	0.0340	0.91	696.4	499.4	55.2	25.0
38	6.782	0.074	-0.2170	0.0140	0.95	534.7	421.1	23.9	14.1
39	7.110	0.222	-0.2980	0.0420	0.88	616.3	444.1	79.4	38.5
40	7.187	0.169	-0.3100	0.0400	0.84	648.1	461.2	55.2	25.1
41	7.454	0.196	-0.3470	0.0410	0.86	776.4	530.2	83.7	37.2
42	5.456	0.116	-0.0630	0.0230	0.88	202.4	188.8	13.9	9.5
43	6.671	0.101	-0.2010	0.0190	0.92	497.1	398.8	30.7	18.6
44	5.680	0.110	-0.0850	0.0190	0.91	240.8	219.2	16.4	11.0
45	7.504	0.274	-0.3940	0.0740	0.85	731.9	474.6	79.9	21.8
46	6.941	0.137	-0.2710	0.0300	0.88	554.7	412.0	41.6	21.3
47	7.280	0.111	-0.3320	0.0230	0.95	675.0	468.5	41.2	18.8
48	5.604	0.037	-0.0820	0.0070	0.98	224.7	205.3	5.0	3.4
49	7.306	0.129	-0.3530	0.0340	0.93	661.5	449.0	36.9	13.4
50	6.905	0.075	-0.2720	0.0160	0.96	533.5	395.9	22.2	11.5
51	7.577	0.165	-0.4320	0.0390	0.96	721.3	448.5	57.3	19.7
52	7.236	0.055	-0.3580	0.0140	0.99	609.5	411.4	14.8	5.5
53	6.671	0.065	-0.2070	0.0130	0.95	490.1	390.6	18.9	11.3
54	7.254	0.175	-0.4070	0.0490	0.90	553.0	353.5	40.7	16.9
55	7.220	0.065	-0.3460	0.0160	0.98	616.1	421.4	18.0	7.0
56	7.305	0.172	-0.3360	0.0370	0.89	685.9	474.1	63.2	28.5
57	7.242	0.174	-0.2660	0.0310	0.83	757.4	565.6	80.5	43.1
58	7.003	0.127	-0.2450	0.0250	0.90	625.4	477.8	46.6	25.2
59	7.474	0.384	-0.3490	0.0730	0.76	789.7	538.5	172.3	76.5
60	7.091	0.113	-0.2510	0.0220	0.94	674.7	512.3	45.2	24.6
61	8.135	0.269	-0.5450	0.0590	0.90	973.3	535.1	132.8	42.3
62	7.205	0.127	-0.3900	0.0340	0.94	548.6	357.5	30.8	12.7
63	6.918	0.095	-0.2340	0.0190	0.92	590.2	456.7	31.9	17.1
64	7.254	0.068	-0.2920	0.0140	0.97	720.8	522.7	28.2	14.3
65	6.658	0.166	-0.2410	0.0360	0.85	447.4	343.3	40.4	20.7
66	7.487	0.197	-0.3830	0.0400	0.91	738.9	485.1	79.8	33.5
67	7.305	0.210	-0.3720	0.0490	0.88	631.7	419.8	68.0	29.6
68	7.475	0.122	-0.3620	0.0250	0.94	766.8	515.4	51.3	22.3
69	6.791	0.161	-0.2160	0.0280	0.88	541.3	427.1	54.6	32.0
70	6.953	0.270	-0.2900	0.0570	0.79	537.2	390.8	78.1	36.4
71	7.123	0.140	-0.4120	0.0390	0.94	480.0	305.2	28.5	11.9
72	7.638	0.093	-0.3880	0.0190	0.98	849.0	554.3	44.1	18.9
73	7.273	0.133	-0.3410	0.0260	0.96	657.1	451.8	49.3	22.2
74	6.753	0.171	-0.2530	0.0380	0.81	478.9	362.8	43.5	22.1
75	8.236	0.234	-0.5760	0.0520	0.94	1000.7	531.2	117.3	35.3

Table A9.5. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
76	7.453	0.238	-0.3790	0.0470	0.92	721.1	475.5	96.2	41.1
77	6.759	0.112	-0.1960	0.0210	0.89	548.6	442.2	37.8	22.5
78	7.445	0.093	-0.3480	0.0190	0.97	767.4	523.4	39.6	18.1
79	6.674	0.166	-0.2490	0.0370	0.85	446.3	339.5	39.2	19.7
80	7.278	0.182	-0.3550	0.0420	0.85	639.2	432.7	58.5	25.1
81	6.808	0.210	-0.2700	0.0440	0.84	486.7	361.9	56.7	29.0
82	6.468	0.318	-0.1610	0.0670	0.49	444.3	372.1	79.3	46.7
83	7.279	0.172	-0.3030	0.0360	0.86	720.7	516.4	67.5	31.7
84	6.847	0.065	-0.2000	0.0140	0.96	593.4	476.3	21.2	11.5
85	7.172	0.286	-0.3340	0.0600	0.82	603.3	417.9	92.1	39.6
86	6.484	0.663	-0.1710	0.1270	0.27	441.9	366.4	169.6	95.4
87	6.736	0.148	-0.2710	0.0360	0.88	451.6	335.4	33.3	16.7
88	6.964	0.125	-0.2170	0.0280	0.90	641.8	505.7	41.4	20.7
89	6.791	0.161	-0.2800	0.0380	0.87	467.4	343.8	38.3	18.3
90	7.362	0.505	-0.4430	0.1290	0.75	567.5	348.7	129.0	45.5
91	7.446	0.167	-0.3270	0.0330	0.88	806.1	562.7	76.3	35.6
92	7.057	0.201	-0.2440	0.0400	0.74	661.4	505.7	77.1	41.2
93	7.071	0.127	-0.2400	0.0250	0.89	676.8	519.8	49.0	25.3
94	8.227	0.159	-0.5320	0.0340	0.96	1098.0	611.9	92.4	31.2
95	6.897	0.200	-0.3180	0.0480	0.86	475.2	334.9	46.6	20.9
96	7.281	0.129	-0.3700	0.0290	0.96	618.8	412.0	41.4	17.4
97	7.114	0.134	-0.2680	0.0270	0.88	663.1	493.9	51.4	26.8
98	6.780	0.113	-0.2650	0.0260	0.92	477.9	357.0	28.3	14.3
99	6.911	0.386	-0.2420	0.0830	0.59	575.0	441.0	122.2	64.6
100	7.974	0.132	-0.4760	0.0290	0.96	971.0	575.7	66.2	23.2
101	7.081	0.148	-0.2520	0.0290	0.88	666.1	505.0	56.7	29.2
102	7.213	0.399	-0.3670	0.0900	0.73	582.3	388.9	116.5	45.3
103	6.565	0.095	-0.2320	0.0210	0.94	416.5	323.0	21.2	11.3
104	7.705	0.319	-0.4230	0.0680	0.80	837.4	525.9	141.7	54.6
105	8.330	0.206	-0.5910	0.0470	0.94	1062.2	554.7	106.5	30.3
106	7.559	0.158	-0.3660	0.0330	0.93	824.7	551.4	70.5	29.9
107	6.941	0.218	-0.2880	0.0420	0.88	533.2	388.7	65.9	31.6
108	7.330	0.279	-0.3590	0.0570	0.85	667.8	450.3	101.6	43.3
109	7.397	0.279	-0.3630	0.0640	0.78	706.6	474.0	100.6	42.4
110	7.334	0.099	-0.3080	0.0220	0.97	753.9	537.5	38.0	16.2
111	8.655	0.265	-0.6530	0.0610	0.93	1275.3	622.2	161.8	40.6
112	7.509	0.182	-0.3420	0.0360	0.89	830.4	570.5	85.9	39.6
113	6.841	0.242	-0.3640	0.0590	0.93	404.7	271.3	44.7	14.9
114	7.422	0.169	-0.3690	0.0330	0.95	715.2	476.9	68.0	29.7

Table A9.5. Continued

Session	u	$s.e.(u)$	v	$s.e.(v)$	R^2	$\widehat{P_{d=10}}$	$\widehat{P_{d=30}}$	$s.e.(\widehat{P_{d=10}})$	$s.e.(\widehat{P_{d=30}})$
115	7.195	0.139	-0.2820	0.0310	0.92	695.5	510.0	50.5	23.6
116	7.174	0.194	-0.3530	0.0450	0.88	578.3	392.3	57.0	25.1
117	7.145	0.092	-0.2650	0.0180	0.94	688.7	514.8	37.1	19.8
118	7.236	0.109	-0.2910	0.0240	0.94	710.7	516.2	40.7	18.7
119	6.830	0.156	-0.1780	0.0280	0.82	613.8	504.6	59.6	36.5
120	6.707	0.218	-0.2650	0.0570	0.71	445.0	332.7	43.3	19.7
121	7.243	0.103	-0.3180	0.0220	0.95	672.8	474.5	37.2	17.2
122	6.940	0.043	-0.2080	0.0090	0.98	639.9	509.3	15.0	8.0
123	7.069	0.061	-0.2850	0.0130	0.98	608.8	444.9	20.4	9.9
124	7.658	0.208	-0.5020	0.0520	0.93	666.6	384.1	62.2	19.6
125	6.815	0.085	-0.2350	0.0170	0.94	530.2	409.5	26.1	14.4
126	7.439	0.240	-0.4600	0.0600	0.90	589.6	355.6	64.4	20.7
127	7.269	0.121	-0.3110	0.0250	0.93	701.6	498.6	47.1	22.4
128	8.207	0.448	-0.6690	0.1190	0.89	786.2	377.0	145.9	33.2
129	7.185	0.096	-0.3900	0.0230	0.97	537.4	350.0	24.8	9.6
130	7.438	0.077	-0.3770	0.0170	0.98	713.3	471.5	28.9	12.2
131	6.936	0.145	-0.2740	0.0330	0.87	547.0	404.7	42.3	21.7
132	6.997	0.085	-0.3740	0.0230	0.97	461.9	306.2	17.7	7.9
133	7.660	0.268	-0.5460	0.0800	0.90	603.7	331.3	56.1	17.4
134	7.352	0.330	-0.4670	0.1000	0.84	532.1	318.7	61.2	23.1
135	7.522	0.147	-0.3920	0.0310	0.94	748.8	486.6	59.0	23.9
136	7.525	0.308	-0.3620	0.0610	0.90	804.5	540.3	139.1	61.1
137	7.039	0.190	-0.2750	0.0440	0.87	606.0	448.1	57.9	27.0
138	7.099	0.102	-0.3170	0.0230	0.95	583.4	411.8	31.5	14.4
139	8.091	0.180	-0.6500	0.0490	0.95	731.2	358.1	53.1	12.7
140	7.403	0.221	-0.5220	0.0680	0.92	492.9	277.6	39.7	17.1
141	7.271	0.102	-0.4590	0.0290	0.97	499.1	301.3	21.7	8.9
142	7.651	0.145	-0.5130	0.0430	0.97	645.6	367.5	33.4	10.4
143	7.402	0.361	-0.4470	0.0960	0.85	586.0	358.6	88.4	27.5
144	8.123	0.511	-0.7740	0.1610	0.85	567.0	242.2	90.7	24.5
145	7.449	0.329	-0.5310	0.1030	0.84	506.5	282.8	59.6	25.7
146	7.818	0.175	-0.4660	0.0380	0.95	850.9	510.2	78.1	28.8
147	7.378	0.193	-0.4880	0.0570	0.90	520.0	304.1	37.9	14.5
148	7.501	0.125	-0.4750	0.0360	0.97	606.9	360.4	28.5	8.9
149	7.652	0.117	-0.5120	0.0340	0.97	647.5	369.0	27.6	7.9
150	8.107	0.187	-0.7710	0.0570	0.98	562.2	241.0	36.4	9.4
151	7.176	0.097	-0.3200	0.0230	0.95	626.1	440.7	29.5	12.5
152	6.913	0.086	-0.2590	0.0200	0.94	553.6	416.5	25.1	13.1

Appendix 10: Fitted line of $\boldsymbol{ln(p) = \hat{u} + \hat{v}\, ln(d)}$ for the relationship between power output $\boldsymbol{p}$ and duration $\boldsymbol{d}$

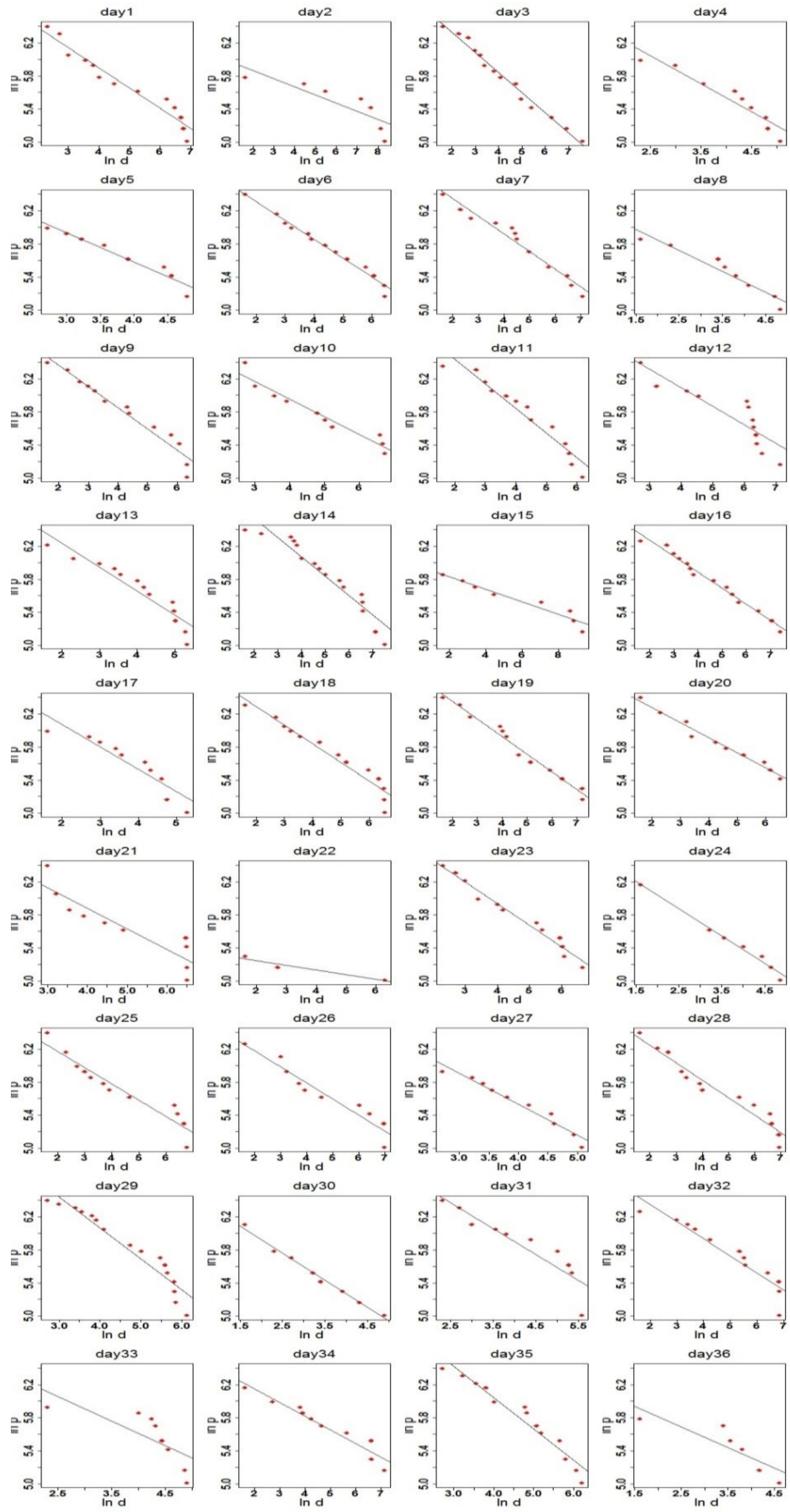

Figure A10.1 Fitted line of $ln(p) = \hat{u} + \hat{v}ln(d)$ for the relationship between power output p and duration d for all sessions for rider 1

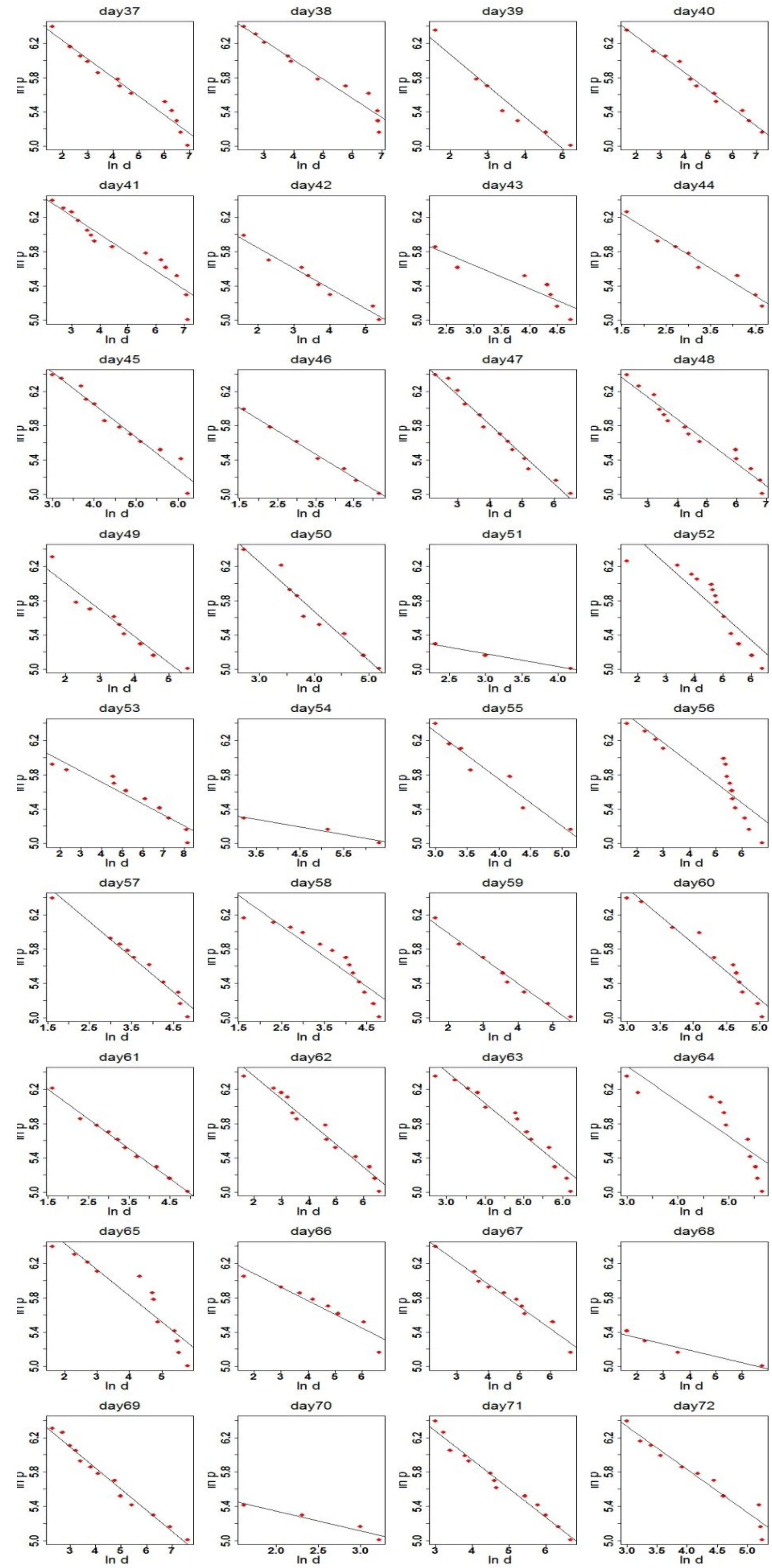

Figure A10.1 Continued

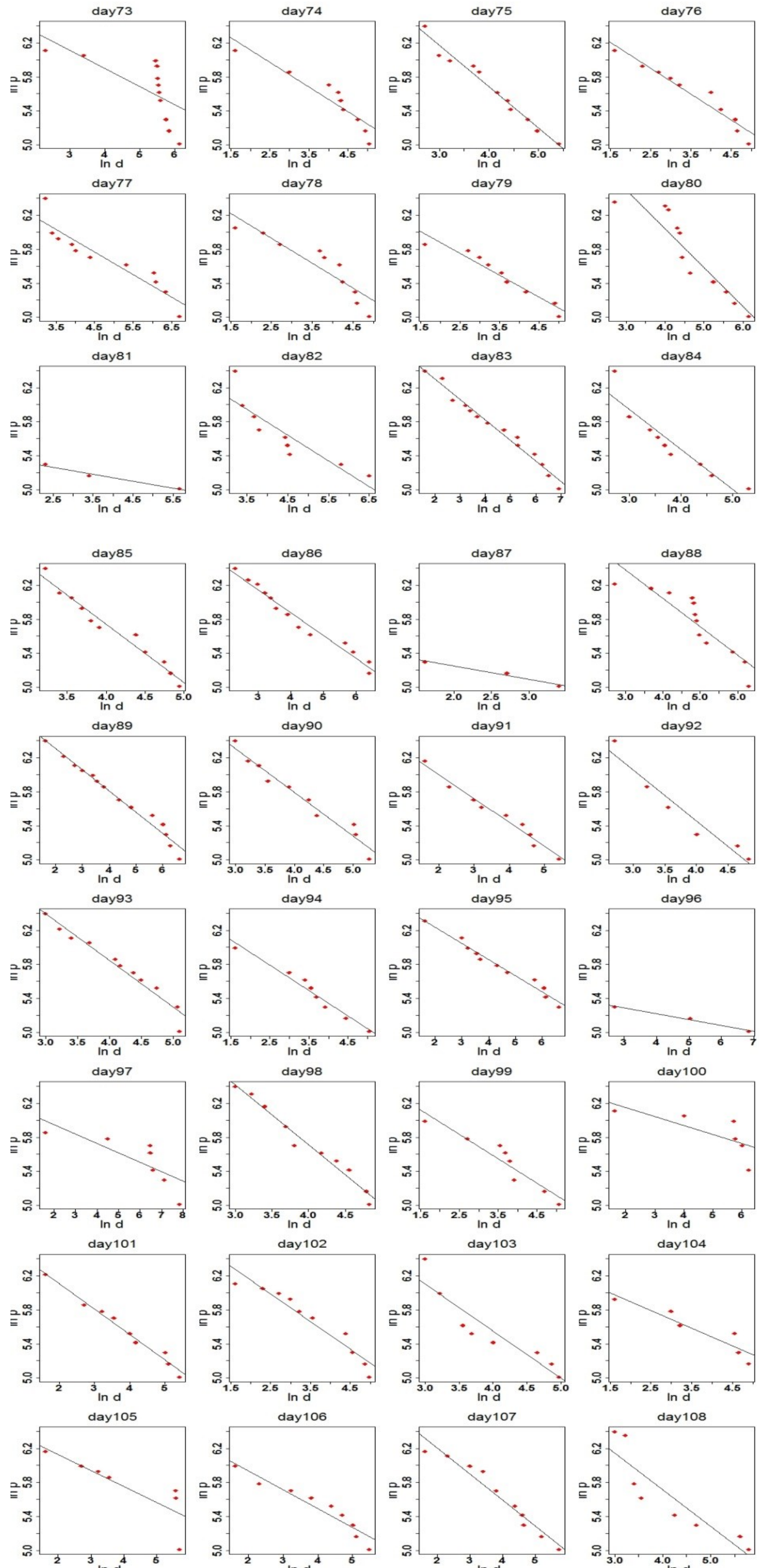

Figure A10.1 Continued

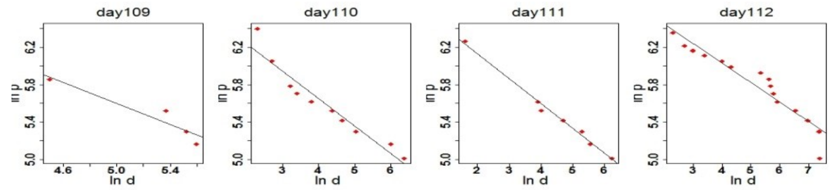

Figure A10.1 Continued

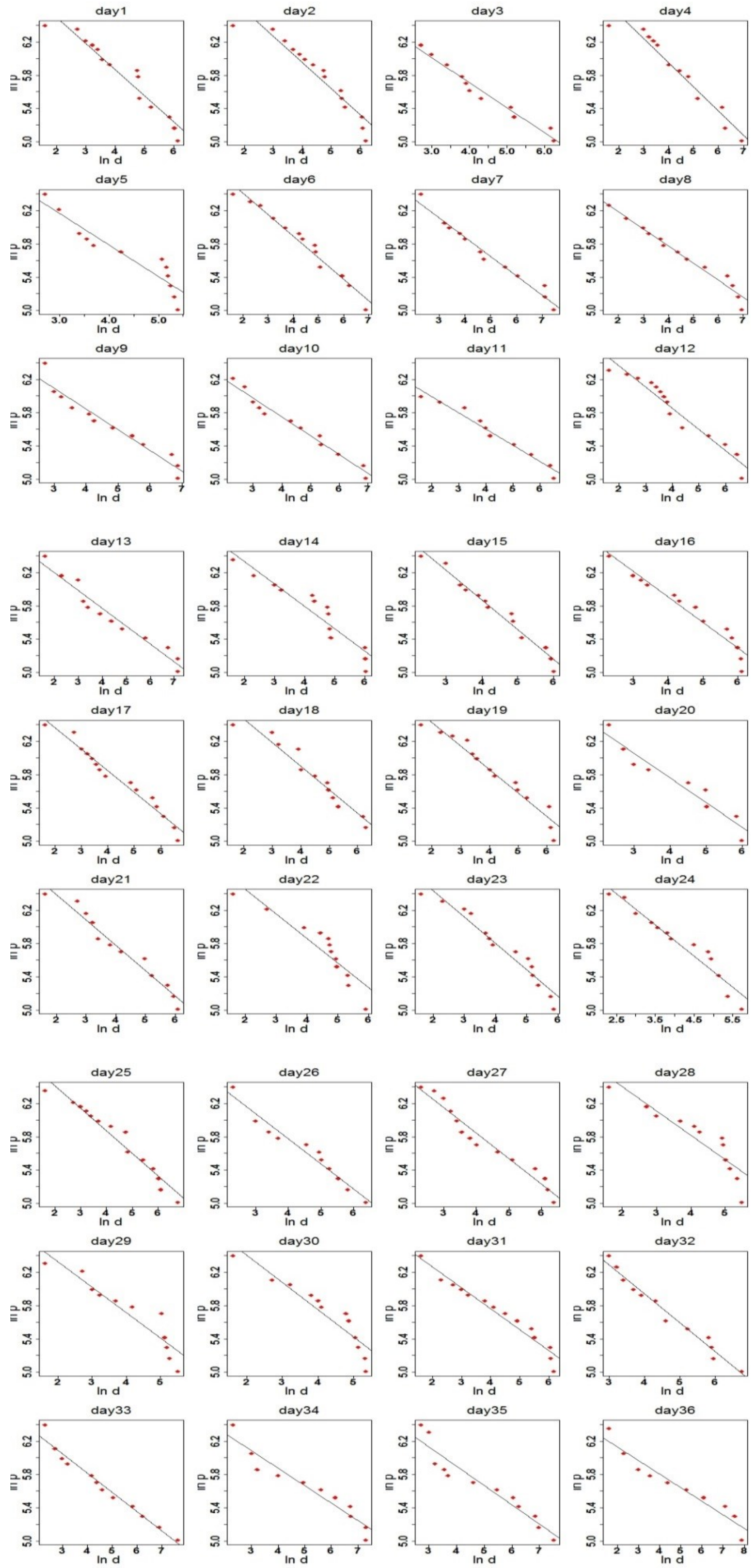

Figure A10.2 Fitted line of $ln(p) = \hat{u} + \hat{v}ln(d)$ for the relationship between power output p and duration d for all sessions for rider 3

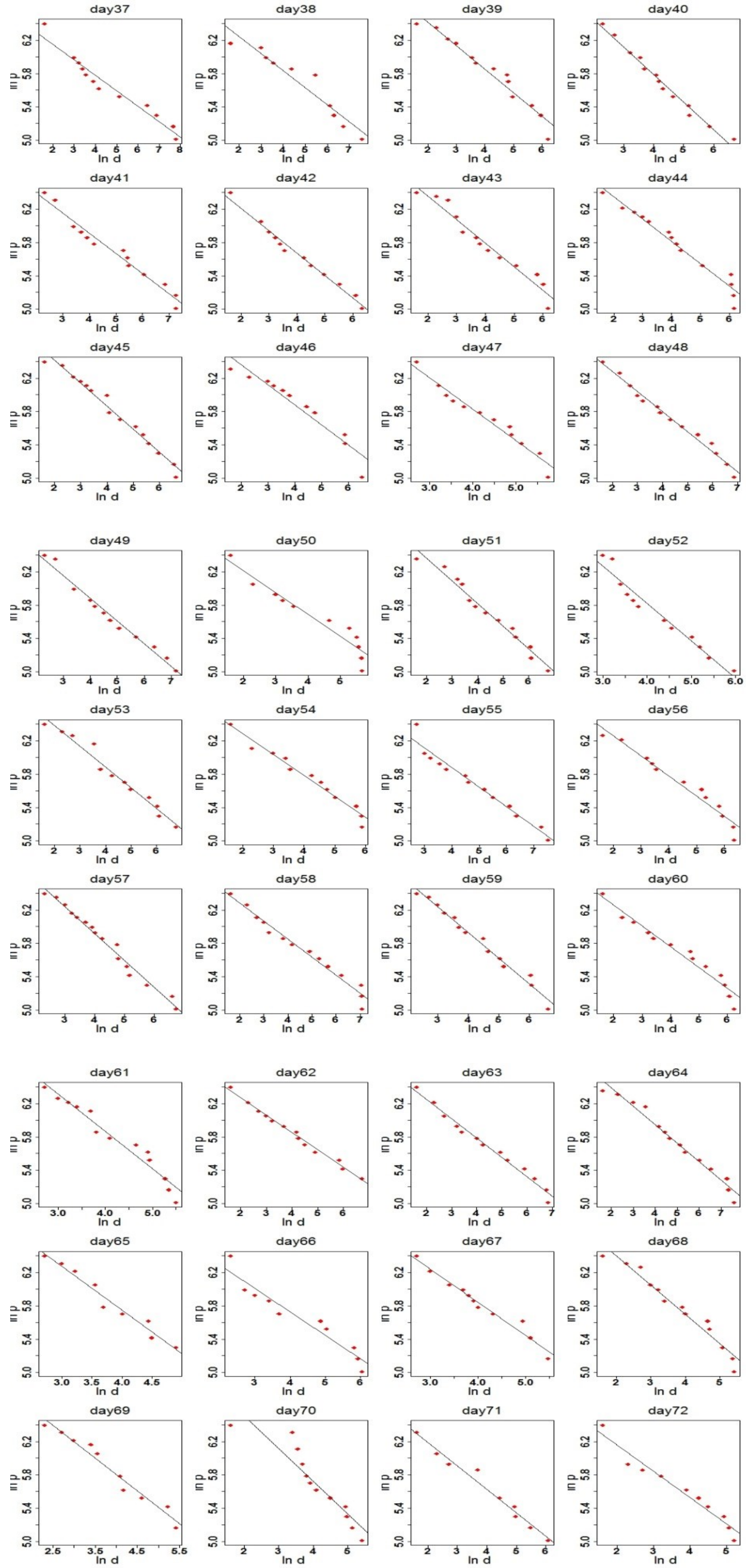

Figure A10.2 Continued

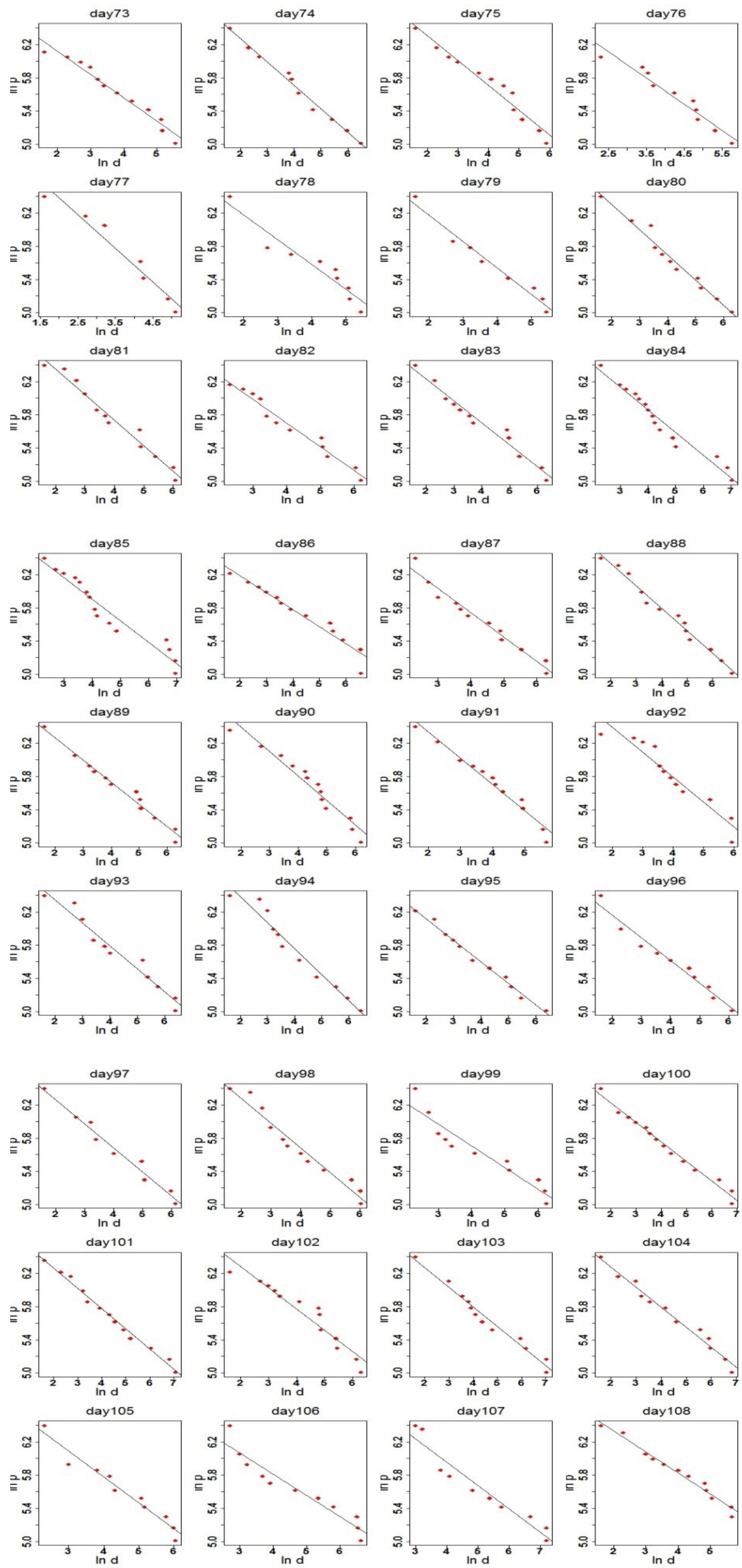

Figure A10.2 Continued

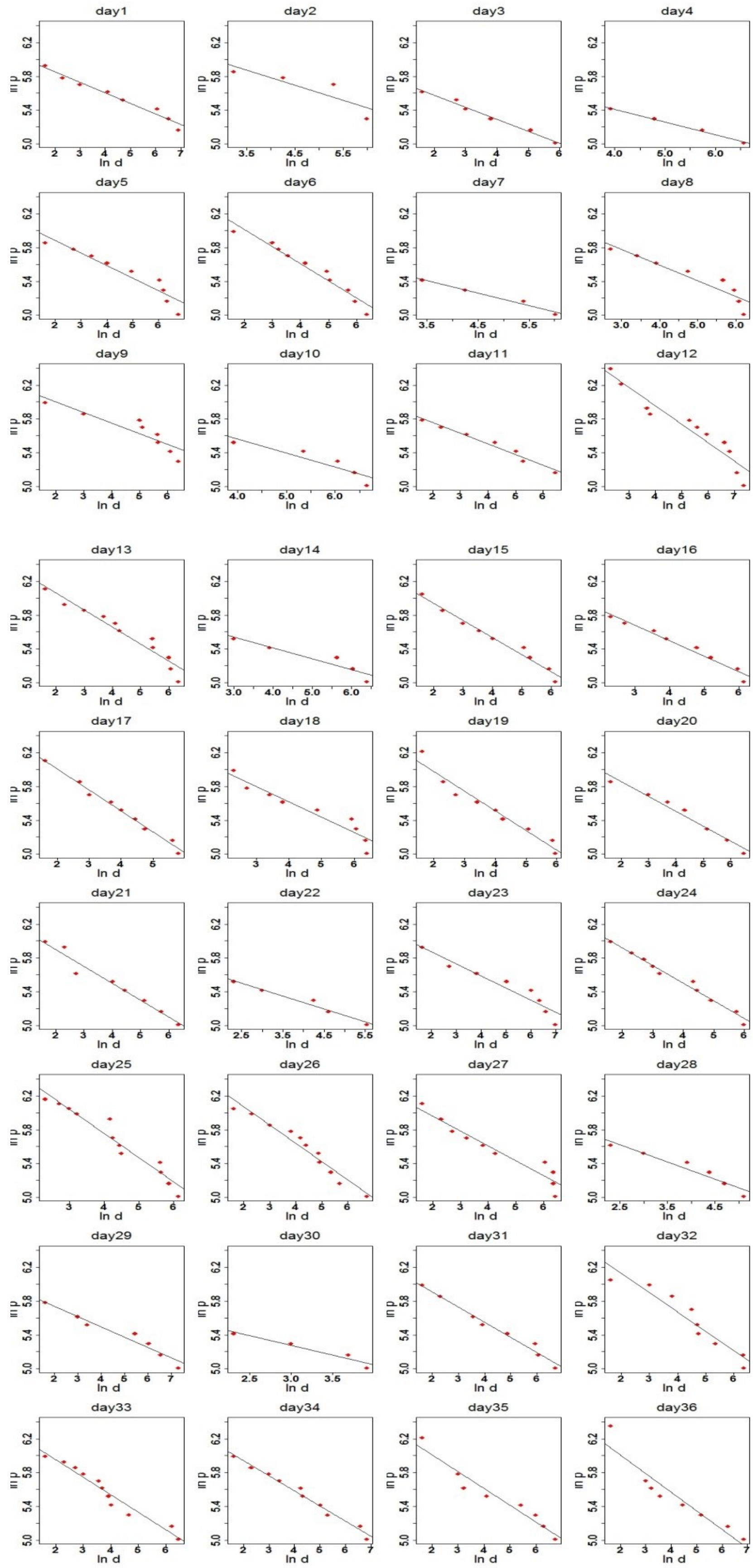

Figure A10.3 Fitted line of $ln(p) = \hat{u} + \hat{v}ln(d)$ for the relationship between power output p and duration d for all sessions for rider 4

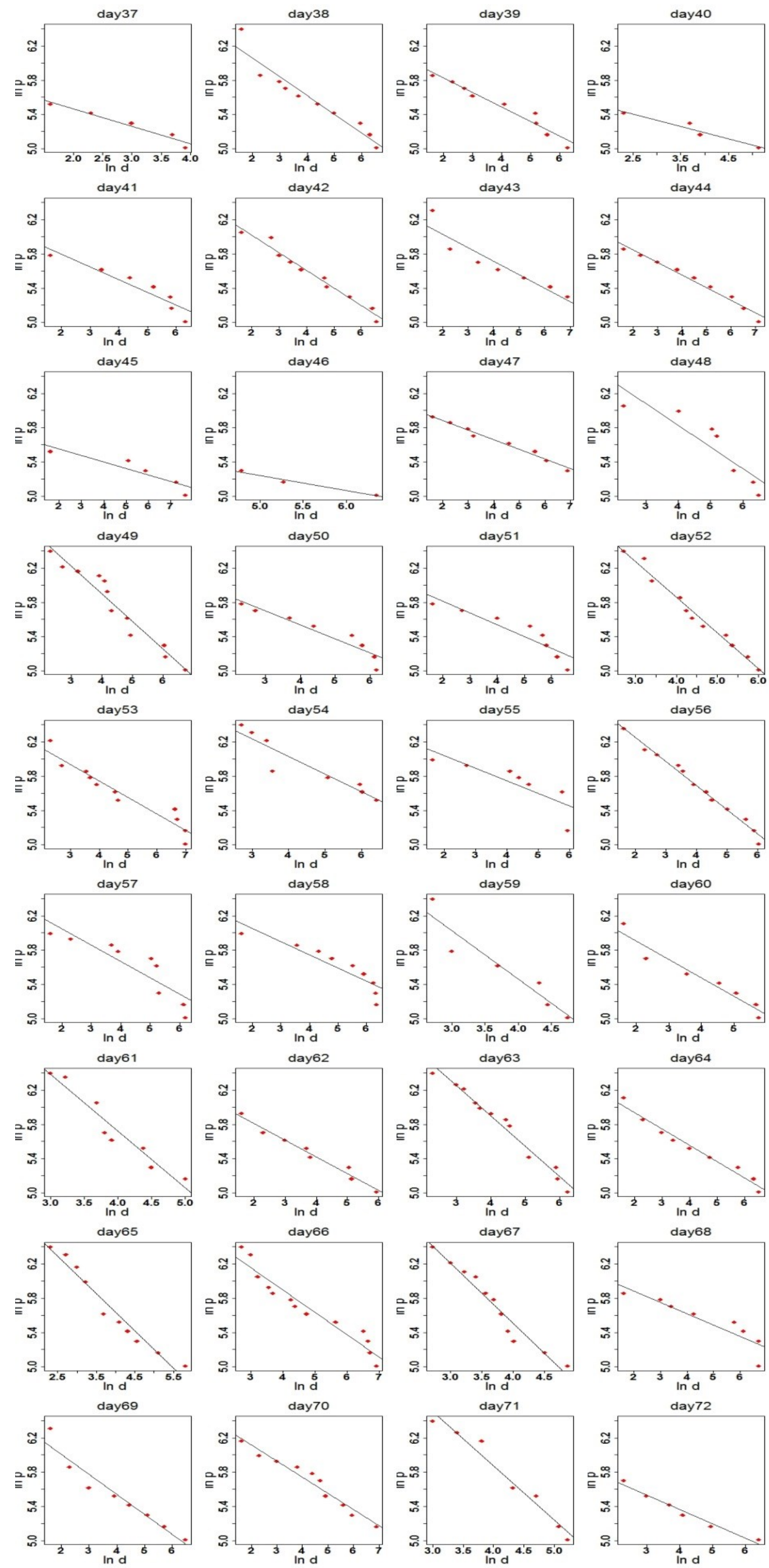

Figure A10.3 Continued

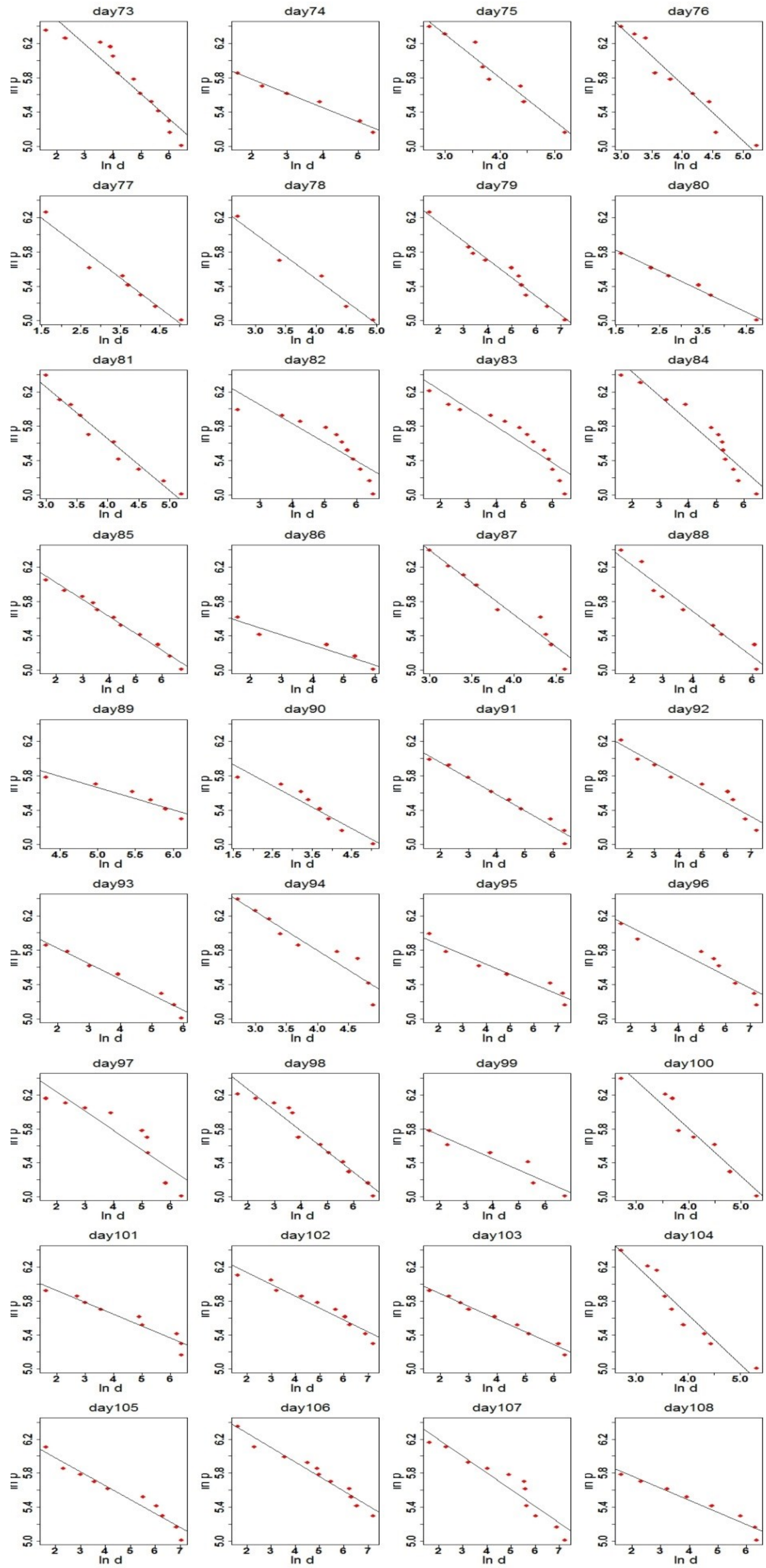

Figure A10.3 Continued

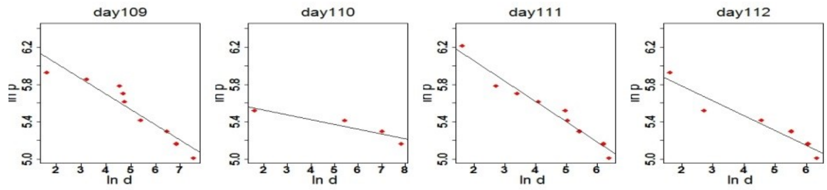

Figure A10.3 Continued

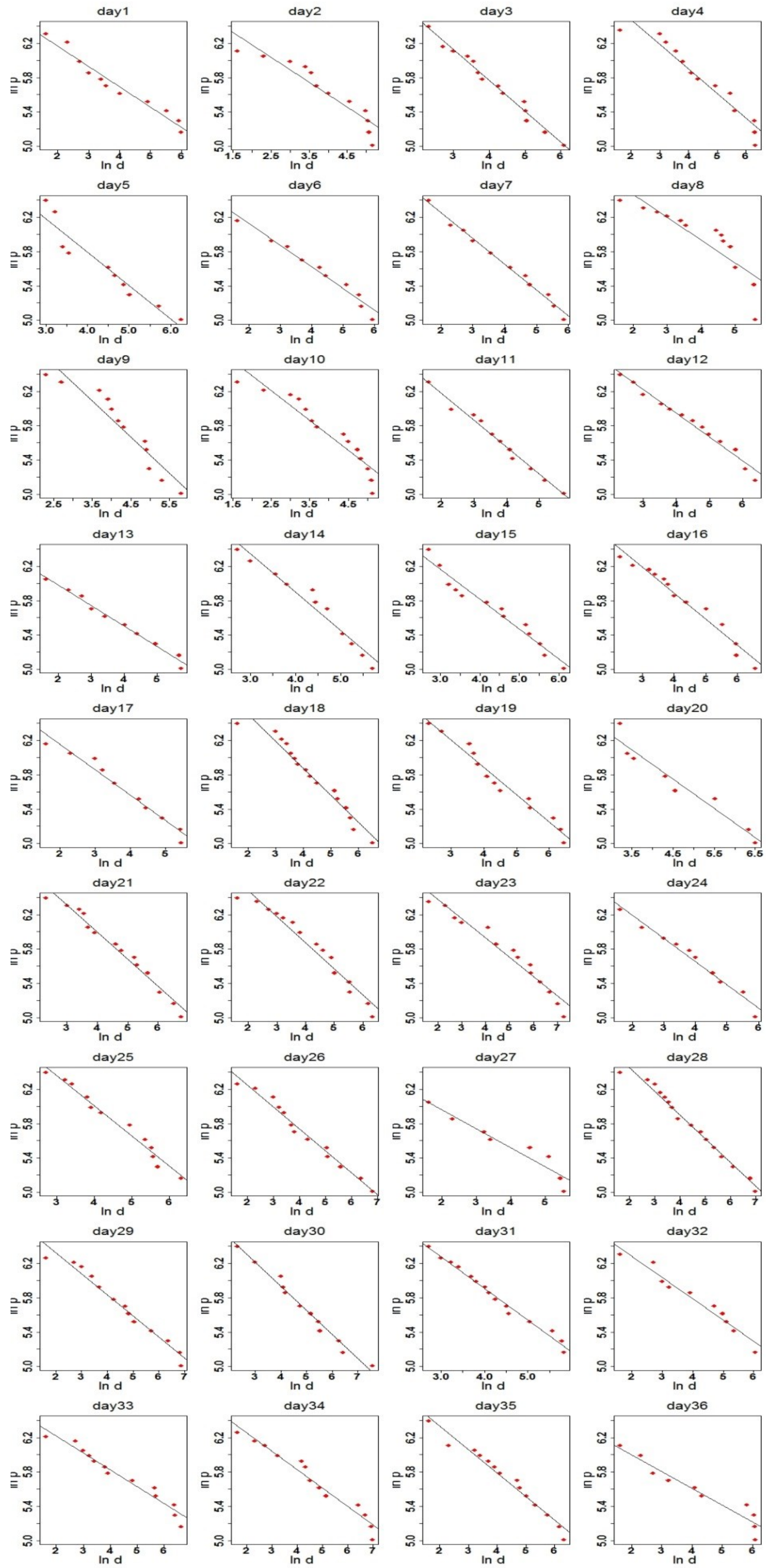

Figure A10.4 Fitted line of $ln(p) = \hat{u} + \hat{v}ln(d)$ for the relationship between power output p and duration d for all sessions for rider 5

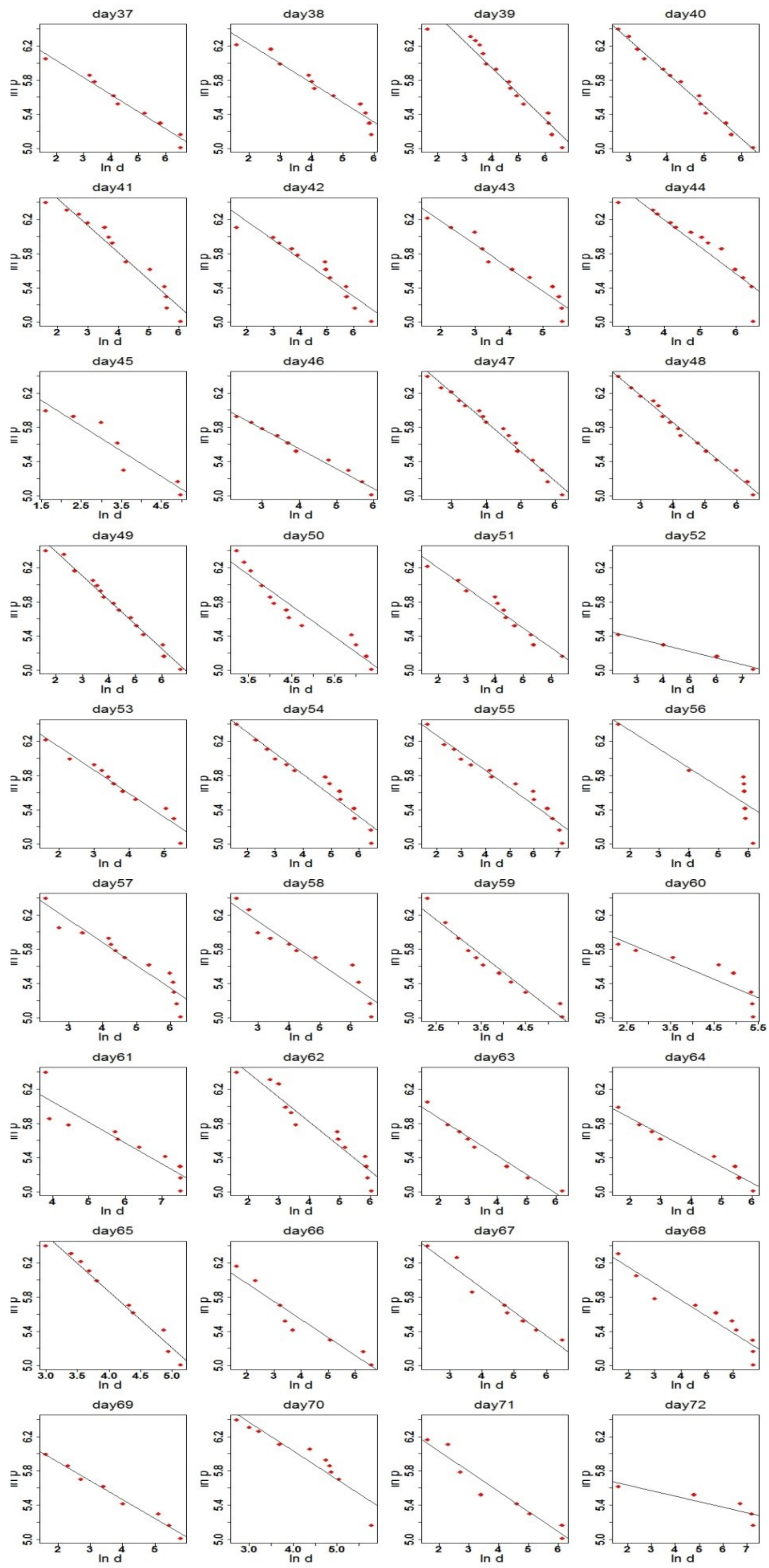

Figure A10.4 Continued

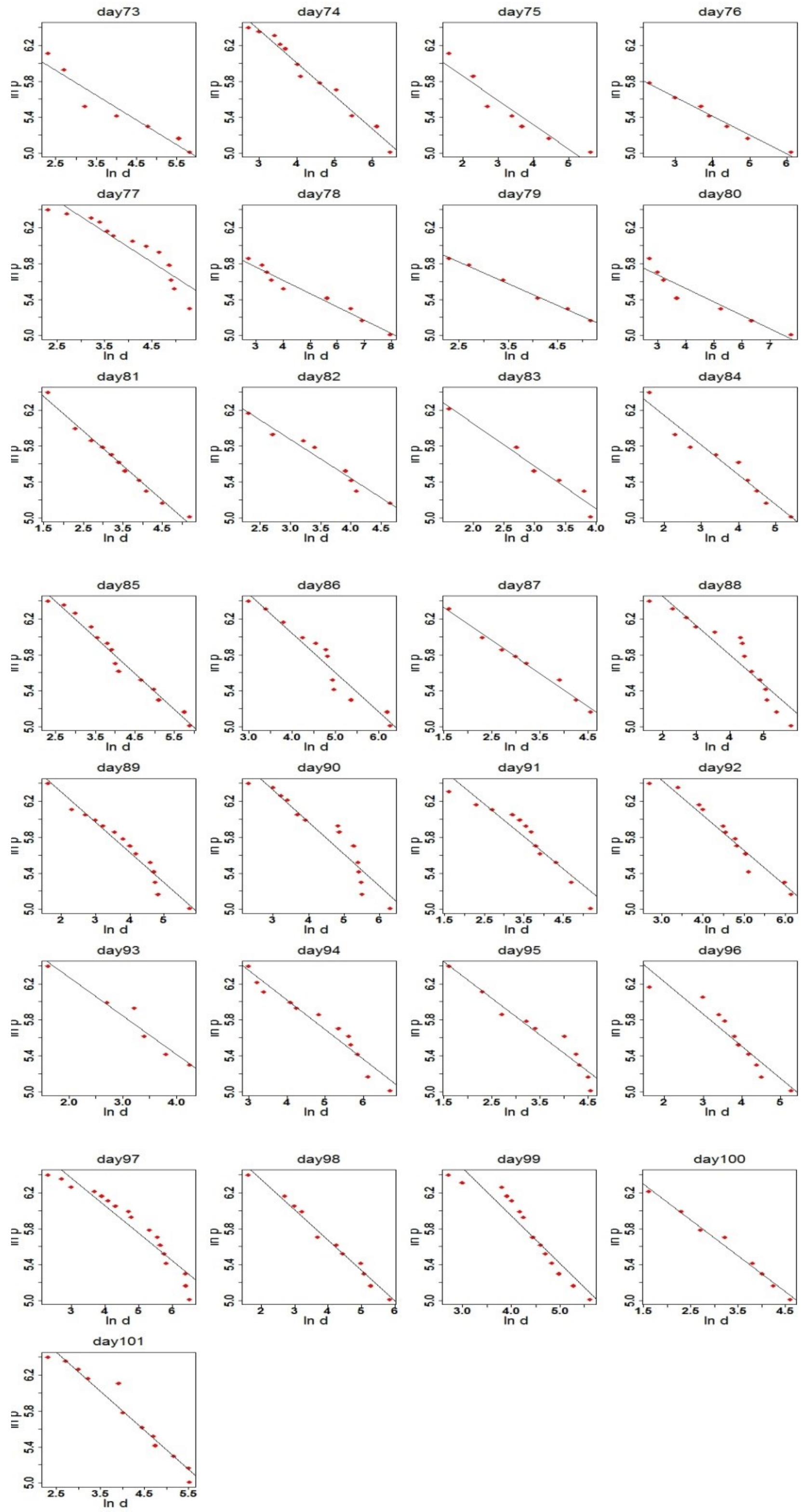

Figure A10.4 Continued

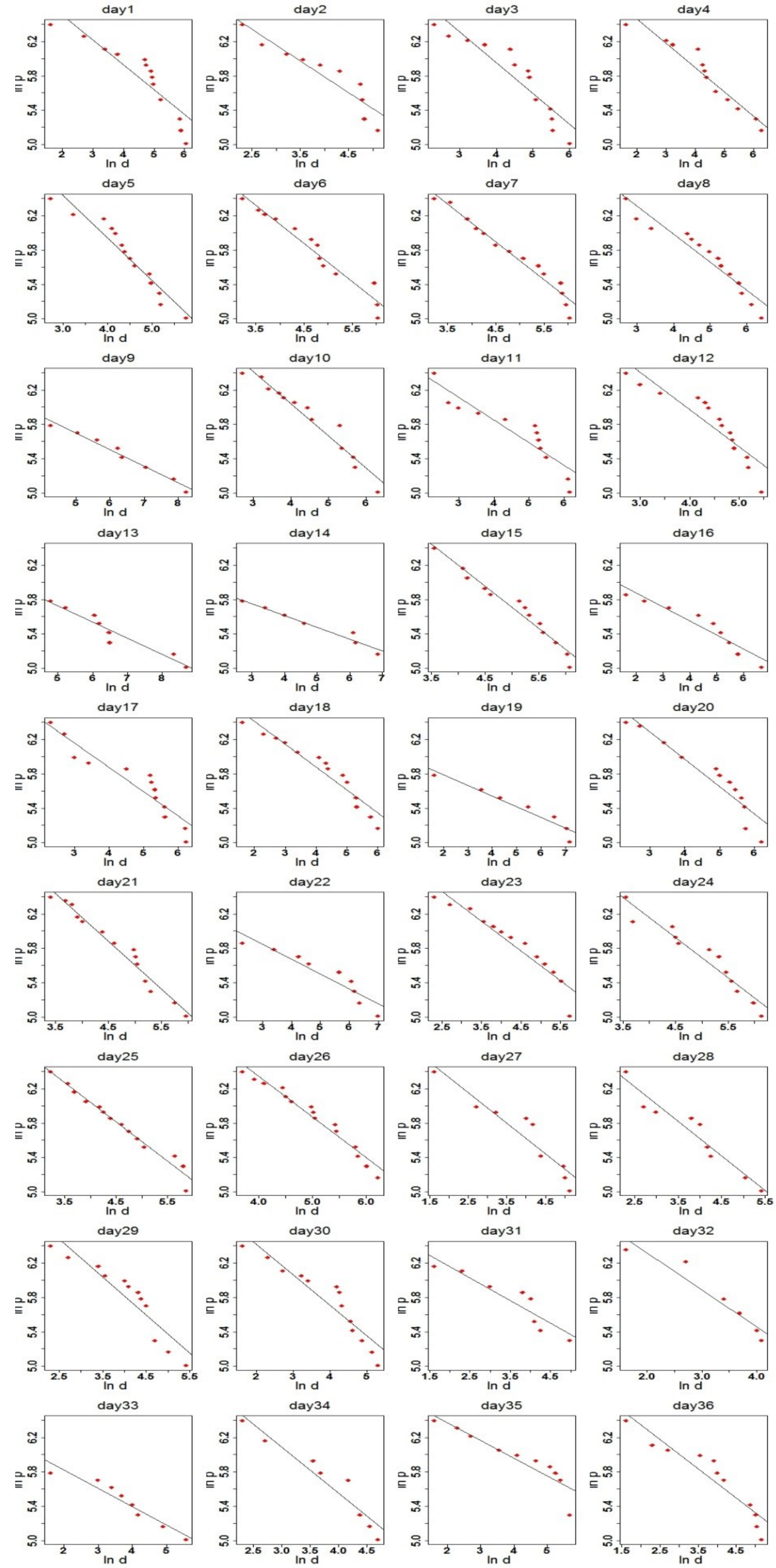

Figure A10.5 Fitted line of $ln(p) = \hat{u} + \hat{v}ln(d)$ for the relationship between power output p and duration d for all sessions for rider 6

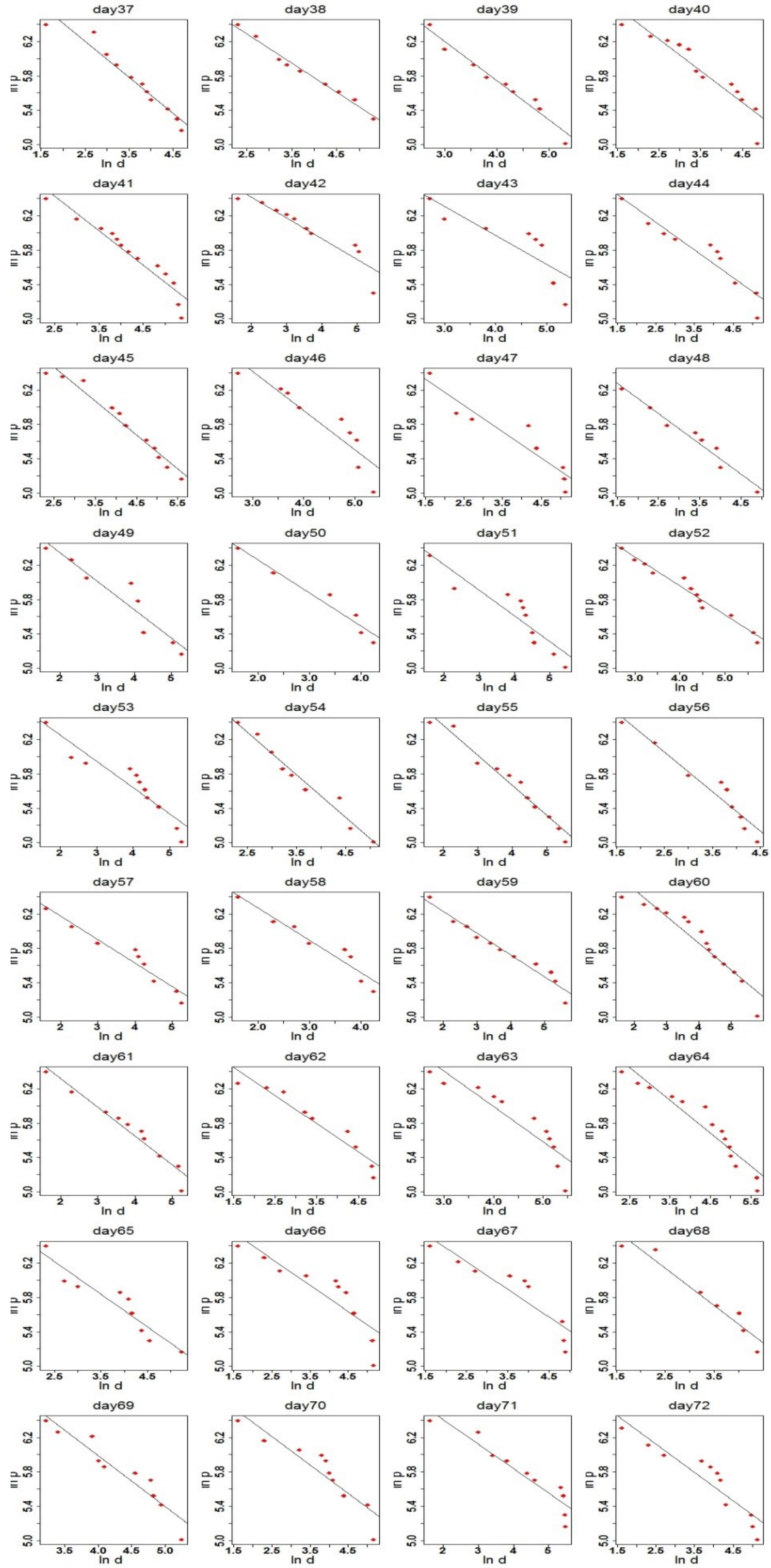

Figure A10.5 Continued

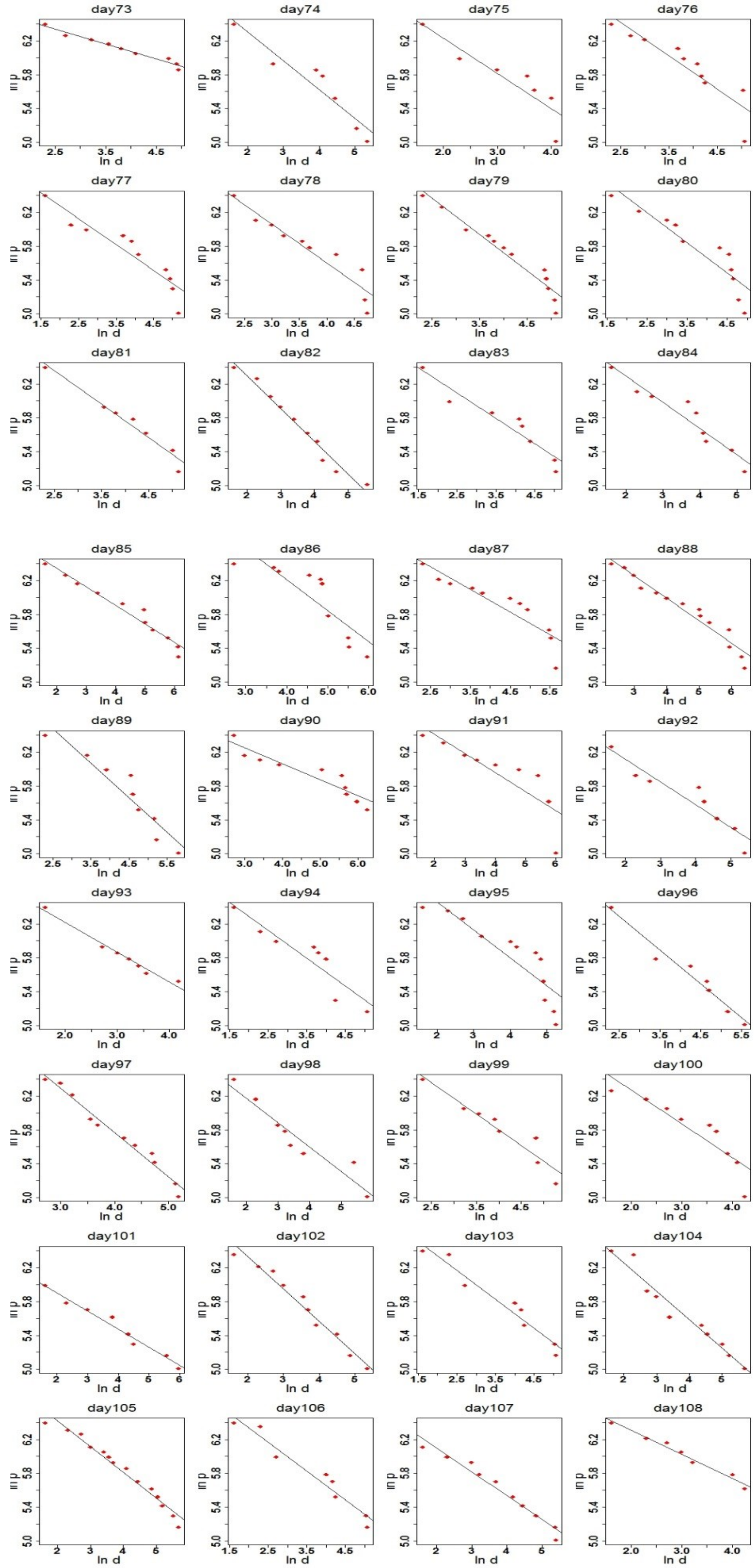

Figure A10.5 Continued

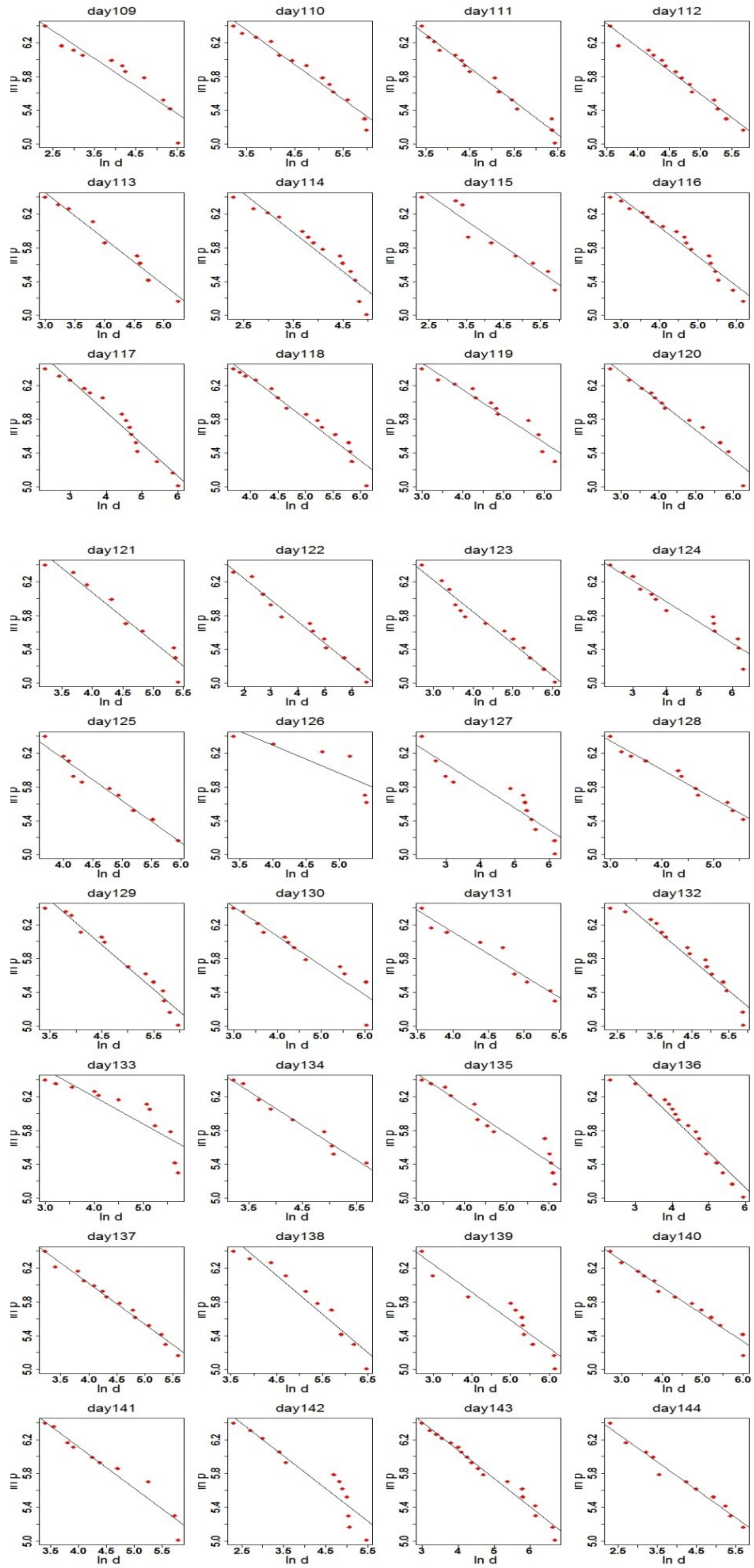

Figure A10.5 Continued

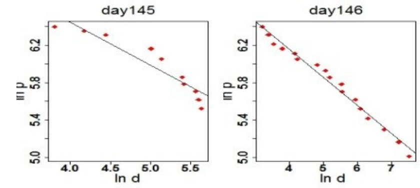

Figure A10.5 Continued

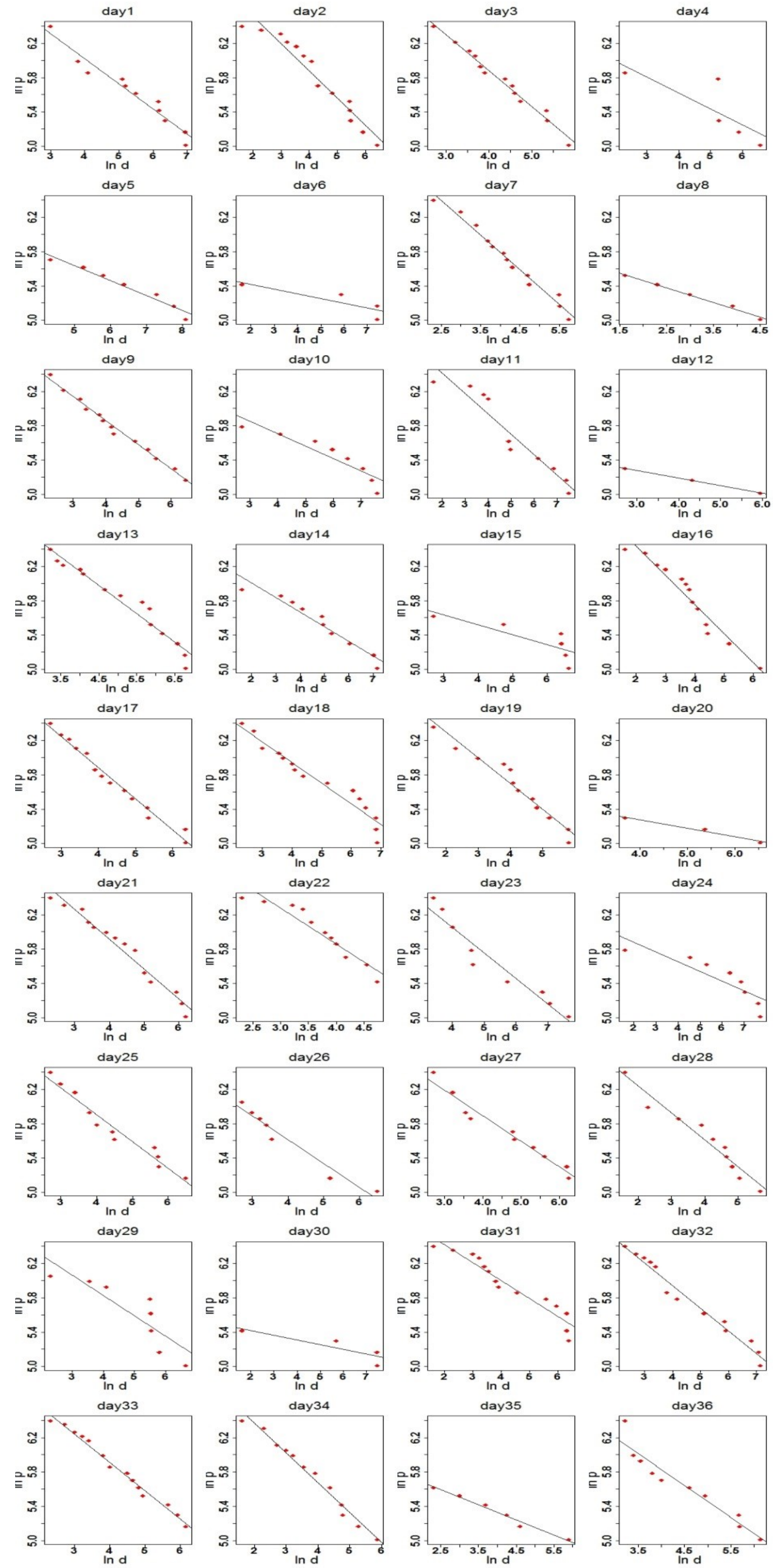

Figure A10.6 Fitted line of $ln(p) = \hat{u} + \hat{v}ln(d)$ for the relationship between power output p and duration d for all sessions for rider 7

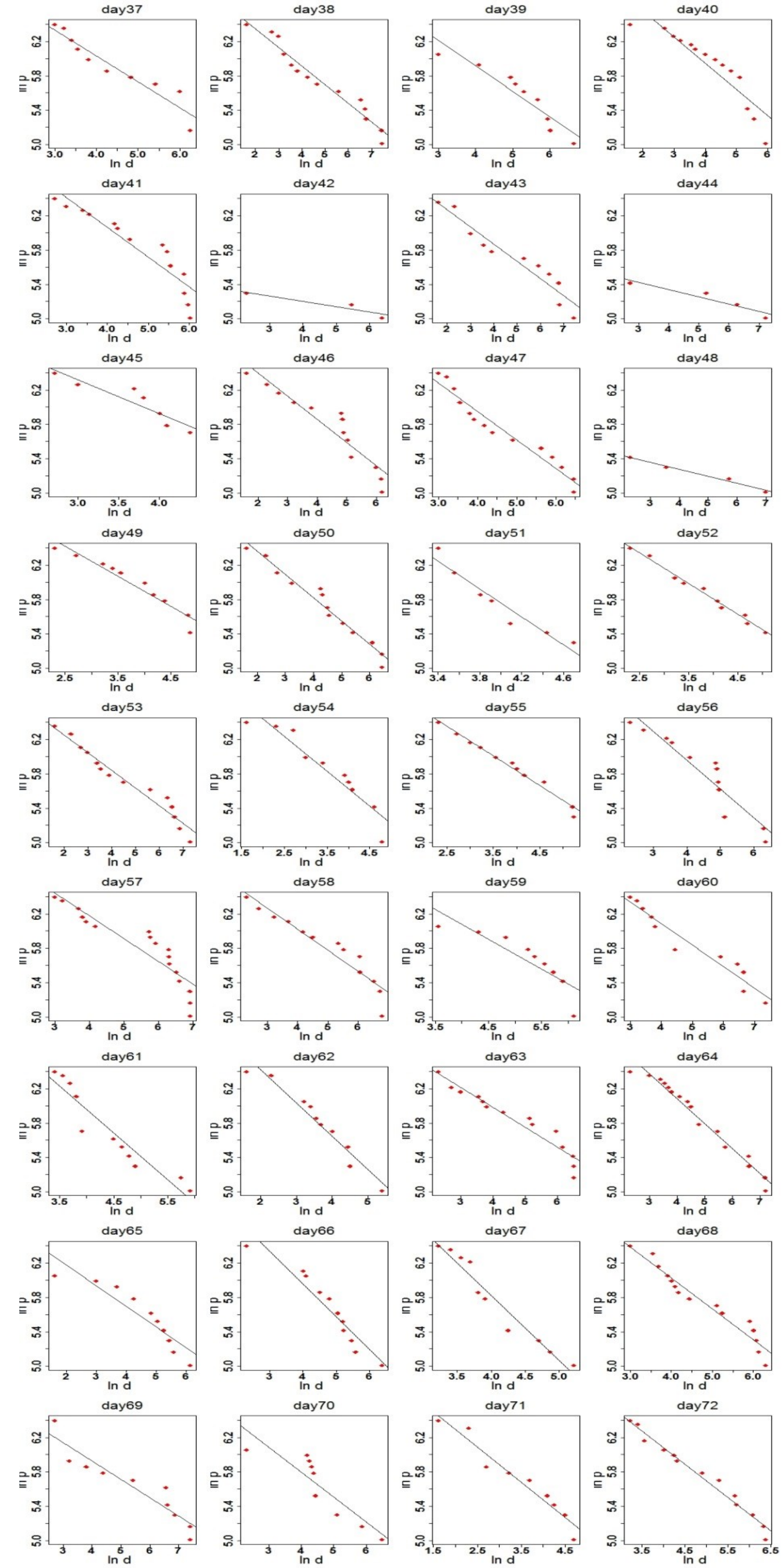

Figure A10.6 Continued

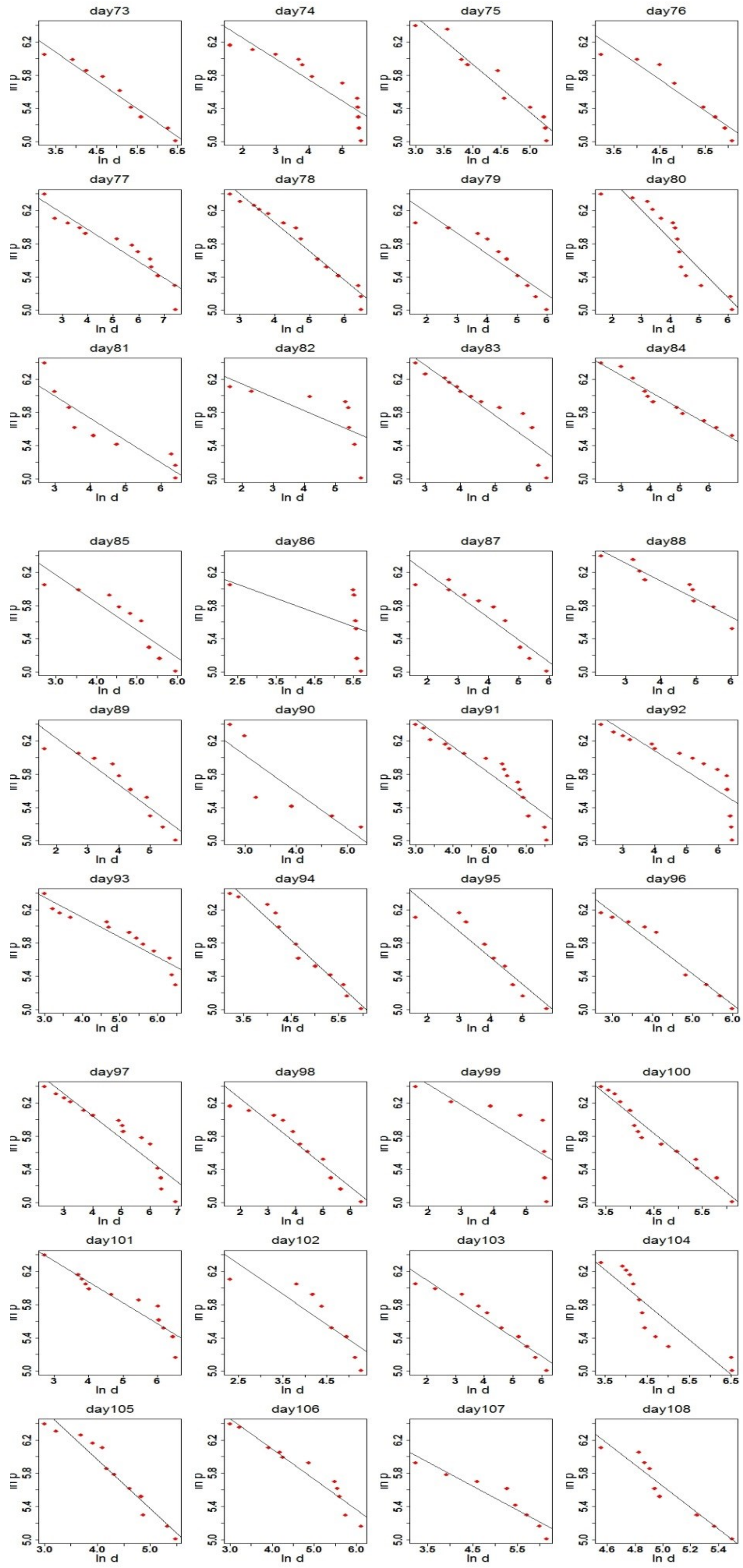

Figure A10.6 Continued

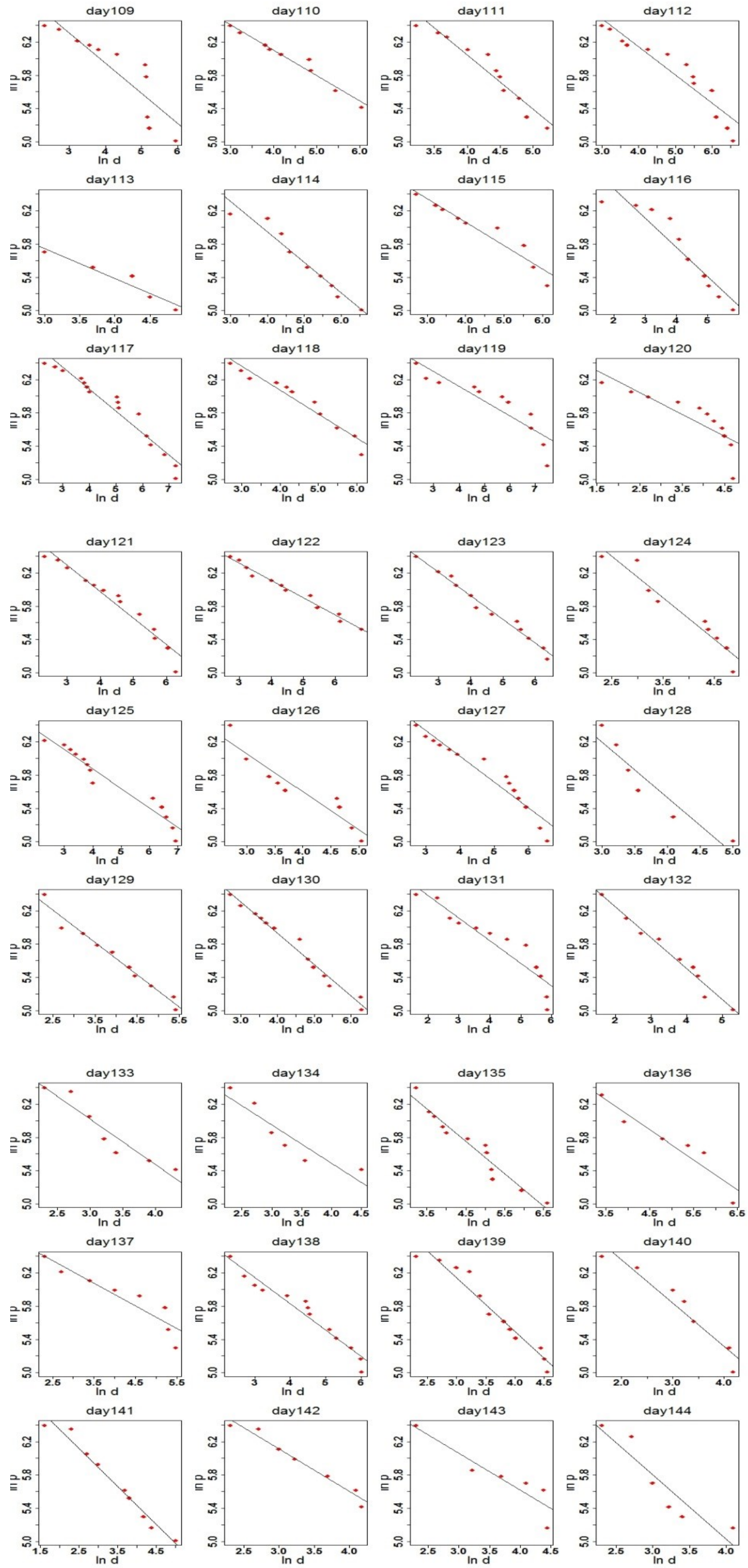

Figure A10.6 Continued

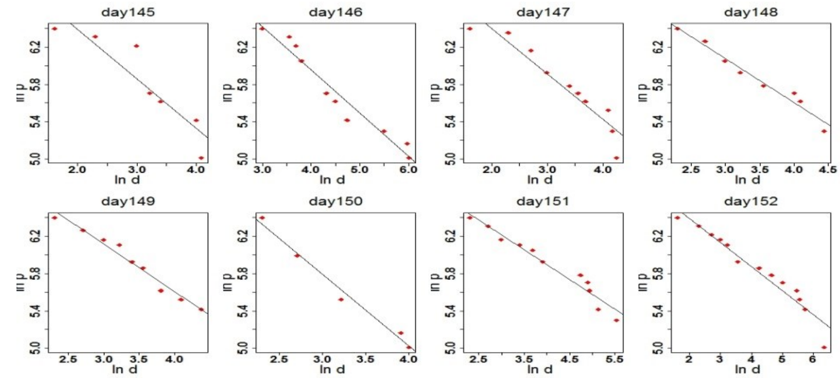

Figure A10.6 Continued

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