

Share valuation: resolving the negative earnings puzzle

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This paper presents a resolution of the apparently illogical observation in a number of papers that higher losses are associated with higher share valuations. We create conditional distributions from a sample of 42,857 firm years on the London Stock Exchange and show that the relationship is illusory. We argue that our model is better aligned with the concept of market efficiency and the problem of differentiating between observations and expectations. More generally, the model developed here can be applied to all studies that seek to explain market prices.

A number of studies (Ashton et al., 2003; Burgstahler and Dichev, 1997; Collins et al., 1999; Darrough and Ye, 2007; Hand, 2000; Jiang and Stark, 2013) observe high market values for high loss making firms. The problem is most clearly expressed by Jiang and Stark (2013) who make the following observation: "The negative relationship between earnings and market value for loss-making firms is a puzzle because it is counter-intuitive, higher losses leading to higher market values does not make intuitive economic sense." (p.113) This note offers a resolution of this apparent puzzle that is of interest to all studies seeking to relate market measures to accounting and economic variables.

THE PUZZLE DEFINED

The importance of accounting income to share prices has been well established in the literature since Ball and Brown (1968). A simple model of market value can therefore be expected to be strongly related to earnings. A basic version by Jiang and Stark (2013) is:

$$MV_t = a + bE_t + BV_t + \varepsilon_t \quad (1)$$

where MV = market value, BV = book value of equity and E = net income to shareholders. Both Burgstahler and Dichev (1997) and Ashton et al. (2003) apply an equivalent model using book value as a deflator:

$$MV/BV_t = a + bE/BV_t + \varepsilon_t \quad (2)$$

In both of these studies, a positive relationship between market to book value and earnings to book value is found except where earnings were negative where the relationship was reversed. An early rationalisation of high market values for high negative earnings was made by Burgstahler and Dichev (1997): "One puzzling aspect of the results ... is that the estimated slope coefficients for E/BV

for low earnings intervals are significantly negative ...one explanation might be that...more negative values of E/BV may be empirically associated with actions which have been taken to improve E/BV in subsequent years and which are already reflected in higher market values" (p.203). This approach was developed by Darrough and Ye (2007) who add "value drivers" such as non-recurring charges, R&D, growth strategy and sustainability. They eliminate the relationship between negative earnings and market value with the major substitutes being R&D and intangible assets. These findings were tested on UK data by Jiang and Stark (2013) using various deflators and a wider set of explanatory variables. They failed to find a robust resolution and observe that there is: "a higher degree of contextualisation of the loss-making firm valuation model" (p.123). On reflection, it should not be too surprising that separate modelling of a sub-sample of the earnings spectrum should prove to be unstable.

It therefore remains curious that empirical testing of equation 2 should produce results that seem so theoretically appealing when earnings are positive and yet so unappealing when earnings are negative.

A REANALYSIS

It is understood in equations 1 and 2 that earnings represents expected earnings. Hence both market value and earnings are expectations about the future. One of the strongest findings in finance arising from weak form market efficiency tests is that share prices follow a martingale; information is randomly better or worse than expected. Therefore, the current price is taken as the best estimate of market expectations (Capinski and Zastawniak, 2011, p.161). It is traditional to assume that the same reasoning can be applied to the explanatory variables. This is a convenient assumption to make, as current earnings can then be taken as best estimates of expected earnings. Unlike share prices, however, there is no theoretical argument to support this assumption. As the original paper (Samuelson, 1965) makes clear, explanatory variables may follow a pattern. The value of the pattern is discounted into the share price and information is information about changes to the expectation of the pattern. The product life cycle is one such example and both Burgstahler and Dichev (1997) and Darrough and Ye (2007) are in effect suggesting patterned earnings of "non capitalized expenditure today - profit tomorrow". It seems reasonable to assume that all firms with high value and high losses have such patterned expectations but most firms are not in that position. Indeed a martingale process might well be an adequate assumption for many if not most firms. Revisiting equation 2 and making expectations explicit, we have:

$$MV/BV_{i,t} = a + b\overline{E/BV}_{i,t} + \varepsilon_{i,t} \quad (3)$$

We agree with the conclusion of Jiang and Stark (2013) that the attempts to replace negative E_i with variables that measure \overline{E}_i are insufficiently robust. We therefore abandon the Markov/martingale assumption for explanatory variables and hence any deterministic relationship between observed earnings and their expectation. Although this relationship cannot be determined at an individual firm level, it seems reasonable to investigate the relationship at a market level. Given the adjustment for size, earnings across the market for any particular market value should be centred around an expected value. Variations

from expectations will obviously occur and are part of the financial language; thus, a "windfall profit" and "writing off exceptional items" and more generally, patterns where the current earnings presage higher returns in the future are all allowable. Note that this does not imply a normal distribution. We nevertheless expect for the market as a whole, earnings to be frequently close to expectations. If those expectations vary with market value, then we would expect that relationship to be positive as in equation 3. We therefore take a Bayesian view and examine earnings conditioned on market value to determine the expectation:

$$MV/BV = a + b\overline{E/BV}|MV/BV + \varepsilon \quad (4)$$

This is a three stage process. Firstly, examine the distributions of $E/BV|MV/BV$ for varying levels of MV to see if there is a concept of centrality in the distribution (typically a single peak triangular distribution). Secondly, measure the central point $\overline{E/BV}|MV/BV$ for all distributions conditioned on value ranges of MV/BV , note that expectation is simply a measure of centrality here. Outliers, from whatever cause, need not influence such estimation. This is a more consistent methodology, since if outliers are regarded as not including earnings in valuation, then it seems contradictory to then include such earnings in a measure of market expectation. Thirdly, regress or more simply plot $\overline{E/BV}|MV/BV$ on MV/BV to examine the relationship. As we know that market values are valued efficiently using the whole of the public information set, it can be inferred that a true outlier of the conditional distribution is by implication responding to elements of the information set other than earnings. In this way, the martingale assumption has been dropped as we no longer assume that current earnings are expectations.

The relationship between an individual earnings observation and its expectation is by this analysis probabilistic to which Bayes theorem applies, assuming a uniform prior:

$$Pr(MV/BV_k|E/BV_j) = \frac{Pr(E/BV_j|M/BV_k)}{\sum_k Pr(E/BV_j|M/BV_k)} \quad (5)$$

Such an approach would not be of particular interest as no serious attempt to value a firm would just use earnings. In Bayesian terms the uniform prior will, rather like *ceteris paribus*, ignore much that is relevant.

Potentially, this reformulation resolves the negative earnings puzzle as E/BV no matter how negative can still credibly belong to a positive market expectation of E/BV if empirically it displays the characteristics of an outlier to a single peaked positive distribution. The implication would be that such earnings did not contribute to estimating future expected earnings; there can be many reasons for this such as R&D, takeovers, patents and intangible assets as identified by Darrough and Ye (2007). Indeed, as we have stated, the efficient markets hypothesis implies the existence of such non earnings related events. In an efficient market, information has to exist to justify not taking earnings into the valuation process; otherwise one is left with the counter-intuitive result as outlined by Jiang and Stark (2013).

EMPIRICAL TESTING

The overall purpose of testing is firstly to validate our sample with respect to previous studies. This we do by following the methodology of previous papers (Burgstahler and Dichev, 1997; Ashton et al., 2003) as in equation 2, and test for the finding of a positive relationship between losses and market value. Then we examine earnings conditioned on market value to determine expected values. Finally, we examine the relation between expected earnings conditioned on market values and market values. The concern here is limited to resolving the puzzle by reformulating equation 2, our analysis is therefore to a sufficient level though our methodology indicates how further research might develop.

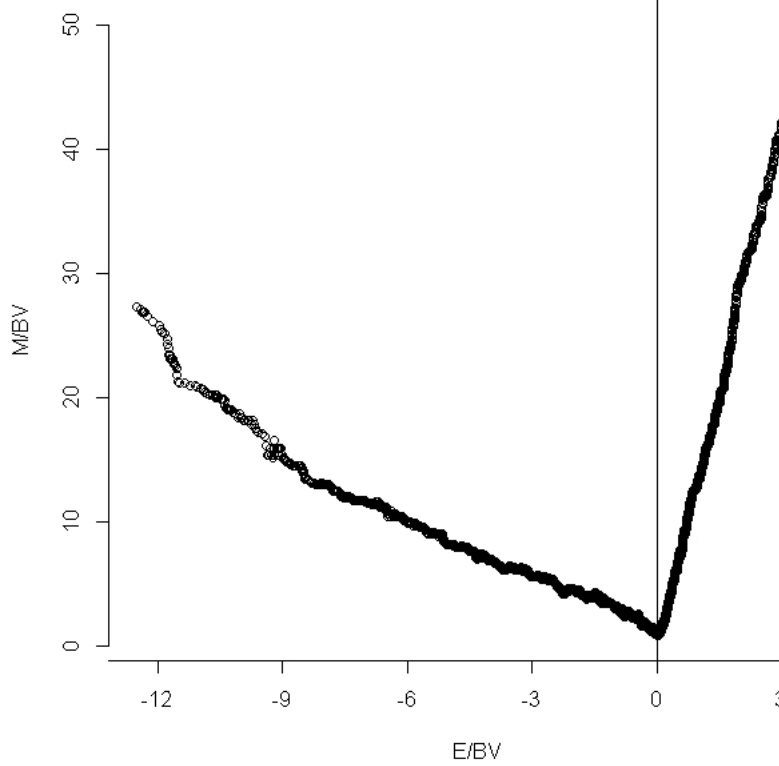
The database consisted of 42,857 firm years quoted on the London Stock Exchange from 1988 to 2012 as stored on Datastream. We followed the definitions of Ashton et al. (2003) who use the same database. Earnings are defined as net income before extraordinary earnings (item 625); book value was defined as common shareholders equity less net intangible assets (item 305 - 344) which is in effect the equity claim on tangible assets, a definition prompted by the claim that such a value represents adaptation value for shareholders (Ashton et al., 2003). In supplementary tests we found that the inclusion or omission of intangible assets did not affect the results. Market value was taken as 100 days after the year end date to get a roughly contemporaneous valuation with the accounting data. A common observation in papers is that the results are not affected by different timings of share dates (e.g. Fama and French (1992)). Data not considered were firm years reporting book values less than £1,000 thus excluding negative M/BV s and very small book values that would potentially report extreme ratios. We are aware of criticisms of Datastream (Ince and Porter, 2006) but point out that with the exception of eliminating de-listed quotes, this paper does not deal with the issues addressed by Ince and Porter (2006). Measurement error will nevertheless always be present. An important feature of this approach is that market expectation is defined as the peak of a distribution and not necessarily the first moment. Extreme results due to measurement error need not influence expectations, the main requirement is therefore that errors are not systematic.

An annual summary of the data is presented in table 1. The parametric measures over the 25 years clearly lack robustness to the extent that the average E/BV is negative in 18 of the 25 years. The standard deviation also shows extreme variation. By contrast the non parametric medians and inter quartile ranges (IQR) are far more stable displaying trends but no extreme variation. From 2000 onwards there appears to be a decline in median earnings to book value and an increase in the IQR for E/BV whereas the median for M/BV is relatively stable but the IQR again increases. It seems that value not recorded in earnings would appear to be a phenomenon that has developed over recent times, a trend also noted in Darrough and Ye (2007). As a result, negative earnings are more likely and this is supported by the number of firms reporting negative earnings being higher in later years. As the parametric measures were clearly very unstable, subsequent analysis was based on the non parametric median and inter quartile range (IQR) as measures of centrality and spread.

Table 1: Sample descriptives for Earnings/Book Value and Market/Book Value

	E/BV		M/BV		E/BV		M/BV	
	median	IQR	median	IQR	mean	st dev	mean	stdev
1988	0.1720	0.1400	1.9268	2.1623	0.2020	0.7285	3.2865	0.7285
1989	0.1582	0.1576	1.5412	1.8026	-0.1455	13.4543	12.8671	365.3271
1990	0.1342	0.1590	1.2187	1.6014	0.0931	1.1585	2.0822	4.3885
1991	0.0997	0.1535	1.2909	1.8366	0.0250	1.2126	2.5259	5.7370
1992	0.0924	0.1523	1.3655	2.0906	-0.0656	1.5604	2.7821	7.7996
1993	0.1006	0.1526	1.8797	2.4187	-0.0303	1.9428	3.3392	5.7970
1994	0.1197	0.1495	1.8941	2.4447	0.0389	2.0490	239.7615	7997.4495
1995	0.1287	0.1508	2.0025	2.6107	0.0748	2.8124	74.6228	965.1773
1996	0.1273	0.1661	2.1533	3.1913	-0.2177	13.3385	55.1198	619.5677
1997	0.1276	0.1761	2.4072	4.3626	-0.8068	30.2968	122.7915	1043.7168
1998	0.1229	0.1825	1.9687	3.6602	0.0768	2.5367	48.4289	556.8971
1999	0.1178	0.2126	2.0538	4.3291	-0.3244	11.8395	62.8185	600.2467
2000	0.0755	0.2768	2.0644	4.0565	-0.4751	12.4393	44.2660	1267.6206
2001	0.0510	0.3492	1.7902	3.0795	-1.0325	10.8594	5.6944	29.0341
2002	0.0431	0.4058	1.5018	2.4918	-0.7448	7.3213	4.9192	22.2006
2003	0.0676	0.2961	2.0876	3.4024	-0.8852	20.6986	15.1354	172.6600
2004	0.0853	0.2883	2.4182	3.7715	-0.6268	19.9467	10.3301	66.5891
2005	0.0809	0.3436	2.8647	4.9173	-1.4127	52.4275	15.6238	204.7153
2006	0.0978	0.3880	2.8959	5.2908	-0.6564	14.3040	14.1806	111.5631
2007	0.0955	0.4147	2.4556	4.4316	-0.4390	7.5080	9.1415	60.2069
2008	0.0442	0.4946	1.5620	3.1579	-0.7775	22.0724	5.7532	27.4816
2009	0.0046	0.4965	1.8174	3.5938	-1.8234	30.7523	14.4933	202.4204
2010	0.0577	0.3886	2.0711	4.5325	-0.8001	21.5259	13.9340	202.9795
2011	0.0371	0.4265	1.8913	4.0624	-2.5217	65.0531	51.8040	1393.4141
2012	0.0443	0.4427	2.0907	4.6814	-0.1088	10.7876	12.2349	89.3506

Figure 1: Plot of running medians of groups of 500 ordered by E/BV

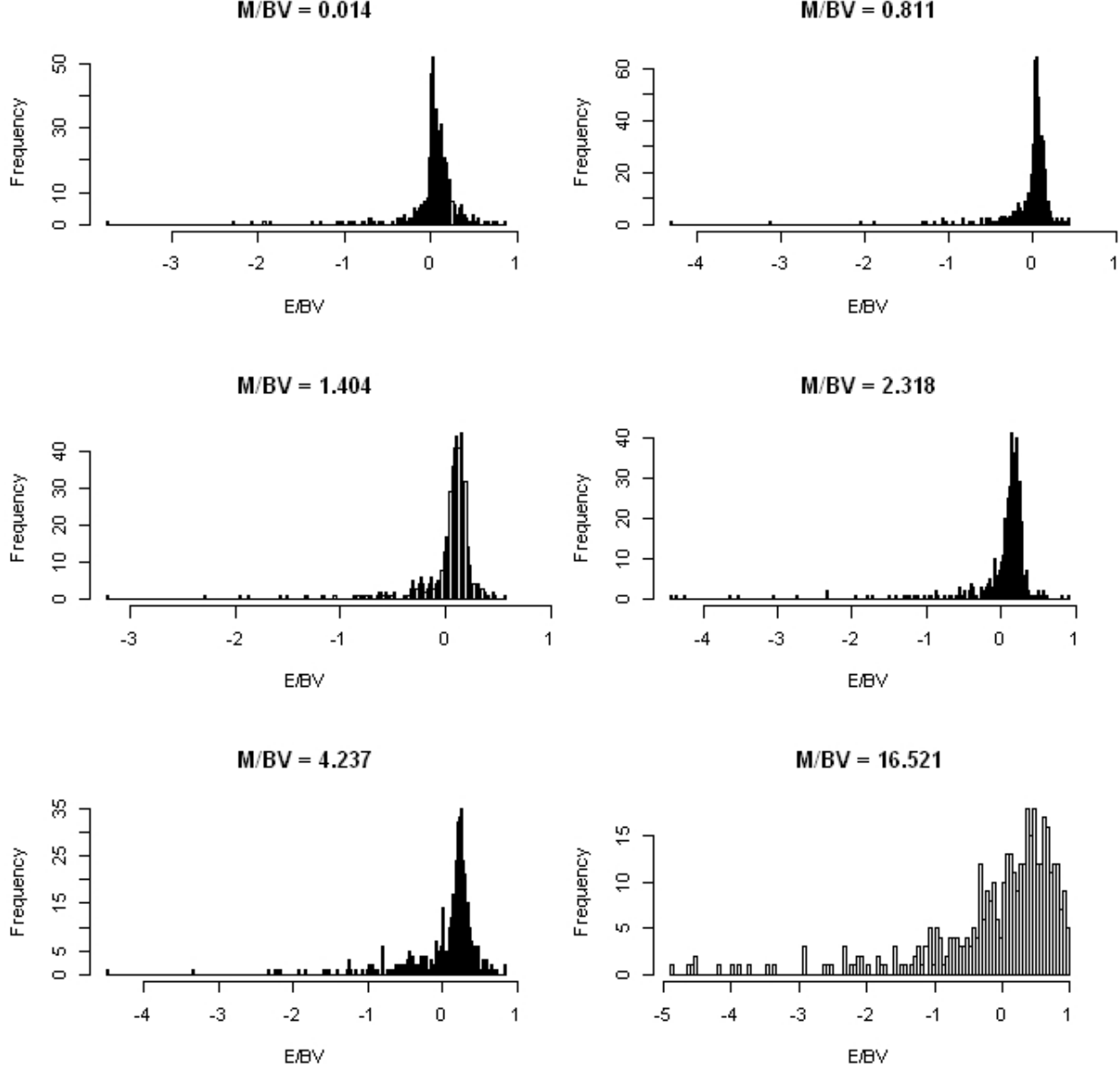


To further validate our sample, we applied equation (2) and compared the results with previous papers, in particular, Burgstahler and Dichev (1997), Ashton et al. (2003) and Darrough and Ye (2007). The sample was ordered by E/BV and plotted against M/BV by creating running medians of both variables with a span of 500¹ as in figure 1. The plot shows a remarkably similar pattern to previous papers². There is a strong negative tail made up of 27% of the sample that reveals the negative relationship that is the basis of the puzzle. The curvature noted in Ashton et al. (2003) and Burgstahler and Dichev (1997) seems to have disappeared, instead there is a remarkable linearity that extends even to the outliers. Using linear regression on non overlapping medians (R^2 of 0.998), the slope of the positive valuations represents an annuity capitalisation rate of 7.9% ie M/BV is greater by a factor of 12.62. The reasonableness of these figures makes it all the more curious that there is such a large and apparently inexplicable negative tail that appears here to be more prominent than previous studies.

¹The highest M/BV group of 357 was omitted to avoid the end problem

²Averages produced a similar but fuzzier picture not reported here.

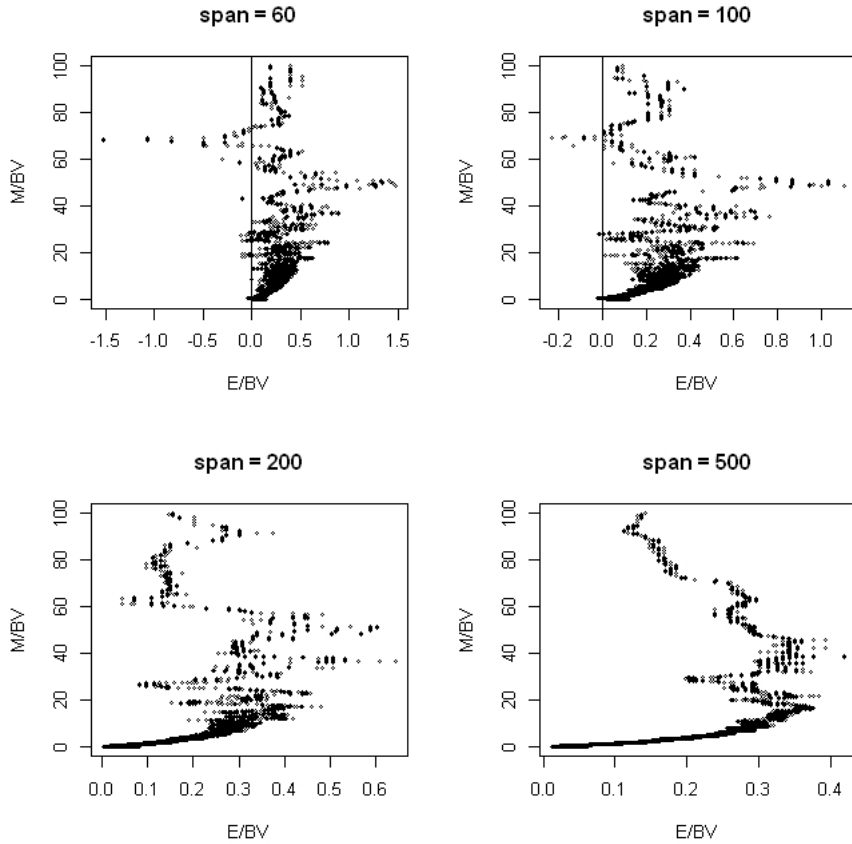
Figure 2: Histogram of groups of 500 E/BV s ordered by M/BV



The first stage of the analysis was to validate the concept of a market expectation. Re-ordering the sample by M/BV , conditional histograms were plotted of 500 E/BV s using 200 bins on a selection of M/BV s across the range. It may be the case that the medians of the grouped E/BV s do not represent some measure of centrality but are merely central observations of rectangular type distributions or some other form lacking in a clear kernel (unimodal) and are likely to be unstable. To illustrate the results, 6 groups are shown in figure

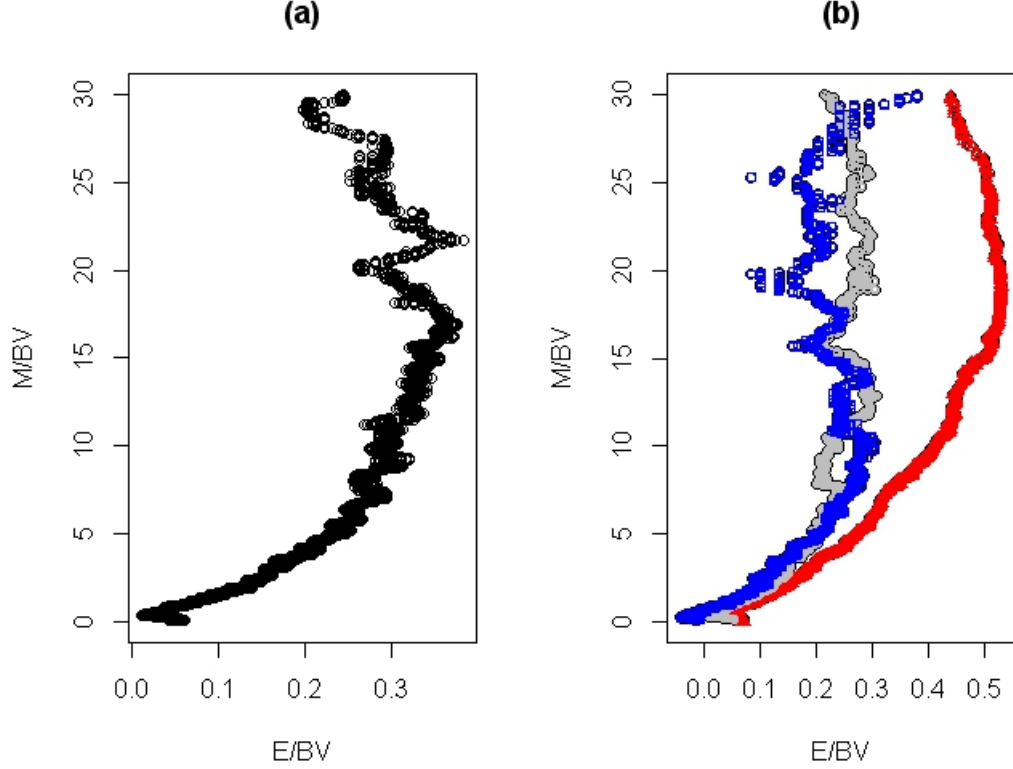
2 at regular intervals starting with the lowest M/BV . The histograms show reasonably well shaped distributions with large left (negative E/BV) tails. It seems clear from figure 2 that there is a clustering of E/BVs around what seems reasonable to regard as the typical value for a given M/BV . The distributions also show that earnings are far less likely to overstate future expectations - the right hand tail. Together the distributions reflect the well established conservative measurement principles in accounting, it is interesting that this bias can be measured in this way.

Figure 3: Plot of running medians of groups ordered by M/BV



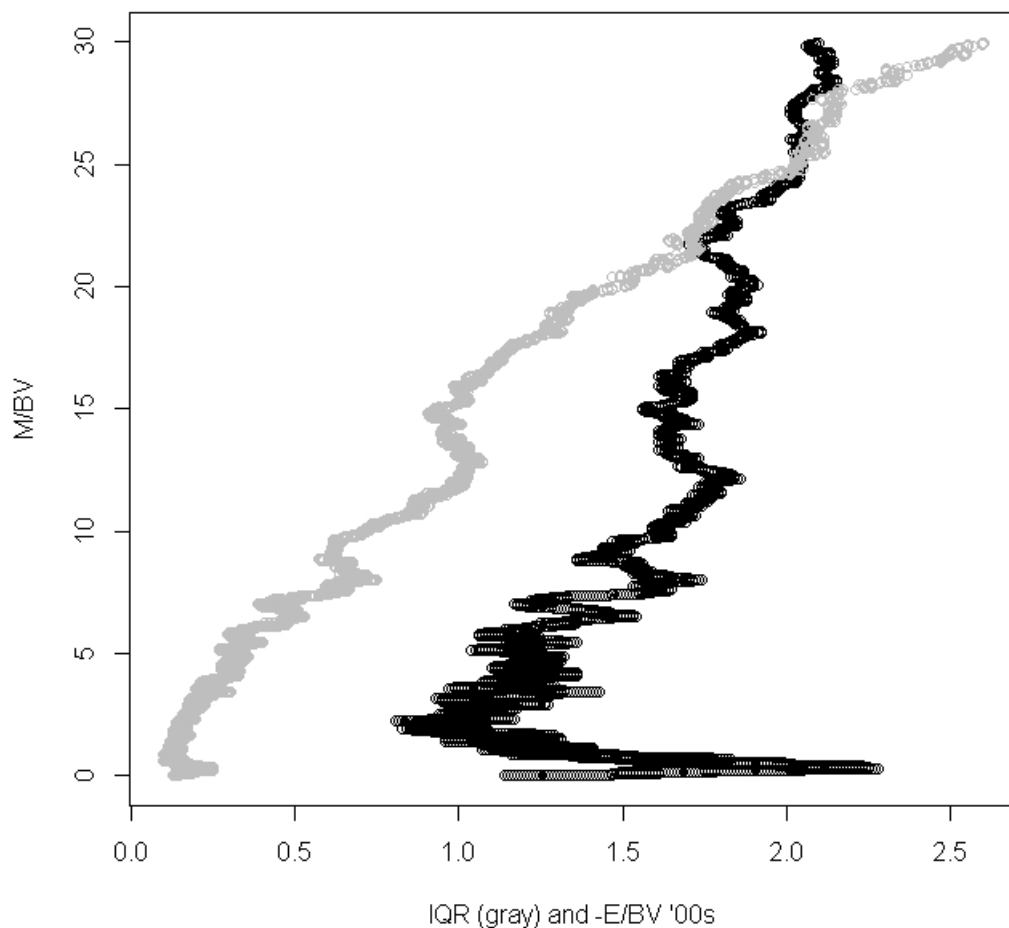
The second stage was to plot the median E/BVs across differing spans of M/BV as in figure 3. The span is critical in tracing a relationship as our results show. In this context it is a measure of the strength of the relationship, the degree of smoothing required. However, even with a span as low as 60, the negative relationship is no longer apparent. Most importantly, however, it is apparent that the plots converge as the span increases, suggesting a reasonably stable relationship. This is further tested in figure 4(b) showing stability over time.

Figure 4: Expected E/BV s conditioned on M/BV (a) 1988 to 2012; (b) red triangle 1988 to 1996, grey circle 1997 to 2005 and blue square 2006 to 2012



The third stage is to interpret the relationship. Taking spans of 500 for clarity, figure 4 shows that from a M/BV of about 0.4 (from more detailed graphs not reported here) to 7 there appears to be a relatively smooth relationship between E/BV and M/BV in the hypothesised direction. The slope of the line is 22.9 (using non overlapping medians, R^2 of 94.4%) an implied discount factor of 4.4%. The curvature noted in Burgstahler and Dichev (1997) and Ashton et al. (2003) becomes more extreme above a M/BV of about 7. From figures 2 and 4 (a), the E/BV at higher M/BV s drifts back to the overall median (as in Table 1). The resulting curvature appears to be part of the declining relevance of earnings as M/BV increases. This is not surprising in that a M/BV of 60 surely implies that the accounts are of little relevance to valuation! From figure 4 (a), below a M/BV of about 0.4, E/BV increases. This effect, we suggest, is also a breaking down of the relationship between E/BV and M/BV . When the market is valuing a company at less than half the tangible assets it seems reasonable to hypothesize that the market has little confidence in the future of the company and hence the current earnings are of little relevance. Figure 4 (b) also shows a remarkable similarity in the relationship over time. The most notable difference is the irregularity at higher levels particularly in the

Figure 5: Negative E/BV s and Inter Quartile Ranges of E/BV s ordered by M/BV



later years. Whereas the studies of Burgstahler and Dichev (1997) and Ashton et al. (2003) cover the 1990's and show, as here, that the relationship is smooth over the range, this cannot be said of later years. Further research is needed to determine what other factors affect market values at these higher market to book values.

The principal observation from figures 2,3 and 4 is that there is no evidence of market expectations of losses and hence no positive relationship between losses and market value. In the equation 4 model, all losses are represented as part of distributions that have a positive market expectation. Figure 5 shows an increasing spread of E/BV as M/BV increases and consequently an increasing

incidence of negative E/BV s. Thus the traditional approach that takes E/BV as an expectation would indeed trace an apparent positive relationship between losses and market value.

CONCLUDING DISCUSSION

The role of earnings in valuing a company is central to both theory and practice. The observation in previous papers that higher losses appear to be related to higher market values is therefore an important anomaly. We have identified the central problem as being the assumption that inter temporal earnings follow a martingale process thus equating observations with expectations. We have based our approach on the thesis that it is allowable for earnings not to be a martingale process (Samuelson, 1965). Exceptionally high valued firms experiencing losses can be expected to return to profitability for reasons that are more than just random variations around an expectation. By creating a series of distributions of earnings conditioned on market value and taking a measure of centrality of the distributions, we in effect detach observation from expectation. Taking the median as the centrality measure isolates outliers from the measurement process and is observably a more robust process (Table 1). As is normal with non parametric analysis (Fox, 2000) this relationship was observed rather than modelled. Tracing the market expectation of E/BV for varying levels of M/BV we show that there are no negative E/BV expectations for reasonable samples sizes and hence no relationship between losses and market value. We find a remarkably linear positive relationship that is stable over time between earnings and market value between a M/BV of 0.4 and 7 which breaks down beyond this range. It seems eminently sensible to regard high levels and very low levels of M/BV to be unrelated to either book value or earnings.

This more flexible interpretation of the role of an observation in the valuation process is very much in accord with what we observe. The huge financial centres across the world have share valuation as one of their central activities. The notion that it can be compressed into a traditional and fairly simple regression equation seems highly ambitious. Relating reported earnings to expectations is, as represented here, a far more complex, richer process.

Finally, our approach is not particular to earnings as an explanatory variable but is general to all variables that use historic values to model market prices. The particular relevance to earnings is the non normal nature of the inter temporal earnings for a significant number of firm years. This resulted in mis-identifying observations as expectations which was the source of the puzzle.

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