Spontaneous Spatial Patterns in Discrete Nonlinear Schrödinger Equations: Ring Cavities and Counter-Propagating Beams

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Discrete nonlinear Schrödinger (dNLS) equations have come to play a pivotal role in optics since their proposal by Christodoulides and Joseph nearly three decades ago [1]. In the context of cavity solitons and pattern formation, dNLS-type approaches have been used to describe light in discrete waveguide arrays confined to ring [2a] and Fabry-Pérot [2b] resonators. The resultant governing equations for the averaged intracavity field turn out to be discrete generalizations of the more familiar continuum models first derived by Lugiato and Lefever [3]. Here, we avoid the ubiquitous mean-field limit (with all its advantages and disadvantages) to focus on spontaneous patterns in two new discrete models where propagation effects have not been eliminated.

(i) Ring cavities. We consider the (dimensionless) complex electric field amplitudes $E_{n,m}$ propagating along the (local-time) longitudinal z axis in waveguide channel (n, m) with period D and a Kerr-type nonlinearity [1]:

$$i\frac{\partial E_{n,m}}{\partial z}+c\left(E_{n+1,m}+E_{n-1,m}+E_{n,m+1}+E_{n,m-1}-4E_{n,m}\right)+\chi L\left|E_{n,m}\right|^{2}E_{n,m}=0, \qquad (1)$$

where c is a coupling constant and χL parametrizes the nonlinear phase shift acquired over a cavity of length L. At the start of every round trip, the cirulating field is subjected to the 'lumped' boundary condition familiar from the corresponding continuum model of McLaughlin *et al.* [4], $E_{n,m}(0) = t_1E_{\text{in}} + r_1\exp(i\delta)E_{n,m}(1)$, capturing periodic pumping by the plane wave E_{in}, losses at the coupling mirror (where $r_1^{\,2} + t_1^{\,2} = 1$), and a common linear mistuning δ for each channel. In this way, emergent cavity phenomena beyond the assumptions of mean-field theory may be investigated. We have derived the uniform states of the discrete system, and performed a linear stability analysis that has uncovered a (periodic-in-K) multiple-minimum Turing threshold instability spectrum [5] – see Fig. 1(a). In this presentation, we will discuss simulation results testing our new theoretical predictions, and demonstrate emergent patterns in one- and two-dimensional discrete arrays [see Fig. 1(b)–(e)].

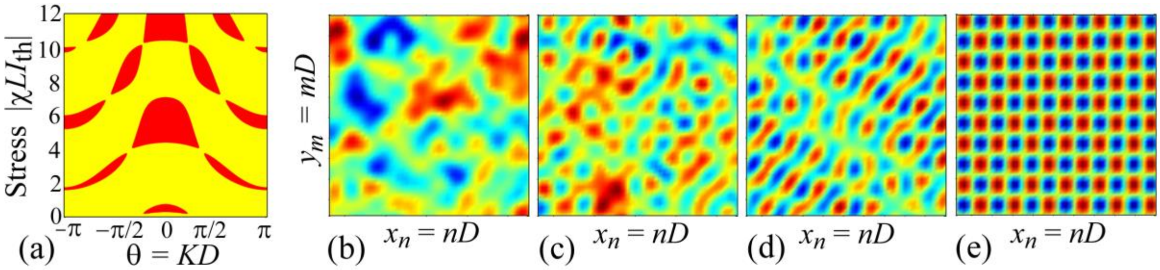

Fig. 1. (a) Multi-Turing instability spectrum [5] for the one-dimensional dNLS ring cavity system in the case of a self-defocusing nonlinearity (where $\chi < 0$, and with I_{th} representing threshold intracavity intensity). (b) – (e) Evolution of a perturbed plane wave state towards a static square pattern for a two-dimensional waveguide array in a ring cavity.

(ii) Counter-propagating beams. Alongside ring cavity geometries, we have re-considered the classic problem of counter-propagating beams [6] but within the coupled-waveguides context. A dNLS-type model has been proposed for describing the evolution of slowly-varying forward and backward envelopes; it is, in essence, a discrete analogue of the continuum system proposed by Firth and Paré. [6]. The time-independent uniform states of the new dNLS system have been derived (subject to equal-intensity constant plane-wave pump fields), and linearization techniques deployed to assess their susceptibility to small-amplitude fluctuations. The perturbative method is reminiscent of that developed for the continuum case [7] and involves a boundary-value problem whose solution requires the exponentiation of a 4×4 matrix. We will report on the multi-Turing threshold instability spectrum [5] that has been uncovered to predict the most-unstable spatial frequency in the discrete counter-propagation system, and present a selection of simulation results to illustrate pattern formation.

References

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