MODELLING AND OPTIMISING THE SPORT AND EXERCISE TRAINING PROCESS

PhD Thesis

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LIST OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

DECLARATION

I declare that the thesis is my original work. No portion of this work has been previously submitted for another degree or qualification of this or any other University.

ABSTRACT

In elite sport, the fundamental aim of training is to improve performance in competition. It should develop the abilities of the athletes to achieve the highest level of performance. The fundamental aim of monitoring in training is to determine whether training is appropriate for an athlete and whether training should be modified. Broadly, the purpose is to control the training program of an athlete to ensure that the maximum level of performance by the athlete is reached at a known competition at a known time in the future.

In this thesis, we aim to model the training process in cycling in particular. Our purpose is to find a quantitative model that coaches and athletes should follow to optimise training in advance of a major competition. To avoid under and over-training, training should be balanced and should support athletes to develop their capabilities. We develop a statistical model to optimise training. This model is based on the relationship between performance and the accumulation of training. To do this, both training and performance must be measured. We establish a new measure of performance based on the relationship between power output and heart-rate, with the appropriate time lag. The measure of the accumulation of training we use is the Banister model proposed in 1975. Then, we relate our performance measure to the accumulation of training. The parameter values of the Banister model are estimated using the method of maximum likelihood. This analysis is done using R statistical packages. Finally, we suggest some points of interest for developing this work in order to optimise a training schedule for an athlete to reach peak performance at a known competition at a given time.

1. INTRODUCTION

1.1 Objectives of research

This thesis is concerned with modelling the training process in sport and exercise, and in cycling in particular. Our purpose is to provide a quantitative model that can be used to optimise training in advance of a major competition. An optimal training program would prevent under-training, overtraining and injury (Meeusen, et al., 2006).

Training is the method by which an athlete improves his or her specific performance and develops individual characteristics according to the requirements of a specific competition (Yin, et al. 2010). Smith (2003a) stated that the training process involves repetition of exercises designed to develop the skills of a rider that lead to increased physical performance. The principal aim of cycling training is to improve and increase the ability of a rider to sustain a power output or speed for a given distance or time.

Training should be balanced. It should support the rider and develop his or her capabilities and allow the athlete to gain the right amount of training. If a rider trains too much, he may get injured or sick but if he trains too little he may get little benefit. Measuring and monitoring the positive and negative effects of training will help coaches and athletes to design their training program in order to maximise performance at a specific time. So, training strategy should be developed to achieve peak performance. To do this, very hard riding and the correct amount of recovery must be combined (Faria, et al., 2005), so that over-training and injury or illness inducing fatique can be avoided (Smith, 2003b). This trade-off between under and over-training was first discussed by researchers in the former East Germany and later developed by Banister et al. (1975). Our purpose is to use a quantitative approach to find the optimum balance between under and over-training. To do so, we develop a statistical model to relate training to performance. To do this, both training and performance must be measured, and we do so using field data relating to power output and heart-rate. This research is the first to use field data to model training and performance in this way.

Ultimately, we intend that measures of training and performance and the statistical model that links them will be used to optimise training: that is, used to determine the training schedule that maximises the performance of an athlete on a particular day in the future. Such a schedule would require a rider to carry out particular tasks at particular times. In practice and theory the schedule would have to be adaptive.

The measure of training we use is an established one based on the concept of accumulated training load; this is broadly an exponentially weighted moving average of the total load on the cardio-vascular system during training of a rider over all time. However, this accumulated training load measure depends on a number of unknown parameters that must be specified for an individual athlete; only then can training be theoretically optimized for this specific athlete.

The measure of performance of an athlete that we use is the estimated heart-rate required by the rider in order to produce power output at a defined, high level. Such a level corresponds to some particular upper percentile (e.g. 75%) of the rider's power output distribution, considered over all time. We then determine those values of the parameters of the accumulated training load measure such that this measure is most closely related to the performance measure. We will explore other measures of performance but ultimately this performance measure is our preferred measure.

Mathematical models of training exist, but it has proved difficult to implement these in a practical context so that training schedules might be optimised. The optimal training strategy to improve functional strength in cycling is still unclear and current practice is rather based on the experience and perception than on sound scientific evidence (Koninckx, et al. 2010). As a result, current practice of training riders relies upon riders' and coaches' intuition and experience, with only limited support from quantitative analyses. This thesis will explore the optimisation of training through the analysis of a large dataset on power output and heart-rate of competitive cyclists.

1.2 The relationship between training and performance

The relationship between training and performance is very important for coaches who look to determine a training program for their riders. Research that has investigated this relationship by using quantitative data can be traced back to the seminal work of Banister, et al. (1975). However, in spite of the time that has elapsed since these early ideas were described, predicting the results of a particular training program is difficult, and in particular predicting performance output from training input remains an unsolved problem (Jobson, et al. 2009). The relationship between training and performance is highly individualised because of a number of factors (Avalos, et al. 2003). These factors include genetic factors, individual training background, psychological factors, technical factors and speciality, and they are very difficult to quantify (Hellard, et al. 2006; Jobson, et al. 2009). However, positive relationships between training and performance and between higher training intensity and performance have been found for individual sports such as swimming and running (Gabbett and Domrow, 2007).

The amount and type of training can positively affect the physical capabilities of a athlete. On the other hand, an athlete is negatively affected by the amount of fatigue that the training itself accumulates in the athlete (Banister, et al. 1975). Qualitative predictions and descriptions of the effect of training have been made. For example, one can observe a rapid improvement in performance when the initial performance is low, but as an athlete becomes fitter and better trained, it becomes more difficult to observe further improvement in performance by continued or more intensive training.

Banister and Calvert (1980) then point out that it is important for a rider to avoid overtraining and injury that may decrease performance. Such arguments have been reinforced by further research (e.g. Borresen and Lambert, 2009). Many studies (e.g. Stewart and Hopkins, 2000; Avalos, et al. 2003; Nimmerichter, et al. 2011) have discussed the relative influence of training. They have found that reactions to training depend on three factors: volume, intensity and frequency of the training sessions (Avalos, et al. 2003). Several methods have been suggested to evaluate exercise intensity during training and competition (Karvonen and Vuorimaa, 1988; Gilman and Wells, 1993; Hopkins, 1991). Borresen and Lambert, (2009) argued that increasing training will improve the sporting performance. However, a random increase in training volume, intensity or frequency may lead to over-training which increases the likelihood of injury.

Avalos, et al., (2003) have discussed the effect of training on performance of 13 elite swimmer over three seasons. They reported significant changes in the impact of training on performance from the first to the third season. The effect of training on performance has been studied in different sports (Millet, et al., 2002), including running (Banister and Hamilton, 1985; Banister, et al., 1986) and swimming (Avalos, et al., 2003; Mujika, et al., 1996). The purpose of this study is to relate training to performance using data collected in the field using a power meter and heart-rate monitor.

1.3 The components of training

Banister et al. (1975) proposed a model that describes the influence of training on performance at any time t. They suggested that in its simplest form, the influence of training is the difference between two components. These components are fitness, which is the positive influence of training, and detriment, which is the negative influence of training. Throughout the training period, the level of training (or readiness to perform) is described as the difference between the accumulated fitness (benefit) and the accumulated detriment (dis-benefit), see figure 1.1. Training load is a combination of three elements. They are intensity, duration and frequency (Smith, 2003a).

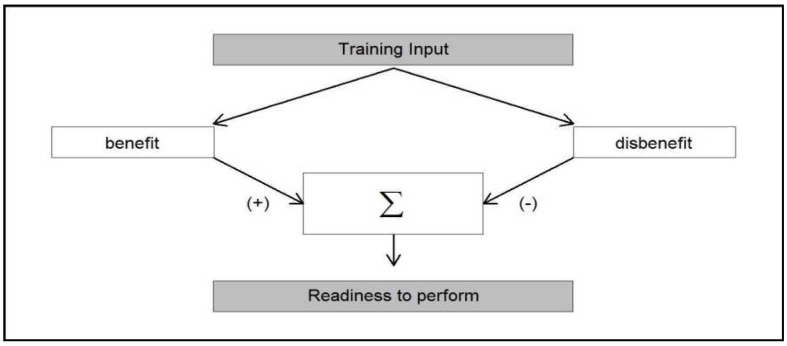

Figure 1.1 The influence of training over a period of time.

1.4 Overtraining symptoms

In this subsection, we give an explanation about overtraining symptoms and the causes and reasons of overtraining. Overtraining is defined as an imbalance between training stress and recovery (Kuipers and Keizer, 1988; Lehman, et al., 1993; Halson and Jeukendrup, 2004). Moreover, it is caused by too much high intensity training and too little recovery time (Fry, et al. 1991). Overtraining symptoms have been defined by Smith, (2003b) as when an athlete is training intensely and shows no improvement in performance. Furthermore, overtraining is described as the incapacity to train and perform over a longer period (Busso, et al., 2002). Borresen and Lambert (2007) stated that ‘heart rate recovery is the rate at which heart rate decreases, usually in the first minute or two, after moderate to heavy exercise’. Heart rate recovery after exercise at similar absolute intensities is faster in trained athletes than untrained ones (Short and Sedlock, 1997). Overtraining can begin as fatigue and then progress to severe symptoms such as sleep problems and lack of motivation (Jeukendrup and Diemen, 1998), so it is important for coaches and athletes to identify overtraining as early as possible to modify training before getting reduction in the athlete's performance.

The number of overtraining symptoms is large (Gleeson, 2002). Fry, et al. (1991) reported over 200 symptoms. Mackinnon, (2000) and Halson and Jeukendrup, (2004) listed some overtraining symptoms as persistent and severe fatigue, poor and declining performance in sport with continued training and frequent illness. In addition, decreased maximum heart rate, decreased oxygen uptake and decreased lactate levels have also reported as overtraining symptoms (Hassmén and Kenttä, 1998) and (Lehman, et al. 1993). However, heart rate monitoring could be used to discover overtraining at early stages and prevent it (Jeukendrup and Diemen, 1998).

There are many causes that can lead to overtraining symptoms. However, there is no single objective marker to identify overtraining syndrome (Mackinnon, 2000). Halson and Jeukendrup, (2004) mentioned that a number of investigations have been carried-out to test the effects of an intensified training period that can lead to overtraining. One of the most important reasons for overtraining symptoms is a dramatic increase of training or competition intensity with insufficient time for recovery (Smith, 2003b; Lehman, et al. 1993; Fry and Kraemer, 1997). Sudden increase in training volume and/or intensity, a heavy competition schedule and monotonous training program are also reported as causes of overtraining (Mackinnon, 2000). Weeks to months of complete rest are required to recover from overtraining symptoms (Mackinnon, 2000).

1.5 Summary and structure

To summarise, the purpose of this thesis is to develop a statistical model that relates training to performance for a particular rider. Training can then be scheduled to maximise performance at a particular competition. This thesis is structured as follows.

In chapter two, we describe the athletes, their data, and how the data were collected. These data are power output and heart rate collected every five seconds for the sessions (training and competition) of ten riders over a period of time. We plot examples of the power output and heart rate series for a number of sessions. In the final part of this chapter, we present the entire history of power output and heart rate data for each rider.

Chapter three discusses the measurement of training and performance. We use in this chapter a measure of training load for a session called the training impulse (TRIMP). Then, we explain the Banister model (proposed by Banister et al. 1975), which is used to measure the accumulation of training given known parameters of the model. The quantified accumulation of training is called the accumulated training effect (ATE). The next part of this chapter describes our performance measure. This measure is based on the relationship between power output and heart rate. Power is related to heart-rate using the entire history of sessions for each rider. In particular, in this relationship, we use a 15 second time-lag between power output and heart rate, and justify this choice of lag.

In chapter four, we estimate the parameters of the accumulated training effect measure. Then, we present our results and discuss statistically and practically the significance of the training effect. We briefly describe how the estimated parameters can then be used to optimise training.

Chapter five summarises some other possible measures of performance, such as average power, normalised power, critical power and a measure based on the concept of the critical power that might be used to determine the Banister model parameters.

The final chapter summarises our work and discusses the limitations of our study. Finally, we present some suggestions for further development and future research.

2. THE STUDY DATA

2.1 Training data

Training data from a number of competitive riders were available to us. These cyclists gave written, informed consent for their data to be used in our study. The study received local ethical committee approval and was carried out according to the principles of the Declaration of Helsinki (World Medical Association, 2013). For each rider, for a number of training sessions typically extending over a 300 day period between December 2006 and September 2007, power output and heart-rate were recorded every five seconds. In the current study, riders are numbered to maintain their anonymity and privacy. The ten riders have mean (standard deviation) age of 36 (9) years, height of 1.79 (0.46) metres, and weight of 74.3 (6.8) kg. The age (years), height (metres) and weight (kilograms) of each rider are shown in Table 2.1. A summary brief description of our data is given in Figure 2.1, Table 2.2 and Figure 2.2. According to Figure 2.1, there is variation among the ten athletes. Each athlete has trained or approximately 50% of the total number of days. Missing data for a particular day might be due to either a lack of recording or there being no ride that day.

Table 2.1 Age, height and weight of the ten riders

Rider	Age (years)	Height (m)	Weight (kg)	Rider	Age (years)	Height (m)	Weight (kg)
1	45	183.0	74.3	6	27	183.7	71.8
2	52	175.0	74.5	7	40	177.5	75.5
3	35	181.0	71.0	8	34	182.0	77.0
4	42	178.5	78.2	9	34	185.5	88.2
5	21	171.4	60.9	10	29	174.5	71.5

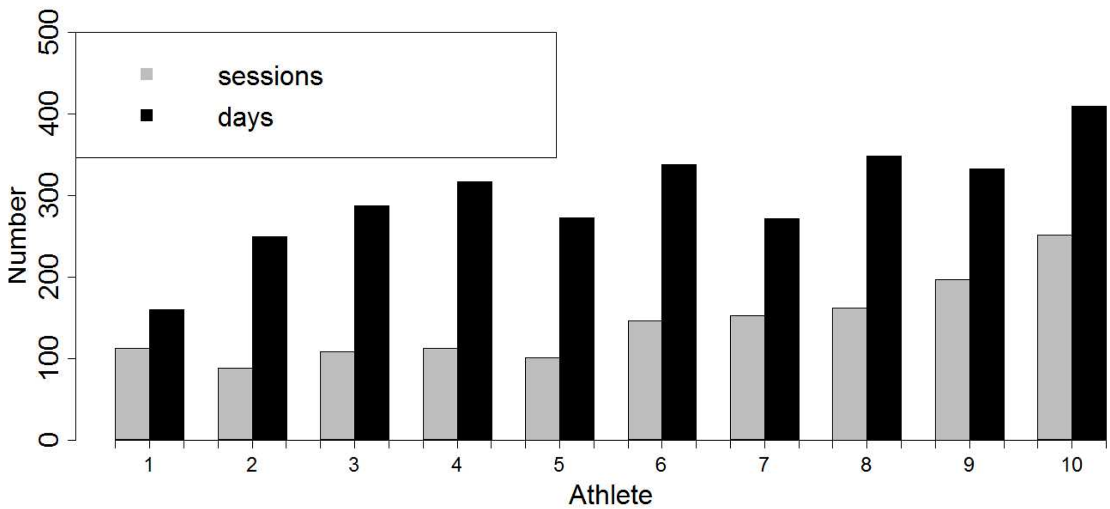

Figure 2.1 The number of sessions and the period in day for each rider's training schedule

Table 2.2 Start date, end date and duration of training schedules for each of the 10 riders

Rider	Start	End	Sampling interval (seconds)	Total training period (days)
1	04/03/2007	11/08/2007	5	160
2	21/11/2006	28/07/2007	5; 15	249
3	19/04/2007	31/01/2008	5	287
4	10/11/2006	23/09/2007	5	317
5	02/11/2006	02/08/2007	5	273
6	27/10/2006	30/09/2007	5; 15	338
7	06/12/2006	04/09/2007	5; 7	272
8	24/10/2006	07/10/2007	5	348
9	01/11/2006	30/09/2007	5	333
10	28/10/2006	12/12/2007	5	410

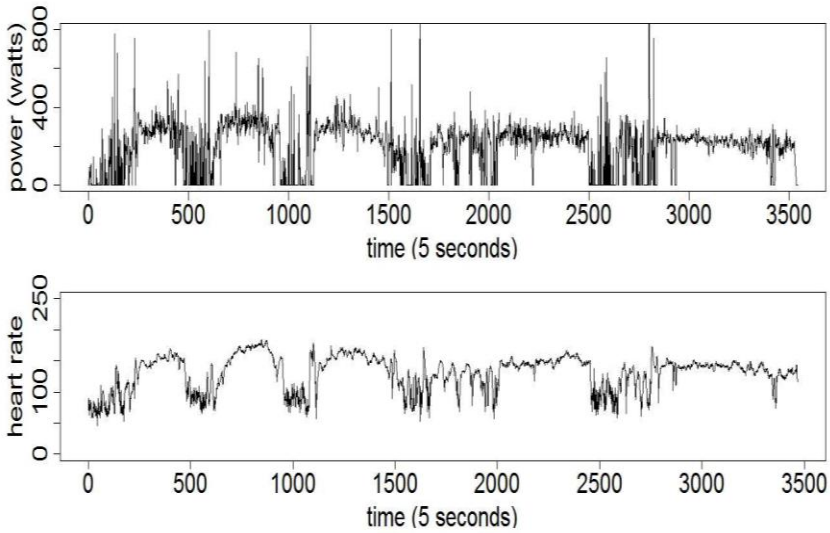

Figure 2.2 An example of power output (Watts) and heart rate (beats per minute, bpm) from a single session for one rider

It should be noted that these data were not collected for the express purpose of our study and so the data collection protocol was not designed by us. The original purpose of the data collection was for the riders to describe their training and to provide useful information for themselves and their coaches in the manner described in Nimmerichter et al. (2011).The idea to use these data in a study of performance output and training input was developed after the data were collected. Therefore, we plan our method according to the available data.

2.2 Heart rate measurement

Heart rate measurement is one of the most popular methods of measuring exercise intensity during a training session. It is measured in beats per minute. Nimmerichter, et al. (2011) mentioned that many studies have used heart rate as a measure of estimating exercise intensity in a variety of sports such as cycling (Lucia, et al. 1999); running (Gilman and Wells, 1993); tennis (Therminarias, et al. 1991); and soccer (Ali and Farrally, 1991). Many researchers referred the basis of this method to the established linear relationship between heart rate and steady-state work rate (Hopkins, 1991; Arts and Kuipers, 1994; Robinson, et al. 1991). There are many devices available that can monitor the heart rate of an athlete when he does exercise (figure 2.3). These monitors have been widely used for different sports over the last two decades (Achten and Jeukendrup, 2003). They are used to determine the exercise intensity of a training session or race. The exercise intensity of a session is one of the most important applications of heart rate monitoring (Achten and Jeukendrup, 2003). These devices can help coaches and athletes to monitor and plan the athletes' training intensity. The first telemetric monitors of heart rate were invented in 1982 and then developed to store heart rate data (Lambert, et al. 1998). These data can be transferred to a computer in order to analyse them and get some information about an athlete. The use of heart rate monitors has been studied in many sports such as cycling, running and soccer (Lambert, et al. 1998). The mean heart rates for each session for each rider are shown in figure 2.4.

Figure 2.3 A chest-worn heart-rate monitor.

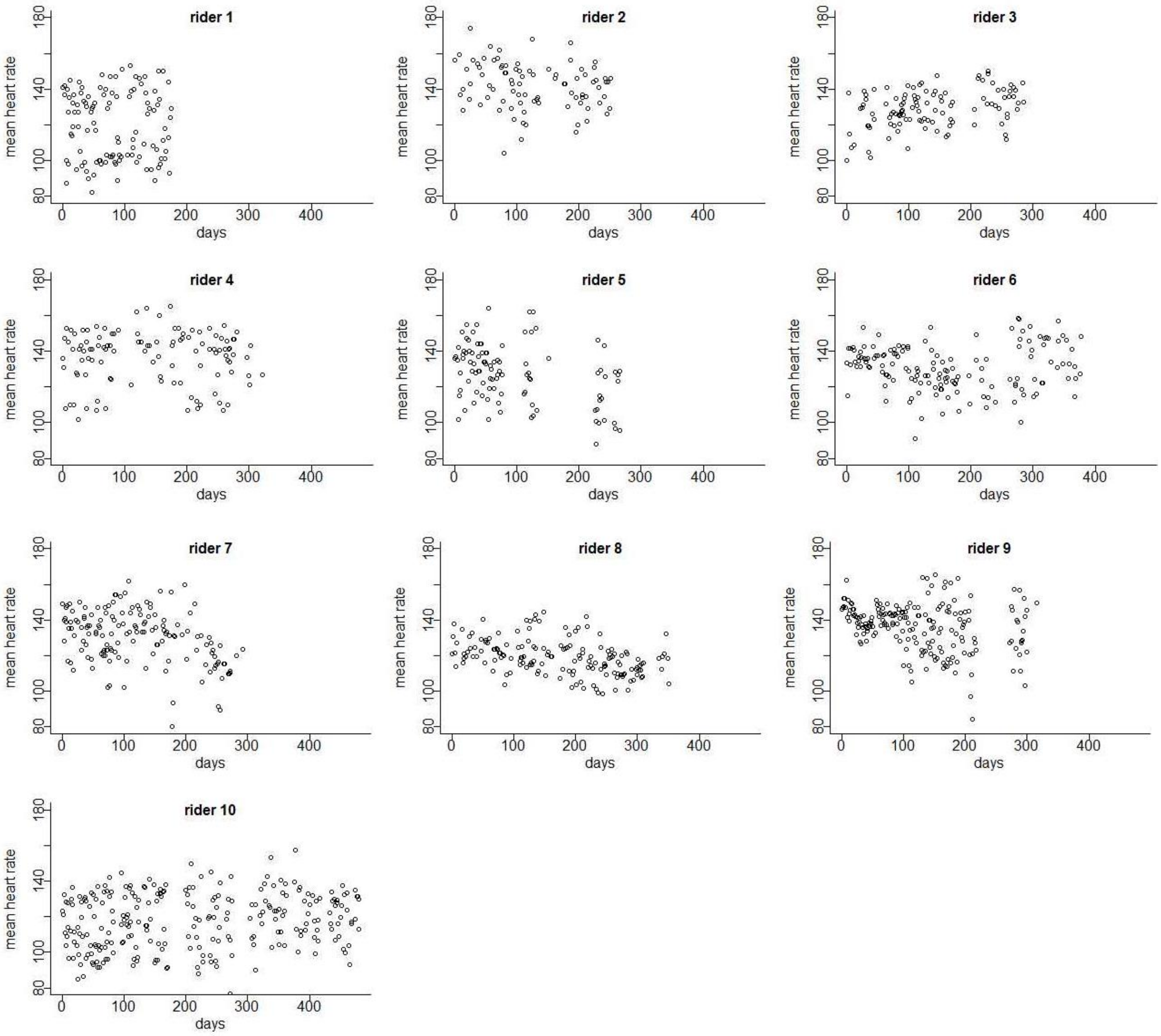

Figure 2.4 Mean heart rates for each session for each rider

2.3 Power output

Power output has become one of the most important measures of monitoring training. It provides a direct and immediate measure of the work rate of a rider (Vogt, et al. 2007 and Jobson, et al. 2009). Now, with mobile cycle ergo-meters (figures 2.5 and 2.6), it is easy to measure and record power output. SRM is a power meter developed by the engineering company Schoberer Rad Messtehnik. This meter provides useful information for coaches and athletes. It calculates power output (Watts), heart rate (beats per minute), cadence (revolutions per minute), speed (miles or kilometres per hour) and temperature (Fahrenheit or Celsius) together at the same time. It also calculates the mean power output up to the current time point. The mean power outputs for each session for each rider are presented in figure 2.7. With those data recorded, training can be examined and coaches and riders can aim to improve their abilities to get better results especially in competitions. The entire training histories of power output and heart rate for each rider are shown in figure 2.8 and figure 2.9.

Figure 2.5 An example SRM power meter crank. This is the SRM Canondale MTB 2x10 model, which weighs 521g and costs €1892 (SRM, 2012).

Figure 2.6 An example SRM data recorder and display. This is the Power Control 7 model, with a battery life of 120 hours (SRM, 2012).

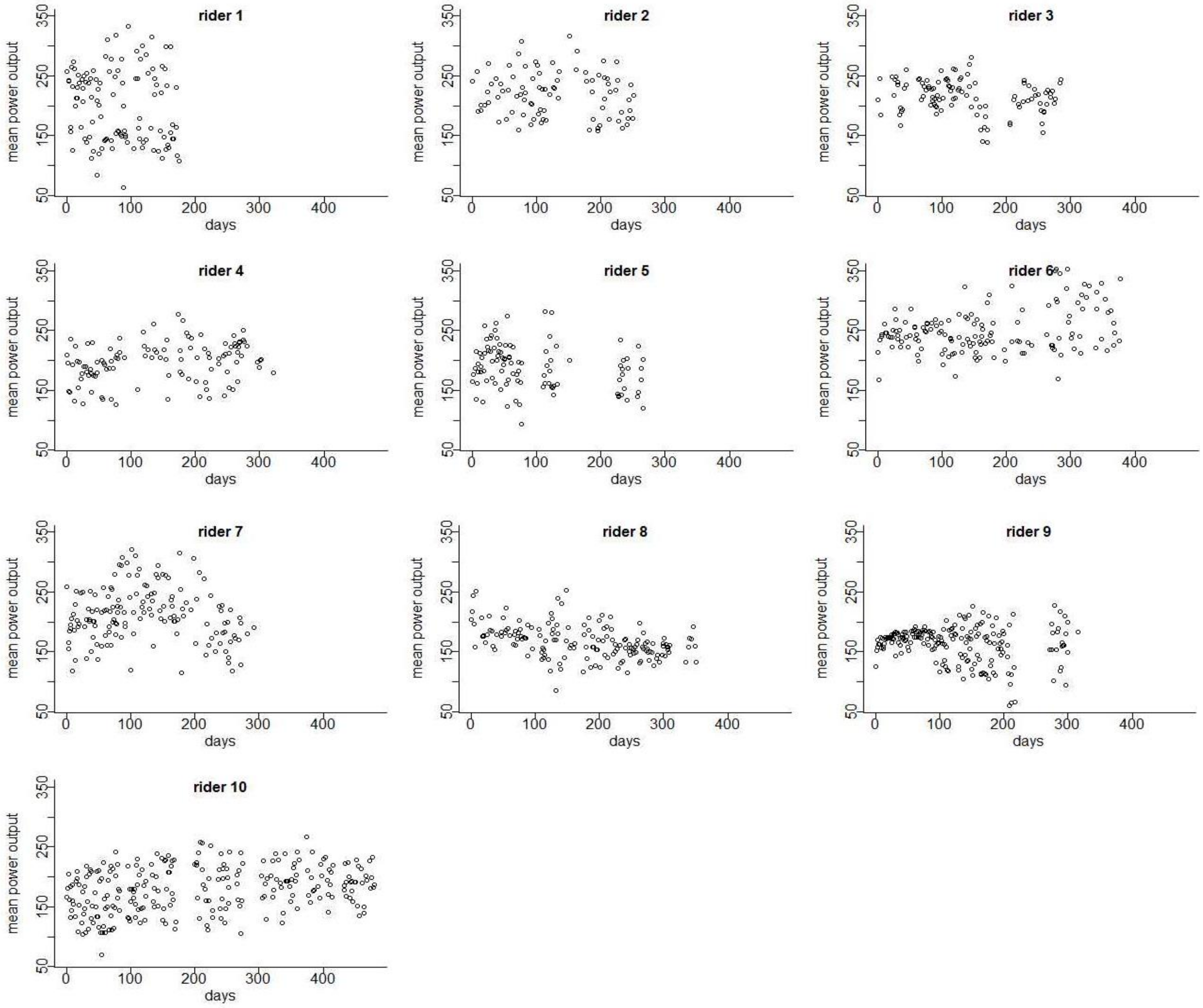

Figure 2.7 Mean power outputs (Watts) for each session for each rider

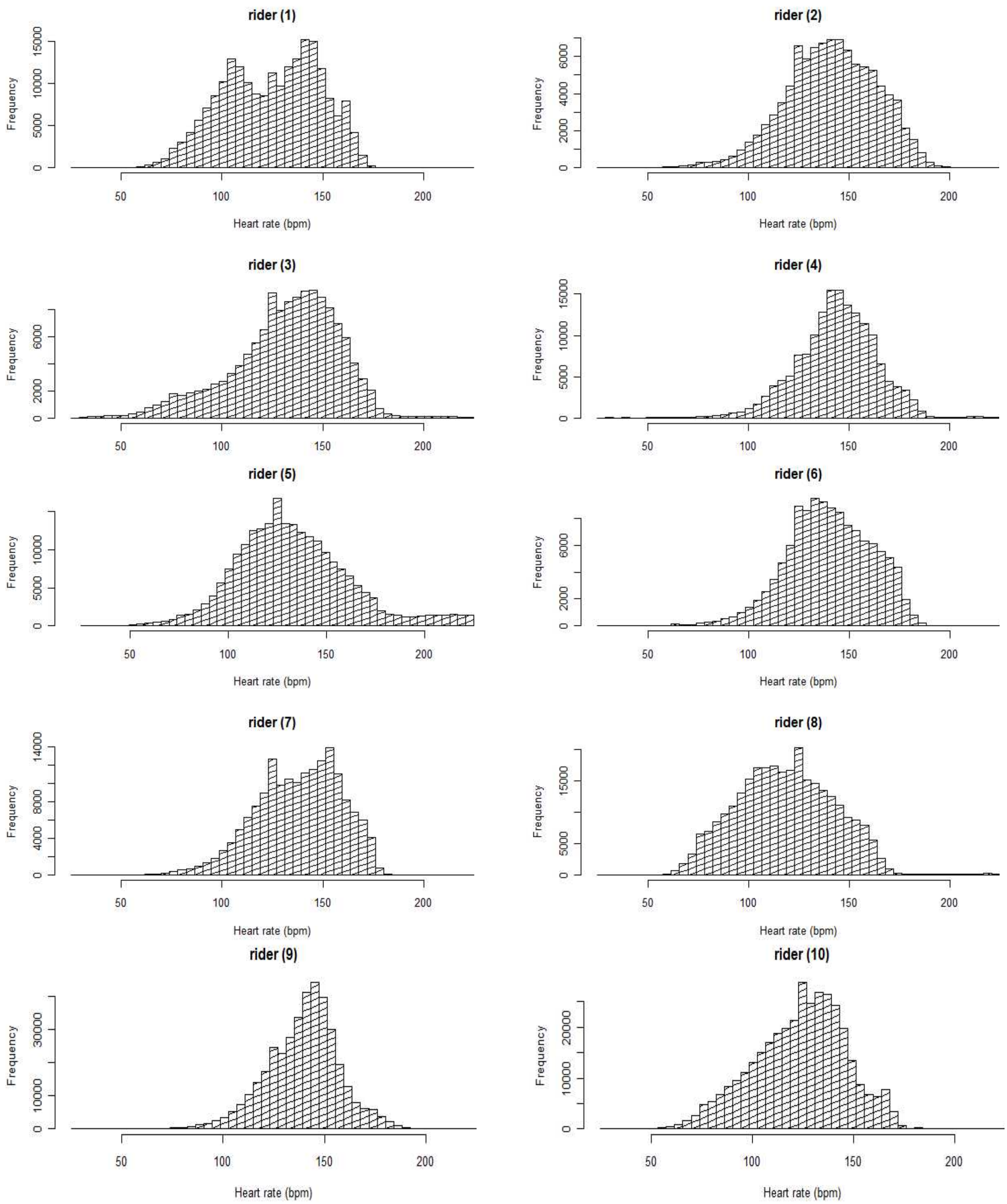

Figure 2.8 The histograms of heart rate data (all sessions) for each rider (1-10)

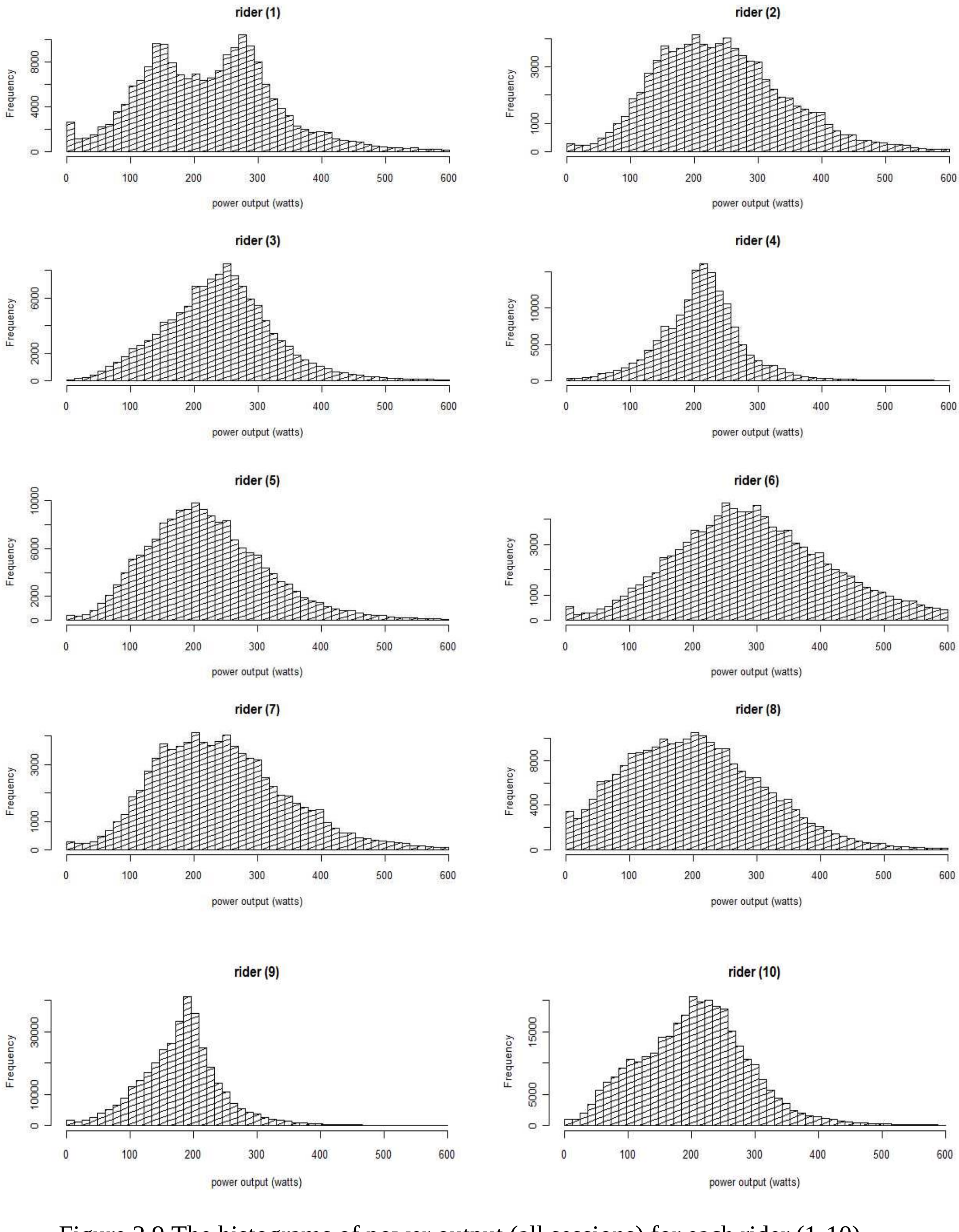

Figure 2.9 The histograms of power output (all sessions) for each rider (1-10)

3. MEASURING TRAINING AND PERFORMANCE

3.1 Introduction

In this chapter, we explain our method for relating training to performance. In order to do this, both training and performance must be measured. We will give a brief explanation of measuring training and measuring performance. We use data on power output and heart rate collected every five seconds during training. The next chapter will then explain how to relate one to the other.

3.2 Measuring training

Banister, et al. (1975) proposed a measure of training that calculates the accumulative effect of all training carried out up to time t. This measure had a number of components. The first one is the measurement of the amount of training for a single session, called the training load of that session. In general, training load of a session can be written as average intensity × duration. In this thesis, we use training impulse (TRIMP) as a measure of the training load for a session. The second component is how training accumulates for a sequence of sessions over time. In the next subsections we will explain these components in detail.

3.2.1 Training load for a session: Training impulse (TRIMP)

The training impulse (TRIMP) is a measure that calculates how hard a rider trains in a single session. The concept of training impulse combines training intensity and training duration into a single measure to provide higher weighting for higher intensity sessions (Akubat and Abt, 2011). This measure is based on heart rate measurements during training (Joosen, et al., 2013). The training impulse (TRIMP) has been used as an indicator of training load during training and competition by several researchers (Morton, et al., 1990; Padilla, et al., 2000). Recently, it has been used for describing training load in professional road cycling to plan training in an appropriate way (Padilla, et al., 2000; Padilla, et al., 2001).

The concept of training impulse was first presented by Banister, et al. (1975) and Banister and Calvert, (1980) as follows

$$TRIMP = T \times \overline{H}$$

where T is the training time in minutes of the session and $\overline{H}$ is the average heart rate of the session (beats per minute). Thus, here, TRIMP is the total number of heart beats during a session. However, using the above formula to calculate TRIMP does not reflect the overall intensity of a session (Akubat and Abt, 2011; Stagno, et al., 2007).

The original formula was modified by Morton, et al. (1990) to include a multiplicative factor that gave greater weight to high-intensity training and it is defined as follows

$$TRIMP = T \times a \times H_{ratio} e^{bH_{ratio}}$$

where

$$H_{ratio} = \left(\frac{HR_{ex} - HR_{rest}}{HR_{max} - HR_{rest}}\right),$$

T is the duration of exercise and HR_{ex} is the average heart rate during the exercise and HR_{rest} is the resting heart rate (the number of heart beats per minute). HR_{rest} should be calculated upon waking and while still lying in bed. The fitter the rider, the lower is his resting heart rate. HR_{max} is the maximal heart rate. Table 3.1 presents the maximum heart rate and the resting heart rate for each rider in our study. These data are recorded in our dataset. The constant a is taken to be 0.64 for males and 0.86 for females (Borresen and Lambert, 2009). The constant b is based on blood lactate and it is taken to be 1.92 in males and 1.67 in females.

There are conflicting views about the values of the TRIMP parameters. The study of Stagno et al (2007) have reported the constants a and b as being 0.1225 and 3.94 for males respectively. They plotted the blood lactate concentration of 8 participants against the fractional elevation in heart rate, and then estimated them by fitting an exponential line.

Figure 3.1 shows the training impulse (TRIMP) for each rider for each session using $a = 0.64, b = 1.92$.

Table 3.1 Maximum and resting heart rate (beats per minute) for each rider

Rider	H_{max}	H_{rest}	Rider	H_{max}	H_{rest}
1	180	45	6	187	39
2	203	48	7	187	49
3	182	45	8	173	42
4	192	42	9	192	53
5	184	42	10	174	42

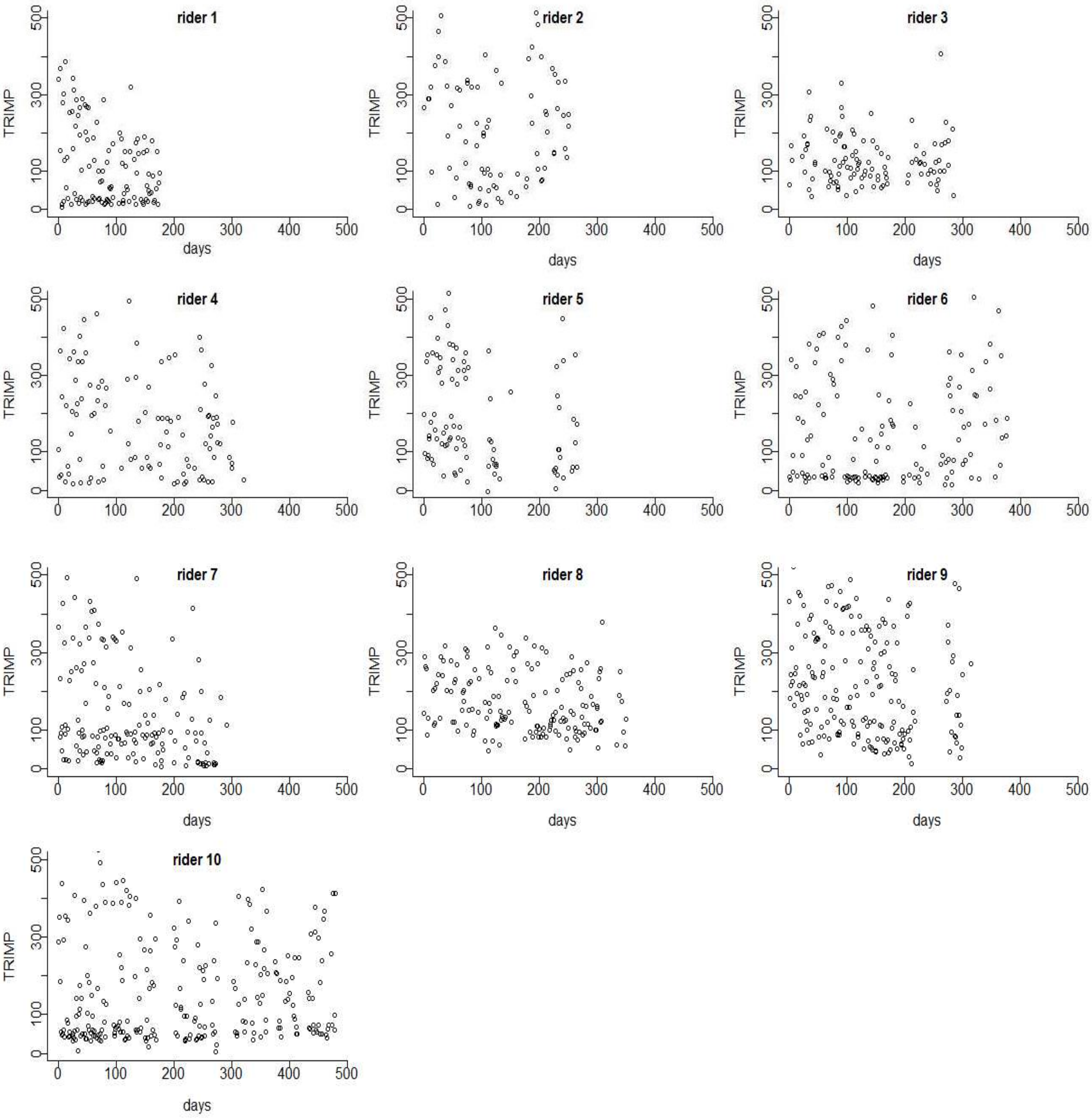

Figure 3.1 Training impulse (TRIMP) for each rider for each session using Borresen and Lambert constants

3.2.2 The accumulation of training (Banister model)

Banister, et al. (1975) proposed a model that quantifies training dose and its effect on performance. This model describes the progress of an athlete in terms of training benefit and detriment. Briefly, it describes individually the exercise dose-response relationship (Clarke and Skiba, 2013). The model of Banister was developed through the study of the training and performance profiles of a top class swimmer over 105 days of training. This model originally considered four components: skills, psychology, cardiovascular and strength. Calvert, et al. (1976) simplified this model to two components which are fitness and detriment.

Hellard, et al. (2006) mentioned some limitations to the Banister model approach. These limitations are "the limited accuracy of the model to predict future performance; the difference between estimated and actual changes in performance; and the poor corroboration of the model with physiological mechanisms" (Hayes and Quinn, 2009). Moreover, this model has many parameters which are hard to estimate especially with noisy data. Additionally, missing data will affect the accumulation of training.

The Banister model has been applied for several sports such as running (Morton, et al., 1990; Wood, et al., 2005), swimming (Hellard, et al., 2006; Hellard, et al., 2005; Mujika, et al., 1996), weight lifting (Busso, et al., 1990) and cycling (Busso, 2003; Busso, et al., 2002; Busso, et al., 1991; Busso, et al., 1997). It has been commonly used to describe the dynamics of training (Hellard, et al., 2005). We will discuss in detail what they have done in the next chapter.

The Banister model defines the accumulated training effect at time t of training sessions occurring up to time *t* as

$$W(t) = w_0 + k_a \sum_{i=1}^{n_t} w_{s_i}\, e^{-(t-s_i)/\tau_a} - k_f \sum_{i=1}^{n_t} w_{s_i}\, e^{-(t-s_i)/\tau_f} \qquad (3.1)$$

where $W(t)$ is the accumulated training effect (ATE) at time t. This can then be interpreted as the readiness-to-perform at time t and hence represents the potential performance at time *t*. s_i is the time at which session i was completed. w_{s_i} is the known training load during session i which is the amount of training that a rider completed during the session (Wallace, et al. 2013). It is defined as a function of $\underline{h_i}$ where $\underline{h_i}$ is the heart rate history for session i alone. One possible candidate for training load is training impulse (TRIMP) which was defined previously. n_t is the number of sessions up to time *t*. w_0 corresponds to the net training effect at time $t = 0$ of sessions in $(-\infty, 0]$. We will call $w_{s_i}e^{-(t-s_i)/\tau_a}$ the training benefit at time t of a session i that took place at time $s_i < t$ and $w_{s_i}e^{-(t-s_i)/\tau_f}$ the training detriment (fatigue) at time t of a session i that took place at time s_i.

Critically, it is the training benefit and training detriment that must be quantified in order to optimise training (Hayes and Quinn, 2009; Taha and Thomas, 2003). The benefit and detriment associated with a particular session decay at different rates depending on the parameters τ_a and τ_f, the fitness and detriment decay time constants, respectively. The decay in both fitness and detriment is assumed to be exponential and in principle, the decay of fitness is slower than the decay of detriment: $\tau_a > \tau_f$. k_a and k_f are the scale constants that control the relative size of the immediate training benefit with respect to the immediate training detriment. Strictly, one or other of these parameters is redundant as the scale of $W(t)$ is arbitrary. Therefore, without loss of generality we will set $k_a = 1$ throughout. Thus $W(t)$ in equation (3.1) is the resultant accumulation of decaying benefits and detriments over time.

Figure 3.2 illustrates the components of the Banister model for a single session, the benefit and dis-benefit (detriment), and the resultant, overall training effect. Notice how the benefit, dis-benefit and resultant decay with time.

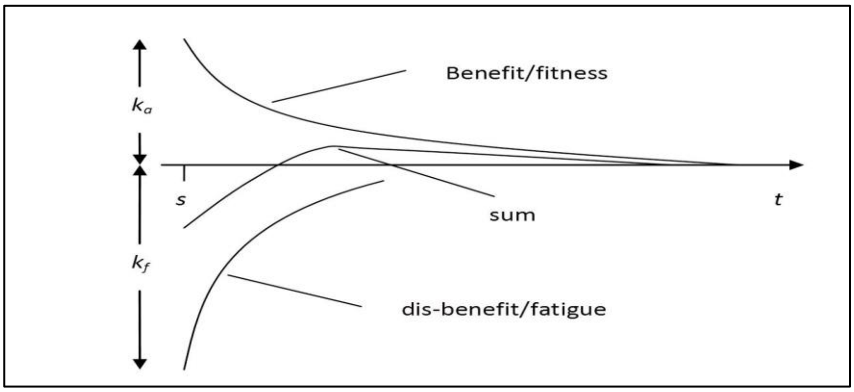

Figure 3.2 The components of the Banister model

In this study, as the Banister model is a nonlinear model we need more data points per parameter than for a linear regression model. This means a large number of observations would be required to use suitable statistical analysis and to get accurate results. An example of the Banister curve for the response to a single session is shown in figure 3.3 with default parameters. Figure 3.4 shows the Banister curve for a progressive training schedule of 200 days with unit training load for each session, and with parameters as in figure 3.3. For a different type of training schedule with similar parameter values of the Banister model see figure 3.5.

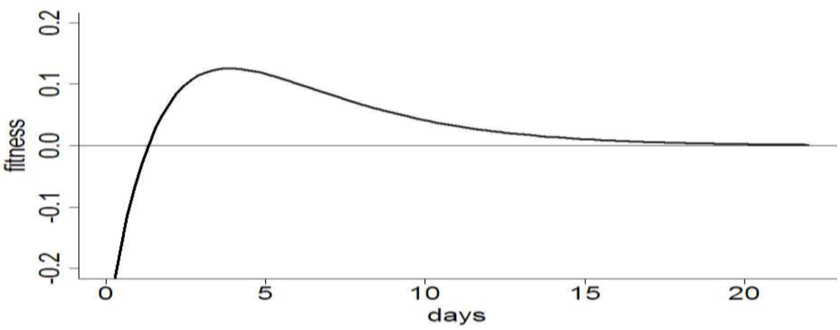

Figure 3.3 An example of the Banister curve for a single session with default parameters ($\tau_a = 3\ days, \tau_f = 2\ days, k_a = 1\ and\ k_f = 1.3$)

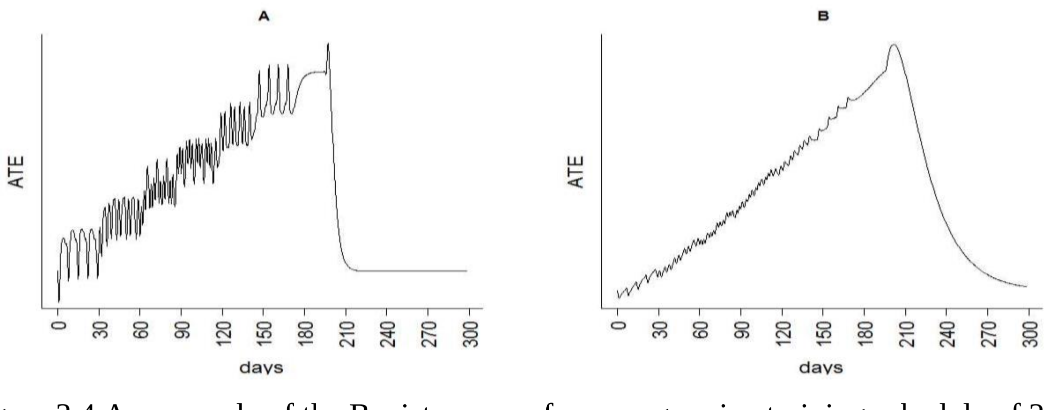

Figure 3.4 An example of the Banister curve for a progressive training schedule of 200 days with unit training load for each session, and with parameters as $\tau_a = 3\ days, \tau_f =$

$2\ days, k_a = 1$ and $k_f = 1.3$, (plot A) and $\tau_a = 20\ days, \tau_f = 10\ days, k_a = 1$ and $k_f = 1.3$, (plot B).

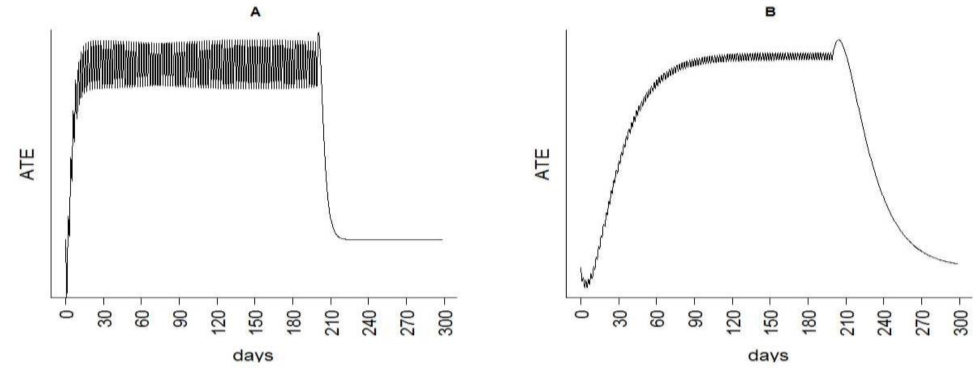

Figure 3.5 Another example of the Banister curve for an every other day training schedule over 200 days with unit training load for each session, and with parameters as $\tau_a = 3\ days, \tau_f = 2\ days, k_a = 1$ and $k_f = 1.3$, (plot A) and $\tau_a = 20\ days, \tau_f = 10\ days, k_a = 1$ and $k_f = 1.3$, (plot B).

3.3 Measuring performance

3.3.1 Introduction

Performance can be measured in a standard way by asking an athlete to swim or ride or run (depending on the type of sport) a particular specified distance. It could be practically defined as maximum peak power or speed, or time to exhaustion for a given speed or power (Smith, 2003a). Larson, et al., (2013) stated that 'In many sports, performance is based on maintaining high-level physical outputs during repeated bouts'. A difficulty with this approach is that performance measurements may be infrequent and they may underestimate actual capability or readiness-to-perform; or the rider may hold something back. However, in our approach we aim to use data from a long period of training in order to measure performance.

Cycling performance can be influenced by two groups (internal and external) of factors (Jeukendrup and Martin, 2001). Some internal factors were reported as training (Hawley and Stepto, 2001; Stepto, et al., 1999), carbohydrate intake (Burke, 2001), and caffeine intake (Costill, et al., 1978; Spriet, et al., 1992). On the other hand, Martin, et al., (1998) and Olds, (2001) noted some external factors such as body position, clothing, bicycle and wheels. Therefore, Jeukendrup and Martin, (2001) studied the comparison of internal and external factors with respect to their influence on the time taken to complete a 40 km time. Additionally, heart rate could be affected by seat position (Price and Donne, 1997). However, seat position has not been investigated in our study due to the relevant information not being available to us. Nonetheless, Schniepp, et al., (2002) stated that cycling performance could be affected by many factors during competition of which cold environmental conditions may be the most influential. This can change muscle blood flow and metabolism. Furthermore, these changes will affect power output and as a result cycling performance will decline. Moreover, Foster, et al., (1996) mentioned that increasing training load by a ten-fold factor is associated with an approximately 10% improvement in performance (Gabbett and Domrow, 2007).

Therefore, measuring performance from data on training is potentially useful. We now discuss how to do this. Firstly in the next section we present the measure that we think is most important. Other possible measures are also discussed in chapter five.

3.3.2 A new measure of performance

In this section, we present a new measure of performance based on the relationship between power output and heart rate collected every five seconds for cyclists during riding. We focus firstly on the relationship between power output and heart rate. Then we propose our new measure of performance based on this relationship.

3.3.2.1 The relationship between power output and heart rate

It is generally accepted that power output is proportional to heart rate excess (the difference between heart rate and resting heart rate). For example, Grazzi et al. (1999) investigated the relationship between power output and heart rate for 290 participants including 500 tests conducted. They found a strong correlation of 0.98 or above for many riders. There is also a delay or time lag between the change in power output and the heart rate response. The literature is less clear on the value of this delay or lag. Jeukendrup and Diemen, (1998) argued its existence for periods of exercise of short duration, as the circulatory system is not able to fully adapt to change in exercise intensity. However, the size of the lag was not indicated. Stirling, et al. (2008) suggested that for both increases and decreases in heart-rate, these changes in heart rate (e.g. 80 to 160 beats per minute) occur over a period of approximately 30-60 seconds. For the data in our study, short term changes in heart-rate tend to be smaller than in the Stirling et al. study. We speculate that for sessions where intensity changes gradually power output will be best explained by a heart rate lag towards the bottom end of the 30-60 second range, or indeed less.

We investigate different lags of some seconds (0, 10, 15, 20 and 30 seconds) between power output and heart rate and find the strongest relationship when the lag is 15 seconds for almost all sessions (see Table 3.2 and Appendix 1).

Figure 3.6 illustrates the power output/heart rate relationship for a single session for rider 3 with lag of 15 seconds. All sessions for rider 3 are shown in figure 3.7. For the other riders for all sessions see Appendix 2.

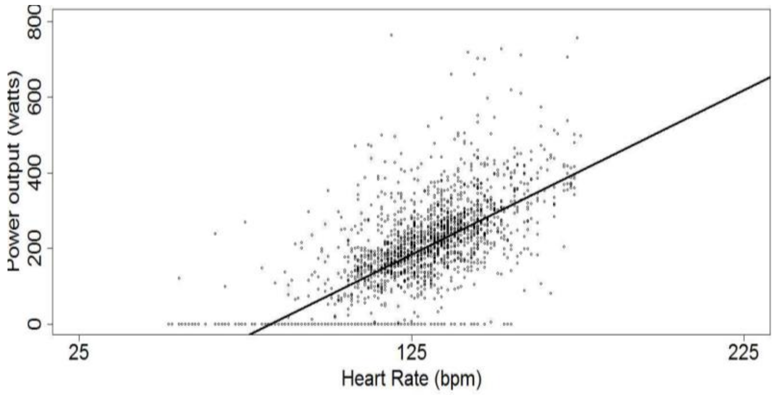

Figure 3.6 Power output against heart rate for a single session with lag =15 seconds

Table 3.2 Sample linear correlation between power output and heart rate for each session for rider 3 with different heart rate lags of seconds (0, 10, 15, 20, and 30 seconds), the strongest correlation for each session is highlighted.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.48	0.66	0.70	0.70	0.67	55	0.48	0.60	0.63	0.63	0.63
2	0.47	0.64	0.68	0.69	0.67	56	0.58	0.71	0.72	0.72	0.66
3	0.46	0.59	0.60	0.61	0.54	57	0.52	0.62	0.63	0.61	0.53
4	0.47	0.61	0.64	0.63	0.59	58	0.26	0.32	0.36	0.36	0.34
5	0.42	0.54	0.57	0.57	0.48	59	0.55	0.67	0.71	0.72	0.70
6	0.46	0.47	0.49	0.49	0.51	60	0.40	0.47	0.49	0.52	0.44
7	0.61	0.69	0.72	0.72	0.71	61	0.61	0.71	0.72	0.69	0.60
8	0.47	0.53	0.54	0.56	0.54	62	0.59	0.67	0.69	0.70	0.66
9	0.32	0.46	0.48	0.48	0.45	63	0.41	0.52	0.56	0.56	0.54
10	0.65	0.72	0.74	0.73	0.70	64	0.39	0.42	0.40	0.36	0.33
11	0.79	0.86	0.87	0.86	0.79	65	0.39	0.48	0.49	0.47	0.42
12	0.62	0.75	0.76	0.74	0.66	66	0.59	0.69	0.72	0.71	0.64
13	0.46	0.53	0.53	0.53	0.48	67	0.45	0.54	0.56	0.55	0.49
14	0.26	0.35	0.39	0.39	0.36	68	0.46	0.60	0.62	0.63	0.61
15	0.35	0.37	0.42	0.41	0.38	69	0.54	0.66	0.69	0.68	0.66
16	0.55	0.68	0.70	0.70	0.67	70	0.52	0.63	0.65	0.65	0.62
17	0.48	0.59	0.60	0.60	0.58	71	0.62	0.69	0.70	0.70	0.67
18	0.46	0.53	0.55	0.57	0.55	72	0.56	0.66	0.67	0.66	0.62
19	0.45	0.61	0.66	0.67	0.66	73	0.51	0.68	0.70	0.71	0.64
20	0.57	0.67	0.69	0.68	0.64	74	0.41	0.55	0.56	0.54	0.49
21	0.47	0.62	0.62	0.62	0.59	75	0.26	0.37	0.41	0.44	0.45
22	0.58	0.67	0.69	0.69	0.67	76	0.49	0.57	0.59	0.59	0.56
23	0.49	0.64	0.64	0.63	0.58	77	0.18	0.26	0.19	0.18	0.16
24	0.53	0.65	0.66	0.66	0.60	78	0.1	0.11	0.11	0.08	0.09
25	0.55	0.66	0.68	0.68	0.68	79	0.46	0.43	0.47	0.47	0.40
26	0.37	0.48	0.50	0.49	0.43	80	0.36	0.50	0.54	0.56	0.57
27	0.17	0.25	0.26	0.26	0.22	81	0.19	0.28	0.37	0.36	0.43
28	0.61	0.67	0.71	0.72	0.72	82	0.54	0.68	0.70	0.72	0.69
29	0.24	0.26	0.24	0.24	0.23	83	0.42	0.57	0.61	0.63	0.59
30	0.17	0.17	0.14	0.15	0.18	84	0.46	0.59	0.61	0.64	0.63
31	0.52	0.61	0.62	0.61	0.56	85	0.39	0.53	0.55	0.54	0.51
32	0.58	0.68	0.69	0.69	0.66	86	0.6	0.66	0.68	0.68	0.67
33	0.6	0.67	0.68	0.67	0.65	87	0.42	0.55	0.58	0.58	0.55
34	0.75	0.80	0.81	0.82	0.79	88	0.47	0.57	0.57	0.57	0.56
35	0.7	0.75	0.76	0.75	0.71	89	0.47	0.60	0.62	0.63	0.60
36	0.83	0.84	0.85	0.85	0.84	90	0.39	0.45	0.44	0.44	0.40
37	0.83	0.84	0.85	0.84	0.83	91	0.36	0.49	0.49	0.50	0.48
38	0.86	0.85	0.87	0.87	0.86	92	-0.05	-0.04	-0.02	-0.04	-0.08
39	0.54	0.67	0.70	0.71	0.68	93	0.52	0.66	0.70	0.70	0.68
40	0.5	0.58	0.59	0.57	0.49	94	0.45	0.61	0.65	0.65	0.63
41	0.58	0.66	0.68	0.67	0.65	95	0.62	0.74	0.74	0.74	0.69
42	0.68	0.74	0.74	0.72	0.68	96	0.61	0.68	0.67	0.67	0.65
43	0.56	0.70	0.73	0.73	0.67	97	0.55	0.66	0.67	0.67	0.64
44	0.47	0.57	0.59	0.60	0.54	98	0.52	0.68	0.71	0.70	0.63
45	0.51	0.61	0.64	0.65	0.62	99	0.48	0.64	0.66	0.66	0.63
46	0.49	0.63	0.67	0.68	0.65	100	0.48	0.62	0.65	0.65	0.61
47	0.53	0.65	0.66	0.65	0.57	101	0.47	0.63	0.67	0.68	0.64
48	0.56	0.70	0.73	0.71	0.65	102	0.54	0.63	0.64	0.65	0.63
49	0.37	0.44	0.46	0.45	0.40	103	0.51	0.64	0.66	0.67	0.66
50	0.49	0.67	0.72	0.72	0.66	104	0.45	0.62	0.64	0.63	0.56
51	0.62	0.67	0.67	0.69	0.69	105	0.59	0.71	0.73	0.72	0.68
52	0.52	0.57	0.57	0.56	0.52	106	0.56	0.69	0.71	0.69	0.63
53	0.29	0.34	0.37	0.36	0.39	107	0.36	0.46	0.53	0.52	0.49
54	0.27	0.37	0.40	0.38	0.34	108	0.46	0.59	0.59	0.59	0.57

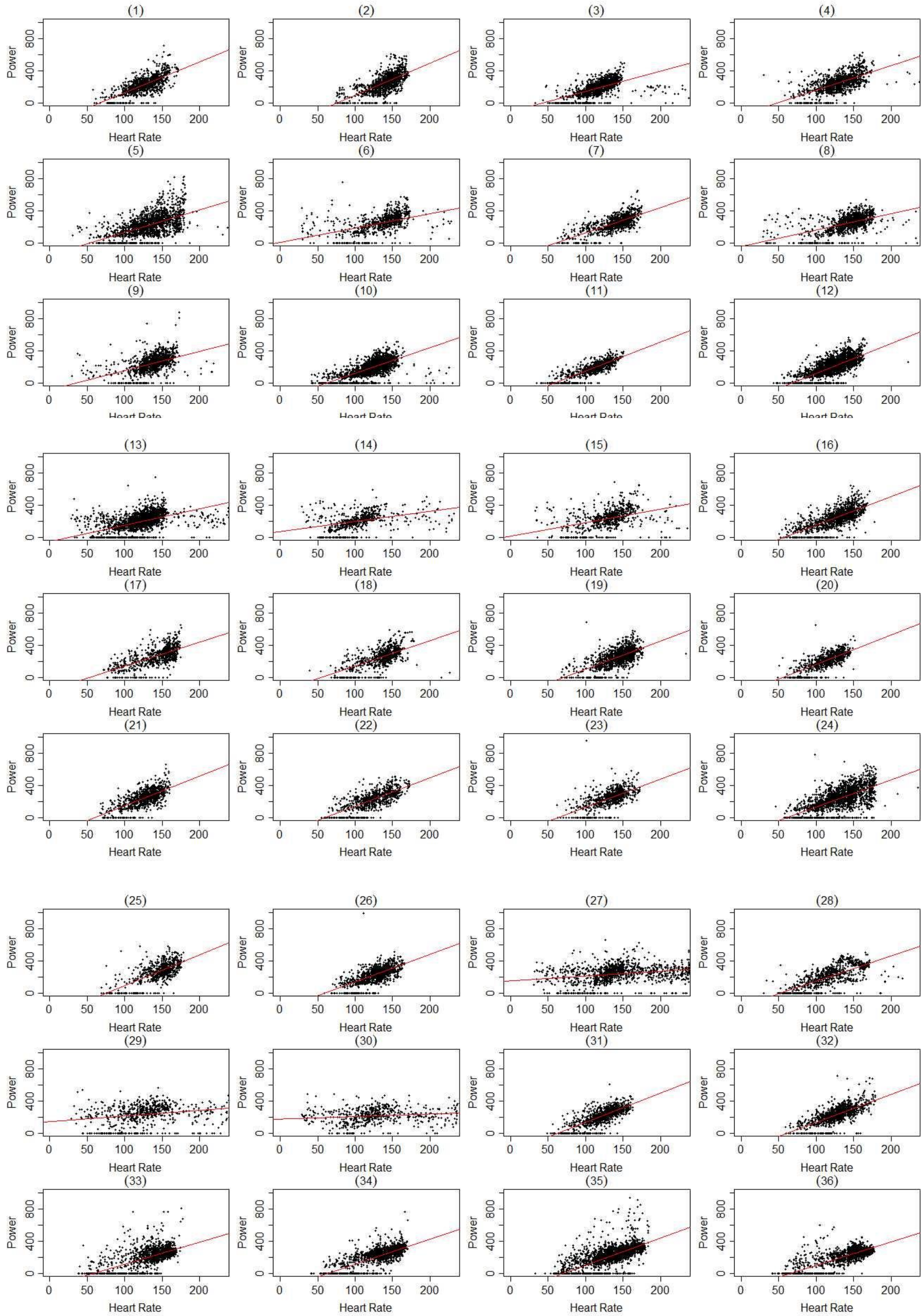

Figure 3.7 Power output against heart rate for all sessions for a specific rider (rider 3) with lag = 15 seconds.

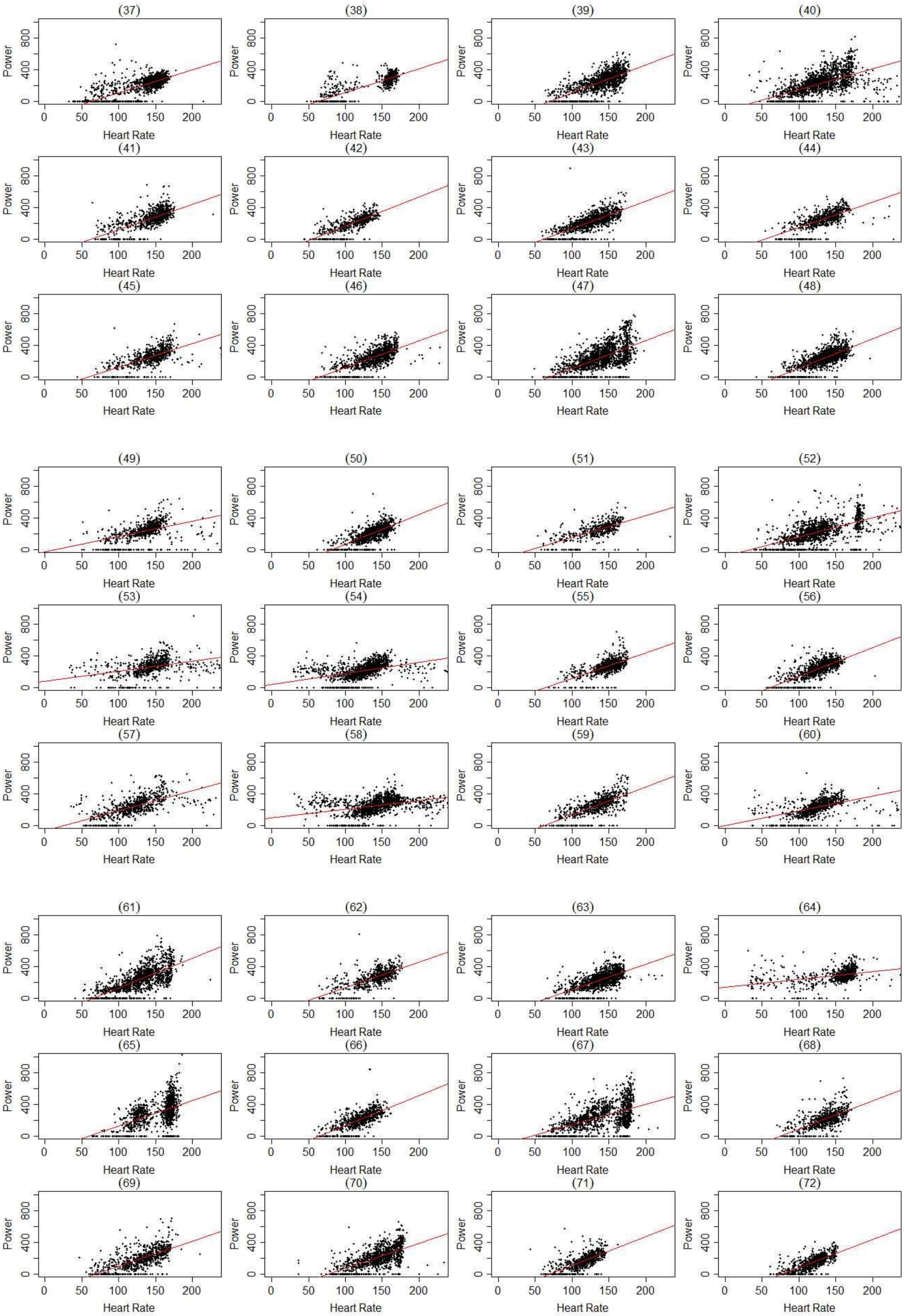

Figure 3.7 Continued.

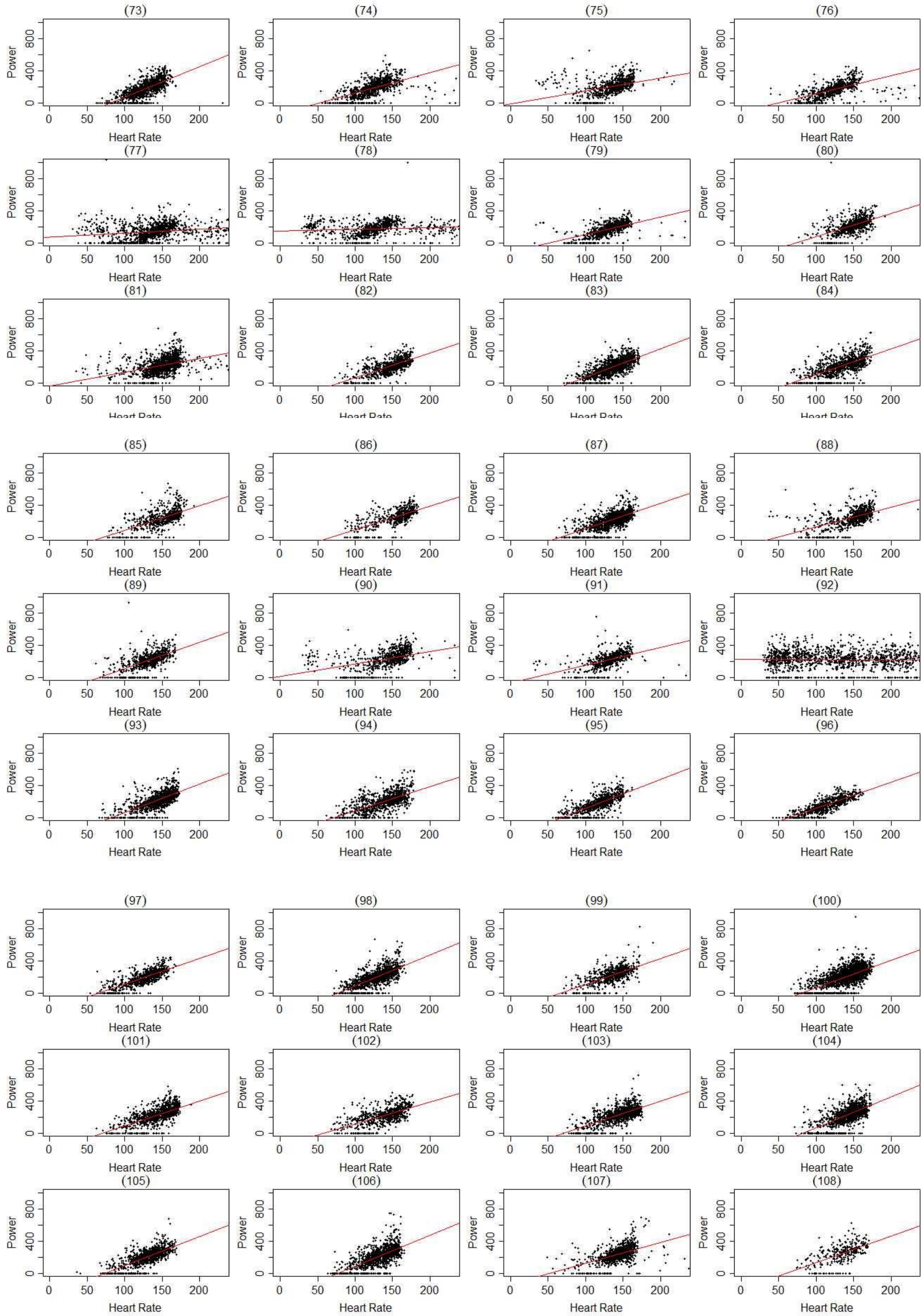

Figure 3.7 Continued.

3.3.2.2 A performance measure based on the relationship between power output and heart rate

For a specific rider of interest, firstly we determine some high percentiles (e.g. the 75th) of power output using the entire training history of the rider. These percentiles divide the ordered data with $q\%$ below it and $(100 - q)\%$ above it e.g. see Figure 3.8.

The appropriate percentile depends on the nature of the competition for which the rider is training. For example, if the race is an endurance race, q should be moderate and if it is a sprint race, q should be high. Selected percentiles of power output for each rider are shown in Table 3.3.

Now, the performance measure for a session that we propose is defined as the expected heart rate (given a linear model that relates power output to heart rate excess) at this power output percentile. It is denoted h_{Pq} in general. For h_{P75} in particular, we show this performance measure for rider 3 for a particular session in Figure 3.9. This performance measure is calculated for all sessions. Figure 3.10 shows the performance measure h_{P75} for each rider for each session. As a rider becomes trained, and all else being equal, we would expect h_{Pq} to decrease. That is, the heart rate required to maintain a specified high power output ought to decrease as a rider becomes fitter. We will relate this measure of performance to the accumulated training effect in the next chapter of this thesis.

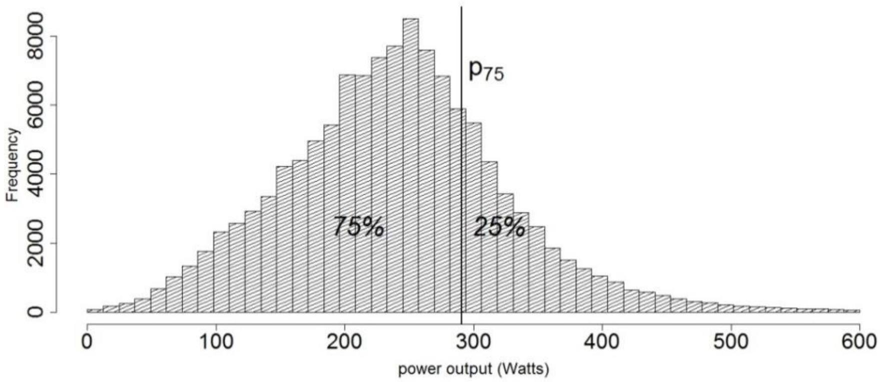

Figure 3.8 The histogram of power output, pooling all sessions for a specific rider (rider 3).

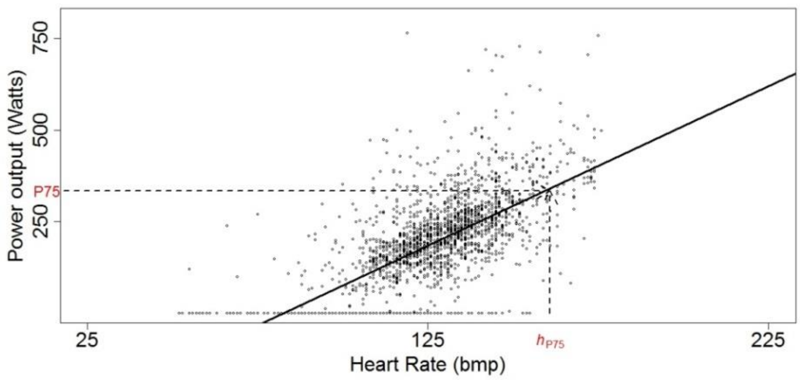

Figure 3.9 The performance measure for a single session for rider 3.

Table 3.3 Selected percentiles of power output for each rider

Rider	p_{50}	p_{75}	p_{90}	p_{95}	p_{99}
1	225	291	360	424	615
2	235	307	387	439	573
3	239	291	347	391	508
4	213	246	289	328	451
5	213	280	350	402	536
6	293	384	488	566	776
7	238	323	405	451	595
8	197	274	350	398	514
9	184	214	257	296	407
10	208	260	312	351	469

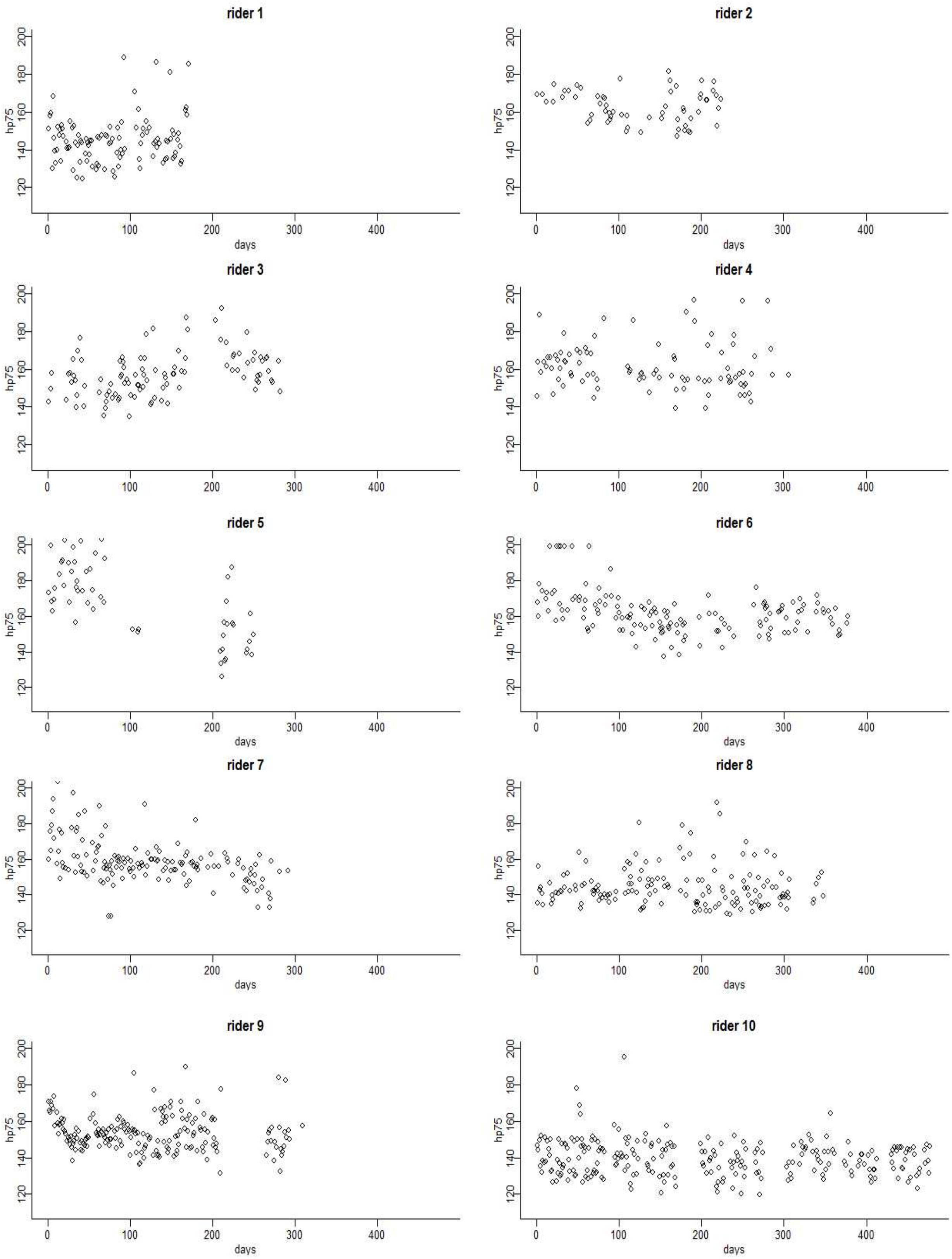

Figure 3.10 The performance measure h_{P75} for each rider for each session

3.4 Summary

In this chapter, we explained how to quantify training and performance using data on power output and heart rate. Firstly, we presented a measure of training load for each session called the training impulse (TRIMP) proposed by Banister, et al. (1975). Then we discussed about the accumulation of training using the Banister model and briefly explain its components. After that, we proposed our performance measure based on the relationship between power output and heart rate with the most appropriate time lag (15 seconds). In the next chapter, we will relate the accumulation of training to this performance measure to determine the Banister model parameters which are required to optimise training.

4. DETERMINING THE PARAMETERS OF THE ACCUMULATED TRAINING EFFECT

4.1 Introduction

Many researchers have used qualitative approaches to relate training to performance (e.g. Avalos, et al. 2003; Grazzi, et al. 1999; Hopkins, 1991; and Stewart and Hopkins, 2000). However, the first person who used a quantitative approach to this issue was Banister. Banister, et al. (1975) proposed a model that describes athletic progress in terms of training benefit and detriment. These authors proposed a system model to relate a profile of athletic performance to a profile of training. Avalos, et al. (2003) used this model in a limited way to consider the relationship between training and performance for 13 competitive swimmers over three seasons, and identified individual and group responses to training. Our aim is to use the same model of Banister et al. (1975) to relate performance to the accumulated training effect, using data collected over a period of training. The model requires two input measurements: 1) a measure of performance; 2) a measure of training load.

The aim of the Banister model is to relate training to performance over time. To optimise training (to maximise performance at a future time), the parameters of the Banister model should be known. Few studies have been able to quantitatively relate training to performance. Nonetheless, a number of interesting previous studies exist.

Mujika, et al., (1996) studied the effect of training on performance for 18 elite swimmers (8 female, 10 male) using different tapers. They minimised the residual sum of squares between real performance measured throughout the training program and modelled performance using the Banister model. The mean (standard error) of the scale parameter values of their Banister model were reported as $k_a = 0.062(0.041)$ and $k_f = 0.128(0.055)$ in arbitrary units. The fitness and detriment decay time constants τ_a and τ_f were given as 41.4(12.5) and 12.4(6.9) days respectively.

Another study for swimming was carried out by Hellard et al. (2006). Nine elite swimmers (5 female, 4 male) participated in their research over a one year. Real performances were measured during actual competitions throughout the study period. They presented real performance over time. The parameter values of the Banister model were estimated for each participant using non-linear least squares between real and modelled performances. The means (standard errors) of these parameters were determined for k_a and k_f as 0.036 (0.038) and 0.050 (0.044) arbitrary units respectively. The mean decay time constants τ_a and τ_f were presented as 38(16) and 19(11) days respectively.

Morton et al. (1990) reported the Banister model parameters in different sport, running in particular. These parameter values were presented as $k_a = 1$ and $k_f = 2$ arbitrary units respectively. The fitness and detriment decay time constants τ_a and τ_f were reported as 45 and 15 days respectively.

In cycling, the Banister model parameters were reported by Busso, et al., (1997). Two subjects participated in the study for 16 weeks. The least squares method was used to determine the model parameters by fitting the model performances using the Banister model to the actual performances recorded during training. The scale parameter values k_a

and k_f were reported as 0.0021 and 0.0078 respectively for subject A and 0.0019 and 0.0073 respectively for subject B. The fitness decay time constants were given as 60 days for both subjects. The detriment decay time constants were reported as 4 days in subject A and 6 days in subject B. Further analysis has been done for cycling by Busso, et al., (2002). They used the Banister model for analysing the effect of increasing training frequency on exercise-induced fatigue using 6 subjects over 15 weeks. The subjects participated for 8 weeks of training period with 3 sessions per week (low-frequency training), one week without training, 4 weeks training with 5 sessions per week (high-frequency training) and then 2 weeks without training. The Banister model parameters were estimated by fitting modelled performances to the measured ones using the least squares method. The main finding of this study was that an increase in training frequency induced changes in the dynamics of response of performance to a single training bout.

In this thesis, our aim is to estimate these parameter values for the Banister model for cycling. Our approach is different from previous studies. We develop a new model to estimate these parameters using training data such as power output and heart rate collected every five seconds. We explain the new approach in the next subsection.

4.2 Estimating the Banister model parameters

We assume a linear relationship between our performance measure and the accumulated training effect, so that the performance on day i, $h_{P75,i}$, is related to the accumulated training effect on day i, ATE_i by

$$h_{P75,i} \sim N(\alpha + \beta . ATE_i, \sigma^2)$$

where σ^2 measures the variability in the performance-training relationship.

However $h_{P75,i}$ is latent (unobserved) and instead of that we observe an estimate from the session power-heart rate data (e.g. figure 3.9). So, we will assume that

$$\hat{h}_{P75,i} \sim N\left(h_{P75,i}, \lambda_i\right).$$

The variance λ_i can be determined from the variability in the power-heart rate relationship and is estimated using the delta method as described later.

So our full model is written as

$$\hat{h}_{P75,i} \sim N(\alpha + \beta . ATE_i, \sigma^2 + \lambda_i)$$

Then the log likelihood function of the above model considered over days, whose $\hat{h}_{P75,i}$ is independent for each day is written

$$\log L = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{n}\log(\lambda_i + \sigma^2) - \frac{1}{2}\sum_{i=1}^{n}\frac{\left(\hat{h}_{P75,i} - \alpha - \beta(ATE_i)\right)^2}{(\lambda_i + \sigma^2)} \qquad (4.1)$$

The variances λ_i, $i = 1, \dots, n$ in the likelihood are specified as follows. In our model the relationship between power output and heart rate is linear, so we can write power output as a function of heart rate as follows

$$P_{75} = a + b . h_{P75}.$$

Hence

$$h_{P75} = \frac{P_{75}-a}{b}.$$

For each session, the parameters a and b and their variances are estimated using least squares. We can then estimate λ_i as follows. In its general form the delta method is

$$var[y(\underline{\hat{\theta}})] = \sum_i \sum_j \frac{\partial y}{\partial \theta_i} \frac{\partial y}{\partial \theta_j} . cov(\hat{\theta}_i, \hat{\theta}_j).$$

So in our case

$$\theta = (\theta_1, \theta_2) = (a, b)$$

and

$$y(\theta) = h_{P75} = \frac{P_{75} - a}{b}, \quad \frac{\partial y}{\partial a} = -\frac{1}{b}, \qquad \frac{\partial y}{\partial b} = -\frac{(P_{75} - a)}{b^2}$$

This leads to

$$var(h_{P75,i}) = \lambda_i \approx \frac{1}{b^2} var(a) + \frac{(P_{75} - a)^2}{b^4} var(b) + \frac{2(P_{75} - a)}{b^3} cov(a, b)$$

The parameters in the above formula are then specified by their estimates

$$var(\hat{h}_{P75,i}) = \hat{\lambda}_i = \frac{1}{\hat{b}^2} var(\hat{a}) + \frac{(P_{75} - \hat{a})^2}{\hat{b}^4} var(\hat{b}) + \frac{2(P_{75} - \hat{a})}{\hat{b}^3} cov(\hat{a}, \hat{b}) \qquad (4.2)$$

The remaining parameters are then estimated by maximising the log likelihood (4.1). These parameters are $\alpha, \beta, \sigma^2, k_f, \tau_a$ and τ_f. Maximisation is carried out using R.

4.3 Determining starting values of our model

The likelihood maximisation process is sensitive to the starting values. To handle this, we developed a procedure to find preliminary estimates of the parameters based on the correlation between the performance measure and the accumulated training effect (ATE) calculated for a number of specific parameter values. These parameter values and the correlations are reported in Appendix 3. We then used response surface methodology with a quadratic function

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{11}x_{11}^2 + a_{22}x_{22}^2 + a_{33}x_{33}^2$$

to find the parameter values that minimise the correlation between h_{P75} and ATE, where y is the correlation between the performance measure and the accumulated training effect (ATE), x_1 is the disbenefit scale parameter k_f and x_2, x_3 are the detriment and fitness decay time constants (τ_f, τ_a) respectively with $k_a = 1$.

Table 4.1 shows the values of the parameters for the Banister model that minimise the correlation between the performance measure and the accumulated training effect (ATE). According to this table, for some riders (3, 8, 9, and 10) we might not expect a strong negative correlation because as figure 3.10 shows the performance measures h_{P75} for those riders do not change clearly over time and we could not relate or link those measures to their accumulated training effect. However, for riders 1 and 7 obvious negative correlations (-0.34,-0.28) respectively are seen. The unused correlation for rider 5 is likely due to observing few data.

Table 4.1 The initial parameter values for the Banister model with the correlation between the performance measure and the accumulated training effect (ATE)

Rider	k_f	τ_f	τ_a	$corr(h_{P75}, ATE)$	Rider	k_f	τ_f	τ_a	$corr(h_{P75}, ATE)$
1	1.7	9	15	-0.34	6	1.5	13	90	-0.11
2	2.8	2	6	-0.12	7	1.7	19	15	-0.28
3	1.8	3	7	-0.10	8	2.1	2	7	0.11
4	2.3	7	13	-0.16	9	1.2	2	13	-0.09
5	1.2	4	30	0.31	10	1.1	18	35	0.04

4.4 Pre-processing the data

In our study, we have some limitations in the data. For instance, we have plenty of variations for some sessions for some riders. Although we give these sessions less weight in our analysis by using $\hat{\lambda}_i$, the estimates of the Banister model parameters for some riders (e.g. 6,10) are still affected. So for these two riders we set their performance measures between their resting heart rate and maximum heart rate to exclude odd sessions.

4.5 Results

Banister, et al. (1975) stated that ‘It has been theorized that the training impulse generates twice as much fatigue in each session as it does fitness’. Since $k_a = 1$ has been proposed to take the value 1, we perform maximum likelihood estimates of the accumulated training effect parameters both when $k_f = 2$ (fixed) and $k_f \neq 2$ (free) and also with performance measures h_{P50} and h_{P75}.

The maximum likelihood estimates of the parameters when $k_f = 2$ are presented in Table 4.2 and Table 4.3 for the performance measures h_{P50} and h_{P75} respectively. The results when $k_f \neq 2$ for performance measures h_{P50} and h_{P75} are seen in table 4.4 and table 4.5.

Our performance measures h_{P50}, h_{P75} are shown in Figures 4.1 and Figure 4.2. For each measure, the accumulated training effect curves are presented. These curves depend on the estimates of the accumulated training effect parameters with $k_f = 2$. Similarly, the corresponding results but for the second case when $k_f \neq 2$ are presented in Figure 4.3 and Figure 4.4.

Table 4.2 Parameter estimates of the model (standard errors) when $k_f = 2$ for performance measure h_{P50}

Rider	σ	τ_a	k_f	τ_f	α	β	$\beta/s.e.(\beta)$
1	6.2 (1.0)	33 (15)	2	0.23 (0.6)	135 (3.0)	-0.0019 (0.0008)	-2.40
2	0.01 (3.9)	19 (4)	2	11 (2.0)	153 (4.0)	0.0200 (0.0050)	4.00
3	3.5 (1.2)	8 (5)	2	3 (1.5)	139 (2.2)	-0.0200 (0.0100)	-2.00
4	0.001 (7.4)	6.1 (3)	2	3.4 (1.4)	148 (2.3)	0.0300 (0.0150)	2.00
5	0.4 (4.6)	199 (81)	2	37 (16.0)	137 (4.0)	-0.0020 (0.0010)	-2.00
6	0.2 (0.1)	180 (73)	2	38 (4.0)	154 (1.0)	-0.0040 (0.0014)	-2.90
7	2.3 (1.2)	163 (66)	2	0.33 (1.4)	149 (2.2)	-0.0012 (0.0004)	-3.00
8	3.3 (0.6)	13.6 (11)	2	9.5 (8.0)	127 (2.0)	0.0040 (0.0040)	1.00
9	4 (0.6)	13.5 (7)	2	2.7 (2.0)	147 (2.0)	-0.0030 (0.0020)	-1.50
10	5.2 (0.5)	92 (41)	2	28 (16.0)	127 (2.0)	-0.0010 (0.0005)	-2.00

Table 4.3 Parameter estimates of the model (standard errors) when $k_f = 2$ for performance measure h_{P75}

Rider	σ	τ_a	k_f	τ_f	α	β	$\beta/s.e.(\beta)$
1	10 (1.3)	35 (17)	2	0.2 (1.0)	153 (4.0)	-0.0025 (0.0012)	-2.10
2	6 (1.4)	12 (5)	2	0.2 (0.5)	172 (5.0)	-0.0072 (0.0040)	-1.80
3	4.6 (1.2)	5.6 (4)	2	2.7 (1.3)	156 (2.0)	-0.0300 (0.0200)	-1.50
4	4 (1.7)	0.8 (0.4)	2	0.5 (0.1)	166 (2.0)	-0.5000 (0.8000)	-0.63
5	20 (5.0)	125 (45)	2	73 (18.0)	159 (7.0)	-0.0070 (0.0013)	-5.40
6	8.5 (1.1)	112 (23)	2	21 (4.6)	183 (4.0)	-0.0077 (0.0022)	-3.50
7	4.4 (1.0)	12.4 (7)	2	6.5 (3.0)	158 (2.0)	-0.0200 (0.0100)	-2.00
8	5.3 (0.7)	6.6 (7)	2	0.1 (3.0)	145 (2.0)	-0.0070 (0.0060)	-1.20
9	5 (0.7)	13.4 (8)	2	3 (2.0)	155 (2.0)	-0.0030 (0.0030)	-1.00
10	6 (0.6)	111 (84)	2	29 (15.5)	141 (2.2)	-0.0010 (0.0009)	-1.11

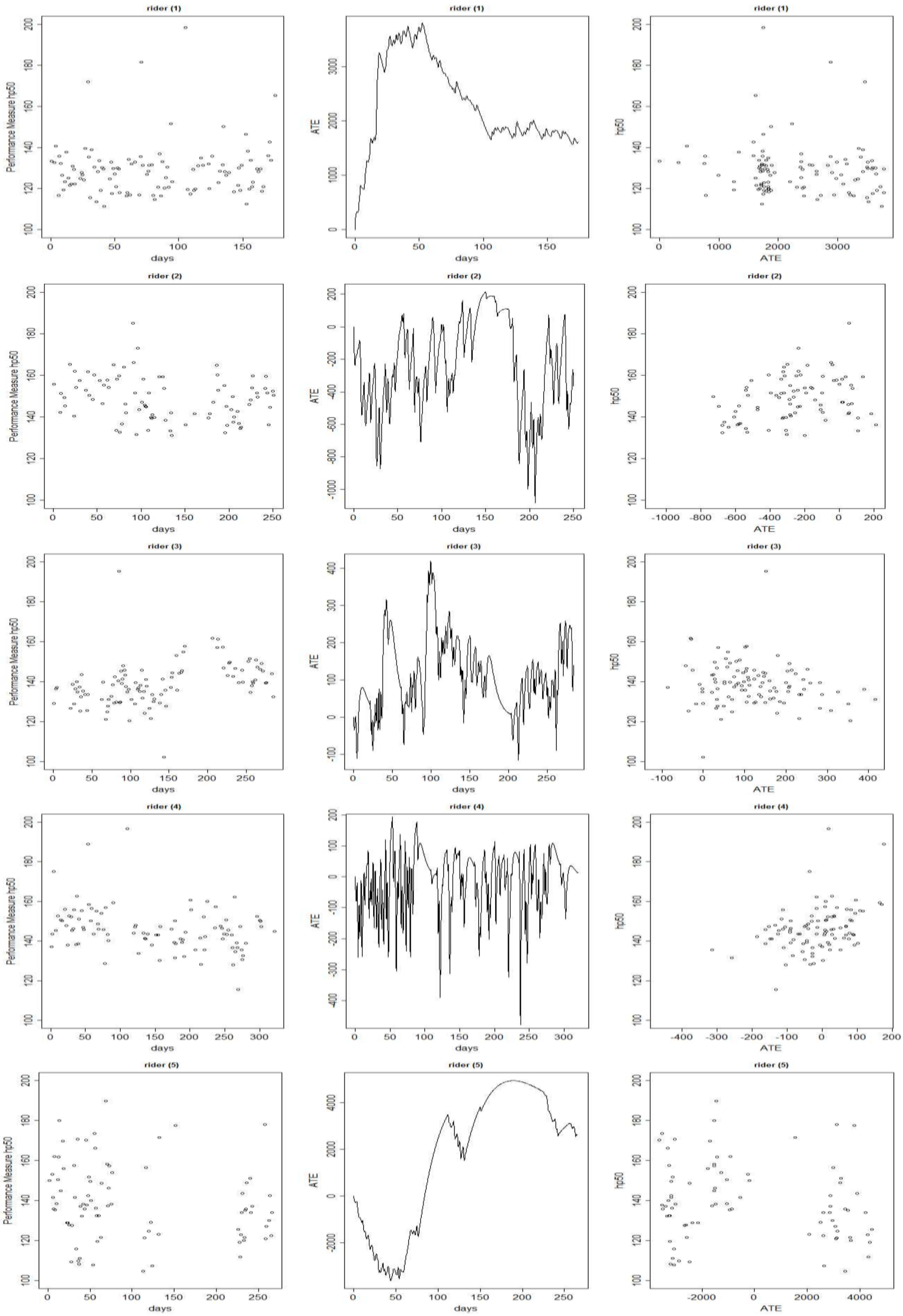

Figure 4.1 Performance measure h_{P50} and the curve of the accumulated training effect over time for each rider when $k_f = 2$

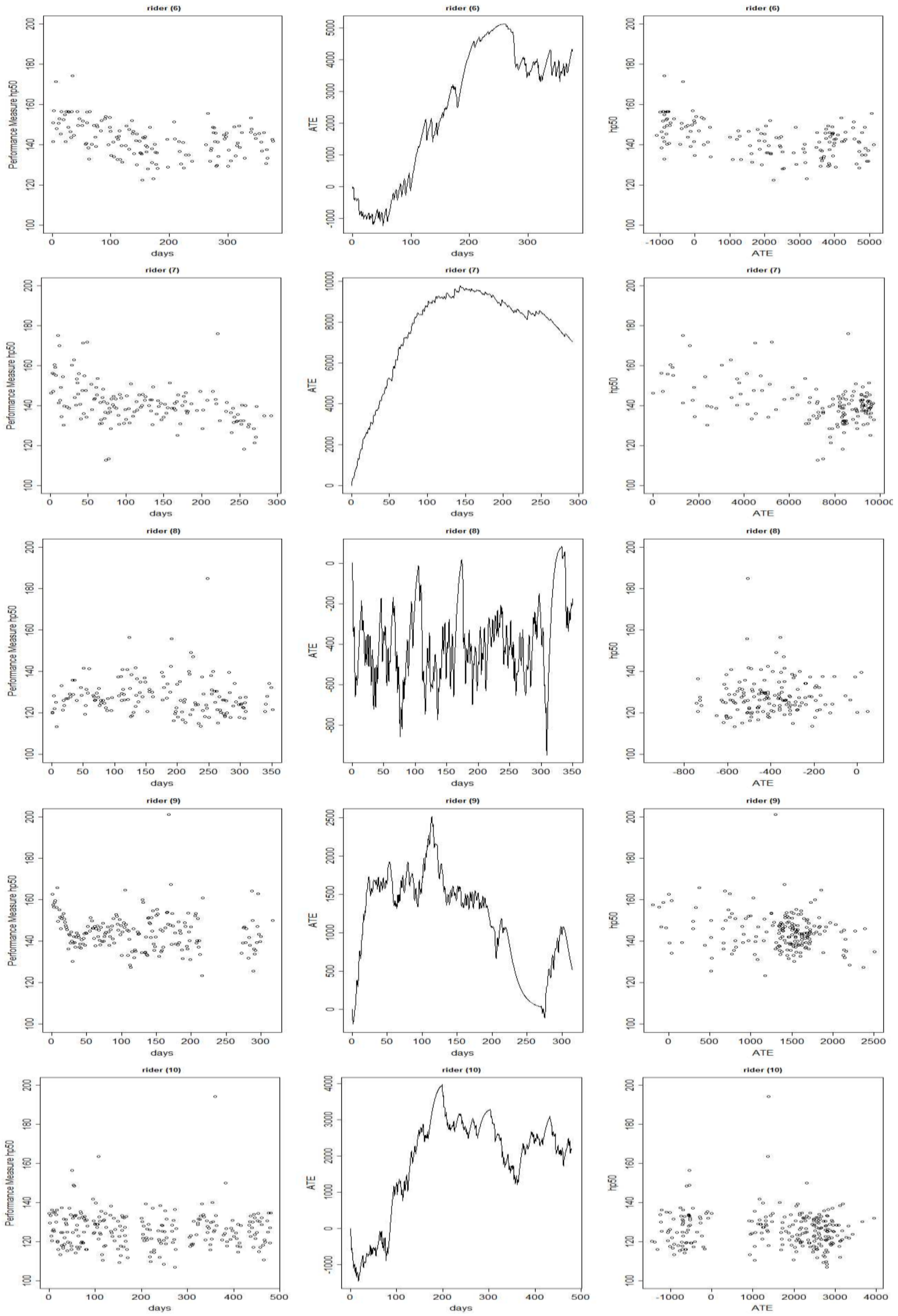

Figure 4.1 Continued.

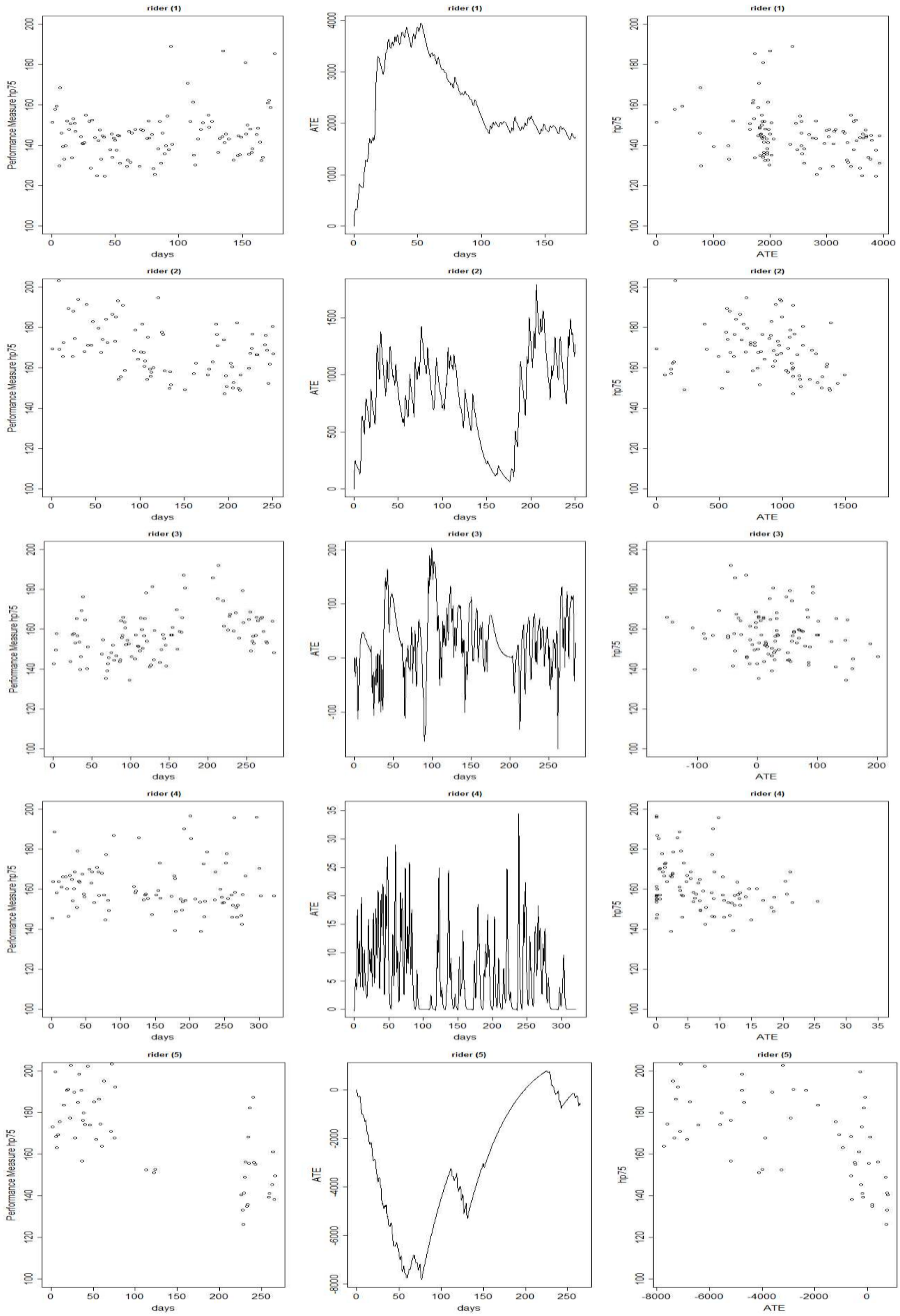

Figure 4.2 Performance measure h_{P75} and the curve of the accumulated training effect over time for each rider when $k_f = 2$

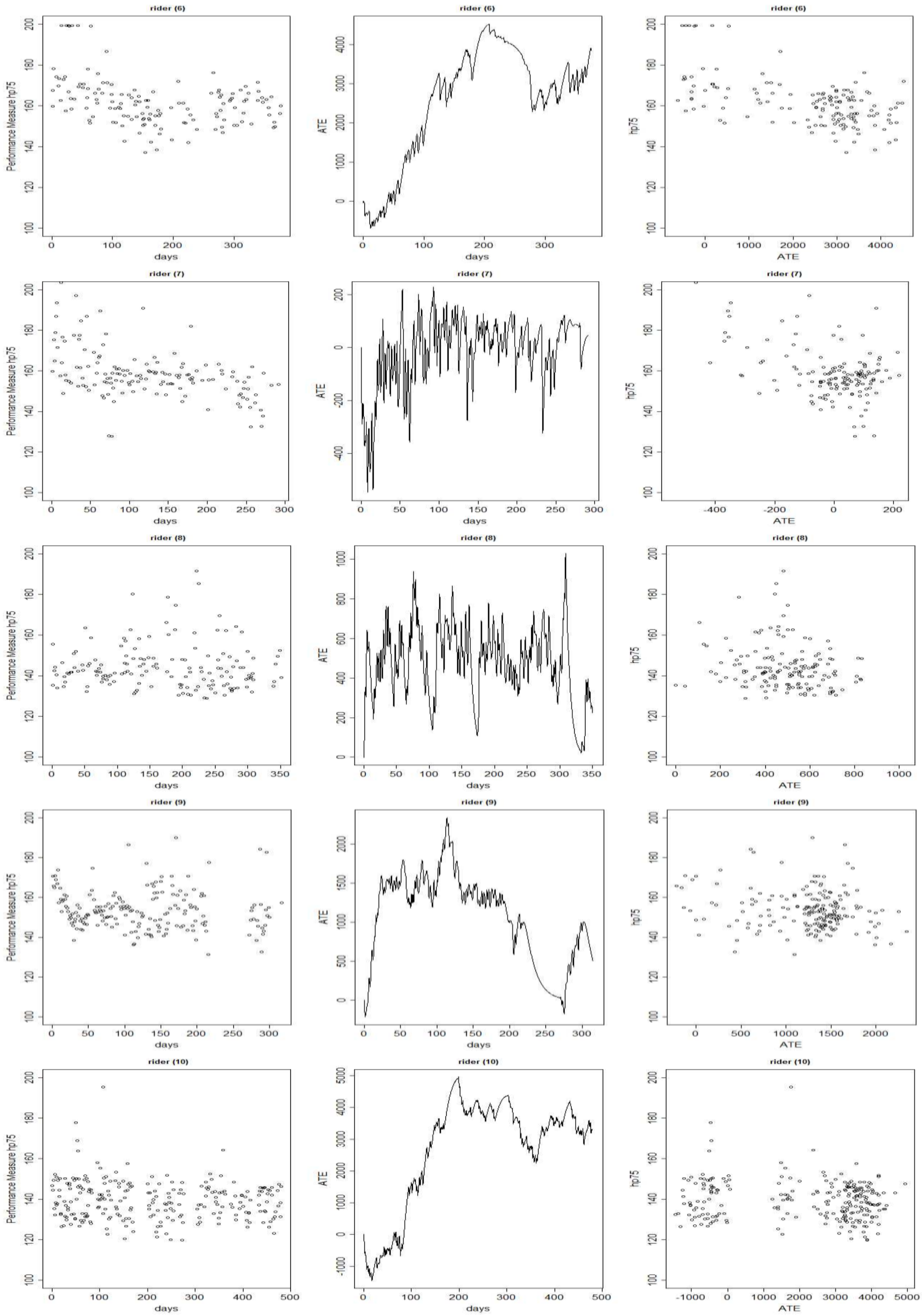

Figure 4.2 Continued.

Table 4.4 Parameter estimates of the model (standard errors) when $k_f \neq 2$ for performance measure h_{P50}

Rider	σ	τ_a	k_f	τ_f	α	β	$\beta/s.e.(\beta)$
1	5.7 (0.9)	32 (19)	2.5 (2.8)	0.1 (5.1)	132 (2.6)	-0.0014 (0.0008)	-1.75
2	3 (1.7)	5 (2)	2.5 (9.3)	0.5 (1.1)	156 (3.0)	-0.0300 (0.0150)	-2.00
3	3.4 (1.2)	10 (14)	3.7 (5.4)	2.1 (1.5)	139 (2.4)	-0.0100 (0.0190)	-0.53
4	2 (2.2)	193 (894)	0.9 (1.5)	99 (345)	152 (3.0)	-0.0022 (0.0240)	-0.92
5	7.8 (2.7)	176 (227)	1.6 (1.5)	72 (69)	137 (4.4)	-0.0012 (0.0027)	-0.44
6	0.7 (0.3)	52 (6.2)	1.4 (0.14)	32 (3.4)	149 (1.5)	-0.0250 (0.0080)	-3.13
7	1.9 (1.6)	83 (42)	4.2 (10.2)	2 (3.8)	146 (3.6)	-0.0014 (0.0005)	-2.80
8	2.4 (0.6)	118 (52)	1.02 (0.04)	112 (51)	128 (2.0)	-0.0230 (0.0400)	-0.58
9	3.7 (0.6)	8.3 (3)	1.04 (0.2)	6.5 (3.4)	148 (2.0)	-0.0160 (0.0380)	-0.42
10	5.2 (0.6)	124 (88)	1.11 (1.1)	41 (45.6)	128 (2.2)	-0.0010 (0.001)	-1.00

Table 4.5 Parameter estimates of the model (standard errors) when $k_f \neq 2$ for performance measure h_{P75}

Rider	σ	τ_a	k_f	τ_f	α	β	$\beta/s.e.(\beta)$
1	5.7 (0.97)	23 (13.5)	2.6 (4.2)	2 (3.8)	149 (3.1)	-0.0031 (0.0019)	-1.63
2	4.5 (1.5)	5.7 (3)	2.8 (28)	0.4 (1.4)	174 (4.0)	-0.0250 (0.0100)	-2.50
3	4.6 (1.3)	8 (32)	3.2 (16)	2.2 (4.7)	156 (3.4)	-0.0140 (0.0900)	-0.16
4	4.8 (1.6)	228 (791)	0.93 (1.2)	67 (144)	164 (3.4)	-0.0011 (0.0004)	-2.75
5	38 (5.8)	89 (139)	7.2 (13)	57 (18)	168 (10.0)	-0.0010 (0.0020)	-0.50
6	8.3 (1.1)	74 (14)	1.2 (0.2)	43 (14)	186 (4.4)	-0.0230 (0.0100)	-2.30
7	4.1 (1.1)	13 (7)	1.9 (1.1)	7 (3.7)	159 (2.2)	-0.0210 (0.0200)	-1.05
8	5.3 (0.7)	6.3 (7)	2.1 (1.7)	0.12 (0.07)	145 (2.3)	-0.0100 (0.0050)	-2.00
9	4.6 (0.7)	9 (3)	1.1 (0.4)	6 (3.6)	156 (2.0)	-0.0100 (0.0100)	-1.00
10	6 (0.6)	96 (72)	1.4 (1.2)	42 (50)	142 (2.4)	-0.0013 (0.0030)	-0.43

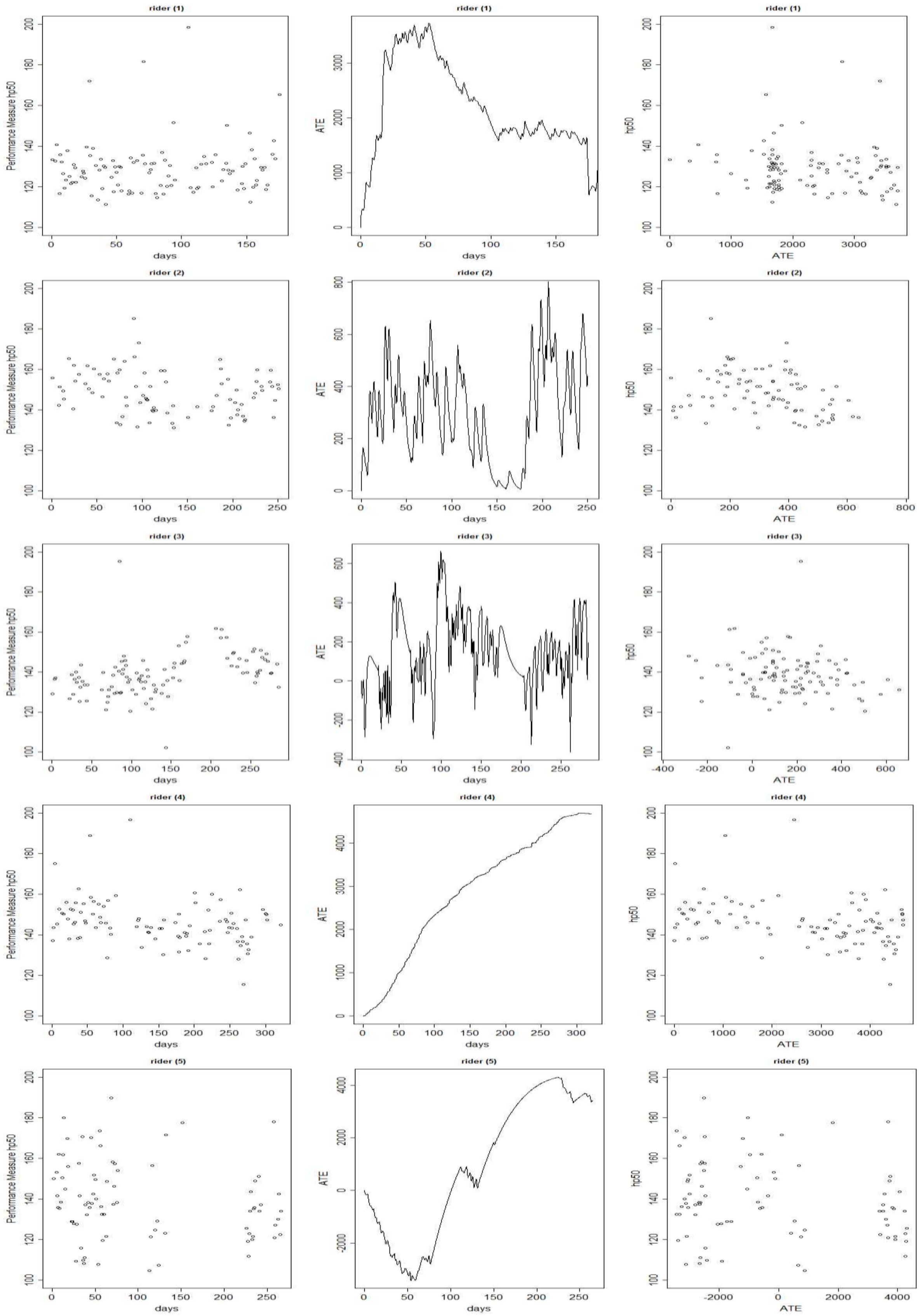

Figure 4.3 Performance measure h_{P50} and the curve of the accumulated training effect over time for each rider when $k_f \neq 2$

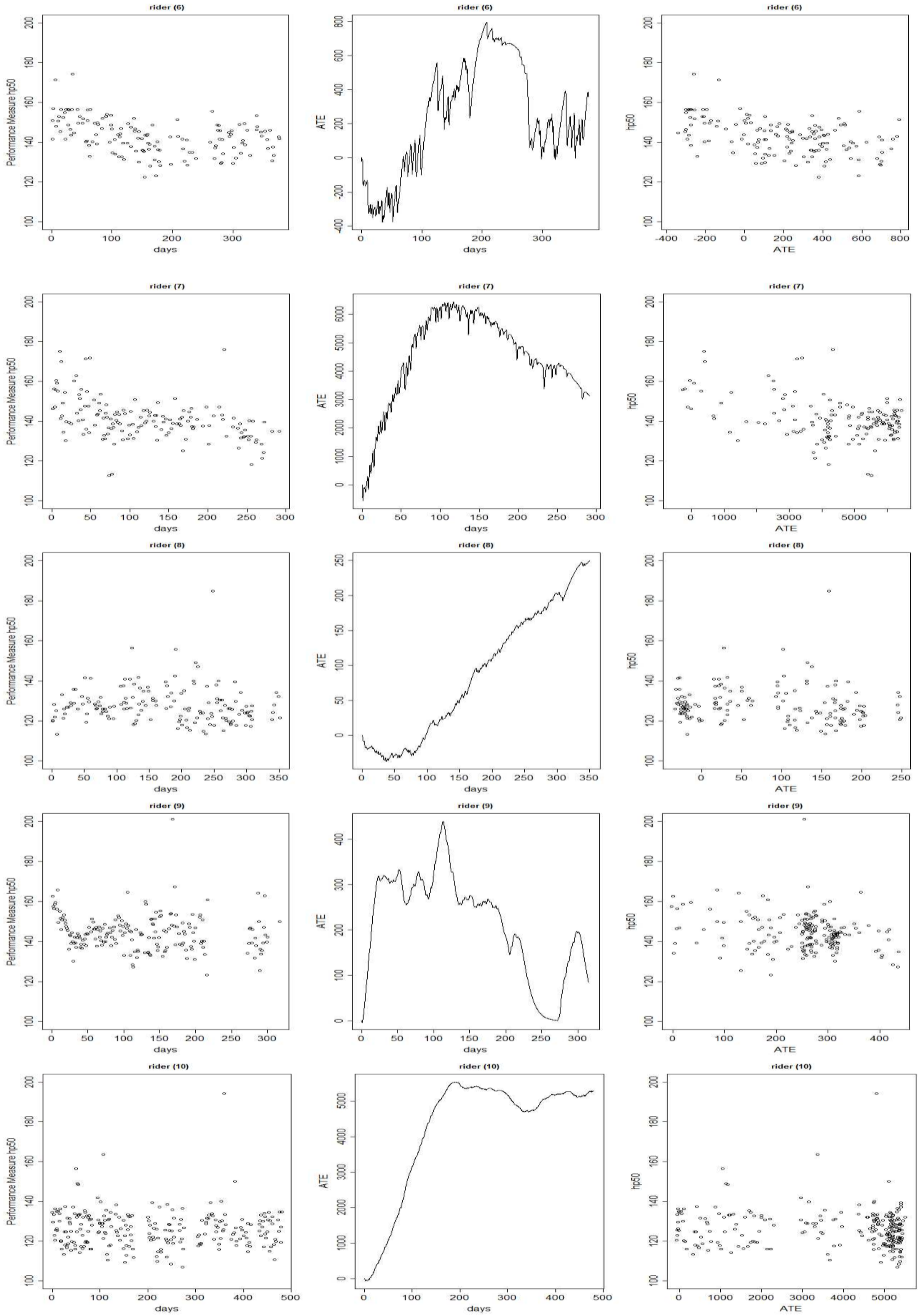

Figure 4.3 Continued.

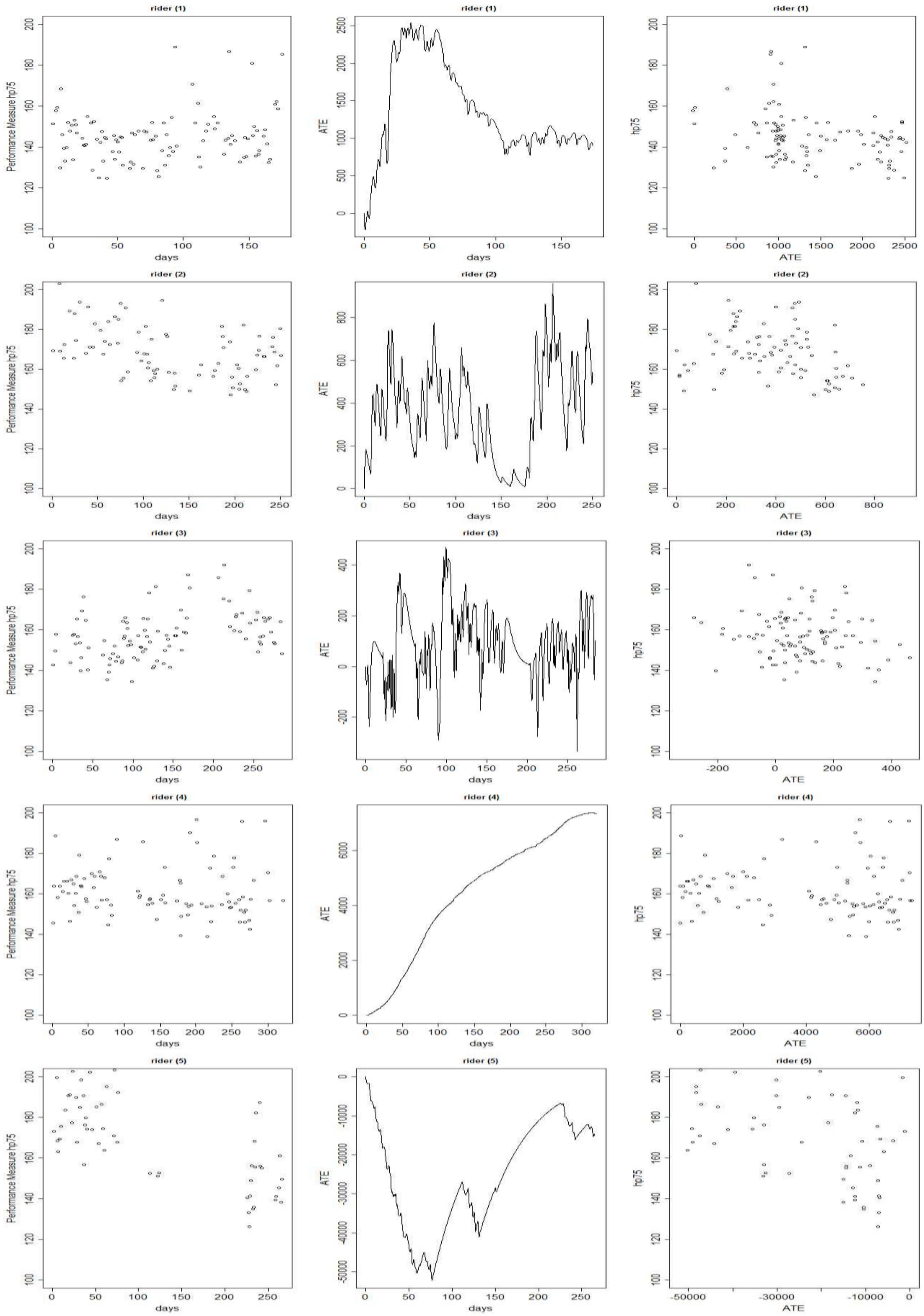

Figure 4.4 Performance measure h_{P75} and the curve of the accumulated training effect over time for each rider when $k_f \neq 2$

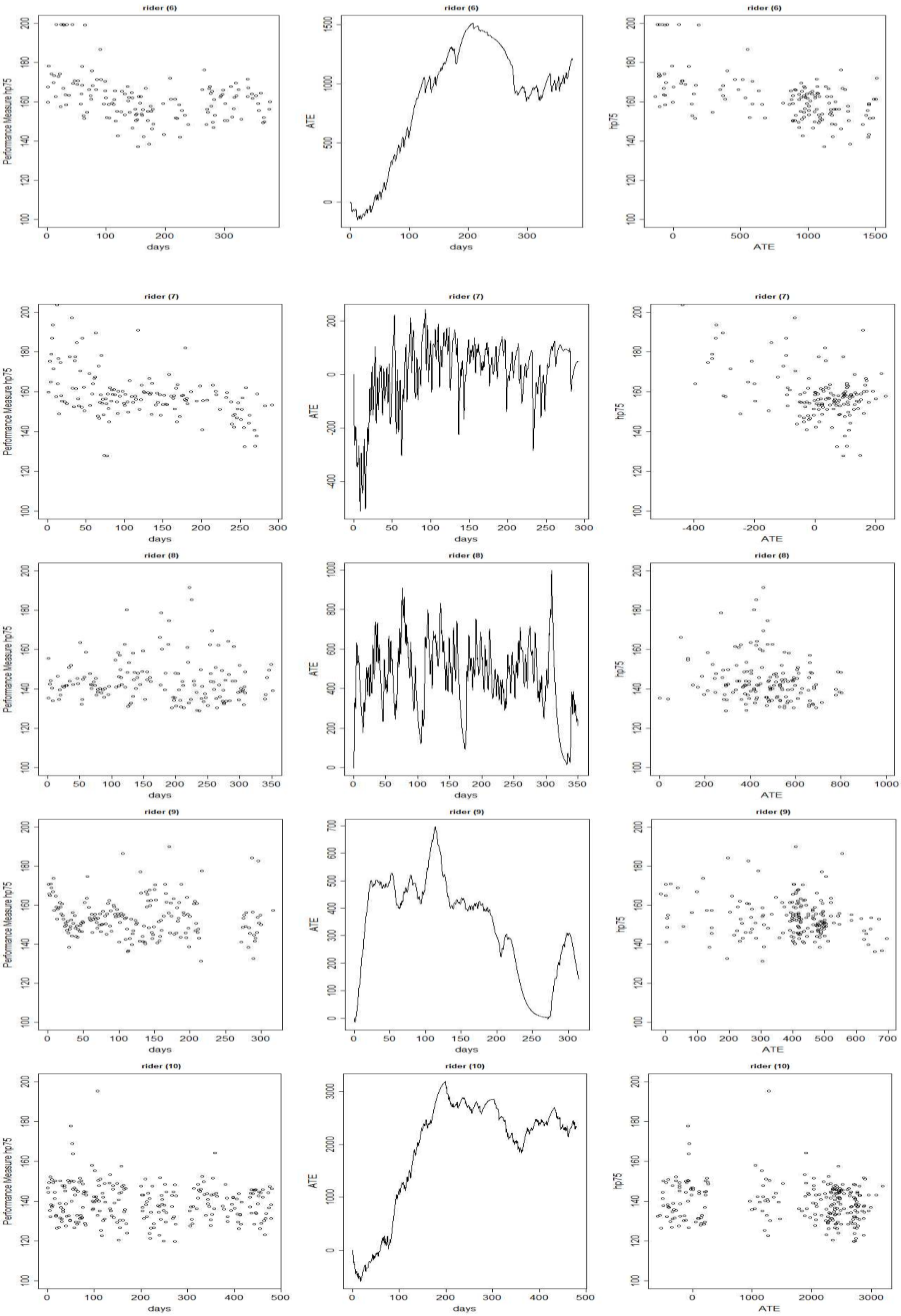

Figure 4.4 Continued.

4.6 The significance of the training effect

4.6.1 The statistical significance of the training effect

We would like to test if there is a significant linear relationship between our performance measure (h_{P50} or h_{P75}) and the accumulated training effect (ATE). As the model between performance measure and the accumulated training effect is assumed to be linear (equation 4.1), we test

$$H_0: \beta = 0 \quad vs \quad H_1: \beta < 0$$

using

$$t_{\hat{\beta}} = \frac{\hat{\beta}}{s.e.(\hat{\beta})}$$

where $s.e.(\hat{\beta})$ is the standard error of the estimator $\hat{\beta}$.

According to tables 4.2, 4.3, 4.4 and 4.5 for cases when $k_f = 2$ and $k_f \neq 2$, we would accept statistically at 5% significance level that there is a significant linear relationship between performance measures (h_{P50}, h_{P75}) and the accumulated training effect (ATE) when $t_{\hat{\beta}} < -1.65$. For instance, for riders 1, 2, 6, and 7 there appears to be evidence of a significant linear relationship between their performance measures and their accumulated training effects when the performance measure is h_{P50} and k_f is allowed to vary freely.

Next, we will consider the practical significance of the training effect using the amount of change in power output from the beginning of training until the rider is most trained.

4.6.2 The practical significance of the training effect

In this subsection, we consider the practical significance of the training effect. To do this we determine the change in power output (power gain) between the start of the training and the point at which the rider is most trained. To calculate this power gain, firstly we use the linear model that relates power output to heart rate using the entire training history of the data $(P = a + b.HR)$. To be more precise, we chose multiple recent sessions (the last two months of the data for each rider) to calculate the coefficients of the model (a, b) because the gradient b is the relevant value now. Table 4.6 shows the values of a and b for each rider with the standard error of each calculated from the last two months of data. We then calculate the change in the accumulated training effect (ATE) which is defined as the difference between the maximum accumulated training effect and the initial accumulated training effect $\Delta_{ATE} = ATE_{max} - ATE_0$ and the corresponding performance measure reduction which is $|\hat{\beta}|. \Delta_{ATE}$. Finally, we determine the power gain as follows

$$\Delta_{P_q} = b \times |\hat{\beta}| \times \Delta_{ATE}$$

where q =50 or 75. This is the power gain at a high heart-rate (defined by the riders' heart rate at a specified power percentile) over the training period.

Table 4.6 The coefficients of the linear model between power output and heart rate calculated from the last two months for each rider with the standard error

Rider	a (s.e.)	b (s.e.)	Rider	a (s.e.)	b (s.e.)
1	-103 (2.3)	2.31 (0.02)	6	-187 (4.5)	3.35 (0.03)
2	-137 (2.6)	2.45 (0.02)	7	-43 (3.0)	1.85 (0.02)
3	-6 (2.6)	1.61 (0.02)	8	-196 (1.9)	3.11 (0.02)
4	65 (2.3)	1.11 (0.02)	9	-147 (2.4)	2.34 (0.02)
5	-43 (2.0)	1.83 (0.02)	10	-109 (2.2)	2.43 (0.02)

To judge the value of the power gain Δ_{P_q} we look at it as a proportion Δ_{P_q}/P_q where P_q is the q^{th} percentile of the power output using the entire training history of the rider. For each rider, Tables 4.7 and 4.8 present the accumulated training effect change from beginning to the maximum ATE. Then we present the change in power output (power gain) when $k_f = 2$. For the other case when $k_f \neq 2$ the results are presented in Tables 4.9 and 4.10. Thus when

$$\frac{\Delta_{\mathrm{p_q}}}{\mathrm{P_q}} > 0.05\ (5\%)$$

we would accept that there is a significant practical effect of the accumulated training effect (ATE) on performance.

Table 4.7 The accumulated training effect change and performance gain for each rider when $k_f = 2$ and for performance measure h_{P50}

Rider	b	β	Δ_{ATE}	$\Delta_{p_{50}}$	P_{50}	$\Delta_{p_{50}}/P_{50}$
1	2.31	-0.0019	3777	17	225	0.08
2	2.45	0.0200	212	10	235	0.04
3	1.61	-0.0200	416	14	239	0.06
4	1.11	0.0300	176	6	213	0.03
5	1.83	-0.0009	4663	8	213	0.04
6	3.35	-0.0250	792	66	293	0.23
7	1.85	-0.0012	9701	22	238	0.09
8	3.11	0.0050	75	1	197	0.01
9	2.34	-0.0030	2464	17	184	0.09
10	2.43	-0.0010	5534	14	208	0.07

Table 4.8 The accumulated training effect change and performance gain for each rider when $k_f = 2$ and for performance measure h_{P75}

Rider	b	β	Δ_{ATE}	$\Delta_{p_{75}}$	P_{75}	$\Delta_{p_{75}}/P_{75}$
1	2.31	-0.0031	2517	18	291	0.06
2	2.45	-0.0072	1507	27	307	0.09
3	1.61	-0.0300	188	9	291	0.03
4	1.11	-0.0900	24	3	246	0.01
5	1.83	-0.0043	694	6	280	0.02
6	3.35	-0.0077	4519	117	384	0.31
7	1.85	-0.0200	218	8	323	0.03
8	3.11	-0.0080	831	21	274	0.08
9	2.34	-0.0030	2278	16	214	0.08
10	2.43	-0.0013	3185	10	260	0.04

Table 4.9 The accumulated training effect change and performance gain for each rider when $k_f \neq 2$ and for performance measure h_{P50}

Rider	b	β	Δ_{ATE}	$\Delta_{p_{50}}$	P_{50}	$\Delta_{p_{50}}/P_{50}$
1	2.31	-0.0020	3627	17	225	0.08
2	2.45	-0.0300	639	47	235	0.20
3	1.61	-0.0100	658	11	239	0.05
4	1.11	-0.0020	4688	12	213	0.06
5	1.83	-0.0012	4304	10	213	0.05
6	3.35	-0.0240	729	59	293	0.20
7	1.85	-0.0010	8583	16	238	0.07
8	3.11	-0.0230	248	18	197	0.09
9	2.34	-0.0100	439	10	184	0.05
10	2.43	-0.0010	3741	9	208	0.04

Table 4.10 The accumulated training effect change and performance gain for each rider when $k_f \neq 2$ and for performance measure h_{P75}

Rider	b	β	Δ_{ATE}	$\Delta_{p_{75}}$	P_{75}	$\Delta_{p_{75}}/P_{75}$
1	2.31	-0.0030	3290	23	291	0.08
2	2.45	-0.0250	786	48	307	0.16
3	1.61	-0.0140	430	10	291	0.03
4	1.11	-0.0011	7355	9	246	0.04
5	1.83	-0.0010	0	0	280	0
6	3.35	-0.0230	1510	116	384	0.30
7	1.85	-0.0020	7376	27	323	0.08
8	3.11	-0.0100	798	25	274	0.09
9	2.34	-0.0100	692	16	214	0.08
10	2.43	-0.0010	4937	12	260	0.05

4.7 Discussion of results

In cycling, a training program should be optimised individually. Each athlete has personal characteristics. So, we should discuss our results athlete by athlete.

For rider (1), slight improvements in his performance measures (h_{P50}, h_{P75}) are seen in figures 4.1 and 4.2. However, the accumulation of training effect of this rider is initially increasing and then decreasing gradually after 60 days for all cases of the study whether the performance measure is h_{P50} or h_{P75} and also whether k_f is free or fixed. Furthermore, a huge difference between the fitness and the fatigue decay time constants is seen in tables 4.2, 4.3, 4.4, and 4.5. However, this rider shows a statistically significant relationship between his performance measures (h_{P50}, h_{P75}) and the accumulated training effect for both cases when $k_f = 2$ and k_f being free. Furthermore, the practical training effect of this rider is improved for all cases whether performance measure is h_{P50} or h_{P75} and also whether k_f is free or fixed.

We did not obtain what we expected for rider (2) when $k_f = 2$ and performance measure h_{P50}. Additionally, a positive increasing relationship between the performance measure and the accumulated training effect is shown in this case $(h_{P50}, k_f = 2\ )$ which is unexpected. So, no overall improvement is apparently seen in this case as the rider becomes tired with continuous training. However, the linear relationship between his performance measure h_{P75} and the accumulated training effect is statistically significant when $k_f = 2$. On the other hand, when k_f is free, he presents better results than when k_f is fixed. The values of the gradient are statistically significant for both performance measures (h_{P50}, h_{P75}). In practical term, this rider shows in table 4.8 a huge improvement of his practical training effect when k_f is free.

Rider (3) has moderate parameter values for fitness and fatigue decay time constants for each case as presented in Tables 4.2, 4.3, 4.4, and 4.5. Although a decreasing relationship between the performance measures and the accumulated training effects are presented in these previous tables, the effect of training for this rider is not statistically significant for almost all cases except when $\left(h_{P50}, k_f = 2\right)$. However, the practical effects of training for this rider appeared to be significant when the performance measure is h_{P50} for fixed and free k_f.

For rider (4), although his performance measure h_{P50} has clearly improved over time as presented in Figure 4.2, he gets tired with continuous training as the relationship between h_{P50} and the accumulated training effect (ATE) is significantly positive. On the other hand, when k_f is free, the optimal scale parameter k_f is less than 1. It should be bigger than 1 as we set $k_a = 1$. However, this rider shows no significant practical and statistical effects from training when $k_f = 2$ if the performance measure is h_{P75}.

Although the results for rider (5) are statistically significant when $k_f = 2$ for both performance measures, we should not take them into account as this rider having many gaps in his data. So, his results might be affected by those gaps. Moreover, poor relationships between his performance measures (h_{P50}, h_{P75}) and the accumulated training effects are seen in Tables 4.4 and 4.5. Practically, when $k_f = 2$ for both performance measures the effect of training is not significant. Furthermore, he does not appear to have improved at all when k_f is free and the performance measure is h_{P75}.

Rider (6) has a large number of sessions recorded at 15-second intervals. This alternative recording provides a lot of variations between power output and heart rate for many sessions. In addition, those variations will affect his performance measure as the performance measure is based on the relationship between power output and heart rate. The results of this rider present an obvious negative relationship between performance measure and the accumulated training effect for all cases whether k_f is free or fixed and also whether the performance measure is h_{P50} or h_{P75}. Furthermore, the difference between fitness and fatigue decay time constants is large when $k_f = 2$ than in the other case when k_f is free. The performance measure of this rider has obviously improved by training for both cases (h_{P50}, h_{P75}). Moreover, the effect of the accumulation of training is also developed. Practically, this rider has a huge improvement due to the effect of training as presented in Tables 4.7, 4.8, 4.9 and 4.10.

For rider (7), a large difference between fitness and fatigue decay time constants is seen in Tables 4.2 and 4.4 for performance measure h_{P50} whether k_f is free or fixed. On the other hand, when the performance measure is h_{P75}, the parameter values of the fitness and fatigue decay times are moderate. Moreover, this rider has the largest improvement in his performance measures (h_{P50}, h_{P75}) among other riders. We would accept that there are statistically significant linear relationships between the performance measures (h_{P50}, h_{P75}) and their accumulated training effects whether k_f is free or fixed. However, this relationship appears to be less significant when $k_f \neq 2$ and the performance measure is h_{P75}. Furthermore, the practical effects of training for this rider appear to be significant as shown in tables 4.7 and 4.9 when the performance measure is h_{P50}.

For rider (8), we did not get statistically what we expected for all cases except when k_f is free and the performance measure is h_{P75}. However, this rider shows an obvious practical improvement in his training effects for almost all cases.

Moderate parameter values of the Banister model are seen in Tables 4.2, 4.3, 4.4 and 4.5 for rider (9). No case has shown a statistical significance of the training effect. However, this rider demonstrates practically significant effects of training for all cases whether k_f is free or fixed for both performance measures (h_{P50}, h_{P75}).

Rider (10) has the greatest number of training sessions amongst all the riders. He displays a slight improvement in his performance measures over time as shown in Figures 4.1 and 4.2. Moreover, his fitness decay time constants are large for all cases whether k_f is free or fixed for both performance measures (h_{P50}, h_{P75}). However, our model is assumed to work statistically and practically better when $k_f = 2$ and the performance measure is h_{P50}.

To summarise, considering both the statistical and practical significances of the training effects, we suggest that the statistical model can be used for optimising training if $\hat{\beta}/s.e.(\hat{\beta})$ is large and negative (≤ -1.65) and also $b \times |\hat{\beta}| \times \Delta_{ATE}$ is relatively large $(\geq 5\%)$. According to previous suggestions and tables 4.2, 4.3, 4.4 and 4.5, our procedure appears to work best when $k_f = 2$ and for performance measure h_{P50}. In general, our model has 50% chance to work if a coach A would use it with rider B. So, we recommend coaches and riders to use power output and heart rate monitors for every single session and do not miss any training session or race.

4.8 Optimising training

Given values of the Banister model parameters, the exercise training optimisation problem can in principle be solved: determine the exercise (training load input) X_t that should be carried out at time *t* for all $0 < t < T$ in order to maximise performance Y_T at time T (given our model of the exercise training effect that relates performance at time *t* to exercise carried out up to *t*). This then corresponds to specifying the TRIMP on each day t from which the ATE at time T will follow given the known parameters.

There are two difficulties: the search space here is very large. Also, many training schedules will be infeasible. Training will also have to consider a lower bound on ATE because in an extreme case overtraining can lead to severe negative consequences for health. In practice, one would expect a limited number of prescribed training schedules to be compared. Furthermore, schedules can be updated dynamically, as training progresses, so that periodically the schedule is reviewed and "re-optimised".

4.9 Summary

In this chapter, we used the maximum likelihood approach to estimate the accumulated training effect parameters. For training to be optimised values of these training parameters must be specified. We studied two cases of the detriment scale constants: when $k_f = 2$ and $k_f \neq 2$. Then we discussed our results statistically and practically in terms of the relationship between our performance measures and the accumulated training effects. We used the surface response methodology to determine the initial values of the accumulated training effect parameters in terms of the correlation between the performance measure and the accumulated training effect. As the rider becomes fitter and well-trained, his heart rate should be lower at a given power. So, we are looking for negative correlation between our performance measure and the accumulated training effect. We produced models for each of the 10 riders for both cases for k_f and for two different performance indicators. Finally, we described in principle how training can be optimised with known training parameter values.

5. OTHER MEASURES OF PERFORMANCE

In this chapter we describe other measures of performance that were considered by us in early development of our ideas. Although we have used h_{Pq} as our fundamental performance measure and presented this in earlier chapters, we summarise other measures of performance to complement the work. There are various candidate summary measures of performance such as average power, normalised power and critical power. These measures of performance have some limitations and we explain them here.

5.1 Average power (AP)

Average power (AP) is a direct measure that describes the data over the duration of training. It can be calculated by summing the observations in a specific period or ride and dividing the total by the number of observations in the set as follows

$$\bar{p} = \sum_{i=1}^{n} \frac{p_i}{n},$$

where p_i is the power value at time point t_i and n is the number of observations. We shall call the interval $[t_1, t_n]$ a session. In the context of our data, typically a single session would be a ride of between 30 minutes and 6 hours duration.

The difficulty with average power as a summary measure of performance is that it does not capture very well power measurements at high intensity. That is, a steady ride at constant power and an interval session that alternates between very high and very low intensity output may have the same average power summary but very different training effects. It is only a useful measure when the training session achieves an approximately constant power output (Jobson, et al. 2009).

5.2 Normalised power (NP)

Normalized power (NP) is a measure that takes more account of high periods of intensity power output during a session. It is argued that normalised power is a better measure and more useful than average power because normalised power describes how hard the rider was riding and we could get the same average power for two rides despite the actual intensity of the rides being quite different (Jobson, et al. 2009).

Normalised power is calculated in four steps. First of all, calculate a 30 second moving average for power because many physiological processes respond to changes in exercise intensity with a constant time of 30 seconds (Jobson, et al. 2009) as follows

$$P_1 = \sum_{i=1}^{6} \frac{p_i}{6} \quad , \qquad P_2 = \sum_{i=2}^{7} \frac{p_i}{6} \ , \dots , P_{n-5} = \sum_{i=n-5}^{n} \frac{p_i}{6}$$

then raise the values to the 4th power as follows

$$(P_1)^4 \ , (P_2)^4 \ , \dots , (P_{n-5})^4$$

and then take the average of all the values in the previous step as follows

$$\bar{P}^* = \frac{(P_1)^4 + (P_2)^4 + \cdots + (P_{n-5})^4}{n-5}$$

finally, calculate the 4th root of the number obtained in the last step as follows

$$NP = \sqrt[4]{\bar{P}^*}.$$

An example of average power and normalised power calculated for a single session is seen in figure 5.1. Normalised power is always higher than average power, unless the training session is at constant power output (AP=NP). In figure 5.2, we show the average power and normalised power for each of the sessions and for each of the riders in our dataset. Correlations between AP and NP for each rider are presented in Table 5.1.We can see that they are highly correlated as Table 5.1 and Figure 5.2 show.

Table 5.1 The correlation between average power (AP) and normalised power (NP) for all riders.

Rider	1	2	3	4	5	6	7	8	9	10
$Corr(NP, AP)$	0.89	0.87	0.93	0.89	0.58	0.78	0.83	0.76	0.84	0.81

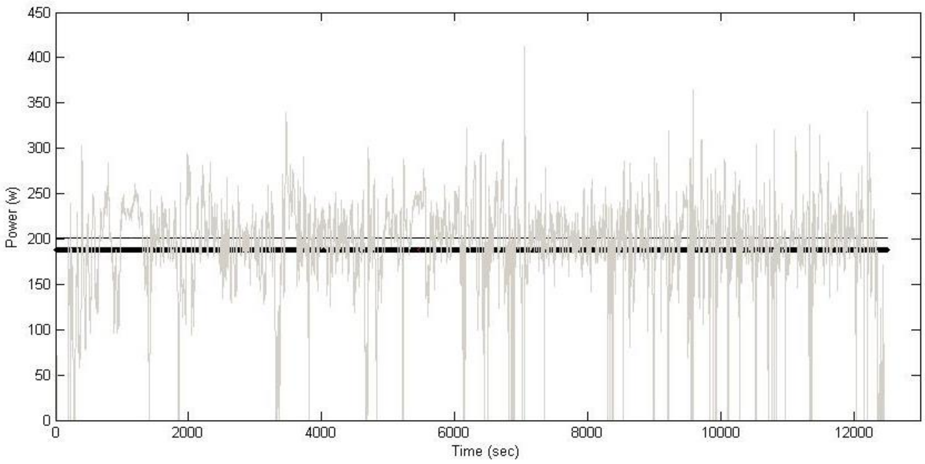

Figure 5.1 An example of average power (▬▬) and normalised power (——) with underlying power measurements sampled every 5 seconds during a single session.

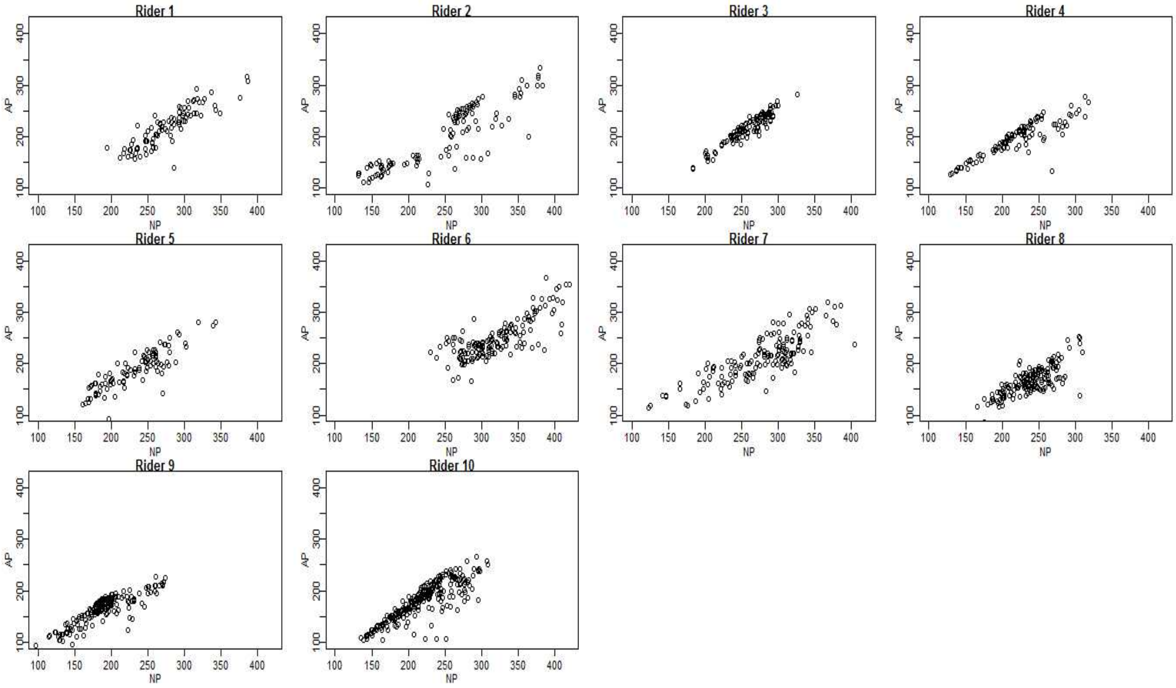

Figure 5.2 Normalised power (NP) against average power (AP) for all riders (1 to 10)

5.3 Critical power (CP)

Critical power (CP) is another method of summarising power output. It is a laboratory based concept and we would like to generalise it to training data by using both power output and heart rate collected during riding. The critical power model summarises individually the relationship between exercise intensity and duration. Briefly, Hill, (1925) suggested the relationship between work rate and time by plotting velocity versus time over several distances in swimming and running (Clarke and Skiba, 2013). Monod and Scherrer, (1965) developed a technique for determining 'the amount of work a muscle can do before being exhausted' and 'the conditions of a fatigueless task' which is called the critical power test (Bull, et al. 2000). They argued that critical power represents the power that could be maintained for a very long time without fatigue, but more recently it has been established as time to exhaustion (Carter, et al. 2005). The work of Monod and Scherrer has been expanded and applied to cycle ergometry by Moritani and his colleagues (Brickley, et al., 2002). This model of critical power has been applied for many sports such as running (Hughson, et al., 1984) and swimming (Wakayoshi, et al., 1992; Wakayoshi, et al., 1993). Jenkins and Quigley, (1992) represented critical power as a linear regression coefficient for maximum work with respect to maximum time as follows

$$W = a + bT$$

where the critical power is the coefficient of linear regression b and a represents energy reserved in the muscle at the start of exercise. They mentioned that critical power closely approximates the power that can be maintained for more than 20 minutes. Other researchers (Jenkins and Quigley, 1992; Hughson, et al. 1984; and Poole, et al. 1988) have plotted power against endurance time and taken the asymptotic value of power as the critical power. Mielke, et al., (2011) tested whether the mathematical linear model of critical power proposed by Moritani et al. (1981) can be used for heart rate to determine critical heart rate (CHR). They found that the relationship between heart rate and time to exhaustion can be described by this model. The basis of the critical power concept is that there is a relationship between power output and the time for which that the power output can be sustained (Hill, 1993). A number of models have described this relationship between power output and time to fatigue (Gaesser, et al., 1995). Vanhatalo, et al. (2007) described this relationship between power output and time to exhaustion by two parameters. These parameters are critical power which represents the highest sustainable work rate and the maximum amount of work that can be performed above critical power. Carter, et al., (2005) discussed whether the intensity of prior exercise modifies the time to exhaustion at critical power in 11 participants (8 males, 3 females). They found that prior heavy exercise can decrease the time to exhaustion at critical power by approximately 10%. According to Walsh, (2000) the fundamental concept of critical power is for describing fatigue and exhaustion. Moreover, Borresen and Lambert, (2009) defined the critical power as an estimate of the maximal power output that can be maintained at a physiological steady state without fatigue. This relationship is shown in figure 5.3.

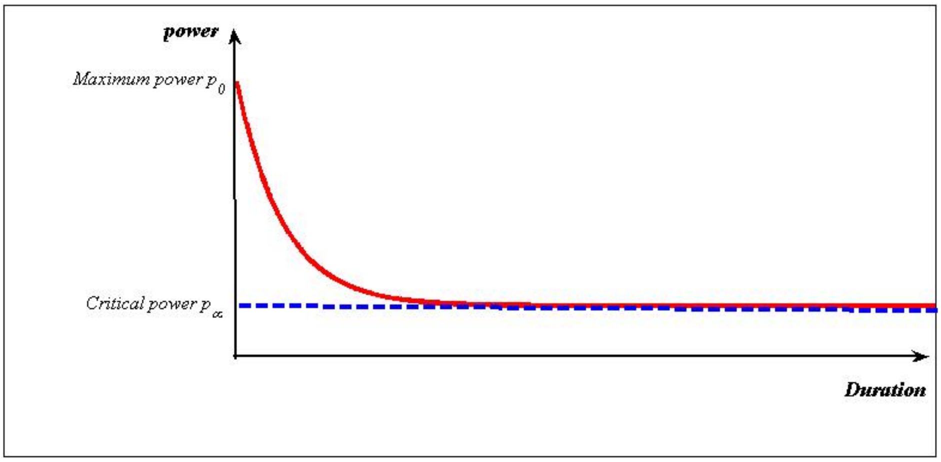

Figure 5.3 The concept of critical power

In the next section, we will present a measure of performance based on the concept of critical power. We will use four different empirical models to fit our data. Then we will use the Akaike information criterion (AIC) which is defined in the next subsection to select the best one.

5.3.1 The Akaike Information Criterion (AIC)

The Akaike information criterion (AIC) is a measure that discusses the best fitting model among several models which are used to fit a set of data. This criterion was first proposed by Hirotugu Akaike in 1974. It is defined as follows

$$AIC = 2k - 2\ln(L)$$

where k is the number of parameters in the statistical model and L is the maximized value of the likelihood function of the estimated model. Then the best fitting model is considered to be the one with the minimum AIC value. However, the sample size is not explicitly taken into account when we use the AIC (Raftery, 1986).

5.4 Another measure of performance based on the critical power concept

5.4.1 Modelling the critical power

In this subsection, we aim to fit critical power models. Bull et al. (2000) have discussed five mathematical regression models to model critical power; two of them are linear and the others are nonlinear. They re-examined the previous findings of critical power using the five different models, comparing the critical power estimates using nine male subjects. In their study, they determined the time to exhaustion during cycle ergometry at the lowest critical power. They reported results consistent with previous studies.

In 1981, Moritani et al. proposed a linear model (the linear-TW model) based on the regression of total work performed (TW) versus time to exhaustion (*t*) as follows

$$TW = AWC + CP.t$$

where AWC is the anaerobic work capacity and CP is the critical power. They found very strong linear relationships ($r^2 = 0.982 - 0.998$) between total work (TW) and time to exhaustion (*t*) for the cycle ergo-meter work bouts.

The second linear model (linear-p model) that could describe the relationship between power output and time to exhaustion is by plotting power output p against the inverse of time as follows

$$P = AWC.(T) + CP$$

where $T = {}^{1}/_{t}$

The third mathematical model is nonlinear. This model was based on the relationship between P and *t*. It is defined by solving the second linear equation (linear-p) for t as follows

$$t = \frac{AWC}{(P - CP)}$$

The fourth nonlinear model (nonlinear-3) is the third model including the parameter maximal instantaneous power (P_{max}). It is defined as follows

$$t = \frac{AWC}{(P - CP)} + k$$

where k is defined by putting $P = P_{Max}$ at $t = 0$.

so

$$k = -\frac{AWC}{(P_{Max} - CP)}$$

then the fourth nonlinear model (nonlinear-3) becomes

$$t = \frac{(AWC)}{(P - CP)} - \frac{(AWC)}{(P_{Max} - CP)}$$

The fifth regression model is an exponential model (EXP) and it is defined as follows

$$P = CP + (P_{Max} - CP).e^{-(t/\tau)}$$

where τ is an undefined time constant.

In the fourth and fifth models, P_{Max} is added to overcome the assumption of infinite power over very short durations. However, the fifth model does not give an estimation of AWC.

We are going here to model critical power by using four different empirical models. Those models are defined as follows

1) $y_i = (p_0 - p_\infty)e^{-\alpha x_i} + p_\infty + \epsilon_i \quad , \quad p_0, p_\infty, \alpha \geq 0 \ , \ \epsilon_i \sim N(0, \sigma^2)$
2) $y_i = \frac{p_0 - p_\infty}{(1+x_i)^\alpha} + \epsilon_i \quad , \quad p_0, p_\infty, \alpha \geq 0 \ , \ \epsilon_i \sim N(0, \sigma^2)$
3) $y_i = \frac{p_0 - p_\infty}{(1+x_i{}^\beta)^\alpha} + \epsilon_i \quad , \ p_0, p_\infty, \alpha \geq 0, \beta > 0 \ , \ \epsilon_i \sim N(0, \sigma^2)$
4) $y_i = p_0 e^{-\alpha x_i} + \epsilon_i \quad , \ p_0, \alpha \geq 0 \ , \ \epsilon_i \sim N(0, \sigma^2)$

where y_i is the fixed level of power and x_i is the largest L such that the time interval of length L, $[t_{j_{k+1}}, \ldots, t_{j_{k+L}}]$ has $y_j \geq y_i$ for all $j \epsilon [t_{j_{k+1}}, \ldots, t_{j_{k+L}}]$ and ϵ_i is the random sampling error of the model. One of the nonlinear models mentioned in the study of Bull et al (2000) is the exponential decay model and that model is the model 1 in our list. Model 4 above $y_i = p_0 e^{-\alpha x_i} + \epsilon_i$, $p_0, \alpha \geq 0$, $\epsilon_i \sim N(0, \sigma^2)$ is a special case of model one $y_i = (p_0 - p_\infty)e^{-\alpha x_i} + p_\infty + \epsilon_i$, $p_0, p_\infty, \alpha \geq 0$, $\epsilon_i \sim N(0, \sigma^2)$ when $p_\infty = 0$. Comparing the fit of these models allows us to test hypothesis $H_0: p_\infty = 0 \ vs \ H_1: p_\infty > 0$.

To fit the models, we use maximum likelihood estimation (MLE). This is a method of estimating the parameters of a statistical model. When applied to a dataset and given a statistical model, MLE provides estimates of the parameters of the model and it is formulated as follows

$$L(\theta | x_1, x_2, \ldots, x_n) \propto f(x_1, x_2, \ldots, x_n | \theta) = \prod_{i=1}^{n} f(x_i | \theta).$$

The log-likelihood is more convenient for estimating parameters and is written as follows:

$$ln\big(L(\theta | x_1, x_2, \ldots, x_n)\big) = \sum_{i=1}^{n} ln\, f(x_i | \theta) + constant.$$

The log likelihood functions of the four models that we tested to fit our data are

Model 1 $$-\frac{n}{2}\log(2\pi) - n\log(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - (p_0 - p_\infty)e^{-\alpha x_i} - p_\infty)^2$$

Model 2 $$-\frac{n}{2}\log(2\pi) - n\log(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - \frac{p_0}{(1+x_i)^\alpha}\right)^2$$

Model 3 $$-\frac{n}{2}\log(2\pi) - n\log(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i - \frac{p_0}{(1+{x_i}^\beta)^\alpha}\right)^2$$

Model 4 $$-\frac{n}{2}\log(2\pi) - n\log(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - p_0 e^{-\alpha x_i})^2$$

5.4.2 Models of critical power with CP varying by rider and by session

Data for ten riders are available to us in the present study. Their power outputs were measured every five seconds during a number of sessions. We should carry out the following procedure to find the critical power models with CP varying by rider. First of all, we regard the CP for a rider as a fixed effect. Then, we specify ten levels of power [50, 100, 150, ..., 500] (watts) and calculate the time (duration) that the respective power output can be sustained for each session for each rider. An example of just one session for each rider is shown in figure 5.4, in which the power level is plotted against the duration. Next, we use the maximum likelihood method to estimate the parameters of the models, assuming common parameter values across sessions for any rider and that sessions are independent. Table 5.2 presents parameter values for the fitted models with their AIC and maximum likelihood values (ML) where y_i is fixed level of power output and x_i is the largest l such that $t_{k+1} \le t \le t_{k+l}$ has $> y_i$. The shaded values are the lowest AIC and the best fitting model for each rider. In table 5.3, we present the estimates of the parameters for all models with the standard error for each parameter for each rider. Finally, figure 5.5 shows the fitted critical power curves for the "best" fitting model, model 1, for each rider of all sessions with the corresponding fitted minimum AIC fixed effects model.

The main findings here are as follows

1) According to the AIC, the best model of the four considered models is model 1 for all riders.
2) There are differences between the short term p_0 (the power that gets a sprinter away from the start blocks or explosive power of a rider) and the long term p_∞ (critical power) for each different rider and that is implicit because each rider has different critical power but how their power decay over time α is strongly similar for each rider as the following table shows.
3) Our results support the concept of a non-zero critical power.
4) There may be a case for considering models in which duration is the response to the fixed explanatory variable, namely power level.

Next, we allow p_0 to vary from session to session. We then find m_k fixed values of p_0 for each rider k where m_k is the number of sessions for rider k. Figure 5.6 indicates the parameter estimates p_0 for each session for each rider with ± 2 standard errors values of p_0 varying by session for each rider are presented in figure 5.7. Figure 5.8 shows p_0 varying by session with smoothing curves for p_0 plotted against the day of training because not every day has a session and we are ultimately interested in the development of riders over time from day to day rather than from session to session.

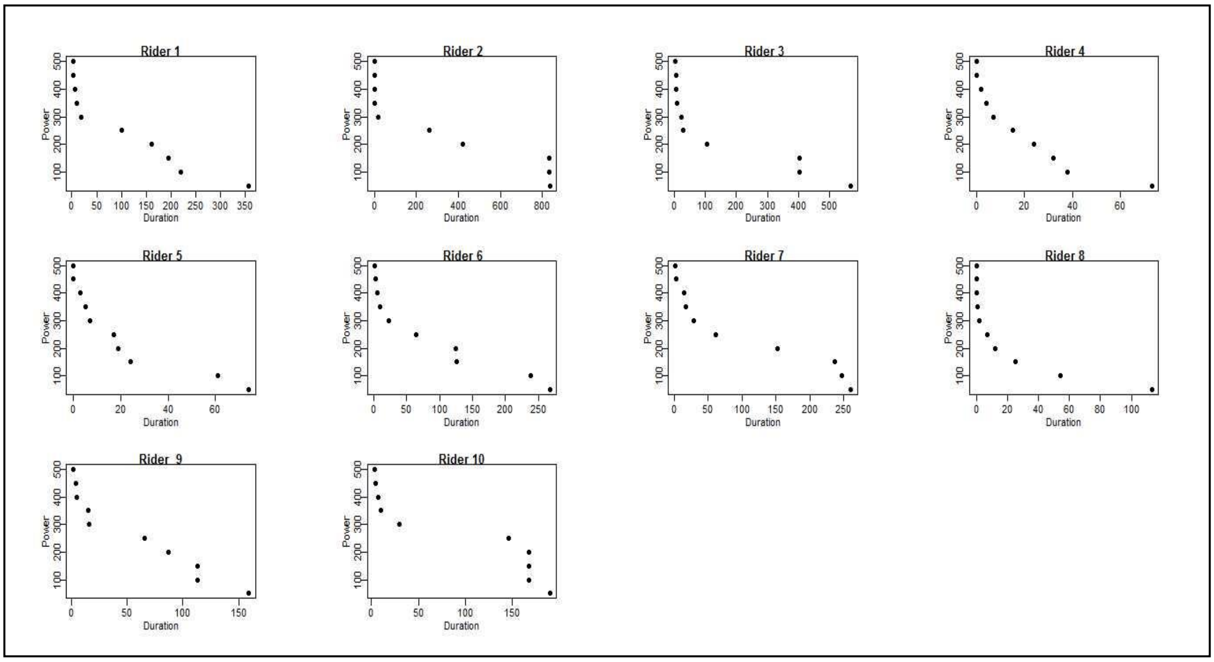

Figure 5.4 Fixed levels of intensity (Watts) versus duration (5 seconds) for one session for each rider (1 to 10)

Table 5.2 Fitted models with their AIC and maximum likelihood values

Rider	Models	Fitted Models	ML	AIC
Rider 1	1	$y = 307e^{-0.033x} + 119$	6579	13166
	2	$y = \frac{476}{(1+x)^{0.209}}$	6719	13444
	3	$y = \frac{464}{(1+x^{2.76})^{0.075}}$	6694	13396
	4	$y = 395\ \mathrm{e}^{-0.0117x}$	6710	13426
Rider 2	1	$y = 370e^{-0.034x} + 90$	4952	9911
	2	$y = \frac{568}{(1+x)^{0.268}}$	5139	10284
	3	$y = \frac{540}{(1+x^{2.61})^{0.1003}}$	5114	10235
	4	$y = 439\ \mathrm{e}^{-0.0194x}$	5013	10031
Rider 3	1	$y = 359e^{-0.033x} + 108$	5843	11694
	2	$y = \frac{645}{(1+x)^{0.297}}$	5981	11969
	3	$y = \frac{617}{(1+x^{1.34})^{0.215}}$	5973	11954
	4	$y = 426\mathrm{e}^{-0.013x}$	6122	12250
Rider 4	1	$y = 299e^{-0.028x} + 104$	6354	12715
	2	$y = \frac{439}{(1+x)^{0.211}}$	6477	12960
	3	$y = \frac{425}{(1+x^{30.6})^{0.0069}}$	6449	12905
	4	$y = 384\ \mathrm{e}^{-0.0096x}$	6465	12936
Rider 5	1	$y = 335e^{-0.03x} + 101$	7050	14107
	2	$y = \frac{519}{(1+x)^{0.237}}$	7237	14480
	3	$y = \frac{501}{(1+x^{2.21})^{0.106}}$	7208	14424
	4	$y = 410\ \mathrm{e}^{-0.013x}$	7221	14449

Table 5.2 Continued.

Rider	Models	Fitted Models	ML	AIC
Rider 6	1	$y = 374e^{-0.036x} + 100$	8605	17219
	2	$y = \frac{596}{(1+x)^{0.276}}$	8841	17688
	3	$y = \frac{556}{(1+x^{2.92})^{0.089}}$	8822	17653
	4	$y = 448\ \mathrm{e}^{-0.099x}$	8687	17379
Rider 7	1	$y = 323e^{-0.025x} + 111$	8900	17807
	2	$y = \frac{512}{(1+x)^{0.211}}$	9103	18212
	3	$y = \frac{506}{(1+x^{3.49})^{0.061}}$	9065	18137
	4	$y = 403\ \mathrm{e}^{-0.0102x}$	9005	18015
Rider 8	1	$y = 385e^{-0.027x} + 86$	8797	17601
	2	$y = \frac{607}{(1+x)^{0.27}}$	9334	18675
	3	$y = \frac{568}{(1+x^{7})^{0.037}}$	9308	18623
	4	$y = 454\ \mathrm{e}^{-0.0165\mathrm{x}}$	8966	17939
Rider 9	1	$y = 308e^{-0.051x} + 99$	11123	22254
	2	$y = \frac{440}{(1+x)^{0.245}}$	11224	22455
	3	$y = \frac{424}{(1+x^{10.6})^{0.023}}$	11191	22389
	4	$y = 386\ \mathrm{e}^{-0.018x}$	11438	22882
Rider 10	1	$y = 323e^{-0.038x} + 109$	14120	28248
	2	$y = \frac{510}{(1+x)^{0.257}}$	14354	28714
	3	$y = \frac{489}{(1+x^{2})^{0.127}}$	14296	28600
	4	$y = 409\ \mathrm{e}^{-0.0153x}$	14524	29054

Table 5.3 Parameter estimates and their standard error (in brackets) for all models and riders

Model	Parameters	Rider 1	Rider 2	Rider 3	Rider 4	Rider 5
		Estimate (Standard Error)				
1	p_0	426.2 (4.73)	459.6 (5.00)	466.8 (3.56)	403.4 (3.44)	435.9 (3.70)
	p_∞	118.8 (5.29)	89.8 (6.36)	108.2 (3.12)	104.0 (4.76)	101.2 (4.18)
	α	0.033 (0.002)	0.034 (0.002)	0.033 (0.0013)	0.028 (0.002)	0.03 (0.0015)
	σ	81.7 (1.72)	67.2 (1.60)	49.06 (1.05)	70.4 (1.49)	65.1 (1.30)
2	p_0	475.7 (1.95)	567.5 (8.69)	644.6 (6.33)	438.5 (4.16)	518.6 (5.33)
	α	0.21 (0.006)	0.27 (0.006)	0.30 (0.004)	0.21 (0.005)	0.24 (0.004)
	σ	92.5 (1.95)	83.2 (1.98)	55.6 (1.19)	78.6 (1.66)	75.6 (1.51)
3	p_0	463.6 (5.89)	539.7 (8.26)	616.8 (8.40)	424.9 (3.76)	501.0 (5.29)
	α	0.075 (0.017)	0.1 (0.024)	0.21 (0.019)	0.007 (0.0006)	0.11 (0.018)
	β	2.8 (0.61)	2.61 (0.61)	1.34 (0.109)	30.1 (2.58)	2.21 (0.35)
	σ	90.5 (1.91)	80.8 (1.93)	55.2 (1.18)	76.6 (1.62)	73.8 (1.47)
4	p_0	395.3 (4.58)	438.6 (4.61)	425.9 (3.41)	384.0 (3.33)	410.1 (3.56)
	α	0.012 (0.0006)	0.019 (0.0006)	0.013 (0.0004)	0.01 (0.0004)	0.013 (0.0005)
	σ	91.7 (1.92)	72.1 (1.72)	63.2 (1.35)	77.8 (1.64)	74.6 (1.49)
Model	**Parameters**	**Rider 6**	**Rider 7**	**Rider 8**	**Rider 9**	**Rider 10**
		Estimate (Standard Error)				
1	p_0	474.2 (4.93)	434.7 (4.89)	470.9 (2.95)	406.4 (2.51)	432.5 (2.40)
	p_∞	100.2 (5.93)	111.4 (5.56)	86.4 (3.77)	98.7 (3.13)	109.4 (2.79)
	α	0.036 (0.002)	0.025 (0.002)	0.027 (0.0008)	0.05 (0.0024)	0.038 (0.001)
	σ	78.0 (1.43)	84.4 (1.53)	55.2 (0.97)	66.6 (1.06)	62.8 (0.88)
2	p_0	596.2 (8.50)	512.1 (6.73)	606.8 (6.54)	440.1 (2.77)	510.4 (3.23)
	α	0.28 (0.006)	0.211 (0.005)	0.27 (0.004)	0.24 (0.004)	0.26 (0.003)
	σ	91.3 (1.67)	96.5 (1.75)	76.9 (1.35)	70.1 (1.11)	68.9 (0.97)
3	p_0	555.5 (8.53)	506.1 (6.25)	567.9 (5.54)	423.9 (2.52)	489.3 (3.45)
	α	0.089 (0.03)	0.06 (0.013)	0.037 (0.004)	0.023 (0.006)	0.127 (0.012)
	β	2.92 (0.86)	3.49 (0.74)	7.0 (0.76)	10.6 (2.57)	2.0 (0.18)
	σ	90.2 (1.66)	93.8 (1.69)	75.7 (1.33)	68.9 (1.10)	67.3 (0.94)
4	p_0	448.2 (4.41)	402.6 (4.06)	454.1 (2.91)	385.5 (2.69)	408.6 (2.33)
	α	0.02 (0.0005)	0.01 (0.0004)	0.02 (0.0003)	0.02 (0.0008)	0.015 (0.0004)
	σ	82.4 (1.51)	90.5 (1.64)	61.3 (1.08)	78.1 (1.24)	73.6 (1.03)

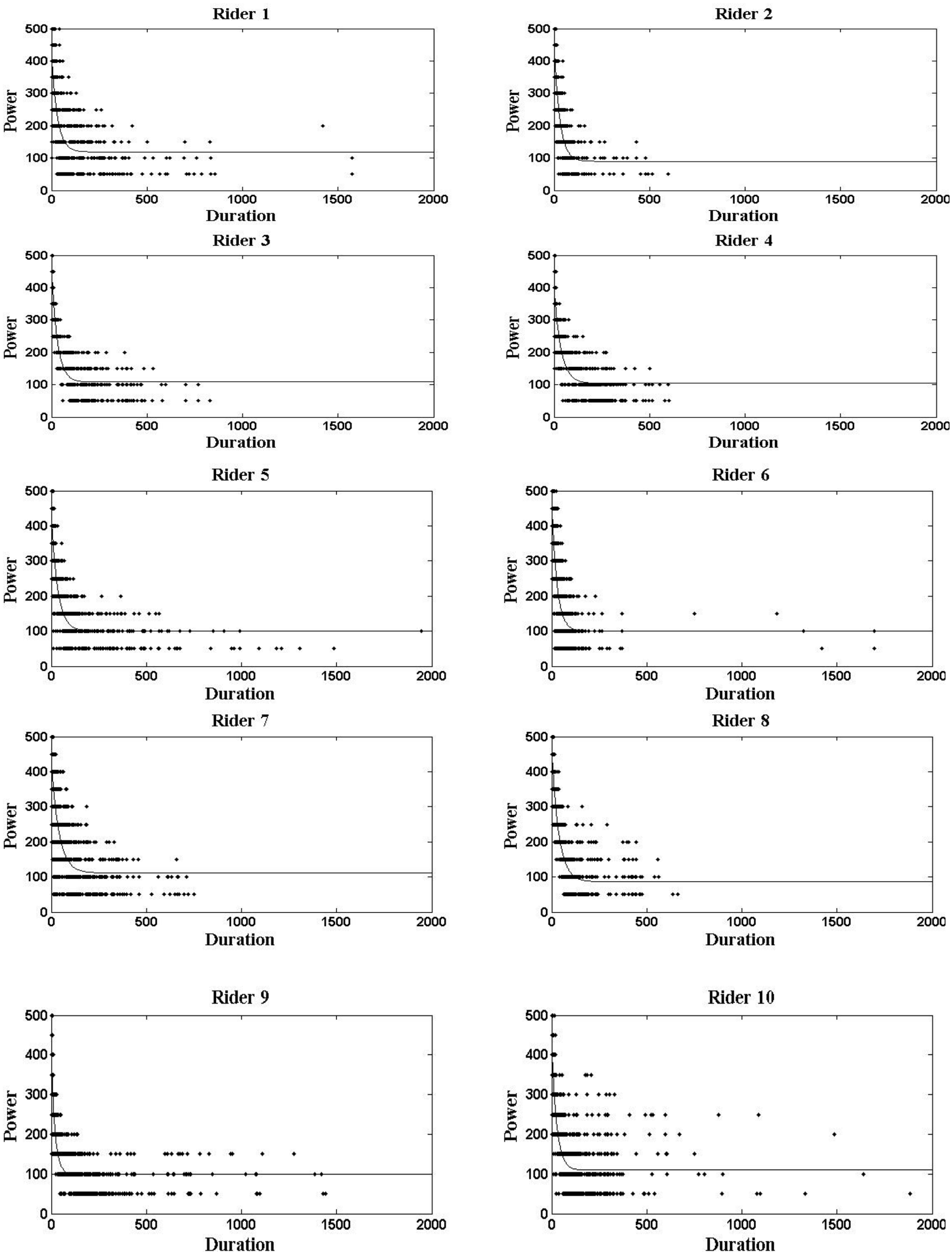

Figure 5.5 Critical power curve for each rider for all sessions with fitted minimum AIC fixed effects model with unit of duration (5 seconds) and unit of power (Watts)

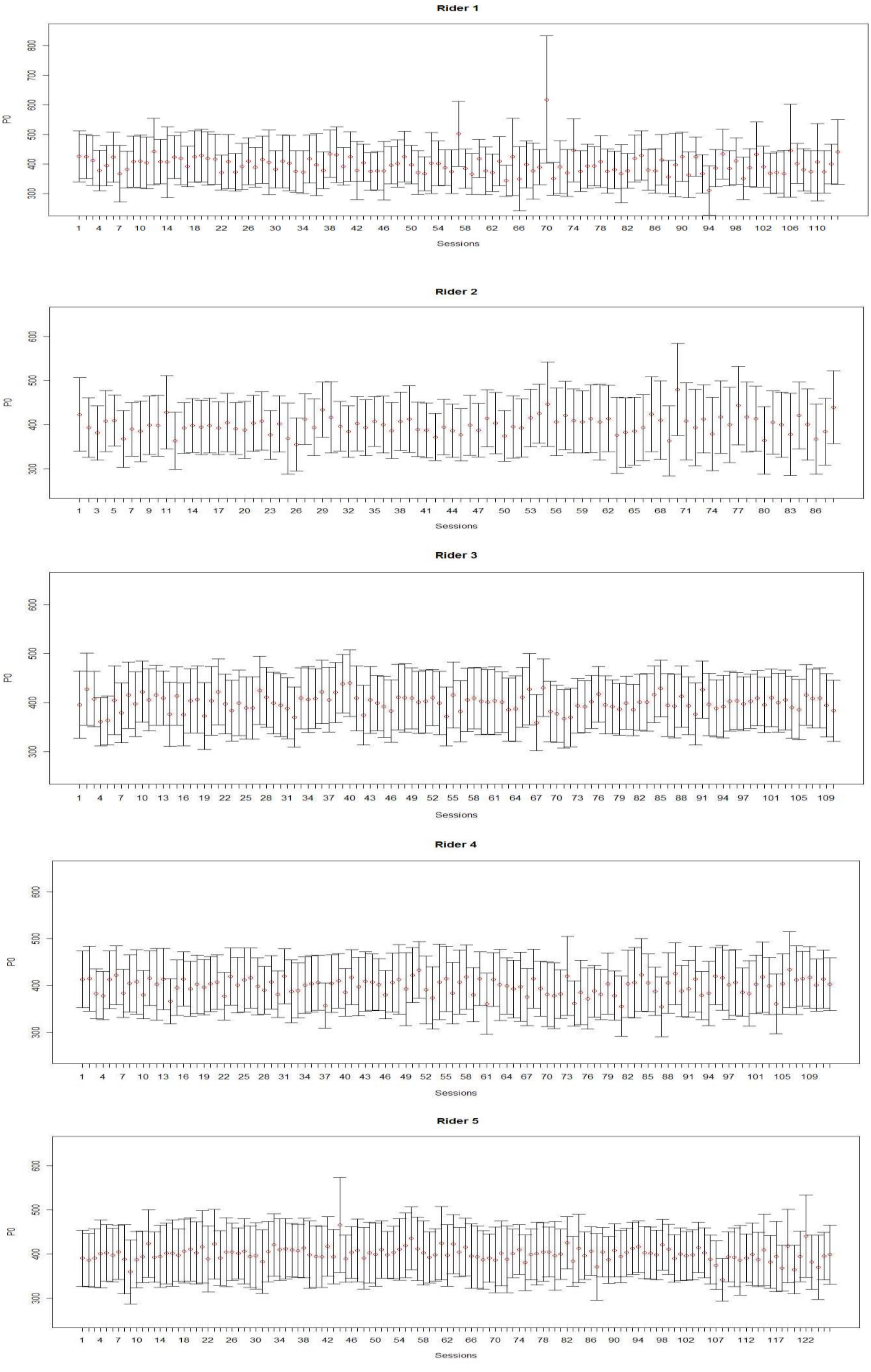

Figure 5.6 The parameter estimates p_0 for each session for each rider ± 2 standard errors

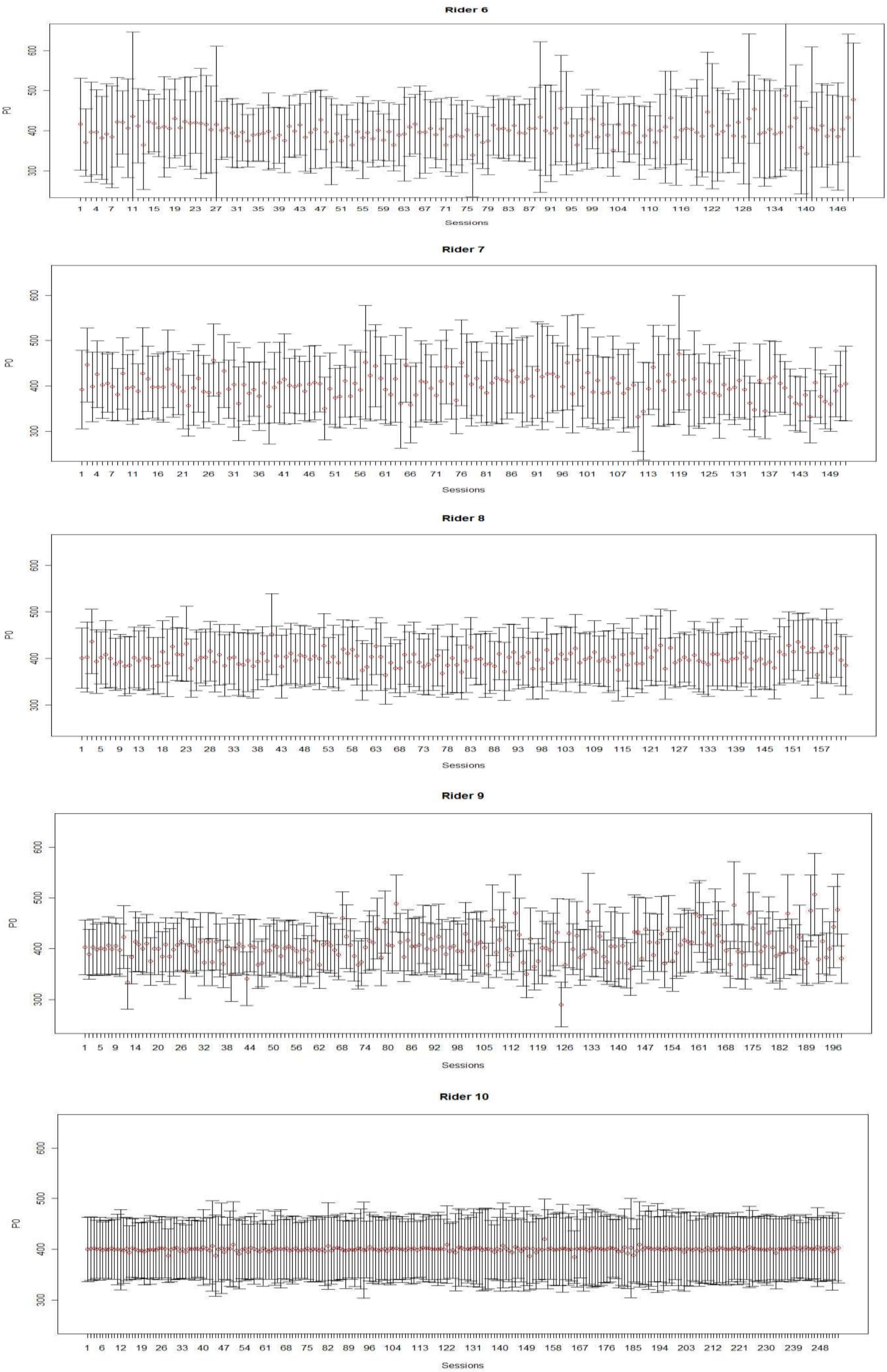

Figure 5.6 Continued**.**

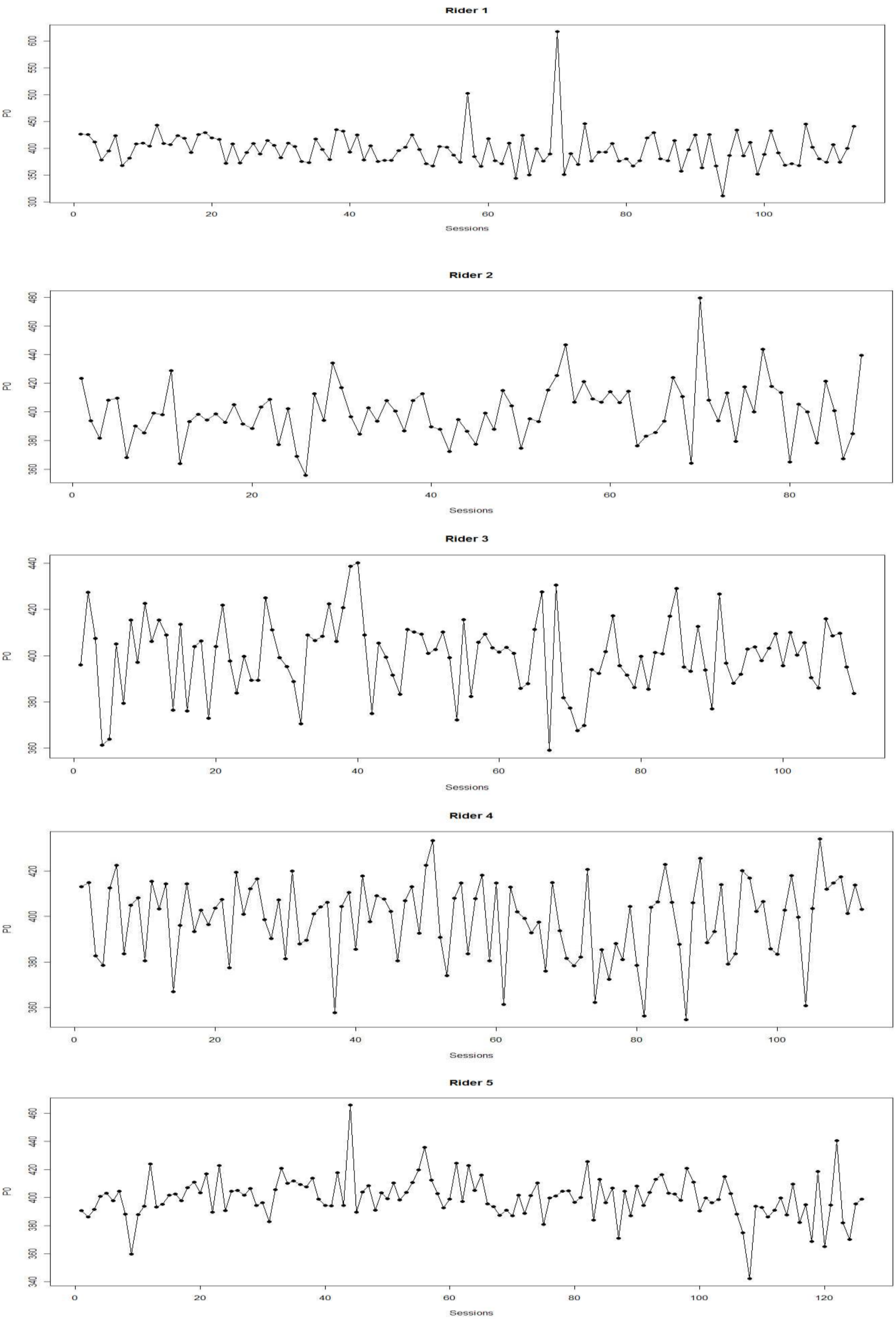

Figure 5.7 p_0 varying by session for all riders

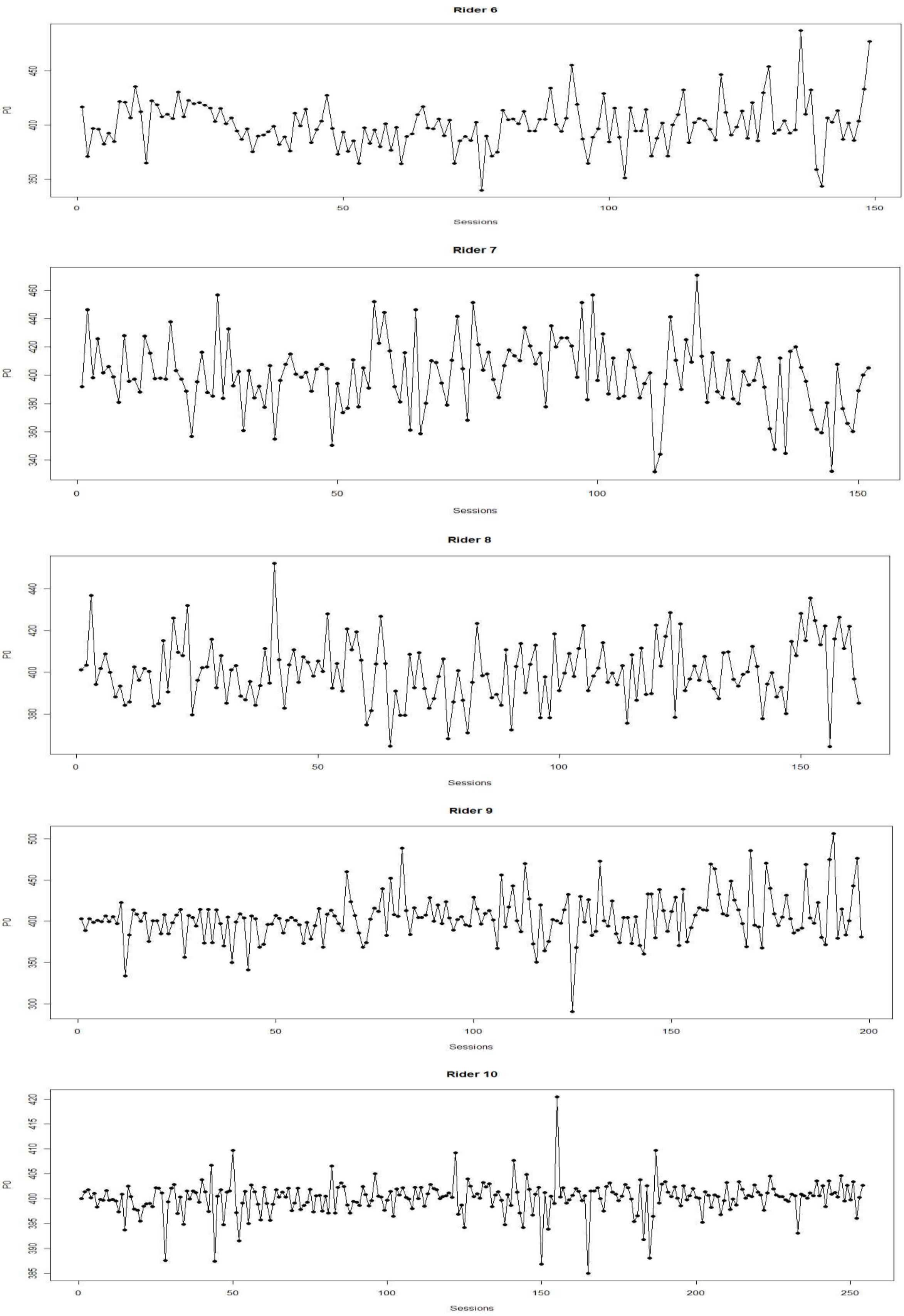

Figure 5.7 Continued.

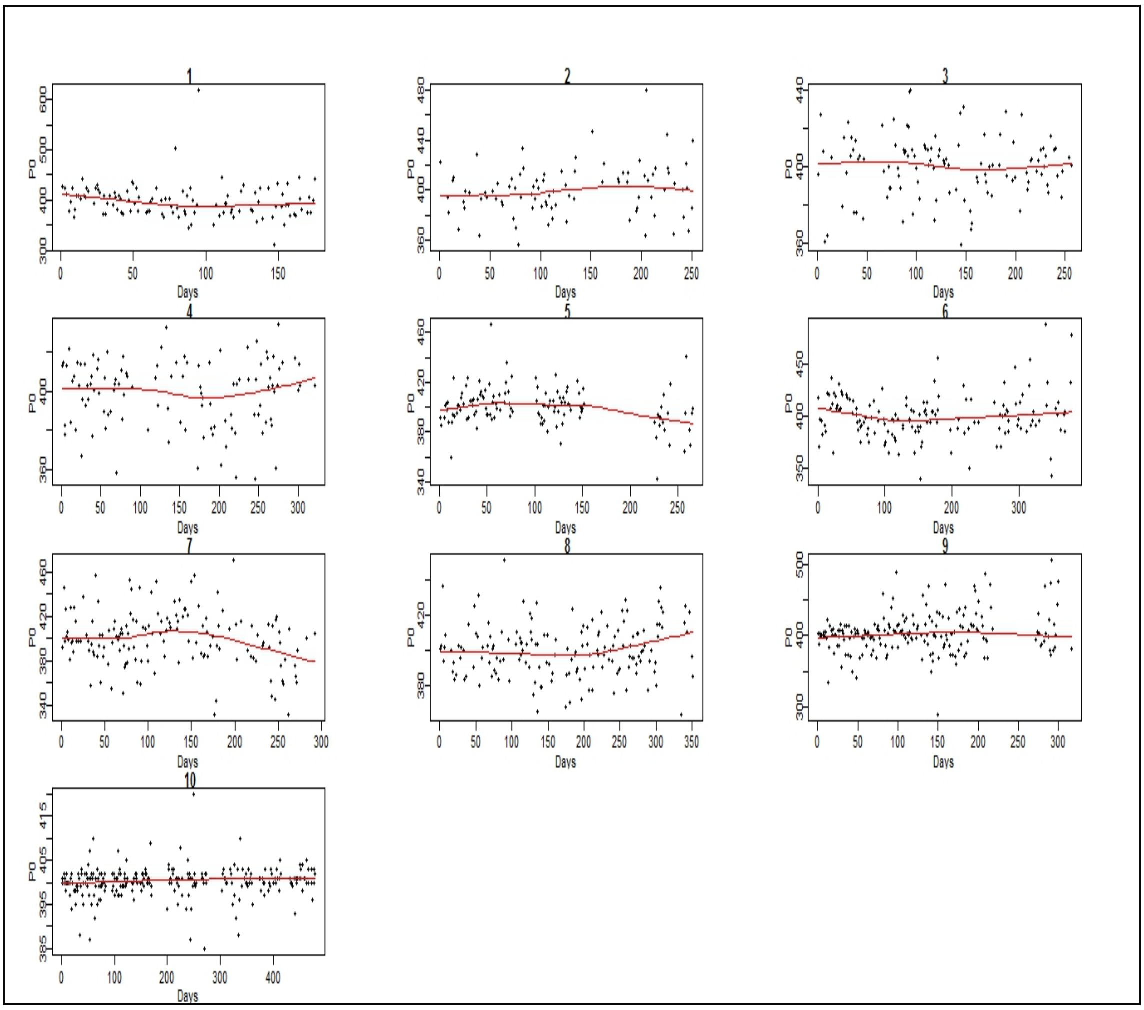

Figure 5.8 p_0 varying by session and smoothing curves for p_0, plotted against day for all riders (1 to 10)

5.4.3 Models of critical heart rate (CH) with CH varying by rider and by session

In this subsection, we use the same models as those defined previously to model heart rate as we have already done with power output to find the estimates of maximum heart rate varying by session. Table 5.4 shows the fitted models of heart rate against days of training with their AIC and maximum likelihood values (ML) where y_i is fixed level of heart rate and x_i is the largest l such that $t_{k+1} \le t \le t_{k+l}$ has $y > y_i$. The shaded values are the lowest AIC and the best fitting model for each rider. The estimates of the parameters of heart rate for all models with their standard errors are presented in table 5.5. Figure 5.9 then shows the critical heart rate curves for each rider for all sessions with fixed effects model fitted model which has minimum AIC. Next, we do the same as we have done for the power output by taking into account session variables and allow h_0 to vary from session to session. Figure 5.10 indicates the parameter estimates of h_0 for each session for each rider with ± 2 standard error while the h_0 values varying by session for each rider are presented in figure 5.11. Finally, h_0 varying by session, with smoothing curves for h_0 plotted against training day for all riders (1 to 10) are shown in figure 5.12.

Table 5.4 Fitted models of heart rate with their AIC and maximum likelihood values

Rider	Models	Fitted Models	ML	AIC
Rider 1	1	$y = 94e^{-0.055x} + 66$	2699	5406
	2	$y = \frac{174}{(1+x)^{0.125}}$	2710	5426
	3	$y = \frac{173}{(1+x^{2.43})^{0.05}}$	2706	5420
	4	$y = 152\ \mathrm{e}^{-0.00102x}$	2808	5622
Rider 2	1	$y = 108e^{-0.007x} + 64$	2144	4296
	2	$y = \frac{224}{(1+x)^{0.165}}$	2180	4367
	3	$y = \frac{224}{(1+x)^{0.163}}$	2180	4369
	4	$y = 152\ \mathrm{e}^{-0.00166x}$	2190	4386
Rider 3	1	$y = 117e^{-0.0037x} + 51$	2533	5073
	2	$y = \frac{204}{(1+x)^{0.148}}$	2635	5277
	3	$y = \frac{198}{(1+x^{5.6})^{0.025}}$	2636	5279
	4	$y = 160\ \mathrm{e}^{-0.0015x}$	2577	5160
Rider 4	1	$y = 102e^{-0.005x} + 65$	2746	5500
	2	$y = \frac{188}{(1+x)^{0.125}}$	2794	5594
	3	$y = \frac{186}{(1+x^{1.8})^{0.0696}}$	2793	5593
	4	$y = 150\ \mathrm{e}^{-0.0011x}$	2799	5603
Rider 5	1	$y = 106e^{-0.0051x} + 59$	2955	5918
	2	$y = \frac{224}{(1+x)^{0.167}}$	3020	6046
	3	$y = \frac{221}{(1+x^{1.23})^{0.134}}$	3019	6047
	4	$y = 150\ \mathrm{e}^{-0.00124x}$	3041	6087

Table 5.4 Continued.

Rider	Models	Fitted Models	ML	AIC
Rider 6	1	$y = 104e^{-0.016x} + 70$	3595	7197
	2	$y = \frac{191}{(1+x)^{0.148}}$	3662	7330
	3	$y = \frac{189}{(1+x^{2.12})^{0.069}}$	3658	7324
	4	$y = 140\ \mathrm{e}^{-0.0017x}$	7090	14185
Rider 7	1	$y = 94e^{-0.0148x} + 77$	3725	7458
	2	$y = \frac{181}{(1+x)^{0.129}}$	3742	7490
	3	$y = \frac{180}{(1+x^{3.14})^{0.041}}$	3739	7485
	4	$y = 142\ \mathrm{e}^{-0.0011x}$	3868	7743
Rider 8	1	$y = 114e^{-0.0036x} + 51$	3560	7129
	2	$y = \frac{199}{(1+x)^{0.145}}$	3741	7488
	3	$y = \frac{196}{(1+x^{1.45})^{0.099}}$	3738	7484
	4	$y = 152\ \mathrm{e}^{-0.00105x}$	3726	7458
Rider 9	1	$y = 107e^{-0.0026x} + 61$	4627	9262
	2	$y = \frac{186}{(1+x)^{0.12}}$	4800	9607
	3	$y = \frac{185}{(1+x^{1.6})^{0.074}}$	4799	9605
	4	$y = 157\ \mathrm{e}^{-0.00074x}$	4759	9523
Rider 10	1	$y = 98e^{-0.0063x} + 64$	5967	11941
	2	$y = \frac{172}{(1+x)^{0.129}}$	6025	12055
	3	$y = \frac{172}{(1+x^{1.94})^{0.07}}$	6021	12049
	4	$y = 148\ \mathrm{e}^{-0.0012x}$	6215	12435

Table 5.5 Parameter estimates and their standard error (in brackets) for all models and riders

Model	Parameters	Rider 1	Rider 2	Rider 3	Rider 4	Rider 5
		Estimate (Standard Error)				
1	h_0	159.3 (2.16)	170.9 (3.59)	168.3 (1.95)	167.4 (2.68)	164.8 (2.07)
	h_∞	65.8 (2.08)	63.5 (3.17)	51.1 (2.19)	65.1 (2.68)	58.7 (2.04)
	α	.005 (.001)	.007 (.001)	.004 (.0002)	.01 (.0004)	.005 (.0004)
	σ	28.8 (0.86)	31.6 (1.07)	24.2 (0.73)	32.6 (0.98)	26.4 (0.74)
2	h_0	173.8 (2.53)	223.9 (6.52)	204.1 (3.59)	188.2 (4.13)	223.9 (4.29)
	α	0.125 (.004)	0.165 (.007)	0.148 (.004)	0.125 (.005)	0.167 (.005)
	σ	29.3 (0.87)	34.3 (1.16)	29.2 (0.88)	35.5 (1.06)	29.2 (0.82)
3	h_0	172.5 (2.48)	223.8 (7.76)	197.7 (3.36)	186.2 (4.18)	221.3 (4.77)
	α	.05 (.02)	0.163 (.05)	0.025 (0.01)	0.07 (0.03)	0.134 (0.032)
	β	2.43 (0.96)	1.00 (0.26)	5.61 (2.27)	1.78 (0.85)	1.23 (0.28)
	σ	29.1 (0.86)	34.3 (1.16)	29.2 (0.88)	35.4 (1.05)	29.2 (0.82)
4	h_0	151.8 (1.94)	151.8 (2.71)	159.5 (1.81)	150.2 (2.11)	150.3 (1.68)
	α	.001 (.000002)	.002 (.0001)	.002 (.00003)	.001 (.00001)	.001 (.00002)
	σ	34.3 (1.00)	35.1 (1.20)	26.2 (0.84)	35.8 (1.08)	30.2 (0.87)

Model	Parameters	Rider 6	Rider 7	Rider 8	Rider 9	Rider 10
		Estimate (Standard Error)				
1	h_0	174.3 (2.51)	170.3 (2.68)	164.7 (1.25)	168.2 (1.53)	161.4 (1.35)
	h_∞	69.9 (1.79)	76.8 (1.76)	51 (1.28)	61.4 (1.32)	63.6 (1.3)
	α	0.016 (0.001)	0.015 (0.002)	0.004 (0.0001)	0.003 (0.0001)	0.006 (0.0004)
	σ	30.1 (0.78)	32.5 (0.83)	19.6 (0.49)	25.9 (0.59)	26.6 (0.53)
2	h_0	190.7 (3.26)	180.9 (2.88)	198.9 (2.22)	186.2 (2.37)	172.4 (1.54)
	α	0.148 (0.005)	0.129 (0.004)	0.145 (0.003)	0.12 (0.003)	0.129 (0.002)
	σ	33 (0.85)	33.3 (0.85)	24.5 (0.61)	30.9 (0.69)	27.8 (0.55)
3	h_0	188.8 (3.22)	179.5 (2.82)	196.3 (2.42)	185 (2.43)	171.8 (1.53)
	α	0.07 (0.03)	0.041 (0.02)	0.099 (0.02)	0.07 (0.03)	0.07 (0.02)
	β	2.12 (0.89)	3.14 (1.36)	1.45 (0.28)	1.6 (0.56)	1.94 (0.63)
	σ	32.8 (0.85)	33.2 (0.85)	24.4 (0.61)	30.8 (0.69)	27.7 (0.55)
4	h_0	140.2 (1.48)	141.6 (1.93)	152.1 (1.16)	157.1 (1.37)	148.2 (1.24)
	α	.002 (.00004)	.001 (.00002)	.001 (.00001)	.001 (.000003)	.001 (.00001)
	σ	28.2 (0.51)	39.3 (1.02)	24.1 (0.63)	29.6 (0.73)	32.3 (0.65)

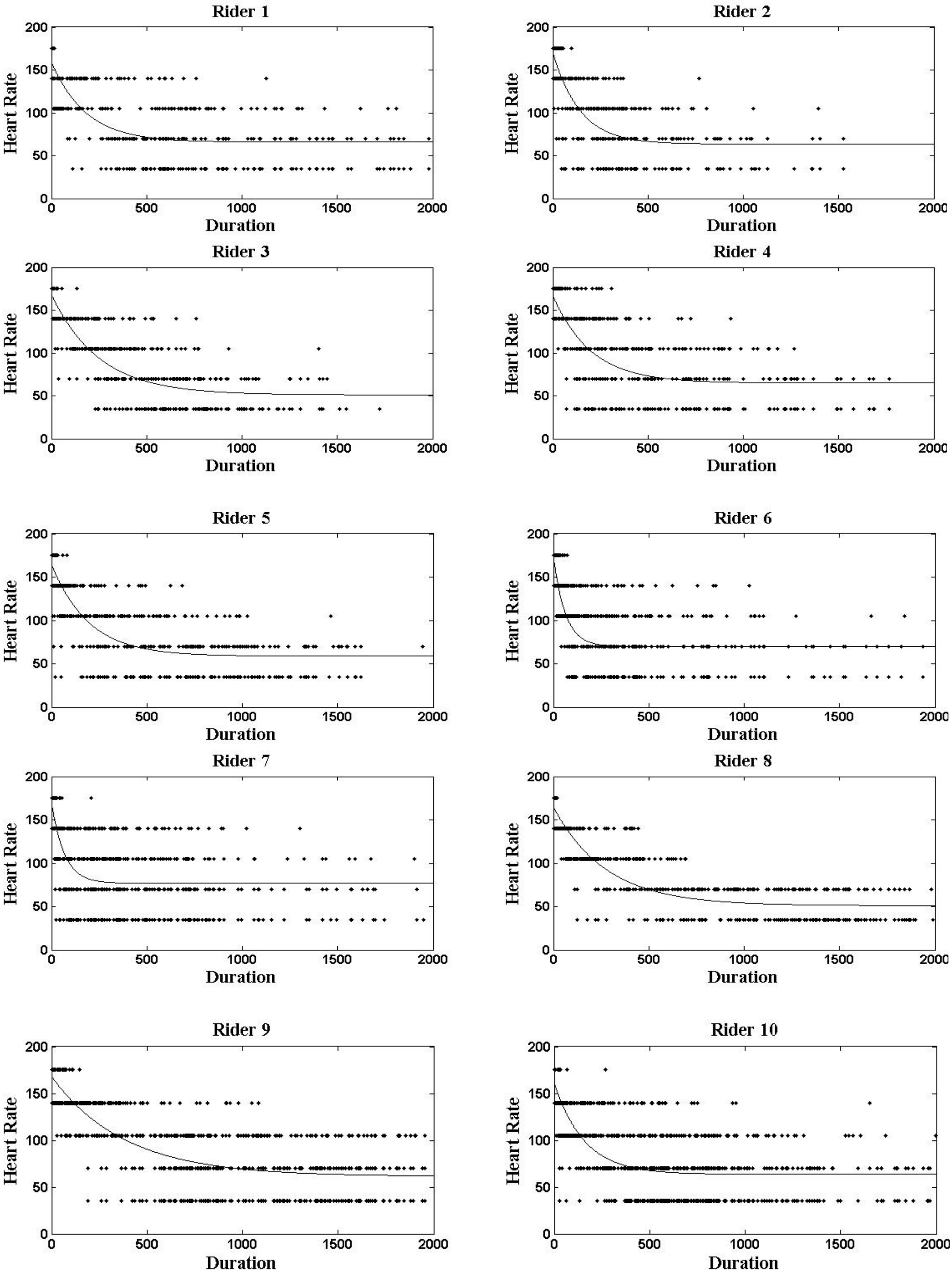

Figure 5.9 Critical heart rate curve for each rider for all sessions with fitted fixed effects model with minimum AIC fixed effects model with unit of duration (5 seconds) and unit of heart rate (beats per minute)

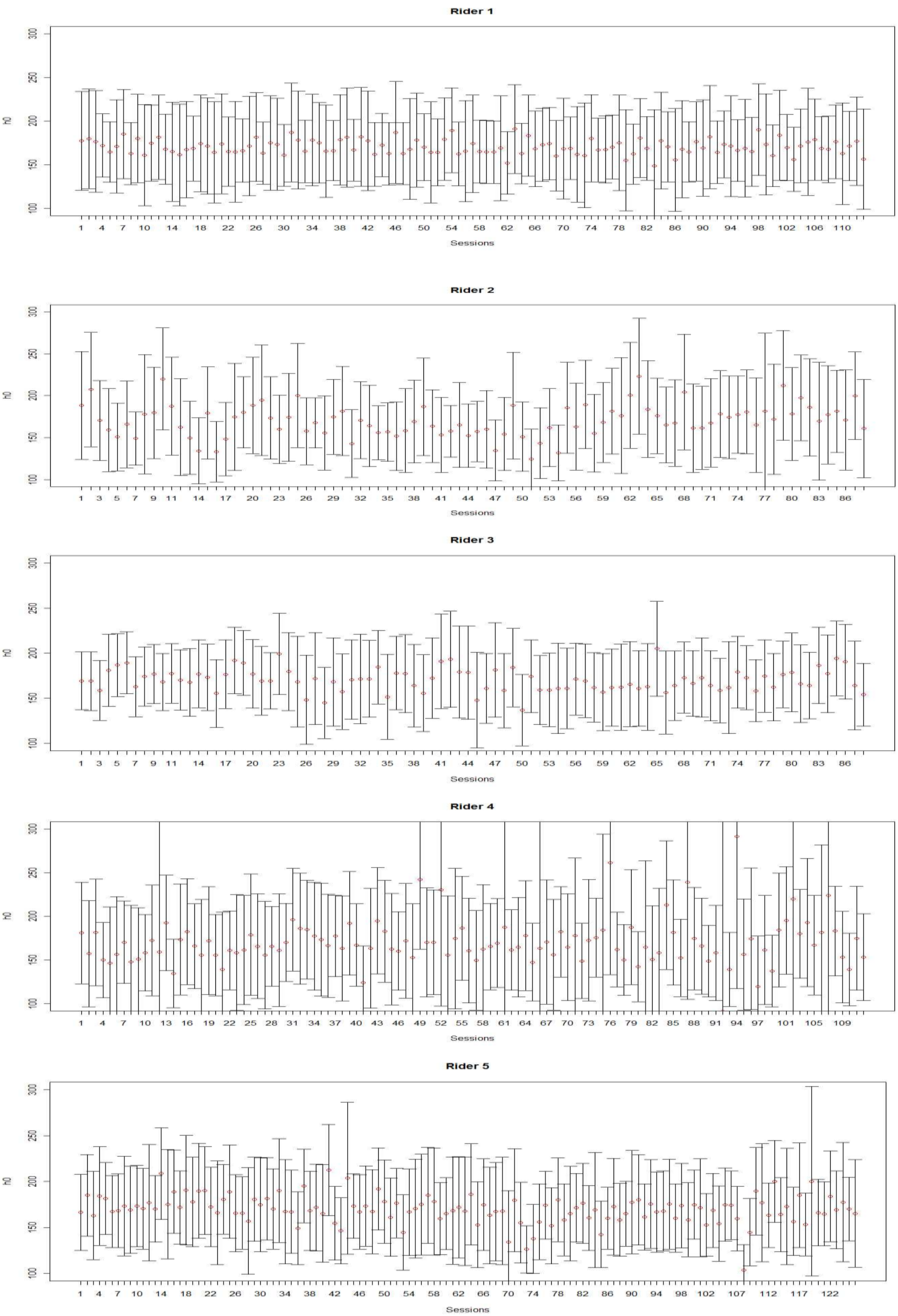

Figure 5.10 The parameter estimates h_0 for each session for each rider ± 2 standard error

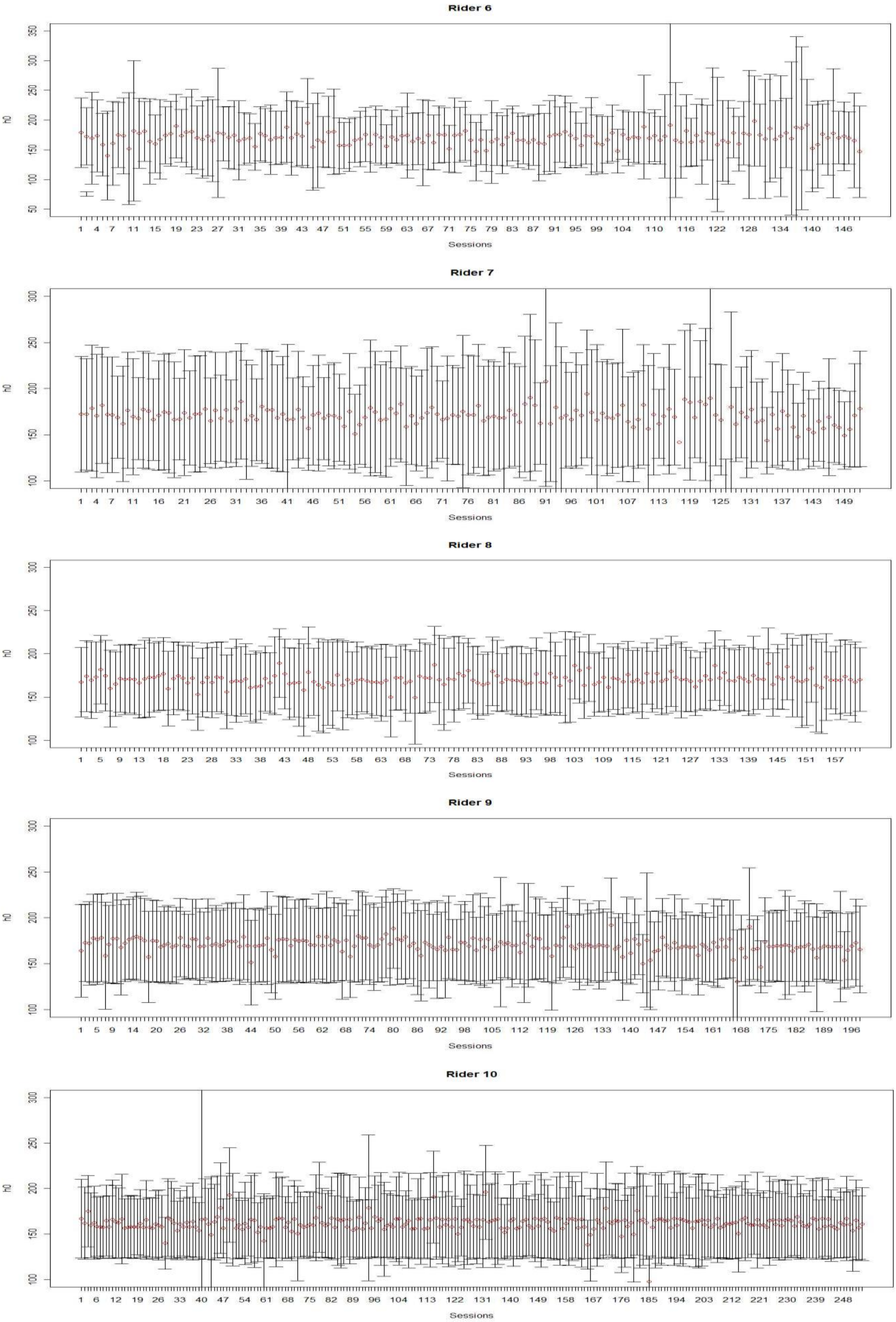

Figure 5.10 Continued.

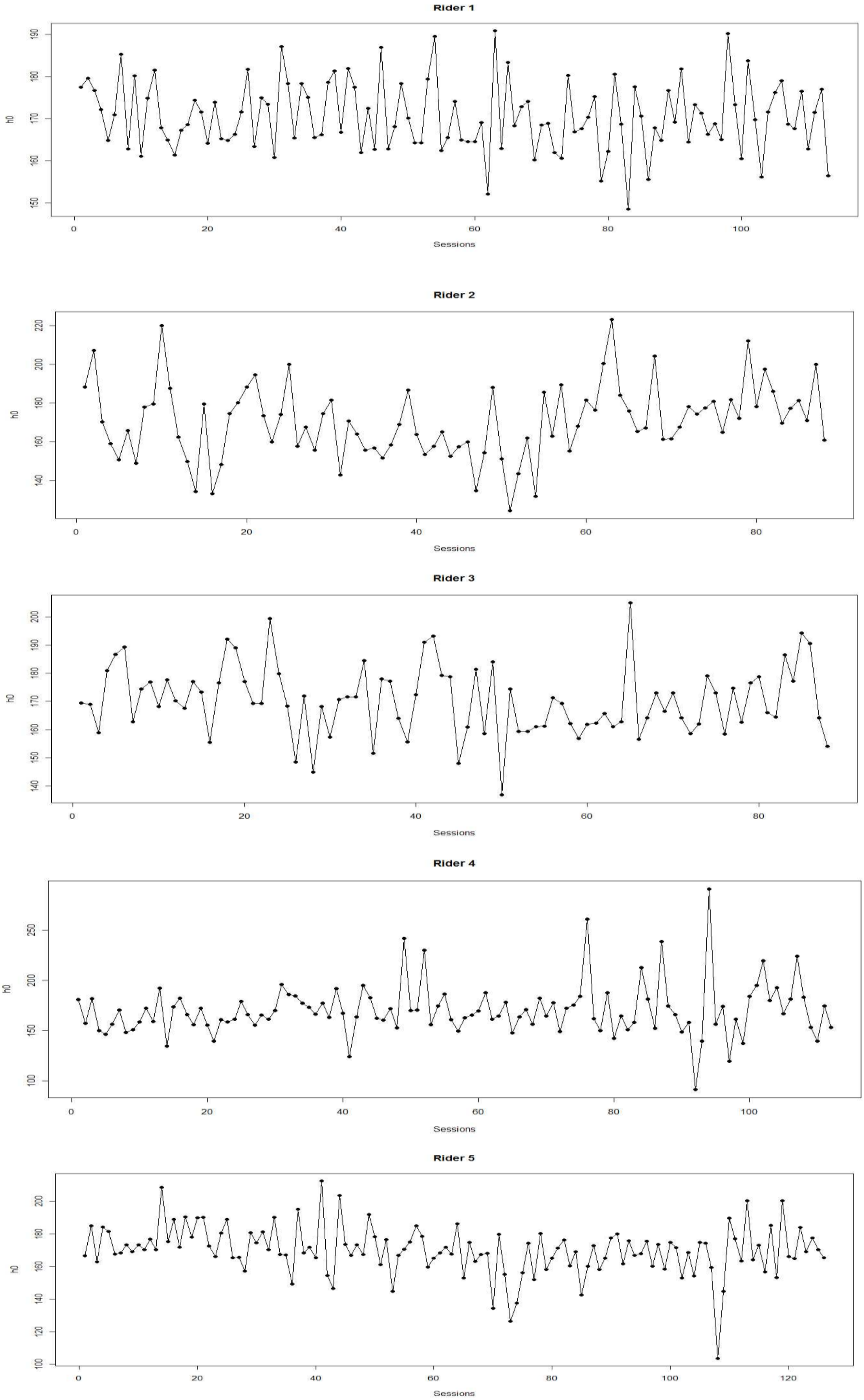

Figure 5.11 h_0 varying by session for all riders

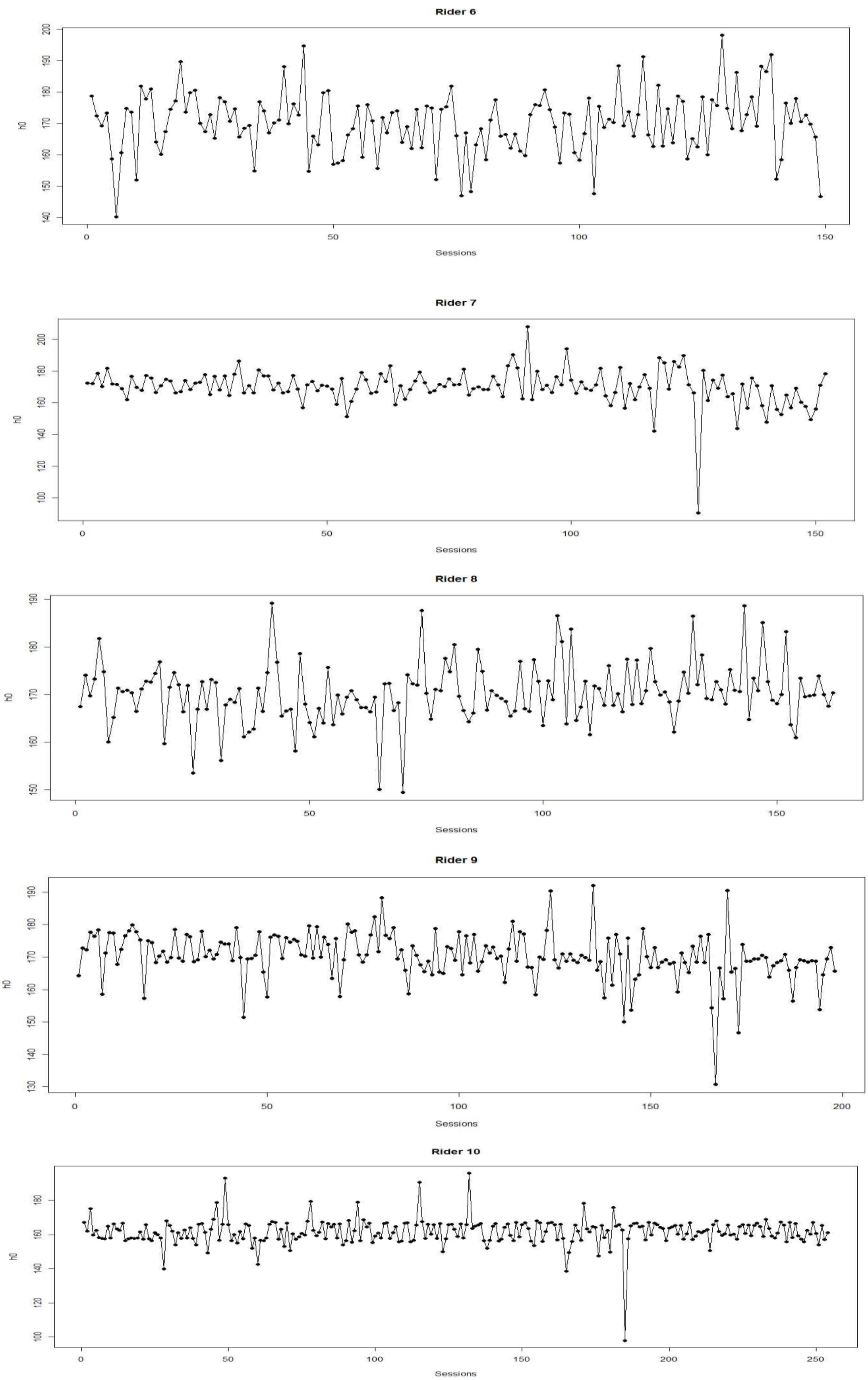

Figure 5.11 Continued.

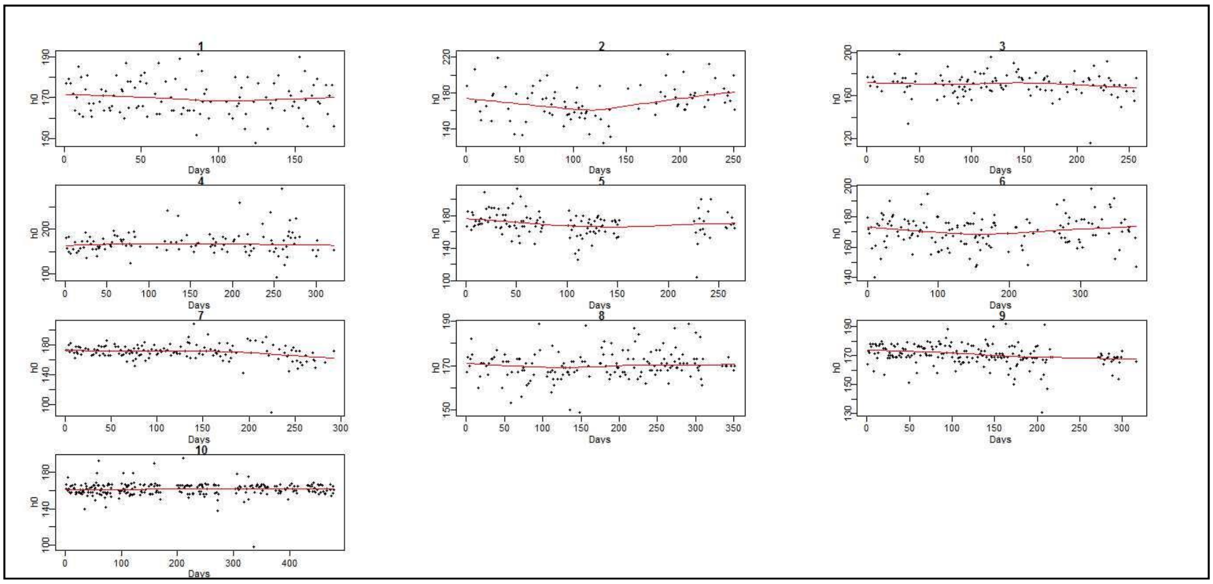

Figure 5.12 $h_0$varying by session, and smoothing curves for h_0 plotted against day for each rider (1 to 10)

5.4.4 Another candidate measure of performance based on the critical power concept

We focus on a candidate measure of performance and now we propose p_0/h_0 as another possible measure of performance based on training data. We focus on p_0/h_0 varying by session and this generates a training session related performance measure. This measure will be related to the accumulated training effect at time t. There is no obvious relationship between p_0 and h_0 as shown in figure 5.13. The relationship between our measure p_0/h_0 and days is seen in figure 5.14. Smoothing splines method is used with two different degrees of smoothing parameters.

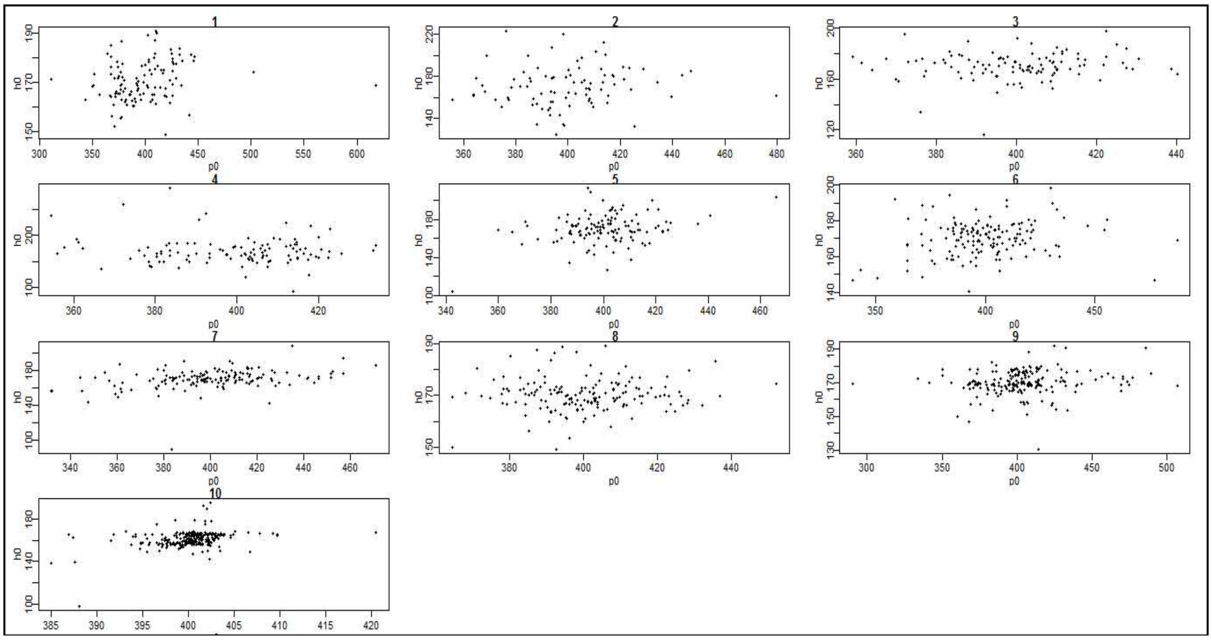

Figure 5.13 h_0 versus p_0 of all riders

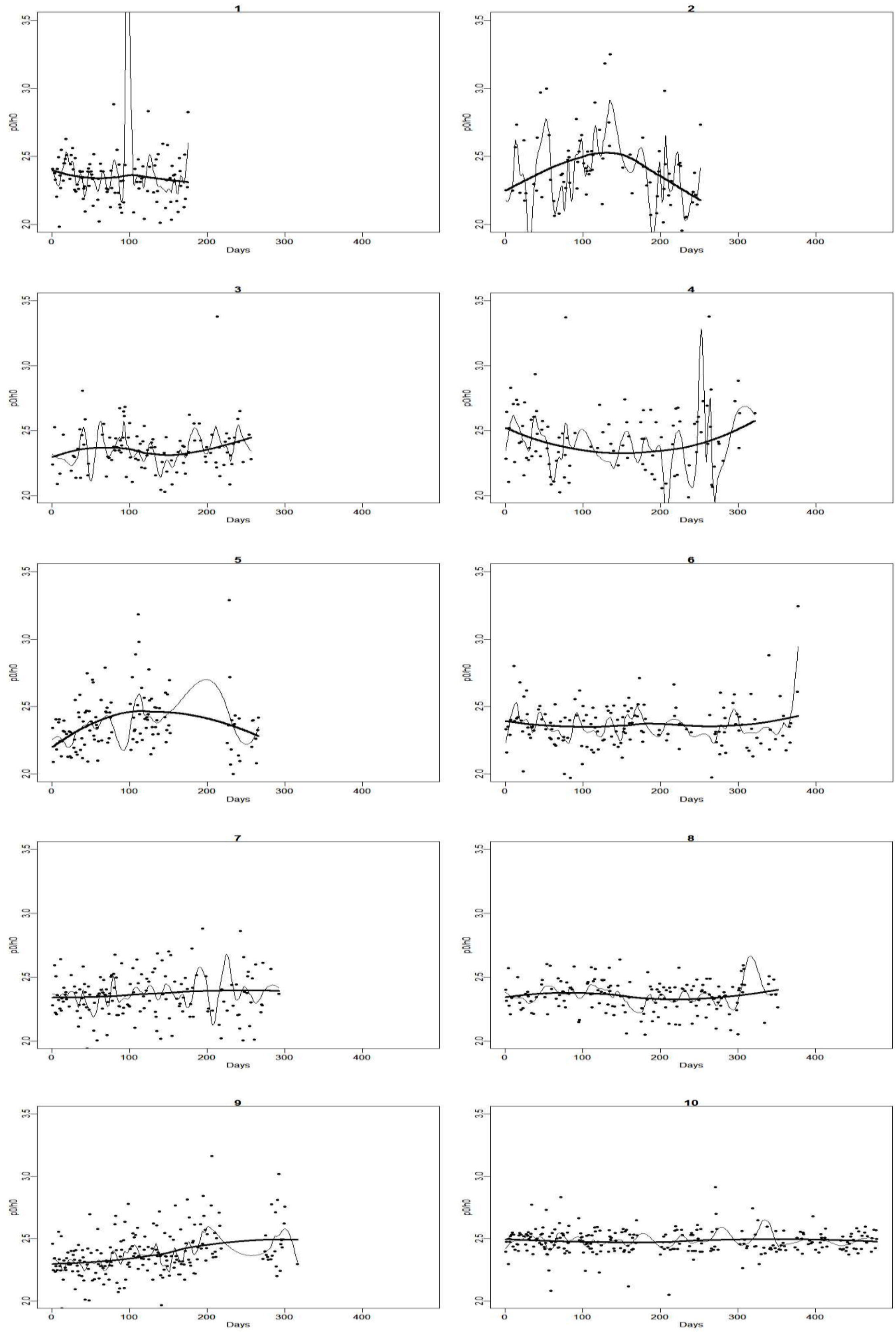

Figure 5.14 p_0/h_0versus day for each rider with smoothing degree = 0.1 (——), and smoothing degree =0.9 (▬▬)

5.5 Summary

In this chapter, we summarised some other possible measures of performance such as average power, normalised power and critical power. We presented different models that fit the critical power concept. Those models were reported in the study of Bull et al. (2000). Then we investigated four different models to model critical power with critical power varying by rider and by session to estimate the maximum power output. The best model that fitted our data was chosen using the Akaike information criterion (AIC). We presented the estimates of maximum power output for all riders for all models. Next, we plotted the estimates of maximum power output for each session for each rider $\pm$ 2 standard error. After that, we have done with heart rate as we have already done with power output to find the estimates of maximum heart rate varying by session. Finally, we suggested another measure of performance based on the concept of critical power.

6. CONCLUSION AND FUTURE WORK

6.1 Summary and conclusion

Training should be balanced between achieving maximum performance and avoiding overtraining. In the thesis, we are interested in modelling and optimising the training process in sport and exercise, and cycling in particular. We use data collected every five seconds during training sessions on power output and heart rate. Our aim is to use these data and provide a method that coaches and riders can follow in order to maximise performance in a major competition. This method is based on the relationship between the accumulation of training and performance. To relate the accumulation of training to performance both of them must be measured. In the first part of this research we describe a measure of training load for a session first proposed by Banister, et al. (1975) called training impulse (TRIMP). Then we describe the accumulation of training. Next we propose a new measure of performance. This measure is based on the relationship between power output and heart rate with the appropriate time lag of seconds (15 seconds). To calculate this performance measure for each session first we determine a particular high percentile of power output using the entire training history of the rider, P_q where q is specified according to the type of competition (sprint or endurance). Under consideration for each session we then calculate this performance measure using a linear model that relates power output to heart rate at this specified power output percentile. After that, we describe the latent variable model that we use to estimate the Banister model parameters using maximum likelihood. Finally, we investigate some other performance measures such as average power, normalised power and critical power.

To conclude, we use field data to estimate parameters of the Banister model. Our methodology is working individually and specifically for each rider. We propose a new measure of performance that can be measured using field data, and we develop a methodology to measure it and relate it to training load that is measured in a standard way. However, for this approach to work, very many performance measurements must be available, because of the high level of noise in the training-performance relationship, and training load measurements must be available for all sessions, otherwise the accumulated training effect is under estimated. Thus, to use the methodology we describe, a rider must quantify his/her training load in every session undertaken and must use a power meter for the majority of sessions.

6.2 Main findings

To model an optimum training schedule, Banister model parameters must be known. So the fundamental aim of this research is to estimate these parameters using field data such as power output and heart rate sampled every 5 seconds. We provided a new model to relate training to performance in order to maximise performance at a known competition. For each rider, the session coefficients a_i and b_i for the power output/heart-rate relationship, (and their estimated variances and covariances) are estimated using the R programming language. We use the 50th and 75th percentiles of power output, ($P50$, $P75$), as the reference power output. Then, the performance measures is calculated for each session i, $(h_{P50,i}, h_{P75,i})$. We estimated the parameters of our model α, β, σ, k_f, τ_a and τ_f for each rider using the maximum likelihood methodology. This procedure is relatively sensitive to starting values so some care is required. The response surface methodology was used to determine the starting values. The estimated parameters of the Banister model vary from rider to rider because of personal characteristics. We analysed 2 cases of performances and also 2 cases of detriment scale parameter. Then, we discussed practically the significance of training effects at 5% level of significance. Our results show that our method worked for some rider and that because of the noise in our data.

6.3 Implications

The Banister model parameter estimates found are similar to those reported in the previous studies for the majority of riders in the sample, (in swimming, $\tau_a = 41.4, \tau_f = 12.4, k_a = 0.062, k_f = 0.128$ (Mujika, et al., 1996), in swimming, $\tau_a = 38, \tau_f = 19, k_a = 0.036, k_f = 0.0.050$ (Hellard, et al., 2006), in running, $\tau_a = 45, \tau_f = 15, k_a = 1, k_f = 2$ (Morton, et al., 1990), in cycling, $\tau_a = 60, \tau_f = 4, k_a = 0.0021, k_f = 0.0078$ for participant A and $\tau_a = 60, \tau_f = 6, k_a = 0.0019, k_f = 0.0073$ for participant B (Busso, et al., 1997)). However, the uncertainty in our estimates is quite large, even though some big datasets of power output and heart rate have been analyzed. Given the uncertainty we observe, it is difficult to recommend their use in the planning of training.

Furthermore, knowledge of training capacity (the lower limit for ATE) is required to plan training. Therefore the Banister model appears to fall short for practical application. Alternatives to the Banister model might be considered. For example, consider the following multi-factorial model in which training influences both performance and the capacity for further training. Suppose that the performance of an athlete at time t, P_{t+1}, depends on his/her training load L_t at time t, subject to diminishing returns: $E(P_{t+1}) = \alpha L_t^{\beta}$ $(\beta < 1)$ (Helland et al., 2006). Here, the effect of training on performance is not persistent and an athlete is "only as good as his last session!". Further, suppose that the athlete's capacity to train at time t, $C_t = L_t^{\max} > L_t$ depends on his/her cumulative training load to date: $L_t^{\max} = L_0 + \sum_{s=1}^{t-1} L_t R_{t-s}$, where R_{t-s} is a training response function e.g. $R_{t-s} = e^{-(t-s)/\tau_a} + k\, e^{-(t-s)/\tau_f}$. In this way, training develops an athlete's capacity to train, through the accumulation of decaying benefits and detriments from past sessions. L_0 is the athlete's baseline capacity for training. A variation on this model might suppose that $L_t^{\max} = \max_{s<t}(L_0, L_t R_{t-s})$ so that an athlete is "only as good as the hardest session he has ever done", accounting for time since that session. This model, and the careful estimation of $h_{Pq,i}$ from the session data will be the focus of our future research on modelling training.

6.4 Limitations of the work

Our method is not completely satisfactory as the Banister model parameters are not always well estimated. So, we have some limitations in our work related to the data. Firstly, the data are very noisy; there is mis-recording as some heart rate values are recorded as zeros. Also, we do not have the training diaries of the riders with such diaries we might know how training has been planned. Finally we might expect that if we monitor a young rider for a period of time (e.g. 2 years), the improvement in his or her training (change in ATE) may be greater and we may obtain better estimates of our model parameters.

6.5 Future work

To optimise a training schedule for coaches and riders, in cycling in particular, Banister model parameters must be known. In this thesis we provide a method to estimate these parameters using training data sampled every five seconds. The next step then is to use these parameter values to choose the best training program in order to maximise performance and avoid over-training at the future time *T*. This is a pure optimisation problem and we do not consider it further here. On the other hand, a further very interesting and useful task would be to obtain new data on a developing rider and to test the estimation procedure in practice.

Another important point to consider in the estimation of the Banister model parameters and hence the optimisation of training is the effect of environmental temperature on the relationship between power output and heart rate. In the study of Lafrenz et al. (2008), ten athletes (comprising cyclists and runners) participated in a study using cycle ergo meters at two different ambient temperatures (22 and 35 Celsius). They found an increase in heart rate by approximately 10 beats per minute in the hot conditions and 3 beats per minute in the cool conditions. So, studying the ambient temperature as a fundamental factor should be taken into account.

Another key point that should be taken into account to optimise training was mentioned by Fitz-Clarke et al. (1991). The key issue is about fatigue. They highlighted a fundamental point of optimising a training schedule. They have used the parameter values of the Banister model presented by Morton, et al., (1990) to derive a formula for the period of time t_n when training should be stopped before a competition in order to maximise performance at a known time in future. This period of time is given by

$$t_n = \frac{\tau_a \tau_f}{\tau_a - \tau_f} ln\left(\frac{k_f}{k_a}\right).$$

where τ_a, τ_f and are 45, 15 days respectively and k_a, k_f are 1, and 2 arbitrary units respectively.

Training within t_n days before competition will increase the amount of fatigue rather than the benefit. So, athletes should avoid training within this period of time immediately prior to competing (Taha and Thomas, 2003).

Another key of optimising training schedule was mentioned by Fitz-Clarke et al. (1991). This key is the time t_g to achieve maximal performance after the completion of a training session. It is given by

$$t_g = \frac{\tau_a \tau_f}{\tau_a - \tau_f} ln\left(\frac{k_f \tau_a}{k_a \tau_f}\right).$$

In conclusion, there is much work that remains to be done. This thesis makes a start at optimising training schedules, and in cycling in particular. We have suggested some key points which should be taken into account to develop this work and contribute to the knowledge.

Appendix 1 Correlations for power output against heart rate at different lags.

Table A1.1 Correlation between power output and heart rate for each session for rider 1 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.36	0.51	0.55	0.57	0.55	57	0.43	0.66	0.71	0.65	0.46
2	0.76	0.78	0.80	0.79	0.79	58	0.56	0.78	0.81	0.79	0.58
3	0.40	0.52	0.56	0.59	0.58	59	0.48	0.64	0.64	0.62	0.46
4	0.44	0.70	0.76	0.75	0.66	60	0.36	0.44	0.44	0.45	0.40
5	0.23	0.41	0.47	0.47	0.43	61	0.53	0.66	0.67	0.64	0.49
6	0.38	0.57	0.66	0.65	0.63	62	0.45	0.64	0.69	0.69	0.61
7	0.57	0.70	0.73	0.74	0.70	63	0.04	0.04	0.02	0.02	0.03
8	0.62	0.75	0.78	0.77	0.69	64	0.31	0.41	0.46	0.50	0.56
9	0.54	0.63	0.66	0.66	0.65	65	0.36	0.48	0.53	0.57	0.64
10	0.43	0.55	0.59	0.58	0.55	66	0.39	0.38	0.50	0.40	0.42
11	0.27	0.39	0.44	0.49	0.58	67	0.41	0.44	0.55	0.47	0.50
12	0.18	0.25	0.28	0.31	0.37	68	0.54	0.56	0.57	0.57	0.57
13	0.45	0.69	0.75	0.76	0.68	69	0.25	0.27	0.35	0.28	0.29
14	0.25	0.32	0.35	0.38	0.43	70	0.53	0.50	0.53	0.51	0.52
15	0.81	0.81	0.82	0.82	0.82	71	0.2	0.27	0.29	0.31	0.29
16	0.30	0.35	0.37	0.39	0.43	72	0.17	0.22	0.24	0.22	0.21
17	0.37	0.57	0.64	0.70	0.62	73	0.44	0.49	0.51	0.52	0.53
18	0.56	0.67	0.71	0.73	0.74	74	0.46	0.62	0.67	0.66	0.56
19	0.55	0.66	0.72	0.71	0.71	75	0.45	0.67	0.72	0.72	0.64
20	0.77	0.82	0.83	0.83	0.83	76	0.22	0.24	0.27	0.29	0.26
21	0.51	0.62	0.66	0.65	0.59	77	0.48	0.60	0.64	0.65	0.65
22	0.50	0.51	0.49	0.48	0.46	78	0.37	0.45	0.48	0.48	0.44
23	0.56	0.66	0.71	0.69	0.66	79	0.13	0.17	0.19	0.18	0.20
24	0.57	0.70	0.71	0.68	0.54	80	0.66	0.71	0.73	0.74	0.75
25	0.42	0.59	0.64	0.66	0.59	81	0.70	0.70	0.75	0.72	0.74
26	0.58	0.72	0.75	0.75	0.68	82	0.36	0.41	0.41	0.41	0.37
27	0.55	0.78	0.84	0.85	0.77	83	0.47	0.58	0.64	0.63	0.63
28	0.55	0.68	0.73	0.72	0.70	84	0.25	0.45	0.48	0.48	0.43
29	0.51	0.63	0.65	0.67	0.65	85	0.32	0.41	0.44	0.47	0.41
30	0.61	0.72	0.75	0.74	0.67	86	0.46	0.61	0.65	0.68	0.66
31	0.72	0.78	0.79	0.79	0.78	87	0.63	0.63	0.62	0.62	0.60
32	0.51	0.63	0.67	0.68	0.65	88	0.54	0.67	0.72	0.76	0.80
33	0.64	0.79	0.81	0.80	0.71	89	0.45	0.54	0.58	0.60	0.60
34	0.62	0.72	0.74	0.74	0.71	90	0.44	0.53	0.54	0.53	0.50
35	0.59	0.70	0.72	0.72	0.69	91	0.62	0.70	0.72	0.71	0.64
36	0.59	0.70	0.74	0.75	0.67	92	0.35	0.52	0.55	0.54	0.50
37	0.74	0.80	0.81	0.81	0.78	93	0.45	0.54	0.56	0.55	0.52
38	0.64	0.72	0.73	0.73	0.70	94	0.56	0.69	0.72	0.71	0.64
39	0.56	0.62	0.63	0.61	0.54	95	0.54	0.66	0.68	0.69	0.66
40	0.69	0.74	0.76	0.76	0.74	96	0.55	0.55	0.55	0.57	0.57
41	0.64	0.72	0.73	0.73	0.69	97	0.61	0.62	0.63	0.64	0.63
42	0.66	0.76	0.75	0.71	0.56	98	0.39	0.45	0.47	0.46	0.45
43	0.51	0.69	0.70	0.66	0.48	99	0.51	0.68	0.70	0.68	0.60
44	0.45	0.60	0.62	0.60	0.47	100	0.73	0.77	0.79	0.81	0.84

Table A1.1 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
45	0.50	0.60	0.62	0.60	0.55	101	0.46	0.58	0.62	0.60	0.59
46	0.54	0.67	0.69	0.67	0.53	102	0.45	0.56	0.61	0.62	0.59
47	0.43	0.49	0.49	0.49	0.46	103	0.29	0.43	0.47	0.48	0.47
48	0.39	0.55	0.57	0.58	0.54	104	0.17	0.29	0.33	0.32	0.30
49	0.61	0.77	0.78	0.76	0.63	105	0.69	0.77	0.79	0.80	0.81
50	0.35	0.45	0.46	0.44	0.39	106	0.52	0.71	0.79	0.77	0.72
51	0.34	0.39	0.41	0.40	0.36	107	0.55	0.73	0.78	0.81	0.79
52	0.73	0.79	0.81	0.83	0.84	108	0.22	0.34	0.40	0.38	0.37
53	0.89	0.91	0.94	0.93	0.93	109	0.33	0.35	0.41	0.37	0.38
54	0.65	0.69	0.69	0.69	0.67	110	0.39	0.44	0.53	0.49	0.51
55	0.38	0.47	0.48	0.47	0.41	111	-0.05	-0.03	0.04	-0.02	-0.01
56	0.51	0.63	0.71	0.69	0.66	112	0.30	0.31	0.36	0.32	0.33

Table A1.2 Correlation between power output and heart rate for each session for rider 2 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.40	0.59	0.64	0.6	0.51	45	0.65	0.65	0.65	0.37	0.26
2	0.25	0.33	0.25	0.13	0.05	46	0.59	0.58	0.59	0.30	0.18
3	0.54	0.50	0.55	0.28	0.25	47	0.69	0.60	0.70	0.36	0.38
4	0.63	0.60	0.63	0.44	0.37	48	0.51	0.61	0.51	0.59	0.51
5	0.41	0.32	0.42	0.15	0.08	49	0.49	0.60	0.49	0.35	0.18
6	0.43	0.46	0.43	0.34	0.31	50	0.35	0.47	0.35	0.27	0.16
7	0.74	0.87	0.74	0.87	0.75	51	0.85	0.90	0.85	0.87	0.78
8	0.30	0.54	0.30	0.24	0.04	52	0.64	0.60	0.66	0.27	0.13
9	0.59	0.58	0.61	0.41	0.31	53	0.57	0.52	0.59	0.18	0.11
10	0.38	0.39	0.41	0.27	0.16	54	0.61	0.72	0.61	0.54	0.18
11	0.64	0.69	0.64	0.44	0.34	55	0.49	0.64	0.72	0.70	0.69
12	0.41	0.43	0.45	0.28	0.24	56	0.50	0.62	0.69	0.66	0.61
13	0.56	0.52	0.56	0.32	0.27	57	0.20	0.26	0.35	0.28	0.30
14	0.53	0.56	0.53	0.10	-0.07	58	0.46	0.62	0.67	0.71	0.75
15	0.47	0.54	0.47	0.26	0.13	59	0.56	0.69	0.73	0.76	0.79
16	0.82	0.91	0.82	0.87	0.79	60	0.37	0.53	0.55	0.53	0.47
17	0.51	0.50	0.52	0.29	0.12	61	0.28	0.46	0.59	0.56	0.60
18	0.47	0.56	0.47	0.26	0.19	62	0.39	0.57	0.62	0.68	0.72
19	0.48	0.52	0.48	0.21	0.09	63	0.46	0.57	0.59	0.58	0.52
20	0.50	0.51	0.53	0.24	0.12	64	0.42	0.55	0.58	0.58	0.53
21	0.42	0.45	0.46	0.31	0.24	65	0.61	0.73	0.74	0.74	0.66
22	0.19	0.28	0.19	0.07	0.12	66	0.46	0.67	0.79	0.76	0.74
23	0.48	0.46	0.49	0.25	0.18	67	0.64	0.75	0.77	0.75	0.68
24	0.30	0.42	0.30	0.26	0.16	68	0.53	0.63	0.68	0.66	0.64
25	0.46	0.48	0.46	0.38	0.32	69	0.36	0.58	0.65	0.69	0.70
26	0.44	0.48	0.44	0.21	0.11	70	0.61	0.75	0.79	0.78	0.75
27	-0.16	0.04	-0.16	0.04	-0.09	71	0.58	0.73	0.76	0.75	0.69
28	0.64	0.66	0.67	0.56	0.40	72	0.30	0.37	0.46	0.43	0.43
29	-0.23	-0.26	-0.23	-0.24	-0.25	73	0.32	0.54	0.57	0.55	0.47
30	0.58	0.58	0.58	0.36	0.27	74	0.53	0.67	0.68	0.66	0.57
31	0.21	0.25	0.27	-0.01	0.05	75	0.50	0.60	0.62	0.60	0.57
32	0.24	0.26	0.24	0.12	0.15	76	0.44	0.61	0.65	0.64	0.60
33	0.56	0.48	0.58	0.15	0.10	77	0.47	0.57	0.63	0.60	0.59
34	0.48	0.53	0.48	0.48	0.33	78	0.51	0.64	0.68	0.68	0.65
35	0.64	0.64	0.66	0.64	0.62	79	0.45	0.58	0.64	0.62	0.59
36	0.36	0.40	0.36	0.13	0.13	80	0.39	0.56	0.60	0.62	0.57
37	0.45	0.48	0.49	0.34	0.24	81	0.43	0.60	0.63	0.62	0.55
38	0.59	0.59	0.59	0.46	0.43	82	0.47	0.62	0.64	0.64	0.59
39	0.60	0.63	0.64	0.48	0.37	83	0.54	0.66	0.69	0.70	0.67
40	0.54	0.49	0.54	0.32	0.15	84	0.35	0.51	0.55	0.54	0.47
41	0.60	0.56	0.61	0.33	0.27	85	0.51	0.64	0.66	0.65	0.59
42	0.55	0.56	0.57	0.24	0.18	86	0.43	0.60	0.67	0.64	0.61
43	0.46	0.47	0.46	0.21	0.17	87	0.38	0.44	0.46	0.45	0.42
44	0.62	0.60	0.62	0.29	0.25	88	0.59	0.71	0.73	0.74	0.71

Table A1.3 Correlation between power output and heart rate for each session for rider 3 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.70	0.75	0.73	0.69	0.69	57	0.36	0.48	0.48	0.50	0.50
2	0.56	0.63	0.66	0.65	0.65	58	0.25	0.24	0.26	0.21	0.21
3	0.43	0.60	0.61	0.58	0.58	59	0.50	0.60	0.61	0.60	0.60
4	0.29	0.27	0.25	0.22	0.22	60	0.19	0.24	0.26	0.21	0.21
5	0.36	0.46	0.47	0.41	0.41	61	0.25	0.37	0.37	0.33	0.33
6	0.46	0.59	0.60	0.59	0.59	62	0.18	0.23	0.19	0.15	0.15
7	0.28	0.29	0.31	0.33	0.33	63	0.26	0.35	0.35	0.35	0.35
8	0.54	0.64	0.64	0.63	0.63	64	0.31	0.44	0.43	0.42	0.42
9	0.53	0.61	0.67	0.65	0.65	65	0.40	0.61	0.64	0.63	0.63
10	0.33	0.24	0.21	0.19	0.19	66	0.47	0.57	0.57	0.53	0.53
11	0.49	0.60	0.61	0.59	0.59	67	0.28	0.38	0.44	0.41	0.41
12	0.79	0.82	0.81	0.81	0.81	68	0.56	0.62	0.62	0.59	0.59
13	0.69	0.77	0.78	0.71	0.71	69	0.55	0.60	0.58	0.55	0.55
14	0.33	0.37	0.35	0.35	0.35	70	0.23	0.25	0.27	0.23	0.23
15	0.36	0.47	0.48	0.45	0.45	71	0.37	0.49	0.50	0.47	0.47
16	0.46	0.56	0.56	0.54	0.54	72	0.52	0.44	0.47	0.42	0.42
17	0.48	0.64	0.64	0.62	0.62	73	0.33	0.34	0.35	0.35	0.35
18	0.35	0.48	0.48	0.45	0.45	74	0.29	0.28	0.27	0.27	0.27
19	0.46	0.57	0.58	0.53	0.53	75	0.27	0.31	0.35	0.37	0.37
20	0.49	0.56	0.55	0.52	0.52	76	0.46	0.52	0.53	0.46	0.46
21	0.44	0.52	0.53	0.52	0.52	77	0.52	0.57	0.51	0.46	0.46
22	0.16	0.16	0.18	0.11	0.11	78	0.15	0.17	0.15	0.06	0.06
23	0.29	0.37	0.33	0.31	0.31	79	0.26	0.36	0.37	0.34	0.34
24	0.55	0.63	0.61	0.57	0.57	80	0.61	0.60	0.55	0.47	0.47
25	0.58	0.66	0.67	0.63	0.63	81	0.50	0.58	0.59	0.53	0.53
26	0.59	0.68	0.67	0.64	0.64	82	0.42	0.53	0.55	0.55	0.55
27	0.61	0.69	0.70	0.65	0.65	83	0.61	0.74	0.78	0.76	0.76
28	0.58	0.57	0.59	0.57	0.57	84	0.46	0.54	0.55	0.54	0.54
29	0.67	0.74	0.73	0.71	0.71	85	0.40	0.53	0.53	0.51	0.51
30	0.45	0.53	0.54	0.49	0.49	86	-0.05	-0.11	-0.09	-0.1	-0.10
31	0.53	0.59	0.57	0.57	0.57	87	0.41	0.52	0.54	0.47	0.47
32	0.41	0.55	0.56	0.54	0.54	88	0.41	0.52	0.53	0.50	0.50
33	0.51	0.66	0.66	0.63	0.63	89	0.62	0.72	0.78	0.80	0.80
34	0.37	0.49	0.50	0.49	0.49	90	0.35	0.33	0.35	0.30	0.30
35	0.28	0.37	0.36	0.34	0.34	91	0.25	0.31	0.33	0.32	0.32
36	0.53	0.60	0.58	0.55	0.55	92	0.81	0.84	0.85	0.84	0.84
37	0.24	0.11	0.07	0.05	0.05	93	0.05	-0.01	-0.01	0.02	0.02
38	0.36	0.48	0.49	0.47	0.47	94	0.34	0.41	0.42	0.40	0.40
39	0.47	0.59	0.56	0.52	0.52	95	0.51	0.60	0.59	0.57	0.57
40	0.28	0.31	0.35	0.33	0.33	96	0.40	0.43	0.44	0.44	0.44
41	0.49	0.56	0.54	0.49	0.49	97	0.23	0.30	0.32	0.33	0.33
42	0.28	0.38	0.40	0.38	0.38	98	0.37	0.38	0.40	0.38	0.38
43	0.36	0.52	0.54	0.48	0.48	99	0.13	0.07	0.10	0.08	0.08
44	0.58	0.65	0.64	0.60	0.60	100	0.35	0.48	0.46	0.41	0.41
45	0.50	0.53	0.54	0.53	0.53	101	0.60	0.69	0.69	0.67	0.67
46	0.25	0.29	0.31	0.28	0.28	102	0.33	0.35	0.36	0.34	0.34
47	0.28	0.40	0.38	0.38	0.38	103	0.11	0.09	0.13	0.11	0.11
48	0.55	0.65	0.68	0.70	0.70	104	0.37	0.46	0.44	0.40	0.40

Table A1.3. Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
49	0.41	0.53	0.53	0.52	0.52	105	0.48	0.53	0.54	0.51	0.51
50	0.27	0.28	0.29	0.33	0.33	106	0.43	0.44	0.46	0.44	0.44
51	0.72	0.78	0.78	0.76	0.76	107	0.44	0.44	0.44	0.42	0.42
52	0.33	0.40	0.41	0.40	0.40	108	0.30	0.36	0.32	0.35	0.35
53	0.45	0.51	0.49	0.48	0.48	109	0.64	0.69	0.72	0.71	0.71
54	0.57	0.64	0.64	0.63	0.63	110	0.33	0.32	0.36	0.29	0.29
55	0.63	0.72	0.73	0.71	0.71	111	0.54	0.63	0.63	0.61	0.61
56	0.37	0.47	0.47	0.48	0.48	112	0.51	0.55	0.48	0.45	0.45

Table A1.4 Correlation between power output and heart rate for each session for rider 5 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.08	0.10	0.11	0.09	0.07	52	0.67	0.66	0.68	0.66	0.65
2	0.45	0.58	0.61	0.58	0.54	53	0.15	0.11	0.09	0.09	0.11
3	0.24	0.33	0.36	0.35	0.30	54	0.26	0.28	0.27	0.26	0.24
4	0.31	0.43	0.45	0.44	0.42	55	0.33	0.35	0.37	0.37	0.34
5	0.54	0.61	0.59	0.56	0.52	56	0.56	0.59	0.61	0.58	0.57
6	0.24	0.33	0.33	0.34	0.31	57	0.39	0.41	0.39	0.38	0.36
7	0.28	0.40	0.42	0.39	0.33	58	0.36	0.39	0.39	0.38	0.36
8	0.36	0.44	0.45	0.46	0.42	59	0.18	0.19	0.21	0.17	0.17
9	0.09	0.17	0.22	0.20	0.20	60	0.56	0.58	0.58	0.57	0.49
10	0.19	0.25	0.27	0.26	0.24	61	0.38	0.42	0.43	0.43	0.40
11	0.18	0.23	0.23	0.22	0.22	62	-0.10	-0.08	-0.10	-0.10	-0.12
12	0.35	0.40	0.41	0.41	0.38	63	-0.11	-0.03	-0.10	-0.10	-0.02
13	0.11	0.18	0.12	0.12	0.07	64	0.17	0.15	0.20	0.13	0.10
14	0.16	0.24	0.27	0.27	0.24	65	0.24	0.31	0.33	0.28	0.24
15	0.22	0.33	0.36	0.38	0.38	66	0.08	0.16	0.17	0.14	0.14
16	0.19	0.19	0.16	0.11	0.08	67	0.13	0.14	0.15	0.13	0.10
17	0.07	0.14	0.17	0.21	0.22	68	0.29	0.26	0.27	0.32	0.28
18	0.25	0.33	0.36	0.34	0.34	69	0.11	0.10	0.13	0.08	0.10
19	0.14	0.25	0.27	0.26	0.21	70	0.15	0.20	0.23	0.20	0.21
20	0.07	0.12	0.11	0.11	0.15	71	0.09	0.11	0.10	0.08	0.03
21	0.17	0.23	0.24	0.19	0.21	72	0.86	0.87	0.88	0.88	0.89
22	0.25	0.36	0.38	0.38	0.33	73	0.02	0.05	0.09	0.04	0.02
23	0.08	0.15	0.17	0.18	0.16	74	0.23	0.26	0.27	0.27	0.26
24	0.28	0.31	0.33	0.29	0.24	75	0.06	0.09	0.03	0.01	0.04
25	0.33	0.39	0.39	0.40	0.35	76	0.19	0.21	0.23	0.20	0.16
26	0.15	0.21	0.25	0.26	0.21	77	0.13	0.18	0.17	0.16	0.15
27	0.33	0.38	0.37	0.36	0.29	78	0.19	0.29	0.27	0.27	0.23
28	0.12	0.21	0.22	0.23	0.22	79	0.39	0.65	0.56	0.50	0.40
29	0.29	0.36	0.39	0.37	0.35	80	0.16	0.24	0.21	0.21	0.18
30	0.14	0.23	0.25	0.26	0.26	81	0.49	0.64	0.65	0.57	0.46
31	0.29	0.38	0.39	0.37	0.36	82	0.46	0.66	0.63	0.59	0.45
32	0.35	0.43	0.46	0.43	0.37	83	0.20	0.49	0.51	0.45	0.19
33	0.13	0.22	0.25	0.25	0.27	84	0.38	0.52	0.54	0.47	0.34
34	0.12	0.10	0.07	0.08	0.06	85	0.41	0.58	0.63	0.60	0.52
35	0.25	0.31	0.33	0.31	0.26	86	0.49	0.61	0.62	0.61	0.58
36	0.28	0.32	0.33	0.34	0.32	87	0.45	0.59	0.58	0.56	0.42
37	0.42	0.46	0.47	0.47	0.45	88	0.63	0.71	0.73	0.67	0.59
38	0.14	0.20	0.17	0.15	0.16	89	0.36	0.45	0.47	0.46	0.42
39	0.27	0.33	0.34	0.33	0.31	90	0.48	0.62	0.64	0.63	0.58
40	0.21	0.30	0.31	0.30	0.29	91	0.44	0.48	0.50	0.47	0.44
41	0.25	0.34	0.36	0.36	0.31	92	0.40	0.43	0.47	0.42	0.41
42	0.11	0.12	0.11	0.09	0.11	93	0.33	0.48	0.47	0.42	0.30
43	-0.03	0.08	0.10	0.07	0.03	94	0.62	0.67	0.68	0.67	0.64
44	0.24	0.25	0.25	0.26	0.27	95	0.13	0.23	0.25	0.19	0.11
45	0.28	0.36	0.38	0.28	0.26	96	0.46	0.55	0.57	0.55	0.40
46	0.29	0.34	0.35	0.36	0.37	97	0.54	0.67	0.68	0.66	0.59
47	0.24	0.32	0.33	0.32	0.28	98	0.31	0.50	0.52	0.51	0.41
48	0.22	0.28	0.27	0.26	0.24	99	0.50	0.59	0.61	0.60	0.55

Table A1.4 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
49	0.20	0.31	0.31	0.29	0.23	100	0.55	0.66	0.64	0.59	0.47
50	0.44	0.49	0.49	0.46	0.44	101	0.61	0.66	0.66	0.65	0.63
51	0.18	0.28	0.31	0.29	0.26						

Table A1.5 Correlation between power output and heart rate for each session for rider 6 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.69	0.78	0.78	0.76	0.71	74	0.70	0.62	0.47	0.41	0.24
2	0.70	0.75	0.77	0.71	0.68	75	0.77	0.57	0.32	0.12	-0.02
3	0.46	0.60	0.63	0.63	0.57	76	0.61	0.59	0.49	0.40	0.33
4	0.25	0.35	0.40	0.38	0.37	77	0.63	0.60	0.41	0.36	0.39
5	0.36	0.49	0.48	0.45	0.36	78	0.50	0.52	0.32	0.08	-0.10
6	0.53	0.66	0.68	0.68	0.63	79	0.65	0.57	0.44	0.38	0.23
7	0.48	0.62	0.63	0.62	0.56	80	0.63	0.48	0.24	0.12	0.05
8	0.55	0.65	0.65	0.63	0.59	81	0.23	0.27	0.29	0.29	0.25
9	0.97	0.96	0.92	0.88	0.79	82	0.60	0.28	0.11	-0.02	-0.05
10	0.61	0.70	0.71	0.71	0.67	83	0.34	0.30	0.36	0.26	0.14
11	0.51	0.62	0.63	0.63	0.56	84	0.57	0.63	0.45	0.32	0.20
12	0.50	0.62	0.64	0.61	0.51	85	0.64	0.58	0.46	0.39	0.32
13	0.95	0.96	0.93	0.90	0.83	86	0.59	0.49	0.34	0.23	0.13
14	0.18	0.22	0.07	0.04	0.05	87	0.58	0.59	0.59	0.45	0.38
15	0.52	0.64	0.66	0.64	0.56	88	0.67	0.64	0.67	0.53	0.47
16	0.54	0.59	0.59	0.53	0.47	89	0.68	0.57	0.43	0.32	0.22
17	0.52	0.62	0.64	0.60	0.49	90	0.61	0.58	0.62	0.50	0.46
18	0.55	0.67	0.67	0.62	0.50	91	0.63	0.57	0.49	0.44	0.45
19	0.24	0.33	0.19	0.31	0.26	92	0.63	0.60	0.65	0.43	0.29
20	0.30	0.35	0.37	0.34	0.28	93	0.78	0.51	0.25	0.11	0.05
21	0.54	0.65	0.66	0.64	0.59	94	0.63	0.56	0.50	0.38	0.23
22	0.53	0.66	0.59	0.51	0.40	95	0.56	0.53	0.44	0.36	0.32
23	0.48	0.63	0.65	0.62	0.53	96	0.68	0.68	0.69	0.67	0.63
24	0.50	0.63	0.65	0.63	0.54	97	0.63	0.61	0.46	0.34	0.18
25	0.54	0.68	0.69	0.68	0.62	98	0.45	0.31	0.13	-0.02	-0.04
26	0.52	0.66	0.69	0.68	0.62	99	0.55	0.58	0.49	0.43	0.33
27	0.53	0.46	0.35	0.27	0.22	100	0.72	0.43	0.23	0.03	-0.13
28	0.43	0.45	0.31	0.19	0.20	101	0.58	0.44	0.21	0.09	0.07
29	0.57	0.57	0.39	0.29	0.17	102	0.64	0.60	0.41	0.29	0.17
30	0.57	0.59	0.48	0.34	0.25	103	0.54	0.44	0.23	0.13	0.07
31	0.75	0.66	0.51	0.41	0.34	104	0.41	0.51	0.27	0.17	0.09
32	0.64	0.57	0.47	0.28	0.15	105	0.62	0.60	0.64	0.52	0.46
33	0.33	0.48	0.28	0.21	0.21	106	0.54	0.44	0.23	0.13	0.07
34	0.62	0.56	0.43	0.32	0.07	107	0.60	0.46	0.24	0.18	0.22
35	0.56	0.56	0.39	0.23	0.14	108	0.70	0.64	0.42	0.23	0.10
36	0.54	0.58	0.46	0.43	0.36	109	0.68	0.70	0.72	0.64	0.51
37	0.39	0.43	0.24	0.14	-0.02	110	0.40	0.50	0.51	0.50	0.46
38	0.49	0.54	0.55	0.31	0.18	111	0.43	0.57	0.58	0.57	0.51
39	0.57	0.54	0.41	0.33	0.27	112	0.45	0.59	0.60	0.57	0.48
40	0.55	0.57	0.39	0.26	0.18	113	0.37	0.51	0.51	0.48	0.41
41	0.54	0.50	0.57	0.35	0.25	114	0.59	0.72	0.74	0.68	0.55
42	0.42	0.41	0.46	0.28	0.21	115	0.68	0.72	0.73	0.68	0.63
43	0.55	0.59	0.47	0.35	0.26	116	0.54	0.62	0.62	0.62	0.59
44	0.53	0.56	0.40	0.35	0.33	117	0.41	0.52	0.53	0.51	0.37

Table A1.5 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
45	0.57	0.60	0.44	0.33	0.19	118	0.61	0.71	0.72	0.70	0.62
46	0.60	0.59	0.46	0.33	0.21	119	0.56	0.62	0.63	0.62	0.61
47	0.65	0.56	0.50	0.51	0.37	120	0.49	0.60	0.61	0.59	0.53
48	0.53	0.42	0.24	0.05	0.02	121	0.45	0.57	0.59	0.58	0.51
49	0.59	0.55	0.44	0.33	0.22	122	0.42	0.62	0.63	0.59	0.40
50	0.50	0.46	0.32	0.23	0.23	123	0.39	0.59	0.61	0.58	0.44
51	0.72	0.62	0.48	0.40	0.26	124	0.58	0.68	0.69	0.64	0.53
52	0.12	0.07	0.13	0.02	0.03	125	0.52	0.63	0.65	0.65	0.61
53	0.62	0.56	0.49	0.40	0.28	126	0.61	0.72	0.73	0.72	0.67
54	0.47	0.48	0.26	0.12	-0.16	127	0.55	0.62	0.62	0.60	0.53
55	0.58	0.61	0.51	0.43	0.25	128	0.48	0.55	0.56	0.53	0.48
56	0.55	0.41	0.23	0.09	-0.09	129	0.44	0.59	0.62	0.62	0.57
57	0.67	0.63	0.52	0.41	0.23	130	0.33	0.49	0.54	0.52	0.48
58	0.71	0.43	0.19	-0.01	-0.12	131	0.50	0.59	0.59	0.58	0.53
59	0.63	0.60	0.44	0.30	0.15	132	0.63	0.71	0.71	0.70	0.67
60	0.63	0.60	0.47	0.37	0.32	133	0.60	0.72	0.73	0.71	0.63
61	0.63	0.60	0.44	0.35	0.25	134	0.37	0.51	0.53	0.50	0.44
62	0.68	0.57	0.42	0.31	0.25	135	0.38	0.55	0.56	0.53	0.44
63	0.49	0.55	0.45	0.36	0.26	136	0.50	0.61	0.62	0.60	0.52
64	0.33	0.38	0.42	0.31	0.22	137	0.44	0.57	0.58	0.55	0.49
65	0.33	0.38	0.38	0.37	0.28	138	0.48	0.59	0.60	0.58	0.53
66	0.30	0.34	0.39	0.30	0.23	139	0.49	0.61	0.63	0.59	0.49
67	0.43	0.52	0.43	0.36	0.20	140	0.59	0.66	0.67	0.63	0.57
68	0.58	0.47	0.26	0.05	-0.09	141	0.48	0.57	0.57	0.55	0.50
69	0.59	0.59	0.44	0.32	0.21	142	0.54	0.66	0.67	0.63	0.49
70	0.71	0.69	0.57	0.42	0.31	143	0.57	0.67	0.68	0.66	0.57
71	0.67	0.62	0.49	0.40	0.28	144	0.45	0.60	0.61	0.52	0.35
72	0.78	0.70	0.59	0.46	0.21	145	0.63	0.75	0.75	0.72	0.65
73	0.69	0.66	0.56	0.48	0.43	146	0.75	0.77	0.78	0.77	0.75

Table A1.6 Correlation between power output and heart rate for each session for rider 7 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.51	0.64	0.65	0.64	0.60	77	0.65	0.72	0.72	0.72	0.68
2	0.52	0.62	0.64	0.66	0.70	78	0.53	0.62	0.64	0.62	0.59
3	0.56	0.64	0.66	0.60	0.56	79	0.69	0.76	0.79	0.81	0.83
4	0.67	0.71	0.78	0.74	0.77	80	0.30	0.38	0.42	0.44	0.47
5	0.54	0.57	0.58	0.59	0.61	81	0.32	0.54	0.55	0.53	0.47
6	0.60	0.64	0.67	0.68	0.71	82	0.78	0.85	0.87	0.89	0.91
7	0.29	0.45	0.48	0.48	0.44	83	0.69	0.75	0.76	0.75	0.73
8	0.75	0.77	0.79	0.76	0.75	84	0.61	0.69	0.71	0.70	0.67
9	0.51	0.61	0.61	0.59	0.53	85	0.63	0.72	0.76	0.79	0.83
10	0.4	0.43	0.49	0.46	0.48	86	0.76	0.80	0.82	0.83	0.85
11	0.64	0.69	0.74	0.71	0.72	87	0.65	0.70	0.72	0.74	0.76
12	0.86	0.87	0.88	0.88	0.86	88	0.69	0.74	0.75	0.74	0.71
13	0.57	0.67	0.67	0.66	0.62	89	0.73	0.79	0.82	0.83	0.86
14	0.61	0.66	0.69	0.71	0.73	90	0.60	0.70	0.65	0.57	0.44
15	0.55	0.55	0.57	0.52	0.46	91	0.60	0.65	0.66	0.66	0.65
16	0.52	0.63	0.65	0.58	0.49	92	0.75	0.78	0.78	0.77	0.74
17	0.37	0.51	0.53	0.50	0.42	93	0.70	0.73	0.73	0.73	0.71
18	0.54	0.61	0.61	0.60	0.55	94	0.52	0.57	0.57	0.56	0.52
19	0.44	0.56	0.62	0.60	0.58	95	0.54	0.64	0.67	0.70	0.73
20	0.36	0.39	0.42	0.40	0.40	96	0.77	0.84	0.86	0.85	0.83
21	0.59	0.66	0.66	0.64	0.60	97	0.70	0.74	0.74	0.74	0.72
22	0.34	0.52	0.54	0.53	0.43	98	0.60	0.69	0.72	0.75	0.79
23	0.48	0.54	0.54	0.53	0.53	99	0.73	0.77	0.79	0.79	0.78
24	0.79	0.81	0.83	0.82	0.83	100	0.58	0.64	0.65	0.64	0.65
25	0.34	0.47	0.49	0.49	0.45	101	0.68	0.72	0.73	0.73	0.72
26	0.39	0.45	0.45	0.45	0.40	102	0.66	0.74	0.76	0.79	0.80
27	0.53	0.64	0.64	0.63	0.59	103	0.78	0.83	0.86	0.86	0.88
28	0.62	0.70	0.67	0.64	0.52	104	0.53	0.65	0.70	0.74	0.78
29	0.65	0.69	0.72	0.72	0.73	105	0.53	0.64	0.65	0.63	0.59
30	0.39	0.21	0.31	0.25	0.21	106	0.67	0.73	0.74	0.73	0.70
31	0.55	0.65	0.67	0.63	0.58	107	0.97	0.96	0.97	0.95	0.92
32	0.44	0.50	0.52	0.53	0.54	108	0.6	0.69	0.72	0.74	0.77
33	0.57	0.67	0.67	0.66	0.61	109	0.61	0.70	0.72	0.69	0.64
34	0.60	0.71	0.71	0.68	0.58	110	0.72	0.77	0.78	0.77	0.74
35	0.48	0.51	0.56	0.48	0.44	111	0.41	0.48	0.50	0.48	0.44
36	0.58	0.68	0.70	0.69	0.64	112	0.70	0.75	0.76	0.76	0.74
37	0.61	0.70	0.72	0.71	0.67	113	0.87	0.90	0.91	0.90	0.88
38	0.64	0.72	0.74	0.73	0.69	114	0.58	0.67	0.71	0.73	0.77
39	0.61	0.69	0.72	0.74	0.78	115	0.67	0.73	0.74	0.74	0.72
40	0.53	0.62	0.63	0.61	0.56	116	0.55	0.64	0.68	0.70	0.72
41	0.48	0.55	0.56	0.55	0.51	117	0.75	0.78	0.79	0.79	0.76
42	0.65	0.66	0.68	0.65	0.58	118	0.71	0.76	0.76	0.76	0.72
43	0.69	0.76	0.77	0.73	0.66	119	0.73	0.77	0.77	0.77	0.75
44	0.54	0.55	0.62	0.53	0.50	120	0.67	0.80	0.83	0.77	0.58
45	0.38	0.55	0.56	0.52	0.46	121	0.72	0.76	0.76	0.76	0.74
46	0.59	0.64	0.63	0.64	0.63	122	0.81	0.85	0.85	0.85	0.83
47	0.37	0.52	0.55	0.55	0.50	123	0.75	0.78	0.78	0.76	0.73
48	0.60	0.64	0.66	0.65	0.65	124	0.54	0.66	0.66	0.60	0.46

Table A1.6 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
49	0.29	0.47	0.45	0.43	0.42	125	0.68	0.76	0.76	0.80	0.81
50	0.52	0.60	0.61	0.60	0.59	126	0.29	0.41	0.44	0.16	-0.01
51	0.23	0.48	0.50	0.46	0.39	127	0.74	0.80	0.82	0.78	0.73
52	0.56	0.57	0.55	0.50	0.41	128	-0.06	-0.03	-0.02	0.07	0.06
53	0.44	0.56	0.59	0.59	0.56	129	0.54	0.64	0.66	0.52	0.35
54	0.31	0.50	0.52	0.47	0.43	130	0.57	0.67	0.67	0.62	0.51
55	0.58	0.63	0.57	0.50	0.35	131	0.68	0.75	0.75	0.73	0.63
56	0.62	0.70	0.72	0.69	0.67	132	0.61	0.73	0.73	0.60	0.35
57	0.64	0.73	0.75	0.72	0.68	133	0.18	0.46	0.48	0.25	0.01
58	0.69	0.75	0.75	0.72	0.67	134	0.33	0.48	0.48	0.28	0.13
59	0.76	0.80	0.81	0.83	0.85	135	0.67	0.73	0.73	0.66	0.56
60	0.42	0.55	0.59	0.58	0.56	136	0.43	0.55	0.56	0.46	0.29
61	0.31	0.46	0.53	0.51	0.5	137	0.70	0.74	0.74	0.68	0.61
62	0.54	0.65	0.62	0.58	0.47	138	0.60	0.67	0.67	0.58	0.48
63	0.75	0.76	0.75	0.75	0.73	139	0.38	0.56	0.56	0.51	0.44
64	0.47	0.52	0.53	0.53	0.53	140	0.57	0.39	0.39	0.24	0.14
65	0.71	0.77	0.85	0.81	0.84	141	0.62	0.71	0.73	0.61	0.46
66	0.46	0.59	0.58	0.57	0.51	142	0.49	0.68	0.67	0.56	0.32
67	0.28	0.48	0.53	0.55	0.55	143	0.39	0.47	0.49	0.31	0.05
68	0.50	0.58	0.58	0.55	0.54	144	0.48	0.52	0.52	0.40	0.21
69	0.37	0.43	0.45	0.45	0.43	145	0.52	0.56	0.57	0.48	0.33
70	0.58	0.71	0.76	0.79	0.81	146	0.66	0.71	0.71	0.64	0.58
71	0.52	0.63	0.58	0.51	0.41	147	0.53	0.61	0.61	0.48	0.35
72	0.50	0.54	0.56	0.56	0.55	148	0.58	0.59	0.60	0.44	0.21
73	0.75	0.82	0.89	0.86	0.88	149	0.46	0.41	0.46	0.28	0.2
74	0.74	0.80	0.82	0.77	0.70	150	0.42	0.50	0.52	0.34	0.25
75	0.44	0.50	0.52	0.52	0.50	151	0.58	0.65	0.65	0.60	0.53
76	0.67	0.72	0.75	0.74	0.76	152	0.80	0.84	0.86	0.81	0.75

Table A1.7 Correlation between power output and heart rate for each session for rider 8 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.73	0.80	0.79	0.76	0.71	82	0.65	0.76	0.76	0.74	0.67
2	0.37	0.44	0.43	0.41	0.37	83	0.76	0.82	0.85	0.87	0.90
3	0.58	0.71	0.71	0.69	0.61	84	0.03	0.09	0.12	0.09	0.08
4	0.54	0.60	0.59	0.57	0.51	85	0.41	0.48	0.51	0.50	0.42
5	0.69	0.77	0.76	0.73	0.64	86	0.31	0.39	0.38	0.39	0.37
6	0.50	0.60	0.58	0.56	0.50	87	0.27	0.33	0.34	0.32	0.31
7	0.64	0.76	0.76	0.75	0.67	88	0.67	0.78	0.78	0.74	0.63
8	0.69	0.79	0.81	0.75	0.63	89	0.65	0.74	0.75	0.73	0.62
9	0.73	0.83	0.84	0.79	0.69	90	0.67	0.78	0.79	0.77	0.69
10	0.63	0.72	0.74	0.67	0.60	91	0.73	0.81	0.82	0.81	0.76
11	0.68	0.78	0.77	0.74	0.65	92	0.68	0.77	0.80	0.83	0.86
12	0.66	0.77	0.78	0.72	0.63	93	0.70	0.78	0.78	0.74	0.63
13	0.62	0.73	0.73	0.70	0.63	94	0.64	0.75	0.78	0.81	0.84
14	0.67	0.76	0.75	0.72	0.64	95	0.75	0.82	0.82	0.81	0.78
15	0.63	0.73	0.74	0.72	0.68	96	0.64	0.76	0.76	0.74	0.67
16	0.63	0.73	0.73	0.71	0.63	97	0.75	0.82	0.82	0.79	0.70
17	0.60	0.69	0.69	0.67	0.62	98	0.63	0.69	0.68	0.67	0.64
18	0.68	0.76	0.76	0.73	0.65	99	0.63	0.72	0.73	0.72	0.69
19	0.63	0.74	0.74	0.72	0.66	100	0.75	0.83	0.84	0.79	0.69
20	0.70	0.78	0.78	0.75	0.68	101	0.43	0.51	0.53	0.54	0.55
21	0.77	0.79	0.84	0.82	0.81	102	0.57	0.68	0.72	0.76	0.79
22	0.72	0.82	0.83	0.78	0.68	103	0.70	0.79	0.79	0.76	0.68
23	0.68	0.76	0.76	0.74	0.68	104	0.31	0.38	0.38	0.36	0.29
24	0.62	0.71	0.73	0.69	0.64	105	0.61	0.72	0.74	0.74	0.68
25	0.54	0.66	0.64	0.63	0.57	106	0.29	0.37	0.43	0.41	0.37
26	0.73	0.75	0.75	0.75	0.74	107	0.49	0.61	0.62	0.59	0.50
27	0.57	0.66	0.66	0.64	0.55	108	0.67	0.73	0.73	0.70	0.63
28	0.70	0.80	0.81	0.76	0.68	109	0.6	0.70	0.72	0.68	0.62
29	0.60	0.70	0.70	0.68	0.59	110	0.73	0.82	0.81	0.77	0.65
30	0.72	0.79	0.79	0.77	0.69	111	0.75	0.81	0.83	0.78	0.69
31	0.70	0.79	0.78	0.75	0.67	112	0.51	0.66	0.68	0.68	0.67
32	0.66	0.76	0.75	0.72	0.63	113	0.63	0.75	0.75	0.72	0.63
33	0.64	0.75	0.74	0.72	0.65	114	0.60	0.72	0.74	0.74	0.70
34	0.66	0.75	0.74	0.71	0.62	115	0.76	0.83	0.83	0.80	0.71
35	0.67	0.76	0.75	0.74	0.67	116	0.65	0.75	0.76	0.75	0.70
36	0.70	0.81	0.81	0.78	0.67	117	0.71	0.82	0.82	0.79	0.70
37	0.70	0.78	0.78	0.76	0.68	118	0.16	0.20	0.21	0.18	0.17
38	0.64	0.75	0.76	0.71	0.61	119	0.74	0.82	0.84	0.77	0.67
39	0.62	0.72	0.75	0.69	0.60	120	0.46	0.54	0.55	0.54	0.49
40	0.72	0.81	0.83	0.77	0.67	121	0.68	0.76	0.76	0.77	0.76
41	0.68	0.77	0.77	0.75	0.68	122	0.64	0.71	0.72	0.70	0.66
42	0.68	0.76	0.76	0.73	0.65	123	0.45	0.50	0.49	0.49	0.45
43	0.71	0.80	0.82	0.77	0.68	124	0.57	0.69	0.72	0.72	0.69
44	0.71	0.77	0.79	0.81	0.83	125	0.65	0.74	0.74	0.73	0.67
45	0.73	0.83	0.83	0.81	0.72	126	0.57	0.64	0.64	0.60	0.51
46	0.84	0.88	0.90	0.88	0.87	127	0.56	0.72	0.72	0.69	0.63
47	0.80	0.86	0.88	0.85	0.81	128	0.71	0.80	0.83	0.77	0.70
48	0.86	0.89	0.92	0.90	0.88	129	0.53	0.49	0.49	0.45	0.39

Table A1.7 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
49	0.69	0.73	0.73	0.73	0.73	130	0.54	0.62	0.64	0.62	0.55
50	0.84	0.86	0.87	0.87	0.85	131	0.69	0.75	0.75	0.72	0.67
51	0.88	0.90	0.91	0.90	0.89	132	0.51	0.59	0.58	0.56	0.48
52	0.84	0.88	0.88	0.88	0.85	133	0.64	0.74	0.75	0.73	0.67
53	0.84	0.87	0.88	0.89	0.90	134	0.71	0.79	0.81	0.80	0.75
54	0.76	0.84	0.86	0.83	0.77	135	0.70	0.79	0.79	0.77	0.70
55	0.39	0.48	0.49	0.47	0.42	136	0.73	0.81	0.83	0.77	0.68
56	0.71	0.78	0.81	0.82	0.84	137	0.71	0.79	0.80	0.80	0.75
57	0.71	0.81	0.83	0.79	0.69	138	0.66	0.76	0.76	0.74	0.63
58	0.74	0.80	0.82	0.83	0.86	139	0.39	0.48	0.49	0.50	0.48
59	0.72	0.81	0.83	0.78	0.71	140	0.59	0.67	0.67	0.65	0.59
60	0.67	0.79	0.79	0.76	0.65	141	0.71	0.77	0.78	0.76	0.66
61	-0.04	0.00	-0.01	-0.03	-0.09	142	0.58	0.65	0.65	0.63	0.56
62	0.67	0.77	0.78	0.76	0.69	143	0.51	0.56	0.56	0.55	0.49
63	0.68	0.74	0.76	0.78	0.81	144	0.62	0.70	0.73	0.70	0.62
64	0.62	0.70	0.72	0.67	0.62	145	0.76	0.84	0.83	0.80	0.71
65	0.58	0.68	0.68	0.67	0.64	146	0.63	0.71	0.71	0.70	0.62
66	0.70	0.81	0.83	0.77	0.68	147	0.48	0.57	0.57	0.56	0.49
67	0.67	0.78	0.77	0.75	0.64	148	0.69	0.77	0.76	0.74	0.68
68	0.62	0.70	0.71	0.70	0.68	149	0.83	0.88	0.91	0.88	0.85
69	0.70	0.80	0.79	0.77	0.67	150	0.87	0.91	0.91	0.91	0.89
70	0.66	0.73	0.75	0.71	0.68	151	0.86	0.89	0.89	0.89	0.87
71	0.52	0.58	0.58	0.57	0.51	152	0.86	0.91	0.92	0.92	0.88
72	0.65	0.77	0.79	0.74	0.63	153	0.88	0.90	0.91	0.90	0.88
73	0.63	0.70	0.73	0.68	0.61	154	0.88	0.90	0.92	0.90	0.88
74	0.66	0.75	0.74	0.72	0.64	155	0.72	0.73	0.74	0.73	0.71
75	0.83	0.88	0.89	0.89	0.86	156	0.09	0.19	0.21	0.22	0.20
76	0.85	0.88	0.89	0.89	0.87	157	0.70	0.79	0.80	0.77	0.69
77	0.48	0.56	0.56	0.53	0.50	158	0.72	0.81	0.81	0.80	0.75
78	0.67	0.73	0.73	0.72	0.68	159	0.57	0.69	0.71	0.68	0.59
79	0.32	0.37	0.39	0.36	0.34	160	0.76	0.82	0.84	0.86	0.88
80	0.46	0.55	0.56	0.53	0.49	161	0.91	0.92	0.93	0.93	0.94
81	0.09	0.15	0.17	0.14	0.10	162	0.69	0.76	0.76	0.75	0.68

Table A1.8 Correlation between power output and heart rate for each session for rider 9 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.60	0.70	0.71	0.66	0.60	100	0.54	0.62	0.62	0.60	0.50
2	0.49	0.57	0.53	0.50	0.41	101	0.52	0.63	0.63	0.62	0.56
3	0.50	0.58	0.59	0.53	0.46	102	0.61	0.73	0.73	0.71	0.63
4	0.41	0.46	0.43	0.40	0.37	103	0.61	0.71	0.72	0.70	0.64
5	0.54	0.64	0.64	0.62	0.57	104	0.62	0.70	0.67	0.62	0.49
6	0.39	0.45	0.46	0.40	0.36	105	0.73	0.75	0.75	0.75	0.74
7	0.43	0.57	0.58	0.58	0.52	106	0.44	0.54	0.55	0.54	0.49
8	0.50	0.61	0.61	0.53	0.40	107	0.50	0.59	0.56	0.50	0.38
9	0.43	0.52	0.54	0.50	0.45	108	0.52	0.65	0.66	0.60	0.49
10	0.56	0.67	0.67	0.65	0.58	109	0.52	0.64	0.63	0.61	0.53
11	0.53	0.67	0.68	0.63	0.53	110	0.49	0.64	0.64	0.59	0.48
12	0.50	0.61	0.61	0.58	0.51	111	0.70	0.76	0.72	0.68	0.55
13	0.49	0.63	0.64	0.62	0.55	112	0.43	0.44	0.46	0.43	0.40
14	0.49	0.62	0.63	0.62	0.55	113	0.47	0.57	0.56	0.54	0.47
15	0.48	0.61	0.63	0.58	0.48	114	0.62	0.66	0.65	0.63	0.59
16	0.55	0.69	0.70	0.68	0.58	115	0.87	0.88	0.89	0.88	0.87
17	0.92	0.94	0.94	0.94	0.93	116	0.54	0.68	0.68	0.66	0.58
18	0.40	0.51	0.51	0.46	0.36	117	0.71	0.70	0.73	0.69	0.69
19	0.49	0.62	0.63	0.57	0.46	118	0.77	0.78	0.78	0.77	0.76
20	0.93	0.95	0.95	0.95	0.94	119	0.50	0.58	0.58	0.57	0.53
21	0.42	0.60	0.61	0.58	0.49	120	0.49	0.67	0.66	0.63	0.53
22	0.56	0.70	0.72	0.70	0.61	121	0.55	0.70	0.72	0.67	0.56
23	0.41	0.52	0.53	0.47	0.39	122	0.52	0.64	0.64	0.61	0.52
24	0.48	0.61	0.61	0.58	0.50	123	0.60	0.71	0.73	0.68	0.59
25	0.52	0.67	0.68	0.62	0.49	124	0.70	0.72	0.72	0.72	0.69
26	0.51	0.58	0.55	0.51	0.43	125	0.68	0.70	0.71	0.69	0.66
27	0.40	0.52	0.54	0.47	0.38	126	0.46	0.52	0.52	0.52	0.45
28	0.52	0.68	0.67	0.64	0.51	127	0.65	0.73	0.71	0.66	0.55
29	0.91	0.93	0.93	0.93	0.92	128	0.58	0.70	0.71	0.70	0.62
30	0.54	0.66	0.65	0.62	0.51	129	0.47	0.62	0.61	0.58	0.49
31	0.88	0.91	0.93	0.90	0.89	130	0.52	0.64	0.62	0.58	0.46
32	0.54	0.67	0.67	0.64	0.55	131	0.54	0.65	0.65	0.64	0.58
33	0.88	0.90	0.91	0.90	0.89	132	0.40	0.54	0.55	0.51	0.41
34	0.61	0.75	0.74	0.71	0.59	133	0.03	0.02	0.07	0.02	0.04
35	0.70	0.78	0.77	0.74	0.65	134	0.55	0.65	0.66	0.66	0.61
36	0.84	0.84	0.84	0.82	0.81	135	0.13	0.17	0.17	0.16	0.09
37	0.55	0.67	0.65	0.61	0.50	136	0.72	0.73	0.72	0.71	0.69
38	0.59	0.70	0.68	0.63	0.53	137	0.42	0.49	0.48	0.46	0.45
39	0.47	0.59	0.57	0.52	0.39	138	0.37	0.38	0.39	0.38	0.37
40	0.57	0.65	0.66	0.62	0.53	139	0.22	0.27	0.29	0.21	0.13
41	0.66	0.75	0.74	0.70	0.61	140	0.47	0.59	0.59	0.58	0.49
42	0.51	0.57	0.52	0.47	0.38	141	0.29	0.38	0.35	0.29	0.20
43	0.49	0.58	0.55	0.50	0.42	142	0.71	0.79	0.80	0.76	0.68
44	0.62	0.72	0.74	0.67	0.58	143	0.57	0.65	0.65	0.63	0.59
45	0.90	0.90	0.91	0.91	0.91	144	0.62	0.70	0.69	0.67	0.62
46	0.80	0.83	0.84	0.83	0.83	145	0.08	0.14	0.18	0.16	0.12
47	0.77	0.75	0.74	0.74	0.73	146	0.13	0.25	0.26	0.26	0.23
48	0.38	0.50	0.52	0.48	0.40	147	0.59	0.73	0.74	0.71	0.61

Table A1.8 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
49	0.44	0.59	0.58	0.54	0.45	148	0.10	0.16	0.14	0.12	0.07
50	0.50	0.66	0.63	0.57	0.45	149	0.60	0.74	0.76	0.74	0.65
51	0.78	0.85	0.85	0.84	0.83	150	0.45	0.52	0.52	0.52	0.49
52	0.42	0.51	0.47	0.44	0.38	151	0.53	0.62	0.59	0.56	0.45
53	0.45	0.54	0.50	0.46	0.36	152	0.59	0.68	0.66	0.64	0.59
54	0.55	0.67	0.64	0.60	0.50	153	0.38	0.45	0.46	0.44	0.38
55	0.49	0.60	0.61	0.57	0.47	154	0.61	0.68	0.65	0.61	0.48
56	0.85	0.83	0.82	0.82	0.81	155	0.20	0.27	0.22	0.15	0.21
57	0.55	0.66	0.66	0.63	0.55	156	0.56	0.65	0.66	0.65	0.58
58	0.88	0.91	0.92	0.92	0.91	157	0.74	0.81	0.85	0.84	0.85
59	0.91	0.91	0.89	0.88	0.86	158	0.56	0.66	0.67	0.66	0.60
60	0.56	0.68	0.67	0.64	0.54	159	0.61	0.71	0.69	0.64	0.54
61	0.82	0.84	0.84	0.83	0.82	160	0.56	0.62	0.64	0.63	0.60
62	0.40	0.58	0.57	0.54	0.43	161	0.77	0.80	0.83	0.82	0.82
63	0.67	0.77	0.76	0.73	0.64	162	0.59	0.69	0.68	0.67	0.56
64	0.50	0.59	0.56	0.53	0.44	163	0.64	0.74	0.72	0.68	0.56
65	0.54	0.64	0.63	0.58	0.49	164	0.63	0.71	0.72	0.71	0.66
66	0.86	0.87	0.88	0.86	0.86	165	0.69	0.75	0.76	0.75	0.71
67	0.53	0.68	0.68	0.65	0.56	166	0.71	0.79	0.79	0.77	0.72
68	0.56	0.69	0.66	0.61	0.50	167	0.70	0.77	0.79	0.72	0.60
69	0.52	0.64	0.62	0.57	0.47	168	0.64	0.69	0.67	0.64	0.48
70	0.89	0.90	0.93	0.91	0.91	169	0.52	0.58	0.59	0.57	0.55
71	0.88	0.91	0.94	0.92	0.93	170	0.73	0.80	0.79	0.74	0.63
72	0.89	0.91	0.93	0.91	0.91	171	0.69	0.79	0.79	0.75	0.62
73	0.75	0.81	0.81	0.79	0.71	172	0.71	0.78	0.75	0.70	0.55
74	0.60	0.72	0.72	0.70	0.63	173	0.44	0.49	0.51	0.47	0.43
75	0.53	0.60	0.57	0.54	0.47	174	0.12	0.16	0.18	0.16	0.13
76	0.71	0.77	0.78	0.77	0.74	175	0.10	0.14	0.15	0.14	0.14
77	0.61	0.70	0.67	0.64	0.58	176	0.57	0.73	0.72	0.69	0.55
78	0.67	0.75	0.76	0.71	0.65	177	0.51	0.62	0.62	0.60	0.54
79	0.85	0.88	0.88	0.87	0.84	178	0.56	0.65	0.65	0.63	0.55
80	0.70	0.77	0.78	0.77	0.73	179	0.61	0.68	0.69	0.68	0.64
81	0.50	0.54	0.53	0.51	0.44	180	0.62	0.71	0.73	0.67	0.56
82	0.67	0.74	0.74	0.72	0.67	181	0.65	0.69	0.72	0.70	0.69
83	0.76	0.83	0.85	0.77	0.66	182	0.58	0.73	0.73	0.70	0.60
84	0.61	0.70	0.69	0.68	0.61	183	0.59	0.67	0.66	0.64	0.56
85	0.70	0.78	0.78	0.77	0.72	184	0.57	0.67	0.67	0.66	0.59
86	0.69	0.76	0.76	0.75	0.69	185	0.27	0.35	0.35	0.34	0.30
87	0.47	0.48	0.48	0.47	0.44	186	0.50	0.60	0.62	0.59	0.55
88	0.66	0.73	0.73	0.72	0.67	187	0.69	0.79	0.77	0.72	0.60
89	0.03	0.06	0.09	0.03	0.02	188	0.79	0.84	0.88	0.87	0.87
90	0.71	0.78	0.79	0.76	0.71	189	0.59	0.72	0.72	0.69	0.57
91	0.80	0.84	0.82	0.80	0.72	190	0.71	0.79	0.78	0.76	0.71
92	0.57	0.65	0.65	0.64	0.61	191	0.54	0.72	0.73	0.70	0.58
93	0.72	0.79	0.79	0.77	0.71	192	0.48	0.58	0.58	0.56	0.50
94	0.70	0.81	0.79	0.75	0.63	193	0.75	0.77	0.76	0.74	0.74
95	0.63	0.69	0.66	0.60	0.46	194	0.74	0.82	0.87	0.85	0.84
96	0.61	0.74	0.74	0.70	0.59	195	0.81	0.84	0.86	0.86	0.86
97	0.60	0.72	0.74	0.73	0.66	196	0.48	0.62	0.62	0.59	0.49
98	0.59	0.68	0.66	0.62	0.50	197	0.46	0.63	0.64	0.63	0.55
99	0.77	0.83	0.83	0.83	0.79						

Table A1.9 Correlation between power output and heart rate for each session for rider 10 with different heart rate lags of seconds (0, 10, 15, 20, 30)

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
1	0.57	0.69	0.71	0.69	0.61	127	0.50	0.58	0.57	0.57	0.54
2	0.52	0.62	0.63	0.62	0.54	128	0.67	0.75	0.75	0.74	0.67
3	0.33	0.43	0.45	0.44	0.40	129	0.43	0.53	0.54	0.53	0.49
4	0.57	0.68	0.69	0.67	0.59	130	0.64	0.68	0.68	0.68	0.67
5	0.43	0.54	0.56	0.56	0.51	131	0.51	0.63	0.65	0.65	0.61
6	0.61	0.69	0.69	0.65	0.53	132	0.59	0.71	0.71	0.70	0.61
7	0.68	0.72	0.69	0.66	0.56	133	0.72	0.75	0.76	0.75	0.73
8	0.48	0.64	0.65	0.62	0.49	134	0.79	0.82	0.82	0.81	0.79
9	0.53	0.59	0.58	0.58	0.54	135	0.71	0.78	0.78	0.74	0.64
10	0.52	0.60	0.62	0.61	0.51	136	0.71	0.79	0.81	0.77	0.66
11	0.62	0.71	0.71	0.68	0.59	137	0.60	0.69	0.70	0.67	0.63
12	0.35	0.55	0.58	0.57	0.50	138	0.52	0.65	0.66	0.65	0.62
13	0.50	0.65	0.67	0.68	0.62	139	0.39	0.53	0.53	0.51	0.45
14	0.48	0.55	0.56	0.55	0.50	140	0.66	0.70	0.69	0.66	0.58
15	0.72	0.82	0.82	0.79	0.65	141	0.67	0.75	0.74	0.71	0.63
16	0.71	0.78	0.73	0.70	0.59	142	0.60	0.68	0.69	0.68	0.63
17	0.53	0.69	0.70	0.68	0.56	143	0.50	0.62	0.62	0.60	0.56
18	0.17	0.21	0.21	0.24	0.22	144	0.73	0.78	0.79	0.72	0.65
19	0.68	0.78	0.78	0.74	0.61	145	0.52	0.62	0.62	0.60	0.48
20	0.56	0.66	0.67	0.62	0.54	146	0.39	0.52	0.54	0.50	0.50
21	0.45	0.73	0.77	0.79	0.69	147	0.6	0.69	0.67	0.64	0.56
22	0.49	0.63	0.68	0.65	0.59	148	0.59	0.64	0.64	0.63	0.60
23	0.57	0.72	0.73	0.70	0.58	149	0.61	0.68	0.68	0.67	0.63
24	0.76	0.82	0.82	0.79	0.67	150	0.55	0.66	0.68	0.67	0.64
25	0.54	0.62	0.63	0.61	0.53	151	0.62	0.70	0.68	0.65	0.59
26	0.47	0.55	0.54	0.52	0.46	152	0.60	0.73	0.74	0.71	0.58
27	0.59	0.74	0.75	0.73	0.59	153	0.56	0.66	0.66	0.65	0.60
28	0.45	0.52	0.51	0.50	0.40	154	0.71	0.76	0.77	0.76	0.73
29	0.33	0.46	0.53	0.50	0.47	155	0.55	0.65	0.67	0.65	0.55
30	0.50	0.58	0.58	0.57	0.52	156	0.51	0.62	0.64	0.63	0.57
31	0.46	0.54	0.52	0.51	0.46	157	0.55	0.62	0.63	0.62	0.61
32	0.68	0.81	0.81	0.77	0.63	158	0.62	0.68	0.67	0.67	0.63
33	0.72	0.79	0.79	0.78	0.71	159	0.67	0.75	0.77	0.77	0.75
34	0.65	0.79	0.81	0.78	0.63	160	0.61	0.70	0.69	0.64	0.53
35	0.46	0.60	0.63	0.63	0.56	161	0.57	0.63	0.64	0.61	0.58
36	0.59	0.69	0.72	0.71	0.61	162	0.48	0.61	0.59	0.55	0.46
37	0.44	0.53	0.53	0.50	0.43	163	0.37	0.51	0.49	0.38	0.27
38	0.69	0.76	0.75	0.73	0.61	164	0.44	0.54	0.55	0.53	0.48
39	0.66	0.76	0.77	0.73	0.57	165	0.55	0.66	0.70	0.69	0.57
40	0.80	0.80	0.82	0.80	0.81	166	0.55	0.62	0.63	0.62	0.60
41	0.60	0.65	0.66	0.65	0.62	167	0.57	0.64	0.62	0.59	0.52
42	0.54	0.64	0.66	0.65	0.57	168	0.45	0.61	0.61	0.55	0.44
43	0.66	0.67	0.67	0.68	0.68	169	0.68	0.73	0.73	0.72	0.70
44	0.44	0.51	0.54	0.55	0.56	170	0.55	0.68	0.68	0.65	0.57
45	0.56	0.65	0.66	0.66	0.60	171	0.55	0.71	0.74	0.73	0.64
46	0.77	0.78	0.78	0.78	0.79	172	0.67	0.71	0.71	0.69	0.65
47	0.75	0.82	0.82	0.78	0.70	173	0.47	0.58	0.59	0.59	0.53
48	0.66	0.72	0.72	0.71	0.67	174	0.62	0.71	0.71	0.69	0.61

Table A1.9 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
49	0.87	0.88	0.90	0.88	0.89	175	0.61	0.67	0.68	0.67	0.66
50	0.77	0.78	0.79	0.79	0.80	176	0.63	0.70	0.72	0.66	0.61
51	0.67	0.77	0.76	0.72	0.60	177	0.56	0.65	0.65	0.64	0.57
52	0.60	0.73	0.75	0.73	0.65	178	0.50	0.58	0.59	0.57	0.53
53	0.60	0.77	0.78	0.73	0.57	179	0.52	0.62	0.63	0.62	0.57
54	0.62	0.71	0.71	0.69	0.62	180	0.58	0.64	0.65	0.64	0.62
55	0.72	0.79	0.81	0.78	0.67	181	0.51	0.59	0.59	0.58	0.54
56	0.66	0.71	0.71	0.70	0.66	182	0.47	0.60	0.62	0.62	0.59
57	0.91	0.91	0.93	0.91	0.92	183	-0.03	0.05	0.08	0.10	0.15
58	0.62	0.75	0.77	0.75	0.62	184	0.47	0.62	0.62	0.57	0.45
59	0.77	0.82	0.84	0.81	0.71	185	0.49	0.57	0.57	0.56	0.51
60	0.90	0.90	0.92	0.90	0.90	186	0.56	0.65	0.66	0.65	0.60
61	0.57	0.73	0.72	0.68	0.55	187	0.80	0.83	0.83	0.82	0.79
62	0.76	0.84	0.84	0.80	0.68	188	0.52	0.66	0.67	0.65	0.57
63	0.55	0.72	0.73	0.69	0.55	189	0.61	0.72	0.74	0.70	0.62
64	0.87	0.87	0.88	0.88	0.88	190	0.78	0.83	0.83	0.79	0.72
65	0.57	0.69	0.70	0.68	0.59	191	0.62	0.70	0.73	0.71	0.67
66	0.37	0.53	0.55	0.55	0.51	192	0.56	0.70	0.71	0.68	0.56
67	0.59	0.71	0.72	0.69	0.54	193	0.48	0.57	0.58	0.57	0.55
68	0.61	0.69	0.69	0.67	0.59	194	0.66	0.71	0.72	0.71	0.67
69	0.63	0.77	0.77	0.73	0.57	195	0.55	0.67	0.68	0.67	0.60
70	0.65	0.72	0.72	0.71	0.64	196	0.28	0.32	0.33	0.30	0.32
71	0.36	0.45	0.46	0.45	0.42	197	0.21	0.25	0.24	0.22	0.22
72	0.57	0.71	0.73	0.71	0.63	198	0.43	0.54	0.54	0.52	0.46
73	0.66	0.77	0.75	0.74	0.66	199	0.61	0.66	0.66	0.65	0.62
74	0.66	0.78	0.79	0.77	0.64	200	0.49	0.57	0.59	0.57	0.55
75	0.59	0.68	0.68	0.65	0.57	201	-0.02	-0.04	-0.05	-0.05	-0.05
76	0.52	0.70	0.72	0.71	0.59	202	0.65	0.73	0.73	0.70	0.63
77	0.47	0.57	0.60	0.59	0.55	203	0.43	0.49	0.50	0.49	0.40
78	0.45	0.60	0.63	0.64	0.55	204	0.15	0.09	0.12	0.12	0.06
79	0.54	0.66	0.67	0.66	0.59	205	0.52	0.60	0.62	0.60	0.57
80	0.52	0.67	0.70	0.69	0.61	206	0.50	0.64	0.66	0.64	0.54
81	0.31	0.33	0.34	0.35	0.36	207	0.65	0.75	0.74	0.71	0.62
82	0.66	0.76	0.77	0.75	0.67	208	0.64	0.69	0.69	0.67	0.63
83	0.52	0.57	0.57	0.57	0.52	209	0.59	0.71	0.71	0.69	0.58
84	0.92	0.92	0.94	0.93	0.93	210	0.54	0.60	0.58	0.56	0.51
85	0.44	0.52	0.52	0.50	0.47	211	0.58	0.63	0.62	0.60	0.54
86	0.58	0.72	0.74	0.71	0.61	212	0.57	0.65	0.66	0.66	0.62
87	0.46	0.55	0.57	0.57	0.52	213	0.55	0.62	0.64	0.59	0.53
88	0.69	0.79	0.79	0.75	0.62	214	0.62	0.71	0.70	0.68	0.60
89	0.63	0.78	0.79	0.76	0.59	215	0.69	0.78	0.78	0.75	0.67
90	0.60	0.68	0.69	0.69	0.64	216	0.65	0.75	0.75	0.69	0.52
91	0.60	0.75	0.75	0.71	0.56	217	0.69	0.73	0.73	0.72	0.70
92	0.51	0.65	0.67	0.67	0.60	218	0.62	0.71	0.68	0.64	0.58
93	0.69	0.81	0.81	0.78	0.63	219	0.55	0.64	0.64	0.62	0.55
94	0.53	0.63	0.64	0.62	0.55	220	0.65	0.74	0.73	0.70	0.58
95	0.55	0.67	0.67	0.64	0.53	221	0.57	0.66	0.67	0.65	0.60
96	0.54	0.61	0.61	0.60	0.57	222	0.52	0.61	0.62	0.60	0.56
97	0.45	0.55	0.56	0.55	0.50	223	0.53	0.66	0.65	0.63	0.58

Table A1.9 Continued.

Session	0 sec	10 sec	15 sec	20 sec	30 sec	Session	0 sec	10 sec	15 sec	20 sec	30 sec
98	0.54	0.68	0.70	0.67	0.56	224	0.57	0.64	0.63	0.62	0.59
99	0.67	0.77	0.77	0.75	0.67	225	0.59	0.71	0.72	0.70	0.61
100	0.59	0.74	0.74	0.72	0.60	226	0.52	0.63	0.63	0.61	0.53
101	0.75	0.80	0.81	0.80	0.79	227	0.43	0.53	0.59	0.58	0.57
102	0.50	0.58	0.58	0.58	0.54	228	0.55	0.61	0.58	0.56	0.52
103	0.63	0.74	0.75	0.73	0.65	229	0.61	0.71	0.72	0.69	0.55
104	0.37	0.50	0.52	0.50	0.41	230	0.52	0.65	0.66	0.65	0.57
105	0.48	0.54	0.54	0.54	0.51	231	0.50	0.61	0.62	0.60	0.53
106	0.52	0.63	0.65	0.63	0.52	232	0.60	0.68	0.66	0.61	0.50
107	0.75	0.80	0.82	0.77	0.67	233	0.61	0.70	0.72	0.65	0.56
108	0.57	0.65	0.66	0.68	0.66	234	0.52	0.61	0.62	0.60	0.54
109	0.63	0.68	0.69	0.69	0.68	235	0.60	0.69	0.69	0.68	0.63
110	0.66	0.78	0.79	0.77	0.66	236	0.45	0.53	0.54	0.54	0.49
111	0.63	0.76	0.75	0.73	0.63	237	0.64	0.75	0.77	0.74	0.61
112	0.66	0.71	0.71	0.71	0.69	238	0.59	0.64	0.63	0.62	0.60
113	0.43	0.49	0.52	0.50	0.48	239	0.67	0.79	0.79	0.75	0.62
114	0.58	0.65	0.66	0.66	0.64	240	0.45	0.54	0.54	0.51	0.44
115	0.51	0.62	0.66	0.66	0.58	241	0.56	0.65	0.66	0.66	0.57
116	0.56	0.62	0.63	0.62	0.59	242	0.61	0.71	0.71	0.66	0.54
117	0.54	0.69	0.71	0.70	0.65	243	0.67	0.77	0.78	0.73	0.59
118	0.69	0.72	0.72	0.72	0.72	244	0.45	0.61	0.64	0.63	0.55
119	0.57	0.65	0.63	0.62	0.54	245	0.55	0.63	0.64	0.60	0.47
120	0.69	0.74	0.76	0.76	0.75	246	0.48	0.56	0.55	0.54	0.51
121	0.71	0.81	0.81	0.76	0.58	247	0.54	0.64	0.65	0.63	0.53
122	0.70	0.80	0.83	0.77	0.64	248	0.55	0.63	0.62	0.60	0.55
123	0.72	0.76	0.76	0.76	0.74	249	0.50	0.62	0.63	0.62	0.57
124	0.44	0.56	0.58	0.58	0.53	250	0.59	0.69	0.68	0.65	0.53
125	0.47	0.57	0.58	0.56	0.53	251	0.48	0.61	0.62	0.63	0.55
126	0.43	0.53	0.55	0.54	0.49						

Appendix 2 Power output against heart rate for all sessions for all riders

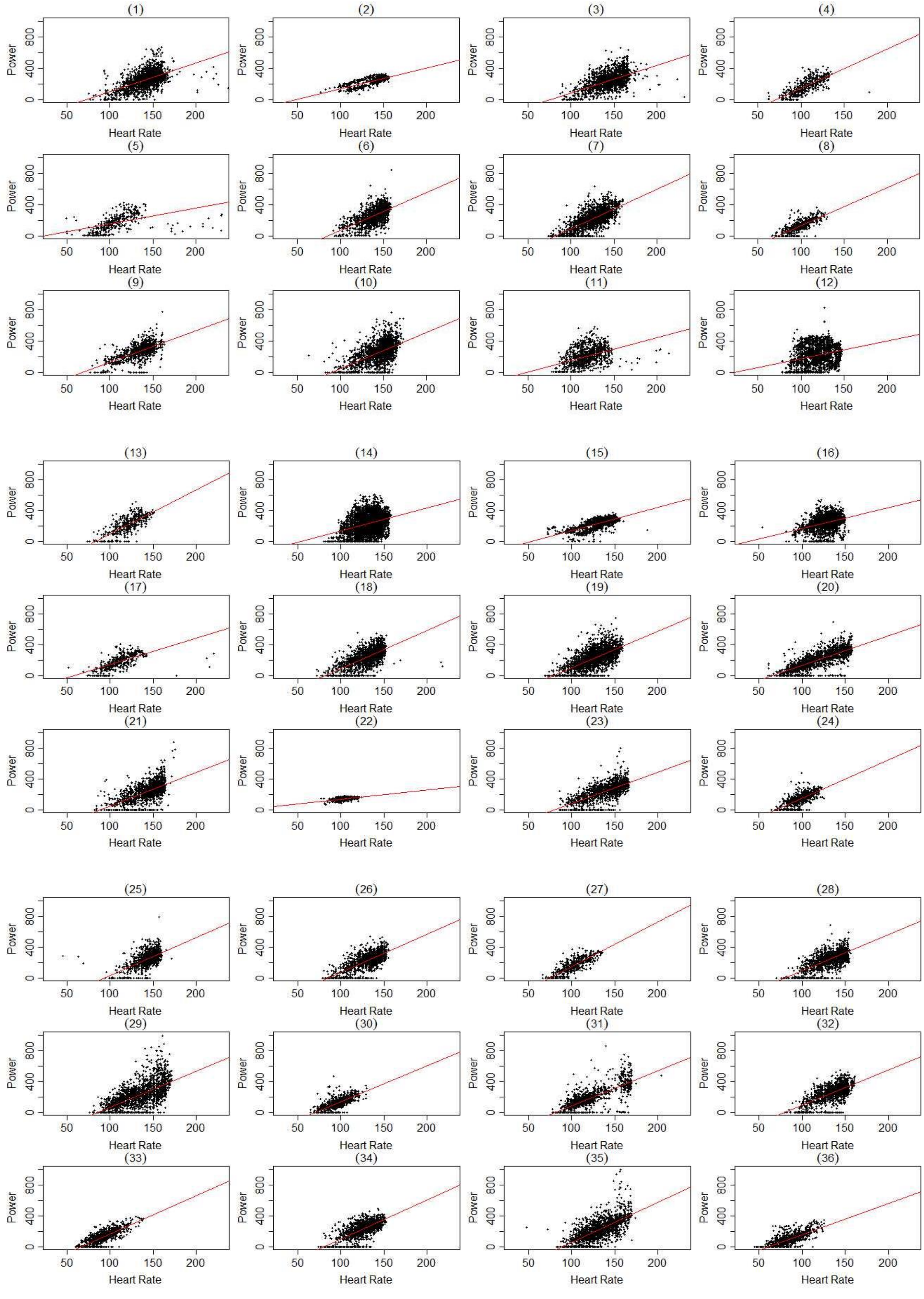

Figure A2.1 The relationship between power output and heart rate for all sessions for rider 1 with shift = 15 seconds.

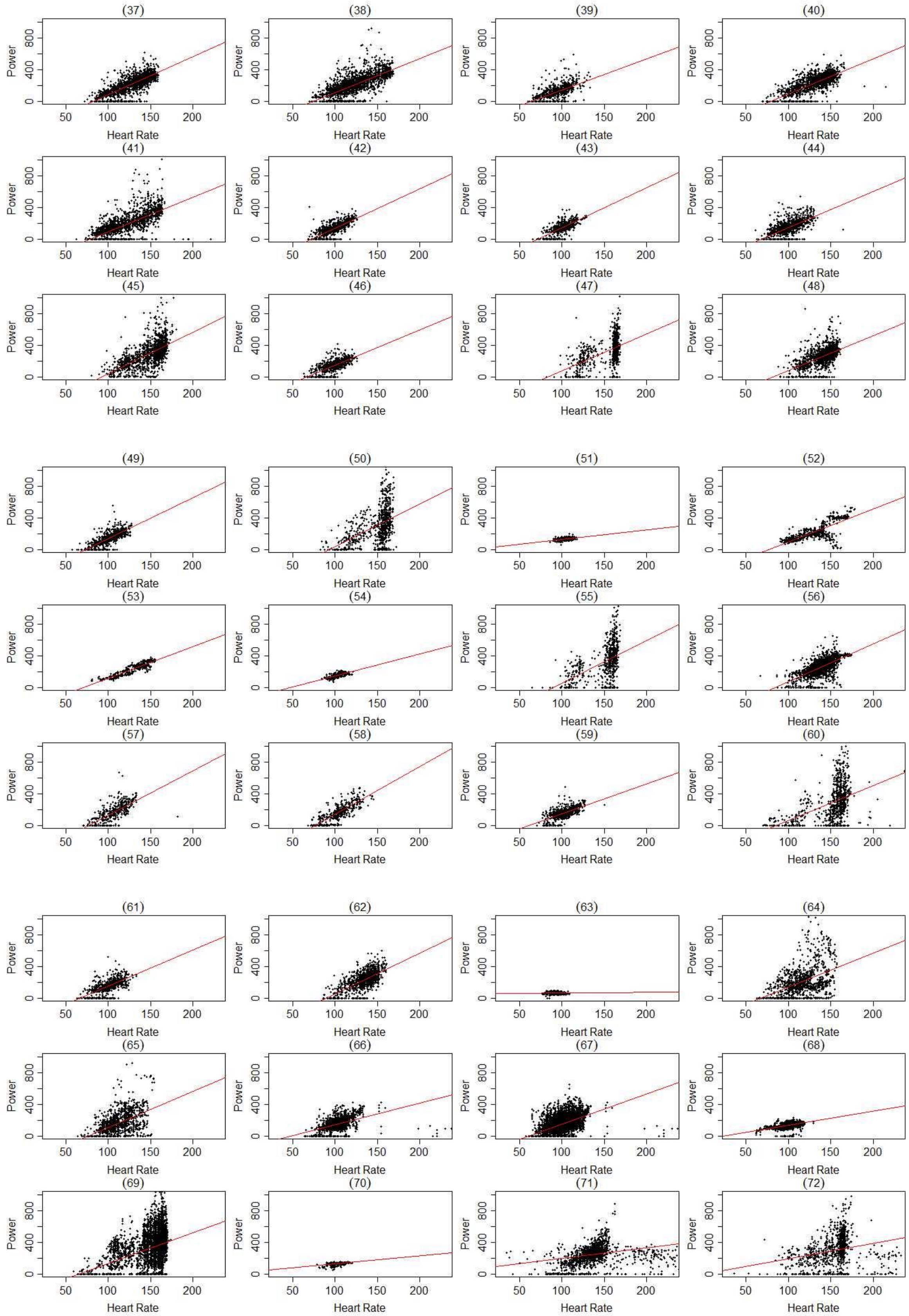

Figure A2.1 Continued.

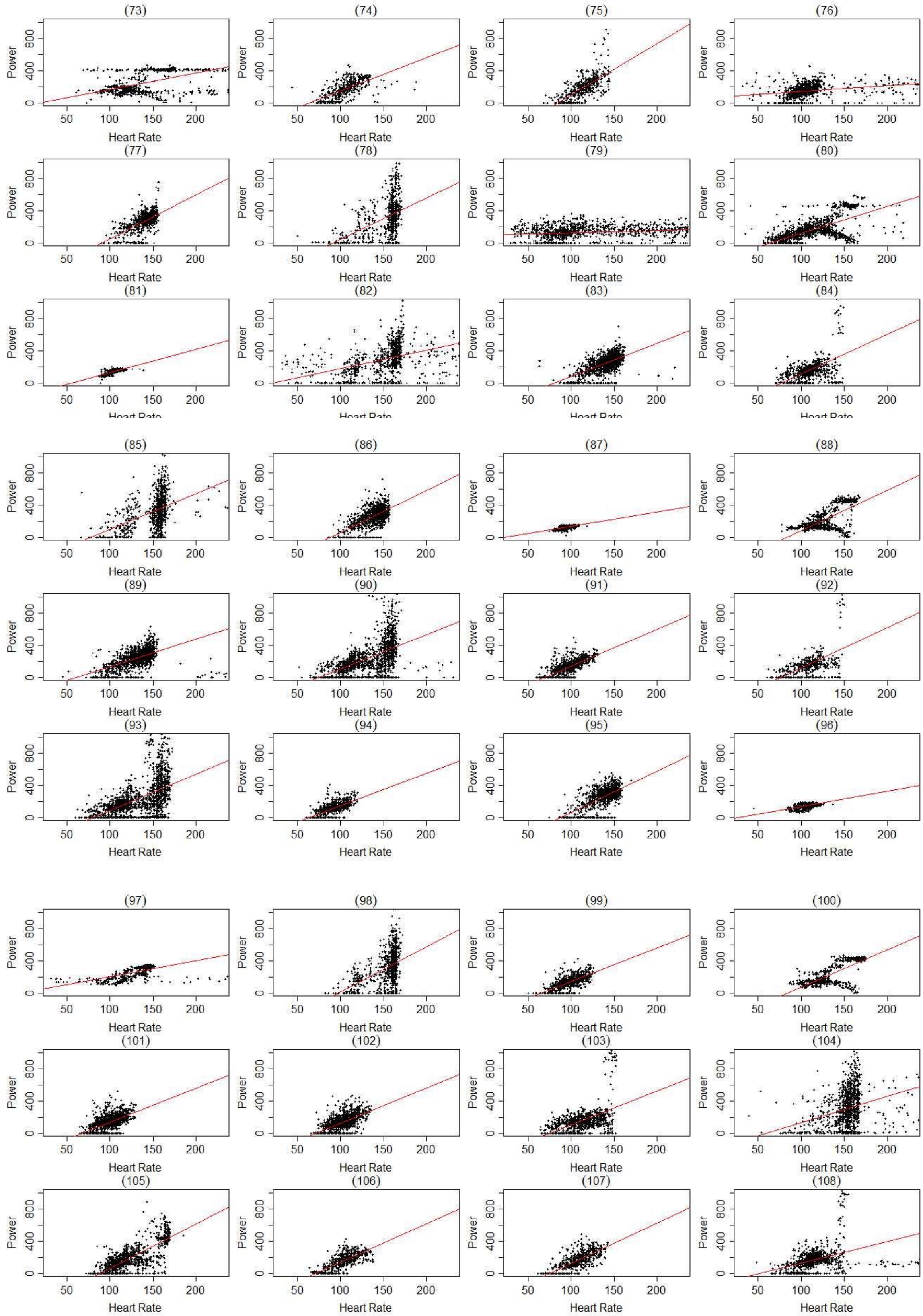

Figure A2.1 Continued.

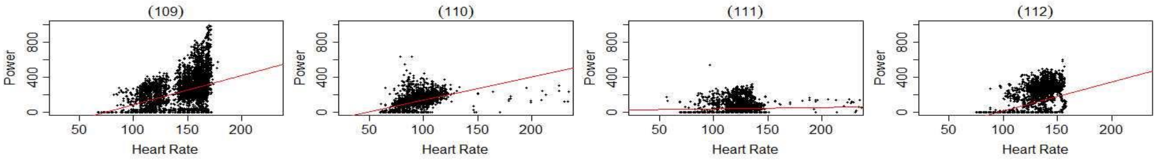

Figure A2.1 Continued.

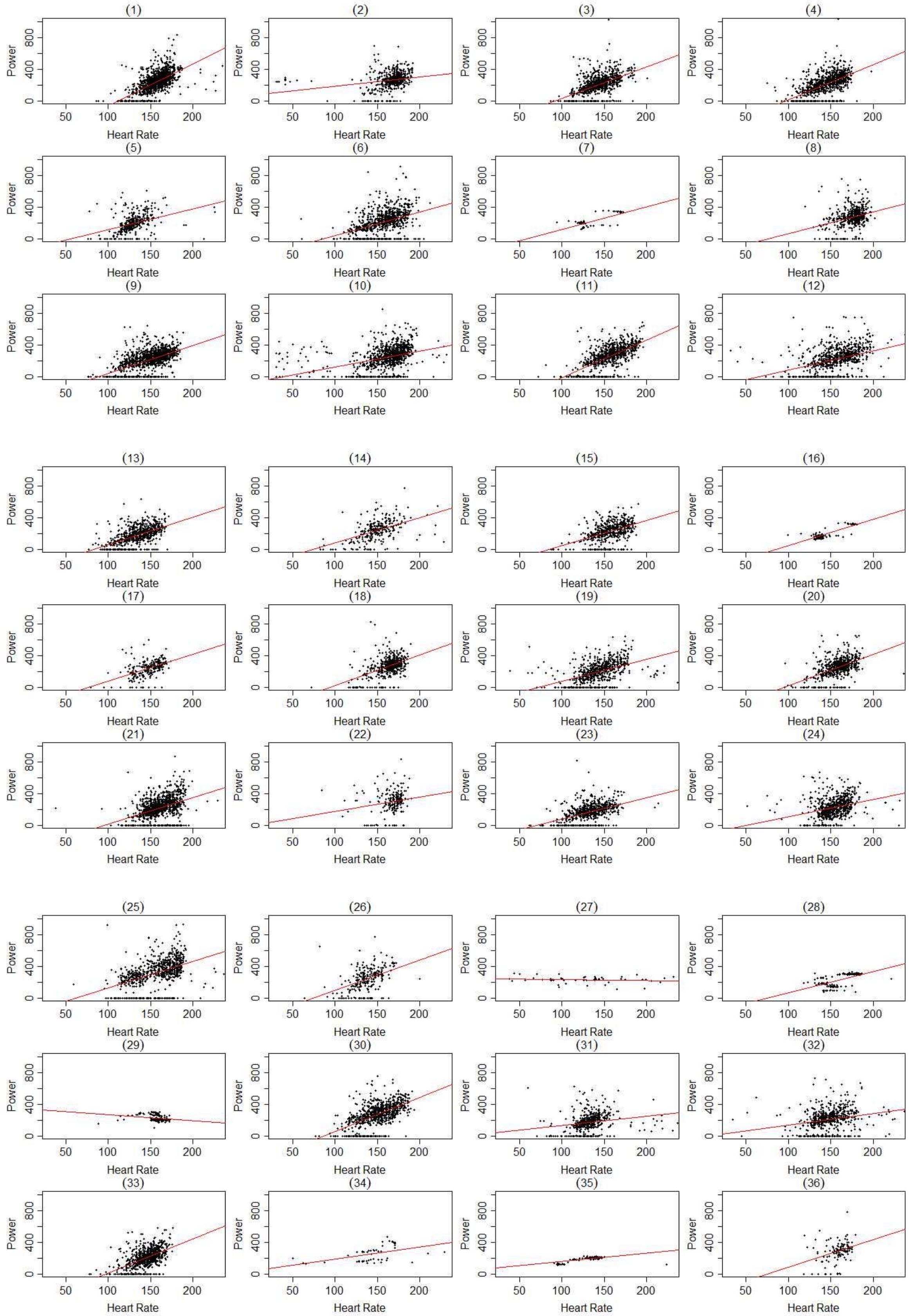

Figure A2.2 The relationship between power output and heart rate for all sessions for rider 2 with shift = 15 seconds.

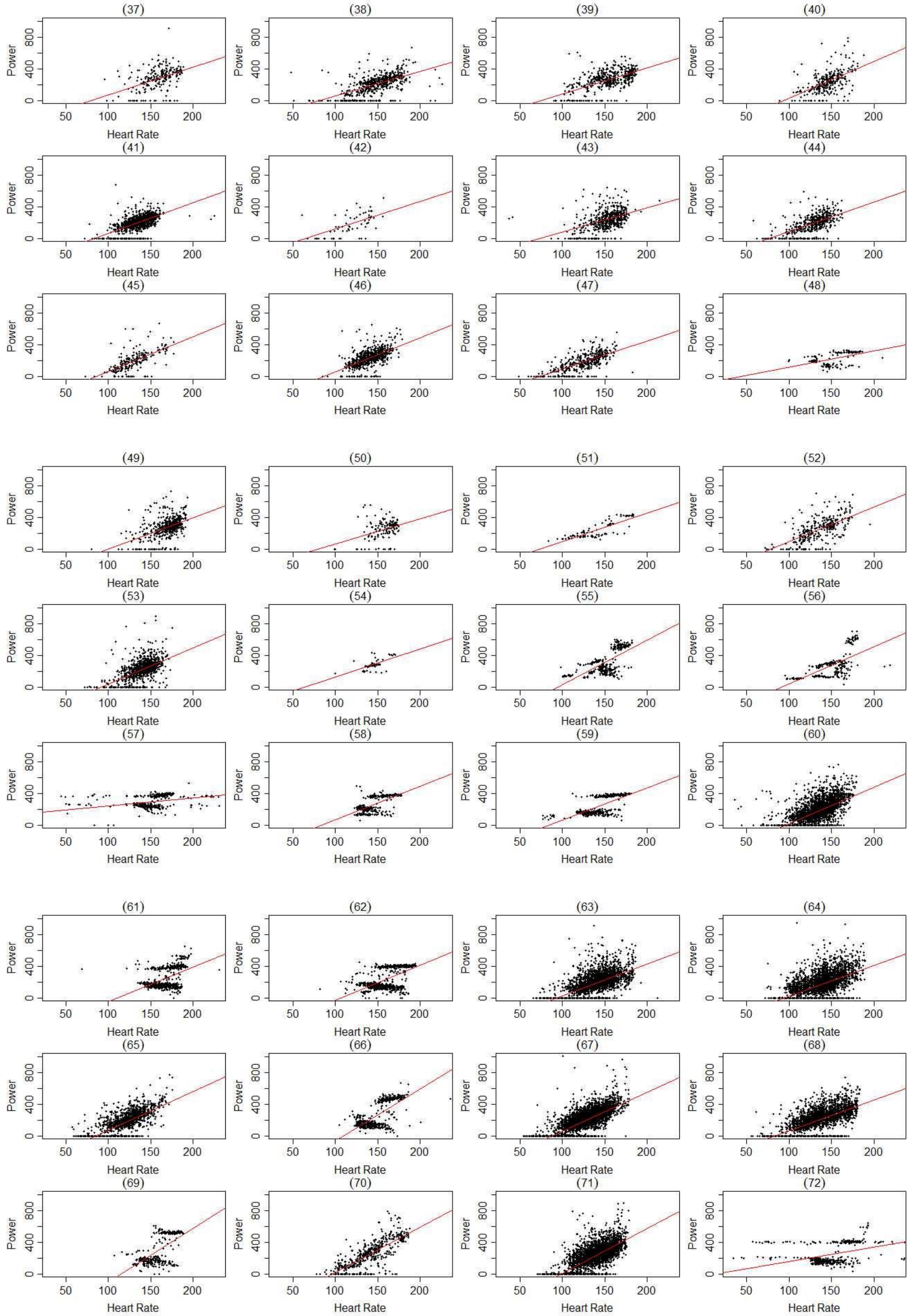

Figure A2.2 Continued.

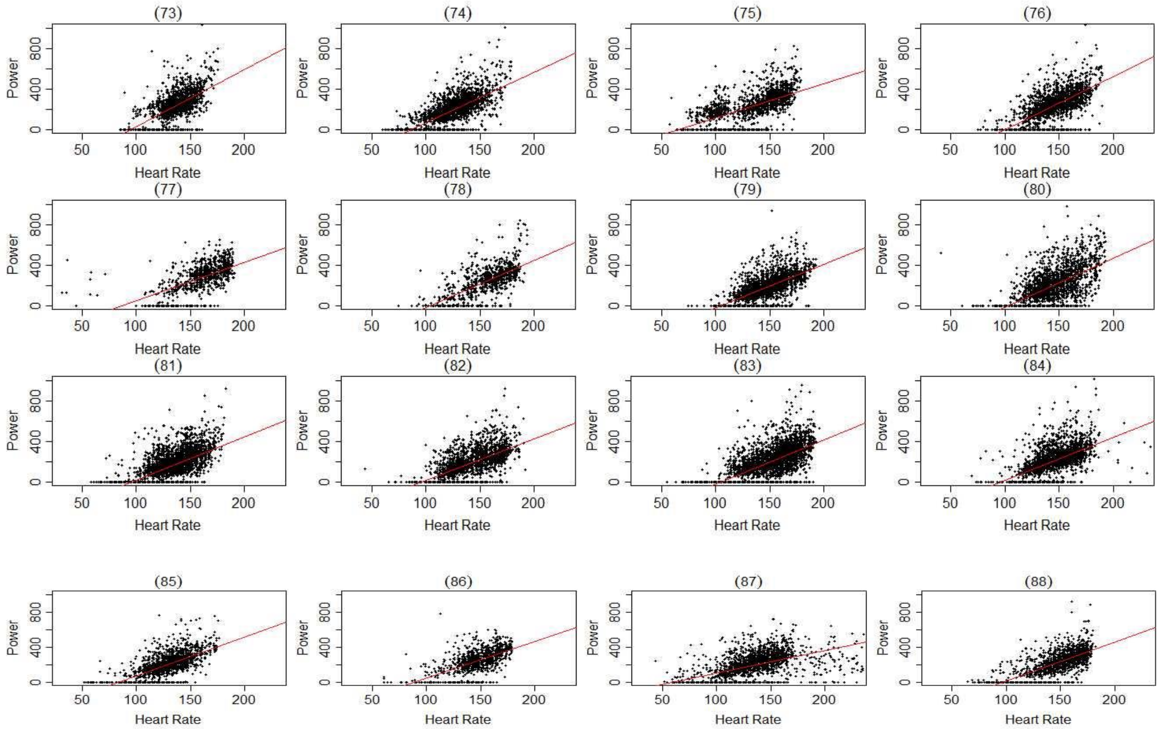

Figure A2.2 Continued.

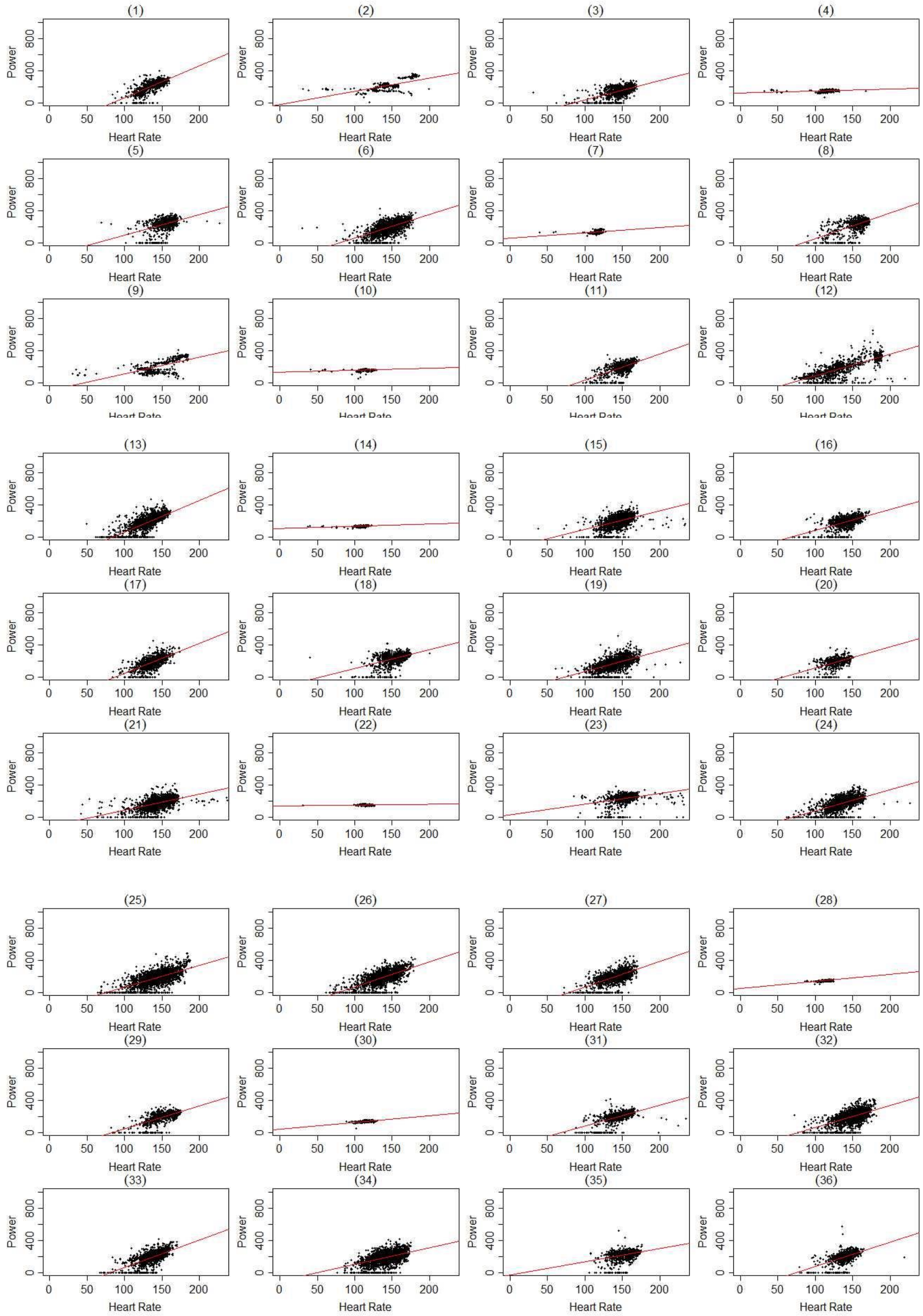

Figure A2.3 The relationship between power output and heart rate for all sessions for rider 4 with shift = 15 seconds.

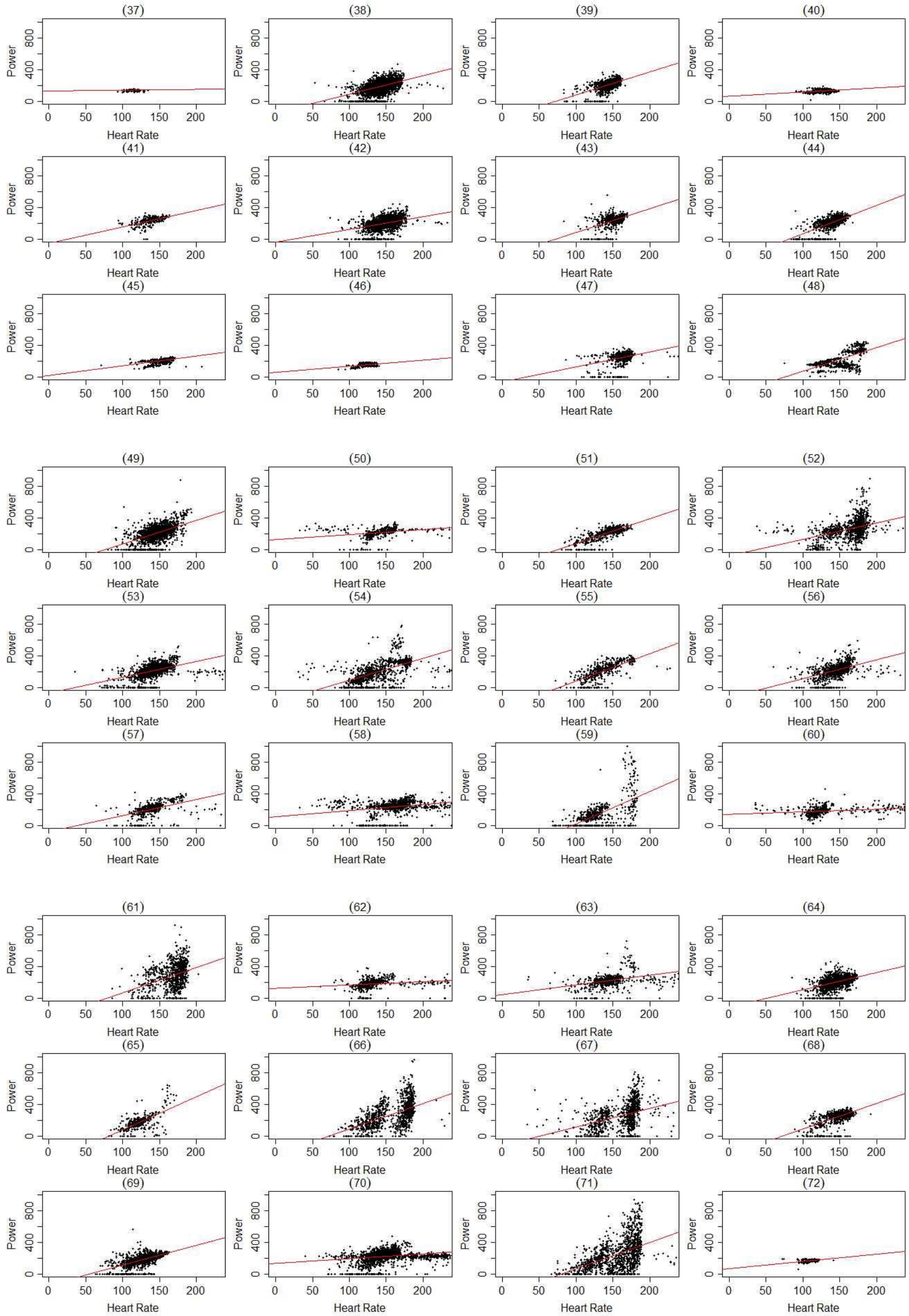

Figure A2.3 Continued.

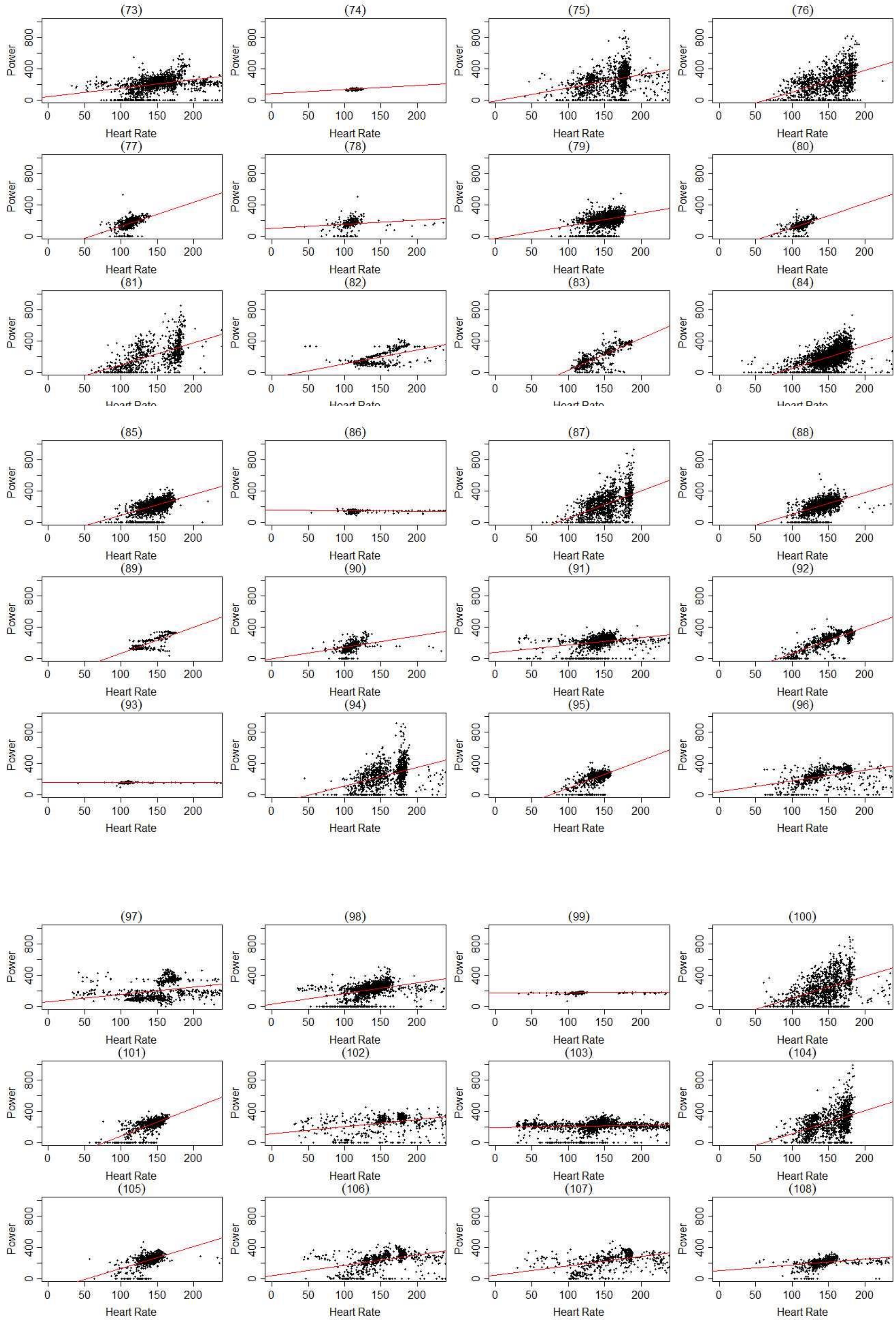

Figure A2.3 Continued.

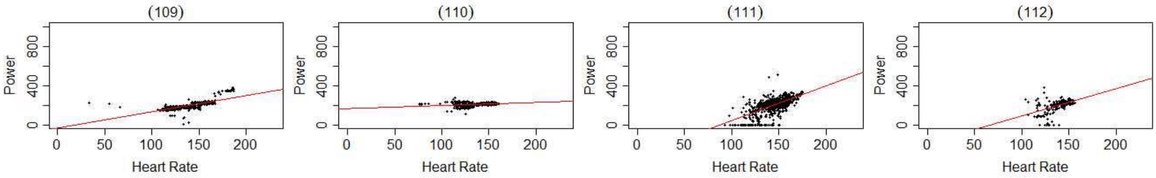

Figure A2.3 Continued.

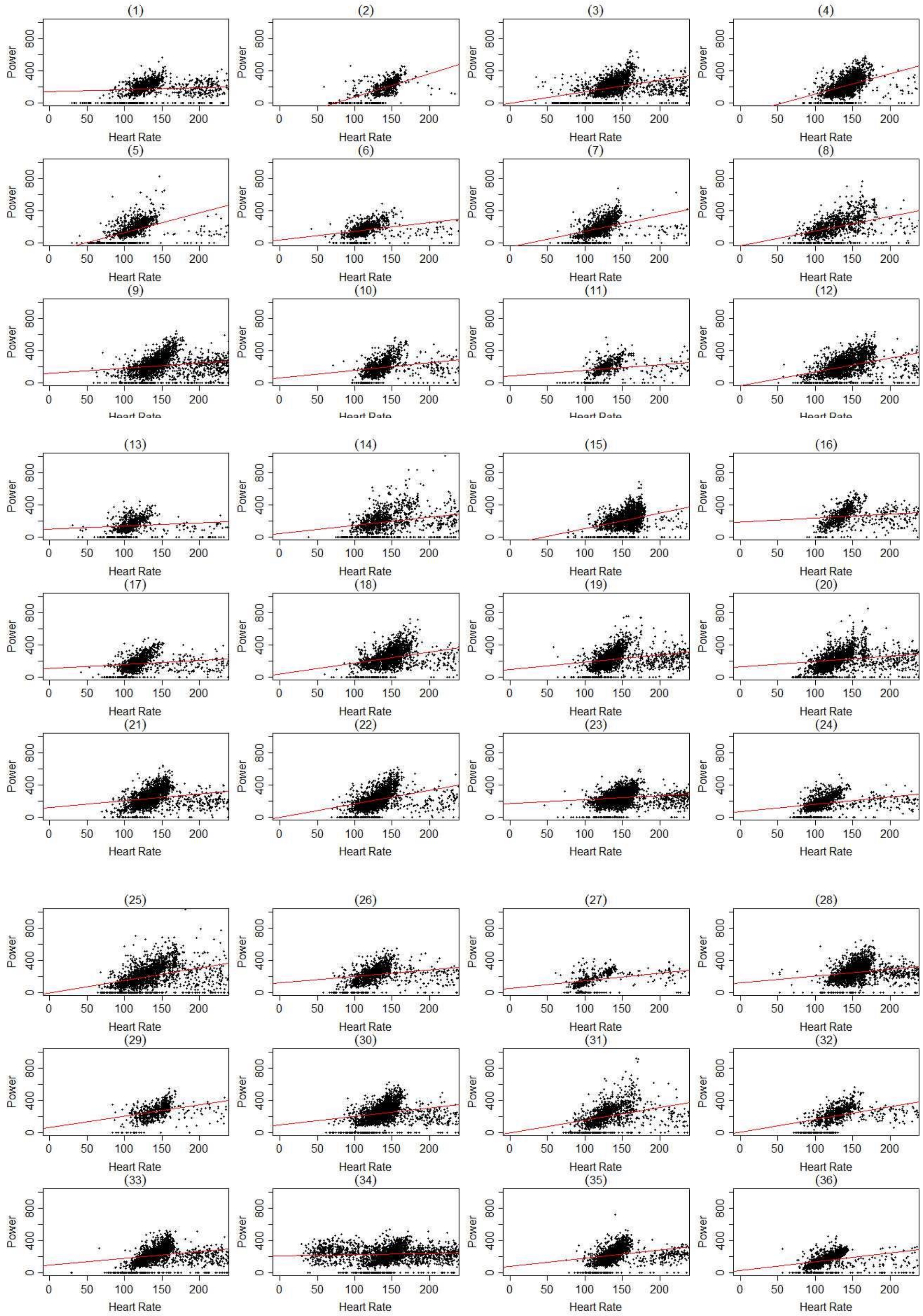

Figure A2.4 The relationship between power output and heart rate for all sessions for rider 5 with shift = 15 seconds.

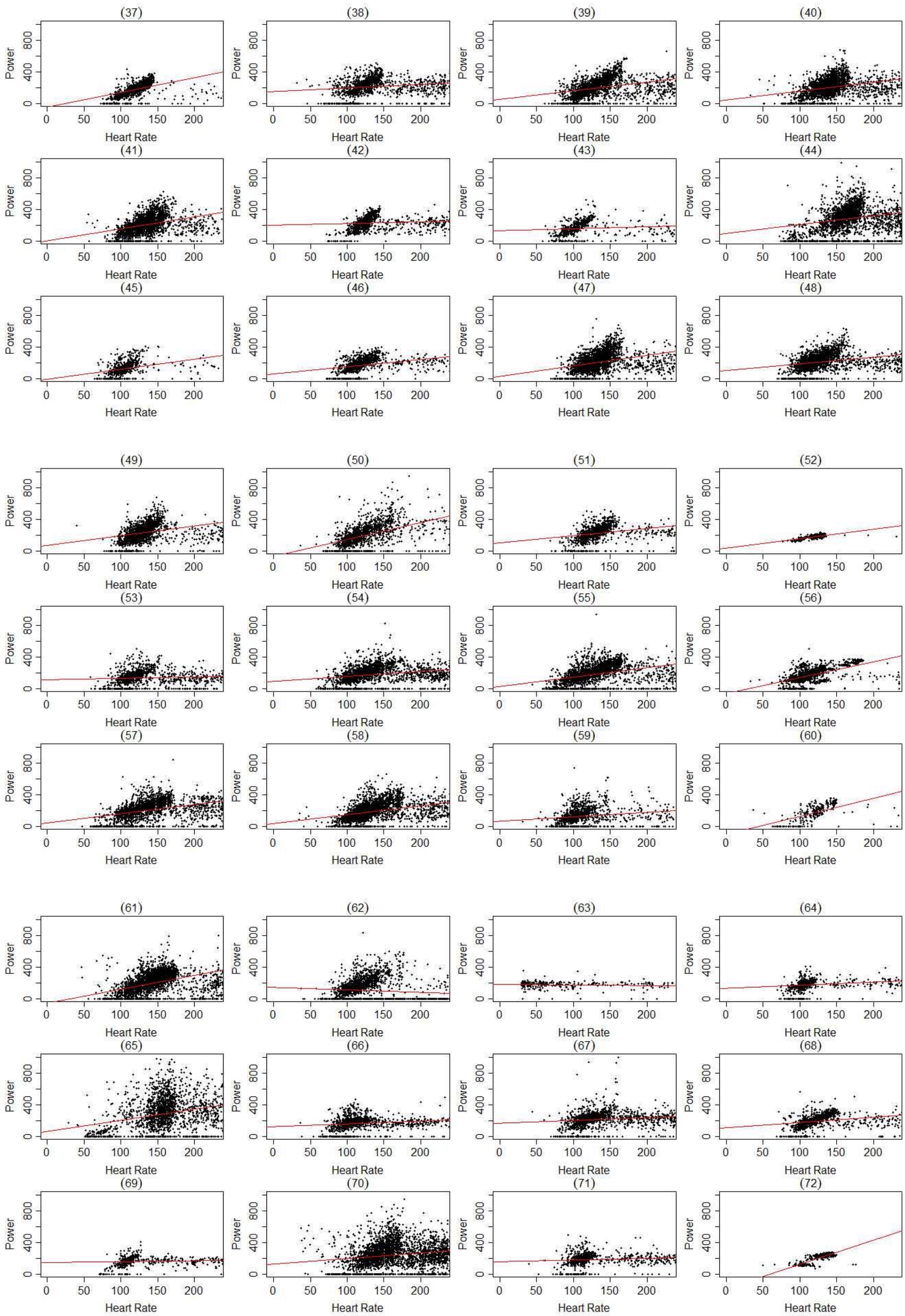

Figure A2.4 Continued.

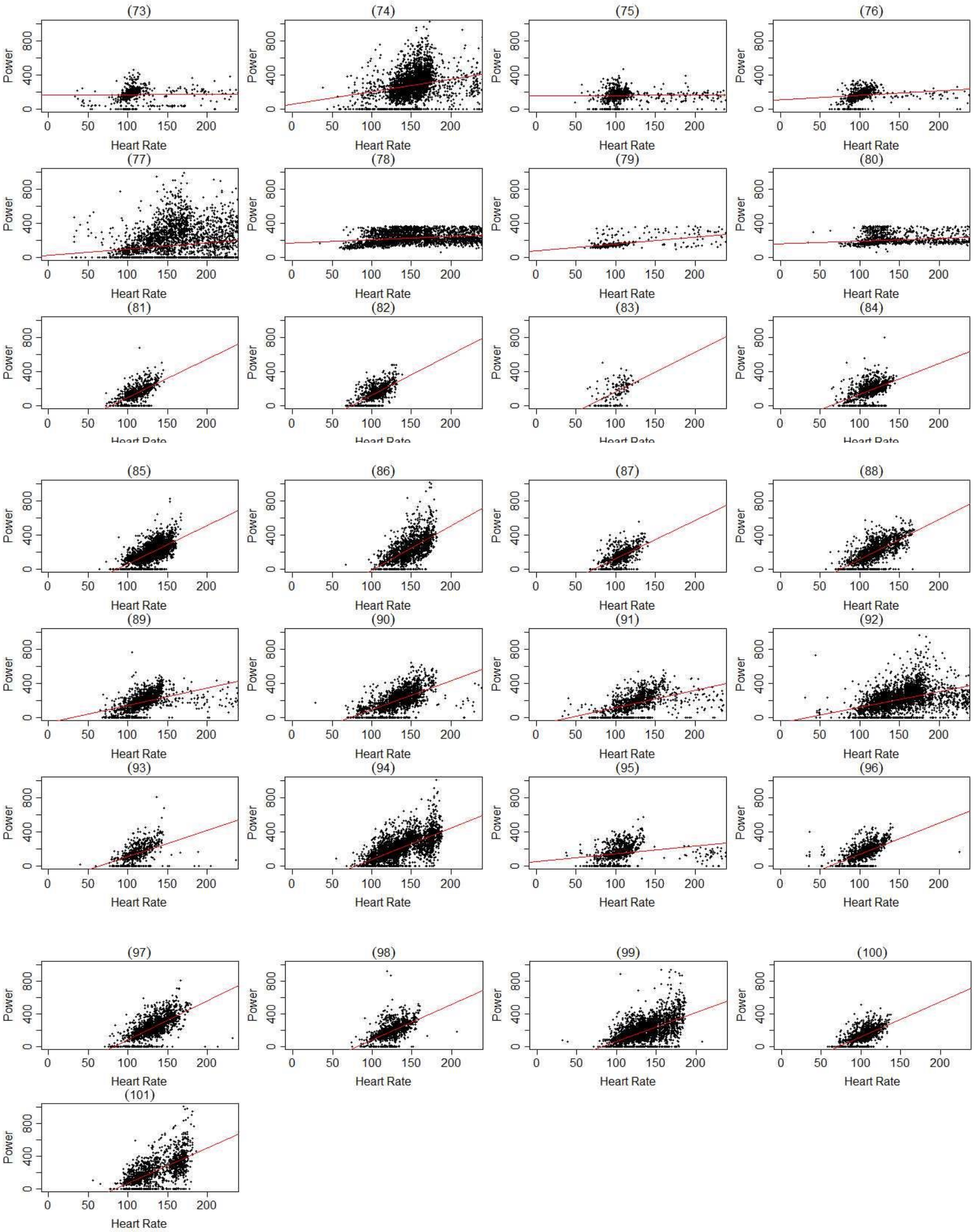

Figure A2.4 Continued.

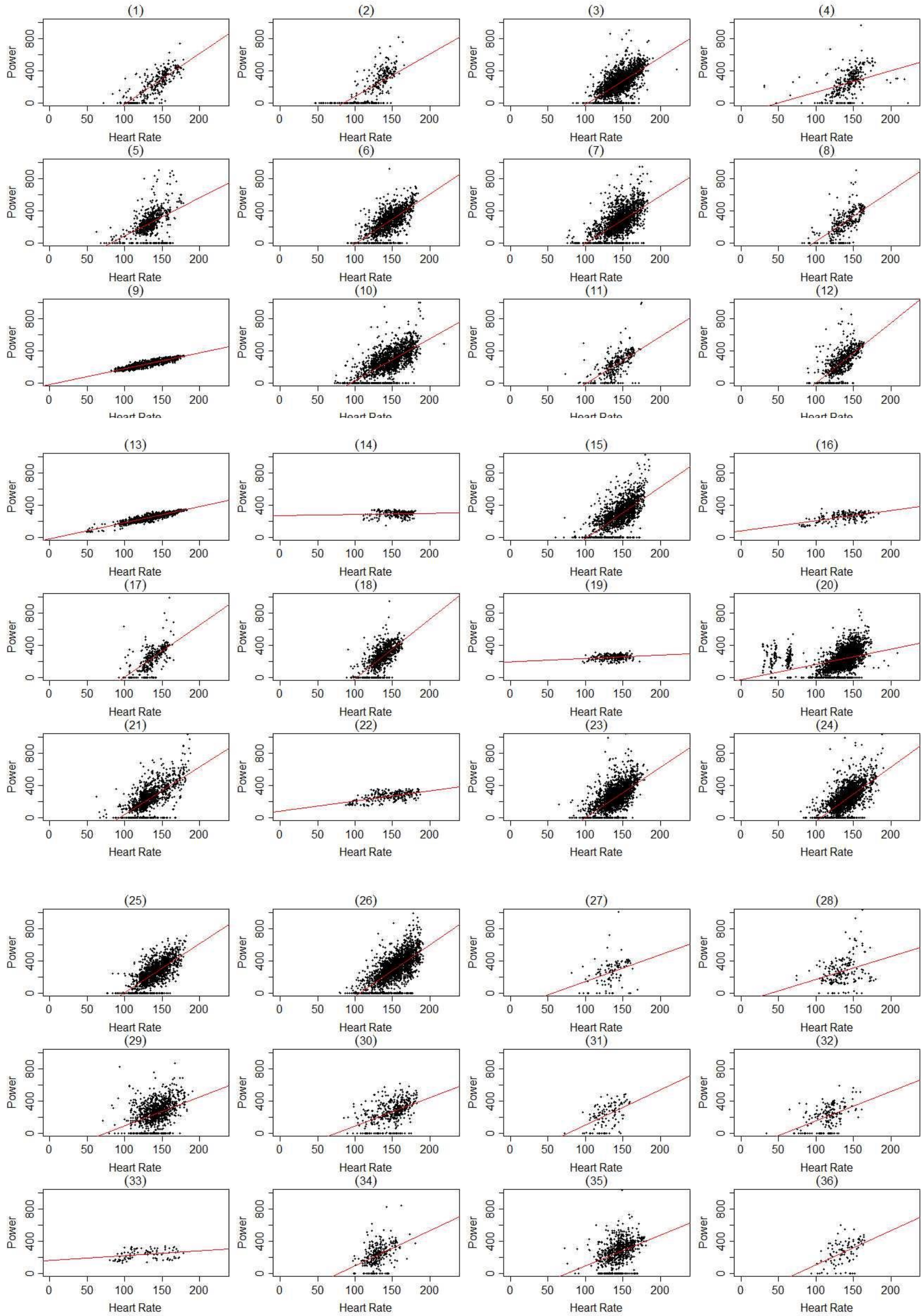

Figure A2.5 The relationship between power output and heart rate for all sessions for rider 6 with shift = 15 seconds.

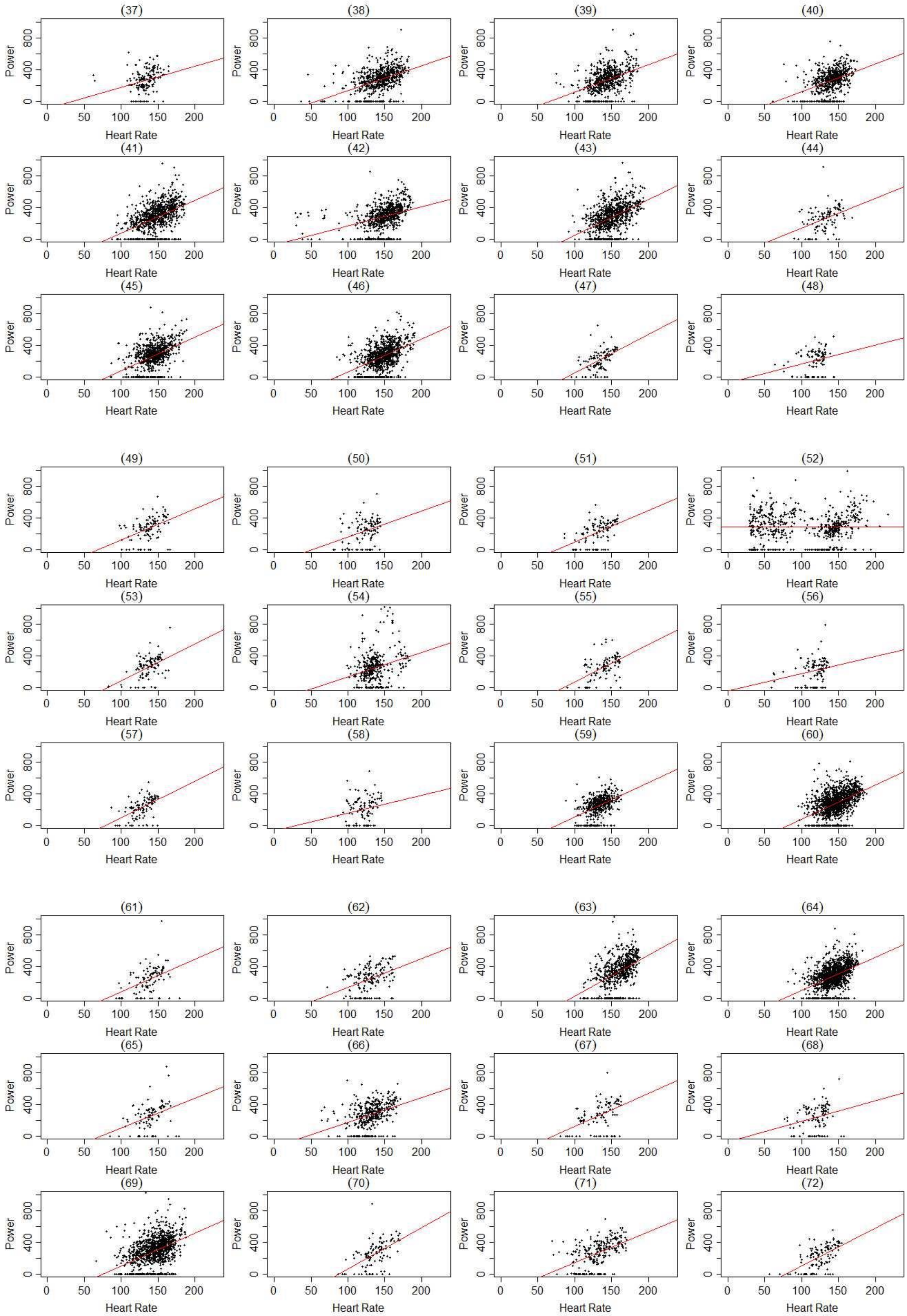

Figure A2.5 Continued.

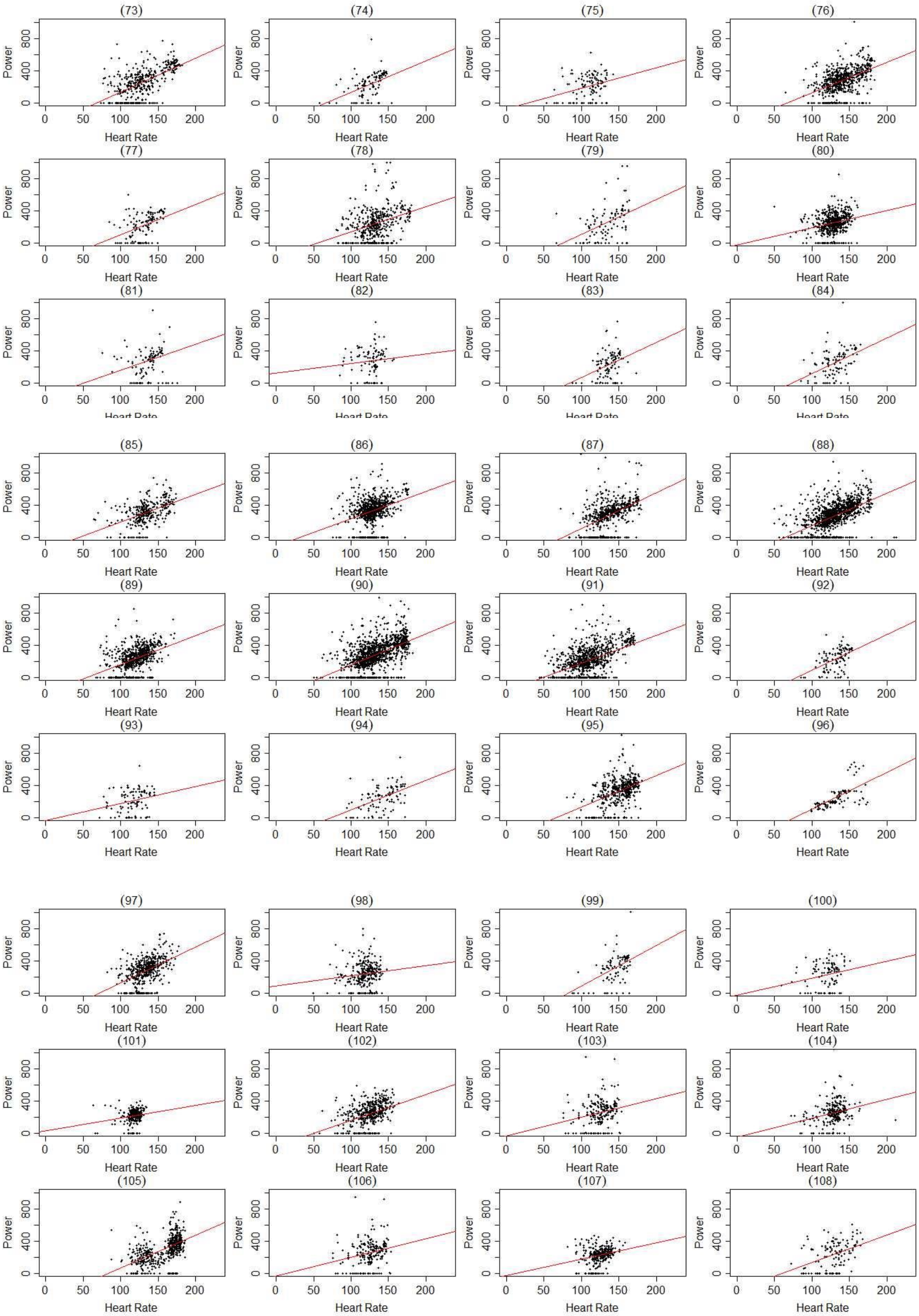

Figure A2.5 Continued.

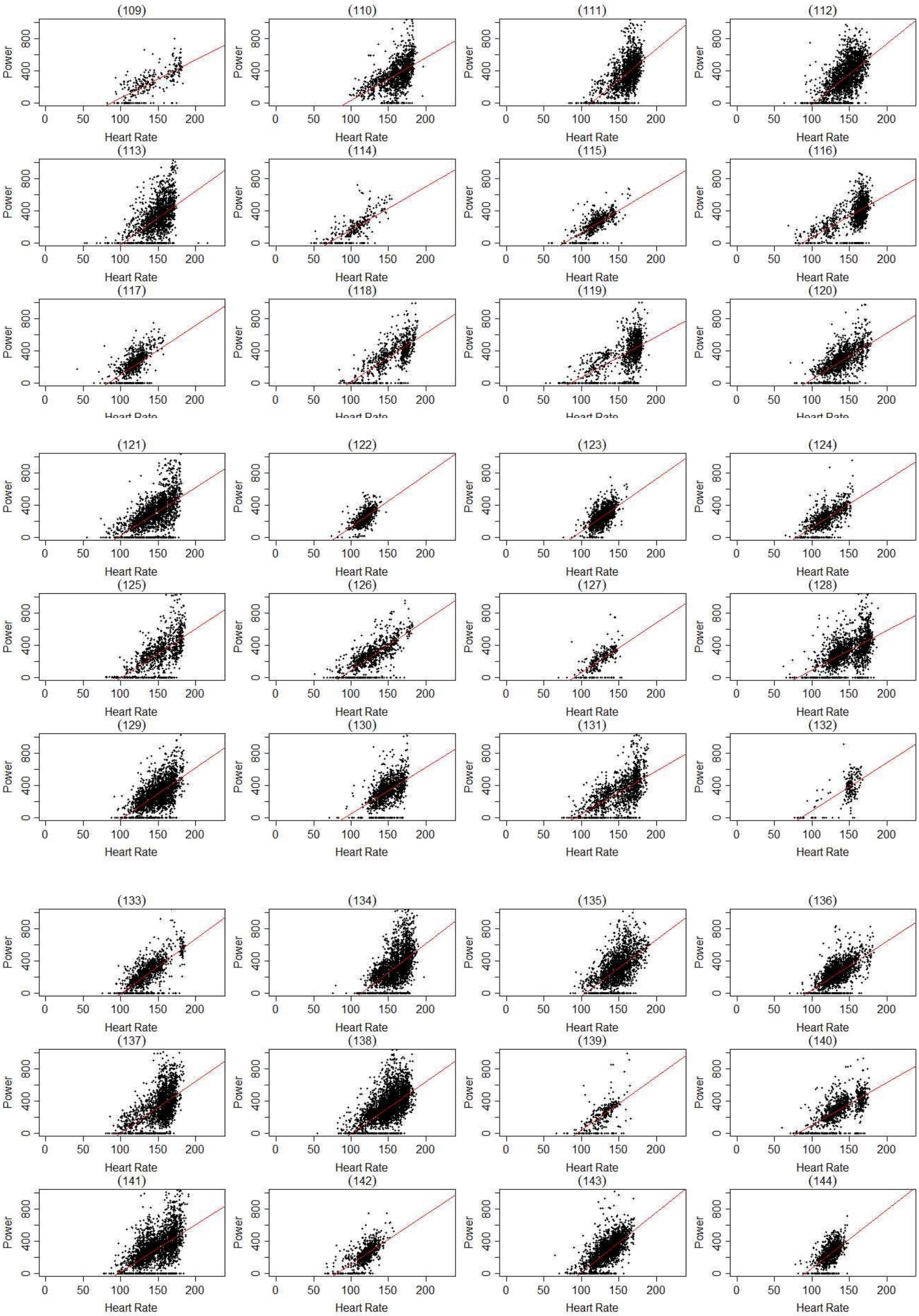

Figure A2.5 Continued.

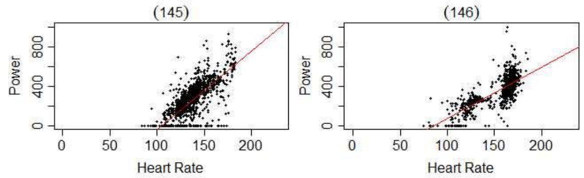

Figure A2.5 Continued.

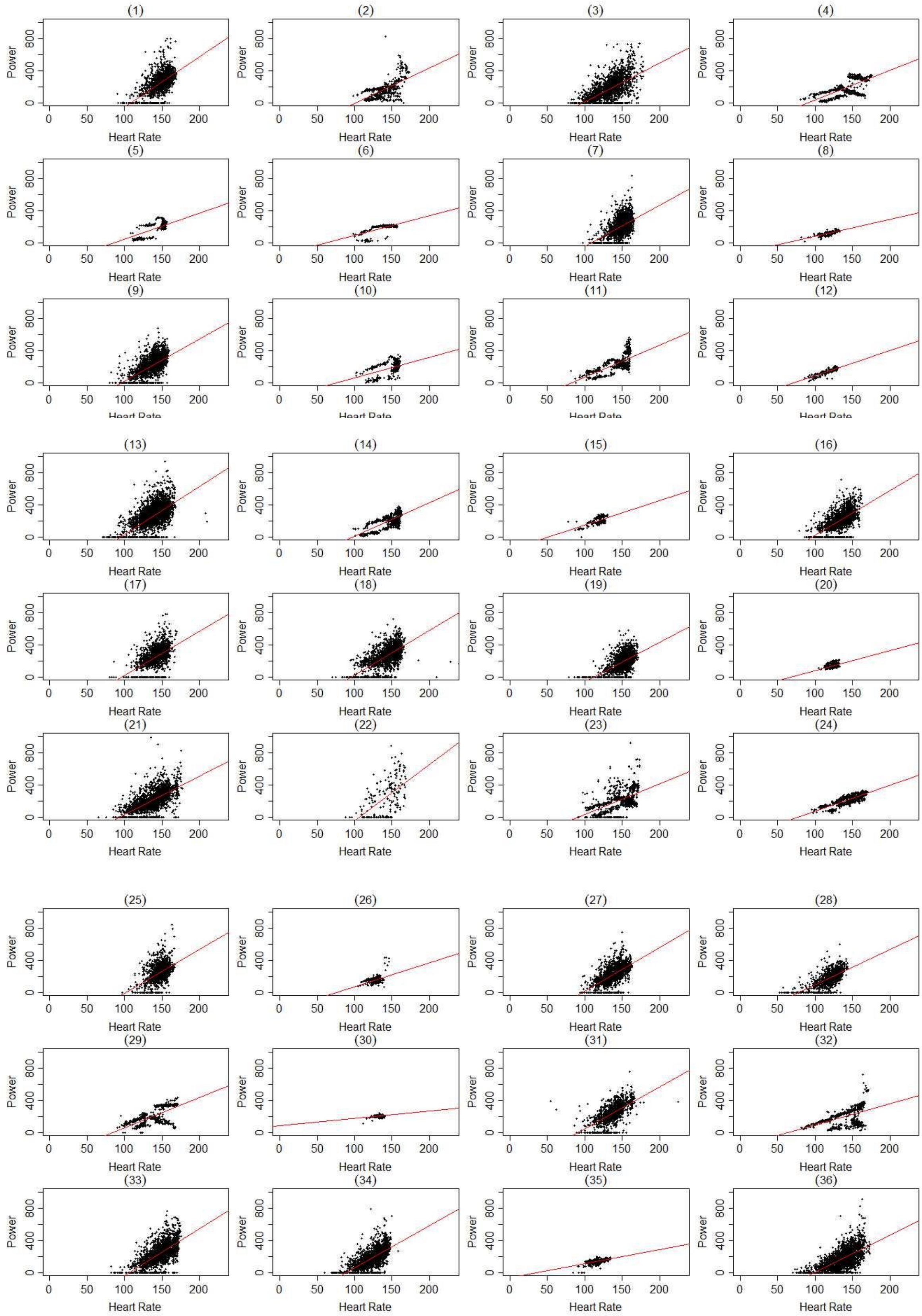

Figure A2.6 The relationship between power output and heart rate for all sessions for rider 7 with shift = 15 seconds.

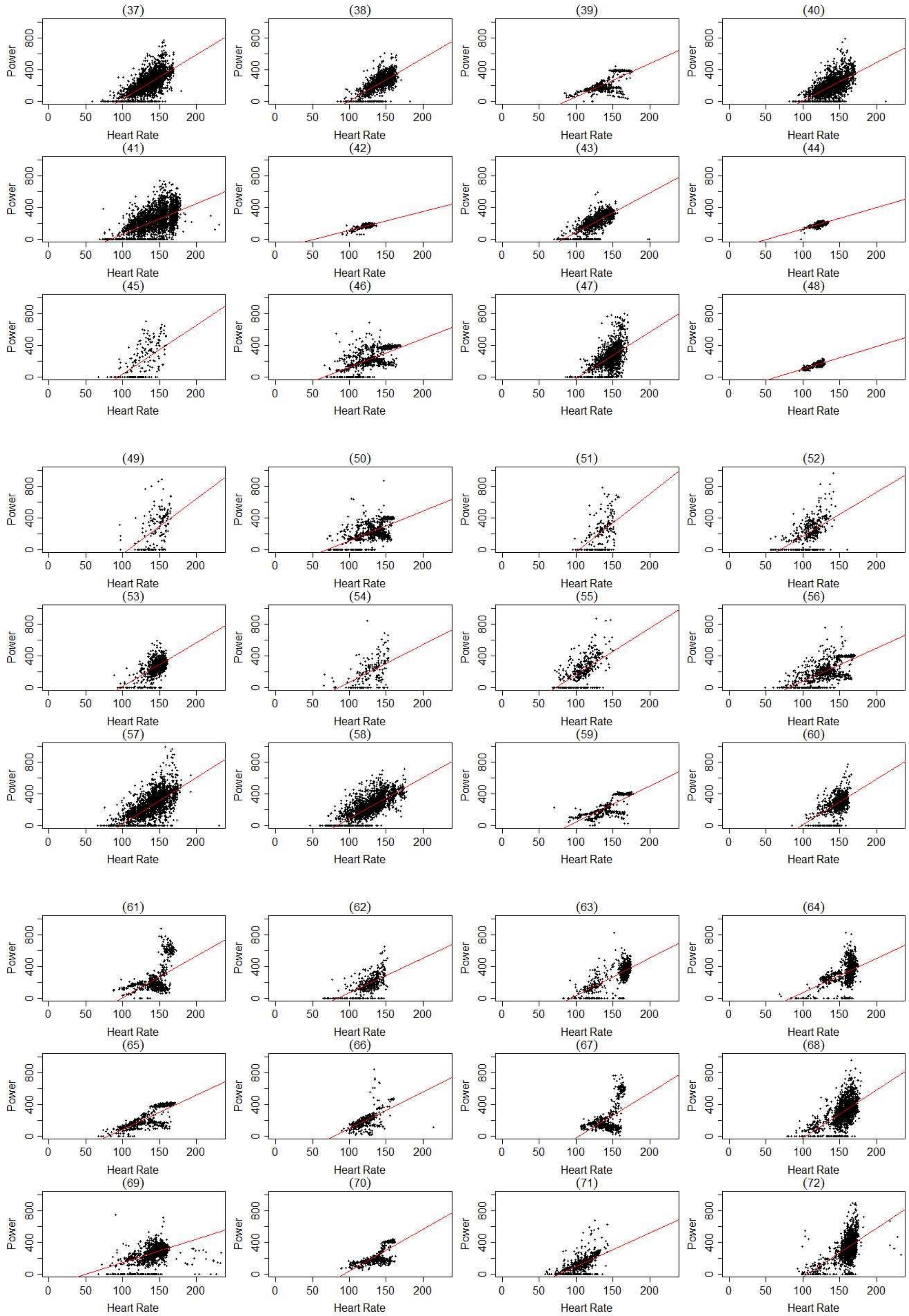

Figure A2.6 Continued.

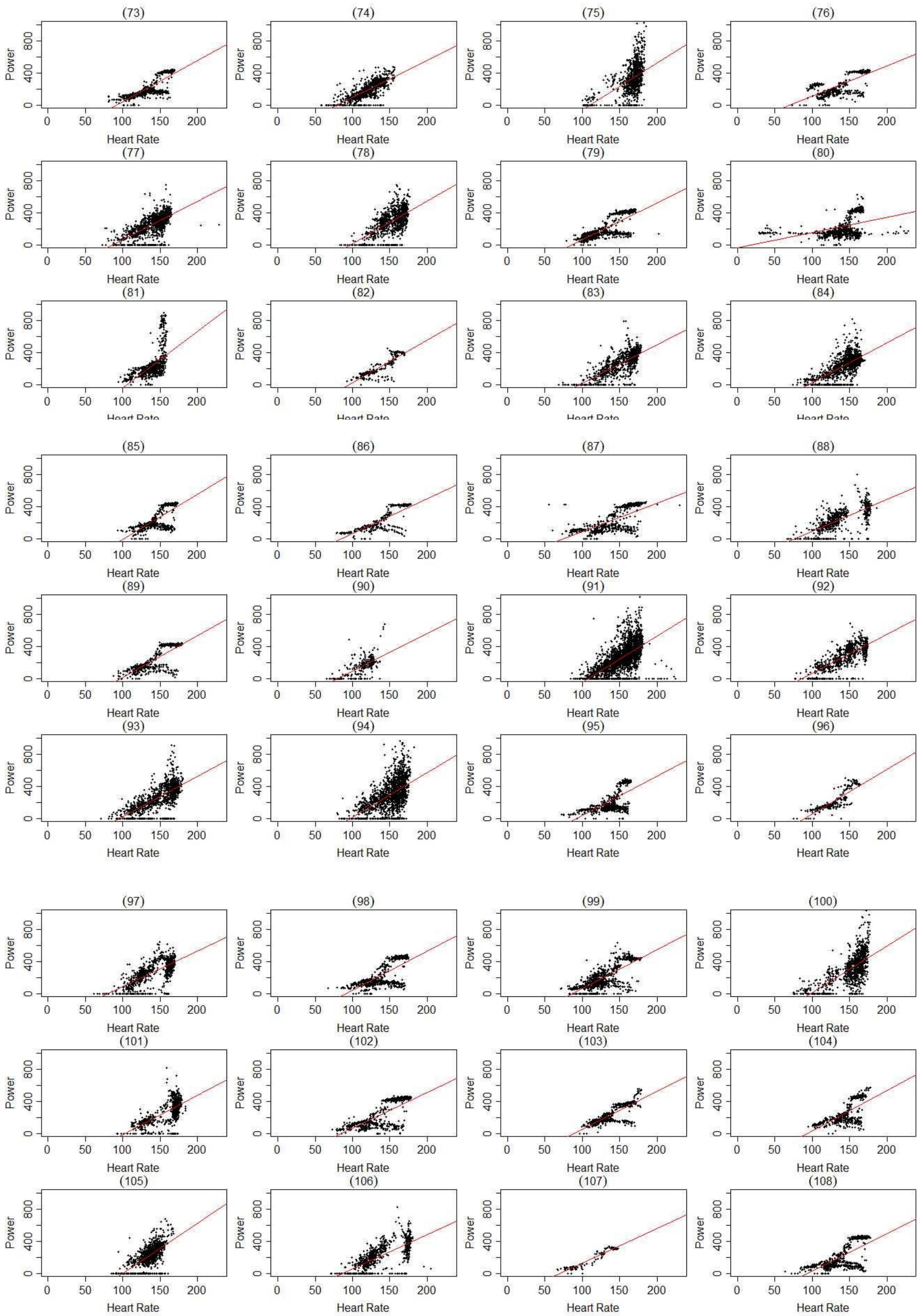

Figure A2.6 Continued.

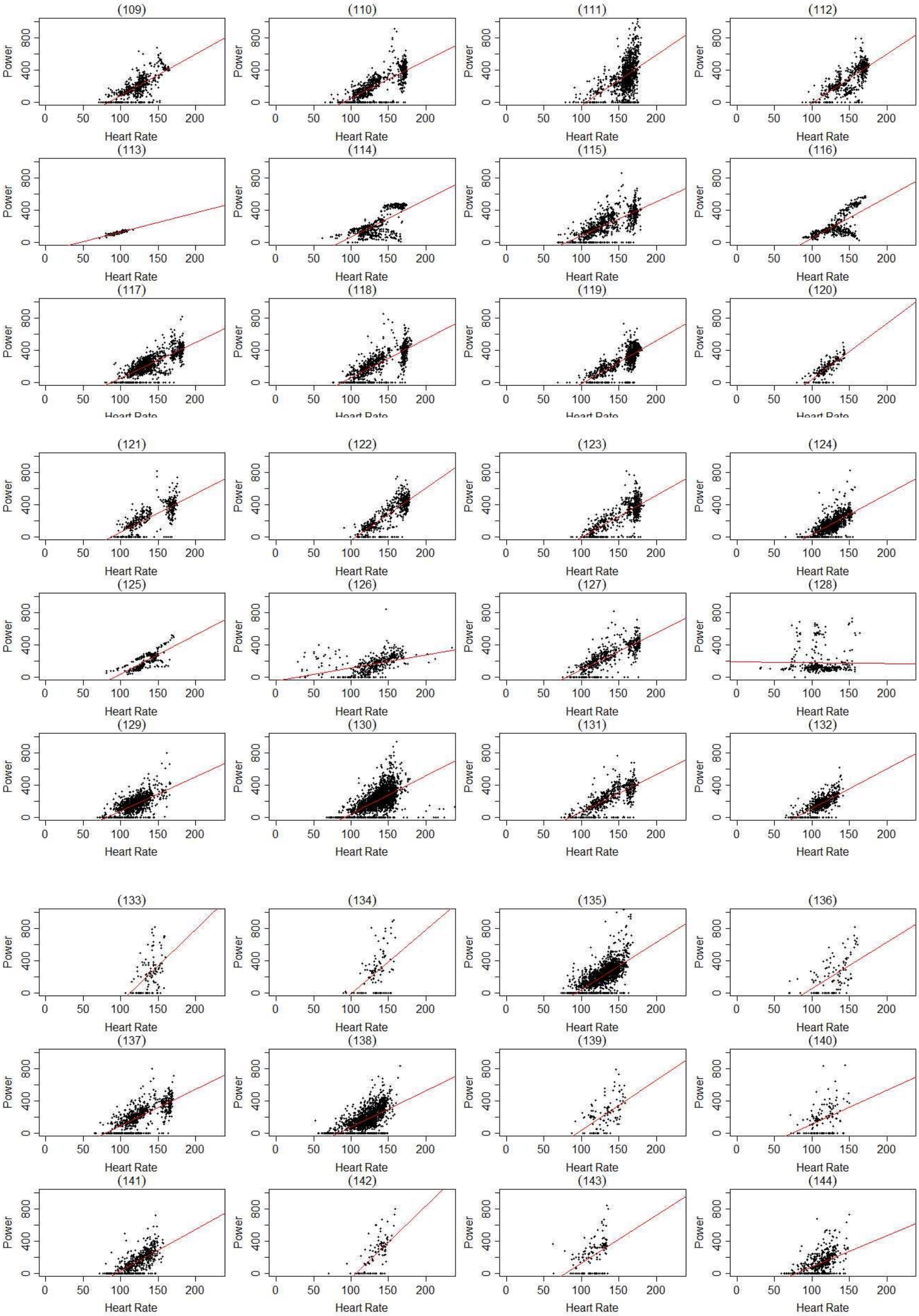

Figure A2.6 Continued.

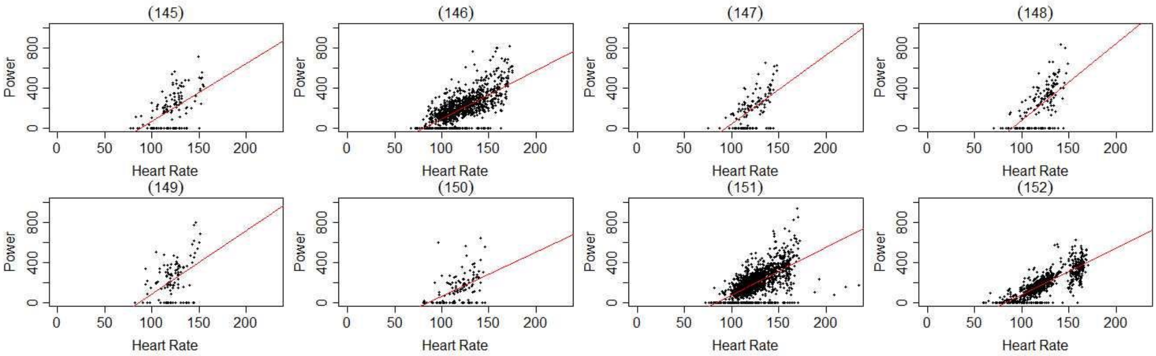

Figure A2.6 Continued.

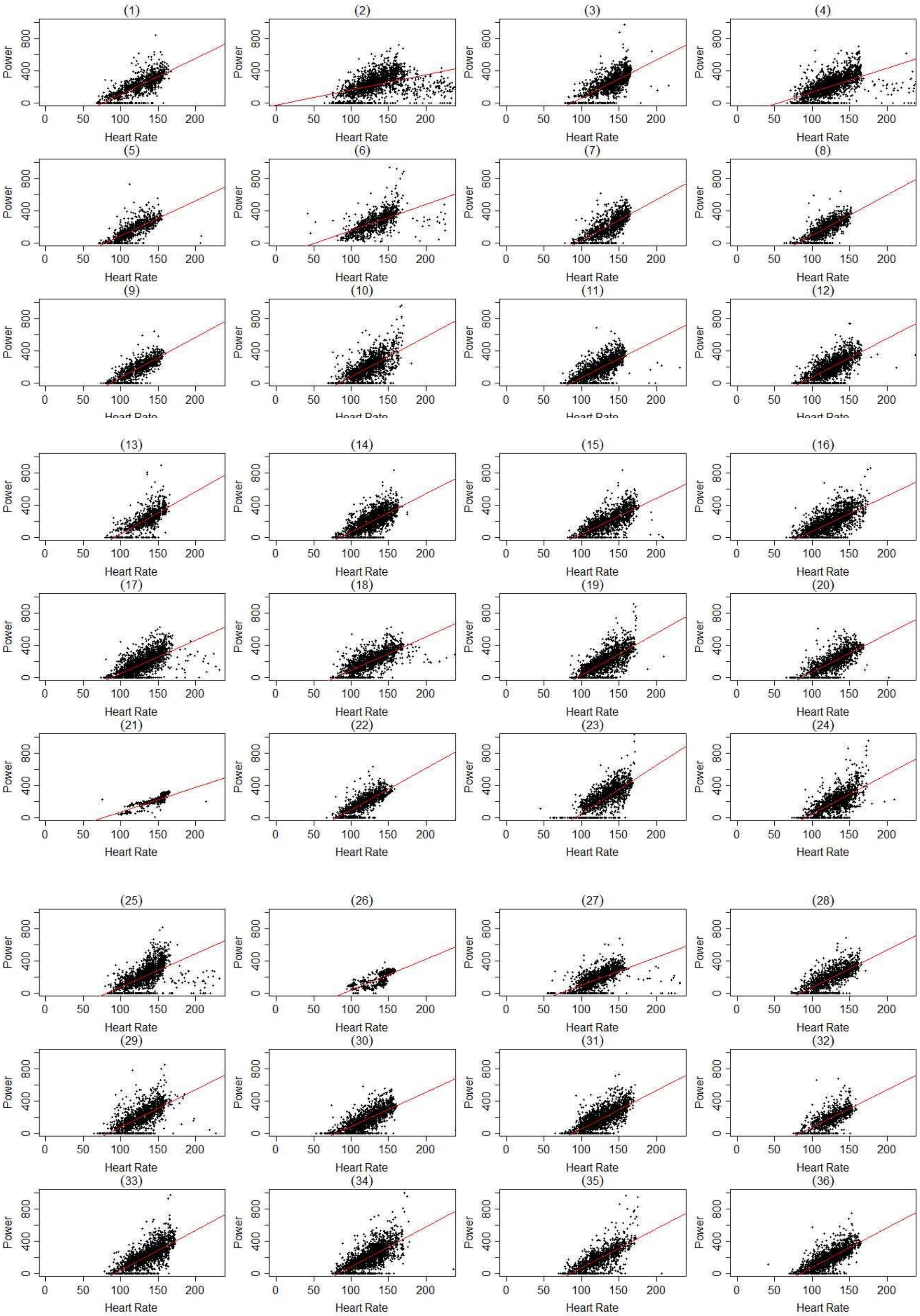

Figure A2.7 The relationship between power output and heart rate for all sessions for rider 8 with shift = 15 seconds.

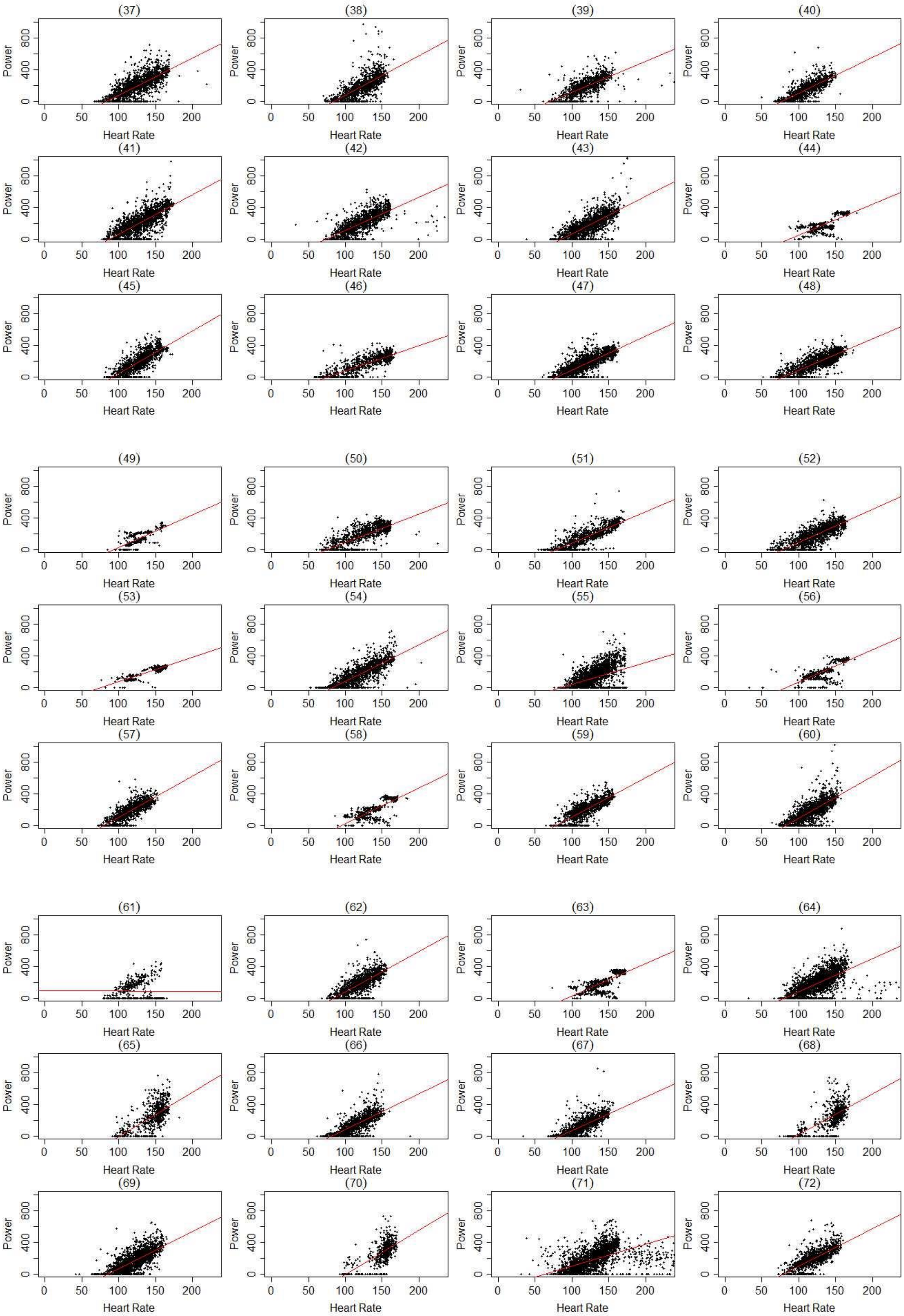

Figure A2.7 Continued.

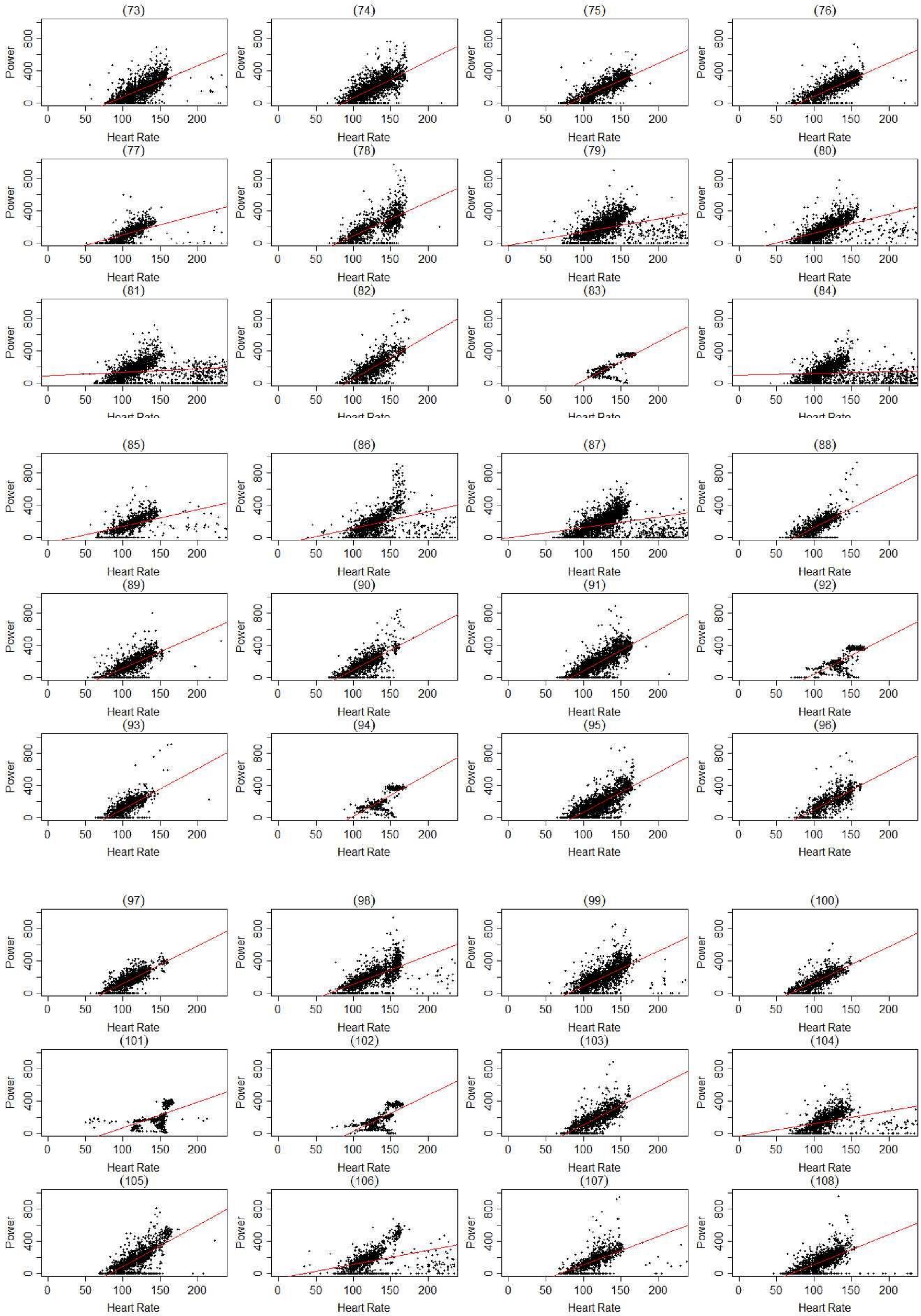

Figure A2.7 Continued.

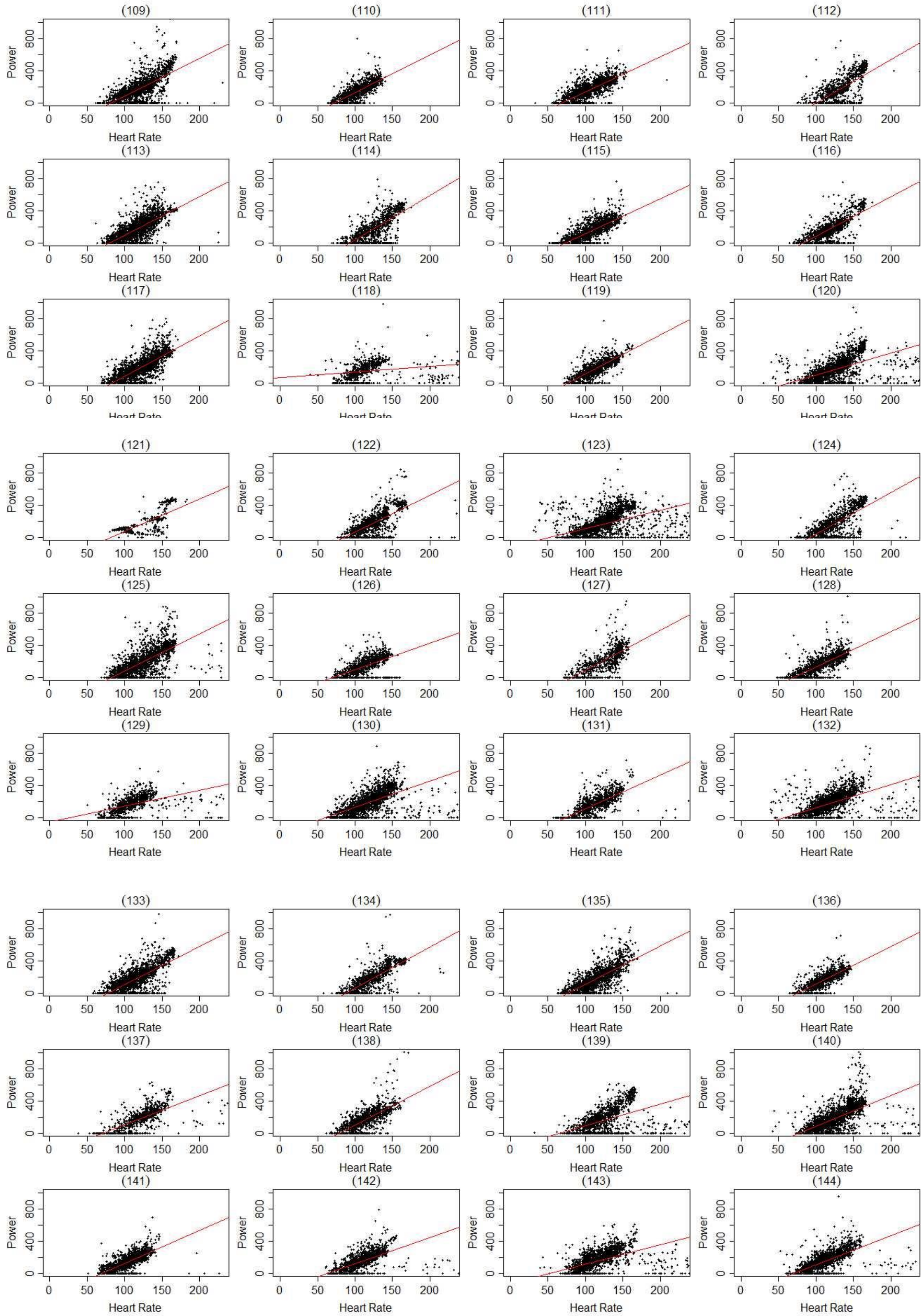

Figure A2.7 Continued.

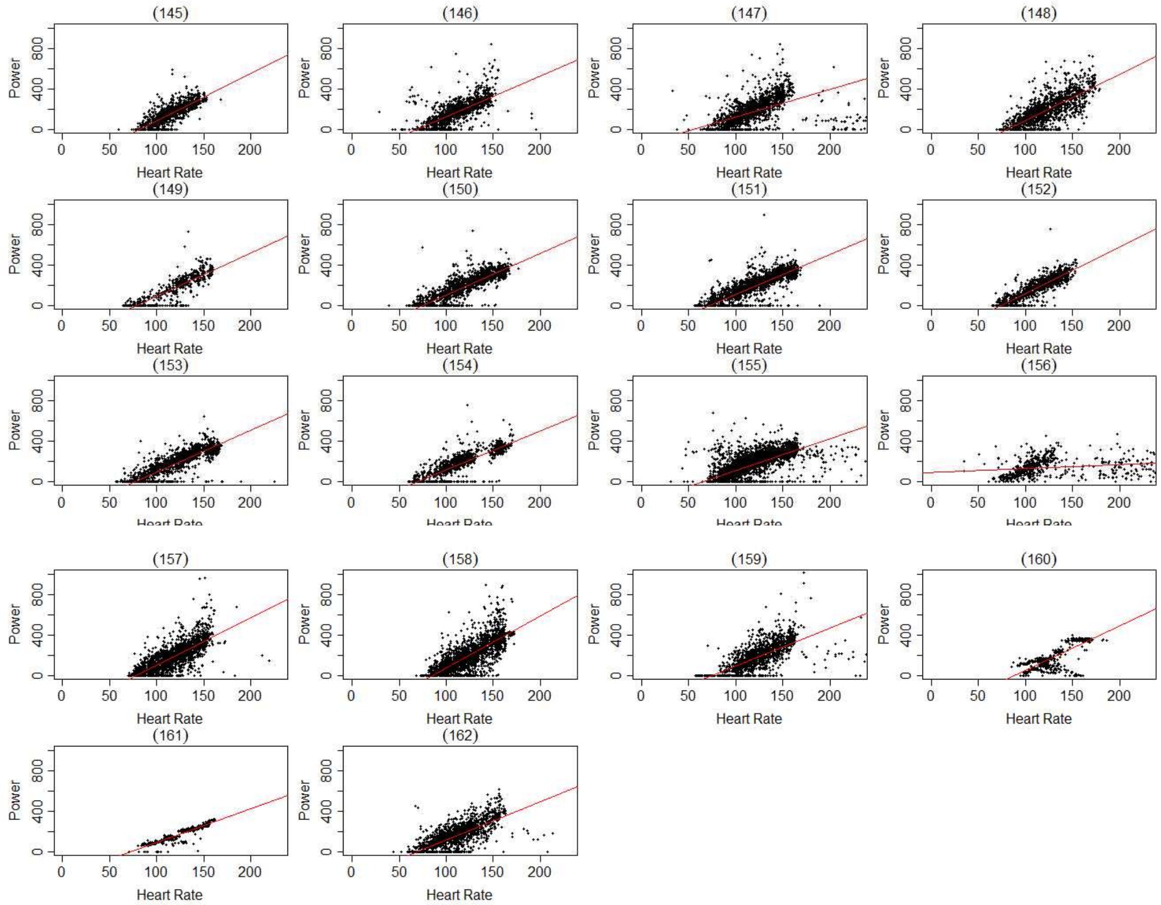

Figure A2.7 Continued.

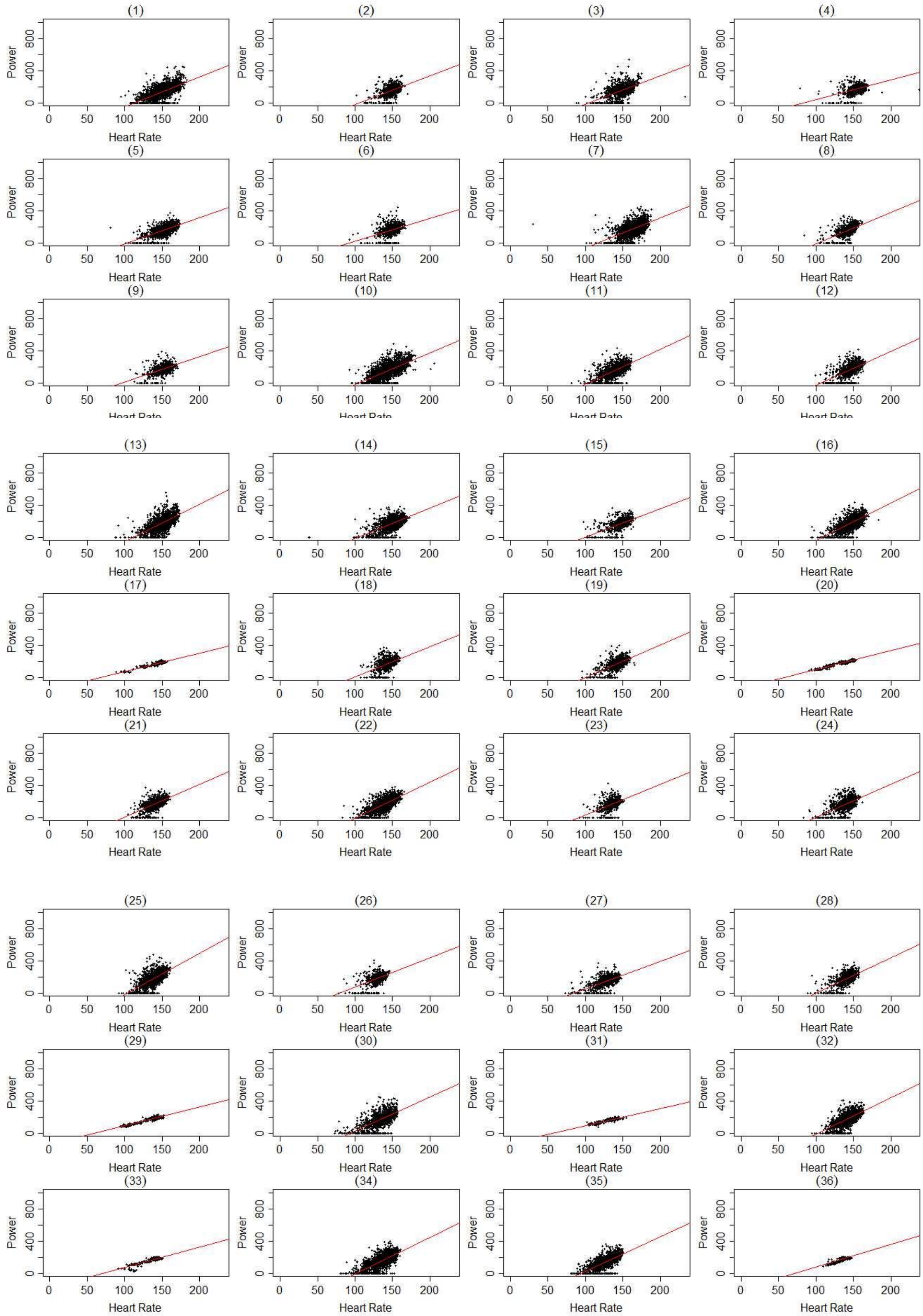

Figure A2.8 The relationship between power output and heart rate for all sessions for rider 9 with shift = 15 seconds.

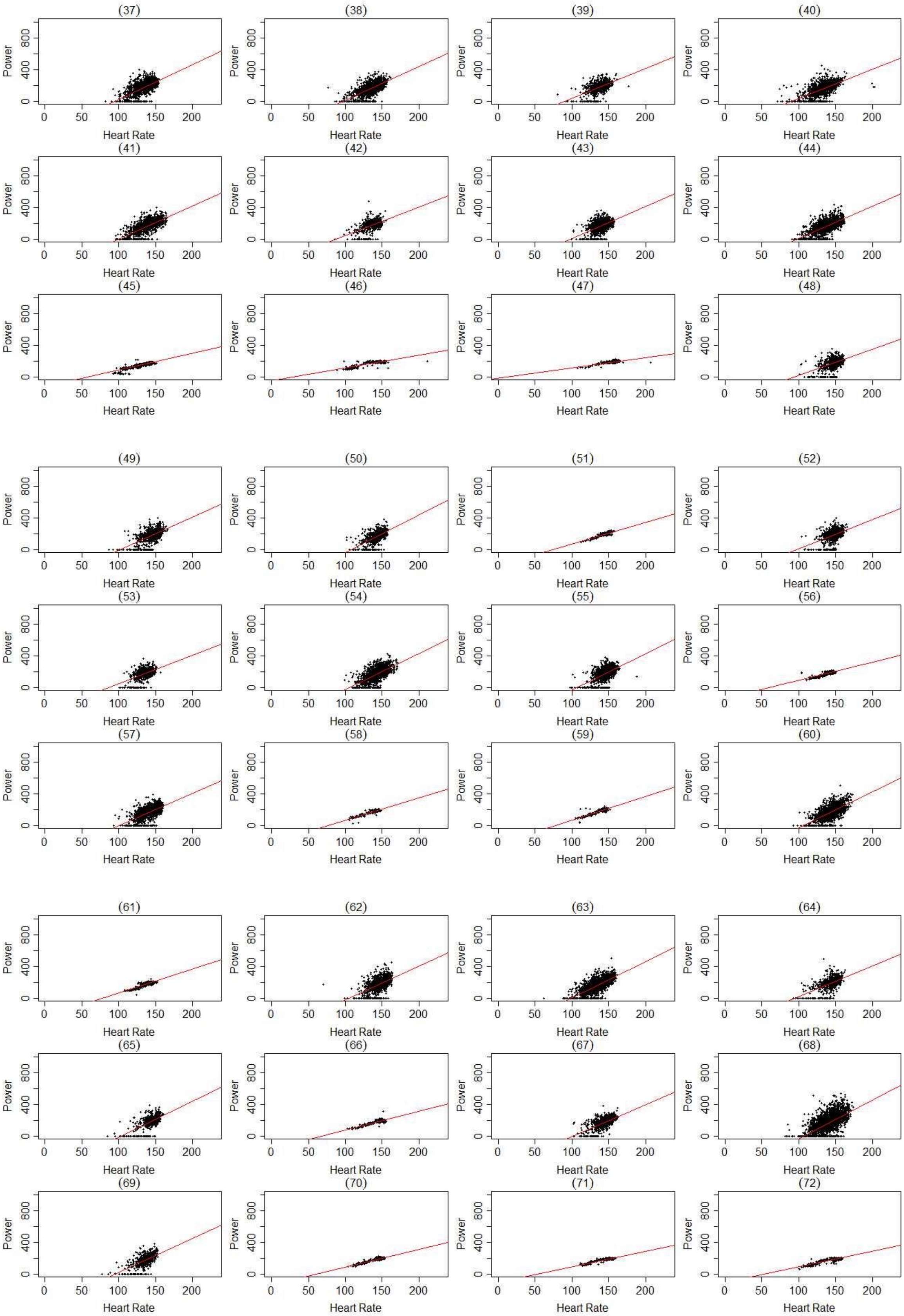

Figure A2.8 Continued.

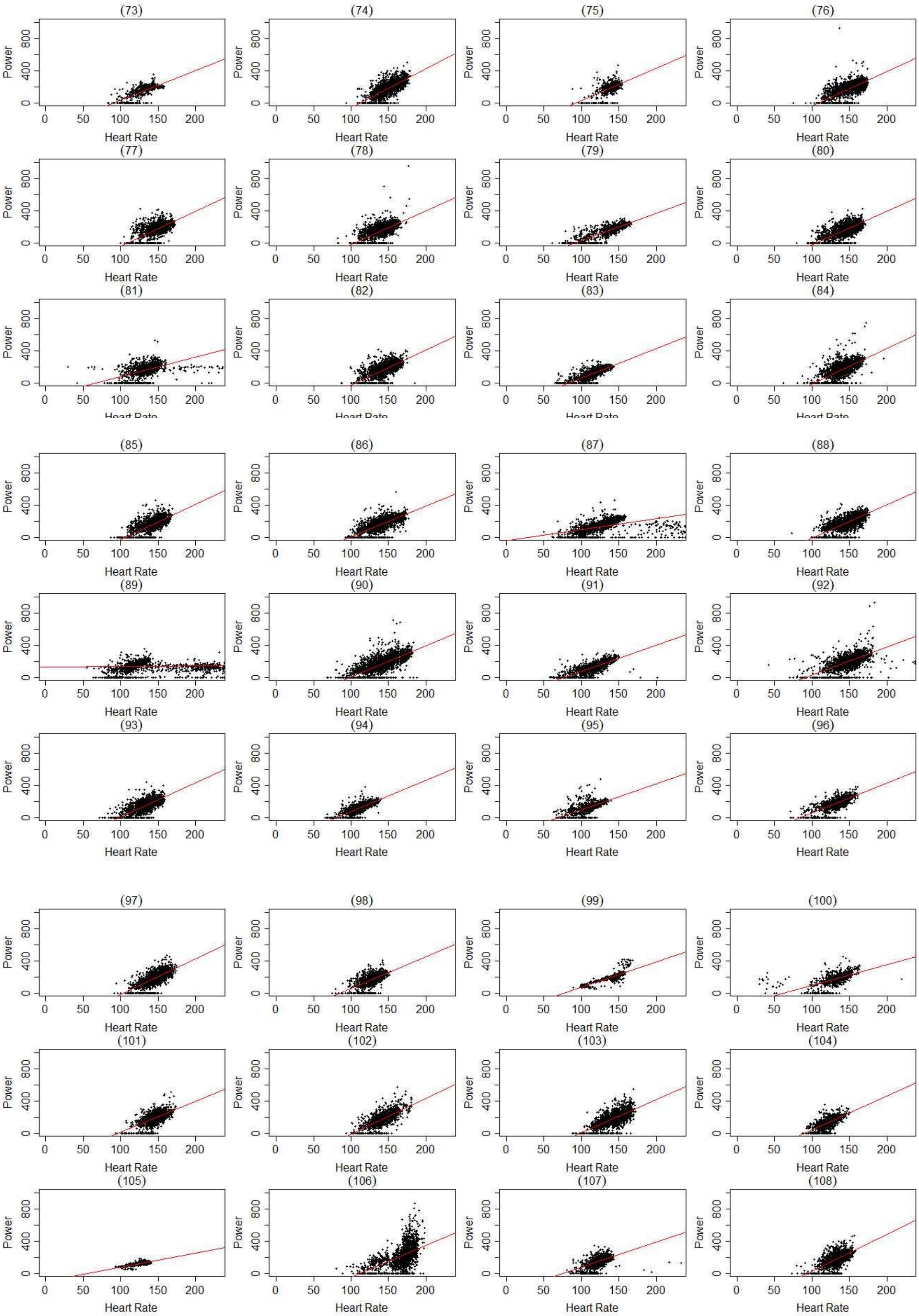

Figure A2.8 Continued.

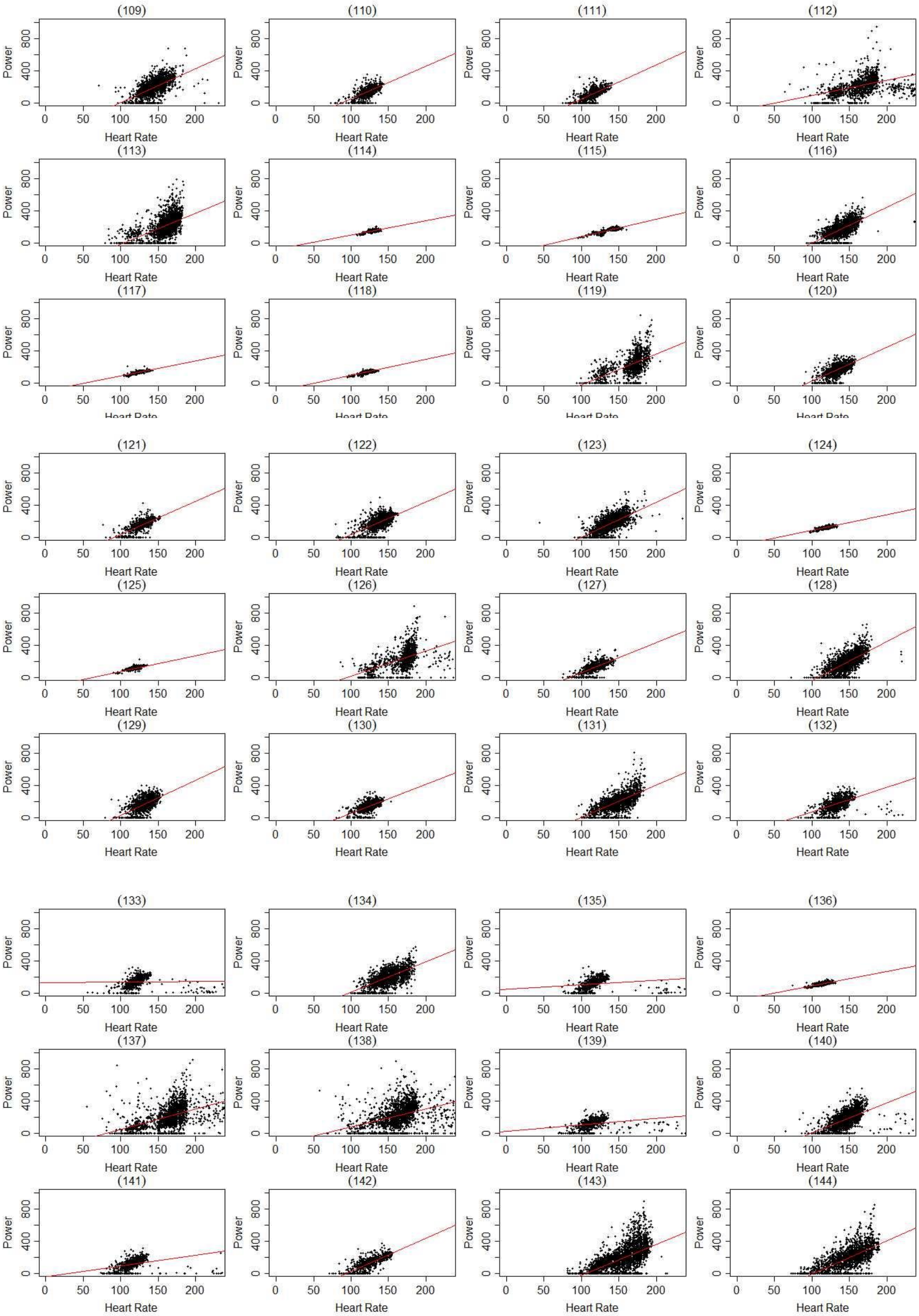

Figure A2.8 Continued.

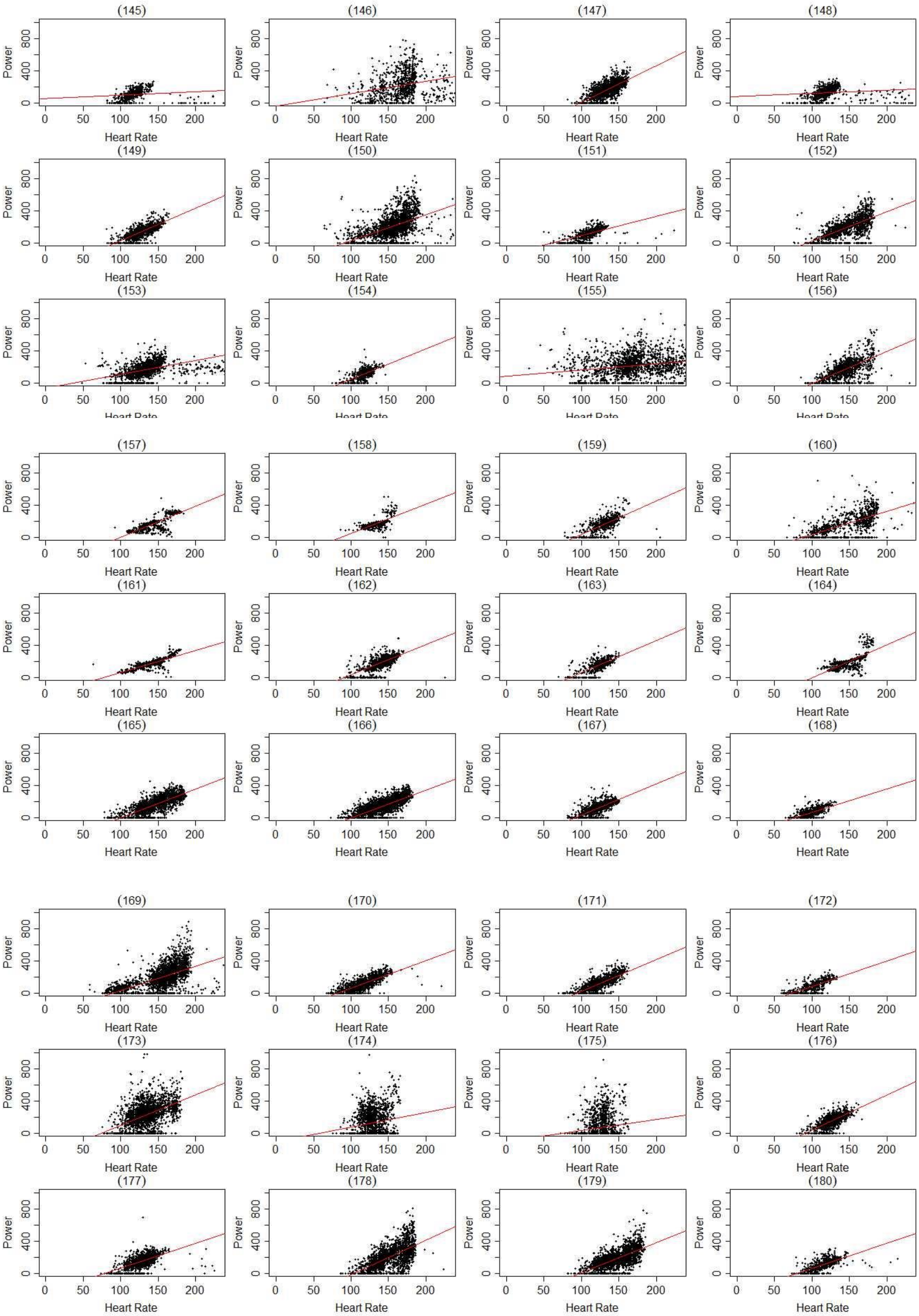

Figure A2.8 Continued.

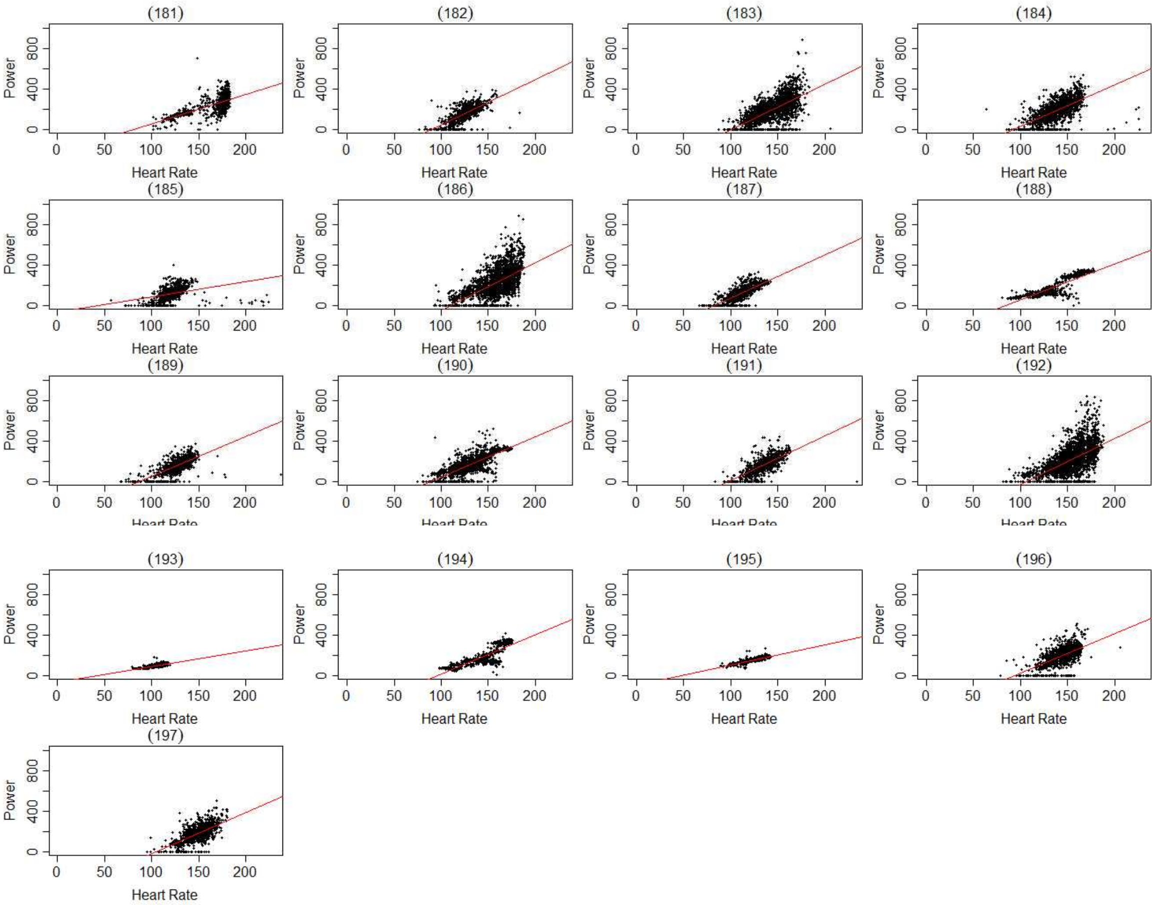

Figure A2.8 Continued.

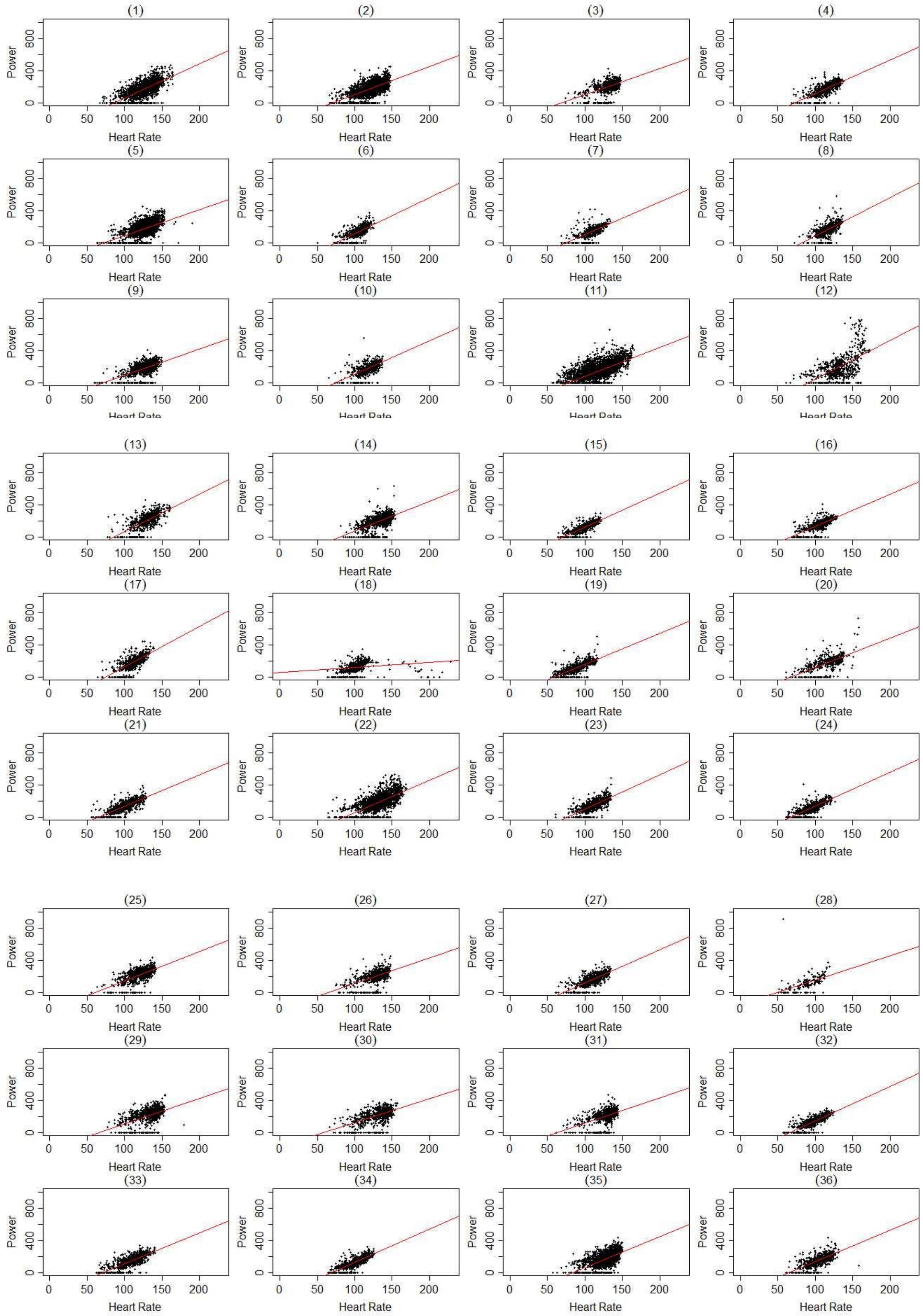

Figure A2.9 The relationship between power output and heart rate for all sessions for rider 10 with shift = 15 seconds.

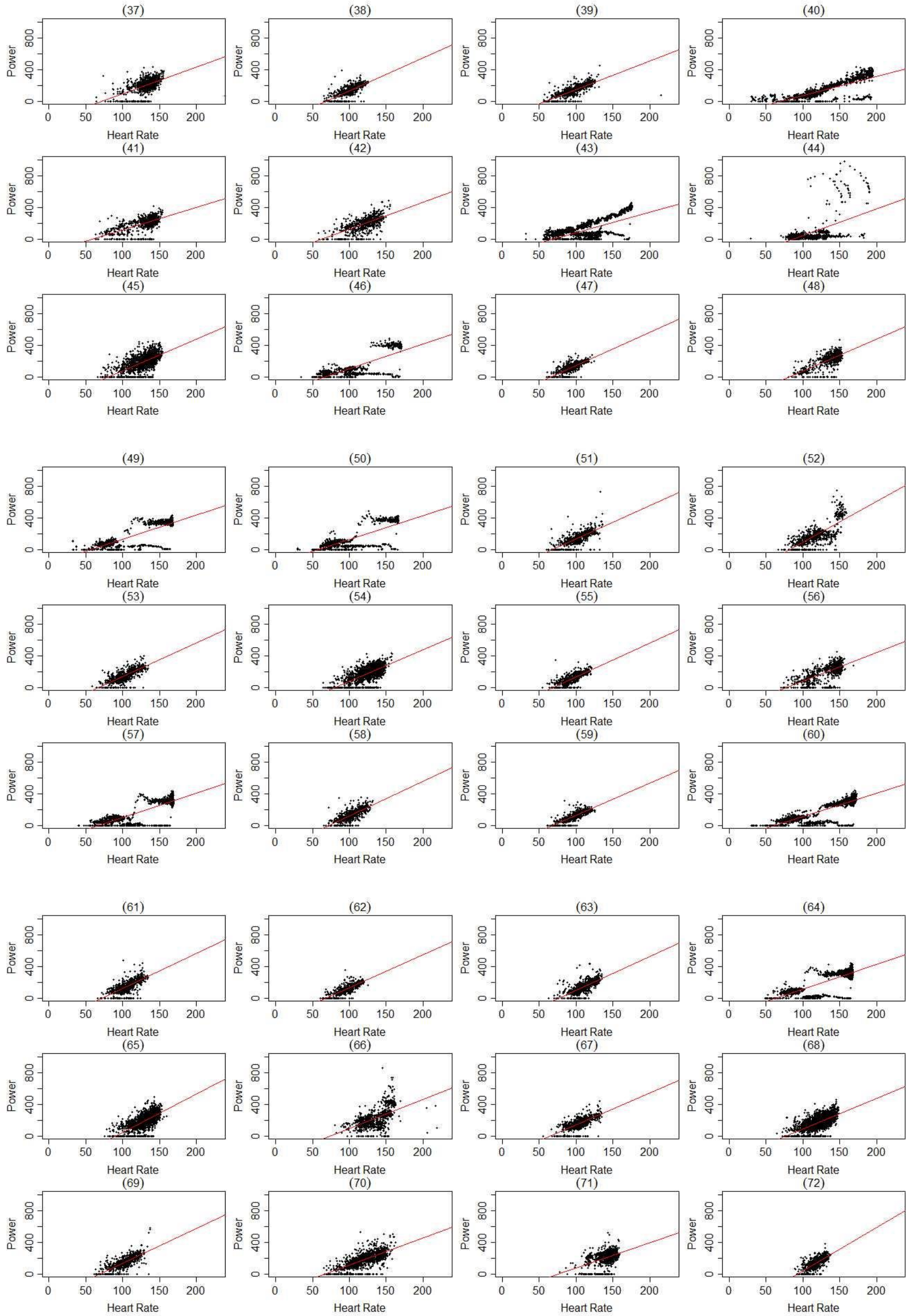

Figure A2.9 Continued.

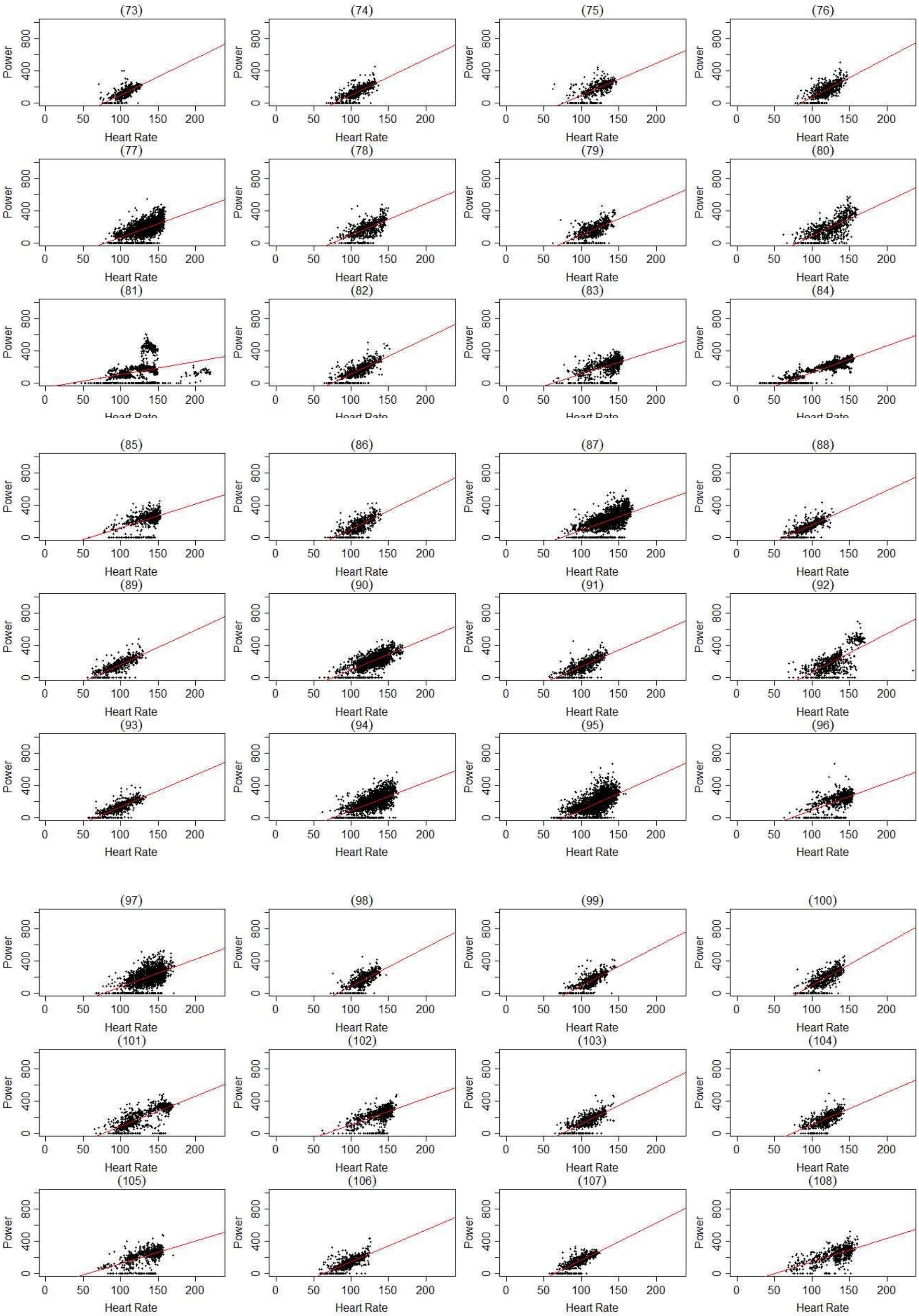

Figure A2.9 Continued.

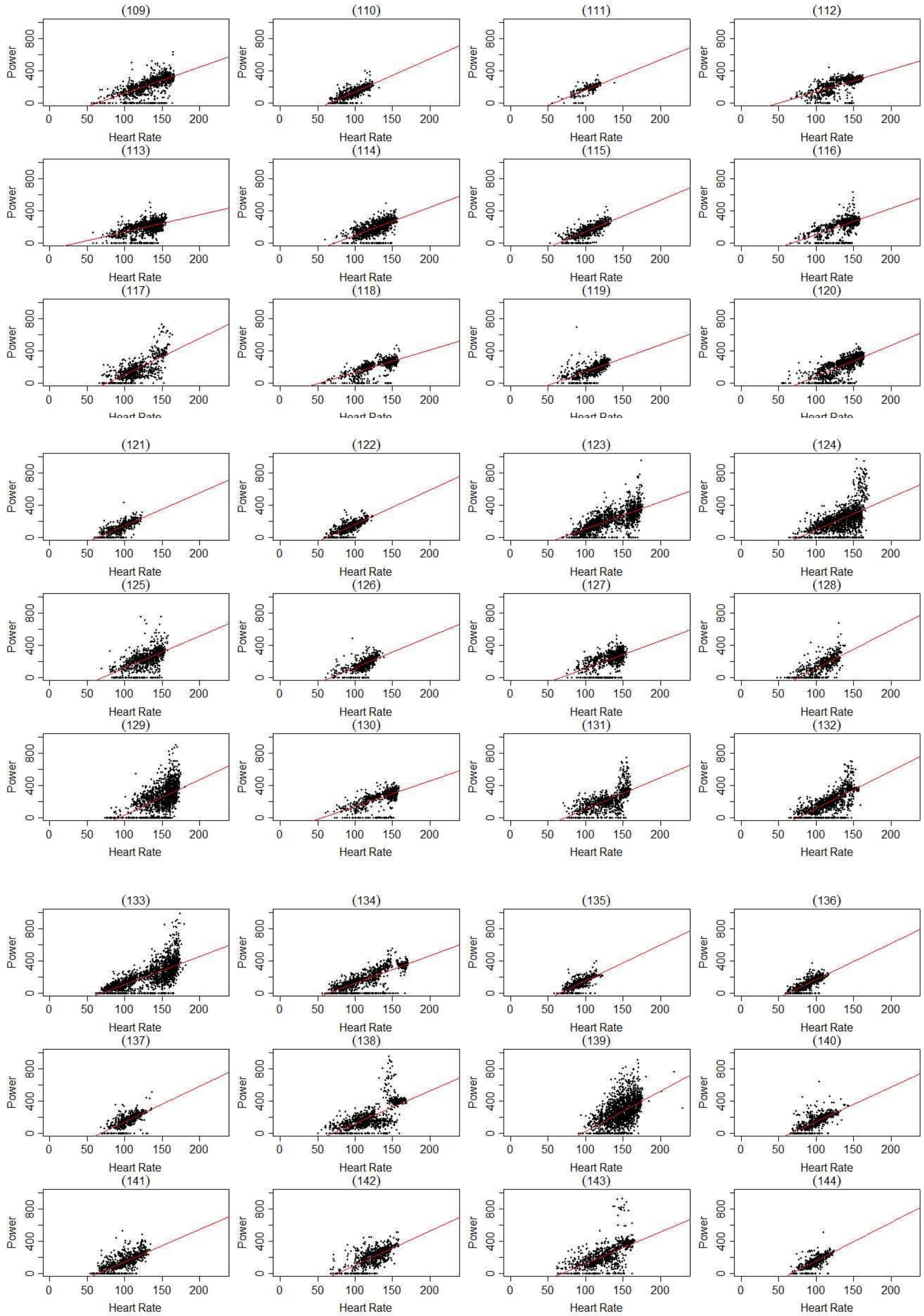

Figure A2.9 Continued.

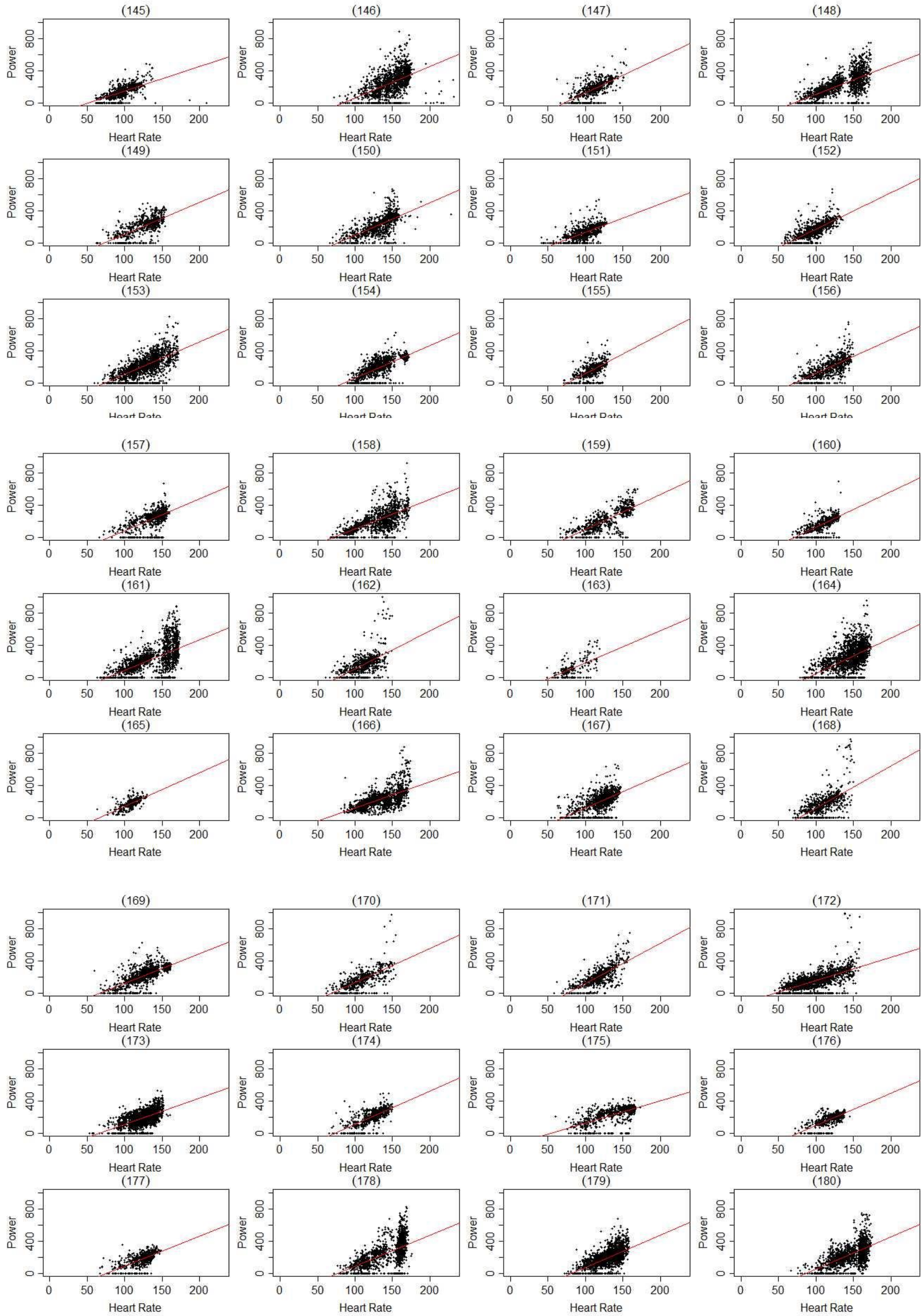

Figure A2.9 Continued.

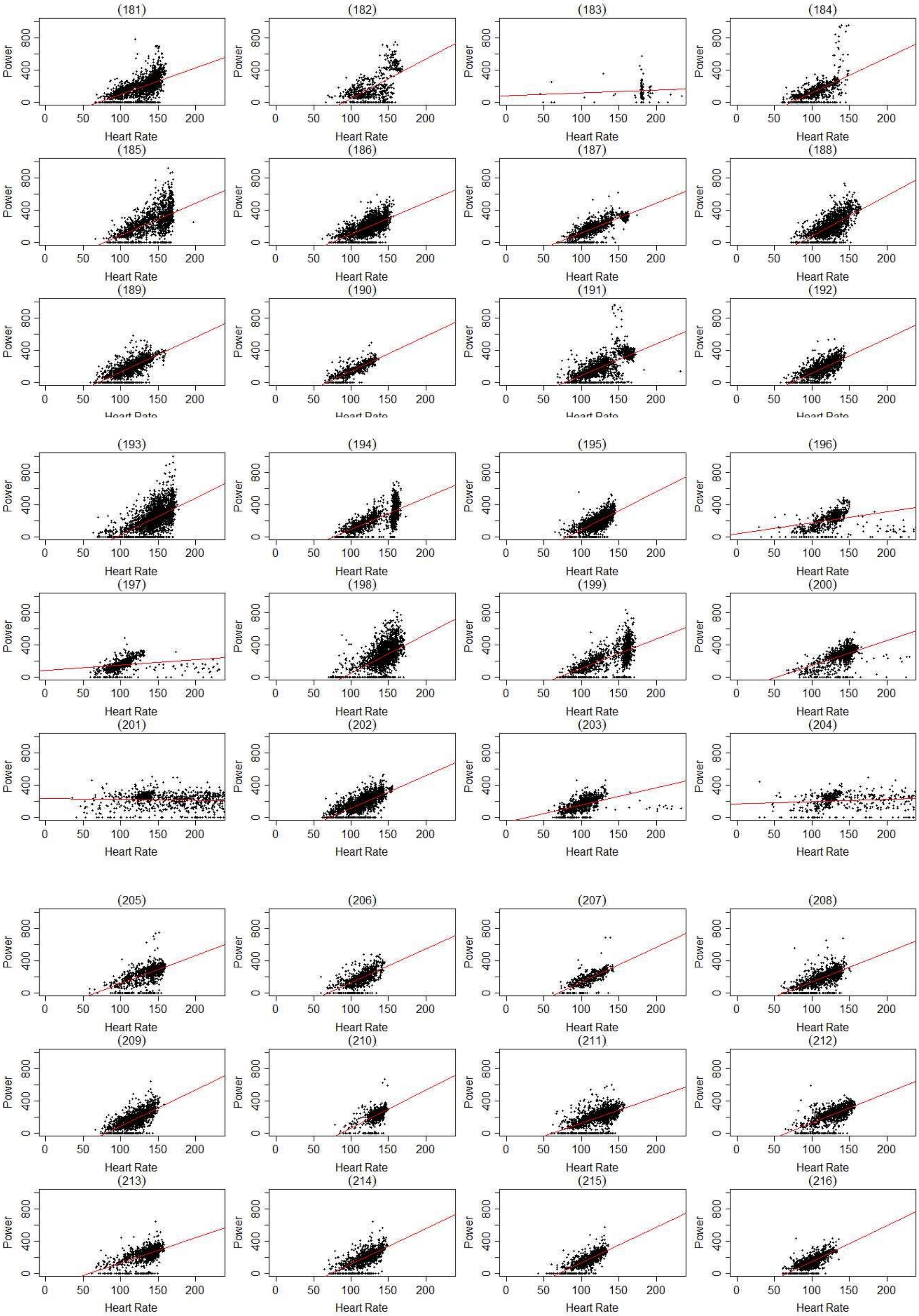

Figure A2.9 Continued.

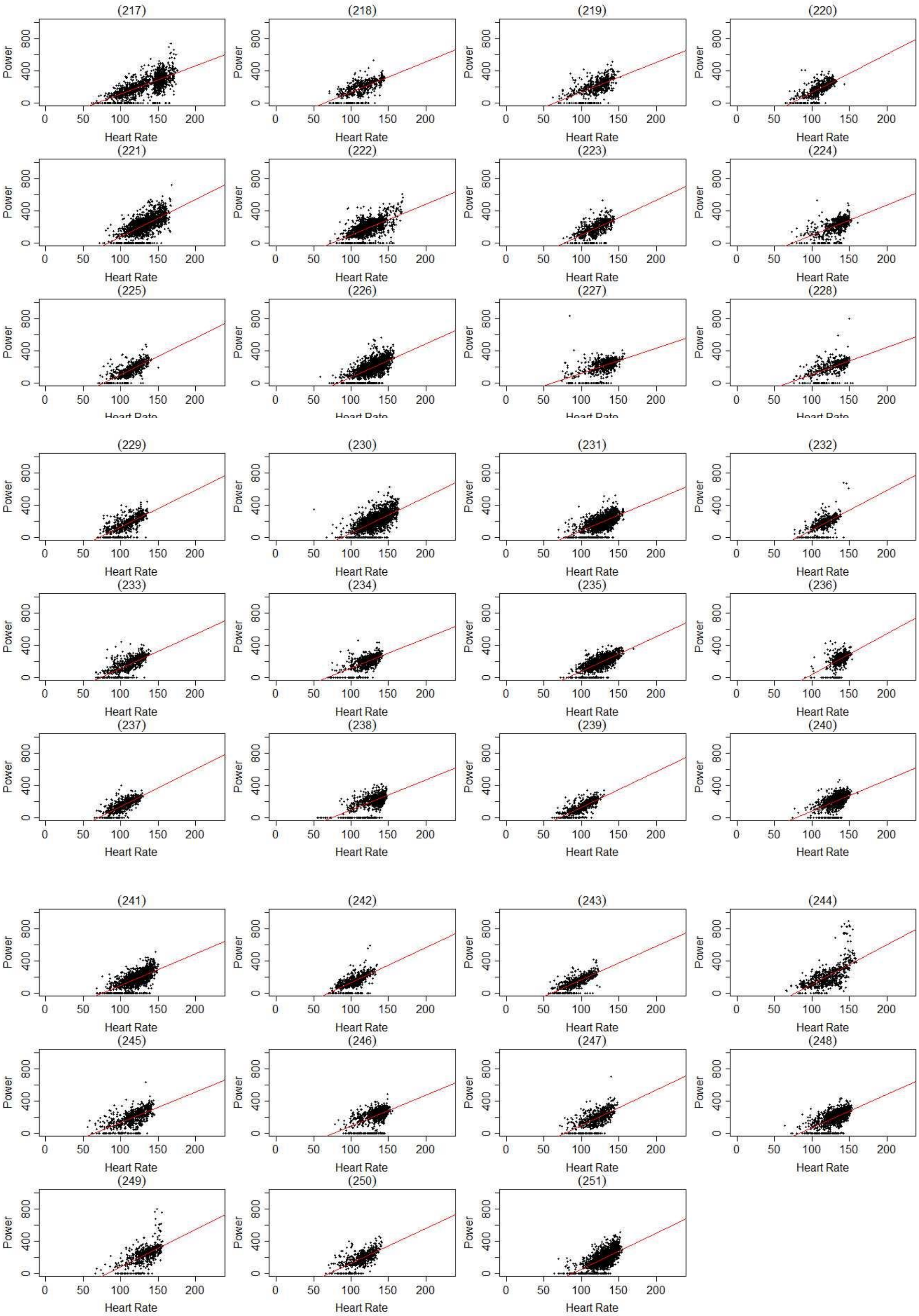

Figure A2.9 Continued.

Appendix 3 Correlations of the ATE and performance measure for various parameter values.

Table A3.1 Preliminary parameter values for the Banister model with the correlation between the performance measure h_{P75} and the accumulated training effect for rider 1

Case	k_a	k_f	τ_f	τ_a	Corr	Case	k_a	k_f	τ_f	τ_a	Corr
1	1	1	1	2	-0.10	40	1	2	3	8	-0.24
2	1	1	1	3	-0.13	41	1	2	4	5	0.10
3	1	1	1	4	-0.15	42	1	2	4	6	0.04
4	1	1	1	5	-0.17	43	1	2	4	7	-0.05
5	1	1	1	6	-0.19	44	1	2	4	8	-0.14
6	1	1	1	7	-0.21	45	1	3	1	2	0.03
7	1	1	1	8	-0.22	46	1	3	1	3	-0.09
8	1	1	2	3	-0.16	47	1	3	1	4	-0.16
9	1	1	2	4	-0.18	48	1	3	1	5	-0.19
10	1	1	2	5	-0.19	49	1	3	1	6	-0.21
11	1	1	2	6	-0.21	50	1	3	1	7	-0.22
12	1	1	2	7	-0.22	51	1	3	1	8	-0.23
13	1	1	2	8	-0.23	52	1	3	2	3	0.06
14	1	1	3	4	-0.20	53	1	3	2	4	0.02
15	1	1	3	5	-0.21	54	1	3	2	5	-0.05
16	1	1	3	6	-0.22	55	1	3	2	6	-0.12
17	1	1	3	7	-0.23	56	1	3	2	7	-0.17
18	1	1	3	8	-0.25	57	1	3	2	8	-0.21
19	1	1	4	5	-0.22	58	1	3	3	4	0.10
20	1	1	4	6	-0.23	59	1	3	3	5	0.06
21	1	1	4	7	-0.25	60	1	3	3	6	0.02
22	1	1	4	8	-0.26	61	1	4	1	7	-0.21
23	1	2	1	2	-0.07	62	1	4	1	8	-0.23
24	1	2	1	3	-0.15	63	1	4	2	3	0.07
25	1	2	1	4	-0.17	64	1	4	2	4	0.05
26	1	2	1	5	-0.19	65	1	4	2	5	0.01
27	1	2	1	6	-0.20	66	1	4	2	6	-0.04
28	1	2	1	7	-0.22	67	1	4	2	7	-0.09
29	1	2	1	8	-0.23	68	1	4	2	8	-0.13
30	1	2	2	3	0.03	69	1	4	3	4	0.10
31	1	2	2	4	-0.10	70	1	4	3	5	0.08
32	1	2	2	5	-0.18	71	1	4	3	6	0.06
33	1	2	2	6	-0.22	72	1	4	3	7	0.03
34	1	2	2	7	-0.23	73	1	4	3	8	-0.01
35	1	2	2	8	-0.25	74	1	4	4	5	0.13
36	1	2	3	4	0.07	75	1	4	4	6	0.11
37	1	2	3	5	-0.02	76	1	4	4	7	0.10
38	1	2	3	6	-0.13	77	1	4	4	8	0.07
39	1	2	3	7	-0.20	78	1	1	1	16	-0.30

Table A3.1 Continued.

Case	k_a	k_f	τ_f	τ_a	Corr	Case	k_a	k_f	τ_f	τ_a	Corr
79	1	1	2	16	-0.31	124	1	0.5	4	16	-0.31
80	1	1	3	16	-0.31	125	1	0.5	1	32	-0.35
81	1	1	4	16	-0.32	126	1	0.5	2	32	-0.35
82	1	2	1	16	-0.30	127	1	0.5	3	32	-0.34
83	1	2	2	16	-0.31	128	1	0.5	4	32	-0.34
84	1	2	3	16	-0.32	129	1	1	1	2	-0.10
85	1	2	4	16	-0.31	130	1	1	1	3	-0.13
86	1	3	1	16	-0.31	131	1	1	1	4	-0.15
87	1	3	2	16	-0.31	132	1	1	1	5	-0.17
88	1	3	3	16	-0.28	133	1	1	1	6	-0.19
89	1	3	4	16	-0.23	134	1	1	1	7	-0.21
90	1	4	1	16	-0.31	135	1	1	1	8	-0.22
91	1	4	2	16	-0.28	136	1	1	2	3	-0.16
92	1	4	3	16	-0.22	137	1	1	2	4	-0.18
93	1	4	4	16	-0.13	138	1	1	2	5	-0.19
94	1	1	1	32	-0.35	139	1	1	2	6	-0.21
95	1	1	2	32	-0.34	140	1	1	2	7	-0.22
96	1	1	3	32	-0.34	141	1	1	2	8	-0.23
97	1	1	4	32	-0.34	142	1	1	3	4	-0.20
98	1	2	1	32	-0.35	143	1	1	3	5	-0.21
99	1	2	2	32	-0.34	144	1	1	3	6	-0.22
100	1	2	3	32	-0.33	145	1	1	3	7	-0.23
101	1	2	4	32	-0.31	146	1	1	3	8	-0.25
102	1	3	1	32	-0.34	147	1	1	4	5	-0.22
103	1	3	2	32	-0.33	148	1	1	4	6	-0.23
104	1	3	3	32	-0.30	149	1	1	4	7	-0.25
105	1	3	4	32	-0.27	150	1	1	4	8	-0.26
106	1	4	1	32	-0.34	151	1	1.2	1	2	-0.11
107	1	4	2	32	-0.31	152	1	1.2	1	3	-0.13
108	1	4	3	32	-0.27	153	1	1.2	1	4	-0.16
109	1	4	4	32	-0.22	154	1	1.2	1	5	-0.18
110	1	0.5	1	2	-0.09	155	1	1.2	1	6	-0.19
111	1	0.5	1	3	-0.12	156	1	1.2	1	7	-0.21
112	1	0.5	1	4	-0.15	157	1	1.2	1	8	-0.22
113	1	0.5	1	5	-0.17	158	1	1.2	2	3	-0.18
114	1	0.5	1	6	-0.18	159	1	1.2	2	4	-0.19
115	1	0.5	1	7	-0.20	160	1	1.2	2	5	-0.20
116	1	0.5	1	8	-0.21	161	1	1.2	2	6	-0.22
117	1	0.5	2	3	-0.13	162	1	1.2	2	7	-0.23
118	1	0.5	2	4	-0.15	163	1	1.2	2	8	-0.24
119	1	0.5	2	5	-0.17	164	1	1.2	3	4	-0.20
120	1	0.5	2	6	-0.19	165	1	1.2	3	5	-0.22
121	1	0.5	1	16	-0.30	166	1	1.2	3	6	-0.23
122	1	0.5	2	16	-0.30	167	1	1.2	3	7	-0.24
123	1	0.5	3	16	-0.30	168	1	1.2	3	8	-0.25

Table A3.1 Continued.

Case	k_a	k_f	τ_f	τ_a	Corr	Case	k_a	k_f	τ_f	τ_a	Corr
169	1	1.2	4	5	-0.19	214	1	1.8	2	4	-0.15
170	1	1.2	4	6	-0.25	215	1	1.8	2	5	-0.20
171	1	1.2	4	7	-0.26	216	1	1.8	2	6	-0.22
172	1	1.2	4	8	-0.27	217	1	1.8	2	7	-0.24
173	1	1.4	1	2	-0.11	218	1	1.8	2	8	-0.25
174	1	1.4	1	3	-0.14	219	1	1.8	3	4	0.05
175	1	1.4	1	4	-0.16	220	1	1.8	3	5	-0.08
176	1	1.4	1	5	-0.18	221	1	1.8	3	6	-0.18
177	1	1.4	1	6	-0.20	222	1	1.8	3	7	-0.23
178	1	1.4	1	7	-0.21	223	1	1.8	3	8	-0.25
179	1	1.4	1	8	-0.22	224	1	1.8	4	5	0.09
180	1	1.4	2	3	-0.14	225	1	1.8	4	6	0
181	1	1.4	4	6	-0.20	226	1	1.8	4	7	-0.12
182	1	1.4	4	7	-0.25	227	1	1.8	4	8	-0.20
183	1	1.4	4	8	-0.27	228	1	2	1	2	-0.07
184	1	1.6	1	2	-0.11	229	1	2	1	3	-0.15
185	1	1.6	1	3	-0.14	230	1	2	1	4	-0.17
186	1	1.6	1	4	-0.16	231	1	2	1	5	-0.19
187	1	1.6	1	5	-0.18	232	1	2	1	6	-0.20
188	1	1.6	1	6	-0.20	233	1	2	1	7	-0.22
189	1	1.6	1	7	-0.21	234	1	2	1	8	-0.23
190	1	1.6	1	8	-0.23	235	1	2	2	3	0.03
191	1	1.6	2	3	-0.05	236	1	2	2	4	-0.10
192	1	1.6	2	4	-0.18	237	1	2	2	5	-0.18
193	1	1.6	2	5	-0.21	238	1	2	2	6	-0.22
194	1	1.6	2	6	-0.22	239	1	2	2	7	-0.23
195	1	1.6	2	7	-0.24	240	1	2	2	8	-0.25
196	1	1.6	2	8	-0.25	241	1	3	3	7	-0.04
197	1	1.6	3	4	0.02	242	1	3	3	8	-0.09
198	1	1.6	3	5	-0.15	243	1	3	4	5	0.12
199	1	1.6	3	6	-0.22	244	1	3	4	6	0.10
200	1	1.6	3	7	-0.25	245	1	3	4	7	0.07
201	1	1.6	3	8	-0.26	246	1	3	4	8	0.03
202	1	1.6	4	5	0.07	247	1	4	1	2	0.05
203	1	1.6	4	6	-0.08	248	1	4	1	3	-0.02
204	1	1.6	4	7	-0.20	249	1	4	1	4	-0.10
205	1	1.6	4	8	-0.25	250	1	4	1	5	-0.16
206	1	1.8	1	2	-0.10	251	1	4	1	6	-0.19
207	1	1.8	1	3	-0.14	252	1	0.5	2	7	-0.21
208	1	1.8	1	4	-0.17	253	1	0.5	2	8	-0.22
209	1	1.8	1	5	-0.19	254	1	0.5	3	4	-0.15
210	1	1.8	1	6	-0.20	255	1	0.5	3	5	-0.18
211	1	1.8	1	7	-0.21	256	1	0.5	3	6	-0.19
212	1	1.8	1	8	-0.23	257	1	0.5	3	7	-0.21
213	1	1.8	2	3	0	258	1	0.5	3	8	-0.23

Table A3.1 Continued.

Case	k_a	k_f	τ_f	τ_a	Corr	Case	k_a	k_f	τ_f	τ_a	Corr
259	1	0.5	4	5	-0.17	271	1	1.4	3	7	-0.25
260	1	0.5	4	6	-0.19	272	1	1.4	3	8	-0.26
261	1	0.5	4	7	-0.21	273	1	1.4	4	5	0.01
262	1	0.5	4	8	-0.23	274	1	2	3	4	0.07
263	1	1.4	2	4	-0.19	275	1	2	3	5	-0.02
264	1	1.4	2	5	-0.21	276	1	2	3	6	-0.13
265	1	1.4	2	6	-0.22	277	1	2	3	7	-0.20
266	1	1.4	2	7	-0.23	278	1	2	3	8	-0.24
267	1	1.4	2	8	-0.24	279	1	2	4	5	0.10
268	1	1.4	3	4	-0.07	280	1	2	4	6	0.04
269	1	1.4	3	5	-0.21	281	1	2	4	7	-0.05
270	1	1.4	3	6	-0.24	282	1	2	4	8	-0.14

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