Lateral load bearing capacity modelling of piles in cohesive soils in undrained conditions; an intelligent evolutionary approach

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Abstract

The complex behaviour of fine-grained materials in relation with structural elements has received noticeable attention from geotechnical engineers and designers in recent decades. In this research work an evolutionary approach is presented to create a structured polynomial model for predicting the undrained lateral load bearing capacity of piles. The proposed evolutionary polynomial regression (EPR) technique is an evolutionary data mining methodology that generates a transparent and structured representation of the behaviour of a system directly from raw data. It can operate on large quantities of data in order to capture nonlinear and complex relationships between contributing variables. The developed model allows the user to gain a clear insight into the behaviour of the system. Field measurement data from literature was used to develop the proposed EPR model. Comparison of the proposed model predictions with the results from two empirical models currently being implemented in design works, a neural network-based model from literature and also the field data shows that the EPR model is capable of capturing, predicting and generalising predictions to unseen data cases, for lateral load bearing capacity of piles with very high accuracy. A sensitivity analysis was conducted to evaluate the effect of individual contributing parameters and their contribution to the predictions made by the proposed model. The merits and advantages of the proposed methodology are also discussed.

Introduction

Deep foundations are used as an effective way of avoiding lower quality soils or transferring large loads to the soil lying underneath the structures. Analysis and design of deep foundations under various loading conditions is widely investigated by researchers in the past few decades. Some research contributions have revealed that solving equations of static equilibrium can be an effective way of designing axially loaded piles, whereas, design of laterally loaded piles will only be possible by solving nonlinear differential equations. Poulos and Davis (1980) implemented a methodology based on elasticity, by adopting a previously developed soil model, to analyse the behaviour of piles. However, their proposed approach was not suitable for the nonlinear analysis of behaviour of soil and pile systems. The analysis of nonlinear soil behaviour has been conducted by Matlock and Reese (1962) and Portugal and Seco e Pinto (1993). Portugal and Seco e Pinto (1993) also utilized the finite element method for numerically predicting the behaviour of laterally loaded piles. This methodology is widely used in analysis and design of deep foundations despite the presence of uncertainties in such predictions due to the variability of soil properties. Semi-empirical methods were also suggested for analysis and design of laterally loaded piles and for predicting their load bearing capacity (e.g., Meyerhof (1976)).

In recent years, artificial neural network (ANN) models have been proposed as alternates to experimental and empirical approaches ((Shahin, et al., 2002); (Guyon & Elisseeff, 2003); (Ozesmi & Ozesmi, 1999)). Goh (1995a) used a back propagation neural network (BPNN) to predict the skin friction of piles in clayey soils. Goh ((1995b); (1996)) showed that artificial neural network models outperform some of the existing empirical models in predicting the ultimate load bearing capacity of timber piles in clay and pre-cast concrete and also steel piles in cohesionless soils. Chan et al (1995) and Teh et al (1997) argues that artificial neural networks have been successful in predicting the static load bearing capacity of piles and their

predations are in agreement with the outcomes of analyses conducted using commercial software CAPWAP (Rausche, et al., 1972). Lee and Lee (1996) utilized neural networks to predict the ultimate bearing capacity of piles based on data simulated using previously suggested models and also in situ pile loading test results. Abu-Kiefa (1998) used a probabilistic neural network model, generalized regression neural network (GRNN), to predict the pile load bearing capacity considering the contributions of the tip and shaft separately and also the total load bearing capacity of piles driven into cohesionless soils. Nawari et al (1999) used neural networks for predicting the axial load bearing capacity of steel piles (including the ones with H cross sectional shape) and also pre-stressed and reinforced concrete piles using both back propagation and generalized regression neural networks. The same authors also predicted the settlement of the top of the drill shaft due to lateral loading of piles with similar methodology based on data from in-situ tests.

Artificial neural networks have mostly been used to predict the vertical load bearing capacity of piles and their performance is usually measured based on the coefficient of correlation (R). Coefficient of correlation is commonly used amongst researchers; however, it is difficult to judge, based on this method, whether the developed model is over-predicting or underpredicting the actual values. As a result, Briaud and Tucker (1988) have strongly emphasized that other statistical criteria should also be implemented along with the coefficient of correlation to evaluate the quality of the predictions of the ANN models created for pile load bearing capacity. To address this issue, Abu-Farsakh (2004) used statistical parameters, mean and standard deviation, calculated for the ratio of predicted pile capacity (Q_p) over the measured pile capacity (Q_m) to evaluate the quality of the predictions of the model.

Das and Basudhar (2006) also suggested an artificial neural network model for predicting lateral load capacity of piles and used similar procedures suggested by Abu-Farsakh (2004) to evaluate their presented model.

The results of previous works have shown that artificial neural network offers great capabilities and advantages in modelling the behaviour of materials and systems. However, it is generally accepted that ANNs also suffer from a number of shortcomings. One of the main shortcomings of the neural network based approach is that the optimum structure of the neural network (e.g., the number of input layers, hidden layers and transfer functions) needs to be identified a priori through a time consuming trial and error procedure. Another main drawback of the neural network approach is the large complexity of the structure of ANN. This is because the neural network stores and represents the knowledge in the form of weights and biases which are not easily accessible to the user. Artificial neural networks are considered as black-box systems as they are unable to explain the underlying principles of prediction and the effect of inputs on the output (Goh, et al., 2005).

A number of investigators have studied the use of connection weights to interpret the contributions of input variables to neural network models ((Wilby, et al., 2003), (Olden & Jackson, 2002), (Olden, et al., 2004)). However, interpretation of weights may still be considered a subject of further research in the future.

In this paper an evolutionary-based data mining approach is proposed to model the bearing capacity of laterally loaded piles in undrained conditions. The evolutionary polynomial regression has been successfully applied to modelling a number of civil engineering materials and systems including torsional strength prediction for reinforced concrete beams (Fiore et al., 2012), stress-strain and volume change behaviour of unsaturated soils (Javadi et al., 2012), stability of soil and rock slopes (Ahangar-Asr et al., 2010), mechanical behaviour of rubber concrete (Ahangar-Asr et al., 2011a) and permeability and compaction characteristics of soil (Ahangar-Asr et al., 2011b). EPR provides a structured and transparent representation of the model in the form of mathematical (polynomial) expressions to describe the complicated behaviour of systems. The proposed methodology overcomes most of the issues

and drawbacks associated with the neural network modelling approach by providing clear insight into the behaviour of the system and the levels of contribution of the influencing parameters in the developed models.

Database

Field measurement data from literature is used to develop and evaluate the proposed EPR model. From among 38 data cases (Rao & Suresh, 1996) 29 cases, representing 80% of the total data, were used to train the EPR model and the remaining cases were kept unseen to EPR during the model development process and were used in the model evaluation stage to examine generalization capabilities of the created model.

Tables 1 and 2 represent the training and testing data sets used in EPR model development and validation stages respectively. The main contributing parameters that affect the lateral load bearing capacity of piles (Q) include the diameter of the pile (D), depth of embedment of the pile in soil (L), eccentricity of load (e) and also undrained shear strength of the soil (Su).

The training and testing data were kept the same as those used in previously developed models (Das & Basudhar, 2006). The purpose was to keep the predictions of the EPR model comparable to the results from those models. However, a statistical analysis was conducted to make sure that the testing data was covered by the ranges of parameter values available in the training data set to prevent extrapolation and to ensure that a statistically consistent combination was used for construction and validation of the EPR model (Ahangar-Asr et al., (2012)).

Evolutionary polynomial regression; methodology and procedure

Evolutionary polynomial regression (EPR) is a data mining technique that integrates numerical and symbolic regression to perform evolutionary polynomial regression. The strategy uses polynomial structures to take advantage of their favourable mathematical properties. The key idea behind the EPR is to use evolutionary search for exponents of polynomial expressions by means of a genetic algorithm (GA) engine. This allows (*i*) easy computational implementation of the algorithm, (*ii*) efficient search for an explicit expression, and (*iii*) improved control of the complexity of the expression generated (Giustolisi & Savic, 2006). EPR is a data-driven method based on evolutionary computing, aimed to search for polynomial structures representing a system. A physical system, having an output y, dependent on a set of inputs X and parameters θ , can be mathematically formulated as:

$$y = F(\mathbf{X}, \mathbf{\theta}) \tag{1}$$

where F is a function in an m-dimensional space and m is the number of inputs. To avoid the problem of mathematical expressions growing rapidly in length with time in EPR the evolutionary procedure is conducted in the way that it searches for the exponents of a polynomial function with a fixed maximum number of terms. During one execution it returns a number of expressions with increasing numbers of terms up to a limit set by the user to allow the optimum number of terms to be selected. The general form of expression used in EPR can be presented as (Giustolisi and Savic, (2006)):

$$y = \sum_{j=1}^{m} F(\mathbf{X}, f(\mathbf{X}), a_j) + a_0$$
(2)

where y is the estimated vector of output of the process; a_j is a constant; F is a function constructed by the process; X is the matrix of input variables; f is a function defined by the user; and m is the number of terms of the target expression. The first step in identification of the model structure is to transfer equation 2 into the following vector form:

$$Y_{N\times 1}(\theta, Z) = \begin{bmatrix} I_{N\times 1} & Z_{N\times m}^{j} \end{bmatrix} \times \begin{bmatrix} a_{0} & a_{1} & \dots & a_{m} \end{bmatrix}^{T} = Z_{N\times d} \times \theta_{d\times 1}^{T}$$
(3)

where $Y_{N\times 1}(\theta, Z)$ is the least squares estimate vector of the N target values; $\theta_{d \times 1}$ is the vector of d=m+1 parameters a_j and a_0 (θ^T is the transposed vector); and $Z_{N\times d}$ is a matrix formed by I (unitary vector) for bias a_0 , and m vectors of variables Z_j . For a fixed j, the variables Z_j are a product of the independent predictor vectors of inputs, $X = \langle X_1 X_2 \dots X_k \rangle$.

In general, EPR is a two-stage technique for constructing symbolic models. Initially, using standard genetic algorithm (GA), it searches for the best form of the function structure, i.e. a combination of vectors of independent inputs, $X_s=1$:k, and secondly it performs a least squares regression to find the adjustable parameters, θ , for each combination of inputs. In this way a global search algorithm is implemented for both the best set of input combinations and related exponents simultaneously, according to the user-defined cost function (Giustolisi & Savic, 2006). The adjustable parameters, a_j , are evaluated by means of the linear least squares (LS) method based on minimization of the sum of squared errors (SSE) as the cost function. The SSE function, which is used to guide the search process towards the best fit model, is:

$$\mathbf{SSE} = \frac{\sum_{i=1}^{N} (y_a - y_p)^2}{N}$$
(4)

where y_a and y_p are the target experimental and the model prediction values respectively. The global search for the best form of the EPR equation is performed by means of a standard GA over the values in the user defined vector of exponents. The GA operates based on Darwinian evolution which begins with random creation of an initial population of solutions. Each parameter set in the population represents chromosomes of the individual's. Each individual is assigned a fitness based on how well it performs in its environment. Through crossover and mutation operations, with the probabilities P_c and P_m respectively, the next generation is created. Fit individuals are selected for mating, whereas weak individuals die off. The mated parents create a child (offspring) with a chromosome set which is a mix of parents' chromosomes. In EPR integer GA coding with single point crossover is used to determine the location of the candidate exponents (Giustolisi and Savic, (2006); Doglioni, (2004)).

The EPR process stops when the termination criterion, which can be either the maximum number of generations, the maximum number of terms in the target mathematical expression or a particular allowable error, is satisfied. A typical flow diagram for the EPR procedure is illustrated in figure 1.

Before starting the evolutionary procedure, a number of constraints can be implemented to control the structure of the models to be constructed, in terms of length of the equations, type of functions used, number of terms, range of exponents, number of generations etc. It can be seen that there is a potential to achieve different models for a particular problem which enables the user to gain additional information (Javadi & Rezania, 2009). By applying the EPR procedure, the evolutionary process starts from a constant mean of output values and as the number of evolutions increases EPR gradually picks up the different participating parameters in order to form equations representing the relationship between contributing and output parameters. Each model is trained and validated using the training and validation stage (during the training process). The level of accuracy at each round of the modelling process is evaluated based on the value of the coefficient of determination (COD) i.e. the fitness function which is defined as:

$$\mathbf{COD} = 1 - \frac{\sum_{N} (\mathbf{Y}_{a} - \mathbf{Y}_{p})^{2}}{\sum_{N} \left(\mathbf{Y}_{a} - \frac{1}{N} \sum_{N} \mathbf{Y}_{a}\right)^{2}}$$
(5)

where Y_a is the actual output value; Y_p is the EPR predicted value and N is the number of data points on which the COD is computed. If the model fitness is not acceptable or the other termination criteria (in terms of maximum number of generations and maximum number of terms) are not satisfied, the current model should go through another evolution in order to obtain a new model.

As discussed in database section above, the data was divided into training and testing sets to be used for training of EPR to develop the desired model and also for validation of the created model and to appraise its generalization capabilities.

The proposed EPR model for prediction of the lateral load bearing capacity of piles (Equation 6) was chosen from among 15 equations developed after the training stage of the EPR modelling process was completed. Some of the developed models did not include all of the considered contributing parameter which were are known to play a significant role in the load bearing capacity of piles (Rao and Suresh, (1996)) and hence had to be removed from the model selection pool. The criteria considered in choosing the final equation from among the remaining equations included: *(i)* The value of coefficient of determination (COD), to ensure that the developed model had the highest possible fitness level; *(ii)* Complexity of the model, to ensure that the selected equation had the least possible number of terms to minimize complexity; and also *(iii)* Sensitivity analysis, so that the suggested model reflected the correct trends, in line with the physical understanding of the problem, in terms of the way each contributing parameter affects the predictions (discussed in detail in the sensitivity analysis section).

$$Q = -\frac{896.56}{D} + \frac{0.14e^3.S_u^3 + 491.87S_u.L^2 - 3.94 \times 10^{-4}(D.L.e.S_u)^2 + 7.28 \times 10^{-4}D^3.e(L.S_u)^2}{L^3} + 45.22$$
(6)

After training, the performance of the trained EPR model was examined using the validation dataset which had not been introduced to EPR during training. Figures 2 and 3 compare the predicted values of the lateral load bearing capacity with the actual field measurement data

used for training and validation stages respectively. The figures show a very good correlation between the predictions of the EPR model and the actual data both for modelling and validation datasets.

In order to further investigate into the capabilities of the developed EPR model, a comparison was made between the model predictions and the predictions of the empirical models proposed by Hansen and Broms (Rao & Suresh, 1996), and also the artificial neural network model presented by Das and Basudhar (2006). Table 3 represents the values of coefficient of determination for all considered models. Figure 4 also shows the comparison of the results between the four model predictions against field measurements. It can be easily seen that the proposed EPR model outperforms the empirical models and provides similar (and in some cases better) predictions to those of the artificial neural network model proposed by Das and Basudhar (2006).

Considering the fact that the design criteria are dictated by codes of practice which are developed based on the specific considerations and regulations in individual countries or regions around the world, the developed model is presented in the way that ensures generality. In other words, the training and testing stages of EPR modelling procedure was completed based on using raw data rather any data affected by any specific code of practice. Therefore, the user will have the capability and choice to apply the proposed model to any design problem considering appropriate recommendations from the code of practice pertaining to the relevant country or region. A similar approach was taken by previous researches that used EPR, ANNs or any other intelligent and/or evolutionary modelling techniques (Faramarzi et al, 2014; Rezania et al., 2011; Ghaboussi et al., 1998; Rao and Suresh, 1996).

Quality of predictions of the proposed model

The statistical parameters including mean and standard deviation for the ratio of the predicted lateral load bearing capacity (Q_p) over field measurement values (Q_m) were calculated to further evaluate the accuracy of the proposed prediction model. In ideal conditions, an accurate and precise model will provide the statistical mean value of unity and the standard deviation of zero. In practice, if this mean value is greater than one, it will be an indication that the model being studied is over-predicting the real conditions and if the statistical mean value is smaller than one, the model will be representing under-predicted results (Abu-Farsakh, 2004).

Table 4 shows the statistical mean and standard deviation values of the ratio of predicted over measured load bearing capacity (Q_p/Q_m) for empirical, artificial neural network and also the proposed EPR models. It can be seen that the empirical method suggested by Broms and also the ANN model over-predict the lateral loading capacity while the model by Hansen provides very large under-predictions. The proposed EPR model also shows very slight under prediction in the same level of the over-prediction of the ANN model. ANN and EPR models are offering almost equal diversions from the actual measurements, however, in different directions. From practical point of view, the use of the EPR model would lead to slightly safer designs.

Cumulative probability was also considered for the predicted over measured load bearing capacity (Q_p/Q_m) to evaluate and compare the performances of the four different models presented in this paper. The values of Q_p/Q_m were arranged from the smallest to the largest and the cumulative probability was calculated using the following equation (Abu-Farsakh, 2004):

$$P = \frac{i}{i+1} \tag{7}$$

where *i* is the order number given from the smallest to the largest of the arranged values of the predicted over measured load bearing capacity ratio (Q_p/Q_m) and *n* is the number of data points. The computed Q_p/Q_m values corresponding to 50% cumulative probability (P_{50}) were considered; less than unity represents under-perdition whilst the values greater that 1 is associated with over-prediction. Best models will be the ones with P_{50} values closest to the unity.

The variation in the ratio (Q_p/Q_m) for all cases is also reflected in 90% cumulative probability (P_{90}) . The model with closest value of P_{90} to unity is considered to be a better model (Abu-Farsakh, 2004). Figure 5 represents the cumulative probability values against the predicted over measured load bearing capacity ratios for all four models considered in this research. Table 5 also shows the P_{50} and P_{90} values for Broms, artificial neural network (Das & Basudhar, 2006), EPR and Hansen models with ANN and EPR models being the closest models to the real conditions followed by the Broms model. The model of Hansen, however, seems to be providing predictions, although with a large safety margin, but far away from actual field measurements.

Sensitivity analysis and discussion

A parametric study was carried out to evaluate the response of the developed model to changes in the contributing input parameters. This was done through a basic approach to sensitivity analysis by fixing all but one input variable to their mean values and varying the remaining one within the range of its maximum and minimum values. The sensitivity analysis was repeated for every contributing parameter with the aim of providing a better understanding of the contribution of individual parameters to the proposed EPR model predictions.

Figures 6 to 9 represent the results of the sensitivity analysis for pile diameter, pile embedded length, eccentricity of loading and undrained shear strength of soil respectively. The diameter

of the pile appears to be the most effective parameter in the lateral load bearing capacity of piles (Figure 6). As expected, increasing the diameter, which would mean a pile with larger perimeter and base areas in contact with the surrounding clay (and hence greater skin resistance and base resistance) would result in higher bearing capacities. Figure 7 shows that pile embedded length is the second most effective parameter in the EPR model. It is correctly shown that an increase in the embedded length of pile, which would again mean greater contact area with clay and greater skin resistance, would improve the lateral load bearing capacity. Figure 9 shows that for a given soil-pile contact surface (i.e. constant diameter and embedded length), any increase in undrained shear strength of the soil would result in higher lateral load bearing capacity; however, according to figure 8, increasing eccentricity of the load would decrease the load bearing capacity, which is also consistent with the expected behaviour of piles under eccentric loading conditions.

Summary and conclusions

Deep foundations can be considered as important structures that are vastly implemented to support heavy structures. Piles are capable of transferring large loads to deeper and stronger layers of soil or rock and also can play the role of reinforcing elements for soils. In some specific but very commonly used cases, like foundations of bridges, transmission towers, offshore structures and other types of large structures, piles are also subjected to lateral loads. Lateral load resistance of piles becomes also extremely important in design of structures that are subject to loading from earthquakes, soil movement or waves.

In this paper, a new approach was presented to develop an evolutionary-based model for predicting lateral load bearing capacity of piles. An EPR model was developed and validated using a field measurement database from literature, created based on tests on model piles. The model prediction results were compared with those of two empirical models and a neural network model as well as the actual measured data. A parametric study was conducted to evaluate the effect of the contributing parameters on the predictions of the proposed EPR model. Comparison of the results showed that the developed EPR model provides very accurate predictions for lateral load bearing capacity of piles. The developed model presents a structured and transparent representation allowing a physical interpretation of the problem that gives the user an insight into the relationship between the lateral load bearing capacity and its various contributing parameters. Sensitivity analysis results also revealed correct relations between contributing parameter.

Analysis of statistical mean and standard deviation, along with cumulative probability function were also utilized to investigate the quality of predictions made by the proposed model. The results clearly showed the robustness of the developed model in providing accurate prediction of lateral load capacity of pile foundations. From practical point of view, the EPR model presented in this paper can be easily implemented into real world problems as it provides more accurate results than existing empirical models that are currently used in routine deep foundation design.

In EPR approach, no pre-processing of data is required and there is no need for normalization or scaling. It is also possible to get more than one model for complex systems. The best model can then be chosen on the basis of its performance on validation set of data that has been kept unseen to the EPR model in the training phase. Predictions made by EPR models based on this data can be used as an unbiased performance indicator of generalization capabilities of the proposed model. Another important advantage of the EPR approach is that as more data becomes available, the quality of the predictions can be easily improved by retraining the EPR model using the new, more comprehensive set of data.

The results presented in this research work showed the robustness of the proposed EPR approach in modelling lateral load bearing capacity of piles in clays in undrained conditions. It was also shown that the developed model is capable of providing a more clear view of the

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lateral load bearing capacity of piles by giving the user a better understanding of the relationships between its contributing parameters and the bearing capacity. The proposed model outperformed the empirical models and also showed equally good and in some cases better performance than the artificial neural network model. As the EPR model provides a structured and transparent representation of the pile lateral load capacity behaviour, it offers a clear advantage to the black box ANN models.

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Diameter	Embedded length	Eccentricity	Undrained shear strength	Lateral load bearing capacity	
<i>D</i> (mm)	<i>L</i> (mm)	<i>e</i> (mm)	<i>Su</i> (kN/m²)	<i>Q_m</i> (N)	
6.35	146.1	19.1	38.8	69.5	
13	260	0	24	225	
12.5	130	0	24	106	
13.5	300	50	3.4	30	
13.5	300	50	4	36	
13.5	300	50	5.5	50	
13.5	300	50	7.2	64	
18	300	50	10	89	
18	300	50	3.4	3	
20.4	300	50	4	46	
12.3	300	50	5.5	44	
18.4	300	50	4	51	
18	300	50	10	116.5	
33.3	300	50	3.4	78.5	
33.3	300	50	5.5	110.5	
12.3	300	50	3.4	29.5	
6.35	139.7	25.4	38.8	65.5	
12.3	300	50	7.2	58	
12.3	300	50	10	81	
18.4	300	50	5.5	65.5	
18.4	300	50	7.2	86.5	
18.4	300	50	10	114	
20.4	300	50	5.5	59.5	
20.4	300	50	7.2	76.5	
20.4	300	50	10	87	
25.4	300	50	7.2	90	
25.4	300	50	10	151.6	
25.4	300	50	3.4	50	
25.4	300	50	5.5	75	

Table 2: Field measurement data for lateral load capacity of piles and contributing parameters (Validation data set)

Diameter	Embedded length	Eccentricity	Undrained shear strength	Lateral load bearing capacity
<i>D</i> (mm)	<i>L</i> (mm)	<i>e</i> (mm)	Su (kN/m²)	<i>Q_m</i> (N)
13.5	190	0	24	128
20.4	300	50	3.4	38
18.4	300	50	3.4	42.5
25.4	300	50	4	58
13	132	33.8	38.8	53
18	300	50	4	49
18	300	50	5.5	65
18	300	50	7.2	87
12.3	300	50	4	35

	1			
Model	COD (%)	COD (%)	COD (%)	
	Training data	Validation data	Empirical models	
Hansen	N/A	N/A	20.21	
Broms	N/A	N/A	63.22	
Artificial neural network	87.09	87.41	N/A	
Evolutionary polynomial regression (EPR)	92.07	87.99	N/A	

Table 3: COD values for empirical, ANN and EPR models

Table 4: Statistical mean and standard deviation for the ratio of predicted over measured load bearing capacity ratio

Madal	Statistical mean			Standard deviation		
Widei	Training	Testing	Total	Training	Testing	Total
Hansen	N/A	N/A	0.5789	N/A	N/A	0.1168
Broms	N/A	N/A	1.1500	N/A	N/A	0.1411
Artificial neural network	1.0390	1.001	1.0308	0.2035	0.1998	0.2017
Evolutionary polynomial regression (EPR)	0.9814	0.9269	0.9685	0.1522	0.1405	0.1495

Table 5: Cumulative probability (%), P_{50} and P_{90} values

Model	P 50	P 90
Hansen	0.595	0.835
Broms	1.124	1.381
Artificial neural network	1.005	1.163
Evolutionary polynomial regression (EPR)	0.960	1.111



Figure 1: Flow diagram for representing the evolutionary polynomial regression procedure



Figure 2: EPR predictions against field measurement values for lateral load bearing capacity values (training data)



Figure 3: EPR predictions against field measurement values for lateral load bearing capacity values (testing data)



Figure 4: Model predictions against field measurements of lateral load bearing capacity values for empirical, ANN and EPR models



Figure 5: Predicted over measured load bearing capacity ratio against cumulative porosity for empirical, ANN and EPR models



Figure 6: Sensitivity analysis of the EPR model (effect of pile diameter)



Figure 7: Sensitivity analysis of the EPR model (effect of pile embedded length)



Figure 8: Sensitivity analysis of the EPR model (effect of lateral load eccentricity)



Figure 9: Sensitivity analysis of the EPR model (effect of undrained shear strength of soil)