Nonlinear Helmholtz wave refraction & Goos-Hänchen shifts in nonparaxial optics: angles and interfaces, solitons and Snell's law

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Abstract

The interaction of self-localized waves with an abrupt interface is a problem of fundamental importance in many branches of physics, engineering, and applied mathematics. Waveguide optics, for instance, is dominated in an essential way by such considerations. There, the full complexity of electromagnetic propagation is conveniently reduced to an equation of the nonlinear Schrödinger (NLS) class. These simplified models are physically intuitive, mathematically tractable, and hold a certain universal appeal.

All their desirable features notwithstanding, theories based on paraxial diffraction are seldom appropriate when angular considerations are of principal interest. Here, we report on our recent analyses of nonlinear optical wave refraction using more general Helmholtz equations, and introduce (for the first time in this single-interface context) a cubic-quintic system response. A Snell law is derived for beams, and its predictions tested by exhaustive computations. New Goos-Hänchen (GH) shift calculations are also detailed for this material regime of nonlinear-interface problem.

Keywords: Helmholtz solitons, beam refraction, Snell's law.

1 Interface Problems

The seminal work of Aceves *et al.* [1] proposed a theory of self-localizing nonlinear optical waves (spatial solitons) at interfaces within an inhomogeneous NLS-type formalism. While their blend of paraxial diffraction and cubic nonlinearity undoubtedly makes for an elegant and instructive analysis, it sacrifices a detailed treatment of oblique-propagation effects. One way to address these inherently nonparaxial considerations is to deploy Helmholtz-type models [2], which retain a simplified scalar character while simultaneously relaxing angular (paraxial) restrictions placed on the spatial spectrum.

Here, our Helmholtz analyses of soliton refraction in cubic systems [2] are extended to capture the quintic term in the nonlinear optical response of the host media [3]. The interaction of a (normalized) electric field envelope u with a planar interface between two dissimilar cubic-quintic materials is described by

$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+i\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}+|u|^2u+\sigma|u|^4u=\left[\frac{\Delta}{4\kappa}+(1-\alpha)|u|^2+(1-\nu)\sigma|u|^4\right]H(\xi,\zeta)u, \quad (1)$$

where (ξ,ζ) denote transverse and longitudinal coordinates, $\kappa \ll \mathcal{O}(1)$ parametrizes the (inverse) beam waist, and σ controls the strength of the quintic response. Mismatches in the linear and nonlinear refractive index are determined by Δ and (α,ν), respectively, while H is a Heaviside unit function that prescribes the position of the interface in the (ξ,ζ) plane.

2 Snell's Law for Spatial Solitons

Using exact solutions of the corresponding homogeneous problem [4], we have derived a Snell law for spatial soliton refraction. Respecting phase continuity at the interface then leads to

$$\gamma n_{01}\cos\theta_{\rm inc}=n_{02}\cos\theta_{\rm ref}, \quad (2)$$

where $\theta_{\rm inc}$ and $\theta_{\rm ref}$ are the incidence and refraction angles (relative to the interface) *in the laboratory frame*, n_{0j} is the linear refractive index of medium j (where $j=1,2$), and γ is a function of the system and incident soliton parameters.

Equation (2) has shown excellent agreement with simulations of model (1) over a wide parameter range (see Fig. 1). The best fit occurs for broader beams (smaller κ values), where amplitude curvature is relatively low. This analysis paves the way for a systematic generalization of our earlier dark soliton-refraction research [5] to regimes involving cubic-quintic nonlinearity [6].

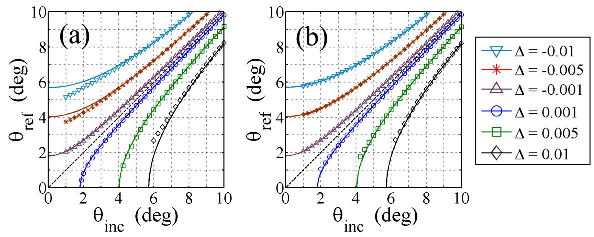

Figure 1: Predictions of the Snell law [see equation (1)] for purely linear interfaces, where the refractive-index mismatch is defined by $\Delta \equiv 1 - (n_{02}/n_{01})^2$ when (a) $\kappa = 2.5 \times 10^{-3}$ and (b) $\kappa = 1.0 \times 10^{-4}$. The cubic and quintic material responses remain unchanged across the boundary.

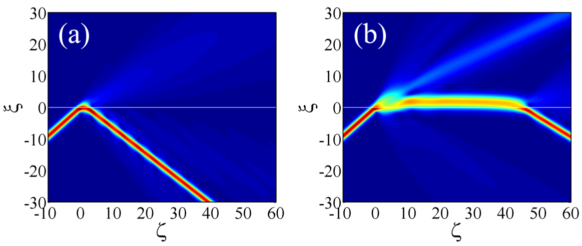

Figure 2: (a) Soliton reflection from a purely-linear interface when $\Delta > 0$, accompanied by the emission of low-amplitude radiation. (b) Giant GH shift when the linear and cubic responses are uniform but where the quintic contribution increases ($\nu = 2$). Both simulations have $\kappa = 2.5 \times 10^{-3}$.

3 Giant Goos-Hänchen Shifts

Equation (2) can be used to make a theoretical approximation for the critical angle of incidence (denoted by θ_c) at a given interface, informing subsequent calculations of GH shifts (see Fig. 2) [7]. By considering the condition $\theta_{ref} = 0$ when $\theta_{inc} = \theta_c$ (so that the refracted beam, in principle, propagates along the boundary), Eq. (2) can be used to show that

$$\tan\theta_c = \sqrt{\frac{\Delta + 2\kappa\rho_0\left[1 - \alpha + \frac{2}{3}\sigma\rho_0(1-\nu)\right]}{1 - \Delta + 2\kappa\rho_0\left(\alpha + \frac{2}{3}\nu\sigma\rho_0\right)}}, \tag{3}$$

where ρ_0 is the (normalized) peak intensity of the incident soliton. Equation (3) provides a reasonable estimate for θ_c, though non-adiabatic processes (e.g., radiation and splitting phenomena) often conspire to reduce its accuracy.

We will conclude with a summary of recent extensive simulations that have investigated the Helmholtz nonparaxial character of GH shifts at (both linear and nonlinear) planar interfaces [8] but in cubic-quintic systems. Qualitatively new predictions are made that appear to have no counterpart in the more familiar purely-cubic scenario [8], including an oscillatory dependence of the GH shift on θ_{inc}. We have also found a strong dependence on the nonparaxial parameter κ, identifying physical (finite beam waist) regimes wherein one would expect paraxial (i.e., NLS-based) modelling to break down.

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