A fast inverse dynamics model of walking for use in optimisation studies

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Abstract

Computer simulation of human gait, based on measured motion data, is a well-established technique in biomechanics. However, optimisation studies requiring many iterative gait cycle simulations have not yet found widespread application because of their high computational cost. Therefore, a computationally efficient inverse dynamics model of 3D human gait has been designed and compared with an equivalent model, created using a commercial multi-body dynamics package. The fast inverse dynamics model described in this paper led to an eight fold increase in execution speed. Sufficient detail is provided to allow readers to implement the model themselves.

Keywords: Fast inverse dynamics; Gait Simulation; Prediction

Notation

a_i	Acceleration of segment *i*'s origin
a_{Ci}	Acceleration of segment *i*'s centre-of-mass
F_i	Force applied to segment *i* at its proximal end (origin)
F_{grrk}	Component *k* of the (right) ground reaction force (*k*=X, Y or Z)
$\sum F_{gr}$	The total ground reaction force
$\sum F_{grk}$	Component *k* of the total ground reaction force (*k*=X, Y or Z)
I_i	Moment of inertia of segment *i*
m_i	Mass of segment *i*
MF	Moment as a result of a distal force
MFs	Sum of the moments resulting from distal forces
MPF	Moment as a result of the proximal force
$MPFs$	Sum of the moments resulting from proximal forces
n_i	Moment applied to segment *i* at its proximal end (origin)
n_{grrk}	Component *k* of the (right) ground reaction moment (*k*=X, Y or Z)
$\sum n_{gr}$	The total ground reaction moment
$\sum n_{grk}$	Component *k* of the total ground reaction moment (*k*=X, Y or Z)
N_i	Euler's equation for segment *i*

${}^{j}P_{i}$	Position of segment *i* origin in segment *j*’s frame
${}^{j}P_{Ci}$	Position of segment *i* centre-of-mass in segment *j*’s frame
$R_j = {}^{pr}_{d}R$	Joint rotation matrix (maps vectors from distal to proximal segment frames)
α	Joint rotation about X axis
β	Joint rotation about Y axis
γ	Joint rotation about Z axis
ω_j	Angular velocity of joint *j*
$\dot{\omega}_j$	Angular acceleration of joint *j*
${}^{k}\omega_{i/j}$	Angular velocity of segment *i* relative to segment *j*, written in segment *k*’s frame
${}^{k}\dot{\omega}_{i/j}$	Angular acceleration of segment *i* relative to *j*, written in *k*’s frame

<u>Subscripts and superscripts</u>

Note that leading superscripts before a vector indicate the frame in which that vector is written.

d	Distal segment
f	Foot
gr	Ground
h	Head
l	Left

larm	Lower arm
p	Pelvis
pr	Proximal segment
r	Right
sh	Shank
st	Stance
sw	Swing
t	Torso
th	Thigh
uarm	Upper arm

Introduction

Computer simulation of human gait (walking or running), based on measured motion data, is a well-established research technique for estimating the forces acting on the body's joints and muscles. Conversely, optimisation of gait kinematics (known as gait prediction) is a relatively new and challenging area of research, which has not yet found widespread application because of its high computational cost (Anderson & Pandy, 2001; Xiang et al., 2010).

Typically, gait prediction is achieved by embedding a forward or inverse dynamics model of human locomotion within an optimisation framework (henceforth referred to as the optimiser). The optimiser is used to represent the coordination of the body's motions by the central nervous system (CNS) based on the assumption that we have evolved to optimise our gait in order, for example, to minimise energy consumption, maximise speed or minimise pain, depending on the situation. The forward dynamics approach to gait prediction is very computationally demanding, with one of the best known examples of this approach requiring 10,000 hours of CPU time to satisfy the terminal conditions (Anderson & Pandy, 2001). Although this well-known study is now rather dated, based on a review of internet sources we estimate that there has been a 10 to 20 fold increase in computational power over the intervening period. As this type of information is very hard to find and to verify, we conservatively assume a 20 fold increase in computation power. This means that the execution time quoted by Anderson & Pandy would reduce to 500 hours which is still very excessive. For this reason, in our previous work, we have chosen to focus on the inverse dynamics approach to gait prediction (Ren, et al., 2007).

In gait prediction, the joint motions can be represented in many ways and well-known curve fitting functions are often chosen, such as polynomials, splines, or a combination of discretisation and interpolation. However, these do not take account of the special features of human walking. Firstly it is periodic and, hence, using functions that explicitly enforce periodicity will avoid having to include this as an optimisation constraint. Secondly, the fundamental frequency of human walking is of the order of 1Hz and over 99% of the power content is below 6Hz (Winter, 2009). As a result, 5^{th} order Fourier series are likely to adequately represent walking, including enforcing periodicity, which means that each joint motion trajectory can be represented by just 11 optimisation parameters. For these reasons, several previous authors have chosen to represent the joint motions using Fourier series (Koopman et al., 1995; Ren et al., 2007). In the case of Ren et al., 2007, this allowed the prediction of a realistic gait even when the initial Fourier coefficients represented standing not walking.

Most previous authors have limited their gait prediction studies by using planar models, because of the complexity and corresponding computational demands of 3D inverse dynamics models. Of those that adopted 3D models, the following limitations can be identified. Koopman et al., 1995, only predicted a small number of unmeasured joint motions. Tlalolini et al., 2010, did not model finite double support periods. Kim et al, 2008, avoided solving the full inverse dynamics problem by adopting an approach that constrains the centre of pressure (COP) to be within the base of support (BOS), thus ensuring that "dynamic equilibrium" is satisfied. However, because the joint moments are not calculated, many optimisation objectives cannot be adopted (e.g. minimisation of mechanical work).

So it is clear that there still remains a challenge to establish a fast inverse dynamics model of 3D human gait that can be used in optimisation based studies. In this paper we describe the design of a bespoke human gait model where computational efficiency has been achieved by adopting a dedicated model structure and calculation sequence that is optimised for human gait, thus avoiding the overheads of general simulation packages that must cater for any model topology.

We have verified this model against an equivalent model, created using a commercial multi-body dynamics package, and compared the execution times of the two models to demonstrate the computational efficiency of our model. Sufficient detail is provided to allow readers to implement the model themselves.

Methods

Although inverse dynamics is less computationally demanding than forward dynamics, in the case of a 3D skeletal model, it is still very important to adopt an efficient solution method. The chain like structure of the model lends itself to a bespoke implementation of the iterative Newton-Euler method, which is well recognised as being particularly efficient (Craig, 2004; Featherstone, 2008; Angeles, 2014) and, therefore, we have adopted this solution approach for the inverse dynamics. This method has a computational complexity of O(n), which means that the calculations required grow linearly with the number of degrees of freedom (n). This compares very favourably to a computational complexity of O(n^4) for a non-iterative approach (i.e. the calculations required grow with n^4).

For the reasons previously discussed, we have chosen to use Fourier series to represent the trajectories of the degrees-of-freedom driving the motions of the 3D skeletal model. This has two benefits, the first of which is that this leads to a relatively small set of optimisation variables (the Fourier coefficients), which reduces computation times. Secondly, Fourier series automatically constrain the motions to be cyclic and continuous.

1. The multi-body model

To maximise computational efficiency whilst maintaining reasonable accuracy in the description of gait kinematics, a compromise was adopted with regard to the number of rigid segments and degrees-of-freedom (DOF). For example, the hands were treated as part of the forearm segments.

Referring to *Figure 1*, the multi-body model has fourteen rigid segments including: the ground, 2 feet, 2 shanks, 2 thighs, pelvis, torso, head, 2 upper-arms, and 2 forearms. Each segment has an attached coordinate frame. For the simple line segments (representing the longitudinal axis of the bone), the origin is located at the proximal end and the Z-axis is determined by the unit vector directed from the distal end to the proximal end. For the torso, the segment origin is located at the lumbosacral joint and the Z-axis is determined by the unit vector directed from the lumbosacral joint to the neck joint. For the pelvis, the segment origin is located at the lumbosacral joint and the Z-axis is determined by the unit vector directed from the mid-point between the two hip joints to the lumbosacral joint. For all segments, the Y-axis points forward when the segment is vertical (i.e. its Z-axis is vertical) and is not rotated about its Z-axis. For all segments, the X axis points to the right when the segment is not rotated about its other axes.

The model has 25 DOF including: a 1-DOF rollover joint between the stance foot and the ground (the 3 ankle coordinates are functions of the foot-ground angle); 2-DOF ankle joints (dorsiflexion and eversion); 1-DOF knee joints; 3-DOF hip joints; a 3-DOF lumbosacral joint; a 3-DOF neck joint; 2-DOF shoulder joints (flexion and abduction); and 1-DOF elbow joints.

2. Joint motions

The joint DOFs are represented by X-Y-Z sequence Euler angles. Each rotation is performed about an axis of the moving system, which is the distal (*d*) segment coordinate frame, starting from an orientation aligned with the reference system, which is the proximal (*pr*) segment coordinate frame. In this context, the pelvis is the most proximal segment and the lower arms, head and feet are the most distal segments. Therefore, each joint's rotation matrix R_j can be calculated from the following expression (Craig, 2004):

$$R_j = {}^{pr}_{d}R_{XYZ}(\alpha,\beta,\gamma) = R_X(\alpha)\,R_Y(\beta)\,R_Z(\gamma)$$

where

$$R_X(\alpha)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & -\sin\alpha\\ 0 & \sin\alpha & \cos\alpha\end{bmatrix};\quad R_Y(\beta)=\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix};\quad R_Z(\gamma)=\begin{bmatrix}\cos\gamma & -\sin\gamma & 0\\ \sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}$$

Then, given the rotation matrix $R_j = {}^{pr}_{d}R = \begin{bmatrix} r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix}$, the joint angular velocity can be calculated as follows (Craig, 2004):

$$\omega_j = {}^{pr}\omega_{d/pr} = \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

where ω_j is the angular velocity of the distal (*d*) segment relative to the proximal (*pr*) segment, expressed in the proximal segment's coordinate frame. The three components of ω_j are given by:

$$
\begin{aligned}
\omega_x &= \dot{r}_{31}r_{21} + \dot{r}_{32}r_{22} + \dot{r}_{33}r_{23} \\
\omega_y &= \dot{r}_{11}r_{31} + \dot{r}_{12}r_{32} + \dot{r}_{13}r_{33} \\
\omega_z &= \dot{r}_{21}r_{11} + \dot{r}_{22}r_{12} + \dot{r}_{23}r_{13}
\end{aligned}
$$

Finally, the angular acceleration vector is simply the derivative of the angular velocity vector. Using the above, the corresponding expressions for each type of anatomical joint can be derived and these are given in the appendix. These dedicated expressions increase computational efficiency in comparison to applying the general analysis described above as is necessary in general simulation packages that must cater for any model topology.

3. Iterative calculation of segment kinematics

The iterative Newton-Euler method has been used for the inverse dynamics calculations. The first stage of this method is to calculate the segment kinematics by iteratively working outwards from one segment to the next, beginning at the stationary reference segment (the ground) and ending at the most distal segments (the swing foot, head and lower arms). The motion of the next segment is calculated from the motion of the previous segment (already calculated) and the motions of the joint DOFs connecting the two segments.

The exact form of the iterative Newton-Euler equations depends on whether the calculations are being performed in a distal to proximal direction or vice versa. Therefore, the following sub-sections deal with the different cases involved in modelling the gait cycle. In most cases (unless for emphasis) the leading superscript is omitted when there is a single subscript and the superscript would be the same as the subscript (e.g. ${}^{d}\omega_d \equiv \omega_d$).

3.1 Stance Leg

For the stance leg the direction of calculation is from the ground (most distal segment) towards the pelvis (most proximal segment). Therefore, referring to *Figure 2*, the general form of the iterative calculations is as follows.

Segment angular velocity:

$$^{pr}\omega_{pr} = {}^{pr}_{d}R\omega_{d} + {}^{pr}\omega_{pr/d} = {}^{pr}_{d}R\omega_{d} - \omega_{j}$$

Segment angular acceleration:

$$^{pr}\dot{\omega}_{pr} = {}^{pr}_{d}R\dot{\omega}_{d} + {}^{pr}_{d}R\omega_{d} \times {}^{pr}\omega_{pr/d} + {}^{pr}\dot{\omega}_{pr/d}$$

$$\rightarrow {}^{pr}\dot{\omega}_{pr} = {}^{pr}_{d}R\dot{\omega}_{d} - {}^{pr}_{d}R\omega_{d} \times \omega_{j} - \dot{\omega}_{j}$$

Acceleration of segment frame origin:

$$a_{pr} = {}^{pr}_{d}Ra_{d} - (\dot{\omega}_{pr} \times {}^{pr}P_{d}) - (\omega_{pr} \times \omega_{pr} \times {}^{pr}P_{d})$$

Acceleration of segment mass centre:

$$a_{Cpr} = a_{pr} + (\dot{\omega}_{pr} \times {}^{pr}P_{Cpr}) + (\omega_{pr} \times \omega_{pr} \times {}^{pr}P_{Cpr})$$

Using the above, the corresponding equations for each segment of the stance leg can be derived.

3.2 Swing leg and upper body

For the swing leg and upper body the direction of calculation is from the most proximal segment (pelvis) to the most distal segment (foot, head, or lower arm). Therefore, the general form of the iterative calculations is as follows.

Segment angular velocity:

$$ {}^{d}\omega_{d} = {}^{d}_{pr}R\,\omega_{pr} + {}^{d}\omega_{d/pr} = {}^{d}_{pr}R(\omega_{pr} + \omega_{j}) $$

Segment angular acceleration:

$$ \begin{aligned} {}^{d}\dot{\omega}_{d} &= {}^{d}_{pr}R(\dot{\omega}_{pr} + \omega_{pr} \times {}^{pr}\omega_{d/pr} + {}^{pr}\dot{\omega}_{d/pr}) \\ \rightarrow {}^{d}\dot{\omega}_{d} &= {}^{d}_{pr}R(\dot{\omega}_{pr} + \omega_{pr} \times \omega_{j} + \dot{\omega}_{j}) \end{aligned} $$

Acceleration of segment frame origin:

$$ a_{d} = {}^{d}_{pr}R\left[a_{pr} + \left(\dot{\omega}_{pr} \times {}^{pr}P_{d} \right) + (\omega_{pr} \times \omega_{pr} \times {}^{pr}P_{d}) \right] $$

Acceleration of segment mass centre:

$$ a_{Cd} = a_{d} + (\dot{\omega}_{d} \times {}^{d}P_{Cd}) + (\omega_{d} \times \omega_{d} \times {}^{d}P_{Cd}) $$

Using the above, the corresponding equations for each segment of the swing leg and upper body can be derived.

*3.3 **Double stance***

To avoid kinematic redundancy during double stance, only one foot is considered to be connected to the ground by the roll-over joint (labelled the stance foot). Therefore, in double stance, the sequence of kinematic calculations is the same as in single stance. From right heel-strike (RHS) to left heel-strike (LHS), the right foot is the stance foot. From LHS to RHS, the left foot is the stance foot.

4. Equations of motion

Segment forces and moments are defined to be the forces and moments acting on a segment at its proximal joint. Therefore, from Newton's third law, the forces and moments acting at distal joints (i.e. belonging to other segments) must be multiplied by −1; hence the minus signs shown in the free-body diagrams (*Figure 3*). Note that, in these diagrams and the accompanying equations of motion, all vectors are assumed to be written in the global frame. Therefore leading superscripts are not shown except for some position vectors. There are three different types of segments in the skeletal model as follows.

4.1 Standard line segment

Most segments within the human body can be modelled as a simple line segment connecting a proximal and a distal joint (*Figure 3a*).

From Newton's second law:

$$F_{pr} - F_d = m_{pr} a_{Cpr}$$

From Euler's equation:

$$n_{pr} - n_d + (-P_{Cpr} \times F_{pr}) + \left[({}^{pr}P_d - P_{Cpr}) \times -F_d \right] = \left[I_{pr}\dot{\omega}_{pr} + (\omega_{pr} \times I_{pr}\omega_{pr}) \right]$$

*4.2 **Torso segment***

As illustrated in *Figure 3b*, the torso segment has been modelled as a quadralateral segment with corners at the four joints connecting it to the pelvis, head and two arms. The lumbo-sacral joint is the proximal joint, the other joints being distal joints.

From Newton's second law:

$$F_t - F_{head} - F_{uarmr} - F_{uarml} = m_t a_{ct}$$

From Euler's equation:

$$n_t - n_{head} - n_{uarml} - n_{uarmr} + (-P_{Ct} \times F_t) + \left[\left({}^{t}P_{head} - P_{Ct} \right) \times -F_{head} \right] + \left[\left({}^{t}P_{uarml} - P_{Ct} \right) \times -F_{uarml} \right] + \left[\left({}^{t}P_{uarmr} - P_{Ct} \right) \times -F_{uarmr} \right] = \left[I_t\dot{\omega}_t + (\omega_t \times I_t\omega_t) \right]$$

*4.3 **Pelvis segment***

As illustrated in *Figure 3c*, the Pelvis segment has been modelled as a triangular segment with corners at the three joints connecting it to the torso and two legs. Being the most proximal segment in the body, all of the joints are distal joints and hence all of the joint forces and moments have minus signs associated with them.

From Newton's second law:

$$-F_t - F_{th_l} - F_{th_r} = (m_p a_{cp})$$

From Euler's equation:

$$-n_t - n_{th_r} - n_{th_l} + (-P_{Cp} \times -F_t) + \left[({}^{p}P_{th_l} - P_{Cp}) \times -F_{th_l} \right] + \left[({}^{p}P_{th_r} - P_{Cp}) \times -F_{th_r} \right] = \left[I_p \dot{\omega}_p + (\omega_p \times I_p \omega_p) \right]$$

5. <u>Iterative kinetics calculations for single stance</u>

The second stage of the iterative Newton-Euler method is to calculate the segment kinetics by iteratively working inwards from one segment to the next, beginning at the most distal segments, furthest from the stationary reference segment (the ground), and ending at the ground. Therefore, in single stance, the calculations begin at the swinging foot, lower arms, and head. This sequence of calculations is executed twice: first to calculate the joint forces; and then again to calculate the joint moments (which depend on the already calculated forces). In the first sequence of calculations, the force acting at the joint connecting the current segment to the next segment is calculated from the other joint forces acting on the current segment (already calculated) and its translational acceleration. In the second sequence of calculations, the moment acting at the joint connecting the current segment to the next segment is calculated from the other joint moments acting on the current segment (already calculated), all of the joint forces acting on the current segment (already calculated), and its angular motion.

Based on this sequence of calculations and the equations of motion presented in the previous sub-section, the following equations can be derived.

5.1 Forces

For the standard line segments found in the upper body and the swing leg, the calculation sequence is from distal segment to proximal segment, leading to the following general equation:

$$F_{pr} = {}^{pr}_{d}RF_{d} + m_{pr}a_{Cpr}$$

Using the above, the corresponding equations for each line segment in the upper body and the swing leg can be derived. Then the torso force can be calculated as follows:

$$F_{t} = {}^{t}_{uarm_r}R(F_{uarm_r}) + {}^{t}_{uarm_l}R(F_{uarm_l}) + {}^{t}_{head}R(F_{head}) + m_{t}a_{Ct}$$

And the stance thigh force ($F_{th_{st}}$) can be calculated as follows:

$$F_{th_{st}} = {}^{th_{st}}_{p}R[-{}^{p}_{th_{sw}}R(F_{th_{sw}}) - {}^{p}_{t}RF_{t} - m_{p}a_{Cp}]$$

Note that *"st"* refers to the stance leg and *"sw"* refers to the swing leg.

For the standard line segments found in the stance leg, the calculation sequence is from proximal segment to distal segment, leading to the following general equation:

$$F_{d} = {}^{d}_{pr}R(F_{pr} - m_{pr}a_{Cpr})$$

Using the above, the corresponding equations for each line segment in the stance leg, and also for the ground, can be derived.

5.2 Moments

For the standard line segments found in the upper body and the swing leg, the calculation sequence is from distal segment to proximal segment, leading to the following general equations:

$$
\begin{aligned}
&n_{pr} = N_{pr} + {}^{pr}_{d}Rn_{d} - MPF_{pr} - MF_{d} \\
&MF_{d} = \left({}^{pr}P_{d} - P_{Cpr} \right) \times - {}^{pr}_{d}RF_{d} \\
&MPF_{pr} = -P_{Cpr} \times F_{pr} \\
&N_{pr} = I_{pr}\dot{\omega}_{pr} + (\omega_{pr} \times I_{pr}\omega_{pr})
\end{aligned}
$$

Using the above, the corresponding equations for each line segment in the upper body and the swing leg can be derived. Then the torso moment can be calculated as follows:

$$
\begin{aligned}
&n_{t} = {}_{uarml}^{t}Rn_{uarml} + {}_{uarmr}^{t}Rn_{uarmr} + {}_{head}^{t}Rn_{head} + N_{t} - MPF_{t} - MF_{uarml} - MF_{uarmr} - MF_{head} \\
&MPF_{t} = -P_{Ct} \times F_{t} \\
&MF_{uarml} = \left[({}^{t}P_{uarml} - P_{Ct}) \times - {}_{uarml}^{t}RF_{uarml} \right] \\
&MF_{uarmr} = \left[({}^{t}P_{uarmr} - P_{Ct}) \times - {}_{uarmr}^{t}RF_{uarmr} \right] \\
&MF_{head} = \left[({}^{t}P_{head} - P_{Ct}) \times - {}_{head}^{t}RF_{head} \right]
\end{aligned}
$$

Then the stance thigh moment ($n_{th_{st}}$) can be calculated as follows:

$$
\begin{aligned}
&n_{th_{st}} = {}^{th_{st}}_{p}R(-{}^{p}_{t}Rn_{t} - {}_{th_{sw}}^{p}Rn_{th_{sw}} - N_{p} + MPF_{p} + MF_{th_{sw}} + MF_{th_{st}}) \\
&MPF_{p} = -P_{Cp} \times - {}^{p}_{t}RF_{t} \\
&MF_{th_{sw}} = \left[({}^{p}P_{th_{sw}} - P_{Cp}) \times - {}_{th_{sw}}^{p}RF_{th_{sw}} \right] \\
&MF_{th_{st}} = \left[({}^{p}P_{th_{st}} - P_{Cp}) \times - {}_{th_{st}}^{p}RF_{th_{st}} \right]
\end{aligned}
$$

Note that *"st"* refers to the stance leg and *"sw"* refers to the swing leg.

For the standard line segments found in the stance leg, the calculation sequence is from proximal segment to distal segment, leading to the following general equations:

$$
\begin{aligned}
n_d &= {}^{d}_{pr}R\left[n_{pr} - N_{pr} + MPF_{pr} + MF_d\right] \\
MPF_{pr} &= -P_{Cpr} \times F_{pr} \\
MF_d &= \left({}^{pr}P_d - P_{Cpr}\right) \times -{}^{pr}_{d}RF_d
\end{aligned}
$$

Using the above, the corresponding equations for each line segment in the stance leg, and also for the ground, can be derived.

6. Iterative kinetics calculations for double stance

One complete gait cycle includes two double stance phases: right double stance (following RHS) and left double stance (following LHS). In walking, the duration of each of these is approximately one tenth of the gait cycle. In the double stance phases, the sequence of kinetics calculations for the upper body is the same as it is in the single stance phases. However, the division of forces and moments between the two stance legs is an indeterminate problem. To resolve this problem, first the total ground reaction force and moment are calculated by applying the Newton-Euler equations to the lower limbs as a whole. Then, to share the total ground reaction force and moment between the two feet, smooth transition assumptions are applied. The details of this process are as follows.

6.1 Calculating the ground reaction forces

During double stance, the sum of the ground reaction forces on both feet is calculated by summing Newton's second law for the pelvis and all segments in both legs as follows:

$$\sum F_{gr} = \{{}^{gr}_{f}Rm_f a_{Cf} + {}^{gr}_{sh}Rm_{sh}a_{Csh} + {}^{gr}_{th}Rm_{th}a_{Cth}\}_{l,r} + {}^{gr}_{p}Rm_p a_{Cp} + {}^{gr}_{t}RF_t$$

where $\sum F_{gr} = F_{grr} + F_{grl}$

Then the right and left ground reaction forces (F_{grr}, F_{grl}) can be calculated from the total ground reaction force ($\sum F_{gr}$) by applying the following smooth transition assumptions:

$$\frac{F_{gr(st)X}}{\sum F_{grX}} = STA(t); \quad \frac{F_{gr(st)Y}}{\sum F_{grY}} = STA(t); \quad \frac{F_{gr(st)Z}}{\sum F_{grZ}} = STA(t)$$

Where (*st*) refers to the stance foot (i.e. the heel-strike foot that has just landed). In this study, the smooth transition assumption, STA(*t*), is a linear function of time with its value changing from 0 to 1 over each double stance period.

6.2 ***Calculating the joint forces in the legs***

Then, starting from the two supporting feet and working upwards, segment by segment, the force at each leg joint can be calculated from the following general equation:

$$F_{pr} = {}^{pr}_{d}RF_{d} + m_{pr}a_{Cpr}$$

Using the above, for both stance legs, the corresponding equations for each segment force (i.e. the force at the segment's proximal joint) can be derived.

6.3 ***Calculating the ground reaction moments***

During double stance, the sum of the ground reaction moments on both feet is calculated by summing Euler's equation for the pelvis and all segments in both legs as follows:

$$\sum n_{gr} = n_{grr} + n_{grl} = \sum_{i=f}^{p} {}^{gr}N_i + {}^{gr}_{t}Rn_t - {}^{gr}MFs - {}^{gr}MPFs$$

$$
\begin{aligned}
&{}^{gr}MPFs = \{{}^{gr}_{f}R(-P_{Cf}\times F_f) + {}^{gr}_{sh}R(-P_{Csh}\times F_{sh}) + {}^{gr}_{th}R(-P_{Cth}\times F_{th})\}_{l,r} + {}^{gr}_{p}R(-P_{Cp}\times -{}^{p}_{t}RF_t)\\
&{}^{gr}MFs = {}^{gr}_{p}R\left[({}^{p}P_{th_l} - {}^{p}P_{Cp})\times -{}^{p}_{th_l}RF_{th_l}\right] + {}^{gr}_{p}R\left[({}^{p}P_{th_r} - {}^{p}P_{Cp})\times -{}^{p}_{th_r}RF_{th_r}\right] + \ldots\\
&\{{}^{gr}_{f}R\left[({}^{f}P_{gr} - {}^{f}P_{Cf})\times {}^{f}_{gr}RF_{gr}\right] + {}^{gr}_{sh}R\left[({}^{sh}P_{f} - {}^{sh}P_{Csh})\times -{}^{sh}_{f}RF_{f}\right] + \ldots\\
&{}^{gr}_{th}R\left[({}^{th}P_{sh} - {}^{th}P_{Cth})\times -{}^{th}_{sh}RF_{sh}\right]\}_{l,r}
\end{aligned}
$$

$$
\begin{aligned}
\sum_{i=f}^{p} N_i = \{&{}^{gr}_{f}R[I_f\dot{\omega}_f + \omega_f\times(I_f\omega_f)] + {}^{gr}_{sh}R[I_{sh}\dot{\omega}_{sh} + \omega_{sh}\times(I_{sh}\omega_{sh})]\\
&+{}^{gr}_{th}R[I_{th}\dot{\omega}_{th} + \omega_{th}\times(I_{th}\omega_{th})]\}_{l,r} + {}^{gr}_{p}R[I_p\dot{\omega}_p + \omega_p\times(I_p\omega_p)]
\end{aligned}
$$

Then the right and left ground reaction moments (n_{grr}, n_{grl}) can be calculated from the total ground reaction moment ($\sum n_{gr}$) by applying the following smooth transition assumptions:

$$
\frac{n_{gr(st)X}}{\sum n_{grX}} = STA(t);\quad \frac{n_{gr(st)Y}}{\sum n_{grY}} = STA(t);\quad \frac{n_{gr(st)Z}}{\sum n_{grZ}} = STA(t)
$$

Where (*st*) refers to the stance foot (i.e. the heel-strike foot that has just landed). In this study, the smooth transition assumption, STA(*t*), is a linear function of time with its value changing from 0 to 1 over each double stance period.

6.4 Calculating the joint moments in the legs

Then, starting from the two supporting feet and working upwards, segment by segment, the moment at each leg joint can be calculated from the following general equation:

$$
\begin{aligned}
n_{pr} &= N_{pr} + {}^{pr}_{d}Rn_{d} - MPF_{pr} - MF_{d} \\
N_{pr} &= I_{pr}\dot{\omega}_{pr} + (\omega_{pr} \times I_{pr}\omega_{pr}) \\
MPF_{pr} &= -P_{Cpr} \times F_{pr} \\
MF_{d} &= \left({}^{pr}P_{d} - P_{Cpr} \right) \times -{}^{pr}_{d}RF_{d}
\end{aligned}
$$

Using the above, for both stance legs, the corresponding equations for each segment moment (i.e. the moment at the segment's proximal joint) can be derived.

Test results

The inverse dynamics model described above has been verified against an identical model created using MathWorks' *Sim Mechanics* software. To achieve model verification, the degrees of freedom, segments, joints and motion inputs were identical. However, it was not possible to simulate double stance using *Sim Mechanics*, because of the indeterminacy problem, or to change the foot in contact with the ground during simulation. Therefore, the model verification applies only over one single stance phase with the right foot in contact with the ground.

There was excellent agreement between the two models. For example, over single stance, the positions of the segment origins were in agreement with a mean error of 10^{-12} mm. The ground reaction forces and moments were in agreement with mean errors of 10^{-14} N and 10^{-14} Nm respectively.

Finally, the execution speeds of the two models were compared. This was done for single stance because the *Sim Mechanics* model could only model single stance. Both models were run on the same PC without compiling the associated MATLAB code. The *Sim Mechanics* model's execution time was 8.5 seconds as compared to 1.1 seconds for the fast inverse dynamics model described in this paper.

Conclusion

A computationally efficient inverse dynamics model of human gait has been designed for use in optimisation based studies requiring many iterative gait cycle simulations. The model has been verified against an equivalent model, created using a commercial multi-body dynamics package, and the execution times of the two models compared. The fast inverse dynamics model described in this paper led to an eight fold increase in execution speed.

The increased computational efficiency is a result of a number of factors including the use of a bespoke model of the human gait cycle, which avoids the overheads associated with general simulation packages that must cater for any model topology. Furthermore, the chain like structure of the model lends itself to a bespoke implementation of the iterative Newton-Euler method, which is well recognised as being a particularly efficient approach.

Disclosure statement

To the best of the authors' knowledge, there are no conflicts of interest.

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Appendix – Joint rotation matrices and angular motions

a) One degree of freedom joints rotating about X axis (Flexion-Extension)

$$ {}^{pr}_{d}R(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_j & -\sin\alpha_j \\ 0 & \sin\alpha_j & \cos\alpha_j \end{bmatrix} $$

$$ \omega_j = {}^{pr}\omega_{d/pr}(\alpha) = \begin{bmatrix} \dot{\alpha}_j \\ 0 \\ 0 \end{bmatrix} $$

$$ \dot{\omega}_j = {}^{pr}\dot{\omega}_{d/pr}(\alpha) = \begin{bmatrix} \ddot{\alpha}_j \\ 0 \\ 0 \end{bmatrix} $$

The joints of this type are foot rollover, knee, and elbow.

b) Two degrees of freedom joints rotating about X and Y axes

$$ {}^{pr}_{d}R(\alpha,\beta) = \begin{bmatrix} \cos\beta_j & 0 & \sin\beta_j \\ \sin\beta_j \sin\alpha_j & \cos\alpha_j & -\sin\alpha_j \cos\beta_j \\ -\cos\alpha_j \sin\beta_j & \sin\alpha_j & \cos\beta_j \cos\alpha_j \end{bmatrix} $$

$$ \omega_j = {}^{pr}\omega_{d/pr}(\alpha,\beta) = \begin{bmatrix} \dot{\alpha}_j \\ \cos\alpha_j \dot{\beta}_j \\ \sin\alpha_j \dot{\beta}_j \end{bmatrix} $$

$$ \dot{\omega}_j = {}^{pr}\dot{\omega}_{d/pr}(\alpha,\beta) = \begin{bmatrix} \ddot{\alpha}_j \\ \cos\alpha_j \ddot{\beta}_j - \sin\alpha_j \dot{\alpha}_j \dot{\beta}_j \\ \sin\alpha_j \ddot{\beta}_j + \cos\alpha_j \dot{\alpha}_j \dot{\beta}_j \end{bmatrix} $$

The joint of this type is the shoulder.

c) Two degrees of freedom joints rotating about X and Z axes

$$
{}^{pr}_{d}R(\alpha,\gamma)=\begin{bmatrix}\cos\gamma_j & -\sin\gamma_j & 0\\ \sin\gamma_j\cos\alpha_j & \cos\alpha_j\cos\gamma_j & 0\\ \sin\alpha_j\sin\gamma_j & \sin\alpha_j\cos\gamma_j & c\alpha_j\end{bmatrix}
$$

$$
\omega_j={}^{pr}\omega_{d/pr}(\alpha,\gamma)=\begin{bmatrix}\dot{\alpha}_j\\ -\sin\alpha_j\dot{\gamma}_j\\ \cos\alpha_j\dot{\gamma}_j\end{bmatrix}
$$

$$
\dot{\omega}_j={}^{pr}\dot{\omega}_{d/pr}(\alpha,\gamma)=\begin{bmatrix}\ddot{\alpha}_j\\ -\cos\alpha_j\dot{\alpha}_j\dot{\gamma}_j-\sin\alpha_j\ddot{\gamma}_j\\ -\sin\alpha_j\dot{\alpha}_j\dot{\gamma}_j+\cos\alpha_j\ddot{\gamma}_j\end{bmatrix}
$$

The joint of this type is the ankle.

d) Three degrees of freedom joints rotating about X, Y and Z axes

$$
{}^{pr}_{d}R(\alpha,\beta,\gamma)=\begin{bmatrix}c\beta_j c\gamma_j & -c\beta_j s\gamma_j & s\beta_j\\ s\alpha_j s\beta_j c\gamma_j+c\alpha_j s\gamma_j & -s\alpha_j s\beta_j s\gamma_j+c\alpha_j c\gamma_j & -s\alpha_j c\beta_j\\ -c\alpha_j s\beta_j c\gamma_j+s\alpha_j s\gamma_j & c\alpha_j s\beta_j s\gamma_j+s\alpha_j c\gamma_j & c\alpha_j c\beta_j\end{bmatrix}
$$

$$
{}^{pr}\omega_{d/pr}(\alpha,\beta,\gamma)=\omega_j=\begin{bmatrix}\dot{\alpha}_j+s\beta_j\dot{\gamma}_j\\ c\alpha_j\dot{\beta}_j-c\beta_j s\alpha_j\dot{\gamma}_j\\ s\alpha_j\dot{\beta}_j+c\alpha_j c\beta_j\dot{\gamma}_j\end{bmatrix}
$$

$$
{}^{pr}\dot{\omega}_{d/pr}(\alpha,\beta,\gamma)=\begin{bmatrix}\ddot{\alpha}_j+c\beta_j\dot{\beta}_j\dot{\gamma}_j+s\beta_j\ddot{\gamma}_j\\ -s\alpha_j\dot{\alpha}_j\dot{\beta}_j+c\alpha_j\ddot{\beta}_j-c\alpha_j c\beta_j\dot{\alpha}_j\dot{\gamma}_j+c\alpha_j\ddot{\beta}_j+s\alpha_j s\beta_j\dot{\beta}_j\dot{\gamma}_j-s\alpha_j c\beta_j\ddot{\gamma}_j\\ c\alpha_j\dot{\alpha}_j\dot{\beta}_j-s\alpha_j c\beta_j\dot{\alpha}_j\dot{\gamma}_j+s\alpha_j\ddot{\beta}_j-c\alpha_j s\beta_j\dot{\beta}_j\dot{\gamma}_j+c\alpha_j c\beta_j\ddot{\gamma}_j\end{bmatrix}
$$

The joints of this type are hip, lumbosacral and neck.

Figure Captions

Figure 1: The multi-body model

Figure 2: Position vectors used in the iterative calculations

Figure 3: Free Body Diagrams: a) Standard line segment; b) Torso; c) Pelvis