Eigenmodes of Next-Generation Unstable Cavity Lasers: Kaleidoscopes, Snowflakes, Pentaflakes, and Fractal Dimension

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We will report on our latest research into fractal lasers (linear systems which involve geometrically-unstable resonators with inherent magnification) [1], introducing two new classes of transverse cavity configuration. These devices are of fundamental interest as table-top generators of tunable fractal light that can be used in a wide range of applications – from optical probes (e.g., increased sensitivity in surface roughness measurements) to secure information encoding/transmission to medical imaging. Moreover, we expect them to play a pivotal role in new Nature-inspired optical architectures and devices (e.g., writing fractal structures into new materials).

Attention will be paid not only to the classic kaleidoscope geometries [2], but also to recently-proposed designs which incorporate a feedback mirror whose outer boundary corresponds to increasing iterations of the von Koch *snowflake* [3] (a six-fold-symmetric iterated function system involving self-similar sequences of equilateral triangles – see Fig. 1) and its five-fold-symmetric counterpart, the von Koch *pentaflake* (constructed in a similar way to the more traditional snowflake, but using isoceles triangles). All three systems can be modelled using a two-dimensional virtual source (2D-VS) method [4], which unfolds the.unstable empty cavity into a equivalent sequence of virtual apertures of increasing size [5]. A selection of mode patterns and eigenvalue spectra will be presented, and convergence issues considered in some detail.

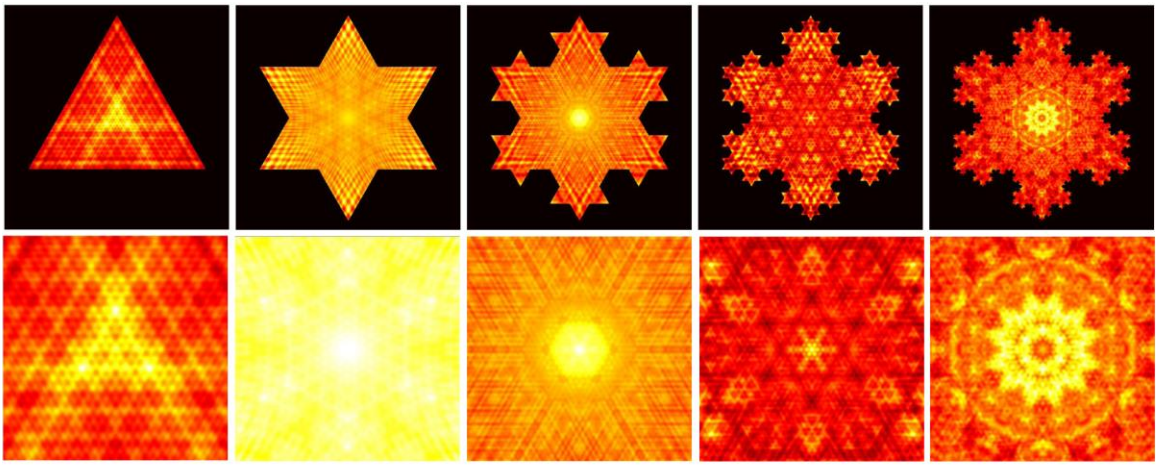

Fig. 1. 2D-VS computations of the lowest-loss mode patterns for a snowflake laser (parameters: $N_{\mathrm{eq}} = 30$ and $M = 1.5$) whose feedback mirror progresses through the first four application of the generator algorithm. The number of edges N_{edge} increases (left to right) geometrically with iteration number n according to $N_{\mathrm{edge}} = 3 \times 4^n$ (so $N_{\mathrm{edge}} = 3, 12, 48, 768, 3072$).

A key issue to be addressed in detail is the fractal dimension of unstable-resonator modes for cavities with arbitrary parameters (i.e., equivalent Fresnel number N_{eq} and round-trip magnification M). Previously, Berry has made similar considerations but only for the lowest-loss modes of kaleidoscope cavities, and in the limit $N_{\mathrm{eq}} \to \infty$ (where asymptotic approximations may be deployed) [6]. We will conclude with a summary of results from the first detailed exploration of fractal dimension in kaleidoscope resonators. Specialist software [7] has been deployed in parallel with our suite of 2D-VS codes to investigate potential anisotropy in the dimension using various different measures: power spectrum, roughness-length, rescaled range, and variogram. One-dimensional cross-sections through the lowest-loss (and several higher-order) mode patterns are computed, and direct comparisons with a strip resonator for the same cavity parameters [8] uncover some intriguing results.

References

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