Dissipative solitons of a spatiotemporal Ginzburg-Landau equation

A. R. Cooper[1], J. M. Christian[1], D. W. Bresnahan[1], G. S. McDonald[1], and P. Chamorro-Posada[2]

[1] *Materials & Physics Research Centre, University of Salford, Greater Manchester M5 4WT, U.K.*

[2] *ETSI Telecomunicación, Universidad de Valladolid, Campus Miguel Delibes Paseo Belén 15, E-47011 Valladolid, Spain*

Complex Ginzburg-Landau (GL) equations play a fundamental role in our understanding of universal wave propagation in generic systems with dispersion, nonlinearity, gain, and loss. Over three decades ago, stationary bright-type solitons were derived by Pereira and Stenflo (PS solitons) for a cubically-nonlinear GL equation [Phys. Fluids, **20**, 1733 (1977)]. Dark-type solutions followed a short time later, introduced by Nozaki and Bekki (NB solitons) [J. Phys. Soc. Jpn. **53**, 5365 (1984)]. In photonics, such self-localizing optical wavepackets can typically arise when group-velocity dispersion is balanced by self-phase modulation, and nonlinear losses (e.g., from two-photon absorption) are compensated by linear gain (e.g., from doping in the host medium). In the corresponding GL model, linear gain tends to introduce instability (growth of infinitesimally-small perturbations) such that all its solutions are rendered unstable in the long term.

In this presentation, we introduce a spatiotemporal generalization of the classic GL equation. Our approach is based on a fully-second order scalar governing equation that omits the traditional prescription based on: (i) assumption of slowly-varying envelopes, and (ii) a Galilean boost to the local time frame (usually introduced for mathematical convenience). As a consequence, our model naturally describes wave phenomena from the laboratory reference frame. Moreover, the velocity combination rule that emerges is reminiscent of that which appears in Einstein's relativistic mechanics [Christian *et al.*, Phys. Rev. Lett. **108**, 034101 (2012)]. Exact bright and dark dissipative solitons have been derived, and their space-time geometry explored in detail. Crucially, the classic PS and NB solutions emerge asymptotically from their spatiotemporal generalizations in a simultaneous multiple limit. Of particular importance is that the inherent instability of the zero-amplitude solution, which plagues waves of the cubically-nonlinear GL equation, persists in the spatiotemporal case. However, we have taken the first steps toward stabilizing our model and preliminary results will be reported.