

## Joint optimisation of inspection maintenance and spare parts provisioning: a comparative study of inventory policies using simulation and survey data

Farhad Zahedi-Hosseini<sup>1</sup>, Philip Scarf<sup>2\*</sup> and Aris Syntetos<sup>3</sup>

<sup>1</sup>*School of Computing, Science and Engineering, University of Salford, Manchester M5 4WT, UK*

<sup>2</sup>*Salford Business School, University of Salford, Manchester M5 4WT, UK, email p.a.scarf@salford.ac.uk*

<sup>3</sup>*Cardiff Business School, Cardiff University, Cardiff CF10 3EU, UK*

*\*corresponding author*

**Abstract:** The demand for industrial plant spare parts is driven, at least in part, by maintenance requirements. It is therefore important to jointly optimise planned maintenance and the associated spare parts inventory using the most appropriate maintenance and replenishment policies. In this simulation-based study, we address this challenge in the context of the random failure of parts in service and the replacement of defective parts at inspections of period  $T$ . Inspections are modelled using the delay-time concept. A number of simultaneous periodic review and continuous review replenishment policies are compared. A paper making plant provides a real context for the presentation of our ideas. We survey practitioners working with such plant to collect real data that inform the values of parameters in the models. Our simulation results indicate that a periodic review policy with ordering that is twice as frequent as inspection is cost optimal in the context of the plant that we study. For the purpose of comparison, we also present and discuss the characteristics of the various policies considered.

**Keywords:** maintenance; spare parts; delay-time; simulation; joint optimisation.

### 1. Introduction

The demand for spare parts for industrial plant is predicated on the operation and maintenance of the plant. Therefore, the planning of spare parts inventory should be driven by operational and maintenance requirements rather than the observation of demand. This is because operation and maintenance schedules provide partial information about the demand for spare parts in advance, and the forecasting of spare parts demand based on historical usage is sub-optimal (Ghobbar and Friend, 2003; Boylan and Syntetos, 2010). Furthermore, maintenance planning that assumes 100% availability of spare parts is also sub-optimal (Sharma and Yadava, 2011). Thus, it is important to coordinate the planning of operation, maintenance and spare parts inventory (Wang and Syntetos, 2011). Many researchers have tackled this coordination of maintenance and inventory separately or sequentially (de Almeida, 2001; Marseguerra et al., 2005; Cheng and Tsao, 2010, de Almeida et al., 2015). However, it has been demonstrated that joint optimisation is superior (in a cost sense) to separately or sequentially optimised policies (Sarker and Haque, 2000).

Therefore, it is important not only to coordinate operation and maintenance planning and spare parts inventory control but also to carry out optimisation jointly. We do just this in this paper, whilst setting aside the question of coordination with operation by supposing that a plant is continuously or regularly operated. We focus on the joint cost-optimisation of planned, periodic inspection

maintenance and each of several periodic and continuous review replenishment policies. We use the delay-time concept to model inspection maintenance. We develop a simulation model in the context of a paper machinery plant. Further, we collect data that inform the values of parameters in the simulation using a survey of practitioners working with such plant. Several replenishment policies are considered so as not to select one arbitrarily, as is the case in almost all joint optimisation studies (Van Horenbeek et al., 2013). Thus we make two contributions: i) this is the first paper to consider a range of inventory replenishment policies in joint optimisation with maintenance planning; and (ii) it develops insights into the characteristics of each policy, not previously addressed in the joint optimisation studies. The use of simulation allows our models to make less simplifying assumptions than is usual with more analytical papers.

This paper is organised as follows. Section 2 critically reviews the joint optimisation literature. Here we aim to justify the use of the delay-time concept and our selection of replenishment policies. In section 3, we describe our industrial problem and model a complex system of multiple components (bearings). In section 4, we discuss our simulation models, including their assumptions and cost factors. Our results and discussion in section 5 precedes the conclusions in the final section.

## 2. Literature review

There are numerous industrial situations where either the replacement of multiple plant items is too costly, prohibiting the block replacement policy, or too critical to bear the risk of replacements at a pre-specified item age (items/units/components are parts of machines or equipment, which may be repaired or replaced). In these circumstances, it is reasonable and rational to inspect periodically and replace only the defective (faulty) parts while retaining the good ones. Such an inspection strategy can be modelled with the delay-time concept (Wang, 2012a) used here and utilised in numerous case studies (e.g. Christer et al., 1995; Pillay et al., 2001; Akbarov et al., 2008; Jones et al., 2010; and Zhao et al., 2015).

Classic maintenance policies (e.g. Barlow and Proschan, 1965) assume 100% availability of spares, implying that spare parts are either highly standardized for easy procurement from suppliers, or so inexpensive that large quantities may be stored to protect against possible stock-outs. Nevertheless, parts are in fact highly customized (and potentially very expensive), and their procurement lead-time cannot be neglected (Brezavscek and Hudoklin, 2003).

A specific question that motivates us is the extent to which particular maintenance policies and particular inventory policies have been jointly optimised. In Table 1 we classify the papers reviewed by Van Horenbeek et al. (2013) and others that have appeared since 2013 in a way that allows us to answer this question. We can see that only Panagiotidou (2014) compares more than one inventory replenishment policy. This reinforces our view that it is timely to consider more than one potential candidate replenishment policy with a particular maintenance policy. While this author also considers inspection maintenance, the model employed (that considers two types of failure: minor and major) is different to ours. This paper aside, the closest in scope to ours are Wang (2011, 2012b), which use the delay-time concept but only consider the  $(R, s, Q)$  inventory policy. The quantities  $R, s$ , and  $Q$ , and  $S$  in Table 1 are the standard inventory control policy parameters, defined in paragraph 2 in section 3.3.

Table 1. Joint maintenance and inventory models in journal papers.

Author (Year)	Maintenance policy		Replenishment policy		Model development		Components in system	
	Age-based	Block-based	Periodic Review	Continuous Review	Analytical	Simulation	Single	Multiple
Sarker and Haque (2000)		♦ Periodic		♦ (s,S)		♦		♦
Chelbi and Ait-Kadi (2001)		♦ Periodic	♦ (R,S)		♦			♦
Yoo et al. (2001)		♦ Periodic	♦ (R,S)		♦			♦
Brezavscek and Hudoklin (2003)		♦ Periodic	♦		♦			♦
Vaughan (2005)	*	♦ Inspection		♦ (s,S)	♦			♦
Ilgin and Tunali (2007)		♦ Periodic		♦ (s,S)		♦		♦
Huang et al. (2008)	*	♦ Periodic	♦		♦			♦
De Smidt-Destombes et al. (2009)		♦ Periodic	♦		♦		♦	
Wang (2011)	#	♦ Inspection (DTM)	♦ (R,s,Q)		♦			♦
Wang (2012b)	#	♦ Inspection (DTM)	♦ (R,s,Q)		♦			♦
Chen et al. (2013)	#	♦		♦ (s,Q)	♦			♦
Panagiotidou (2014)	#	♦ Inspection	♦ (R,S)	♦ (s,S)	♦			♦
Gan et al. (2015)	#	♦	♦		♦		♦	
Jiang et al. (2015)	#	♦ Periodic	♦ (R,S)		♦			♦
Samal and Pratihari (2015)	#	♦ Periodic	♦		♦			♦
Alrabghi and Tiwari (2016)	#	♦ Periodic		♦ (s,Q)		♦		♦

\* Not listed in the Van Horenbeek et al. (2013) review as one of the joint optimisation of maintenance and inventory control provisioning papers.  
# Not included in the latest review paper by Van Horenbeek et al. (2013) since they were published after its submission date (September 2010).

Furthermore, our literature review indicates that in joint optimisation studies: i) few researchers have considered age-replacement, most have considered block replacement, and the literature on inspection-based maintenance is growing; (ii) many researchers continue to use analytical models with restrictive assumptions rather than simulation; (iii) interest in models for multi-unit systems is growing considerably; and finally iv) periodic and continuous review replenishment policies are used almost equally.

### 3. Modelling methodology

#### 3.1. Problem description

We consider a specific industrial plant situation (a paper mill), and in particular we develop several simulation models for jointly optimising the inspection maintenance and the inventory policy for bearings, which are critical components in the plant. Bearings are used extensively in many types of plant and can result in very high costs due to unexpected and catastrophic failures. Folger et al. (2014a, 2014b) describe four conditions under which a bearing might not reach its maximum life: (i) improper handling and installation; (ii) inadequate lubrication; (iii) contamination; and (iv) force, speed, and temperature overload. The consequences of damage to a bearing system in industrial machinery can be very significant, including general risks to safety, cost of replacement and repair, and downtime.

The situation we describe is not a case study. Rather, it is an idealised context that is closely informed by a survey conducted by the authors. In this survey, we asked experts about their experience of paper making machinery in general and the critical components of these machines. A questionnaire (appendix A) was used to obtain information about: inspections; replacements; failure replacements; costs and lead-times; possible defect arrival patterns, delay times, and their distributions; current maintenance policy(s) used; and the policy(s) used for the replenishment of spares for critical components. Nine questionnaires of the 15 distributed were returned. The respondents were three experienced maintenance and inventory control researchers and six paper machine manufacturers. Where available, we have indicated the range of values of the parameters

that were provided. The information obtained from the survey ensured that our models and simulation experiments were realistic, and were the basis for the costs and parameter values used in our models.

In the models (the idealised context) that are informed by this survey, we suppose that:

- The monitoring of bearing condition is carried out by a third party (external specialists).
- Bearing condition is only reported periodically by the third party following processing of the raw condition data. This is typical of condition monitoring arrangements in modern plant (Wang and Wang, 2015). In this way, condition monitoring and the reporting of bearing condition in particular incurs cost but no plant downtime.
- Bearings that are reported as defective are replaced; this intervention has a non-zero downtime.
- Failed bearings are replaced immediately if spare parts are available since we suppose that such events cause an immediate unscheduled stop to the plant and production downtime.
- At failure, only failed bearings are attended to, so that the plant returns to operation promptly. Thus there is no inspection of bearings at failure events.

These assumptions describe the system in broad terms. Next, we describe the inspection model in detail, followed by the inventory model and then the model parameters.

Note, in this paper, the terms bearings, components, items, and spare parts are used interchangeably.

### 3.2. The inspection maintenance model and its assumptions

We suppose that the system has  $n$  identical bearings that are subject to deterioration. In our (complex) system model, multiple concurrent defects are possible and the failure process of a bearing has two-stages, according to the delay-time concept. In the first stage, a bearing is good and working normally. Then in the second stage a bearing is working but defective. The second stage terminates with a failure. The length of the first stage is the *time-to-defect arrival*, and the length of the second stage is the *delay-time*. If inspection is carried out during this second stage, it is assumed that defective items are identified and replaced, as depicted in Figure 1. Only defective items identified at inspections are replaced, rather than replacing all items regardless of their age or condition.

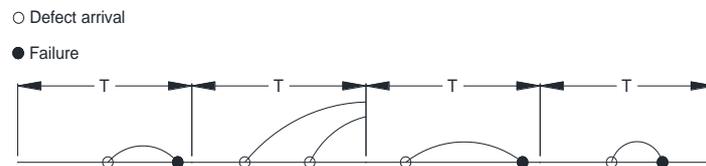


Figure 1. Defect arrivals and failure occurrences in a complex system of multiple components.

All bearings are inspected in parallel every  $T$  time units, and defective bearings are replaced preventively. On failure, failed bearings are replaced and only failed bearings are replaced. We assume the system is in a state of suspension while replacement is carried out. Therefore, defects do not grow and the bearings do not age during replacement downtime, and defects and failures can only arise whilst the plant is operating. The system is assumed to be operating under steady-state

conditions. Any operational loss due to defects other than inspection, replacement and failure are ignored. These are standard assumptions in inspection models (Wang, 2011).

Times between defect arrivals are assumed to be independent, exponentially distributed, consistent with the delay-time model of a complex system (see for example, Wang, 2012b), with a defect arrival rate,  $\lambda$ , (intensity) of 0.05 per week. This value and the others that follow are based on our survey. Note, it is only necessary to specify the defect arrival rate for the collection of identical bearings as a whole, as we do here. Nonetheless, this quantity can be determined as the product of the number of bearings and the “per-bearing” defect arrival rate. The number of identical bearings in a paper rolling machine is typically large ( $>100$ ) (Wang, 2011). The delay-time has a Weibull distribution with **scale and shape** parameters,  $\alpha = 10$ ,  $\beta = 3$  **respectively** (implying a mean delay time of 8.93 weeks). The downtimes due to each replacement and failure are  $d_r = 4$  hours = 0.024 weeks (range 1-6 hours) and  $d_f = 9$  hours = 0.054 weeks (range 1-36 hours).

### 3.3. *The inventory control model and its assumptions*

Paper machinery typically have many identical bearings. We suppose that inventory planning is concerned only with these bearings. That is, we will consider inventory policy for a single stock keeping unit. We make the following assumptions. The demand for bearings is generated through two routes. Failures of parts in service occur between inspections, which generate intermittent single-unit demands. And, every  $T$  time units, at scheduled inspections, all defective bearings are identified and preventively replaced (provided there are enough spares), generating ‘lumpy’ demand. Demand is satisfied from the existing inventory or by expediting an emergency order.

We will compare several periodic and continuous review inventory policies (see e.g. Muller, 2011; Silver et al., 2016). As depicted in Figure 2, these policies include: (i) the periodic  $(R, S)$  policy, where every  $R$  time units (the review period) an order is placed to raise the inventory position to level  $S$ ; (ii) the periodic  $(R, s, S)$  policy, where every  $R$  time units, an order is placed to raise the inventory position to level  $S$  provided the inventory position has reached or fallen below the re-order level  $s$ ; (iii) the periodic  $(R, s, Q)$  policy, where every  $R$  time units an order of  $Q$  units is placed provided the inventory position is less than or equal to  $s$ ; (iv) the continuous  $(s, S)$  policy, where an order is placed to raise the inventory position to level  $S$  when the inventory position falls to or below level  $s$ ; and finally (v) the continuous  $(s, Q)$  policy, where  $Q$  units are ordered when the inventory position falls to or below level  $s$ .

In the five policies illustrated in Figure 2, orders are placed at points A, C, E and G, for example, and arrive at points B, D, F and H respectively, after a lead-time,  $L$ . It is important to note that although the same arbitrary demand profile has been used for all five replenishment policies in Figure 2, the ordering outcome is different in each case. Also, in this illustration  $L < R$  for simplicity, but this restriction is not imposed in our model.

We seek values of the decision variables that minimise the long run expected cost per unit time or cost-rate,  $C(T)$ . The set of decision variables depends on the exact inventory policy considered. In all cases, the joint policy contains the decision variable  $T$ , the inspection interval. Throughout our analysis, the unit of time is one week. This is an arbitrary unit that is convenient for the reporting of the results.

Based on our survey, we set the lead-time  $L$  at 3 weeks (range 2-6 weeks) and the shortage emergency delivery lead-time,  $L_{sh}$ , fixed at 1 day (range 1-10 days). Further, orders are placed at the beginning of each order placing day for the periodic policies, or immediately (when triggered) under the continuous replenishment policies. Orders arrive at the beginning of each order receipt day but before reviewing the current inventory if it coincides with an order placing day. The last two assumptions are for modelling purposes. For the periodic review policies, we set  $T = kR$  for  $k > 0$ . We set the order cost including normal delivery,  $C_o$ , to £100, the cost-rate of inventory holding,  $C_h$ , to 1% of item cost per week, the cost of one bearing (item cost) is  $C_u = £2,000$  (range £1000-4000), and finally the emergency shipment cost is  $C_{sh} = £1,000$  per emergency order (range £500-1200).

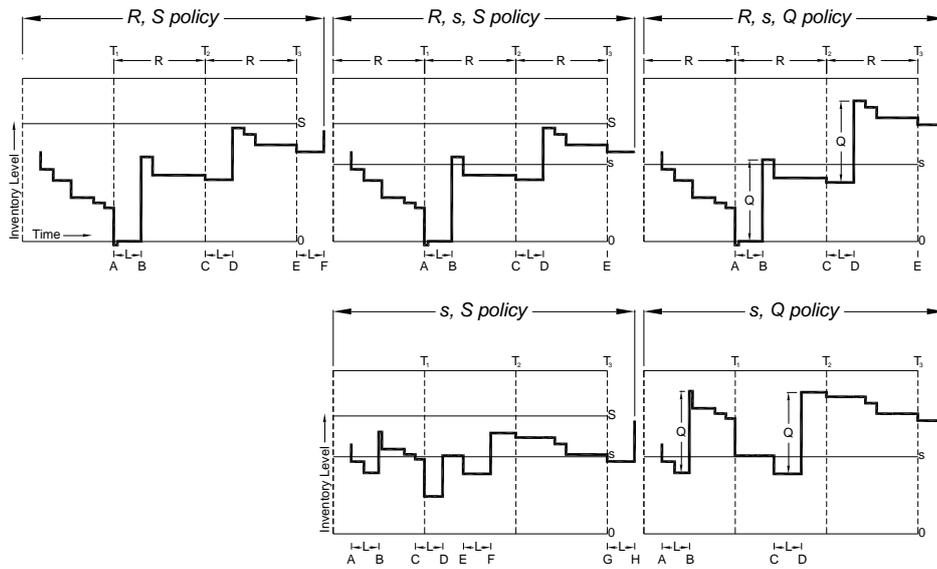


Figure 2. Inventory positions of the periodic and continuous review inventory policies.

### 3.4. Costs and downtime specifications

The cost of preventive replacement (per item),  $C_r$ , and the cost of failure replacement (per item),  $C_f$ , are set on the basis of 3 maintenance technicians at  $C_m = £60$  per maintenance technician per hour. Further, the cost-rate of machine downtime is  $C_d = £1,000$  per hour. One maintenance technician assists the external specialists during their visit for an 8 hour shift, performing data analysis and reporting. Therefore, the cost of inspection:  $C_i = 1,000 + 8 C_m = £1,480$ .

There are two preventive maintenance (PM) inspection renewal costs depending on whether spare parts are immediately available. PM replacement cost ( $C_r$ ) includes 3 maintenance technicians for 4 hours. If spare parts are available, then the cost includes downtime and manpower, hence:  $C_r = 4 C_d + 12 C_m = £4,720$ . Otherwise, there are two extra costs, namely, the shortage shipment cost and the downtime cost while the emergency shipment is in transit, hence:  $C_r = 4 C_d + 12 C_m + C_{sh} + C_d L_{sh} = £29,720$ .

Similarly, there are two failure renewal costs. Failure replacement cost ( $C_f$ ) includes 3 maintenance technicians for 9 hours. If spare parts are available, then the cost includes the costs of downtime and manpower, hence:  $C_f = 9 C_d + 27 C_m = £10,620$ . Otherwise, there are two extra

costs, namely, the shortage shipment cost and the downtime cost while the emergency shipment is in transit, hence:  $C_f = 9 C_d + 27 C_m + C_{sh} + C_d L_{sh} = \text{£}35,620$ .

#### 4. Simulation modelling

Using a modular approach, we developed simulation models for the periodic and continuous review inventory control policies of interest. ProModel, a process-based discrete-event simulation language (see e.g. Harrell et al., 2011) was used to model a system with a single machine (but extendible to multiple machines or lines) as a continuous production system with the consideration of all major assumptions and cost figures given in sections 3.1 to 3.4.

##### 4.1. Construction of the model framework and the minimum system requirements

The development of any model using the ProModel programming environment requires the use of *LEAP: Locations, Entities, Arrivals, and Processing*. *Locations*, which may be single or multiple capacity, are fixed positions in the system where *entities* may wait to be processed, such as, machines, queues, or buffer storage areas. *Entities* are objects that enter into, flow through and depart from the system, such as parts, defects and failed items. *Arrivals* describe the precise pattern: timing; quantity; frequency; and location of *entities* entering into the system. And finally, *processing* defines the exact route that an *entity* follows, from entry, to leaving the system, including all programming logic. The *processing* also includes operations, their durations, and the *resources* needed. Although, the simplest model needs to have at least *LEAP* described, any further model sophistication almost certainly requires the use of other *modules* and/or development of special programming routines. In our models *variables*, *attributes*, *resources* and *path networks* were used extensively. We developed *subroutines* for generating delay-times for the failure and replacement processes, and for calculating various costs in the model (e.g. see appendix B). We also set up various *macros* to enable the easy alteration of decision variables.

##### 4.2. Input parameters, output analysis, model scenarios and optimisation

The simulation models are non-terminating. Settings are inputted, including the warm-up period, number of replications and simulation run time. Macro set-ups enable the instant changing of decision variables. The output report includes data and graphs, and the cost-rate,  $C(T)$ .

Prior to analysis, the *warm-up* period, the number of *replications* and the *length of run* were validated. The Time Series method and Welch's method with a window length of 5 (see e.g. Robinson, 2004; Banks, 2010) were used to determine the warm-up period. A conservative 300 weeks warm-up was used to provide a very high degree of confidence that the system had reached steady-state. Further analysis showed that seven replications provided a consistent confidence interval of 95% for output variables. Various run lengths were explored, and a 5,000-week length was deemed appropriate to ensure a high degree of confidence in the convergence of the simulation results. The computation time to run through all seven replications was 17 minutes. To find the optimal policy, we integrated our simulation models with SimRunner (see SimRunner User Guide, 2010), which uses a sophisticated optimisation algorithm (Kim et al., 2012).

Appendix C presents the flow chart of the general simulation model, showing the nine modelling routines developed for different aspects of the model. Further information regarding the details of our simulation models may be obtained from the corresponding author upon request.

## 5. Results analysis and discussion

### 5.1. Joint optimisation

Five principal joint inventory-maintenance policies were considered: three models with periodic review  $(R, S, T = R)$ ,  $(R, s, S, T = R)$  and  $(R, s, Q, T = R)$  and two with continuous review  $(s, S, T)$  and  $(s, Q, T)$ . In addition, three variant policies were modelled, namely:  $(R, S, T = 2R)$ ;  $(R, s, S, T = 2R)$ , and  $(R, s, Q, T = 2R)$ . While we restrict the presentation here to the cases  $T = kR$  with  $k = 1, 2$ , many other values for  $k$  were investigated but found to be cost-sub-optimal for the range of parameter values we use. Figure 3 and Table 2 illustrate that among all joint policies modelled, the  $(R, S, T = 2R)$  policy has the lowest cost-rate (total cost per unit time), inspecting the bearings in the plant every 10 weeks and ordering spares every 5 weeks. Note, the  $(R, s, S, T = 2R)$  policy is equivalent to the cost-minimal policy because  $S^* - s^* = 1$ . (Here,  $S^*$  and  $s^*$  are the optimum values of  $S$  and  $s$  respectively.) Globally, the second and third best policies (lowest cost-rate) are also the  $(R, S, T = 2R)$  policy, inspecting every 11 weeks and ordering every 5.5 weeks, and the  $(s, S, T)$  policy inspecting every 11 weeks. Clearly, under the cost-optimal policy, more frequent inspections are performed (every 10 compared to every 11 weeks), which will potentially identify more defects (if any) in the system and trigger their replacements, thus reducing failures and ultimately reducing cost. This trade-off between the marginal decreased cost of additional failures and the marginal increased cost of additional inspection lies at the heart of the maintenance decision problem. Similarly in the inventory decision problem, the cost of stock-outs is traded-off with the cost of inventory. Thus, more orders might be placed or more stock might be held to reduce the possibility of stock-outs, depending on the relative sizes of the order cost and the holding cost. In the joint optimisation problem, where the inspection period is an integer multiple of the order period, more frequent ordering can by implication potentially reduce the frequency of bearing failures, and this appears to be the case here.

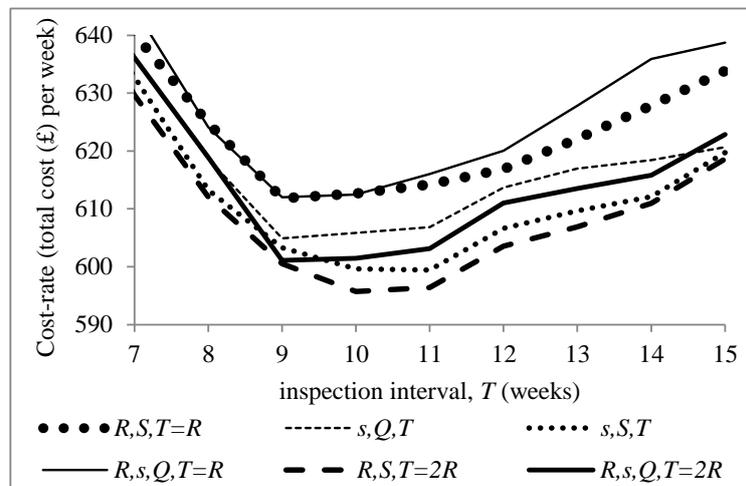


Figure 3. The cost-rate as a function of inspection interval.

Table 2. Cost-rate (£ per week) for various policies and their decision variables. Shaded results are lowest cost-rate for each policy. Bold, shaded is overall the policy with lowest cost-rate.

	$(R,S,T=R)$	$(R,s,S,T=R)$	$(R,s,Q,T=R)$	$(s,Q,T)$	$(s,S,T)$	$(R,S,T=2R)$	$(R,s,S,T=2R)$	$(R,s,Q,T=2R)$
$T$	cost-rate $S$	cost-rate $s S$	cost-rate $s Q$	cost-rate $s Q$	cost-rate $s S$	cost-rate $S$	cost-rate $s S$	cost-rate $s Q$
5	698.07 3	698.07 2 3	704.70 2 2	693.30 1 1	689.47 1 2	704.72 3	704.72 2 3	709.52 2 2
6	660.35 3	660.35 2 3	664.08 2 2	654.05 1 2	652.92 1 2	651.64 2	653.44 2 3	654.22 1 2
7	640.27 3	640.27 2 3	644.57 2 2	636.19 1 2	632.76 1 3	629.64 3	629.64 2 3	636.09 1 2
8	624.79 3	624.79 2 3	624.12 2 2	618.60 1 2	613.26 1 3	612.04 3	612.04 2 3	618.88 1 2
9	611.81 3	611.81 2 3	612.00 2 2	604.92 1 2	603.27 1 3	600.53 3	600.53 2 3	601.08 1 2
10	612.64 4	612.64 2 3	612.48 2 2	605.86 2 2	599.65 2 3	<b>595.69 3</b>	<b>595.69 2 3</b>	601.45 2 2
11	614.21 4	614.21 2 3	616.04 2 2	606.83 2 2	599.41 2 3	596.38 3	596.38 2 3	603.15 2 2
12	616.87 4	616.87 2 3	620.06 2 2	613.68 2 2	606.60 2 3	603.55 3	603.55 2 3	611.00 2 2
13	622.11 4	622.11 2 3	627.82 2 2	616.96 2 2	609.64 2 3	606.88 3	606.88 2 3	613.53 2 2
14	627.84 4	627.84 2 3	635.90 2 2	618.43 2 2	612.12 2 3	610.97 3	610.97 2 3	615.81 2 2
15	633.85 4	633.85 2 3	638.71 2 2	620.66 1 2	619.71 2 3	618.62 3	618.62 2 3	622.85 2 2

## 5.2. Insights into the characteristics of different replenishment policies

We now look at a number of the policies in Table 2 in more detail in Figure 4. Figure 4(a) demonstrates that in general the  $(s,S,T)$  policy has the highest ordering cost-rate since it can potentially place more orders at both inspections and at failures. This is also partly because  $s = 2$  and  $S = 3$ , whence an order is always triggered when the stock level drops by one unit. This is the  $(S - 1, S)$  policy, a special case of the  $(s, S)$  policy. Our conclusion is supported by Figure 4(f) since the mean number of spares ordered per order is lowest for this policy. Note, while the  $(s, Q)$  and  $(s, S)$  inventory policies are equivalent for unit-sized transactions (whence  $Q = S - s$ ), we can see in Table 2, for example, that the cost-rate for  $(s = 1, S = 2, T)$  is less than the cost-rate for  $(s = 1, Q = 1, T)$  for the same  $T$ . This is because in our model, while failure demands are always unit-sized (the probability of two or more failures occurring together is zero), demand at an inspection may be greater than unit-sized (when more than one bearing is found to be defective).

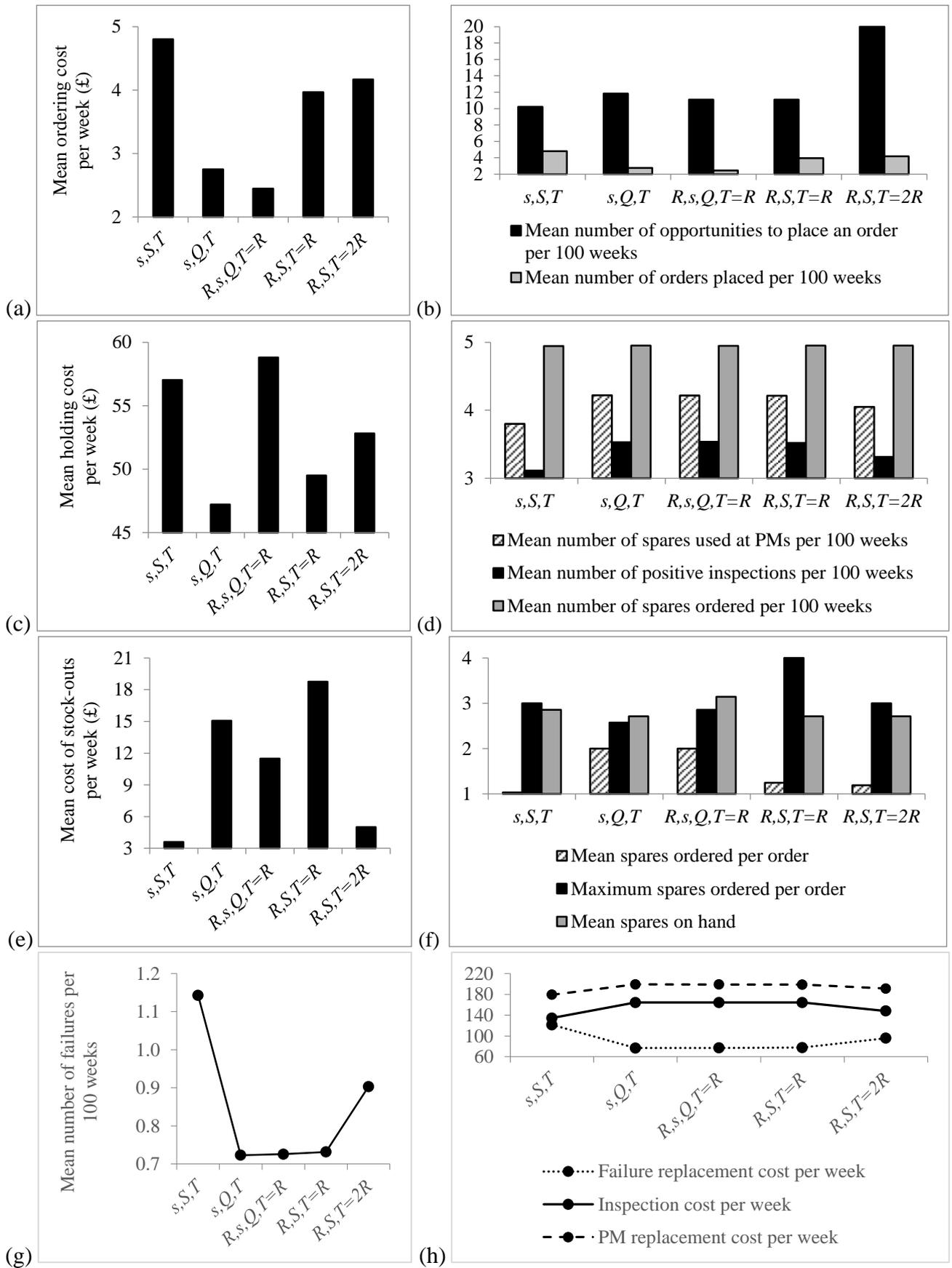


Figure 4. For the optimum policy in each class of inventory policies: (a) order cost-rate; (b) holding cost-rate; (c) stock-out cost-rate; (d) order point statistics; (e) PM statistics; (f) order size statistics; (g) failure rate; and (h) maintenance cost-rates. Data collected over a simulation of 5,000 weeks.

Continuing the discussion of ordering cost-rate, furthermore, at the optimal interval, the  $(R, s, Q, T = R)$  policy orders two spares per order every time (since  $Q = 2$ ), compared to  $(R, s, S, T = R)$  which orders on average a little over one spare per order. So the former policy must have a lower order cost-rate since it places fewer orders. We would also expect  $(R, S, T = 2R)$  to have a higher order cost-rate than  $(R, S, T = R)$  (Figure 4(a)) since the former potentially places orders twice as frequently as the latter. Due to their nature, one would expect the continuous review policies to generate at least as many opportunities to place orders as periodic review policies. This is demonstrated in Figure 4(b), except for one policy. The number of opportunities for placing orders for  $(s, S, T)$  is lower than  $(R, S, T = R)$  only because its optimal inspection interval is longer - 11 weeks compared to 9 weeks. Figure 4(d) illustrates that the number of items replaced at PM intervals was lowest for  $(s, S, T)$ . There is understandably a direct association between the number of items replaced and the number of positive inspections (an inspection is deemed to be positive if at least one defect is found).

Similar observations to those made about order cost-rates can be made about holding cost-rates and stock-out cost-rates (Figures 4(c) and 4(e) respectively). A more interesting observation is that inventory costs seem to be traded off. Thus considering Figures 4(a), (c) and (e), we can see that where the holding cost-rate is high, the stock-out cost-rate is low, and vice versa, except for  $(R, s, Q, T = R)$ , where they are both high but compensated by the low order cost-rate for this policy. Where  $(R, S, T = 2R)$  does appear to outperform the other policies is in the additional opportunities it offers for replenishment, even if it does not necessarily use them (Figure 4(b)).

Figure 4(d) confirms that the usage rate of spare parts is **similar** for all policies, as expected, since in the long run (at steady state) the consumption of parts is **most** influenced by the rate of arrival of defects. This implies the rather obvious but important observation: if one wants to reduce inventory costs, then first and foremost, one should use more reliable (better quality) parts, a point made in Scarf and Cavalcante (2012).

Figure 4(d) also shows variation between the policies in the number of spares used at inspections. Thus, those policies with more positive inspections per unit time use more spares at inspections (and the mean number of spares used per inspection is approximately 1.2). This variation is balanced by the variation in failure: while  $(s, S, T)$  and  $(R, S, T = 2R)$  have the lowest positive inspection rates, their failure rates are the highest. A higher cost of failure might then lead to a different policy ranking. Also, if one were remanufacturing spare parts, one might prefer a policy with fewer failures per unit time.

This then brings us to failures and stock-outs and policy risks. The  $(s, S, T)$  policy might be perceived as a low risk policy since it has the lowest stock-out cost. However, it has the largest failure rate. The optimal policy,  $(R, S, T = 2R)$ , has a very low stock-out rate and a moderate failure rate, although its stock-out rate is much lower than inferior policies (70% lower) and its failure rate is marginally higher (23% higher). The variation in failure rates is due almost entirely to the variation in the inspection interval; a longer inspection interval will result in more failures.  $(s, S, T)$ ,  $(R, S, T = 2R)$ , and the other three policies in Figure 4(g) with optimal inspection intervals of 11, 10, and 9 weeks have failure rates of 1.14, 0.9 and 0.73 per 100 weeks respectively. The other much smaller contribution will come from stock-out rate variation, where a policy with more stock-outs will ceteris paribus have fewer failures because of plant stoppages due to stock-outs.

We now make two final points. The first is that the variation in stock-out cost-rates across the policies is relatively large (Figure 4(e)). The  $(R, S, T = R)$  policy has a stock-out cost that is four times the optimal policy. We would expect this since the latter has the potential to place twice as many orders. The second point is that generally the stock-out cost-rates are much lower than the failure cost-rates (Figure 4(e) vs Figure 4(h)).

Thus, in summary, first and foremost it is the failure rate (and equivalently defect arrival rate) that has the greatest influence on the choice of policy, followed by the emergency order (stock-out) cost. Furthermore, although the cost-rates for the jointly optimised policies are quite similar across the range of policies, the optimal values of decision variables for each policy can be quite different, so that the components of the cost-rate can be different. Thus, the different policies, at their optimal settings, place different demands on inventory.

### 5.3. Sensitivity Analysis

Sensitivity to various parameters for the optimal policy  $(R, S, T = 2R)$  is illustrated in Figure 5. The behaviour of the cost-rate with respect to the defect arrival intensity (Figure 5(a)) is as expected (cost-rate 63% and 166% of the baseline for  $0.5\lambda$  and  $2\lambda$  respectively). The optimal times between inspections behave as expected, thus as the intensity of defect arrivals increases the inspection frequency increases. Similarly, as the scale parameter of the Weibull delay-time distribution  $\alpha$  is reduced (Figure 5(b)), the cost-rate rises as expected since the mean delay-time decreases and defects develop into failures more quickly (cost-rate 89% and 116% of the baseline at  $0.5\alpha$  and  $2\alpha$ ). Also, the optimal interval first decreases as  $\alpha$  decreases and then increases as  $\alpha$  decreases further. This effect is as expected for the delay time model, since  $T \rightarrow \infty$  as  $\alpha \rightarrow 0$  (in the limit, when the delay time is zero, inspection cannot prevent failure, so there is no purpose to inspection). However, in extending the delay-time, minimal effect is displayed when inspection is very frequent. The cost-rates for different machine downtime cost-rates (Figure 5(c)) are 78% and 141% of the baseline for  $0.5C_d$  and  $2C_d$  respectively. The optimal  $T$  also behaves as expected, but with the greatest impact when the cost is reduced by 50% and inspection is infrequent. Finally, the cost-rates for  $0.5C_i$  and  $2C_i$  are 91% and 115% of the baseline respectively (Figure 5(d)). Varying  $C_i$  has the greatest effect when inspection is frequent and the optimal intervals move in the expected direction.

The cost-rates for different unit costs,  $0.5C_u$  and  $2C_u$ , are 87% and 124% of the baseline. The reduction in the unit cost does not seem to have any effect on  $T$ , especially when the cost is decreased (Figure 5(e)). Figures 5(f) and 5(g) suggest that halving or doubling the cost of ordering or holding spares have minimal effect on the cost-rate. In fact, for  $0.5C_{sh}$  and  $2C_{sh}$  this is true for every inspection interval. Further, Figure 5(h) suggests that halving the order lead-time has a smaller effect than doubling it, and that any effect is negligible when inspection is infrequent. This is because when the inspection interval is very large, it matters little if the lead-time is large.

Finally, the sensitivity of  $S$  was investigated when  $T = 10$  and  $R = 5$ . The cost-rate for this scenario was £595.69 per week. When  $S$  is reduced from 3 (the cost-optimal quantity) to 2, the weekly cost was increased, as expected, but only by 3.0%. Similarly, when  $S$  was increased from 3 to 4, the weekly cost was increased, again as expected, but only by 2.6%.

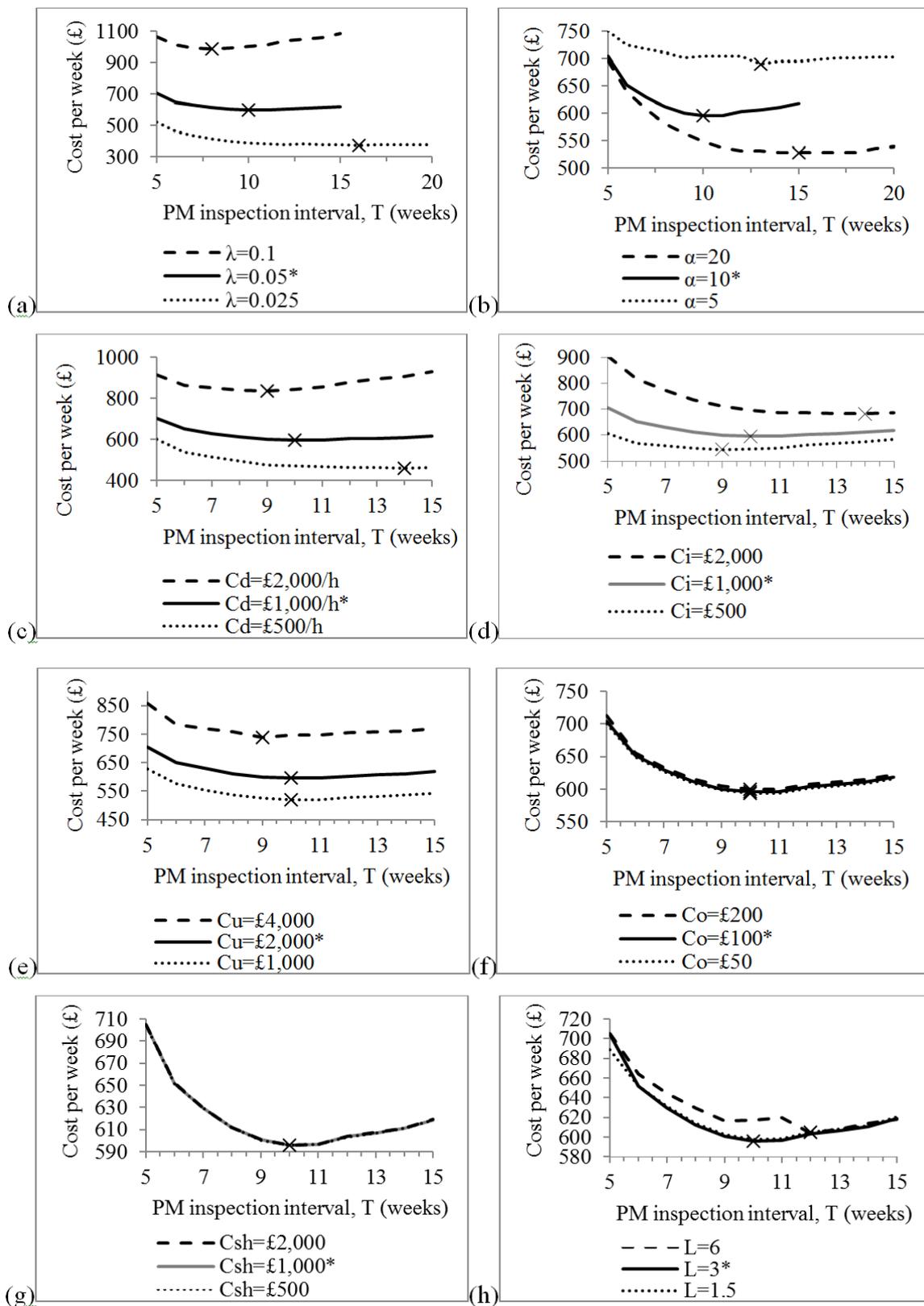


Figure 5. The sensitivity of the cost-rate for the optimal ( $R, S, T = 2R$ ) policy to various parameters ( $x$ =minimum;  $*$ =baseline): (a) defect arrival rate; (b) failure delay-time; (c) **cost-rate of machine downtime**; (d) inspection cost; (e) item cost; (f) ordering cost; (g) shortage shipment cost; (h) order lead-time.

## 6. Conclusions and further work

We have developed several simulation models for a complex system with multiple identical bearings. We have jointly optimised the planned maintenance inspection interval based on the delay time concept, and the spare parts inventory policy. To our knowledge, this is the first paper that compares a number of periodic and continuous review replenishment policies, and analyses their efficacy when joined to the inspection policy. Our objective was not only to find the cost-minimal policy across the range of policies, but also to illustrate the characteristics of each policy, so that engineers might be guided about the suitability of these policies across a range of criteria that may be appropriate for particular industrial contexts. The particular context that motivated our study is the maintenance of bearings in a paper mill.

Joint models require complex mathematical formulations and it may not be possible to solve these analytically except for limited situations and/or with simplifying assumptions. We have made use of simulation, and since simulation is not an optimization technique, we integrated simulation with an optimization tool.

For the policies considered, we make the following conclusions in the context of the plant that we study. It is cost-optimal to order twice as frequently as to inspect. More frequent ordering, and hence inspection, increases planned costs and in compensation decreases unplanned costs due to failure and downtime. The cost-optimal policy is also a relatively low risk policy since it is associated with the lowest stock-out cost-rate. The sensitivity analysis to different parameters for the optimum policy gives results that are expected, and the defect arrival rate is the principal determinant of policy, followed by the emergency order (stock-out) cost. Finally, while the cost-rates are similar across the range of policies, the components of the cost-rates are quite different because the policy decision variables are different, and so the different policies, at their optimal settings, place different demands on inventory.

The models we have developed can be extended in several ways to model more realistic industrial situations, including: imperfect inspection whereby false positive and false negative inspections are present (Berrade et al., 2012); postponed replacement of defective components; variable replenishment lead-times; modelling of dependent and/or non-identical multi-unit systems; the formulation of a cost-effective spare parts ordering policy based on historic data and *dynamic forecasting* of spare parts demand; and multi-line parallel non-identical systems.

## References

- [1]. Akbarov, A., Wang, W., Christer, A. H. (2008). "Problem identification in maintenance modelling: a case study." *International Journal of Production Research* 46: 1031-1046.
- [2]. Alrabghi, A., Tiwari, A. (2016). "A novel approach for modelling complex maintenance systems using discrete event simulation." *Reliability Engineering and System Safety* 154: 160-170.
- [3]. Banks, J. (2010). *Discrete-event system simulation*. Upper Saddle River: Pearson.
- [4]. Barlow, R. E., Proschan, F. (1965). *Mathematical Theory of Reliability*. Wiley, New York.
- [5]. Berrade, M. D., Cavalcante, C. A. V., Scarf, P. A. (2012). "Maintenance scheduling of a protection system subject to imperfect inspection and replacement." *European Journal of Operational Research* 218: 716-725.
- [6]. Boylan, J. E., Syntetos, A. A., 2010. "Spare parts management: a review of forecasting research and extensions." *IMA Journal of Management Mathematics* 21: 227-237.
- [7]. Brezavscek, A., Hudoklin, A. (2003). "Joint optimization of block replacement and periodic-review spare-provisioning policy." *IEEE Transactions on Reliability* 52: 112-117.

- [8]. Chelbi, A., Ait-Kadi, D. (2001). "Spare provisioning strategy for preventively replaced systems subjected to random failure." *International Journal of Production Economics* 74: 183-189.
- [9]. Chen, L., Ye, Z. S., Xie, M. (2013). "Joint maintenance and spare parts provisioning policy for k-out-of-n systems." *Asia-Pacific Journal of Operational Research* 30: 1350023 1-21.
- [10]. Cheng, Y. H., Tsao, H. L. (2010). "Rolling stock maintenance strategy selection, spares parts' estimation, and replacements' interval calculation." *International Journal of Production Economics* 128: 404-412.
- [11]. Christer, A. H., Wang, W. (1995). "A delay time based maintenance model of a multi-component system." *IMA Journal of Maths. Applied in Business and Industry* 6: 205-222.
- [12]. de Almeida, A. T. (2001). "Multicriteria decision making on maintenance, spares and contracts planning." *European Journal of Operational Research* 129: 235-241.
- [13]. de Almeida, A. T., Ferreira, R. J. P., Cavalcante, C.A.V. (2015). A review of the use of multicriteria and multi-objective models in maintenance and reliability. *IMA Journal of Management Mathematics* 26: 249-271.
- [14]. de Smidt-Destombes, K. S., Van Der Heijden, M. C., Van Harten, A. (2009). "Joint optimisation of spare part inventory, maintenance frequency and repair capacity for k-out-of-N systems." *International Journal of Production Economics* 118: 260-268.
- [15]. Folger, R., Rodes, J., Novak, D. (2014a). "Bearing Killer: preventing common causes of bearing system damage – part 1." *Maintenance & Engineering* 14: 12-15.
- [16]. Folger, R., Rodes, J., Novak, D. (2014b). "Bearing Killer: preventing common causes of bearing system damage – part 2." *Maintenance & Engineering* 14: 8-10.
- [17]. Gan, S., Zhang, Z., Zhou, Y., Shi, J. (2015). "Joint optimization of maintenance, buffer, and spare parts for a production system." *Applied Mathematical Modelling* 39: 6032-6042.
- [18]. Ghobbar, A. A., Friend, C. H. (2003). "Evaluation of forecasting methods for intermittent parts demand in the field of aviation: A predictive model." *Computers and Operations Research* 30: 2097-2114.
- [19]. Harrell, C., Ghosh, B. K., Bowden, R. O. (2011). *Simulation using ProModel*. Third Edition. McGraw Hill.
- [20]. Huang, R., Meng, L., Xi, L., Liu, C. (2008). "Modeling and analyzing a joint optimization policy of block-replacement and spare inventory with random-leadtime." *IEEE Transactions Reliability* 57: 113-124.
- [21]. Ilgin, M. A., Tunali, S. (2007). "Joint optimization of spare parts inventory and maintenance policies using genetic algorithms." *International Journal of Advanced Manufacturing Technology* 34: 594-604.
- [22]. Jiang, Y., Chen, M., Zhou, D. (2015). "Joint optimization of preventive maintenance and inventory policies for multi-unit systems subject to deteriorating spare part inventory." *Journal of Manufacturing Systems* 35: 191-205.
- [23]. Jones, B., Jenkinson, I., Yang, Z., and Wang, J. (2010). "The use of Bayesian network modelling for maintenance planning in a manufacturing industry." *Reliability Engineering and System Safety* 95: 267-277.
- [24]. Kim, W. K., Yoon, K. P., Kim, Y., Bronson, G. J. (2012). "Improving system performance for stochastic activity network: A simulation approach." *Computers & Industrial Engineering* 62: 1-12.
- [25]. Marseguerra, M., Zio, E., Podofillini, L. (2005). "Multiobjective spare part allocation by means of genetic algorithms and Monte Carlo simulation." *Reliability Engineering and System Safety* 87: 325-335.
- [26]. Muller, M. (2011). *Essentials of Inventory Management*. Second Edition. AMACOM.
- [27]. Panagiotidou, S. (2014). "Joint optimization of spare parts ordering and maintenance policies for multiple identical items subject to silent failures." *European Journal of Operational Research* 235: 300-314.
- [28]. Pillay, A., Wang, J., Wall, A. D., Ruxton T. (2001). "A maintenance study of fishing vessel equipment using delay-time analysis." *Journal of Quality in Maintenance Engineering* 7: 118-128.
- [29]. Robinson, S. (2004). *Simulation: The practice of model development and use*. Wiley.
- [30]. Samal, N. K., Pratihari, D. K. (2015). "Joint optimization of preventive maintenance and spare parts inventory using genetic algorithms and particle swarm optimization algorithm." *International Journal of System Assurance Engineering and Management* 6: 248-258.
- [31]. Sarker, R., Haque, A. (2000). "Optimization of maintenance and spare provisioning policy using simulation." *Applied Mathematical Modelling* 24: 751-760.
- [32]. Scarf, P. A., Cavalcante, C.A.V. (2012). Modelling quality in replacement and inspection maintenance. *International Journal of Production Economics* 135: 372-381.
- [33]. Silver, E. A., Pyke, D. F., Thomas, D.J. (2016). *Inventory Management and Production Management in Supply Chains*. Fourth Edition. CRC Press.
- [34]. "SimRunner User Guide". (2010). Orem, Utah. ProModel Corporation.

- [35].Sharma, A., Yadava, G. S. (2011). "A literature review and future perspectives on maintenance optimization." *Journal of Quality in Maintenance Engineering* 17: 5-25.
- [36].Van Horenbeek, A., Buré, J., Cattrysse, D., Pintelon, L., Vansteenwegen, P. (2013). "Joint maintenance and inventory optimization systems: A review." *International Journal of Production Economics* 143: 499-508.
- [37].Vaughan, T. S. (2005). "Failure replacement and preventive maintenance spare parts ordering policy." *European Journal of Operations Research* 161: 183-90.
- [38].Wang, W. (2011). "A joint spare part and maintenance inspection optimisation model using the delay-time concept." *Reliability Engineering and Systems Safety* 96: 1535-1541.
- [39].Wang, W. (2012a). "An overview of the recent advances in delay-time-based maintenance modelling." *Reliability Engineering and System Safety* 106: 165-178.
- [40].Wang, W. (2012b). "A stochastic model for joint spare parts inventory and planned maintenance optimisation." *European Journal of Operational Research* 216: 127-139.
- [41]. Wang, W., Syntetos, A. A. (2011). "Spare parts demand: Linking forecasting to equipment maintenance." *Transportation Research Part E* 47: 1194-1209. Wang W., Wang, H. (2015). Preventive replacement for systems with condition monitoring and additional manual inspections. *European Journal of Operational Research* 247: 459-471
- [42].Yoo, Y. K., Kim, K. J., Seo, J. (2001). "Optimal joint spare stocking and block replacement policy (cost modelling of spare stocking and block replacement)." *International Journal of Advanced Manufacturing Technology* 18: 906-909.
- [43].Zhao, F., Wang, W., Peng, R. (2015). "Delay-time-based preventive maintenance modelling for a production plant: a case study in a steel mill." *Journal of the Operational Research Society* 66: 2015-2024.

**Appendix A. Survey questionnaire - Collecting data from maintenance researchers and paper machine manufacturers.**

Information	Explanation	Value
<u>Unit cost of the critical spare part</u>	What is the unit cost of the critical spare part (£)? Although the model can be expanded in the future, our current model assumes that there is one critical part that when it fails will result in the immediate downtime of the machinery/equipment.	
<u>Inspection</u>	How long does it take to carry out a Planned Maintenance inspection for the machinery/equipment, from start to finish, to identify if one or more units (instances) of the critical part need to be replaced (excluding the time it takes for replacing each critical part) (Sec/ Min/ Hour/ Day)?	
	How many operators/technicians (resources) are needed for carrying out the <u>inspection</u> activity, in order to determine if one or more critical spare part(s) need to be replaced?	
	What is the cost of using each operator/technician (£ per Hour/ Day)?	
<u>Replacement</u>	How long does it take to actually replace a single critical spare part after identifying, at inspection, that the part needs to be replaced (Sec/ Min/ Hour/ Day)?	
	How many operators/technicians are needed for the replacement of a unit (an instance) of the critical spare part?	
	If different from the data supplied in the 'inspection' section above, what is the cost of using each operator/technician (£ per Hour/ Day)?	
<u>Failure replacement</u>	How long does it take to replace a single unit (instance) of the critical spare part as a result of Corrective Maintenance, i.e. when the critical part fails (Sec/ Min/ Hour/ Day)?	
	How many operators/technicians are needed for the replacement of a single unit of the critical part?	
	If different from the data supplied in the 'inspection' section above, what is the cost of using each operator/technician (£ per Hour/ Day)?	
<u>Downtime cost</u>	How much does it cost the company (£ per Hour/Day) for the loss of production (or sales) while the machinery/equipment is <u>down</u> due to one of the following reasons? <ul style="list-style-type: none"> <li>• the inspection process</li> <li>• the replacement of a critical part during Preventive maintenance</li> <li>• the replacement of a critical part during Corrective maintenance</li> <li>• waiting for shortages to arrive</li> </ul>	
<u>Ordering cost</u>	How much does it cost to order the critical spare part during normal production times (not ordering shortages), excluding the unit cost of the part (£)?	
<u>Holding cost</u>	How much does it cost to hold a single unit of inventory of the critical spare part in stock (£ per unit per Day/Week/Month/Year)?	
<u>Order Delivery lead time</u>	How long does it take for an order of the critical spare part to arrive, from the time that an order is placed until it arrives (Day/Week)?	
	How variable is this lead time? Can this be estimated, or does it conform to a known statistical distribution. For example, conforming to a Normal distribution, with a mean lead time of 7 days and a standard deviation of 1 day!	
<u>Shortage shipment cost</u>	How much extra (excluding the unit cost) does it cost to replenish a critical spare part in emergency (£)?	
<u>Shortage delivery lead time</u>	How long does it take for a shortage (or emergency) order of the critical spare part to arrive, from the time that the order is placed until the time it arrives (Day/Week)?	
<u>Fault (defect) distribution &amp; parameters</u>	Do you know how often faults (defects) related to the critical spare part occur? Does the pattern of occurrences follow a known statistical distribution? Alternatively, can you provide a time-series history of fault arrivals?	
<u>Failure distribution &amp; parameters</u>	Do you know if the time between failures of the critical part follow a known statistical distribution?	
<u>Currently, how often is the machinery/equipment likely to be inspected in connection with this critical spare part?</u>		
<u>What is the normal policy for replenishing the inventory of this critical spare part? Periodic or Continuous Review?</u>		

## ***Appendix B. The ProModel code for the failure occurrence routine.***

```
# generate the arrival of the next defect
ORDER 1 PreDefectE TO DefectArrivalL
REAL FailureRandom, FailureTimelapse
INC FailureGeneratedV
  //DISPLAY "Possible Failures= ", FailureGeneratedV
  //DISPLAY "Failure Frequency= ", FailureFrequencyV
FailureTimelapse=(10*7)*(-LN(1-FailureRandom))**(1/3)
  //DISPLAY "Failure Timelapse = ", FailureTimelapse
StartTimeA=CLOCK()
WAIT FailureTimelapse
ProlongingSUB
  # if the machine is currently down, delay the failure >> 'suspend its arrival'
INT ShortageInFailure
IF SpOnHandV=0 THEN
  ShortageInFailure=1
IF DowntimeOnFailureV=1 THEN
  WAIT FailureReplacementDowntimeDurationV+
    (ShortageInFailure*SpShortageDeliveryLeadTimeV)
  # if the machine is currently being inspected, delay the failure >> 'suspend its arrival'
INT ShortageInInspection
IF SpOnHandV=0 AND CONTENTS(FailureOccurrenceL)>0 THEN
  ShortageInInspection=1
IF InspectionOnFailureV=1 THEN
  WAIT (InspectionDurationV)+
    (CONTENTS(FailureOccurrenceL)*PMReplacementDowntimeDurationV)+
    (ShortageInInspection*SpShortageDeliveryLeadTimeV)
INC FailureArrivedTotalV
  //DISPLAY "Failure ", FailureArrivedTotalV
  # machine is going down
DOWN Leg1GoDown, 999
  # when the warmup period is over and all variables are reset,
  # there might be a 'generated' failure in its 'timelapse'
  # if so, do the following
IF CLOCK(>WarmUpDurationV AND FailureGeneratedV=0 THEN
  FailureGeneratedV=1
```

**Appendix C. Flowchart of the general simulation procedure.**

