# Two Bioinspired Mobile Manipulators with Walking and Rolling Locomotion 

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#### Abstract

This paper deals with two novel structures for mobile robots. The original inspiration of the robots comes from a salamander and a specific kind of spiders. Our robots have some especial moving capabilities causing to increase the robot maneuverability. Indeed, the capability of rolling motion is added to ordinary quadruped robots. This capability causes increment in maneuvering of the robots. Manipulators can be embedded into the robots to add the ability of transferring materials into the shell and conducting some tasks such as repairing. In this paper, after inspection of motion principles of the rolling robots, their dynamic equations are derived. Different simulations of two Bioinspired mobile robots are presented in order to scrutinize various capabilities of the proposed designs. Walking capabilities of the robots as well as their advantages are to be discussed in detail. The comprehensive simulation results of the robots in various motion modes are presented. Finally the first prototype is introduced to verify the motion mechanisms. Keywords: Bionics; rolling robot; quadruped robot; Lagrange equations.


## 1 Introduction

Designing the mobile robots using rolling motion for their displacement has been attended by many researchers in recent years. The main reason of this attention can be attributed to less energy consumption and more smooth motion [1, 2]. In nature, we can observe utilization of rolling motion in some living creatures as well. By looking towards it, perhaps some lessons regarding rolling can be learned and applied to rolling robots [2]. For animals this motion is fruitful especially in cases needing quick reactions such as hunting and escaping. By millions years of evolution, salamander and spider are two of these animals having the ability of using this motion to avoid from danger.
"Web-toed Salamander", Fig. 1a, is an especial kind of salamander living in steep mountainous area [3]. This 10 cm creature disguises itself like a wheel when feels fear or danger. This disguising gives salamander the possibility of rolling on steep rocks and makes it easy to flee quickly. Fig. 1b shows a species of spiders called "Namib golden wheel spider" [4]. This invertebrate also uses rolling trick to flee when an intruder attacks. In case of intrusion into its nest, the spider is able to roll with a speed about $1 \mathrm{~m} / \mathrm{s}$ or twenty rpm and escape from danger.

We inspired from the motions conducted by the mentioned salamander and spider and introduced two new mobile robots. The designs of the two structures named cylindrical quadruped robot ( CQR ) and spherical quadruped robot (SQR). In fact, CQR and SQR are combined models of the quadruped robots and cylindrical and spherical ones. As shown in Fig. 2, the robots have rolling, climbing and walking motion modes that we are going to deal with their explanations. These two robots compose of four legs that each leg has two links. On each leg two motors are suggested. The first motors are located in the joints of the two links and the second ones are located in the joints between each leg and frame. The main frame of the first robot consists of two hollow hemi-cylinders attached to each other by a revolute joint. A motor is also considered in the joint between two hemi-cylinders.

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Turning this motor causes the motion of the two hemi-cylinders toward each other and closing the frame. In this case, the frame of the robot is in the cylindrical shape. For the second robot, two hemispheres instead of two hemicylinders are used similar to the first robot. The hemispheres are attached to each other by the joints and a motor. In the front part of the robots, the manipulators can be installed.

By these manipulators, the robots can do transformational operations of transferring burdens into the robot or taking off them from inside of the frame towards outside. In Fig. 3, the structural configurations of the robots can be observed. As shown in Fig. 4a, the first prototype of the SQR was implemented also in our mechatronics laboratory. As shown in this figure, the robot has a symmetrical structure causing the robot to utilize the complete area of the spherical shell for locomotion. The prototype was used to verify the equations and prove some capabilities of the robots. It, as shown in Fig. 4b, consists of four pendulums as four legs of the robot.

It is worth noting that the prototype is implemented only for the verification of the rolling movement and it has not been considered for walking motion yet. Hence, as shown in Fig. 4b, the prototype has only four DC motors and we neglect the linkage motors of the legs. There is a joint between this two hemispheres and a spring which is used to open the shell. The servo which is shown in the figure is used to connect two hemispheres of the shell. A tendon lies between this joint and shaft of the servo and helps to close the joint when the shaft starts to rotate. As shown in Fig. 4 b , the stators of the DC motors are connected to the shell of the robot and their rotors are connected to the pendulums. There are also four weights connected to the end of the pendulums to increase the driving moments. By turning the motors, the pendulums start to turn and so the robot move.

In the last decade, there were several researches in the field of the robots using rolling mode as their motion mechanism. Many of these researches were about offering novel mechanisms for the propulsion mechanisms (PMs) of robots. In wheel-based robots for instance the robots use a car with one, two, three or four wheels inside. The propulsion of the robots comes from the movement of the cars. Indeed, the robot moves when its center of mass (CM) is displaced by car displacement inside the robot. The robot implemented by Halm et al. was one of the first samples of these types of the robots. Their robot consisted of a plastic spherical shell and an Internal Driving Unit (IDU). The IDU contained a driving wheel steered by a steering axis, one control box in center and one small passive wheel with spring. By rotation of the wheel and consequently IDU, the CM of the robot was transferred and hence, the robot began to move [5]. The robot developed by Bicchi et al. consisted of two wheels toy car located in a spherical case. When the car was starting to move, the CM of the robot was displaced and thus, the robot was generating a straight or curved motion depend on the motion of the car inside [6, 7]. For more information about the motion of the rolling robots refer to [8-12] and [18-26].

In this paper, after introducing and explaining the designs of the robots, their dynamic equations are derived. The rest of the paper includes consequently an explanation of our proposed designs, a description on the rolling capabilities of our robots and mathematical formulation of the dynamics of the robots.

## 2 Dynamics of the robots

As declared, SQR and CQR , are able to create rolling motion distinguishing them from other quadruped robots. To create rolling motion, as illustrated in Figs. 5 and 6, the robots transfer the CM using the leg motions and consequently move themselves. Their ways of motion are so that after closing the main frames and legs, the frames are located on the floor. After that, the four motors which are responsible for joining the legs to the frames begin to rotate one side. By moving the motors, the joined legs to them begin to rotate in one direction. The path of moving legs is so that by moving two front links to outside and two back links to inside, the main frames of robots roll. In this situation, as explained in the dynamic analysis section, by moving of the legs, the CM of the robots is transferred to create a moment for forward motion. It is clear that the rate of this moment depends on the weights of the links. By the increments in the weights of the links, the produced moment increases. The results of this type of rolling motion reveal smoother motion with lower swings (oscillations) during the movement.

In this section, the dynamic equations of $C Q R$ and motion model of SQR are achieved. The well-known Lagrange and Newton-Euler formulations are applied to obtain the equations. The CQR and SQR are designed to move on the earth or other planets similar to some other mobile robots. In order to control the robots on trajectories,
it is required to have dynamic models of the robots. In this section, we deal with motion modeling and dynamic analyses. At the first sub-section, Lagrangian equations are used to derive the dynamic equations of the CQR. In the Lagrangian formulation, it is required to formulate the potential and kinetic energies of the system. In the next subsection, although we can use the procedure applied in deriving the dynamic equation of the CQR, we try to obtain the motion model of the SQR via another way.

### 2.1 Dynamics of the CQR

We suppose the universal coordinate system XYZ is fixed on the ground and obtain the dynamic relations of the robot with respect to the main coordinate axes. To obtain dynamic formulae of the robot, we consider that:

$$
\begin{equation*}
\psi_{n}=\theta_{n}+\phi_{n}, \quad n=1,2,3,4 . \tag{1}
\end{equation*}
$$

where, $\psi_{n}$ and $\phi_{n}$, as shown in Fig. 7, are respectively the angle of the leg orientation with respect to the horizontal plane and the angle between plane and leg attachment point. $\theta_{n}$ are the angles between the surfaces passing from geometric center of the robot and the attachment point of the leg with the cylinder $p_{n}^{\prime}$ and legs. The potential energies of the cylinder and legs are presented in (2).

$$
\left\{\begin{array}{l}
U_{s}  \tag{2}\\
U_{1} \\
U_{2} \\
U_{3} \\
U_{4}
\end{array}\right\}=-\mathrm{g}\left[\begin{array}{c}
0 \\
\mathrm{~m}\left(\mathrm{R}_{1} S_{\phi_{1}}+\mathrm{l} S_{\psi_{1}}\right) \\
\mathrm{m}\left(\mathrm{R}_{1} S_{\phi_{2}}+1 S_{\psi_{2}}\right) \\
\mathrm{m}\left(-\mathrm{R}_{1} S_{\phi_{3}}+1 S_{\psi_{3}}\right) \\
\mathrm{m}\left(-\mathrm{R}_{1} S_{\phi_{4}}+\mathrm{l} S_{\psi_{4}}\right)
\end{array}\right]
$$

where, $U_{s}$ is the potential energy of the cylinder with respect to the center of the coordinate frame 0 and $U_{i}$ is the potential energy of $i^{\text {th }}$ leg with respect to the center of the coordinate frame 0 . The sine was shown by $S$ in (2) and $\mathrm{R}_{1}$ denotes the external radius of the shell. The symbol 1 , as shown in Fig. 7, is the length between CM of the leg and $p_{n}^{\prime}, \mathrm{m}$ is the mass of each leg and g denotes the gravity. Moreover, $p_{n}$ represent the CMs of the legs. From Fig. 7, we can get:

$$
\begin{equation*}
\vec{V}_{p_{n}}=\vec{V}_{o}+\vec{V}_{p_{n}^{\prime}}+\vec{V}_{p_{n} / \boldsymbol{p}_{n}^{\prime}}, \quad n=1,2,3,4 \tag{3}
\end{equation*}
$$

where, $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{\boldsymbol{n}}}$ are the velocity vectors of CMs of the legs, $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{o}}$ is the velocity vector of geometric center of the robot, $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{\boldsymbol{n}}^{\prime}}$ are the velocity vectors of $p_{n}^{\prime}$ with respect to the geometric center of the robot and $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{\boldsymbol{n}} / \boldsymbol{p}_{n}^{\prime}}$ are the relative velocity vectors of $p_{n}$ with respect to the $p_{n}^{\prime}$. From Fig. 7, we can write:

$$
\begin{align*}
& \overrightarrow{\boldsymbol{V}}_{\boldsymbol{o}}=\left[\begin{array}{l}
V_{O x} \\
V_{O y} \\
V_{O z}
\end{array}\right]=\omega_{S} \mathrm{R}_{1}\left[\begin{array}{c}
C_{\beta} \\
0 \\
S_{\beta}
\end{array}\right],  \tag{4}\\
& \overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{1}^{\prime}}=\left[\begin{array}{l}
V_{p_{1}^{\prime} x} \\
V_{p_{1}^{\prime} y} \\
V_{p_{1}^{\prime} z}
\end{array}\right]=-\omega_{s} \mathrm{R}\left[\begin{array}{c}
S_{\phi} C_{\beta} \\
C_{\phi} \\
S_{\phi} S_{\beta}
\end{array}\right] \tag{5}
\end{align*}
$$

where, $C$ is acronym of cosine, $V_{O x}, V_{O y}, V_{O Z}$ are projections of $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{O}}$ wrt $\mathrm{OXYZ}, \omega_{s}$ is the angular velocity of the shell and $\beta$ denotes the robot deviation angle from the straight line about Y axis. R and $\beta$ are shown in Fig. 7. The
projections of $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{1}^{\prime}}$ in three main directions are $V_{p_{1}^{\prime} x}, V_{p_{1}^{\prime} y}, V_{p_{1}^{\prime} z}$. To obtain $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{\mathbf{1}} / \boldsymbol{p}_{1}^{\prime}}$, we need to calculate $\overrightarrow{\boldsymbol{l}}_{\boldsymbol{1}}$ and the angular velocity vectors as following:

$$
\begin{align*}
& \overrightarrow{\boldsymbol{l}}_{\mathbf{1}}=\left[\begin{array}{l}
l_{1 x} \\
l_{1 y} \\
l_{1 z}
\end{array}\right]=1\left[\begin{array}{l}
C_{\psi_{1}} C_{\beta} \\
-S_{\psi_{1}} \\
C_{\psi_{1}} S_{\beta}
\end{array}\right]  \tag{6}\\
& \overrightarrow{\boldsymbol{\omega}}_{\mathbf{1}}=\left[\begin{array}{c}
\omega_{1 x} \\
\omega_{1 y} \\
\omega_{1 z}
\end{array}\right]=\omega_{1}\left[\begin{array}{c}
-S_{\beta} \\
0 \\
C_{\beta}
\end{array}\right] \tag{7}
\end{align*}
$$

Where $\overrightarrow{\boldsymbol{l}}_{\boldsymbol{1}}$ is a vector which $p_{1}^{\prime}$ and $p_{1}$ are the start point and end point of it respectively and as already mentioned the length of this vector is equal to 1 . The projections of $\overrightarrow{\boldsymbol{l}}_{\boldsymbol{i}}$ are $l_{1 x}, l_{1 y}, l_{1 z}$ respectively and $\omega_{1 x}, \omega_{1 y}, \omega_{1 z}$ are the projections of angular velocity vector of the motor 1 denoted by $\overrightarrow{\boldsymbol{\omega}}_{\mathbf{1}}$ in (7). From (6) and (7) we can calculate $\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{\mathbf{1}} / \boldsymbol{p}_{\mathbf{1}}^{\prime}}$ as:

$$
\overrightarrow{\boldsymbol{v}}_{\boldsymbol{p}_{1} / p_{1}^{\prime}}=\left[\begin{array}{c}
V_{\left(p_{1} / p_{1}^{\prime}\right) x}  \tag{8}\\
V_{\left(p_{1} / p_{1}^{\prime}\right) y} \\
V_{\left(p_{1} / p_{1}^{\prime}\right) z}
\end{array}\right]=\overrightarrow{\boldsymbol{\omega}}_{\mathbf{1}} \times \overrightarrow{\boldsymbol{l}}_{\mathbf{1}}=\mathrm{l} \omega_{1}\left[\begin{array}{c}
C_{\beta} S_{\psi_{1}} \\
-C_{\psi_{1}} \\
S_{\beta} S_{\psi_{1}}
\end{array}\right]
$$

Then, the velocity vector of the leg 1 can be formulated from (3), (4), (5) and (8) as:

$$
\overrightarrow{\boldsymbol{V}}_{p_{1}}=\left[\begin{array}{c}
V_{p 1 x}  \tag{9}\\
V_{p 1 y} \\
V_{p_{1 z}}
\end{array}\right]=\left[\begin{array}{c}
\omega_{s} \mathrm{R}_{1} C_{\beta}-\omega_{s} \mathrm{R} S_{\phi} C_{\beta}+1 \omega_{1} C_{\beta} S_{\psi_{1}} \\
-\omega_{s} \mathrm{R} C_{\phi}-1 \omega_{1} C_{\psi_{1}} \\
\omega_{s} \mathrm{R}_{1} S_{\beta}-\omega_{s} \mathrm{R} S_{\phi} S_{\beta}+1 \omega_{1} S_{\beta} S_{\psi_{1}}
\end{array}\right]
$$

So from (9), we get:

$$
\begin{gather*}
K_{1}=\frac{1}{2} \mathrm{~m}\left|\overrightarrow{\boldsymbol{V}}_{\boldsymbol{p}_{1}}\right|^{2}=\frac{1}{2} \mathrm{~m}\left[\left(\omega_{s} \mathrm{R}_{1} C_{\beta}-\omega_{s} \mathrm{R} S_{\phi} C_{\beta}+\mathrm{l} \omega_{1} C_{\beta} S_{\psi_{1}}\right)^{2}+\left(-\omega_{s} \mathrm{R} C_{\phi}-\mathrm{l} \omega_{1} C_{\psi_{1}}\right)^{2}+\left(\omega_{s} \mathrm{R}_{1} S_{\beta}-\omega_{s} \mathrm{R} S_{\phi} S_{\beta}+\right.\right. \\
\left.\left.\mathrm{l} \omega_{1} S_{\beta} S_{\psi_{1}}\right)^{2}\right] \tag{10}
\end{gather*}
$$

where, $K_{1}$ is the kinetic energy of the leg 1, and the relations of kinetic energies of other legs can be achieved as before. For the kinetic energies of the cylinder, we can write:

$$
\left\{\begin{array}{c}
K_{s}  \tag{11}\\
T_{s}
\end{array}\right\}=\frac{1}{2} \omega_{s}^{2}\left\{\begin{array}{c}
M_{s} \mathrm{R}_{1}^{2} \\
J_{s}
\end{array}\right\}
$$

where, $K_{s}$ and $T_{s}$ are the translational and rotational energies of the cylinder, respectively. From Lagrange equations, we can write:

$$
\begin{align*}
& L=K_{s}+T_{S}-U_{s}+\sum_{n} K_{i}-\sum_{n} U_{i}  \tag{12}\\
& \frac{d}{d t}\left(\frac{\partial L}{\partial \dot{q}}\right)-\left(\frac{\partial L}{\partial q}\right)=Q_{i} \tag{13}
\end{align*}
$$

The following equation is obtained from (12) and (13):

$$
\left\{\begin{array}{c}
T_{n}  \tag{14}\\
-T+T_{f}
\end{array}\right\}=\frac{d}{d t}\left\{\begin{array}{c}
\frac{\partial L}{\partial \omega_{n}} \\
\frac{\partial L}{\partial \omega_{s}}
\end{array}\right\}-\left\{\begin{array}{c}
\frac{\partial L}{\partial \theta_{n}} \\
\frac{\partial L}{\partial \phi}
\end{array}\right\}, n=1,2,3,4
$$

where, $L, q, \dot{q}$ and $Q_{i}$ are respectively Lagrangian, generalized coordinate, derivative of generalized coordinate and generalized force. Moreover, $T$ is the virtual torque applied to the center of geometry of the robot. $T_{f}$ denotes the friction torque between the robot and surface which the robot rolls on. Hence, from (2), (10), (11), (12) and (14), the applied torque on the leg 1 by the motor 1 can be presented as (15):

$$
\begin{gather*}
T_{1}=\mathrm{m} \times \mathrm{l}\left[\alpha_{s}\left(\mathrm{R}_{1} C_{\beta}^{2} S_{\vartheta}-\mathrm{R} C_{\beta}^{2} S_{\phi} S_{\vartheta}-\mathrm{R} C_{\phi} C_{\vartheta}+\mathrm{R}_{1} S_{\beta} C_{\beta} S_{\vartheta}-\mathrm{R} S_{\beta} S_{\phi} C_{\beta} S_{\vartheta}\right)+\alpha_{1} \mathrm{l}+\mathrm{g} C_{\vartheta}-\omega_{s} \omega_{1}\left(\mathrm{R}_{1} C_{\beta}^{2} C_{\vartheta}-\right.\right. \\
\left.\mathrm{R} C_{\beta}^{2} S_{\phi} C_{\vartheta}+\mathrm{R} C_{\phi} S_{\vartheta}+\mathrm{R}_{1} S_{\beta} C_{\beta} C_{\vartheta}-\mathrm{R} C_{\beta}^{2} S_{\phi} C_{\vartheta}\right] \tag{15}
\end{gather*}
$$

where, $\alpha_{s}$ and $\alpha_{1}$ are the angular accelerations of the shell and motor 1 , respectively and $\vartheta=\theta_{1}+\phi$. Similar results can be obtained for the other legs. If the robot moves in a straight trajectory we can write:

$$
\begin{align*}
& \phi_{1}=\phi_{2}=\phi_{3}=\phi_{4}=\phi  \tag{16}\\
& \beta=0 \tag{17}
\end{align*}
$$

So, from (15), (16) and (17), we get:

$$
\begin{equation*}
T_{1}=\mathrm{m} \times \mathrm{l}\left[\alpha_{s}\left(\mathrm{R}_{1} S_{\vartheta}-\mathrm{R} S_{\phi} S_{\vartheta}-\mathrm{R} C_{\phi} C_{\vartheta}\right)+\alpha_{1} \mathrm{l}+\mathrm{g} C_{\vartheta}-\omega_{s} \omega_{1}\left(\mathrm{R}_{1} C_{\vartheta}-\mathrm{R} S_{\phi} C_{\vartheta}+\mathrm{R} C_{\phi} S_{\vartheta}\right)\right] \tag{18}
\end{equation*}
$$

The other torques can be obtained by similar procedures. In (14), the mentioned friction torque consists of two essential parts. One is the Coulomb friction which is almost constant and does not depend on the velocity. The other is the viscous friction that is proportional to the angular velocity of the robot. So, we can write $T_{f}$ as:

$$
\begin{equation*}
T_{f}=T_{c}+\gamma_{v} \omega_{s} \tag{19}
\end{equation*}
$$

where, $T_{c}$ is the torque due to Coulomb friction and can be positive or negative depending on the direction of the angular velocity vector of the robot, and $\gamma_{v}$ is the coefficient of the viscous friction. To verify our calculations, we apply a virtual torque to the center of geometry of the robot. A similar method to obtain $T_{1}$ is applied to formulate the virtual torque $T$. The only difference is that the derivatives are respect to $\omega_{s}$ and $\phi$ instead of $\omega_{n}$ and $\theta_{n}$.

### 2.2 Motion model of the SQR

Here, we are going to obtain the motion model of the SQR. Although, by minor change in the parameters and the resultant changes in the governing equations, we can obtain the dynamic model of the SQR, in this sub-section, we try to obtain the motion model of the SQR via another way. As mentioned in the previous section, inertial coordinate frame attached to the surface is denoted by OXYZ and coordinate frame o'x'y'z' is established on the geometric center of the robot. The robot coordinate system is parallel with the world coordinate system and does not rotate with the robot rotation. As shown in the previous sub-section and Fig. 8, $\phi$ is the rolling angle of the ball measuring the rotational distance from an origin and $\theta_{m c}$ is the tilt angle of CM of the robot. As shown in Fig. 8, this CM is shown by $m c$. As mentioned before and shown in Fig. 9, the robot has four same legs and same components. Thus, we can write:

$$
\begin{equation*}
\mathrm{m}_{1}=\mathrm{m}_{2}=\mathrm{m}_{3}=\mathrm{m}_{4}=\mathrm{m}_{1} \tag{20}
\end{equation*}
$$

where, $\mathrm{m}_{1}, \mathrm{~m}_{2} \ldots \mathrm{~m}_{4}$ are the masses of the four legs and their components. Therefore, we have:

$$
\begin{equation*}
\mathrm{M}_{\mathrm{mc}}=\mathrm{m}_{1}+\mathrm{m}_{2}+\mathrm{m}_{3}+\mathrm{m}_{4}=4 \mathrm{~m}_{1} \tag{21}
\end{equation*}
$$

where, mc is the equivalent mass of the four legs and their components and $M_{m c}$ denotes the mass of mc. The driving torque of the robot can be given by:

$$
\begin{equation*}
\tau=\mathrm{I} \alpha_{s} \tag{22}
\end{equation*}
$$

where, I is the inertial moment of the SQR and $\alpha_{s}$ is its angular acceleration where can be given by:

$$
\begin{equation*}
\alpha_{s}=\frac{d \omega_{s}}{d t}=\frac{d^{2} \phi}{d^{2} t} \tag{23}
\end{equation*}
$$

by (25) and Fig. 8, we get:

$$
\begin{equation*}
\left|l_{c} \times \mathrm{M}_{\mathrm{mc}} \mathrm{~g}\right|=\mathrm{I} \alpha_{s} \tag{24}
\end{equation*}
$$

where, $l_{c}$ is the distance between contact point p and mass center mc. Also from Fig. 8, the driving torque can be written as:

$$
\begin{equation*}
\left|l_{c} \times \mathrm{M}_{\mathrm{mc}} \mathrm{~g}\right|=r_{\mathrm{mc}} \mathrm{M}_{\mathrm{mc}} \mathrm{~g} \sin \theta_{\mathrm{mc}} \tag{25}
\end{equation*}
$$

In Fig. $8, o_{s}$ is the geometric center of the ball, $p$ is the contact point where the robot contacts with the ground, $r_{m c}$ is the distance between the geometric center of the SQR and the mass center mc and $\theta_{\mathrm{mc}}$ represents the tilt angle of the mc with respect the vertical line.
The position of the mc can be calculated as:

$$
\left\{\begin{array}{l}
x_{\mathrm{mc}}^{\prime}=1 / 4\left(x_{1}^{\prime}+x_{2}^{\prime}+x_{3}^{\prime}+x_{4}^{\prime}\right)  \tag{26}\\
y_{\mathrm{mc}}^{\prime}=1 / 4\left(y_{1}^{\prime}+y_{2}^{\prime}+y_{3}^{\prime}+y_{4}^{\prime}\right) \\
z_{\mathrm{mc}}^{\prime}=1 / 4\left(z_{1}^{\prime}+z_{2}^{\prime}+z_{3}^{\prime}+z_{4}^{\prime}\right)
\end{array}\right.
$$

where, $x^{\prime}{ }_{n}, y^{\prime}{ }_{n}$ and $z^{\prime}{ }_{n}$ are the coordinates of cms of the legs shown in Figs. 9 a and 9 b , and $x^{\prime}{ }_{\mathrm{mc}}, y^{\prime}{ }_{\mathrm{mc}}$ and $z_{\mathrm{mc}}^{\prime}$ are the coordinates of mc. Here, by using Fig. 8, we have:

$$
\begin{equation*}
r_{m c}=\sqrt{x_{\mathrm{mc}}^{\prime 2}+y_{\mathrm{mc}}^{\prime 2}+z_{\mathrm{mc}}^{\prime 2}} \tag{27}
\end{equation*}
$$

As we know, the inertial moment of a spherical hollow shell can be calculated as:

$$
\begin{equation*}
I_{\text {ball }}=2 / 5 M_{\text {ball }} \frac{R_{1}^{5}-R_{2}^{5}}{R_{1}^{3}-R_{2}^{3}} \tag{28}
\end{equation*}
$$

where, $I_{\text {ball }}$ and $M_{\text {ball }}$ are the inertial moment and the mass of the shell, respectively. As shown in Fig. 8, $R_{1}$ and $R_{2}$ are the outer and inner radii of the shell, respectively. As mentioned, the four legs and their components were equivalent by mc. The equivalent inertial moment of the four legs and their components can be calculated as:

$$
\begin{equation*}
\mathrm{I}_{i n}=\mathrm{m}_{1}\left(r_{1}^{2}+r_{2}^{2}+r_{3}^{2}+r_{4}^{2}\right) \tag{29}
\end{equation*}
$$

where, $r_{n}$ is:

$$
\begin{equation*}
r_{n}=\sqrt{x_{n}^{\prime 2}+y_{n}^{\prime 2}+z_{n}^{\prime 2}} \tag{30}
\end{equation*}
$$

As shown in Fig. 8, $\theta_{\mathrm{mc}}$ can be calculated as:

$$
\begin{equation*}
\theta_{\mathrm{mc}}=\tan ^{-1}\left(x_{\mathrm{mc}}^{\prime} / y_{\mathrm{mc}}^{\prime}\right) \tag{31}
\end{equation*}
$$

The inertial moment of the robot can be obtained by:

$$
I=\mathrm{I}_{\mathrm{ball}}+I_{\text {in }}=2 / 5 \mathrm{M}_{\mathrm{ball}} \frac{\mathrm{R}_{1}^{5}-\mathrm{R}_{2}^{5}}{\mathrm{R}_{1}^{3}-\mathrm{R}_{2}^{3}}+\mathrm{m} \sum\left(r_{1}^{2}+r_{2}^{2}+r_{3}^{2}+r_{4}^{2}\right)(32)
$$

Hence, from (23), (25) and (32), we can get:

$$
\begin{equation*}
\frac{d^{2} \phi}{d^{2} t}-\frac{r_{\mathrm{mc}} \mathrm{M}_{\mathrm{mc}} \mathrm{~g}}{I} \sin \theta_{m c}=0 \tag{33}
\end{equation*}
$$

As the outer shape of the robot is spherical, there is a needle contact between shell and ground causing a low friction in the contact point p . We have a bit changing in the above results by considering the friction. By import the friction in the previous equations, we can get:

$$
\begin{equation*}
r_{m c} \mathrm{M}_{\mathrm{mc}} \mathrm{~g} \sin \theta_{\mathrm{mc}}-T_{f}=I \alpha_{s} \tag{34}
\end{equation*}
$$

So, by considering the friction, (33) can be rewrite as:

$$
\begin{equation*}
\frac{d^{2} \phi}{d^{2} t}+\frac{T_{f}}{I}-\frac{r_{\mathrm{mc}} \mathrm{M}_{\mathrm{mc}} \mathrm{~g}}{I} \sin \theta_{\mathrm{mc}}=0 \tag{35}
\end{equation*}
$$

### 2.3 Uphill motion

We can also compute the maximum slope of the uphill motion of the robots. As shown in Fig. 10, the kinematic maximum angle of the slope can be calculated easily. As depicted in this figure, it can be deduced that in the uphill motion there are two moments in opposite directions that can be calculated as:

$$
\begin{equation*}
\left|l_{c} \times \mathrm{M}_{\mathrm{mc}} \mathrm{~g}\right|=\mathrm{M}_{\mathrm{mc}} \mathrm{~g}\left(r_{\mathrm{mc}} \sin \theta_{\mathrm{mc}}-\mathrm{R}_{1} \sin \gamma_{\text {slope }}\right) \tag{36}
\end{equation*}
$$

where, $\gamma_{\text {slope }}$ is the slope of the hill. Since, we consider the robots move uphill, so the driving moment of the robot must be larger than or equal to the zero. So, from (36) we get:

$$
\begin{align*}
& \mathrm{M}_{\mathrm{mc}} \mathrm{~g}\left(r_{\mathrm{mc}} \sin \theta_{\mathrm{mc}}-\mathrm{R}_{1} \sin \gamma_{\text {slope }}\right) \geq 0  \tag{37}\\
& \sin \gamma_{\text {slope }} \leq \frac{r_{\mathrm{mc}}}{\mathrm{R}_{1}} \sin \theta_{\mathrm{mc}} \tag{38}
\end{align*}
$$

Hence, the maximum angle of the slope that the robot can move on it is:

$$
\begin{equation*}
\gamma_{\text {slope }}=\sin ^{-1}\left(r_{\mathrm{mc}} / \mathrm{R}_{1}\right) \tag{39}
\end{equation*}
$$

It is worth noting that to obtain this maximum angle, we suppose that there is no slip in the robot motion.
By comparison of the motion of the SQR with pendulum robots, we found that more abilities in motions come from the legs of the SQR. For more explanation, it can be declared that in motion of the SQR, as shown in Figs. 11a and $b$, there are at least two legs in contact with the ground. As shown in the figures, in rolling of these robots, there are three contact points unlike the ordinal rolling robots having one contact point. The two more contact points can decrease the oscillation of motion and increase the hill motion ability of the robot by increasing the contact friction between the robot and surface. In walking mode our robots have a motion mode similar to the motion of ordinary quadruped mobile robots. The robots in both close and open states have the ability to perform walking motion. For more information about the motion of the quadruped and walking robots refer to [13-17]. Robots applying rolling motion for their displacement usually have two spherical and cylindrical shapes [18-27]. To create the rolling motion, the robots should change their situations from open state to the closed one; to do this, as shown in Fig. 12, while the main halves of the frames of the robots are closed by the motor between the two halves, the second links of each leg are gathered on the first link simultaneously.

## 3 Motion simulation and experiment

The results of the simulations and experiments are presented in this section. First, we discuss on the motion of the CQR. In Fig. 13a, b and c, we apply three low, medium and fast constant rotational speeds of the motors, respectively. In this figures, agreements are observed between the simulation results and those obtained from (15). As illustrated in Fig. 13a, the robot is applied a low constant rotational speed of the motors. The figure shows fluctuations in motions of the robot; that these fluctuations decrease as the velocity increases, as presented in Figs. 13 b and 13 c . It should be noted that although there are fluctuations in the robot motions, the dominant behavior of the angular velocity of the robot is a constant value. Figs. 14a and b display the results coming from the robot simulation and deduced in (33). In the figures, we plotted the first and second terms separately. The overlap between them reveals the accuracy of the equation. Furthermore, in these figures, we try to simulate the proposed robots in two distinct angular accelerations. As shown in these figures, there are acceptable coincidences between the simulation results (blue line) and analytic results (green lines).

An AVR controller main board is applied to control the motors and to perform the commands transferred from a PC as a command station. The commands transferred from command station are received by a Bluetooth serial adapter module and then they are sent to AVR main board in order to issue appropriate orders. Based on the commands, the main board controls the speeds of the motors. The robot is equipped with four encoders as sensors. The feedback signals of the sensors are transferred to the main board in order to compute the errors and send the appropriate orders. A rechargeable battery provides the required energy of the robot. The electrical components and structural information of the prototype are presented in Table. 1 and 2 respectively. The accuracy of the results of a circular trajectory is illustrated in Fig. 15a. This figure shows the results of circular trajectories with (a) $\lambda=15.3^{\circ}$, (b) $\lambda=12.1^{\circ}$, (c) $\lambda=8.3^{\circ}$ and (d) $\lambda=5.7^{\circ}$, respectively. The agreements between simulation results and analytic relations are remarkable as shown in this figure. In this figure, there is also an experimental trajectory of the SQR prototype for $\lambda=5.7^{\circ}$. As shown in this figure, there are some deviations between the circular trajectory obtained from analytic formulae and experimental trajectory of the prototype. The deviation may be eliminated by using an appropriate controller. In Fig. 15b, a square trajectory of the robot is shown. As depicted in this figure, the robot easily and immediately can change its path including $90^{\circ}$ bends. For having a square path like this, we should apply the following functions for the motors:

Motor $2=\operatorname{IF}($ time-19: 1, 5, $\operatorname{IF}($ time-38: $1,5, \operatorname{IF}($ time-57:-1, 5, $\operatorname{IF}($ time- $76:-1,5,0)))$ ).

Motor $3=\operatorname{IF}($ time-19:-1,5, IF (time-38: $1,5, \operatorname{IF}($ time-57:1,5, IF( time-76:-1, 5, 0 ) ) ) ).
Motor $4=\operatorname{IF}($ time-19: $-1,5, \operatorname{IF}($ time- $38:-1,5, \operatorname{IF}($ time-57:1,5, $\operatorname{IF}($ time- $76: 1,5,0))$ ).

This is one of the important characteristics of the robot motion capabilities. Furthermore, as shown in Fig. 15a and b, the robot has some deviations from the desired path indicating the robot requirements for a motion controller.

## 4 Conclusion

Two new designs of mobile robots were offered in this paper. In our robots, we tried to hybridize the rolling and walking robots to use all advantages of these two types. By using rolling motion, the robots had smoother and more monotonous motions and lower energy consumptions. Moreover, by using walking motion, they could overcome the obstacles and had stable motion as well as higher abilities in downhill and uphill motions rather than other rolling robots. A relatively comprehensive studied on different propulsion mechanisms of the rolling robots were reviewed and discussed. Then the concepts of the motions of the robots were described. Furthermore, their dynamic and motion models in different situations and applications were developed. The results corresponding to the software and analytic simulations of two Bioinspired mobile robots were compared with each other in detail in order to verify the efficiency of the robots and accuracy of the obtained governing equations. The implementation of the prototype was reported to scrutinize some basic motions of the robots to validate the proposed motion mechanisms. In the near future, we are going to develop our research via application of feedback controllers to control the motions of the robots.

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