Spatiotemporal vector solitons in nonlinear optical systems with $\chi^{(3)}$ susceptibility

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Summary

A new model is proposed for describing coupled optical waves propagating in a system with linear dispersion (both group-velocity and spatial contributions) and $\chi^{(3)}$ nonlinear susceptibility. The modulational instability problem is solved in full, and its predictions are related to the analysis of new exact analytical vector soliton families.

Spatiotemporal model equations

Recently [1], we considered a model of scalar nonlinear optical pulse propagation that captures the novel physical context of spatial dispersion as introduced by Biancalana and Creatore [2]. Frame-of-reference considerations and coordinate transformations play a central role in our approach (for instance, the velocity combination rule is akin to that in relativistic kinematics). The assumption of slowly-varying envelopes is omitted (since it can frustrate the description of spatial dispersion phenomena), and we do not use the standard device of Galilean-boosting to the local time frame (since it hinders, rather than helps, the analysis).

Menyuk's classic coupled nonlinear Schrödinger (NLS)-type model has proved to be a theoretical mainstay for describing the propagation of optical waves in Kerr-type systems [3]. Here, we report on recent research that combines our spatiotemporal formalism [1] with (two-component) vector geometries. In normalized units, envelopes u_j (with j = 1 and 2) satisfy the following pair of coupled wave equations in the laboratory frame:

$$\kappa_j \frac{\partial^2 u_j}{\partial \zeta^2} + i\left(\frac{\partial u_j}{\partial \zeta} + \alpha_j \frac{\partial u_j}{\partial \tau}\right) + \frac{s_j}{2}\frac{\partial^2 u_j}{\partial \tau^2} + \left(|u_j|^2 + \sigma |u_{3-j}|^2\right) u_j = 0, \qquad (1)$$

where τ / ζ denote time / longitudinal coordinates, the (inverse) group speed of wave j is described by α_j, spatial / group-velocity dispersion (GVD) is parametrized by κ_j / s_j, and the strength of the wave coupling (cross-phase modulation) is determined by σ.

Modulational instabilities

The continuous wave (cw) vector solution to model (1) is of the form $u_j(\tau,\zeta)$ = $u_{0j}\exp[i(\Omega_j\tau + k_j\zeta)]\exp(-i\zeta/2\kappa_j)$, where u_{0j} are amplitudes (taken to be real, without loss of generality), Ω_j denote frequency shifts, and the propagation constants are $k_j \equiv \pm[1 + 4\kappa_j(u_{0j}^2 + \sigma u_{03-j}^2) - 4\kappa_j\Omega_j(\alpha_j + s_j\Omega_j/2)]^{1/2}/2\kappa_j$ [the $\pm$ flags propagation in the forward (+) or backward (–) longitudinal sense]. Linear analysis, deployed to test the robustness of these uniform states against small perturbations, yields a modulational instability (MI) spectrum that is described by the roots of an eighth-degree polynomial equation. The general features of the MI gain curve can be somewhat complicated (see Fig. 1), depending critically on the interplay between system parameters and cw intensities.

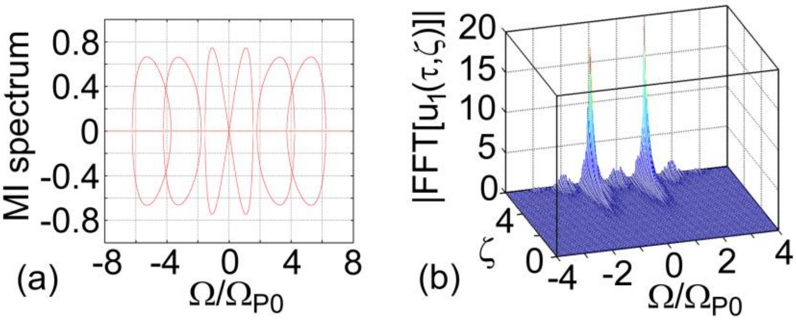

Fig. 1. (a) An induced-MI spectrum for cw solutions of model (1) with $s_1 < 0$ (normal GVD) and $s_2 > 0$ (anomalous GVD) when the coupling parameter is $\sigma = 2/3$. The spectrum is far more complicated than the single bow-tie structure found for the NLS equation. (b) Spontaneous MI for a system with $s_1 > 0$ and $s_2 > 0$ [FFT axis in arbitrary units of 100].

Exact spatiotemporal vector solitons

Exact analytical vector solitons have been derived for model (1), which include bright-bright, bright-dark, and dark-dark families. These new solutions are non-trivial generalizations of their more familiar NLS-type counterparts [1,3–5], with much more intricate and subtle space-time structure (e.g., in terms of solution existence conditions). Multi-parameter asymptotic analysis has proved that under the assumption of slowly-varying envelopes, where $|\kappa_j\partial^2 u_j/\partial\zeta^2| << |\partial u_j/\partial\zeta|$ [so that all contributions from first term in Eq. (1) can be safely neglected simultaneously], one may Galilean-boost from the laboratory to a local-time frame travelling at the average group speed. In that unique frame, all predictions from Eq. (1) are in full agreement with Menyuk's classic system [3] (as must be the case). Our latest investigations are focusing on spatiotemporal analyses of linear and nonlinear birefringence [3,7].

Simulations of model (1) can be performed using a vectorized modification of the difference-differential method used for the numerical integration of related nonlinear wave equations [8]. When used as initial conditions, traditional NLS-type vector solitons may evolve into the stationary solutions of model (1) (see Fig. 2). These results suggest that the new solitons are surrounded by wide basins of attraction.

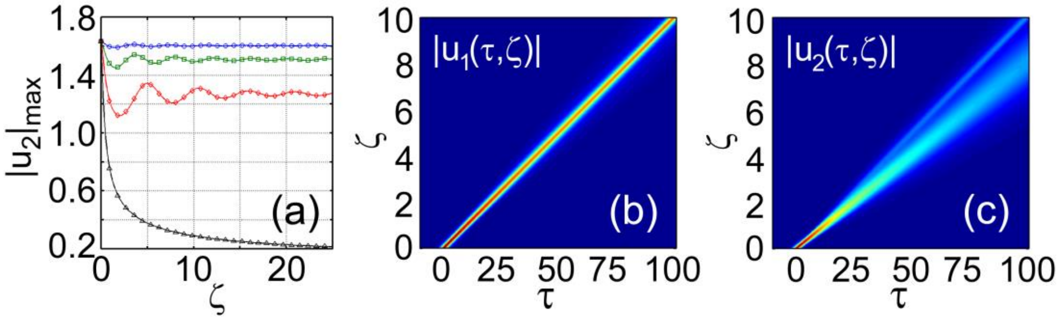

Fig. 2. Perturbed initial-value problem for model (1) involving bright-bright solitons. (a) Evolution of the peak amplitude in u_2 as perturbation strength increases. Evolution of (b) $|u_1|$ and (c) $|u_2|$ for a strongly perturbed bright-bright soliton in the (τ,ζ) plane [corresponding to the black curve in (a)].

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