A new approach to thermo-mechanical modelling of the behaviour of unsaturated soils

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Abstract

A new data mining approach is presented for modelling of the stress-strain and volume change behaviour of unsaturated soils considering temperature effects. The proposed approach is based on the evolutionary polynomial regression (EPR), which unlike some other data mining techniques, generates a transparent and structured representation of the behaviour of systems directly from raw experimental (or field) data. The proposed methodology can operate on large quantities of data in order to capture nonlinear and complex relationships between contributing variables. The developed models allow the user to gain a clear insight into the behaviour of the system. Unsaturated triaxial test data from literature was used for development and verification of EPR models. The developed models were also used (in a coupled manner) to produce the entire stress path of triaxial tests. Comparison of the EPR model predictions with the experimental data revealed the robustness and capability of the proposed methodology in capturing and reproducing the constitutive thermo-mechanical behaviour of unsaturated soils. More importantly, the capability of the developed models in accurately generalising the predictions to unseen data cases was illustrated. The results of a sensitivity analysis showed that the models developed from data are able to capture and represent the physical aspects of the unsaturated soil behaviour accurately. The merits and advantages of the proposed methodology are also discussed.

Introduction

Over the past decades thermal effects in soils have been the focus of much interest. The basic soil parameters like liquid limit, plastic limit, specific gravity, and compaction characteristics are considered to be affected by temperature variations. Temperature effects on liquid and plastic limits were first investigated by Youssef et al (1961). Similar investigations were also conducted by Lagurous (1969), Wang et al (1990) and Towhata et al (1994). Hogentogler (1936) performed compaction tests in the laboratory on several predominantly clay soils and reported that as the temperature increases and causes the optimum moisture content to decrease, the maximum dry unit weight increases accordingly. Burmister (1964) also reported similar results.

The effects of temperature on the volume change behaviour of saturated soils have been investigated by e.g. Campanella and Mitchell (1968), Plum and Esrig (1969), Habibagahi (1973), Demars and Charles (1982), Houston et al (1985), Eriksson (1989), Hueckel and Baldi (1990), Towhata et al (1993), Boudali et al (1994), Tanaka (1995), Crilly (1996), Fox and Edil (1996), Delage et al (2000) and Graham et al (2001). It has been shown that heating normally consolidated and lightly overconsolidated soils under constant effective stress induces volumetric contraction; whereas, cooling the same type of soil causes swelling; (e.g. see Paaswell (1967); Campanella and Mitchell (1968); Plum and Esrig (1969); Baldi et al (1988); Hueckel and Baldi (1990); Towhata et al (1993); Boudali et al (1994); Delage et al (2000)). Experimental results have indicated that the rate of consolidation of clays increases with the increasing temperature (e.g. Paaswell (1967); and Towhata et al (1993)). Paaswell (1967) showed that in a given effective stress condition, the greater the increase in temperature, the greater the volumetric contraction. He showed that the volumetric contraction decreases with increasing overconsolidation ratio and turns into expansion at large overconsolidation ratios. Similar results were also reported by Plum and Esrig (1969); Baldi et al (1988); Hueckel and Baldi (1990); Towhata et al (1993); Delage et al (2000).

The behaviour of normally consolidated soils under cycles of heating and cooling was investigated by Campanella and Mitchell (1968), Plum and Esrig (1969), Demars and Charles (1982), Hueckel and Baldi (1990) and Towhata et al (1993). The experimental results showed that the volumetric contraction of normally consolidated soils caused by heating under constant effective stress could not be recovered by subsequent cooling. Investigations by Campanella and Mitchell (1968), Plum and Esrig (1969), Houston et al (1985), Towhata et al (1993), and Fox and Edil (1996) also revealed that temperature affects the primary consolidation as well as the secondary compression.

Temperature-induced pore water pressure was investigated by a number of researchers (e.g. Campanella and Mitchell (1968); Plum and Esrig (1969); Hueckel and Baldi (1990); Hueckel and Pellegrini (1992); Tanaka (1995) and Graham et al (2001)). General results have shown that the pore water pressure increases with increase in temperature and decreases when the temperature drops. Heating induced failure in saturated soils was also investigated by Hueckel and Baldi (1990).

Some studies have focused on the effects of temperature on the shear strength and the stress and strain characteristics of saturated soils. Experimental results reported by Hueckel and Baldi (1990) and Graham et al (2001) showed that temperature had no effect on the critical state line in the deviator stress/mean effective stress plane. Lingnua (1993) and Houston et al (1985) studied the uniqueness of the critical state line in the deviator stress/mean effective stress plane and reported a small shift in the critical state line with changes in temperature. The shrinkage of yield locus with increasing temperature was observed in the experimental results of Hueckel and Baldi (1990), Tanaka et al (1997), Cui et al (2000) and Graham et al (2001).

Sherif and Burrous (1969) and Maruyama (1969) studied the effects of temperature on shear strength in unconfined compression tests on normally consolidated saturated clays. Lagurous (1969) also carried out unconfined compression tests at different temperatures on compacted soil specimens at optimum moisture content. Hueckel and Baldi (1990) conducted drained triaxial tests on overconsolidated Pontida silty clay samples, which had been heated under drained condition. The results showed that an increase in temperature lowered the peak shear strength and reduced the dilation of the samples towards the critical state. Lingnau et al (1995) performed consolidated undrained triaxial compression tests on lightly overconsolidated sand-bentonite specimens. Kuntiwattanakul et al (1995) also conducted several consolidated undrained triaxial tests along different heating and consolidation paths.

A number of hydro-thermo-mechanical models have been proposed over the past decades to represent the behaviour of unsaturated soils. Philip and De Vries (1957) introduced a model representing the coupled heat and moisture transfer in rigid porous media under the combined gradients of temperature and moisture. De Vries (1958) included moisture and latent heat storage in the vapour phase, and the advection of sensible heat by water in their previous model. Sophocleous (1978), Milly (1982), Thomas and King (1991) and Thomas and Sansom (1995) modified the Philip and De Vries (1957) model. Ewen and Thomas (1989) and Thomas and Li (1997) validated the theory presented by Philip and De Vries (1957) both in the laboratory and in the field, revealing reasonable agreement between the theoretical analyses and the laboratory/field results.

Geraminegad and Saxena (1986) developed a model considering the effect of matrix deformation on moisture, heat and gas flow through the porous media. Similar formulations were also presented by Thomas and He (1997), Gawin et al (1995), and Zhou et al (1998).

Booker and Smith (1989) and Britto et al (1989) investigated the consolidation of soil and distribution of pore-water pressure around hot cylinders buried in saturated clay.

Khalili and Loret (2001) presented an alternative theory for heat and mass transport through deformable unsaturated porous media. They extended their previous work (Loret & Khalili, 2000) on fully coupled isothermal flow and deformation in variably saturated porous media to include thermal coupling effects. Wenhua et al (2004) presented a thermo-hydro-mechanical (THM) constitutive model for unsaturated soils. The influences of temperature on the hydro-mechanical behaviour in unsaturated soils were considered in this model. Francois and Laloui (2008) introduced an unconventional constitutive model for unsaturated soils. Bishop's effective stress framework was adopted that included a number of intrinsic thermo-hydro-mechanical connections to represent the stress state in the soil.

Another thermo-hydro-mechanical (THM) model for unsaturated soils was proposed by Dumont et al (2010). In this research the effective stress concept was extended to unsaturated soils with the introduction of a capillary stress. A thermo-elastic-plastic model was also suggested by Uchaipichat (2005) for unsaturated soils based on the effective stress principle by taking the thermo-mechanical and suction coupling effects into account. Uchaipichat and Khalili (2009) published the results of an experimental investigation on thermo-hydro-mechanical behaviour of unsaturated silt. They conducted an extensive array of isothermal and non-isothermal tests including temperature controlled soaking and desaturation, temperature and suction controlled isotropic consolidation, and suction controlled thermal loading and unloading tests. Khalili et al. 2010 derived an expression for the skeletal thermal expansion coefficient of homogenous porous media. They showed that the porous skeleton as a whole experiences the same thermal strain as that of the solid grains.

In this paper a data mining approach is presented for modelling of thermo-mechanical behaviour of unsaturated soils. Models are developed, based on evolutionary polynomial regression (EPR) to predict the coupled thermal and mechanical behaviour of unsaturated soils. The results from the experimental investigations on compacted samples of silt using triaxial apparatus at different temperatures (Uchaipichat & Khalili, 2009) were used for developing and evaluating the EPR models. The input parameters of the model were considered to be the over consolidation ratio, mean net stress, initial suction, temperature, initial degree of saturation, axial strain, deviator strain and volumetric strain and the models were developed to predict the stress-strain status of the soil in response to an increment in the axial strain.

The developed models were validated using cases of data that had been kept unseen to the EPR during the modelling process, in order to investigate the generalisation capabilities of the developed models. Proposed models were coupled to predict the entire stress paths for unseen cases that were not used in the training stage of the model development process. The EPR model predictions were compared with experimental measurement to evaluate the model performance in predicting the stress-strain behaviour of soils and the level of accuracy of the predictions. A sensitivity analysis was conducted to investigate the effects of contributing parameters on the developed EPR model predictions, and to examine the consistency of the performance of the models with general engineering understanding of the thermo-mechanical behaviour of unsaturated soils.

Evolutionary polynomial regression

Evolutionary polynomial regression (EPR) is a data mining technique that integrates numerical and symbolic regression. The strategy uses polynomial structures to take advantage of their favourable mathematical properties. The key idea behind the EPR is to use evolutionary search for exponents of polynomial expressions by means of a genetic algorithm (GA) engine. This allows *(i)* easy computational implementation of the algorithm, *(ii)* efficient search for an explicit expression, and *(iii)* improved control of the complexity of the expression generated (Giustolisi & Savic, 2006). EPR is a data-driven method based on evolutionary computing, aimed to search for polynomial structures representing a system. A physical system, having an output y, dependent on a set of inputs X and parameters θ, can be mathematically formulated as:

$$y = F(\mathbf{X}, \boldsymbol{\theta}) \tag{1}$$

where F is a function in an m-dimensional space and m is the number of inputs. To avoid the problem of mathematical expressions growing rapidly in length with time, in EPR the evolutionary procedure is conducted in the way that it searches for the exponents of a polynomial function with a fixed maximum number of terms. During one execution it returns a number of expressions with increasing numbers of terms up to a limit set by the user to allow the optimum number of terms to be selected. The general form of expression used in EPR can be presented as (Giustolisi & Savic, 2006):

$$y = \sum_{j=1}^{m} F(\mathbf{X}, f(\mathbf{X}), a_j) + a_0 \tag{2}$$

where y is the estimated vector of output of the process; a_j is a constant; F is a function constructed by the process; X is the matrix of input variables; f is a function defined by the user; and m is the number of terms of the target expression. The first step in identification of the model structure is to transfer equation 2 into the following vector form:

$$Y_{N\times 1}(\theta, Z) = \begin{bmatrix} I_{N\times 1} & Z^{j}_{N\times m} \end{bmatrix} \times \begin{bmatrix} a_0 & a_1 & \ldots & a_m \end{bmatrix}^T = Z_{N\times d} \times \theta^T_{d\times 1} \tag{3}$$

where $Y_{N\times 1}(\theta,Z)$ is the least squares estimate vector of the N target values; $\theta_{d\times 1}$ is the vector of d=m+1 parameters a_j and a_0 (θ^T is the transposed vector); and $Z_{N\times d}$ is a matrix formed by I (unitary vector) for bias a_0, and m vectors of variables Z_j. For a fixed j, the variables Z_j are a product of the independent predictor vectors of inputs, X = <X_1 X_2 … X_k>.

In general, EPR is a two-stage technique for constructing symbolic models. Initially, using standard genetic algorithm (GA), it searches for the best form of the function structure, i.e. a combination of vectors of independent inputs, X_s=1: k, and secondly it performs a least squares regression to find the adjustable parameters, θ, for each combination of inputs. In this way a global search algorithm is implemented for both the best set of input combinations and related exponents simultaneously, according to the user-defined cost function (Giustolisi & Savic, 2006). The adjustable parameters, a_j, are evaluated by means of the linear least squares (LS) method based on minimization of the sum of squared errors (SSE) as the cost function. The SSE function, which is used to guide the search process towards the best fit model, is:

$$\mathbf{SSE} = \frac{\sum_{i=1}^{N} (y_a - y_p)^2}{N} \tag{4}$$

where y_a and y_p are the target experimental and the model prediction values respectively. The global search for the best form of the EPR equation is performed by means of a standard GA over the values in the user defined vector of exponents. The GA operates based on Darwinian evolution which begins with random creation of an initial population of solutions. Each parameter set in the population represents chromosomes of the individuals. Each individual is assigned a fitness based on how well it performs in its environment. Through crossover and mutation operations, with the probabilities Pc and Pm respectively, the next generation is created. Fit individuals are selected for mating, whereas weak individuals die off. The mated parents create a child (offspring) with a chromosome set which is a mix of parents' chromosomes. In EPR integer GA coding with single point crossover is used to determine the location of the candidate exponents (Giustolisi & Savic, 2006).

The EPR process stops when the termination criterion, which can be either the maximum number of generations, the maximum number of terms in the target mathematical expression or a particular allowable error, is satisfied. A typical flow diagram for the EPR procedure is illustrated in Figure 1.

Database

Results from triaxial experiments on samples of an unsaturated soil reported by Uchaipichat and Khalili (2009) were used to develop the EPR-based models. These experiments were conducted at constant suction, constant temperature and constant water content stress paths including: *i)* temperature and suction controlled isotropic loading tests, *ii)* temperature controlled desaturation tests, *iii)* suction controlled thermal loading tests, *iv)* constant water content thermal loading tests, and *v)* temperature and suction controlled shear strength tests.

The tests were performed on silt samples compacted in the laboratory. The soil samples were obtained from the Bourke region of New South Wales, Australia. The index properties of the soil are presented in Table 1.

Data preparation

Results from 27 temperature- and suction-controlled triaxial shear tests were used to develop models to predict the shear and volumetric behaviour of the considered unsaturated soil. All the tests were conducted in modified triaxial equipment (Uchaipichat & Khalili, 2009). The temperature and matric suction values varied from 25 to 60°C and 0 to 300 kPa, respectively. Cell pressures of 50, 100 and 150 kPa were used in the experiments. The implemented testing procedure was consolidated drained and the deviatoric stress was applied by increasing the axial stress while the cell pressure was kept constant.

The total number of cases in the database was divided into training and testing datasets. From the created database 22 cases (approximately 80%) were used to train and develop the EPR models while the remaining 5 cases (about 20%) were kept unseen to EPR during model construction and were used to validate the developed models. It was checked to make sure that all parameter values in the testing data sets were within the range of data chosen to be used for training and developing the EPR models to avoid extrapolation.

A statistical analysis was performed on the data to select the most statistically consistent training and testing sets to be utilized in the development of the presented models. The aim of the analysis was to ensure that the statistical properties of the data in each of the subsets were as close to the others as possible and thus represented the same statistical population. The mean and standard deviation values were calculated for every single contributing parameter and for the training and testing datasets for each combination and the one for which these statistical values were the closest in the training and testing data sets was chosen to be used in training and testing stages in the EPR model development process.

EPR models

A typical scheme to train most of the data mining-based material models for soils includes an input set providing the model with information relating to the current state units (e.g., current stresses and strains) and then a forward pass through the model that yields the prediction of the next expected state of stress or strain relevant to an input strain or stress increment (Ghaboussi, et al., 1998). Due to the incremental nature of soil stress–strain modelling in practical applications, this scheme has been utilized in this research. The EPR models have nine input parameters as summarized in Table 2. Axial strain, volumetric strain and deviator stress are updated independently and incrementally during the training and testing stages of the model development process based on the outputs relating to the previous increment of the axial strain. The output parameters are deviator stress and volumetric strain corresponding to the end of the incremental step.

Two separate models were developed for deviator stress (q) and volumetric strain (ε_v). Constraints were implemented to control the structure of the models in terms of the length and complexity, type of implemented functions, number of terms, range of the exponents used and also the number of generations to complete the evolutionary process. As the modelling process progressed the accuracy level at every stage was evaluated using the coefficient of determination (COD) as the fitness equation (Eq. 5).

$$\mathbf{COD} = 1 - \frac{\sum_{N} (\mathbf{Y}_a - \mathbf{Y}_p)^2}{\sum_{N} \left(\mathbf{Y}_a - \frac{1}{N} \sum_{N} \mathbf{Y}_a \right)^2} \tag{5}$$

where Y_a is the actual output value; Y_p is the EPR predicted value and N is the number of data points on which the COD is computed. If the model fitness is not acceptable or the other termination criteria (in terms of maximum number of generations and maximum number of terms) are not satisfied, the current model should go through another evolution in order to obtain a new model.

After completion of the modelling process, models were developed for deviatoric stress and volumetric strain. From among the developed models some did not include all the defined parameters as inputs to the equations (the parameters that are known to affect the thermo-mechanical behaviour of soils) and hence were removed. The remaining were considered and compared in terms of the robustness of the equations based on the coefficient of determination, sensitivity analysis and also the level of complexity of the equations and the best models satisfying all these criteria were chosen as final models. Equations 6 and 7 represent the selected EPR models for deviator stress and volumetric strain respectively.

$$q_{i+1} = \frac{0.06q_i^2 - 3.81E(-4)Su_iT^2Sr_i\varepsilon_a + 6.73OCR^6\varepsilon_a}{OCR^3q_i} + \frac{1455.02\varepsilon_a}{P_{net}Sr_iq_i} - 5.05Sr_i^3 + \frac{23.11Su_iSr_i\Delta\varepsilon_aP_{net}^2q_i - 8.67E(5)OCR^3\varepsilon_a}{P_{net}^3q_i} + 0.13\varepsilon_{v_i}{}^2 + 0.91q_i + \frac{378.26\varepsilon_{v_i}\Delta\varepsilon_a - 0.07OCR\varepsilon_a^2q_i}{Tq_i} - 0.1T - 0.12OCRq_i\Delta\varepsilon_a + 48.87\Delta\varepsilon_a + 19.81 \tag{6}$$

$$\varepsilon_{v_{i+1}} = \frac{1.06E(-3)Sr_iq_i\Delta\varepsilon_a}{OCR\varepsilon_a} + 0.83\Delta\varepsilon_a + 0.98\varepsilon_{v_i} - 0.05\varepsilon_{v_i}\Delta\varepsilon_a - 4.44E(-3)q_i\Delta\varepsilon_a + \frac{1.31E(-7)Su_i^3Sr_i^2\varepsilon_a - 0.98T + 1.09E(-3)T^3 - 9.15E(-4)T^3Sr_i}{T^2} + \frac{9.87E(-7)Sr_iq_i^3 - 4.09}{P_{net}} + \frac{2.52E(-4)\varepsilon_{v_i}{}^2 - 0.89Sr_i^2\Delta\varepsilon_a}{Sr_i} - 2.24E(-4)q_i + 0.01P_{net}\Delta\varepsilon_a + 0.1 \tag{7}$$

Figures 2 to 4 show deviator stress-axial strain and volumetric strain-axial strain curves predicted using the EPR models (Equations 6 and 7) against the experimental results for the data used in training of the models with Figure 2 showing the worst predicted data case. After training, the performance of the developed EPR models was verified using 5 sets of validation data which had not been introduced to EPR during training. The purpose of validation was to examine the generalisation capabilities of the developed models to conditions that were not seen by the model during the training phase. Figures 5 to 7 show predictions made by the developed EPR models against the experimental data for testing datasets. The COD values of the EPR models are given in Table 3.

The results show the remarkable capabilities of the developed EPR models in capturing, predicting and also generalising the shear and volume change behaviour of unsaturated soils considering temperature effects.

Predicting entire stress paths using developed EPR models

The EPR models (equations 6 and 7) were also used to predict the entire stress paths, incrementally, point by point, in $q:\varepsilon_a$ and $\varepsilon_v:\varepsilon_a$ spaces. The results from three different sets of (testing) data (that were unseen to EPR during the model development stage) were utilized to evaluate the ability of the developed models to predict the complete thermo-mechanical behaviour of unsaturated soil during the entire stress paths. The values of overconsolidation ratio, confining stress, initial suction, temperature and initial degree of saturation were kept constant throughout the tests. The other contributing parameters were updated at each incremental step of axial strain, considering the values corresponding to the previous increment and the outputs of the models in response to the axial strain increment. Figure 8 illustrates the procedure followed for updating the input parameters and building the entire stress paths for the shearing stage of a triaxial test. For a prescribed increment of axial strain, $\Delta\varepsilon_a$, the values of q_{i+1}, $\varepsilon_{v,i+1}$ are calculated using the EPR models. For the next increment, the values of $\varepsilon_{a,i}$, q_i and $\varepsilon_{v,i}$ are updated as:

$$q_i = q_{i+1}$$

$$\varepsilon_{v,i} = \varepsilon_{v,i+1}$$

$$\varepsilon_{a,i} = \varepsilon_{a,i} + \Delta\varepsilon_a$$

In this way the second points on the curves are predicted. The incremental procedure is continued until all the points on the curves are predicted and the curves are established.

Figures 9 to11 show the comparison between the three complete curves predicted using the EPR models following the above incremental procedure and the actual experimental data. It should be noted that the data for these tests were not introduced to EPR during the model development process.

The predicted results are in a very close agreement with the experimental results and considering the fact that the entire curves have been predicted point by point and the errors of prediction of the individual points are accumulated, it can be easily seen that EPR models were able to predict the complete stress paths with a high degree of accuracy which can be an indication of the robustness of the developed EPR framework for modelling thermo-mechanical behaviour of unsaturated soils.

Sensitivity analysis

A parametric study was carried out on a validation set of data to evaluate the response of the models to changes in input parameters. All the input parameters but the one being examined were kept constant and the model predictions for three different values (within the maximum and minimum values of the parameter in the database within the available range of data) of the parameter under study were investigated.

Figures 12 to 15 show the results of the parametric study conducted to investigate the effect of changes in confining pressure, suction, degree of saturation and temperature on the developed models.

As expected, any increase in the values of the confining pressure and suction in the soil sample causes the shear strength of the soil and also the volumetric strain to increase (Figures 12 and 13). Any increase in the degree of saturation of the soil will cause the soil suction to decrease and will result in lower shear strength and also expansion in the soil. This effect was also correctly predicted by the presented EPR models (Figure 14). The developed model for deviator stress also correctly predicted the critical state shear strength (Figure 15a) which was expected to be independent of temperature (Baldi (1990); Graham et al (2001)). The slight effect of temperature on the volumetric strain is also predicted by the developed EPR model (Figure 15b).

The results of the parametric study indicated that the developed EPR models have been able to capture and predict the underlying physical patterns of soil thermo-mechanical behaviour.

Discussion and conclusions

Evolutionary Polynomial Regression was used to develop models to predict shear and volumetric behaviour of unsaturated soils considering the temperature effects. In the developed methodology, EPR provides more than one model for complex behaviour of materials and systems. This allows the user to choose the best possible models on the basis of their complexity and performance in predicting the expected behaviour of the material. Predictions made by EPR models based on unseen data, are also an unbiased performance indicator of generalization capabilities of the proposed models.

Experimental triaxial test data was used to develop and verify the proposed models in this study. After training, the generalization capabilities of the developed models were evaluated by verification of their performance against unseen sets of data. The results revealed the efficiency and robustness of the proposed methodology in successfully capturing and accurately predicting the highly complicated thermo-mechanical behaviour of unsaturated soils. Furthermore, it was shown that the developed models are also able to accurately predict the entire stress paths in a triaxial test, point-by-point and by following an incremental procedure.

A parametric study was conducted to assess the sensitivity of the developed models to variations of the individual contributing parameters. The results showed that the EPR models were capable of capturing and predicting the patterns of soil thermo-mechanical behaviour and the effects of the contributing parameters (confining pressure, suction, initial degree of saturation and temperature) on the shear and volumetric behaviour of unsaturated soils.

Another interesting feature of EPR approach is that as more data becomes available, the quality of the model predictions can be improved by retraining with the more comprehensive set of data.

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Table 1. Index properties of the silt used in the tests (Bourke silt)

Properties	Values
Liquid Limit (%)	20.5
Plastic Limit (%)	14.5
Specific Gravity	2.65
Air Entry Value (kPa)	18
Maximum dry unit weight from standard proctor test (kN/m^3)	18.8
Optimum moisture content from standard proctor test (%)	12.5

Table 2. Parameters involved in the developed incremental EPR models*

Contributing parameters	Model output
$OCR, P_{net}, Su_i, T, Sr_i, \varepsilon_a, q_i, \varepsilon_{vi}, \Delta\varepsilon_a$	q_{i+1}
	$\varepsilon_{v,i+1}$

* OCR = overconsolidation ratio ; P_{net} = mean net stress (kPa); Su_i = initial suction (kPa); T = temperature (°C); Sr_i = initial degree of saturation; ε_a = axial strain; q_i = deviator stress (kPa); ε_{vi} = volumetric strain; $\Delta\varepsilon_a$ = axial strain increment; q_{i+1} = deviator stress corresponding to the next increment of axial strain (kPa); $\varepsilon_{v,i+1}$ = volumetric strain corresponding to the next increment of axial strain.

Table 3. Coefficient of determination values for the presented models

Equation	COD values for training (%)	COD values for testing (%)
Deviator stress	99.85	99.44
Volumetric strain	99.99	99.86

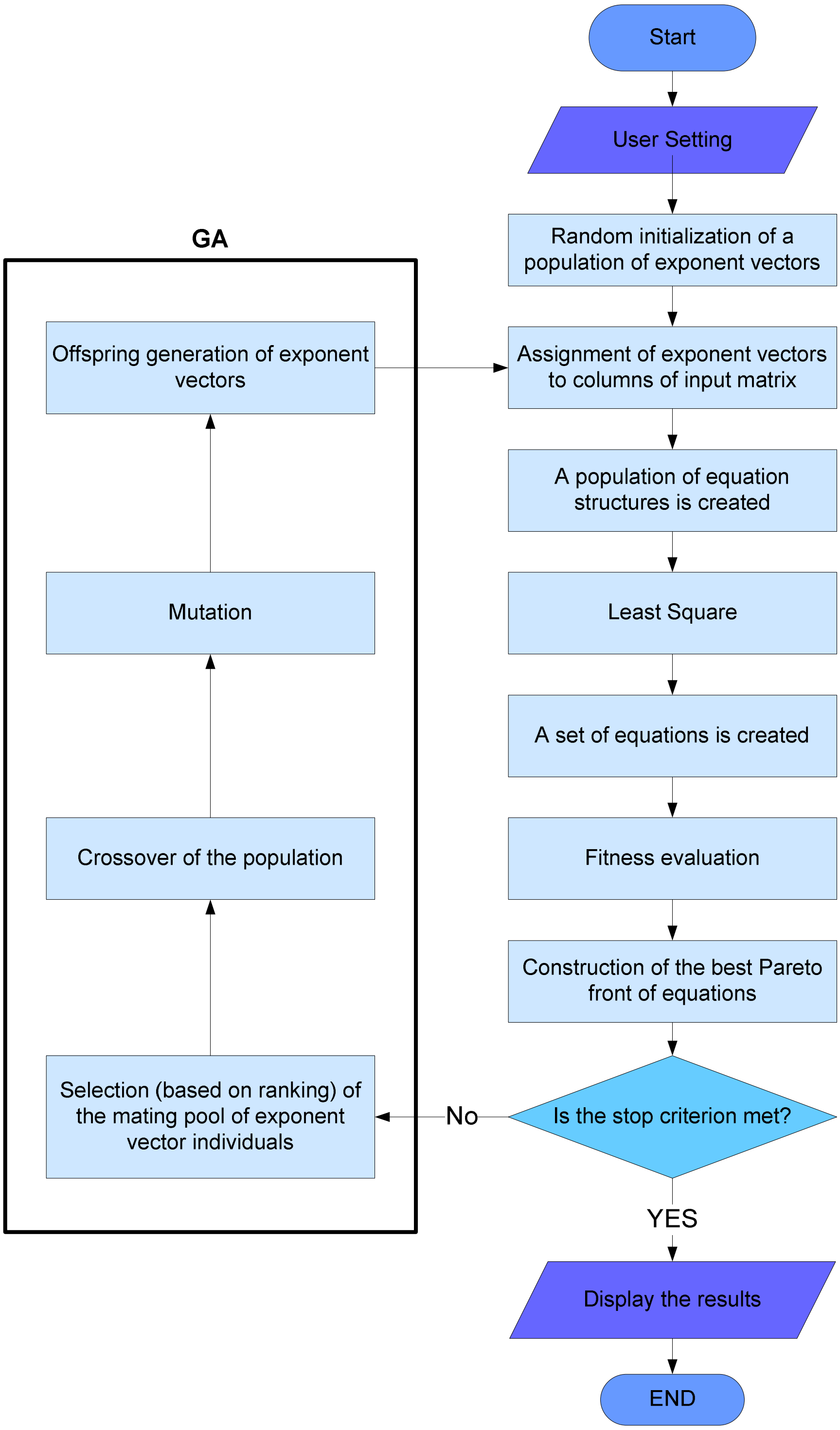
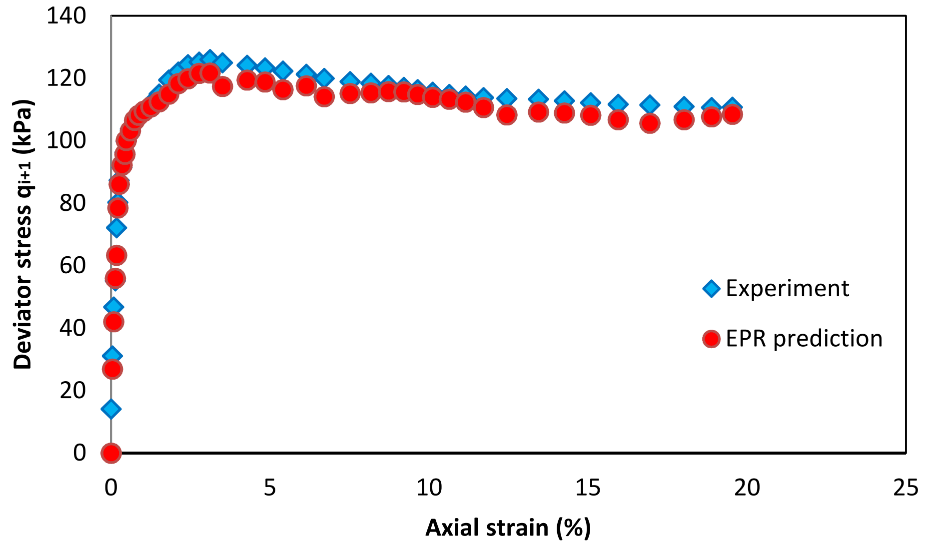


Figure 1: Flow diagram for representing the evolutionary polynomial regression procedure



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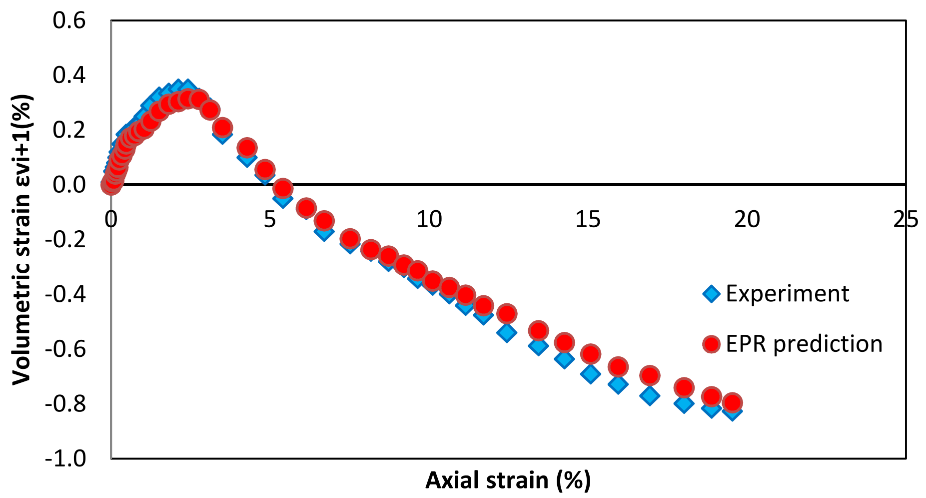
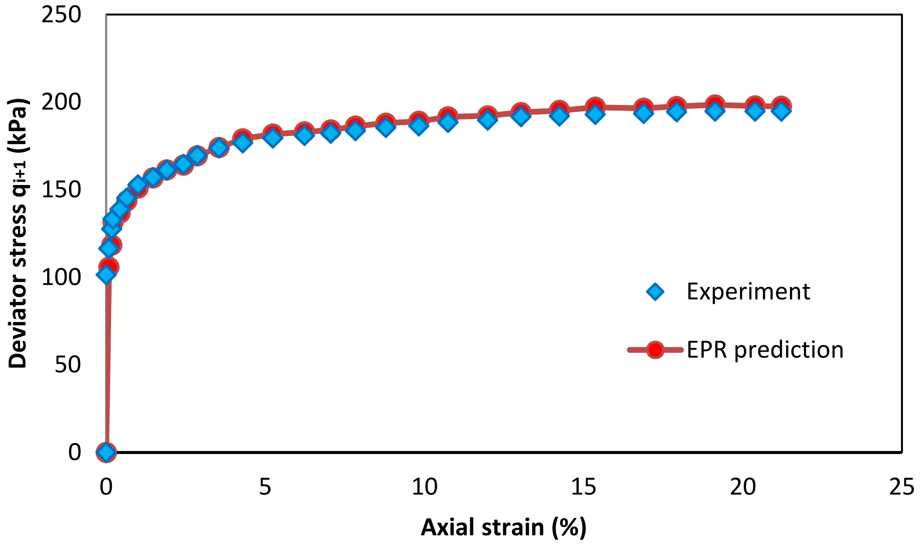


(b)

Figure 2: Comparison between the EPR model predictions with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=4, Mean net stress=50 kPa, T=25°C)



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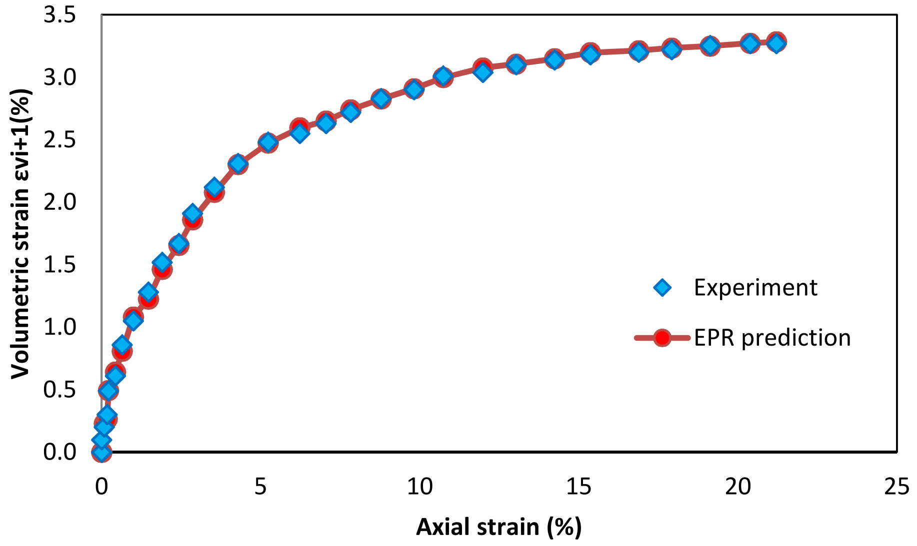

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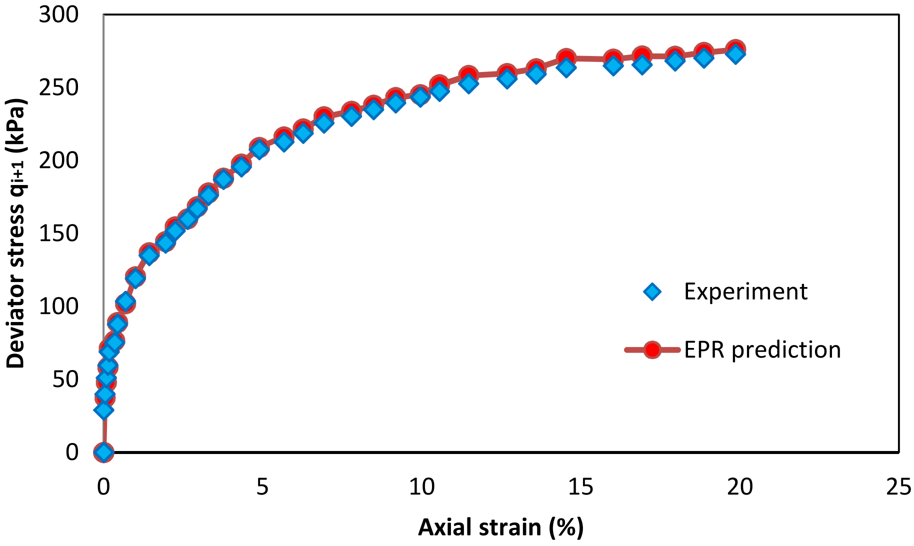
Figure 3: Comparison between the EPR model predictions with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=2, Mean net stress=100 kPa, T=40°C)



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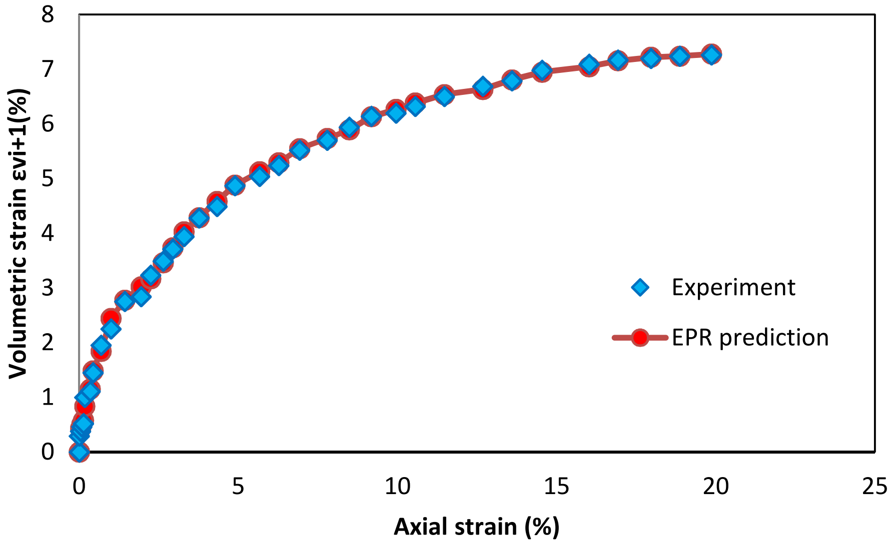
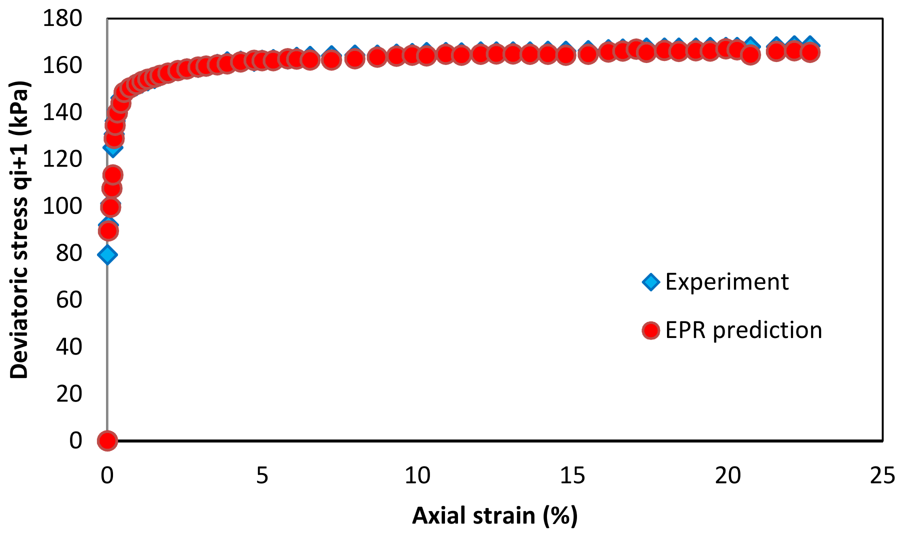


(b)

Figure 4: Comparison between the EPR model predictions with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=1.33, Mean net stress=150 kPa, T=60°C)



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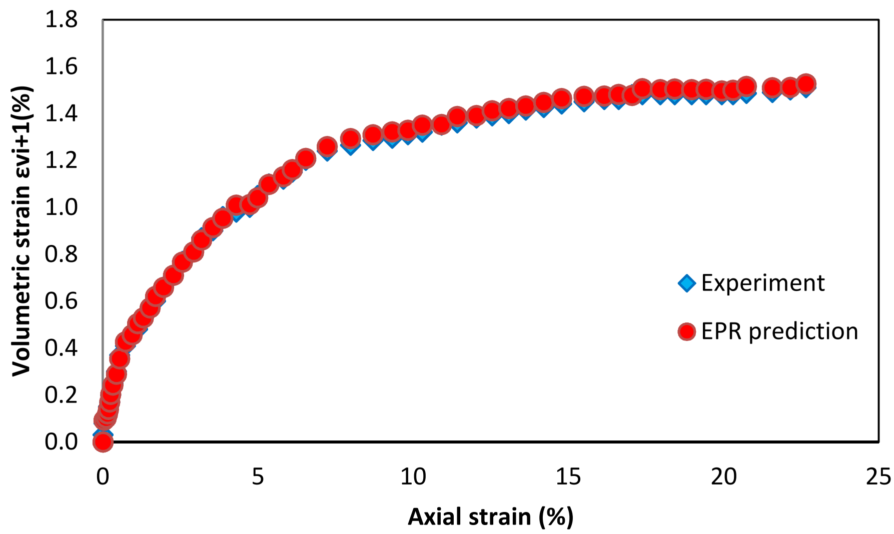

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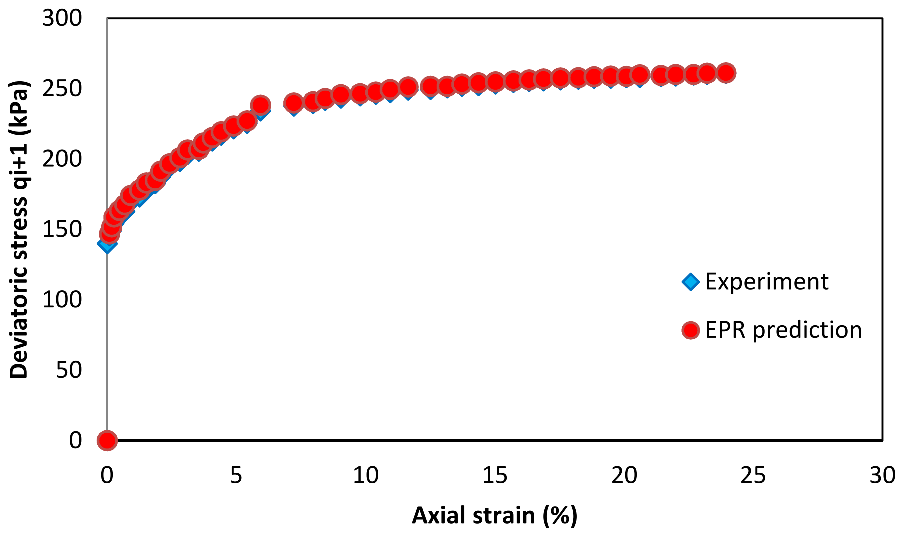
Figure 5: Comparison between the EPR model validation predictions with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=4, Mean net stress=50 kPa, T=40°C)



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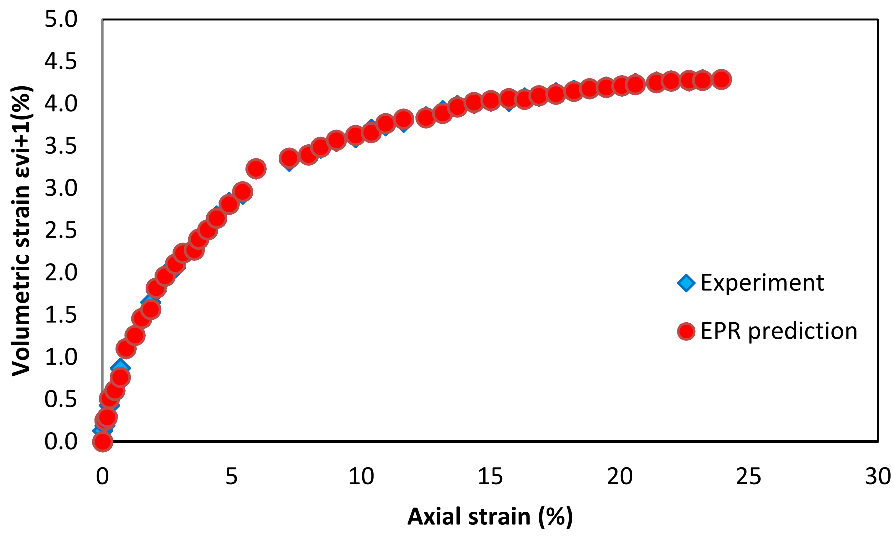

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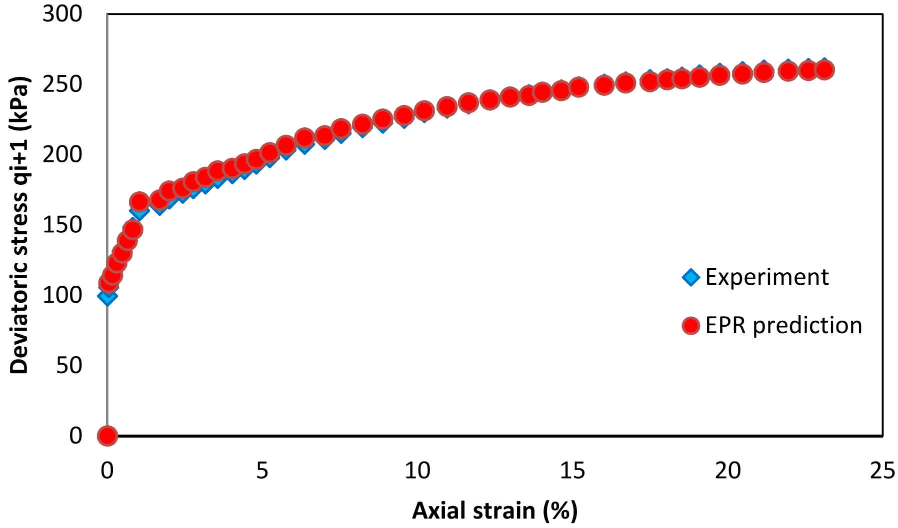
Figure 6: Comparison between the EPR model validation predictions with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=2, Mean net stress=100 kPa, T=25°C)



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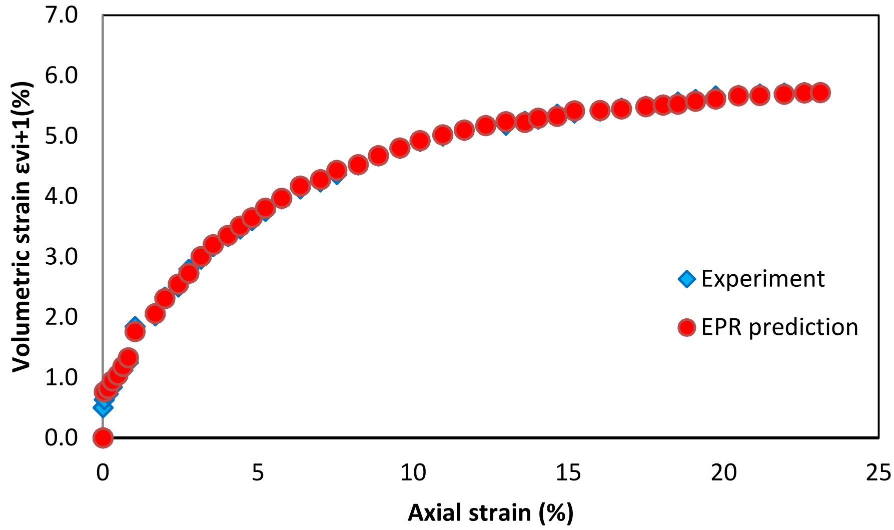


(b)

Figure 7: Comparison between the EPR model validation predictions with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=2, Mean net stress=100 kPa, T=60°C)

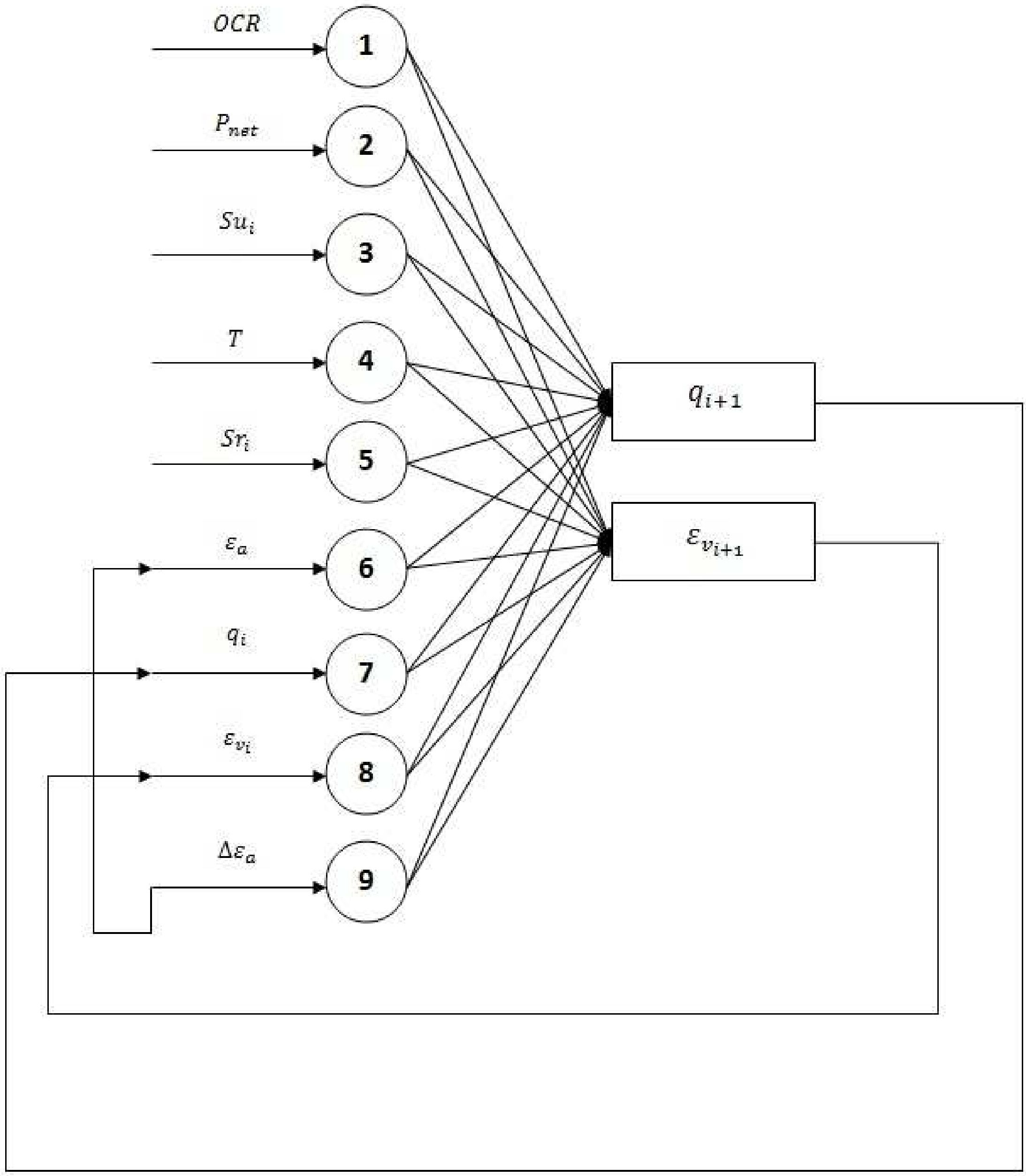
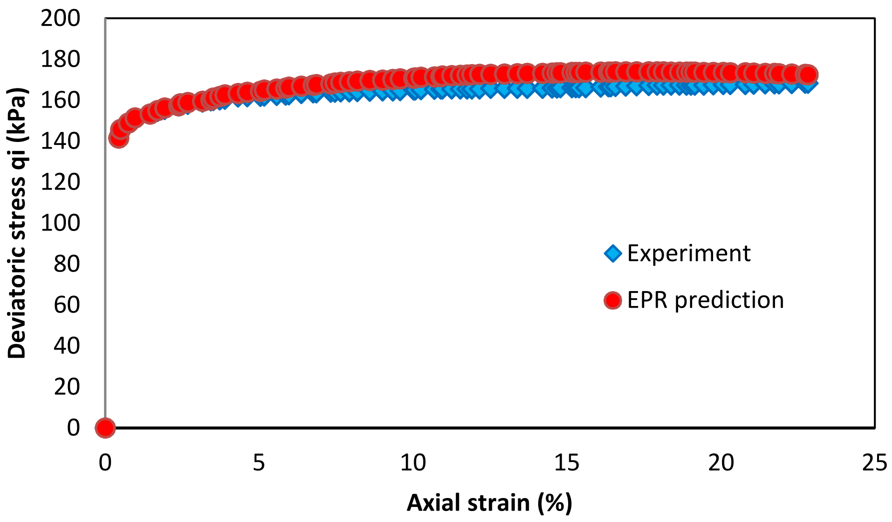


Figure 8: Incremental procedure for predicting the entire stress path



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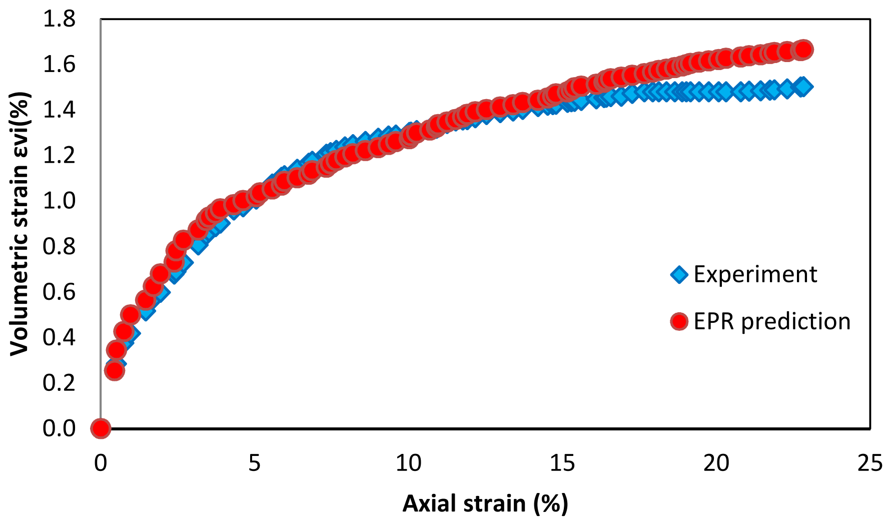
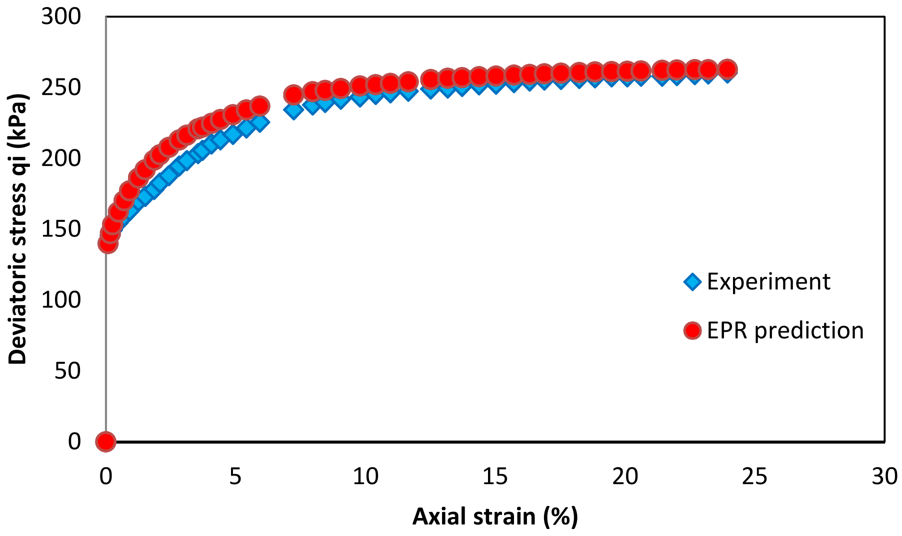


(b)

Figure 9: Comparison between the EPR model predictions (point-by-point predictions of entire stress paths) with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=4, Mean net stress=50 kPa, T=40°C)



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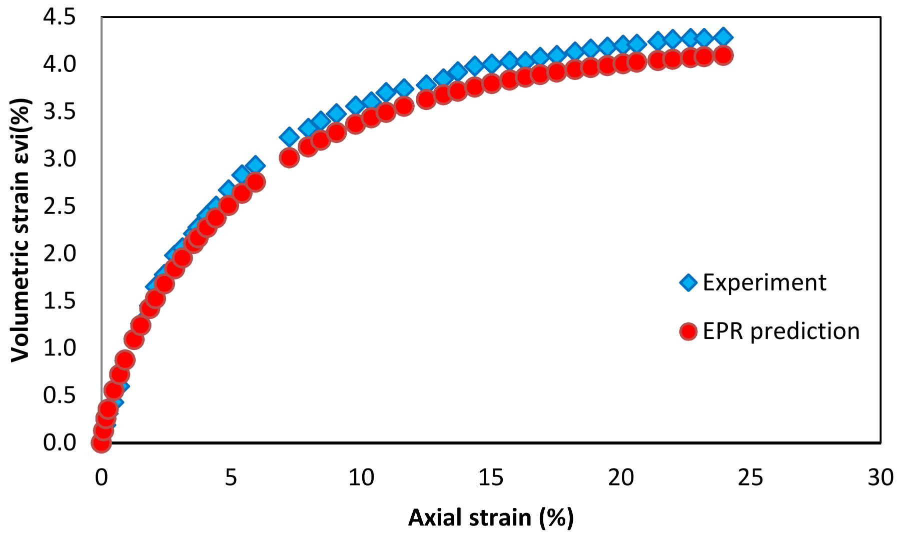
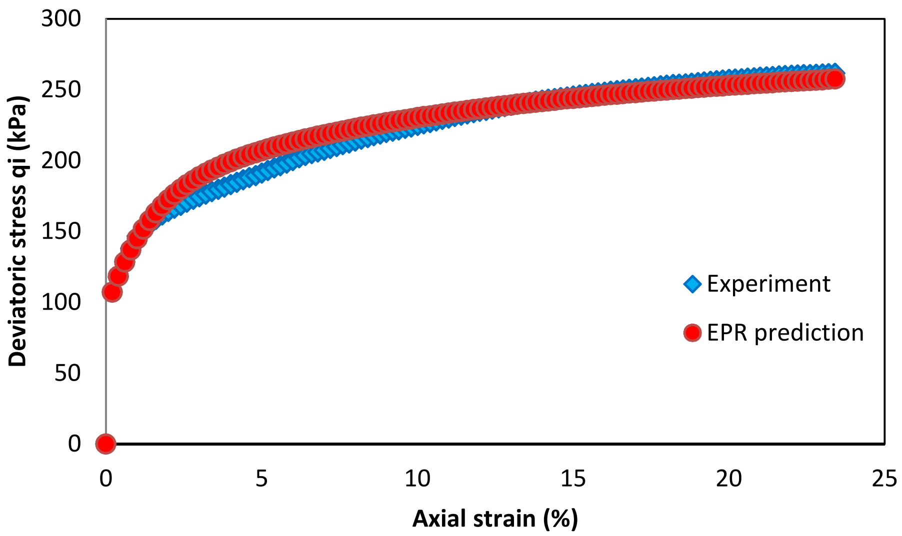


(b)

Figure 10: Comparison between the EPR model predictions (point-by-point predictions of entire stress paths) with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=2, Mean net stress=100 kPa, T=25°C)



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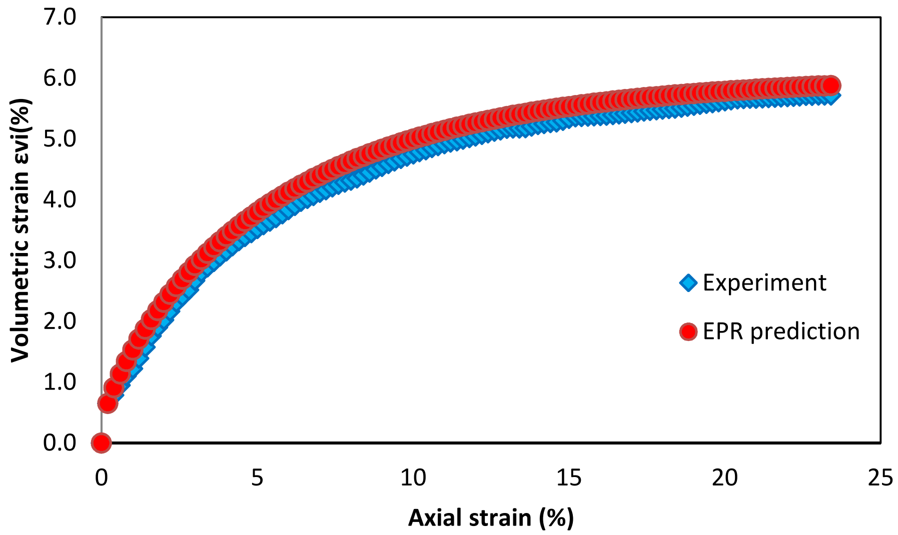
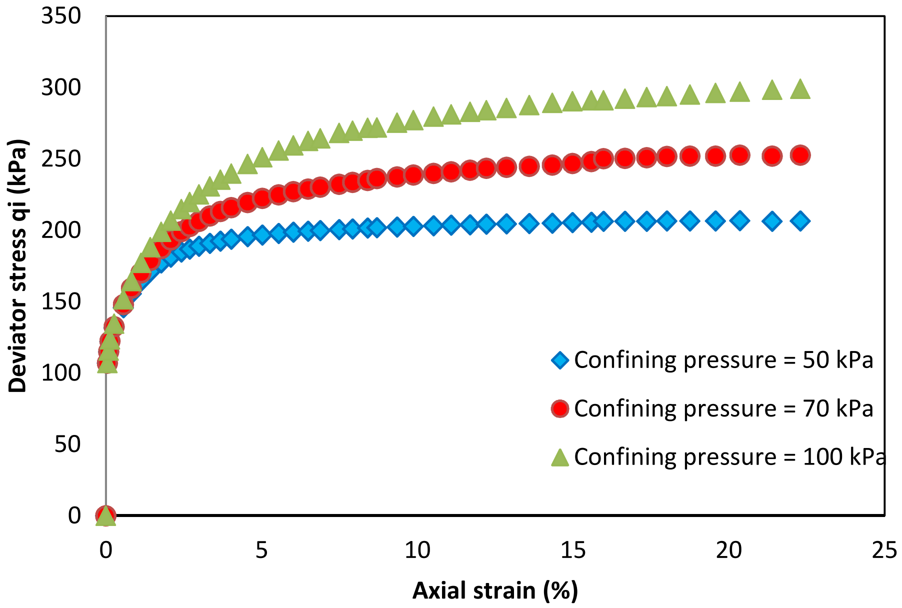


(a)

Figure 11: Comparison between the EPR model predictions (point-by-point predictions of entire stress paths) with experimental data for deviator stress (a) and volumetric strain (b) – (OCR=2, Mean net stress=100 kPa, T=60°C)



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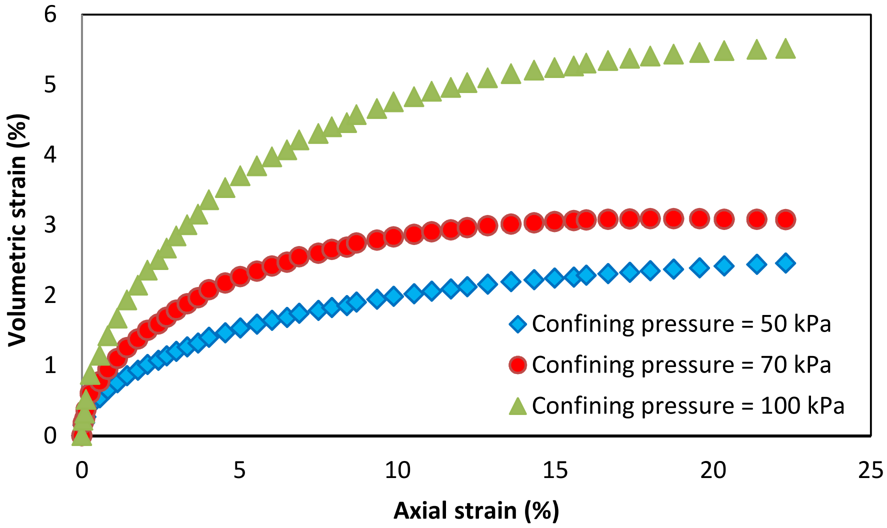
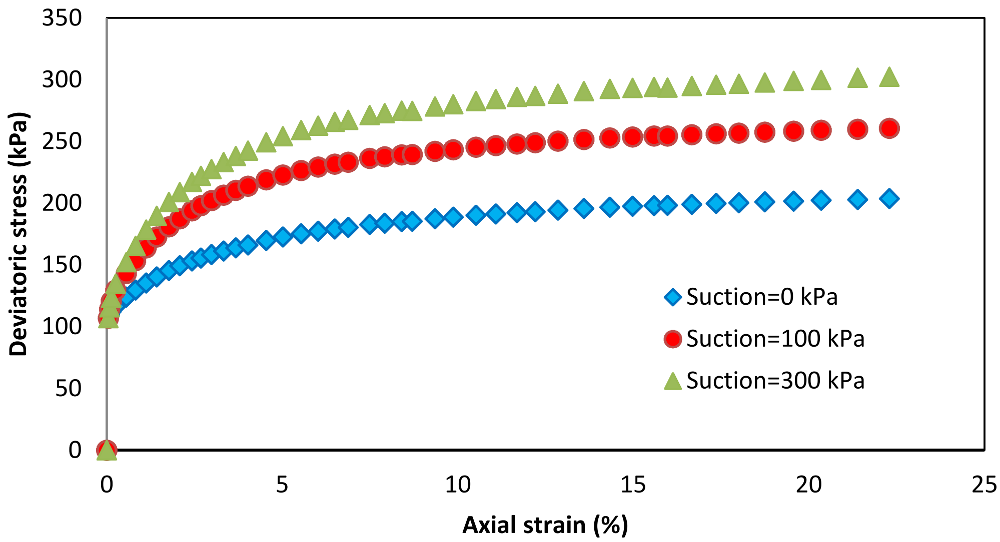


(b)

Figure 12: Effect of changes in confining pressure on (a) deviatoric stress and (b) volumetric strain model predictions



(a)

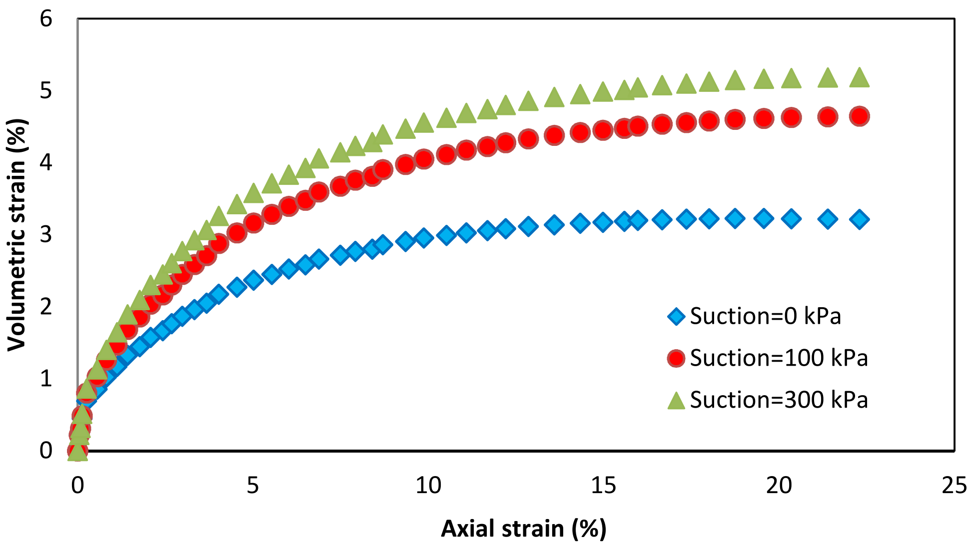


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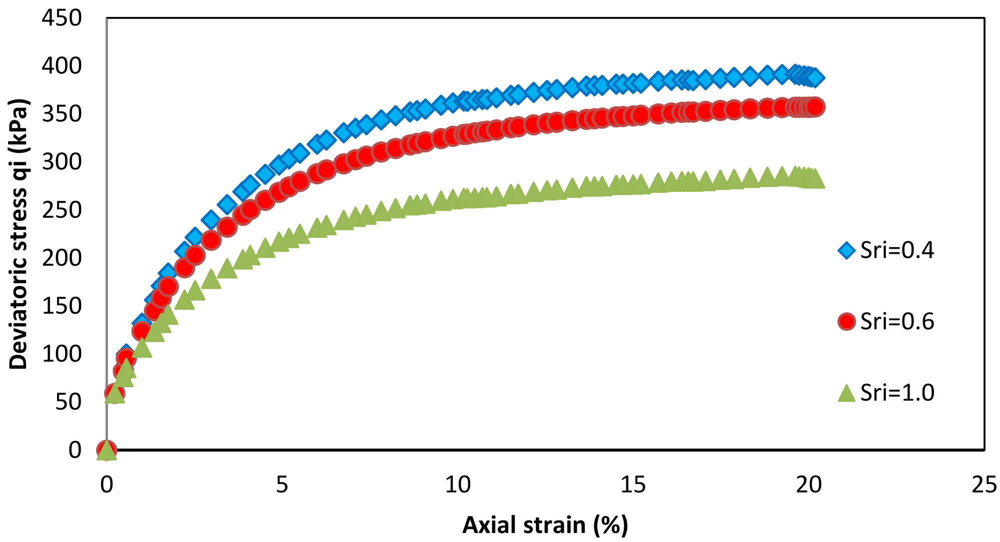
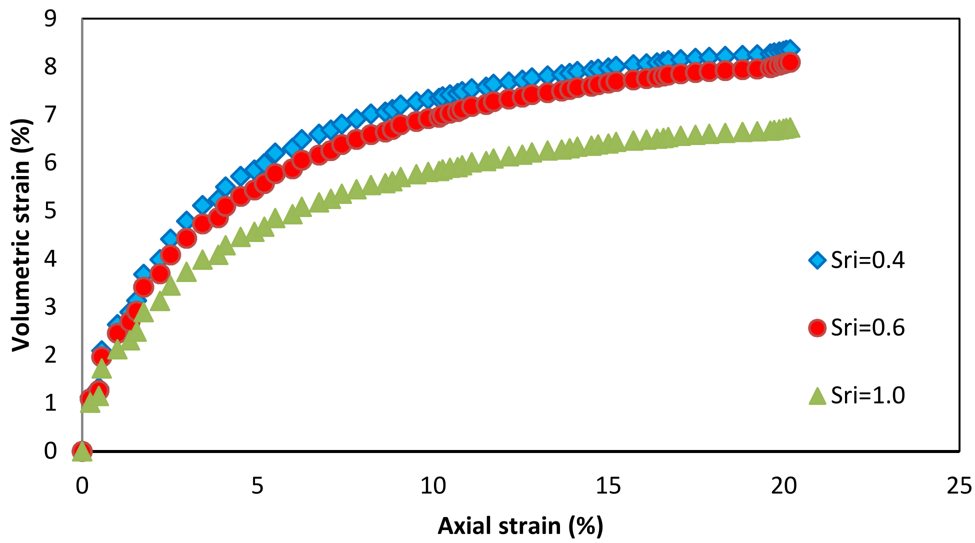
Figure 13: Effect of changes in suction on (a) deviatoric stress and (b) volumetric strain model predictions

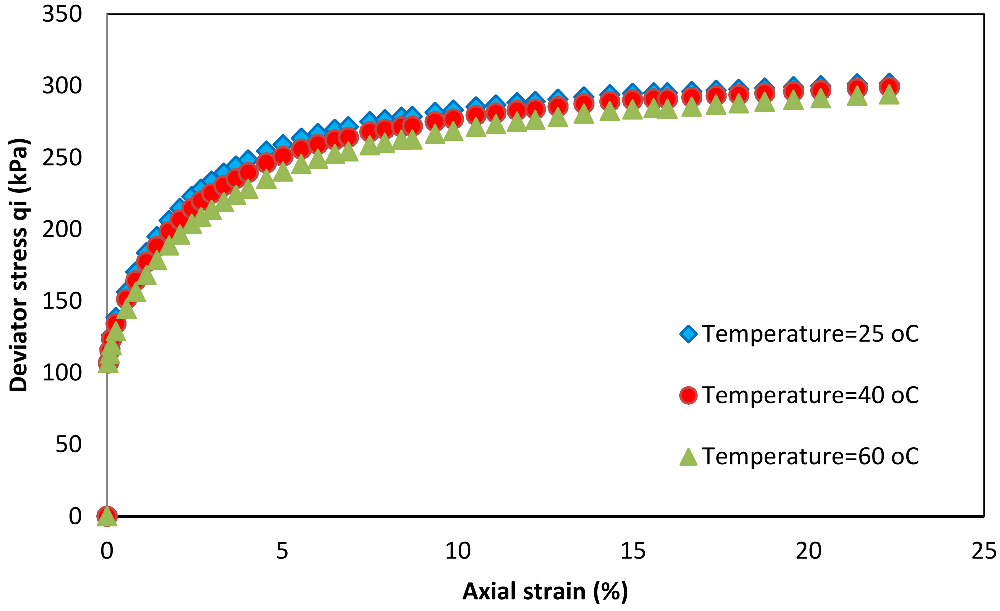


(a)

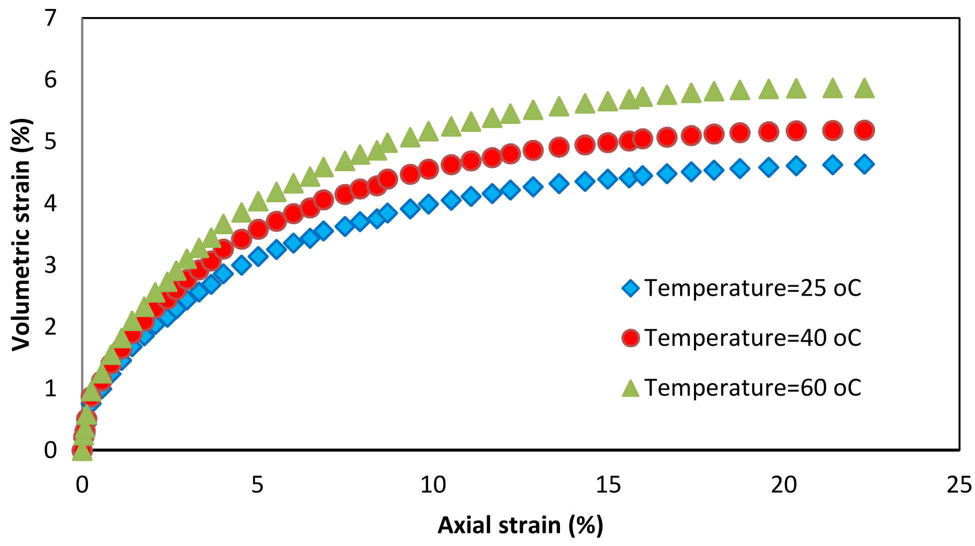


(b)

Figure 14: Effect of changes in degree of saturation on (a) deviatoric stress and (b) volumetric strain model predictions



(a)



(b)

Figure 15: Effect of changes in temperature on (a) deviatoric stress and (b) volumetric strain model predictions