Optical snowflakes:
from Fresnel diffraction to a new class of unstable resonator

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Unstable cavity lasers are linear systems with inherent magnification. It has long been known that the eigenmodes of unstable strip resonators have fractal characteristics [Karman and Woerdman, Opt. Lett. **23**, 1901 (1998)], possessing proportional levels of detail across decimal orders of spatial scale. Kaleidoscope lasers are generalizations of the strip resonator to fully two-dimensional (2D) transverse geometries where the feedback mirror has the shape of a regular polygon [McDonald *et al.*, JOSA B **17**, 524 (2000)]. Here, we propose a new class of unstable resonator: *the snowflake laser*. This novel system has a feedback mirror whose shape matches a classic fractal curve – the von Koch snowflake (an iterated function system involving self-similar sequences of equilateral triangles). As such, we have now designed a cavity whose eigenmodes are inherently fractal, and where successive round trips involve the interplay of that fractal light beam with a fractal aperturing element.

In this presentation, we show how the 2D virtual source (2D-VS) method deployed for kaleidoscope geometries [Huang, Christian, and McDonald, JOSA A **23**, 2768 (2006)] can be applied to modelling the snowflake laser. A key development has been an exact analytical reformulation of the Fresnel diffraction problem for snowflake apertures using a line integral [Hannay, J. Mod. Opt. **47**, 121 (2000)]. In contrast to the traditional Fox-Li approach (based upon paraxial ABCD matrix modelling and fast Fourier transforms), the 2D-VS approach permits entire families of eigenmodes to be obtained from a single calculation (from lowest loss through the hierarchy of higher-order modes). Furthermore, arbitrary cavity parameters (equivalent Fresnel number and round-trip magnification) may be specified and patterns calculated to any prescribed accuracy. A selection of mode patterns and eigenvalue spectra will be reported for increasing iterations of the von Koch snowflake (as the aperture tends towards a fully-developed fractal), and computational challenges highlighted.