A general inspection and opportunistic replacement policy for one-component systems of variable quality

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Abstract: We model the influence of opportunities in a hybrid inspection and replacement policy. The base policy has two phases: an initial inspection phase in which the system is replaced if found defective; and a later wear-out phase that terminates with replacement and during which there is no inspection. The efficacy of inspection is modelled using the delay time concept. Onto this base model, we introduce events that arise at random and offer opportunities for cost-efficient replacement, and we investigate the efficacy of additional opportunistic replacements within the policy. Furthermore, replacements are considered to be heterogeneous and of variable quality. This is a natural policy for heterogeneous systems. Our analysis suggests that a policy extension that allows opportunities to be utilised offers benefit, in terms of cost-efficiency. This benefit is significant compared to those offered by age-based inspection or preventive replacement. In addition, opportunistic replacement may simplify maintenance planning.

Keywords: Maintenance modelling, reliability, delay time, mixtures.

1 BACKGROUND

Preventive maintenance is widely accepted as an effective way of reducing the total cost of ownership of industrial assets (Xia et. al 2015). Maintenance management of industrial systems consists mostly of a variety of maintenance strategies, such as preventive maintenance, corrective maintenance, inspections and so on (Lee and Cha, 2016; De Almeida et al., 2015). The main objectives of maintenance management are related to increasing the reliability and availability of systems and to reducing the cost of maintenance (Berrade et al., 2013; Zheng, et al., 2016).

In a production system, some stops can create opportunities to do preventive maintenance at a lower cost or with less disruption than scheduled preventive maintenance (e.g. maintenance of a bottling sub-system on a soft-drinks production line when cold water supply is lost due to pump failure, Wang et al,, 2000). Based on this idea, opportunistic maintenance policies have been developed (e.g. Dekker and Smeitink, 1991, Zheng, 1995; Tan and Kramer, 1996; Mohamed-Salah et al., 1999; Budai et al., 2006; Laggoune et al., 2010; Xia et al, 2016, Zhang and Zeng, 2017; Xia et al, 2017a). Opportunistic maintenance has been applied to several technical systems such as: wind turbines (e.g. Ding and Tian, 2011; Shafiee et al., 2015; Yildirim et al., 2017); gas turbine and compressor systems (e.g. Hu and Zhang, 2014); feed-water pump systems in nuclear power plants (e.g. Nilsson et. al., 2009); cogeneration systems (Cavalcante and Lopes, 2015); port transportation systems (Xia et al., 2017b); and railway infrastructure (e.g. Garambaki et. al., 2016). Often, opportunities arise from economic and structural interdependencies among components or parts that

constitute these technical systems (Dekker and Smeitink, 1991). However, opportunistic maintenance can be considered as distinct from grouped maintenance policies (Wildeman et al., 1997) that exploit similar interdependencies but which consider maintenance for groups of components or parts or sub-systems (Peng and Zhu, 2017). Grouped policies aim to optimise maintenance for each in the set of components (Vu et al., 2015), whereas opportunistic maintenance optimises maintenance for one or a few components using stoppages that arise due to others.

Also interesting is that opportunities might be considered in the context of inspection maintenance, modelled through the delay time concept of Christer (1999). This connection is not well developed in the literature and there are few articles that address it (Wang and Christer, 2003; Berrade et al., 2017). We build on this connection in this paper, developing a model that is more general than those researched to date.

For this new model, we develop expressions for the long run cost per unit time (Ross, 1996) or cost-rate. The models consider a single-component system that is periodically inspected up to age $K\Delta$ (and replaced if it is defective), replaced at opportunities after age *S*, and replaced preventively at age *T*. Furthermore, components arise from a heterogeneous population (Scarf et al., 2009, Scarf and Cavalcante, 2010) in a way that represents variability in the quality of components or maintenance workmanship (e.g. between different, competing suppliers). The new policy is natural in these circumstances. Furthermore, because inspection has the function to reveal defects and as we are dealing with heterogeneous population, the larger is the proportion of weak components, the more important is the role of the inspection. The hybrid policy is a natural one in these circumstances as it has similarities to "burn-in" policies (Zhang et al., 2014). For the new policy we develop, we describe its behaviour over a range of model parameter settings that typically arise in practical applications.

The layout of this paper is as follows. First, we explicitly describe the system, the failure model, and the maintenance policy, and their assumptions. Cost-rates for the two policies are then developed and their respective graphical representations are illustrated. We analyse the cost-effectiveness of proposed policies by comparing the cost-rate resulting for policies that are special cases of the proposed policies. A numerical example illustrates the performance of the different policy variations for a set of cost and reliability parameters. We finish with concluding remarks.

2 THE MAINTENANCE POLICIES

2.1 *Description of the technical system*

In maintenance modelling, the first thing that we should observe is the potential for practical contribution of a proposed model. Thus, the process of the construction of a model should begin with the observation of engineering practice, including an analysis of the feasibility of application (Scarf, 1997). One may observe specific situations for which appropriate maintenance policies are limited. This is the case for maintenance of a system composed of components from a heterogeneous population, which as already stated arises in many different contexts and where there exist decision problems regarding, for example, supplier selection (Berrade et al., 2012), quality of maintenance (Scarf and Cavalcante, 2012), reliability (Castet and Saleh, 2010), and analysis of failure warranty data (Attardi et al., 2005; Lee et al., 2016).

With this in mind, we consider a single component system that comprises a component and a socket which together perform an operational function (Ascher and Feingold, 1984). The component can be in one of three states, good, defective or failed, and the time in the good state, *X*

(time to defect or fault arrival), arises from a mixture distribution $F_X(t) = pF_1(t) + (1-p)F_2(t)$. Here, p is a mixing parameter, so that components arise from a mixed population of "weak" and "strong" sub-populations. F_1 and F_2 follow any increasing failure rate (IFR) distributions, for example, Weibull distributions with characteristic lives η_1, η_2 and shape parameters $\beta_1, \beta_2 > 1$. We denote the corresponding density and reliability function by f_X and $\bar{F}_X$. This notion of a mixed population of weak and strong components is a natural consideration in the context of the hybrid policy that we develop in this paper and define in section 2.2.

The system is a critical system so that failures are immediately revealed. Inspection on the other hand determines whether the system is good or defective. Replacement of the system corresponds to replacement of the component and renewal of the system. Events occur that provide opportunities for preventive replacement. Practically, these may arise in broadly two ways: those that are external to the system, such as temporary falls in demand; and those that arise in a multi-component system of which the system of interest is one part. In the latter, we assume the single component system is stochastically independent of the rest of the system, so that the failure process of the single component system is independent of that of the rest of the system. The rest of the system is conceptually a complex system for which stoppages (opportunities) arise according to a Poisson process with rate μ. External events are conceptually the same.

When the system is in the defective state, it continues to perform its operational function (e.g. a noisy but functioning bearing). The time in the defective state, H (the delay time) has density $f_H(h)$ and distribution function $F_H(h)$, and X and H are statistically independent. Opportunities are independent of X and H. At replacement, the system age is set to zero. Thus replacement is renewal, and throughout the paper replacement and renewal are synonymous. We will also use the terms component and system interchangeably.

2.2 *The maintenance policies*

The principal policy is as follows. From new, the system is inspected every Δ time units until $K\Delta$ or a defect is found at inspection or a failure occurs, whichever occurs soonest. Inspections are perfect in that the true state of the system is revealed at inspection. Further if the system survives beyond age $K\Delta$, then inspection ceases, and the system is replaced on failure or at age T or at the first opportunity that arises after age S ($S < T$), whichever occurs soonest. Replacements are instantaneous. The policy has four decision variables: Δ, K, S, and T (Figure 1). The cost parameters are defined in Table 1, which shows the principal notation.

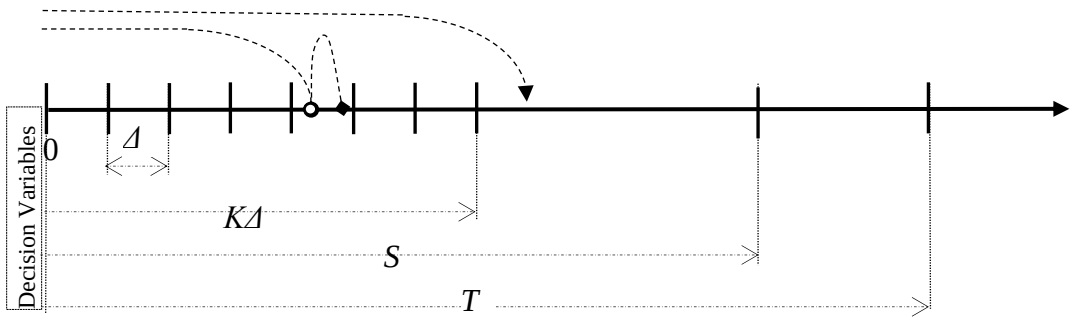


Figure 1. Graphical representation of the policy. A defect arises at ○ at age x leading to failure ● h time units later; ▼ represents the arrival of an opportunity at time z.

Table 1. Notation

Δ	Inspection interval

K	Number of inspections
S	Age threshold for opportunistic replacement
T	Age limit for preventive replacement
M	In policy 2, M is such that $S = M\Delta$
$F_X(.)$	Time to defect arrival distribution
$F_H(.)$	Delay time distribution
β_i	Weibull shape parameter for sub-population *i*
η_i	Weibull characteristic life for sub-population *i*
p	Mixing parameter
λ	Reciprocal of the mean of the delay time distribution
μ	Rate of arrival of opportunities
C_{I}	Cost of an inspection
C_{R}	Cost of a replacement of a defective component and cost of a preventive replacement of a component at age *T*
C_{F}	Cost of a replacement of a failed component
C_{O}	Cost of a preventive replacement at an opportunity, $C_{\mathrm{O}} < C_{\mathrm{R}} < C_{\mathrm{F}}$
$E(U_k)$	Expected cost of a renewal cycle for policy k, $k = 1,2$.
$E(V_k)$	Expected length of a renewal cycle for policy k, $k = 1,2$.
$C_{k,\infty}$	Cost-rate for policy k, $k = 1,2$

The innovation of this model is the consideration of the age threshold for opportunistic replacement *S*, whereby replacement is carried out at opportunities that arise during the wear-out phase $(K\Delta, T]$. In this way, the policy takes greater care in the initial life of an equipment and then in mid-life utilises opportunities for more cost-effective replacement. We call this policy 1 and study this in detail in the paper.

We also study a special case of policy 1 for which $T = K\Delta$, so that inspection is carried out through the entire life of the system, and $S = M\Delta$ $(0 < M \le K)$, so that opportunities that arise after the *M*-th inspection are utilised for replacement. This policy has three decision variables, Δ, *K*, and *M*. We call this policy 2.

Many other special cases of the principal policy arise. If $S = T$, then we have the policy proposed by Scarf et al.(2009), and opportunities are not utilised. If $K = 0$, then there is no inspection phase, and policy corresponds to opportunistic age based replacement (Scarf and Deara, 1998). If $K = 0$ and $S = T$, then the policy is age based replacement with age replacement limit *T* (Barlow and Proschan, 1966). With, $K = 0$ and $T = \infty$ we have the opportunistic replacement policy (Dagpunar, 1996). If $(K = \infty, S = T = \infty)$ then we have a pure inspection policy; this is the single component delay time model (Christer, 1987).

Thus, articulating these special cases demonstrates that the policy has the flexibility to model inspection, and age based and opportunistic replacement. But more than this, in any practical situation one can let the cost parameters, the failure model parameters and the opportunity-rate determine which maintenance policy is most cost-efficient. Furthermore, for example, one can

determine the relative cost-efficiency of inspection and non-inspection, or the relative cost-efficiency of a policy with an ultimate age limit for replacement (K,Δ,S,T) and without $(K,\Delta,S,T=\infty)$.

The motivation for policy 2 is related to application in practice. The management of policy 2 is relatively simple, because it demands only the monitoring of the number of inspections since new, so that if an opportunity arises after the *M*-th inspection but before the final inspection at $K\Delta$ it can be utilised. By definition, policy 1 must have a lower cost-rate that policy 2 (at their respective optima). However, the simpler management of policy 2 (or any other special case policy for that matter) may compensate for the increased cost.

3 CALCULATION OF THE COST-RATE

3.1 *Policy 1*

To develop the cost-rate, we calculate the probabilities of all renewal scenarios. The policy has four decision variables: Δ, the inspection interval; *K*, the number of inspections in the inspection phase; *S*, the age threshold for opportunistic replacement; *T*, the age limit for replacement. The policy is fully defined in section 2.2. We can characterize the events related to four different kinds of renewal scenarios: scenarios related with failure (Figure 2); scenarios related with preventive replacement (Figure 3), and finally, scenarios related with opportunities (Figure 4).

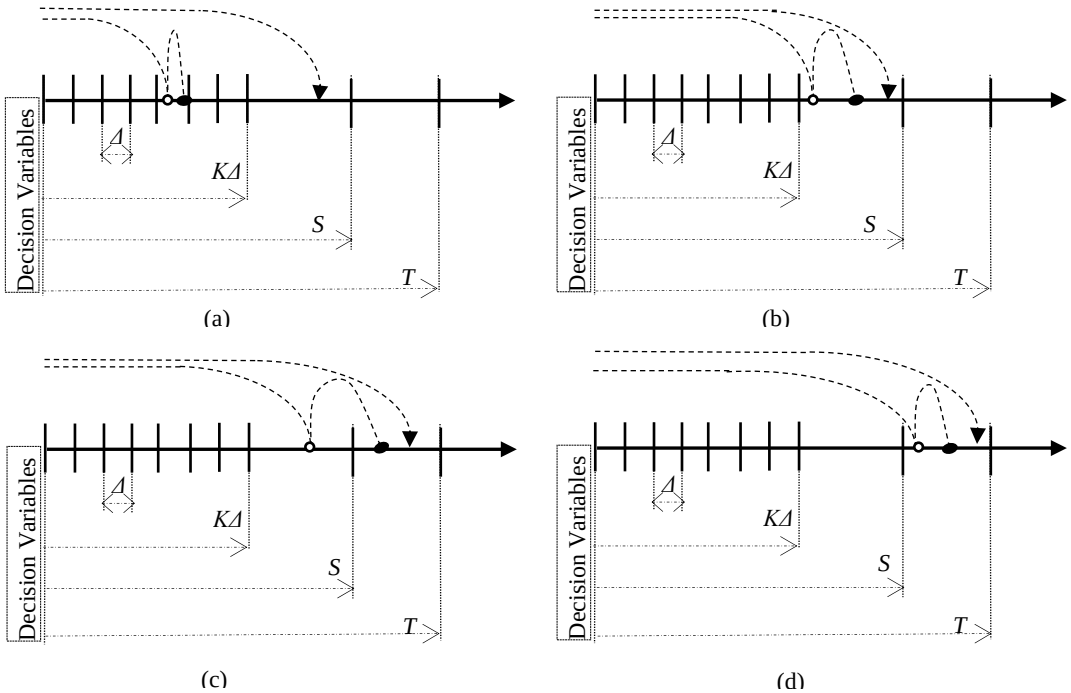


Figure 2. Graphical representation of renewal on failure: (a) during the inspection phase, (b) in the wear out phase prior to the opportunity threshold *S*, (c) after the opportunity but before an opportunity arises and where the defect arises before *S*, and (d) the same but defect arising after *S*. ○ denotes a defect arrival, ● denotes a failure. and ▼ the arrival of an opportunity.

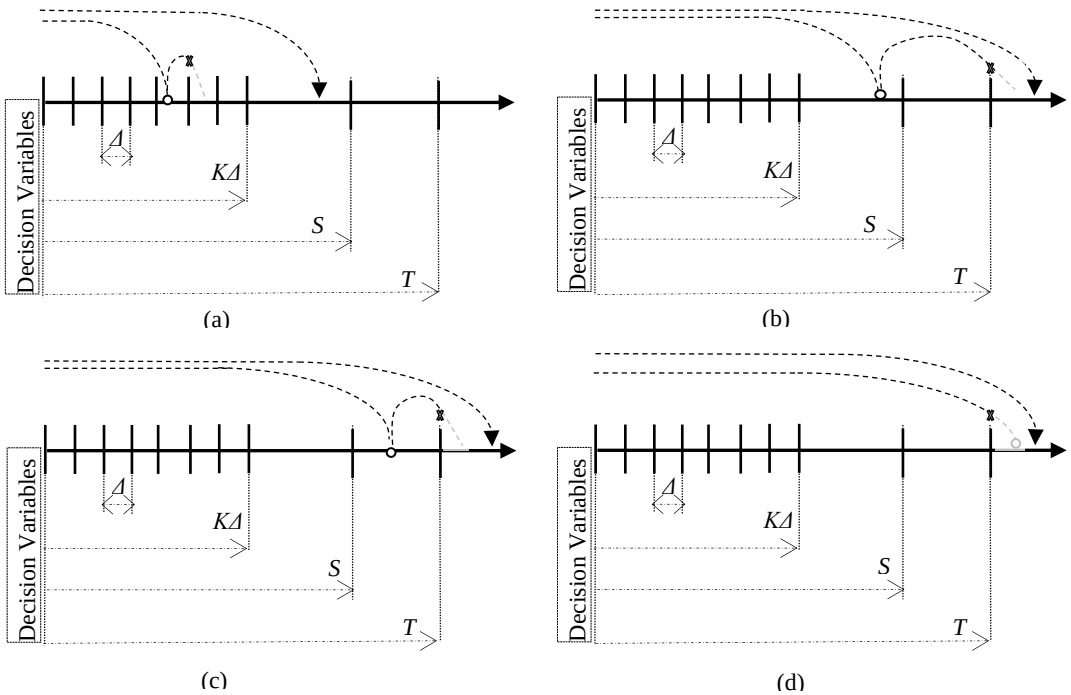


Figure 3. Graphical representation of renewal at preventive replacement: (a) at inspection, (b) at *T* with defect arising before *S*, (c) at *T* with defect arising after *S* and before *T*, and (d) at *T* with no defect arising before *T*.

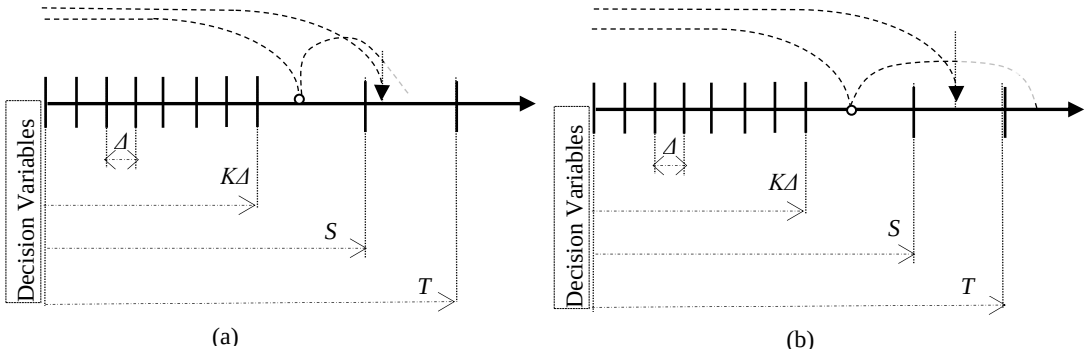


Figure 4. Graphical representation of renewal at an opportunity: (a) where the notional failure occurs after *S* and before *T*, and (b) where the notional failure occurs after *T*.

A failure can occur in four different ways, wherein there is renewal on failure: during the inspection phase $[0, K\Delta]$ (Figure 2a); after the inspection phase but before the limit age for opportunities, *S* (Figure 2b); after the age limit for opportunities *S* and with a defect that precedes *S* (Figure 2c) and with a defect that follows *S* (Figure 2d).

The preventive replacement scenario also occurs in four different ways: at an inspection (Figure 3a); at the age limit for replacement *T* when a defect occurs before *S* (Figure 3b); at *T* when a defect occurs after *S* (Figure 4c); and at *T* when no defect arises (Figure 3d).

Renewal at an opportunity (that brings advantages associated with economy of scale of cost or availability of resources or zero additional downtime) arises in many ways, for example: wherein the opportunity arrives at age *z* and a defect arises in $(K\Delta, S)$ preceding a notional failure that

would have occurred but for the opportunity (Figure 4a); and similarly but where the notional failure would not have occurred because preventive replacement at T precedes it and the opportunity is timely because the system is defective (Figure 4b).

Moving now to the calculations themselves, firstly, renewal occurs when a defect is found at any inspection during the inspection phase $[0, K\Delta]$. The probability of a renewal due to defect found at the *i*-th inspection is, for $i = 1, \ldots, K$,

$$P_{\mathrm{D}_i} = \int_{(i-1)\Delta}^{i\Delta} \bar{F}_H(i\Delta - x)\, f_X(x)\, \mathrm{d}x .$$

This equation does not depend on S or T because $T > S > K\Delta$. This case corresponds to a defect arising in the *i*–th inspection interval and surviving to (not causing a failure by) the end of the interval, whereupon the defect is found (perfect inspection) and the component replaced (renewal).

Renewal occurs also if the system fails during the inspection phase. The probability of failure in the *i*-th inspection interval is, for $i = 1, \ldots, K$,

$$P_{\mathrm{F}_i} = \int_{(i-1)\Delta}^{i\Delta} F_H(i\Delta - x)\, f_X(x)\, \mathrm{d}x .$$

Renewal on failure may also occur in the wear out phase $(K\Delta, T)$, with probability given by

$$\begin{aligned} P_{\mathrm{F}_{K+1}} = & \int_{k\Delta}^{S} F_H(S-x)\, f_X(x)\, \mathrm{d}x \\ & + \int_{k\Delta}^{S} \int_{S-x}^{T-x} \mathrm{e}^{-\mu(x+h-S)}\, f_H(h)\; f_X(x)\, \mathrm{d}h\mathrm{d}x + \int_{S}^{T} \int_{0}^{T-x} \mathrm{e}^{-\mu(x+h-S)}\, f_H(h)\; f_X(x)\, \mathrm{d}h\mathrm{d}x. \end{aligned}$$

The first term corresponds to a defect arising after $K\Delta$ that in turn leads to failure before S. The second term corresponds to a defect arising after $K\Delta$ that in turn leads to failure beyond S (but before T) and no opportunity arising between S and the age at failure $x + h$; this is the probability $\mathrm{e}^{-\mu(x+h-S)}$. The final term corresponds to a defect arising after S that in turn leads to failure (before T) and no opportunity arising between S and the age at failure $x + h$.

Replacement at an opportunity only occurs if an opportunity arrives before the time of failure or before the age of replacement, whichever is soonest, so that the probability of replacement at an opportunity is

$$\begin{aligned} P_{\mathrm{O}} = & \int_{k\Delta}^{S} \left\{ \int_{S-x}^{T-x} (1 - \mathrm{e}^{-\mu(x+h-S)})\, f_H(h)\, \mathrm{d}h + (1 - \mathrm{e}^{-\mu(T-S)})\, \bar{F}_H(T-x) \right\} f_X(x)\, \mathrm{d}x \\ & + \int_{S}^{T} \left\{ \int_{0}^{T-x} (1 - \mathrm{e}^{-\mu(x+h-S)})\, f_H(h)\; \mathrm{d}h + (1 - \mathrm{e}^{-\mu(T-S)})\, \bar{F}_H(T-x) \right\} f_X(x)\mathrm{d}x \\ & + (1 - \mathrm{e}^{-\mu(T-S)})\, \bar{F}_X(T). \end{aligned}$$

Preventive replacement (renewal) at T occurs if and only if there is no defect before $K\Delta$ and a defect, if it arises after $K\Delta$, survives to T, and no opportunities arise in $[S, T]$. This occurs with probability given by

$$P_{\mathrm{R}} = \mathrm{e}^{-\mu(T-S)} \left\{ \int_{k\Delta}^{T} \bar{F}_H(T-x)\, f_X(x)\, \mathrm{d}x + \bar{F}_X(T) \right\}.$$

With each renewal event there is a cost, and the expected cost of a renewal cycle is the sum of the products of the costs and their respective probabilities, so that

$$E\{U_1(K,\Delta,S,T)\} = \sum_{i=1}^{K}\left\{(C_{\mathrm{R}} + iC_{\mathrm{I}})P_{\mathrm{D}_i} + (C_{\mathrm{F}} + (i-1)C_{\mathrm{I}})P_{\mathrm{F}_i}\right\} + (C_{\mathrm{F}} + KC_{\mathrm{I}})P_{\mathrm{F}_{K+1}} + (C_{\mathrm{O}} + KC_{\mathrm{I}})P_{\mathrm{O}} + (C_{\mathrm{R}} + KC_{\mathrm{I}})P_{\mathrm{R}}.$$

The expected cycle length can be derived in a similar manner. We obtain

$$\begin{aligned}E\{V_1(K,\Delta,S,T)\} = {} & \sum_{i=1}^{K}\left\{i\Delta \times P_{\mathrm{D}_i} + \int_{(i-1)\Delta}^{i\Delta}\int_{0}^{i\Delta-x}(x+h)f_H(h)\ f_X(x)\,\mathrm{d}h\mathrm{d}x\right\} \\ & + \int_{k\Delta}^{S}\int_{0}^{S-x}(x+h)f_H(h)\ f_X(x)\,\mathrm{d}h\mathrm{d}x \\ & + \int_{k\Delta}^{S}\int_{S-x}^{T-x}(x+h)\,\mathrm{e}^{-\mu(x+h-S)}f_H(h)\ f_X(x)\,\mathrm{d}h\mathrm{d}x + \int_{S}^{T}\int_{0}^{T-x}(x+h)\,\mathrm{e}^{-\mu(x+h-S)}f_H(h)\ f_X(x)\,\mathrm{d}h\mathrm{d}x \\ & + \int_{k\Delta}^{S}\left\{\int_{S-x}^{T-x}\left\{\int_{0}^{x+h-S}(z+S)\mu\,\mathrm{e}^{-\mu z}\mathrm{d}z\right\}f_H(h)\,\mathrm{d}h + \bar{F}_H(T-x)\left\{\int_{0}^{T-S}(z+S)\mu\,\mathrm{e}^{-\mu z}\mathrm{d}z\right\}\right\}f_X(x)\,\mathrm{d}x \\ & + \int_{S}^{T}\left\{\int_{0}^{T-x}\left\{\int_{0}^{x+h-S}(z+S)\mu\,\mathrm{e}^{-\mu z}\mathrm{d}z\right\}f_H(h)\ \mathrm{d}h + \bar{F}_H(T-x)\left\{\int_{0}^{T-S}(z+S)\mu\,\mathrm{e}^{-\mu z}\mathrm{d}z\right\}\right\}f_X(x)\mathrm{d}x \\ & + \bar{F}_X(T)\left\{\int_{0}^{T-S}(z+S)\mu\,\mathrm{e}^{-\mu z}\mathrm{d}z\right\} + T\times P_{\mathrm{R}}.\end{aligned}$$

We can make use of the renewal reward theory (see e.g. Ross, 1996) to specify the long run cost per unit of time (or cost-rate) $C_{1,\infty}(K,\Delta,S,T) = E(U_1)/E(V_1)$ which we use as the objective function to determine the optimum values of the decision variables.

3.2 *Policy 2*

This policy is a special case of policy 1 and has three decision variables: Δ, the inspection interval; and M and K, where $M\Delta$ is the age threshold for opportunistic replacement and $K\Delta$ is the age limit for replacement. The policy is fully defined in section 2.2. For policy 2, the calculation of the cost-rate is similar to policy 1 in principle. Without developing the preliminary calculations, we write down the expected cost of a cycle and the expected cycle length thus:

$$\begin{aligned}E\{U_2(M,K,\Delta)\} = {} & \sum_{i=1}^{M}\int_{(i-1)\Delta}^{i\Delta}\left\{(C_{\mathrm{R}} + iC_{\mathrm{I}})\bar{F}_H(i\Delta-x) + (C_{\mathrm{F}} + (i-1)C_{\mathrm{I}})F_H(i\Delta-x)\right\}\mathrm{d}F_X \\ & + \sum_{i=M+1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left\{(C_{\mathrm{R}} + iC_{\mathrm{I}})\bar{F}_H(i\Delta-x)\,\mathrm{e}^{-\mu(i-M)\Delta} + (C_{\mathrm{F}} + (i-1)C_{\mathrm{I}})\int_{0}^{i\Delta-x}\mathrm{e}^{-\mu(x+h-M\Delta)}\,\mathrm{d}F_H\right\}\mathrm{d}F_X \\ & + \sum_{i=M+1}^{K}(C_{\mathrm{O}} + (i-1)C_{\mathrm{I}})\int_{(i-1)\Delta}^{i\Delta}\left\{\mathrm{e}^{-\mu(i-1-M)\Delta}\int_{0}^{i\Delta-x}(1-\mathrm{e}^{-\mu(x+h-i\Delta)})\mathrm{d}F_H\right\}\mathrm{d}F_X \\ & + \sum_{i=M+1}^{K}(C_{\mathrm{O}} + (i-1)C_{\mathrm{I}})\int_{(i-1)\Delta}^{i\Delta}\left\{\bar{F}_H(i\Delta-x)\,\mathrm{e}^{-\mu(i-1-M)\Delta}(1-\mathrm{e}^{-\mu\Delta})\right\}\mathrm{d}F_X \\ & + \sum_{i=M+1}^{K}(C_{\mathrm{O}} + (i-1)C_{\mathrm{I}})\bar{F}_X(K\Delta)\,\mathrm{e}^{-\mu(i-1-M)\Delta}(1-\mathrm{e}^{-\mu\Delta}) + (C_{\mathrm{R}} + KC_{\mathrm{I}})\bar{F}_X(K\Delta)\,\mathrm{e}^{-\mu(K-M)\Delta},\end{aligned}$$

(where here $\mathrm{d}F_X$ and $\mathrm{d}F_H$ are shorthand for $f_X(x)\,\mathrm{d}x$ and $f_H(h)\,\mathrm{d}h$ respectively), and

$$
\begin{aligned}
E\{V_2(M,K,\Delta)\} = {} & \sum_{i=1}^{M}\int_{(i-1)\Delta}^{i\Delta}\left\{i\Delta\bar{F}_H(i\Delta-x)+\int_0^{i\Delta-x}(x+h)f_H(h)\,\mathrm{d}h\right\}\mathrm{d}F_X \\
& + \sum_{i=M+1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left\{i\Delta\bar{F}_H(i\Delta-x)\,\mathrm{e}^{-\mu(i-M)\Delta}+\int_0^{i\Delta-x}(x+h)\,\mathrm{e}^{-\mu(x+h-M\Delta)}\,\mathrm{d}F_H\right\}\mathrm{d}F_X \\
& + \sum_{i=M+1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left\{\int_0^{i\Delta-x}\left\{\int_0^{x+h-M\Delta}(z+M\Delta)\mu\,\mathrm{e}^{-\mu z}\,\mathrm{d}z\right\}\mathrm{d}F_H\right\}\mathrm{d}F_X \\
& + \sum_{i=M+1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left\{\bar{F}_H(i\Delta-x)\int_0^{i\Delta-M\Delta}(z+M\Delta)\mu\,\mathrm{e}^{-\mu z}\,\mathrm{d}z\right\}\mathrm{d}F_X \\
& + \bar{F}_X(K\Delta)\left\{K\Delta\,\mathrm{e}^{-\mu(K-M)\Delta}+\int_0^{(K-M)\Delta}(z+M\Delta)\mu\,\mathrm{e}^{-\mu z}\,\mathrm{d}z\right\},
\end{aligned}
$$

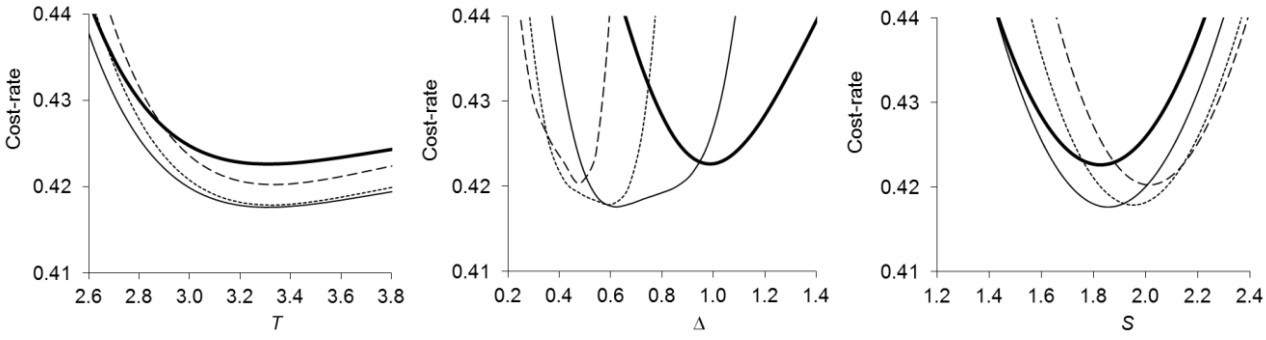
and the long run cost per unit of time (cost-rate) is $C_{2,\infty}(K,M,\Delta) = E(U_2)/E(V_2)$.

4 NUMERICAL EXAMPLE

Our purpose now is to investigate the effectiveness of the proposed maintenance policies. For the sake of this investigation we consider the cost of preventive replacement as our unit cost ($C_{\mathrm{R}} = 1$), so that all the other costs are relative to C_{R}. The parameters values used here are, for example, typical of commuter train components, such as train traction motor bearings (Scarf and Cavalcante, 2010) and power switches (Berrade et al., 2013). The results for policy 1 are shown in Table 2 and Figure 5.

Table 2. For policy 1, optimum values of decision variables and cost-rate for various values of the model parameters. Unit cost is the cost of preventive replacement, C_{R}. Time unit is one year.

Case	β_1	η_1	β_2	η_2	p	μ	λ	C_{I}	C_{O}	C_{F}	Δ	S	T	K	Cost-rate
1	2.5	0.8	5	3.6	0.1	2	1	0.03	0.5	5	0.61	1.86	3.28	2	0.418
2	1.5	0.8	5	3.6	0.1	2	1	0.03	0.5	5	0.49	2.00	3.32	4	0.421
3	5	0.8	5	3.6	0.1	2	1	0.03	0.5	5	0.97	1.83	3.31	1	0.405
4	2.5	0.4	5	3.6	0.1	2	1	0.03	0.5	5	0.34	1.85	3.9	2	0.414
5	2.5	1.6	5	3.6	0.1	2	1	0.03	0.5	5	1.38	1.68	3.27	1	0.392
6	2.5	0.8	2	3.6	0.1	2	1	0.03	0.5	5	0.32	2.56	15.22	8	0.565
7	2.5	0.8	5	3.6	0	2	1	0.03	0.5	5	--	1.65	--	0	0.316
8	2.5	0.8	5	3.6	0.2	2	1	0.03	0.5	5	0.52	2.15	3.39	4	0.498
9	2.5	0.8	5	3.6	0.1	0	1	0.03	0.5	5	0.47	3.07	3.07	6	0.533
10	2.5	0.8	5	3.6	0.1	1	1	0.03	0.5	5	0.62	1.62	3.01	2	0.461
11	2.5	0.8	5	3.6	0.1	4	1	0.03	0.5	5	0.62	2.19	3.79	3	0.390
12	2.5	0.8	5	3.6	0.1	2	0.5	0.03	0.5	5	1.03	2.04	3.88	1	0.354
13	2.5	0.8	5	3.6	0.1	2	2	0.03	0.5	5	0.43	1.72	3.02	3	0.485
14	2.5	0.8	5	3.6	0.1	2	∞	0.03	0.5	5	--	1.56	∞	0	0.693
15	2.5	0.8	5	3.6	0.1	2	1	0.015	0.5	5	0.46	1.94	3.28	4	0.395
16	2.5	0.8	5	3.6	0.1	2	1	0.05	0.5	5	0.99	1.84	3.32	1	0.432
17	2.5	0.8	5	3.6	0.1	2	1	0.03	0.25	5	0.56	1.59	3.54	2	0.316
18	2.5	0.8	5	3.6	0.1	2	1	0.03	1	5	0.48	3.12	3.12	6	0.533
19	2.5	0.8	5	3.6	0.1	2	1	0.03	0.5	2.5	1.01	2.20	4.85	1	0.323
20	2.5	0.8	5	3.6	0.1	2	1	0.03	0.5	10	0.31	1.85	2.81	6	0.526



(a) (b) (c)

Figure 5. For policy 1, cost-rate versus (a) age at preventive replacement *T*, (b) interval between inspections Δ, (c) threshold age for opportunity *S*, for K =1 (——), K =2 (———), K =3 (-----); K=4 (— —); other decision variables held in turn at their respective optimum values; parameter values as case 1: $\beta_1 = 2.5$, $\eta_1 = 0.8$, $\beta_2 = 5$, $\eta_2 = 3.6$, $p = 0.1$, $\mu = 2$, $\lambda = 1$, $C_{\mathrm{I}} = 0.03$, $C_{\mathrm{O}} = 0.5$, $C_{\mathrm{F}} = 5$, $C_{\mathrm{R}} = 1$.

Figure 5 shows that the cost-rate is most sensitive to the age limit for opportunities, *S*, (a 10% deviation from the optimum policy increases the cost-rate by 10% approximately) and least sensitive to *T* (a 10% deviation increases the cost-rate by 2.5%). This immediately indicates that utilising opportunities offers a significant cost advantage. Also, Figure 5b indicates that the inspection decision variables, K and Δ, interact in a way that preserves the length of the inspection period, $K\Delta$, as we might expect.

The effect of heterogeneity in the population through the mixing parameter, *p*, has a large influence on the results for policy 1. The use of the value $p = 0.1$ in the base case is justified in Scarf and Cavalcante (2012), and values in the range 0.15-0.2 have been suggested for pump installations (Gales, 2015). Overall, we see in Table 2 that as *p* increases, inspection intensifies, with more inspections more often. Component heterogeneity demands flexibility of the policy as it adapts to different levels of heterogeneity, from intense inspection to no inspection at all. Scarf et al. (2009) showed that as the distributions that model the different lifetime sub-populations separate the optimum policy can handle this separation by combining what is more effective for each sub-population: intense inspections at the beginning of system life, like a burn-in process; and non-action until the limit for preventive maintenance *T*. This can be observed in the cases 1, 7, and 8 of the Table 2. Introducing the age threshold for opportunities increases the adaptability of the policy further still. As the cost of an opportunistic replacement, C_{O}, decreases (cases 18 to 1 to 17), we can see a clear change in the best policy: the inspection frequency decreases and the number of inspections decreases while the age limit for preventive replacement, *T*, increases, so that the policy becomes less intrusive with less inspection and with a longer window of opportunity (S,T). Also, as opportunities become more frequent (case 11), the age limit for preventive replacement, *T*, increases. Thus, increasing opportunities tends to postpone preventive maintenance, depending on the opportunity parameters; when C_{O} is small and μ is large, $T = \infty$, so that preventive replacement is no longer necessary. It is interesting to note that this effect is also observed when the arrival of the defects for strong components is less predictable (small β_2, case 6). Thus, sometimes the best policy is only to await an opportunity. This may also provide additional benefit for maintainers because opportunities may arise from scheduled interventions on other parts of the

same plant. Nonetheless, this demonstrates further the greater flexibility of this opportunistic policy over the policy without opportunity. Finally, we can see that when $C_O = C_R$ (case 18) or $\mu = 0$ (case 9), the window of opportunity reduces to zero, as we would expect.

The mean delay time $1/\lambda$ also has an important influence on the results. If the power of the observation or diagnosis of a defect becomes restricted, reflected in a decreasing mean delay time (increasing λ, case 12 to 1 to 13 and 14), the age threshold to enjoy an opportunity *S*, decreases, wherein opportunities for replacement are utilised in earlier life. Thus, opportunistic maintenance can be used to compensate for less precision in knowledge about the state of the system. In the limit (case 14), when there is no information about defects (zero delay time), the best policy is opportunistic replacement. Further, it is interesting that the influence of λ on *K** is non-monotonic; initially *K** increases with λ, but then for very large λ inspection becomes ineffective.

Finally, in relation to Table 2, we make some brief points. Comparison of cases 1-5 shows that the base case is an interesting case. Sensitivity to cost parameters is somewhat predictable: greater failure cost (case 19 to 1 to 20) leads to less inspection; cheaper inspection cost (cases 16 to 1 to 15) leads to more inspections.

Table 3 compares the full policy (policy 1) with a number of restricted policies, including policy 2, which itself allows for opportunities to be utilised but in a manner that is easier to manage. These comparisons are presented for some of the more interesting cases. Broadly speaking, it is the cost-rate comparisons that are most interesting here. These demonstrate the comparative economic benefits of these competing policies. Cost-rate differences in the base case are quite large (Figure 6). Here, both the pure inspection policy and the pure age based replacement policy are cost-inefficient (approximately 50% more expensive), but that an opportunistic policy is closer to the most flexible policy. Of course, this is to an extent determined by the relative cost of replacement at opportunity C_O. Nonetheless, it underlines the usefulness of this policy extension. Another point is that policy 2 appears more sensitive than policy 1 to some parameter changes. An example can be seen on cases 19 and 20, where the variation in the cost of failure leads to a large change in the policy: from $K = 3$, and $M = 2$ in the base case to $K = 8$ and $M = 6$ when C_F doubles.

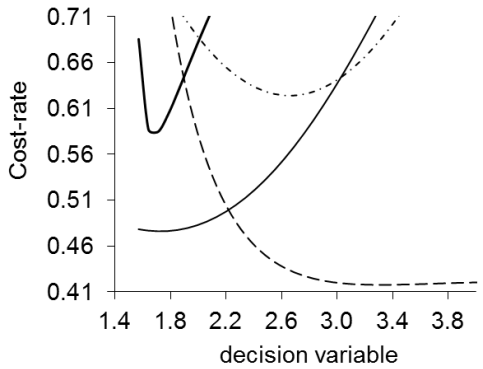


Figure 6. Cost-rate for various special cases of policy 1: versus Δ for pure inspection policy, *K*=∞, (——);versus *T* for age based replacement, *K*=0, *S*=*T*, (– • –); versus *S* for opportunistic replacement, *K*=0, *T*= ∞, (——);versus *T* for policy 1 with *K* =2 (— —). Other decision variables are held in turn at their respective optimum values; parameter values as case 1 (see Table 2).

Table 3. Comparison of policies 1 and 2 and special cases: policy (K,Δ,T) (Scarf et al., 2009); opportunistic replacement (Dagpunar, 1996); pure delay time inspection (Christer, 1999), age based replacement; and corrective (failure based) replacement. Comparisons are made for some of the numbered cases of Table 2.

	Policy 1						Policy 2						Policy (K, Δ,T)					Opportunistic Replacement K=0, T= ∞		Pure inspection K=∞, S=∞		Age based replacement K=0, T=S		Corrective replacement K=0,T= S=∞
case	Δ	S	T	K	KΔ	Cost-rate	Δ	M	MΔ	K	KΔ	Cost-rate	Δ	T	K	KΔ	Cost-rate	S	Cost-rate	Δ	Cost-rate	T	Cost-rate	Cost-rate
1	0.61	1.86	3.28	2	1.21	0.418	1.02	2	2.04	3	3.06	0.427	0.47	3.07	6	2.82	0.533	1.73	0.476	0.25	0.581	2.66	0.624	1.236
7	-	1.65	-	0	0	0.316	1.75	1	1.75	2	3.50	0.318	2.32	2.74	1	2.32	0.460	1.65	0.316	0.26	0.546	2.56	0.478	1.161
8	0.52	2.15	3.39	4	2.08	0.498	0.76	3	2.28	4	3.04	0.520	0.34	3.29	9	3.06	0.589	1.82	0.647	0.24	0.623	2.76	0.780	1.321
9	0.47	3.07	3.07	6	2.81	0.533	0.39	1	0.39	8	3.13	0.545	0.47	3.07	6	2.81	0.533	-	1.236	0.25	0.581	2.66	0.624	1.236
11	0.61	1.86	3.28	2	1.21	0.418	1.02	2	2.04	3	3.07	0.426	0.47	3.07	6	2.82	0.533	1.73	0.476	0.25	0.581	2.66	0.624	1.236
12	1.03	2.04	3.88	1	1.03	0.354	1.12	2	2.24	3	3.35	0.365	0.73	3.29	4	2.92	0.473	1.92	0.385	0.37	0.499	2.89	0.533	0.991
14	0.46	1.89	3.13	4	1.84	0.458	0.97	2	1.94	3	2.92	0.469	0.39	2.95	7	2.73	0.573	1.67	0.529	0.21	0.641	2.57	0.673	1.347
15	0.46	1.93	3.27	4	1.83	0.391	1.01	2	2.02	3	3.03	0.411	0.21	3.47	16	3.36	0.480	1.73	0.476	0.16	0.496	2.66	0.624	1.236
16	0.99	1.84	3.32	1	0.99	0.432	1.03	2	2.06	3	3.10	0.443	0.84	2.92	3	2.52	0.562	1.73	0.476	0.34	0.649	2.66	0.624	1.236
17	0.56	1.59	3.54	2	1.12	0.316	0.98	2	1.96	3	2.93	0.346	0.47	3.07	6	2.82	0.533	1.22	0.360	0.25	0.581	2.66	0.624	1.236
18	0.48	3.12	3.18	6	2.86	0.533	0.46	6	2.76	7	3.21	0.543	0.47	3.07	6	2.82	0.533	2.15	0.661	0.25	0.581	2.66	0.624	1.236
19	1.01	2.20	4.85	1	1.01	0.323	1.18	2	2.36	3	3.54	0.338	1.00	3.46	3	3.00	0.441	2.15	0.331	0.50	0.464	3.17	0.461	0.618
20	0.31	1.85	2.81	6	1.85	0.526	0.33	6	1.98	8	2.63	0.552	0.29	2.81	9	2.61	0.631	1.22	0.720	0.16	0.726	2.32	0.881	2.472

The benefit of policy 1 over other policies can be analysed for individual parameters. Figure 7 shows the percentage cost-reduction for different values of μ (Figure 7a) and p (Figure 7b) for each policy, with other parameters at values in the base case. Figure 7a shows that as the opportunity rate increases the cost-benefit of policy 1 increases, except in comparison to the pure opportunistic replacement policy, as we would expect. The advantage over policy 2 is only slight however, indicating that this simpler policy may be the most appropriate for practice. For Figure 7b the picture is complicated. Nonetheless, it appears that while the cost-benefit of inspection increases with increasing p, the utilisation of opportunities has decreasing cost-benefit. This is likely because for large p (poor quality replacement at its most extreme) inspection becomes the dominant maintenance action. Thus utilisation of opportunities then is relatively more important provided p is not too large.

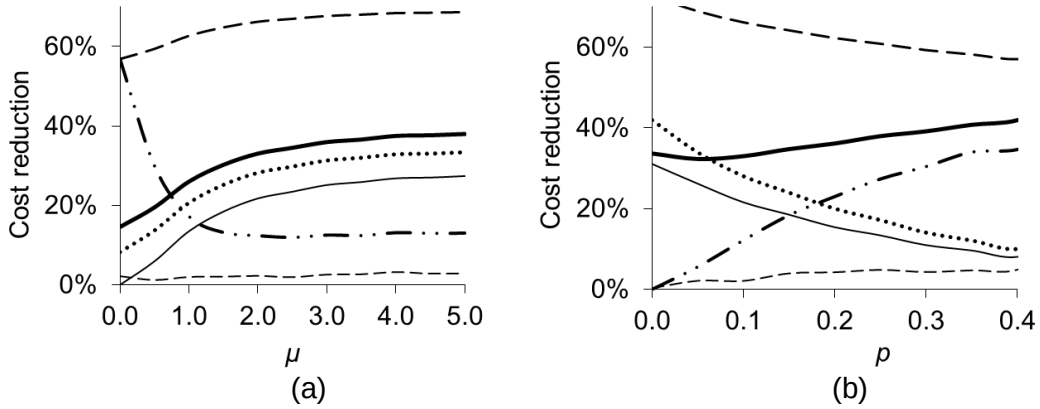


Figure 7. Cost reduction (%) for policy 1 compared to policies that are special cases: (a) as a function of μ, (b) as an function of p. Policy 2 ($S=M\Delta$, $T=K\Delta$) (– – –); opportunistic replacement (K=0, T= ∞) (—••); age based replacement (K=0, $S=T$) (———); corrective maintenance (K=0, T= ∞, S=∞) (– – –); pure inspection (K=∞) (••••); policy (K, Δ,T) ($S=T$) (——) .

5 CONCLUSION

We analyse an opportunistic replacement policy that is a generalisation of a hybrid inspection and replacement policy proposed by Scarf et al. (2009). In addition to preventive replacement at the end of the wear-out phase at age T, the policy allows preventive replacement to take place opportunistically, at a cost discount, any time after age S. Such opportunities may arise as the results of stoppages, planned or unplanned, to a plant of which the system under consideration is a part. In this way, the opportunistic policy may simplify maintenance planning since such opportunities may arise with less uncertainty than scheduled age-based replacements. This is because age-based replacements do not occur periodically, unlike block replacements. The hybrid policy is a natural one where replacements are of variable quality and in the extended policy we persist with this notion of heterogeneous component lifetimes.

We determine the long run cost per unit time (cost-rate) for the policy and a simpler hybrid opportunistic policy. We illustrate these policies using a numerical example. In part, we compare, in cost-rate terms, the principal policy with a number of policies that are special cases including the hybrid policy, the simpler hybrid opportunistic policy, age-based replacement; pure inspection, and pure opportunistic replacement.

While notionally the system of interest is a one-component system, implicitly this is part of a large multi-component system, and it is this greater part that generates the opportunities. Thus, the policy and model we propose are applicable to and offers benefits for the maintenance of multicomponent systems.

Summarising some of the finer points of detail about the proposed policy, we find that when the cost of opportunistic replacement is small relative to the cost of preventive replacement at the end of the wear-out phase and the rate of arrival of opportunities is relatively high, preventive replacement is no longer necessary. This is not surprising. Also, when the arrival of the defects for strong components is relatively unpredictable, the best policy may also be to only await an opportunity. Furthermore, we observe that when there is little information about defects the best policy is opportunistic replacement. On the other hand, if the quality of replacements is poor then the utilisation of opportunities is relatively less important.

Finally, we note that the cost advantage of the principal policy, policy 1, over the simpler hybrid opportunistic policy, policy 2, is only small, indicating that the simpler policy may be the most appropriate for practice.

For further research, we may focus on deeper analysis of the trade-off between the theoretical effectiveness and the ease of application in practice of a maintenance policy, by investigating in particular some further strategies that make maintenance policies more applicable. Another direction to consider is the use of multi-criteria analysis for applications to service systems, where the consequences of failure go beyond the cost dimension.

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