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IMA Journal of Management Mathematics (2015) Page 1 of 21
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Reliability equivalence factors for a series–parallel system of components with exponentiated Weibull lifetimes Q1

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We study reliability equivalence factors of a system of independent and identical components with exponentiated Weibull lifetimes. The system has n subsystems connected in parallel and subsystem i has m_i components connected in series, $i = 1, \ldots, n$. We consider improving the reliability of the system by (a) a reduction method and (b) several duplication methods: (i) hot duplication; (ii) cold duplication with perfect switching; (iii) cold duplication with imperfect switching. We compute two types of reliability equivalence factors: survival equivalence factors and mean equivalence factors. Although our methods adapt for more general lifetime models, we use the exponentiated Weibull distribution because it is flexible and enables comparisons with other reliability equivalence studies. The example we present demonstrates the potential for applying these methods to address specific questions that arise when attempting to improve the reliability of simple systems or simple configurations of possibly complex subsystems in many diverse applications. Q6

Keywords: series–parallel system; exponentiated Weibull distribution; reliability equivalence factor; reduction method; duplication method.

1. Introduction

The concept of reliability equivalence factors was introduced by Råde (1993a,b). He applied this concept to simple systems that consist of one component or two components connected in series or parallel. Later, Sarhan (2000, 2005) and Sarhan *et al.* (2008) applied this concept to more general systems. Most of the designs considered have components with exponential lifetime distributions although some studies applied this concept to other lifetime distributions, such as the Weibull distribution, El-Damcese (2009), gamma distribution, Xia & Zhang (2007), exponentiated exponential distribution, Abdelkader *et al.* (2013) and recently Burr-type X distribution, Migdadi & Al-Batah (2014).

There are two main methods for improving a system's design. The first method is reduction, which involves improving the reliability of the system by reducing the failure rate by a factor ρ for some of the system components, where $\rho \in (0, 1)$. This can be achieved by replacing standard components with more expensive, higher-quality components. The second method for improving a system's design is redundancy duplication, which involves adding extra components in parallel to existing system components. There are three ways to add extra components to the system: hot duplication; cold duplication with perfect switch; cold duplication with imperfect switch. Sometimes, and for many different reasons such as high cost and space limitation, it is impossible to improve the reliability of the system by the redundancy duplication method. Reliability equivalence factors refer to the factors by which the failure rates of some of the system's components must be reduced in order to attain equality of the reliability of the system with that of a better system. Such information can then provide useful input for planning various maintenance strategies as discussed by Percy *et al.* (2010). Q7

Series–parallel and parallel–series system configurations are the building blocks for more complicated systems, and an understanding of the analytical processes and optimal strategies involved for these systems enables and informs arbitrary generalization to complex situations. However, only one of these is needed to illustrate the methodology and we choose the series–parallel system here. In this study, we also assume that all the system's components are independent and follow the exponentiated Weibull distribution of Mudholkar & Srivastava (1993) with identical parameters. We choose this distribution because it includes all common shapes of hazard function and because its hazard and reliability are elementary functions. In particular, it includes the monotone hazard function of the Weibull distribution but also permits bathtub and inverted bathtub hazard functions. Special cases of the exponentiated Weibull distribution include the Weibull, exponentiated exponential and Burr-type X distributions mentioned above. Firstly, we compute the reliability function and the mean time to failure (MTTF) of the original system. Secondly, we compute the reliability functions and MTTFs of the systems following improvement according to reduction, hot duplication and cold duplication (perfect and imperfect) methods. Thirdly, we equate the reliability function and the MTTF of the system improved according to the reduction method with the reliability function and the MTTF of the system improved according to each of the duplication methods to determine the reliability equivalence factors. Finally, we illustrate the results obtained with an application example by presenting summary tables and figures. This paper expands considerably upon some preliminary ideas that Alghamdi & Percy (2014) presented, by investigating both survival and mean reliability equivalence factors (MREFs) for a series–parallel system, and both hot and cold duplication methods.

2. Series–parallel system

The system we consider here is shown in Fig. 1 and consists of n subsystems connected in parallel, where subsystem i consists of m_i components that are connected in series for $i = 1, 2, \ldots, n$. Such a system is usually referred to as a series–parallel system (El-Damcese, 2009). Q8

We assume that the lifetimes of all the system's components are independent and follow the exponentiated Weibull distribution with identical parameters; see Mudholkar & Srivastava (1993) and Lai (2014). The exponentiated Weibull distribution generalizes well-known lifetime distributions including exponential, Rayleigh and Weibull, and has the desirable properties of flexibility and tractability noted earlier.

Under this assumption, the reliability function for each component j ($j = 1, 2, \ldots, m_i$) in subsystem i ($i = 1, 2, \ldots, n$) is given by

$$r(t) = 1 - (1 - \mathrm{e}^{-\alpha t^{\beta}})^{\theta} \tag{1}$$

for $t \geqslant 0$, as the lifetimes of components are unaffected by failures of other components. Now define $R_i(t)$ to be the reliability function of subsystem i. This then takes the form

$$R_i(t) = \{r(t)\}^{m_i} \tag{2}$$

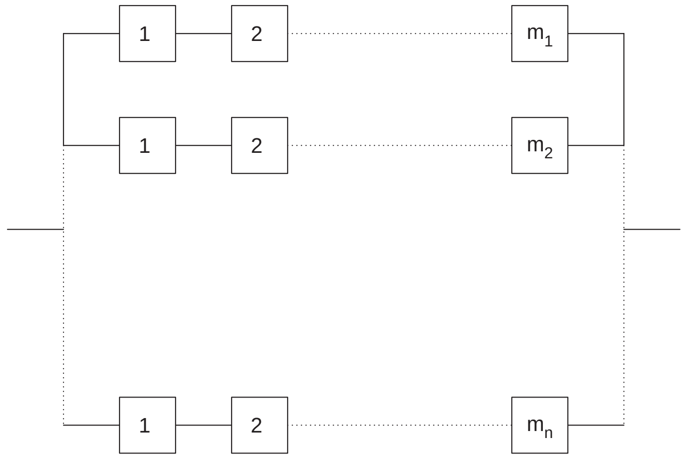

FIG. 1. Series–parallel system.

for $t \geqslant 0$, so the reliability function of the series–parallel system is

$$R(t) = 1 - \prod_{i=1}^{n}\{1 - R_i(t)\} \tag{3}$$

for $t \geqslant 0$, and the MTTF of the series–parallel system is given by

$$\text{MTTF} = \int_0^{\infty} R(t)\,\mathrm{d}t. \tag{4}$$

3. Designs of improved systems

The two main approaches for improving a system are reduction methods and standby redundancy (duplication) methods. The latter comprise two variations, hot duplication and cold duplication. Furthermore, cold duplication can be performed with perfect switch or imperfect switch. In this section, we derive the reliability function and the MTTF, primarily for the series–parallel system, when improved according to the methods identified above.

3.1 *Reduction method*

As mentioned in the introduction, the reliability of a system can be improved by reducing the failure rate for some of the system’s components by a factor $\rho \in (0, 1)$. For the exponentiated Weibull distribution, reducing only the scale parameter α reduces the failure rate. Here, we consider reducing α for a set A of the system’s components by a factor $\rho \in (0, 1)$, in order to reduce the failure rate (hazard function) for the whole system. This is a logical procedure for the exponentiated Weibull distribution.

Define a_i $(i = 1, 2, \ldots, n)$ to be the number of components in subsystem i whose failure rate is reduced, so $a_i \in \{0, 1, \ldots, m_i\}$ and the cardinality of the set of improved components is $|A| = \sum_{i=1}^{n} a_i$. By comparison with Equation (2), we see that the reliability function $R_i^{(A)}(t)$ of subsystem i is then given by

$$R_i^{(A)}(t) = \{1 - (1 - \mathrm{e}^{-\rho\alpha t^\beta})^\theta\}^{a_i}\{1 - (1 - \mathrm{e}^{-\alpha t^\beta})^\theta\}^{m_i - a_i}$$

for $t \geqslant 0$ from Equation (1) and by comparison with Equation (3), since the components are connected in series. Then the reliability function of the system takes the form

$$R^{(A)}(t) = 1 - \prod_{i=1}^{n}\{1 - R_i^{(A)}(t)\}$$

since the subsystems are connected in parallel. We can then compute the MTTF of this series–parallel system as

$$\mathrm{MTTF}^{(A)} = \int_0^\infty R^{(A)}(t)\,\mathrm{d}t.$$

3.2 *Duplication methods*

Now we obtain the corresponding reliability measures of the system when it is improved by duplication. We derive the reliability function and the MTTF, primarily for the series–parallel system, when improved according to the hot duplication method and the cold duplication methods with perfect and imperfect switches.

3.2.1 *Hot duplication method.* This means that some of the system components are duplicated in parallel by similar components. We assume that, in the hot duplication method, each component of the set B is augmented by introducing a new but identical component in the same subsystem.

Let b_i $(i = 1, 2, \ldots, n)$ be the number of components in subsystem i whose reliability is improved according to the hot duplication method, so $b_i \in \{0, 1, \ldots, m_i\}$ and $|B| = \sum_{i=1}^n b_i$. The reliability function $R_i^{(B)}(t)$ of subsystem i is given by

$$R_i^{(B)}(t) = \{1 - (1 - \mathrm{e}^{-\alpha t^\beta})^{2\theta}\}^{b_i}\{1 - (1 - \mathrm{e}^{-\alpha t^\beta})^\theta\}^{m_i - b_i}$$

for $t \geqslant 0$ from Equation (1), since the components are connected in series. Then the reliability function of the whole system takes the form

$$R^{(B)}(t) = 1 - \prod_{i=1}^{n}\{1 - R_i^{(B)}(t)\}$$

for $t \geqslant 0$, and the MTTF of this series–parallel system can then computed as

$$\mathrm{MTTF}^{(B)} = \int_0^\infty R^{(B)}(t)\,\mathrm{d}t.$$

3.2.2 *Cold duplication method with perfect switch.* This approach to improving system reliability means that a similar component is connected with an original component in such a way that it is activated immediately upon failure of the original component. For this aspect of our analysis, the cold duplication method assumes that each component of a set C is improved by introducing a new but identical component with a perfect switch. The switch immediately transfers load to the standby component when the original component fails, which means the switch operation time is negligible.

Let c_i $(i = 1, 2, \ldots, n)$ be the number of components in subsystem i, whose reliability is improved according to the cold duplication method with perfect switch, so $c_i \in \{0, 1, \ldots, m_i\}$ and $|C| = \sum_{i=1}^{n} c_i$. Let $s_1(t)$ be the reliability function of each component whose reliability is improved according to cold duplication with perfect switch. Regarding a definition of cold duplication with perfect switch, we can describe this improvement as a renewal process with only one renewal (Gamiz *et al.*, 2011). Using the convolution technique, the reliability function of each component whose reliability is improved according to cold duplication with perfect switch can be derived as follows:

$$s_1(t) = 1 - \int_0^t \frac{-\mathrm{d}r(x)}{\mathrm{d}x}[1 - r(t-x)]\,\mathrm{d}x, \tag{5}$$

where $r()$ is the reliability function for the exponentiated Weibull lifetime distribution presented in Equation (1). By comparison with Equation (2), we see that the reliability function $R_i^{(C)}(t)$ of subsystem i is given by

$$R_i^{(C)}(t) = \{s_1(t)\}^{c_i}\{1 - (1 - \mathrm{e}^{-\alpha t^{\beta}})^{\theta}\}^{m_i - c_i}$$

for $t \geqslant 0$, from Equation (1), since the components are connected in series. Then the reliability function of the system takes the form

$$R^{(C)}(t) = 1 - \prod_{i=1}^{n}\{1 - R_i^{(C)}(t)\}$$

for $t \geqslant 0$, and $s_1(t)$ as defined in Equation (5), since the subsystems are connected in parallel. We can then compute the MTTF of this series–parallel system as

$$\mathrm{MTTF}^{(C)} = \int_0^{\infty} R^{(C)}(t)\,\mathrm{d}t.$$

3.2.3 *Cold duplication method with imperfect switch.* This approach to improving system reliability means that a similar component is connected with an original component by a cold standby via a random switch having a constant failure rate. For this aspect of our analysis, the cold duplication method assumes that each component of a set D is improved by introducing a new but identical component with an imperfect switch.

Let d_i $(i = 1, 2, \ldots, n)$ be the number of components in subsystem i, whose reliability is improved according to cold duplication with imperfect switch, so $d_i \in \{0, 1, \ldots, m_i\}$ and $|D| = \sum_{i=1}^{n} d_i$. Let $s_2(t)$ be the reliability function of each component whose reliability is improved according to cold duplication with imperfect switch. Following the same technique that we used for cold duplication with perfect switch but with the extra condition that the switch is not 100% reliable, Billinton & Allan (1992), we have

$$s_2(t) = 1 - \int_0^t \frac{-\mathrm{d}r(x)}{\mathrm{d}x}[1 - r(t-x)s_3(x)]\,\mathrm{d}x, \tag{6}$$

where $r()$ was defined in Equation (1), and $s_3()$ is the reliability function for the imperfect switch. The imperfect switch is chosen to have a constant failure rate λ, which means that it has an exponential lifetime distribution with parameter λ and so

$$s_3(t) = \mathrm{e}^{-\lambda t}. \tag{7}$$

The reliability function $R_i^{(D)}(t)$ of subsystem i is given by

$$R_i^{(D)}(t) = \{s_2(t)\}^{d_i}\{1 - (1 - \mathrm{e}^{-\alpha t^{\beta}})^{\theta}\}^{m_i - d_i}$$

for $t \geqslant 0$, from Equation (1), since the components are connected in series. Then the reliability function of this series–parallel system takes the form

$$R^{(D)}(t) = 1 - \prod_{i=1}^{n}\{1 - R_i^{(D)}(t)\}$$

for $t \geqslant 0$ and $s_2(t)$ as defined in Equation (6), since the subsystems are connected in parallel. We can then compute the MTTF of this series–parallel system as

$$\mathrm{MTTF}^{(D)} = \int_0^{\infty} R^{(D)}(t)\,\mathrm{d}t.$$

4. Reliability equivalence factors

According to El-Damcese (2009), 'A reliability equivalence factor is a factor by which a characteristic of components of a system design has to be multiplied in order to reach equality of a characteristic of this design and a different design regarded as a standard'.

We compute two types of reliability equivalence measures. The first type involves survival reliability equivalence factors (SREFs) and these are determined from the reliability or survival function. The second type involves MREFs and these are determined from the MTTF.

4.1 *Survival reliability equivalence factors*

The idea of SREFs is to assess what degrees of intervention are required to establish equivalence between the reliability functions of a system whose reliability is improved according to one of the duplication methods and a system whose reliability is improved according to the reduction method.

That is, to derive the SREFs, we have to solve the following set of equations:

$$R^{(A)}(t) = R^{(H)}(t) = \omega, \quad H = B, C, D \tag{8}$$

for the appropriate reduction factor ρ and time fractile t corresponding to a specified reliability requirement ω. The system of equations in (8) has no closed-form solutions for our problem and we perform the calculations numerically using a mathematical package.

4.2 *Mean reliability equivalence factors*

The idea of MREFs is to assess what degrees of intervention are required to establish equivalence between the MTTF of a system whose reliability is improved according to one of the duplication methods and a system whose reliability is improved according to the reduction method.

That is, to derive the MREFs, we have to solve the following set of equations:

$$\mathrm{MTTF}^{(A)} = \mathrm{MTTF}^{(H)}, \quad H = B, C, D \tag{9}$$

for the appropriate reduction factor ρ. The system of equations in (9) also has no closed-form solutions and can be solved using a mathematical package. We used Matlab and Mathcad to derive and compare both sets of results for SREFs and MREFs.

5. Numerical analysis and results

Suppose that we have a series–parallel system consisting of two subsystems connected in parallel as shown in Fig. 2. It is easy to imagine systems that display this structure. For example, one of the authors travels to work by train on one of two routes, which comprise two and three stages, respectively, each of which is vulnerable to random failures. The first subsystem that we consider here has two components connected in series and the second subsystem has three components connected in series. This means that $n = 2$, $m_1 = 2$, $m_2 = 3$ and the total number of components is $m = 5$. All of the system's components are assumed to be independent and identically distributed, with lifetimes that behave according to an exponentiated Weibull distribution with parameters $\alpha = 1$, $\beta = 2$ and $\theta = 3$. We define:

1. $A_k^{(i,j)}$, $i = 0, 1, 2$, $j = 0, 1, 2, 3$ and $k = i + j$, to represent a reduction method that requires us to reduce the failure rate of i components from the first subsystem and j from the second subsystem;
2. $B_k^{(i,j)}$, $i = 0, 1, 2$, $j = 0, 1, 2, 3$ and $k = i + j$, to represent hot duplication methods when i components are added to the first subsystem and j to the second subsystem;
3. $C_k^{(i,j)}$, $i = 0, 1, 2$, $j = 0, 1, 2, 3$ and $k = i + j$, to represent cold duplication methods with perfect switch when i components are added to the first subsystem and j components are added to the second subsystem;
4. $D_k^{(i,j)}$, $i = 0, 1, 2$, $j = 0, 1, 2, 3$ and $k = i + j$, to represent cold duplication methods with imperfect switch when i components are added to the first subsystem and j components are added to the second subsystem.

For this scenario, in Tables 1–3 the SREFs for hot and cold (perfect and imperfect) duplication are calculated using Matlab according to the above formulae where ω is chosen to be $0.1, 0.5, 0.9$ and the imperfect switch has a constant failure rate $\lambda = 0.05$. For more discussions based on the results presented in Tables 1–3, the following conditions may be observed:

- Reducing the failure rate of one component in the second subsystem (which we denote as $A_1^{(0,1)}$) by setting $\rho = 0.7238$ improves the reliability of the system like adding one component to the second subsystem (which we denote as $B_1^{(0,1)}$) according to a hot duplication method where the reliability function of the system is chosen to be $\omega = 0.1$; see Table 1. Q9
- Reducing the failure rate of each component belonging to the set $A_5^{(2,3)}$ of the system components by setting $\rho = 0.9040$ improves the reliability of the system like adding a set $C_1^{(0,1)}$ of components to the system according to a cold duplication method with perfect switch where the reliability function of the system is chosen to be $\omega = 0.5$; see Table 2.

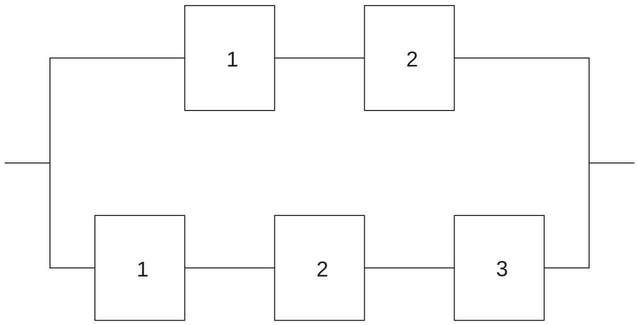

FIG. 2. Series–parallel system consisting of two subsystems connected in parallel.

- Reducing the failure rate of each component belonging to the set $A_5^{(2,3)}$ of the system components by setting factor $\rho = 0.2177$ improves the reliability of the system like adding a set $D_5^{(2,3)}$ of components to the system according to a cold duplication method with perfect switch where the reliability function of the system is chosen to be $\omega = 0.9$; see Table 3.
- Missing values of the SREFs mean that it is not possible to reduce the failure rate for the set A of components in order to improve the system reliability to be equivalent with the system reliability that can be obtained by improving the sets B, C, D of components according to duplication methods.
- In the same manner, one can interpret the other results presented in Tables 1–3.

Tables 4–6 present the MREFs for hot and cold (perfect and imperfect) duplication. Based on the results presented in those tables, we see that the following conditions hold:

- The modified system that can be obtained by improving the set $H_1^{(0,1)}$, where $H = B, C, D$ of the system components, according to hot and cold (perfect and imperfect) duplication has the same MTTF of that system which can be obtained by reducing the failure rate of each component belonging to the set $A_1^{(0,1)}$ by factors $\rho = 0.614, 0.134, 0.226$, respectively.
- Empty cells of MREFs mean that it is not possible to reduce the failure rate of the set A components in order to improve the MTTF of the system to be equivalent with the MTTF of the system that can be obtained by improving the sets B, C, D of components according to the duplication methods.
- In the same manner, one can interpret the other results presented in Tables 4–6,

Table 7 presents the MTTF of the modified systems assuming hot and cold duplication methods, the latter with perfect and imperfect switch, assuming a constant failure rate $\lambda = 0.05$. The MTTF of the original system is 1.172. From this table, one can conclude that

$$\mathrm{MTTF} < \mathrm{MTTF}^{(B)} < \mathrm{MTTF}^{(D)} < \mathrm{MTTF}^{(C)}.$$

Figure 3 explains the improvement strategies to calculate the SREFs. Figure 4 presents reliability functions of the original and some modified systems. From these figures, one may observe that, for this scenario:

TABLE 1 *Hot SREFs*

	ω	$B_1^{(0,1)}$	$B_2^{(0,2)}$	$B_3^{(0,3)}$	$B_1^{(1,0)}$	$B_2^{(1,1)}$	$B_3^{(1,2)}$	$B_4^{(1,3)}$	$B_2^{(2,0)}$	$B_3^{(2,1)}$	$B_4^{(2,2)}$	$B_5^{(2,3)}$
	0.1	0.7238	0.4111	—	—	—	—	—	—	—	—	—
$A_1^{(0,1)}$	0.5	0.6009	—	—	—	—	—	—	—	—	—	—
	0.9	0.4519	—	—	—	—	—	—	—	—	—	—
	0.1	0.8657	0.7330	0.6047	0.6482	0.6108	0.5591	0.4930	0.4250	0.4134	0.3944	0.3648
$A_2^{(0,2)}$	0.5	0.8173	0.6203	0.4006	0.6483	0.5501	0.4239	0.2429	0.2666	0.1961	—	—
	0.9	0.7803	0.4800	—	0.6188	0.4345	—	—	—	—	—	—
	0.1	0.9111	0.8251	0.7445	0.7714	0.7482	0.7167	0.6774	0.6384	0.6320	0.6216	0.6057
$A_3^{(0,3)}$	0.5	0.8807	0.7603	0.6444	0.7767	0.7206	0.6554	0.5836	0.5910	0.5712	0.5444	0.5096
	0.9	0.8597	0.6998	0.5234	0.7675	0.6807	0.5790	0.4623	0.5035	0.4720	0.4312	0.3783
	0.1	0.9182	0.8163	0.6981	0.7403	0.7042	0.6517	0.5804	0.5022	0.4884	0.4654	0.4290
$A_1^{(1,0)}$	0.5	0.8111	0.5830	0.2579	0.6173	0.4929	0.3029	—	—	—	—	—
	0.9	0.7162	—	—	0.4671	—	—	—	—	—	—	—
	0.1	0.9336	0.8459	0.7381	0.7773	0.7438	0.6943	0.6255	0.5487	0.5350	0.5122	0.4760
$A_2^{(1,1)}$	0.5	0.8677	0.6963	0.4697	0.7226	0.6279	0.4953	0.2879	0.3159	0.2322	—	—
	0.9	0.8204	0.5318	—	0.6713	0.4839	—	—	—	—	—	—
	0.1	0.9451	0.8730	0.7848	0.8167	0.7894	0.7491	0.6937	0.6327	0.6219	0.6041	0.5762
$A_3^{(1,2)}$	0.5	0.9013	0.7773	0.6259	0.7959	0.7295	0.6419	0.5283	0.5410	0.5062	0.4552	0.3808
	0.9	0.8732	0.6922	0.3914	0.7749	0.6667	0.5078	0.1574	0.3384	0.2208	—	—
	0.1	0.9537	0.8945	0.8248	0.8497	0.8284	0.7976	0.7565	0.7129	0.7055	0.6932	0.6744
$A_4^{(1,3)}$	0.5	0.9222	0.8286	0.7224	0.8423	0.7940	0.7331	0.6600	0.6679	0.6467	0.6173	0.5780
	0.9	0.9030	0.7753	0.6084	0.8318	0.7587	0.6643	0.5433	0.5876	0.5539	0.5086	0.4473
	0.1	0.9594	0.9095	0.8532	0.8731	0.8560	0.8315	0.7991	0.7647	0.7588	0.7491	0.7341
$A_2^{(2,0)}$	0.5	0.9085	0.8090	0.7070	0.8230	0.7747	0.7169	0.6511	0.6580	0.6395	0.6141	0.5807
	0.9	0.8697	0.7185	0.5488	0.7828	0.7003	0.6026	0.4894	0.5295	0.4988	0.4590	0.4071

(*continued*).

TABLE 1 *Continued.*

	ω	$B_1^{(0,1)}$	$B_2^{(0,2)}$	$B_3^{(0,3)}$	$B_1^{(1,0)}$	$B_2^{(1,1)}$	$B_3^{(1,2)}$	$B_4^{(1,3)}$	$B_2^{(2,0)}$	$B_3^{(2,1)}$	$B_4^{(2,2)}$	$B_5^{(2,3)}$
	0.1	0.9634	0.9167	0.8617	0.8813	0.8645	0.8401	0.8073	0.7722	0.7661	0.7562	0.7407
$A_3^{(2,1)}$	0.5	0.9235	0.8332	0.7333	0.8463	0.8004	0.7433	0.6757	0.6829	0.6635	0.6366	0.6009
	0.9	0.8954	0.7612	0.5929	0.8201	0.7441	0.6483	0.5297	0.5726	0.5399	0.4966	0.4390
	0.1	0.9669	0.9239	0.8720	0.8907	0.8747	0.8512	0.8193	0.7846	0.7785	0.7685	0.7530
$A_4^{(2,2)}$	0.5	0.9352	0.8563	0.7649	0.8679	0.8268	0.7742	0.7099	0.7169	0.6980	0.6715	0.6355
	0.9	0.9144	0.8008	0.6489	0.8513	0.7859	0.7004	0.5879	0.6296	0.5979	0.5548	0.4952
	0.1	0.9700	0.9308	0.8831	0.9004	0.8856	0.8640	0.8344	0.8020	0.7963	0.7869	0.7723
$A_5^{(2,3)}$	0.5	0.9443	0.8762	0.7968	0.8863	0.8507	0.8050	0.7486	0.7548	0.7381	0.7147	0.6826
	0.9	0.9283	0.8336	0.7071	0.8756	0.8211	0.7500	0.6559	0.6909	0.6644	0.6280	0.5771

TABLE 2 *Cold SREFs with perfect switch*

	ω	$C_1^{(0,1)}$	$C_2^{(0,2)}$	$C_3^{(0,3)}$	$C_1^{(1,0)}$	$C_2^{(1,1)}$	$C_3^{(1,2)}$	$C_4^{(1,3)}$	$C_2^{(2,0)}$	$C_3^{(2,1)}$	$C_4^{(2,2)}$	$C_5^{(2,3)}$
	0.1	0.1409	—	—	—	—	—	—	—	—	—	—
$A_1^{(0,1)}$	0.5	0.1208	—	—	—	—	—	—	—	—	—	—
	0.9	0.0774	—	—	—	—	—	—	—	—	—	—
	0.1	0.6631	0.1749	—	0.1809	0.1370	—	—	—	—	—	—
$A_2^{(0,2)}$	0.5	0.6984	0.1207	—	0.3541	0.1095	—	—	—	—	—	—
	0.9	0.7302	0.0917	—	0.5010	0.0917	—	—	—	—	—	—
	0.1	0.7808	0.5209	0.2476	0.5230	0.5097	0.4413	0.2470	0.2087	0.2087	0.2085	0.2000
$A_3^{(0,3)}$	0.5	0.8067	0.5580	0.2036	0.6240	0.5568	0.4380	0.2011	0.1779	0.1779	0.1771	0.1550
	0.9	0.8298	0.6054	0.1534	0.7092	0.6054	0.4576	0.1468	0.1388	0.1379	0.1334	0.1015
	0.1	0.7543	0.1654	—	0.1756	0.0853	—	—	—	—	—	—
$A_1^{(1,0)}$	0.5	0.6771	—	—	0.1194	—	—	—	—	—	—	—
	0.9	0.6450	—	—	0.0622	—	—	—	—	—	—	—
	0.1	0.7901	0.2274	—	0.2355	0.1766	—	—	—	—	—	—
$A_2^{(1,1)}$	0.5	0.7680	0.1421	—	0.4174	0.1288	—	—	—	—	—	—
	0.9	0.7756	0.1000	—	0.5535	0.1000	—	—	—	—	—	—
	0.1	0.8272	0.4051	—	0.4097	0.3792	0.1904	—	—	—	—	—
$A_3^{(1,2)}$	0.5	0.8285	0.4817	—	0.5948	0.4794	0.1396	—	—	—	—	—
	0.9	0.8428	0.5539	—	0.7043	0.5539	0.1000	—	—	—	—	—
	0.1	0.8579	0.5689	0.2485	0.5715	0.5546	0.4679	0.2479	0.2089	0.2089	0.2088	0.2001
$A_4^{(1,3)}$	0.5	0.8666	0.6324	0.2076	0.7019	0.6310	0.4928	0.2049	0.1797	0.1797	0.1789	0.1558
	0.9	0.8806	0.6898	0.1654	0.7834	0.6898	0.5380	0.1573	0.1475	0.1464	0.1410	0.1040
	0.1	0.8797	0.6473	0.3151	0.6495	0.6351	0.5567	0.3144	0.2656	0.2656	0.2654	0.2545
$A_2^{(2,0)}$	0.5	0.8482	0.6271	0.2483	0.6884	0.6259	0.5097	0.2453	0.2170	0.2170	0.2161	0.1892
	0.9	0.8416	0.6281	0.1783	0.7275	0.6281	0.4848	0.1711	0.1622	0.1612	0.1562	0.1196

(*continued*).

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TABLE 2 *Continued.*

	ω	$C_1^{(0,1)}$	$C_2^{(0,2)}$	$C_3^{(0,3)}$	$C_1^{(1,0)}$	$C_2^{(1,1)}$	$C_3^{(1,2)}$	$C_4^{(1,3)}$	$C_2^{(2,0)}$	$C_3^{(2,1)}$	$C_4^{(2,2)}$	$C_5^{(2,3)}$
	0.1	0.8879	0.6511	0.3151	0.6533	0.6384	0.5581	0.3144	0.2656	0.2656	0.2654	0.2545
$A_3^{(2,1)}$	0.5	0.8696	0.6504	0.2485	0.7143	0.6492	0.5239	0.2454	0.2171	0.2171	0.2161	0.1893
	0.9	0.8716	0.6739	0.1808	0.7696	0.6739	0.5247	0.1731	0.1637	0.1626	0.1573	0.1198
	0.1	0.8969	0.6614	0.3154	0.6638	0.6483	0.5647	0.3146	0.2656	0.2656	0.2655	0.2545
$A_4^{(2,2)}$	0.5	0.8885	0.6851	0.2517	0.7469	0.6839	0.5553	0.2484	0.2186	0.2186	0.2176	0.1899
	0.9	0.8946	0.7237	0.1939	0.8080	0.7237	0.5829	0.1845	0.1732	0.1720	0.1657	0.1225
	0.1	0.9060	0.6850	0.3344	0.6872	0.6723	0.5904	0.3337	0.2819	0.2819	0.2817	0.2701
$A_5^{(2,3)}$	0.5	0.9040	0.7267	0.3033	0.7811	0.7257	0.6099	0.2995	0.2651	0.2651	0.2639	0.2311
	0.9	0.9117	0.7694	0.2843	0.8396	0.7694	0.6518	0.2731	0.2591	0.2575	0.2495	0.1914

TABLE 3 *Cold SREFs with imperfect switch* ($\lambda = 0.05$)

	ω	$D_1^{(0,1)}$	$D_2^{(0,2)}$	$D_3^{(0,3)}$	$D_1^{(1,0)}$	$D_2^{(1,1)}$	$D_3^{(1,2)}$	$D_4^{(1,3)}$	$D_2^{(2,0)}$	$D_3^{(2,1)}$	$D_4^{(2,2)}$	$D_5^{(2,3)}$
	0.1	0.2157	—	—	—	—	—	—	—	—	—	—
$A_1^{(0,1)}$	0.5	0.2401	—	—	—	—	—	—	—	—	—	—
	0.9	0.2494	—	—	—	—	—	—	—	—	—	—
	0.1	0.6755	0.2153	—	0.2060	0.1666	—	—	—	—	—	—
$A_2^{(0,2)}$	0.5	0.7113	0.2246	—	0.3866	0.1876	—	—	—	—	—	—
	0.9	0.7425	0.2460	—	0.5255	0.2218	—	—	—	—	—	—
	0.1	0.7886	0.5356	0.2578	0.5320	0.5182	0.4506	0.2570	0.2153	0.2153	0.2153	0.2059
$A_3^{(0,3)}$	0.5	0.8146	0.5784	0.2241	0.6381	0.5693	0.4530	0.2192	0.1910	0.1910	0.1895	0.1637
	0.9	0.8370	0.6233	0.2206	0.7205	0.6187	0.4761	0.1945	0.1893	0.1834	0.1679	0.1158
	0.1	0.7657	0.2287	—	0.2149	0.1508	—	—	—	—	—	—
$A_1^{(1,0)}$	0.5	0.6920	—	—	0.2269	—	—	—	—	—	—	—
	0.9	0.6628	—	—	0.2439	—	—	—	—	—	—	—
	0.1	0.8006	0.2816	—	0.2692	0.2162	—	—	—	—	—	—
$A_2^{(1,1)}$	0.5	0.7793	0.2661	—	0.4541	0.2221	—	—	—	—	—	—
	0.9	0.7867	0.2772	—	0.5786	0.2502	—	—	—	—	—	—
	0.1	0.8358	0.4375	—	0.4297	0.3990	0.2217	—	—	—	—	—
$A_3^{(1,2)}$	0.5	0.8367	0.5191	—	0.6164	0.5027	0.2159	—	—	—	—	—
	0.9	0.8503	0.5827	—	0.7187	0.5756	0.2410	—	—	—	—	—
	0.1	0.8647	0.5875	0.2590	0.5829	0.5655	0.4797	0.2582	0.2156	0.2156	0.2156	0.2061
$A_4^{(1,3)}$	0.5	0.8727	0.6545	0.2306	0.7161	0.6447	0.5110	0.2250	0.1938	0.1938	0.1923	0.1648
	0.9	0.8861	0.7066	0.2508	0.7930	0.7023	0.5584	0.2173	0.2106	0.2032	0.1836	0.1202

(*continued*).

TABLE 3 *Continued.*

	ω	$D_1^{(0,1)}$	$D_2^{(0,2)}$	$D_3^{(0,3)}$	$D_1^{(1,0)}$	$D_2^{(1,1)}$	$D_3^{(1,2)}$	$D_4^{(1,3)}$	$D_2^{(2,0)}$	$D_3^{(2,1)}$	$D_4^{(2,2)}$	$D_5^{(2,3)}$
	0.1	0.8852	0.6631	0.3281	0.6593	0.6444	0.5678	0.3271	0.2740	0.2740	0.2740	0.2621
$A_2^{(2,0)}$	0.5	0.8547	0.6462	0.2730	0.7013	0.6377	0.5248	0.2671	0.2330	0.2330	0.2313	0.1998
	0.9	0.8484	0.6452	0.2494	0.7383	0.6409	0.5029	0.2223	0.2168	0.2107	0.1942	0.1361
	0.1	0.8932	0.6673	0.3281	0.6634	0.6481	0.5694	0.3271	0.2740	0.2740	0.2740	0.2621
$A_3^{(2,1)}$	0.5	0.8755	0.6706	0.2734	0.7275	0.6616	0.5403	0.2674	0.2331	0.2331	0.2313	0.1998
	0.9	0.8774	0.6908	0.2596	0.7796	0.6865	0.5442	0.2291	0.2229	0.2161	0.1979	0.1366
	0.1	0.9019	0.6782	0.3285	0.6741	0.6583	0.5765	0.3274	0.2741	0.2741	0.2741	0.2621
$A_4^{(2,2)}$	0.5	0.8937	0.7049	0.2784	0.7594	0.6961	0.5728	0.2720	0.2353	0.2353	0.2335	0.2007
	0.9	0.8994	0.7389	0.2902	0.8167	0.7350	0.6022	0.2529	0.2454	0.2370	0.2147	0.1414
	0.1	0.9106	0.7011	0.3482	0.6972	0.6820	0.6021	0.3471	0.2908	0.2908	0.2908	0.2781
$A_5^{(2,3)}$	0.5	0.9085	0.7442	0.3334	0.7920	0.7365	0.6259	0.3262	0.2845	0.2845	0.2824	0.2440
	0.9	0.9158	0.7820	0.3892	0.8467	0.7788	0.6679	0.3507	0.3426	0.3335	0.3086	0.2177

TABLE 4 *Hot mean equivalence factors*

	$B_1^{(0,1)}$	$B_2^{(0,2)}$	$B_3^{(0,3)}$	$B_1^{(1,0)}$	$B_2^{(1,1)}$	$B_3^{(1,2)}$	$B_4^{(1,3)}$	$B_2^{(2,0)}$	$B_3^{(2,1)}$	$B_4^{(2,2)}$	$B_5^{(2,3)}$
$A_1^{(0,1)}$	0.614	—	—	—	—	—	—	—	—	—	—
$A_2^{(0,2)}$	0.825	0.634	0.421	0.643	0.555	0.441	0.286	0.289	0.241	0.150	—
$A_3^{(0,3)}$	0.885	0.768	0.651	0.773	0.722	0.661	0.590	0.591	0.573	0.548	0.515
$A_1^{(1,0)}$	0.843	0.647	0.387	0.657	0.556	0.415	0.115	0.117	0.066	—	—
$A_2^{(1,1)}$	0.883	0.728	0.513	0.736	0.653	0.536	0.357	0.360	0.301	0.188	—
$A_3^{(1,2)}$	0.910	0.793	0.640	0.799	0.738	0.655	0.540	0.542	0.510	0.462	0.390
$A_4^{(1,3)}$	0.928	0.838	0.728	0.843	0.797	0.739	0.664	0.665	0.645	0.618	0.58
$A_2^{(2,0)}$	0.923	0.834	0.733	0.838	0.796	0.742	0.676	0.677	0.660	0.635	0.602
$A_3^{(2,1)}$	0.934	0.852	0.753	0.856	0.815	0.763	0.696	0.697	0.679	0.654	0.619
$A_4^{(2,2)}$	0.943	0.870	0.779	0.873	0.836	0.788	0.723	0.724	0.707	0.682	0.647
$A_5^{(2,3)}$	0.950	0.886	0.805	0.889	0.856	0.813	0.756	0.757	0.742	0.719	0.687

TABLE 5 *Cold mean equivalence factors with perfect switch*

	$C_1^{(0,1)}$	$C_2^{(0,2)}$	$C_3^{(0,3)}$	$C_1^{(1,0)}$	$C_2^{(1,1)}$	$C_3^{(1,2)}$	$C_4^{(1,3)}$	$C_2^{(2,0)}$	$C_3^{(2,1)}$	$C_4^{(2,2)}$	$C_5^{(2,3)}$
$A_1^{(0,1)}$	0.134	—	—	—	—	—	—	—	—	—	—
$A_2^{(0,2)}$	0.692	0.162	—	0.288	0.129	—	—	—	—	—	—
$A_3^{(0,3)}$	0.802	0.549	0.208	0.590	0.543	0.442	0.205	0.181	0.180	0.179	0.157
$A_1^{(1,0)}$	0.710	—	—	0.163	—	—	—	—	—	—	—
$A_2^{(1,1)}$	0.780	0.202	—	0.359	0.162	—	—	—	—	—	—
$A_3^{(1,2)}$	0.832	0.464	—	0.541	0.450	0.167	—	—	—	—	—
$A_4^{(1,3)}$	0.867	0.619	0.214	0.665	0.611	0.490	0.211	0.184	0.184	0.182	0.159
$A_2^{(2,0)}$	0.862	0.636	0.256	0.676	0.630	0.525	0.252	0.222	0.222	0.220	0.193
$A_3^{(2,1)}$	0.878	0.655	0.257	0.696	0.648	0.538	0.253	0.223	0.223	0.221	0.193
$A_4^{(2,2)}$	0.894	0.683	0.263	0.724	0.676	0.564	0.259	0.227	0.227	0.225	0.196
$A_5^{(2,3)}$	0.907	0.720	0.310	0.757	0.714	0.611	0.306	0.270	0.270	0.267	0.234

- improving the reliability of all components according to cold duplication with perfect switch gives the best system;
- for the same number of components $R(t) < R^{(B)}(t) < R^{(D)}(t) < R^{(C)}(t)$ where $\lambda = 0.05$;

Figures 5 and 6 present the behaviour of MTTF against the appropriate reduction factor ρ. It seems from these two figures that the following conditions holds:

- MTTFs non-decreasing with decreasing ρ for all possible sets A.

TABLE 6 *Cold mean equivalence factors with imperfect switch*

	$D_1^{(0,1)}$	$D_2^{(0,2)}$	$D_3^{(0,3)}$	$D_1^{(1,0)}$	$D_2^{(1,1)}$	$D_3^{(1,2)}$	$D_4^{(1,3)}$	$D_2^{(2,0)}$	$D_3^{(2,1)}$	$D_4^{(2,2)}$	$D_5^{(2,3)}$
$A_1^{(0,1)}$	0.226	—	—	—	—	—	—	—	—	—	—
$A_2^{(0,2)}$	0.704	0.223	—	0.316	0.179	—	—	—	—	—	—
$A_3^{(0,3)}$	0.810	0.567	0.236	0.602	0.554	0.456	0.229	0.199	0.198	0.195	0.167
$A_1^{(1,0)}$	0.723	—	—	0.224	—	—	—	—	—	—	—
$A_2^{(1,1)}$	0.790	0.280	—	0.393	0.224	—	—	—	—	—	—
$A_3^{(1,2)}$	0.840	0.498	—	0.562	0.473	0.223	—	—	—	—	—
$A_4^{(1,3)}$	0.873	0.639	0.244	0.677	0.624	0.507	0.236	0.204	0.203	0.200	0.170
$A_2^{(2,0)}$	0.867	0.654	0.288	0.688	0.641	0.539	0.280	0.245	0.244	0.240	0.205
$A_3^{(2,1)}$	0.884	0.673	0.291	0.708	0.660	0.553	0.282	0.246	0.245	0.241	0.206
$A_4^{(2,2)}$	0.898	0.701	0.299	0.735	0.688	0.580	0.290	0.251	0.250	0.246	0.209
$A_5^{(2,3)}$	0.911	0.736	0.349	0.767	0.724	0.626	0.339	0.297	0.296	0.291	0.250

- Reducing the failure rate of one or two components from the first subsystem gives a better system than that obtained by reducing the failure rate of one or two components in the second subsystem; see Fig. 5. This means that improving a component from the subsystem with the smaller number of components is better than improving a component from the subsystem with the larger number of components.
- Reducing the failure rates of all components in the system gives the best system; see Fig. 6.
- It is not possible to reduce the failure rate of the sets $A_2^{(1,1)}$ or $A_2^{(0,2)}$ of the system components to reach the MTTF which we can achieve by improving the sets $B_5^{(2,3)}$ or $C_3^{(1,2)}$ of the system components according to hot duplication and cold duplication with perfect switch, respectively, see Fig. 5.
- Reducing the failure rate of three components in the second subsystem (which we denote as $A_3^{(0,3)}$) by setting $\rho = 0.236$ improves the MTTF of the system like adding three components to the second subsystem (which we denote as $D_3^{(0,3)}$) according to the cold duplication method with imperfect switch; see Fig. 6 and compare with Table 6.
- Reducing the failure rate of one components in the first subsystem and two components in the second subsystem (which we denote as $A_3^{(1,2)}$) by setting $\rho = 0.390$ improves the MTTF of the system like adding two components in the first subsystem and three components in the second subsystem (which we denote as $B_5^{(2,3)}$) according to the hot duplication method; see Fig. 6 and compare with Table 4.
- Improving a number of components selected from two subsystems, with equal numbers if they are even, gives a better system than that obtained by improving the number of components selected from the same subsystem or selected from the two subsystems with unequal numbers; see Fig. 6.

TABLE 7 *Mean times to failure of the modified systems*

	$\{0_1, 1_2\}$	$\{0_1, 2_2\}$	$\{0_1, 3_2\}$	$\{1_1, 0_2\}$	$\{1_1, 1_2\}$	$\{1_1, 2_2\}$	$\{1_1, 3_2\}$	$\{2_1, 0_2\}$	$\{2_1, 1_2\}$	$\{2_1, 2_2\}$	$\{2_1, 3_2\}$
Hot	1.202	1.244	1.305	1.242	1.266	1.299	1.347	1.346	1.360	1.381	1.413
Cold perfect	1.230	1.381	2.104	1.347	1.387	1.499	2.120	2.255	2.257	2.266	2.420
Cold imperfect ($\lambda = 0.05$)	1.228	1.366	1.984	1.338	1.377	1.481	2.013	2.150	2.155	2.173	2.343

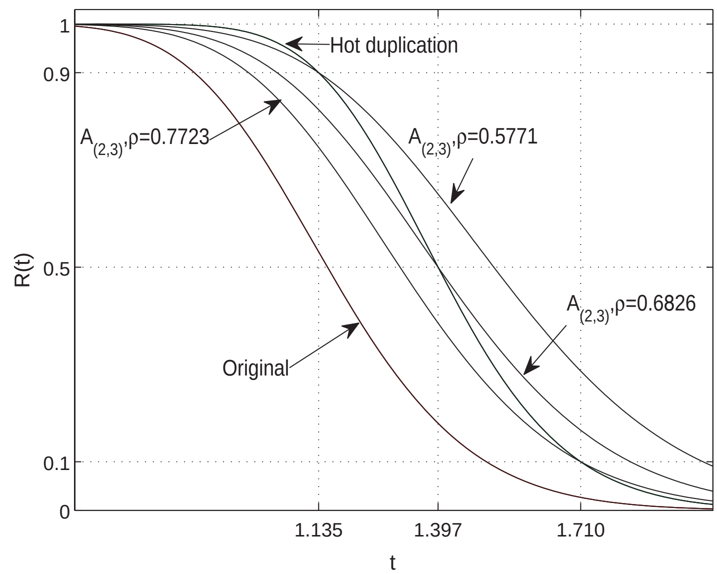

FIG. 3. Use of SREFs to recommend system improvement strategies.

Q12

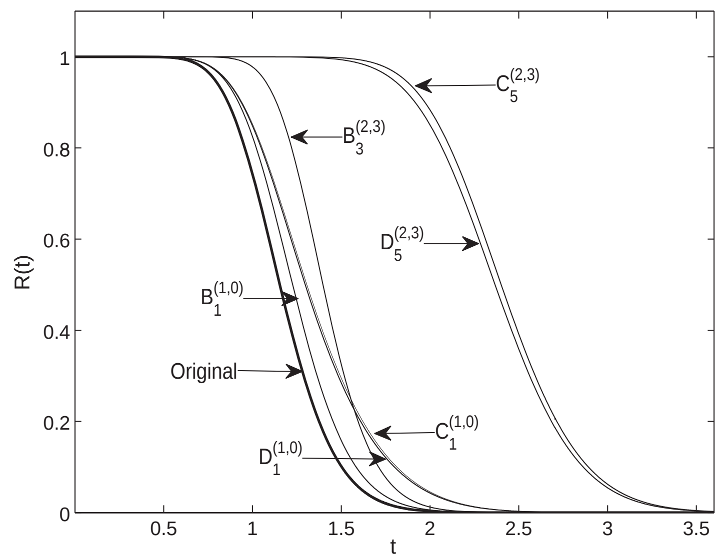

FIG. 4. Reliability function of the original and some modified systems.

This numerical example clearly generates interesting conclusions for this particular system and distributional assumptions. More importantly though, it demonstrates the potential for applying these methods to other system structures. It also illustrates how to address specific questions that arise when attempting to improve the reliability of simple systems or simple configurations of possibly complex subsystems in many diverse applications.

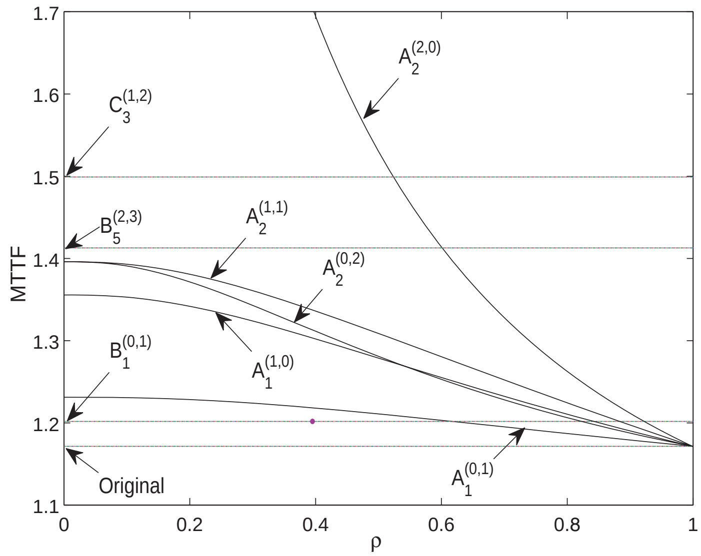

FIG. 5. The behaviour of MTTF against ρ, when $|A| \leqslant 2$.

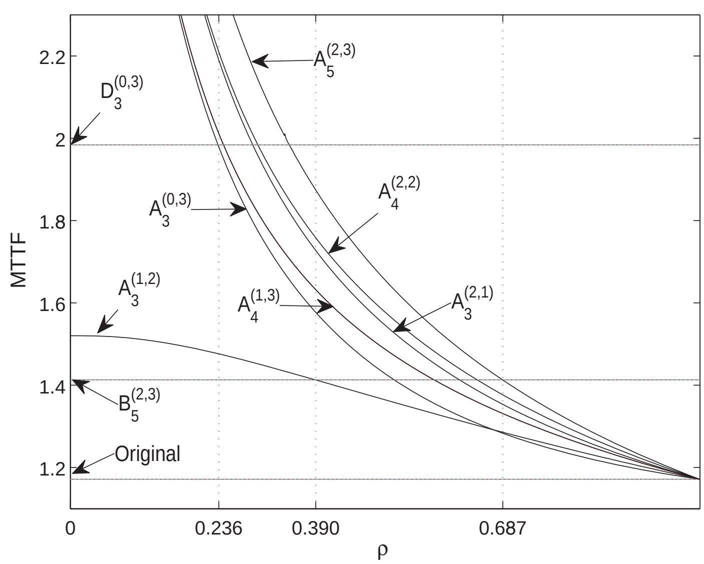

FIG. 6. The behaviour of MTTF against ρ, when $|A| > 2$.

6. Conclusions

In this paper, we evaluate both the system reliability function and the system MTTF in order to study the reliability equivalence factors for series–parallel systems. These system structures arise often in business and industry and the methodology adapts readily for other forms including parallel–series systems and more complex networks. All the system components are assumed to be independent and identically distributed, according to an exponentiated Weibull distribution, on account of its flexibility and tractability for practical purposes. We discuss four different methods to improve such a system: reduction, hot duplication and cold duplication with perfect or imperfect switch.

We derive analytical results for both survival and MREFs of these systems. Some numerical results are then presented for a representative system in order to illustrate how one can apply the theoretical results obtained and to compare the various approaches in this context. Accordingly, detailed recommendations are discussed for improving the system considered in this paper. Although it would be inappropriate to extrapolate these results to other system structures from only this numerical example, we make some interesting observations which suggest patterns that might arise more generally.

We have also identified several extensions of this study that might be worthy of future exploration, including comparisons with parallel–series formats and analysis of other important system structures, equivalent systems with non-identical components and simpler systems with dependent components. The methods described in this paper adapt readily to deal with all these other scenarios.

Perhaps in conjunction with a meta-analysis of the growing literature on reliability equivalence, we also plan to perform a sensitivity analysis to assess how robust these results are to mis-specifications of lifetime distributions. Another aspect of cold duplication also has practical benefits. This is when the standby component deteriorates during storage with a constant failure rate, so that it may not function correctly when replacing the original failed component. We are currently investigating practical evidence to motivate and justify such an analysis of random switch operation times and variations of this scenario and hope to publish our results in due course. Q10

Funding

Q11

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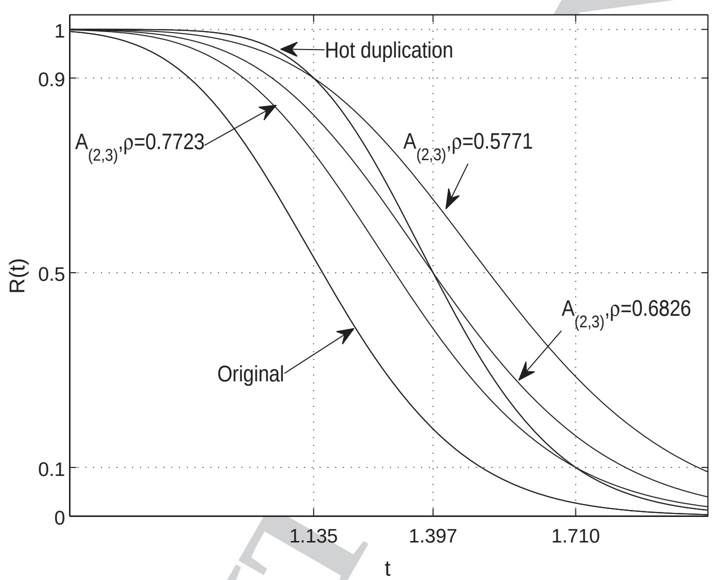
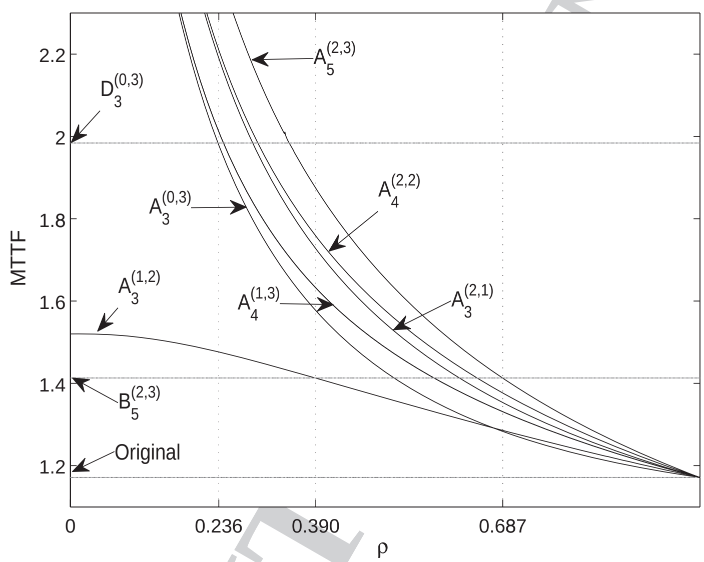

Hot duplication
A(2,3),ρ=0.7723
A(2,3),ρ=0.5771
A(2,3),ρ=0.6826
Original
R(t)
t
1
0.9
0.5
0.1
0
1.135
1.397
1.710



MTTF
2.2
2
1.8
1.6
1.4
1.2
0
0.236
0.390
0.687
ρ
$A_5^{(2,3)}$
$D_3^{(0,3)}$
$A_4^{(2,2)}$
$A_3^{(0,3)}$
$A_3^{(1,2)}$
$A_4^{(1,3)}$
$A_3^{(2,1)}$
$B_5^{(2,3)}$
Original