Unstable resonators with polygon and von Koch-type boundary conditions: virtual source modelling of fractal eigenmodes

J. M. Christian[1,*], I. Begleris[2], S. J. Wickham[1], G. S. McDonald[1], J. G. Huang[3]

[1]Materials & Physics Research Centre, University of Salford, Greater Manchester M5 4WT, UK
[2]Computational Modelling Group, University of Southampton, Highfield SO17 1BJ, UK
[3]Faculty of Advanced Technology, University of South Wales, Pontypridd CF37 1DL, UK
*Email: j.christian@salford.ac.uk

Abstract

We will report on our latest research into modelling fractal lasers (linear systems that involve geometrically-unstable resonators with inherent magnification), and propose two new classes of cavity configuration. These devices are of fundamental theoretical interest as table-top generators of tunable fractal light that can be used in a wide range of applications. Moreover, we expect them to play a pivotal role in new Nature-inspired optical architectures and designs.

The virtual source theory of classic kaleidoscope lasers will be reviewed, and we show how that semi-analytical method can be applied to novel cavity designs which incorporate a feedback mirror whose outer boundary corresponds to iterations of the von Koch *snowflake* (an iterated function system involving self-similar sequences of equilateral triangles) and its isosceles counterpart, the von Koch *pentaflake* . A range of new numerical results will be given, including calculations of mode patterns, eigenvalue spectra, and detailed computations of fractal dimension measures.

Keywords: Unstable resonators, virtual source theory, snowflake, pentaflake.

1 Fractal Lasers

Unstable cavity lasers involve linear resonators with inherent magnification whose eigenmodes possess fractal characteristics (that is, proportional level of details spanning decades of spatial scale). The physical origin of such multi-scale patterns in strip resonators (systems comprising a single transverse dimension) has been explained by considering repeated diffraction of the circulating cavity field at the feedback mirror (which subsequently plays a key role in determining mode properties) [1].

The term *kaleidoscope laser* has been coined to describe similar systems with two transverse dimensions where the feedback mirror has the shape of a regular polygon (e.g., an equilateral triangle) [2]. This complicated boundary-value problem (which involves non-orthogonal edges in the aperturing element) gives rise to mode patterns that have a remarkable beauty and complexity [3]. Here, we propose two new classes of unstable resonator that involve fractal (rather than regular) boundary conditions: *snowflake* and *pentaflake* systems.

2 Virtual Source Modelling

A confocal unstable resonator is fully described by two parameters: the equivalent Fresnel number N_{eq} and the round-trip magnification M. Southwell's virtual source (VS) method unfolds the cavity into a plane wave diffracting through a sequence of $N_{\mathrm{S}} = \log(250N_{\mathrm{eq}})/\log M$ apertures, each of which has a characteristic size [4]. The modes of the cavity correspond to a linear superposition of the edge waves from each of these fictitious apertures.

Previously, we have applied a two dimensional (2D) VS approach to find the empty-cavity eigenmodes $V(\mathbf{X})$ of kaleidoscope lasers across the feedback mirror, where

$$V(\mathbf{X}) = \epsilon \left[\frac{E_{N_{\mathrm{S}}+1}(\mathbf{X_C})}{\alpha^{N_{\mathrm{S}}}(\alpha - 1)} - \sum_{m=1}^{N_{\mathrm{S}}} \alpha^{-m} E_m(\mathbf{X}) \right], \tag{1a}$$

$E_m(\mathbf{X})$ is the edge-wave pattern from virtual aperture m, $\mathbf{X_C}$ is an arbitrary point on the boundary of the feedback mirror (typically a vertex), and ϵ is a Heaviside function (equal to 1 in the domain of the feedback mirror, and 0 otherwise). The mode eigenvalue α, obtained by solving the high-order polynomial equation

$$\alpha^{N_{\mathrm{S}}+1} + \sum_{m=0}^{N_{\mathrm{S}}} \left[E_m(\mathbf{X_C}) - E_{m+1}(\mathbf{X_C}) \right] \alpha^{N_{\mathrm{S}}-m} = 0, \tag{1b}$$

plays the role of a formal expansion parameter. Each individual root of Eq. (1b) thus describes

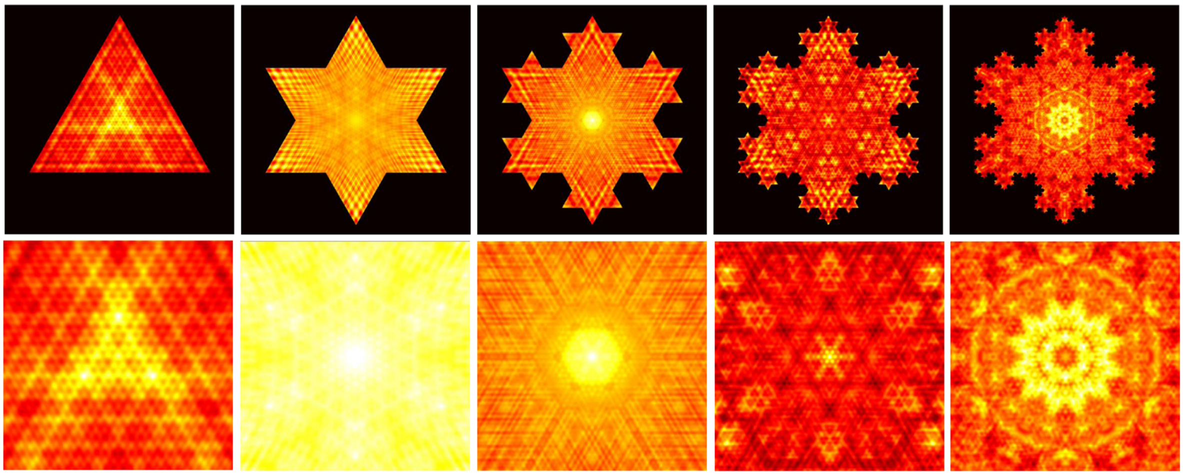

Figure 1: Snowflake laser modes (top row) and corresponding magnification of the central portion (bottom row). For iteration number $n = 0, 1, 2, 3$ and 4 (left to right), there are $N = 3 \times 4^n = 3, 12, 48, 192$, and 768 edges to the feedback mirror (a computationally-intensive problem).

an eigenmode of the unstable resonator. In this way, the virtual source formalism provides a hierarchy of solutions whose round-trip losses are related to $|\alpha|$ (and where the lowest-loss mode corresponds to the largest value of $|\alpha|$). In contrast, *ABCD* (paraxial) matrix modelling in combination with fast Fourier transforms computes only a single mode per application.

In this presentation, we show how our 2D-VS theory can also be applied to find the modes of snowflake (see Fig. 1) and pentaflake resonators. The approach requires detailed knowledge of the constituent edge waves, which are typically found using a line-integral method [5].

3 Modes & Fractal Dimension

A key issue to be addressed in detail is the fractal dimension of unstable-resonator modes for cavities with arbitrary N_{eq} and M parameters. Previously, Berry [6] has made similar considerations but only for the lowest-loss modes of kaleidoscope cavities, and in the limit $N_{\mathrm{eq}} \to \infty$ (where asymptotic approximations may be deployed). We will conclude with a summary of results from the first detailed exploration of fractal dimension in kaleidoscope systems. Specialist software [7] has been deployed in parallel with our suite of 2D-VS codes to investigate potential anisotropy in the dimension using various different measures. Cross-sections through the lowest-loss (and a set of higher-order) mode patterns are computed, and direct comparisons with a strip resonator for the same cavity parameters [1] uncover some intriguing results.

References

[1] G. H. C. New, M. A. Yates, J. P. Woerdman, and G. S. McDonald, Diffractive origin of unstable resonator modes, *Optics Communications* **193** (2001), pp. 261–266.

[2] G. P. Karman, G. S. McDonald, G. H. C. New, and J. P. Woerdman, Fractal modes in unstable resonators, *Nature* **402** (1999), p. 138.

[3] G. S. McDonald, G. P. Karman, G. H. C. New, and J. P. Woerdman, The kaleidoscope laser, *Journal of the Optical Society of America B* **17** (2000), pp. 524–529.

[4] W. H. Southwell, Virtual-source theory of unstable resonator modes, *Optics Letters* **6** (1981), pp. 487–489.

[5] J. G. Huang, J. M. Christian, and G. S. McDonald, Fresnel diffraction and fractal patterns from polygonal apertures, *Journal of the Optical Society of America A* **23** (2006), pp. 2768–2774.

[6] M. V. Berry, Fractal modes of unstable lasers with polygonal and circular mirrors, *Optics Communications* **200** (2001), pp. 321–330.

[7] BENOIT 1.3, TruSoft International Inc., www.trusoft-international.com.