Diffraction of fractal optical fields by simple apertures

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Summary

We propose and investigate, for the first time to our knowledge, an entirely new regime in wave physics – *the diffraction of fractal waves from simple apertures*. A selection of new analyses and physical predictions will be given for experimental geometries when the illuminating field has structure on multiple spatial scale-lengths.

Weierstrass fractal illumination

Berry's seminal work from over three decades ago [1] established that plane waves scattering from infinitely-wide complex objects (e.g., a transparent mask with a random fractal phase modulation) may acquire fractal characteristics in their statistics. Here, we consider the diametrically-opposing paradigm in complexity: the diffraction of a *fractal wave* from a *simple object*. We report on very recent research results concerning the scattering of fractal light from simple apertures. Attention is paid to two classic configurations that underpin both theoretical and experimental studies of diffraction: (i) a single infinite edge, and (ii) a single infinite slit. Classic analyses consider normally-incident plane-wave illumination, and the corresponding diffraction patterns are well known.

The novelty of our approach lies in accommodating an incident optical field that possesses a very broad spatial bandwidth (i.e., a waveform whose spatial spectrum extends over decimal orders of pattern scale-length). We consider an input wave in the form of a Weierstrass fractal that has a straightforward optical interpretation:

$$\frac{U_{\mathrm{in}}(\xi)}{U_0} = 1 + \frac{\varepsilon}{2}\sum_{n=0}^{\infty}\frac{1}{\gamma^{(2-D)n}}\left\{\exp\left[i\left(K_n\xi+\phi_n\right)\right]+\exp\left[-i\left(K_n\xi+\phi_n\right)\right]\right\}, \qquad (1)$$

where U_0 is the plane wave amplitude and ε is the strength of the fractal modulation. The n^{th} spectral component has a spatial frequency $K_n = (2\pi/\Lambda)\gamma^n$ (with $\gamma > 1$ and Λ is a fundamental scale-length), while its phase ϕ_n may be deterministic or random. Note that the Weierstrass fractal in Eq. (1) has a largest pattern scale-length $2\pi/K_0 = \Lambda$ but no small-scale cut-off [3]. Typically, the degree of complexity in $U_{\mathrm{in}}(\xi)$ is determined by $1 < D < 2$, where $D = 2$ describes an area-filling pattern (see Fig. 1).

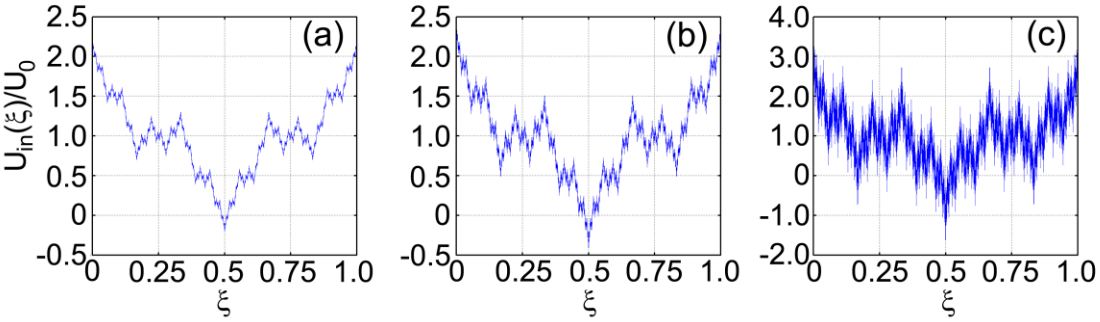

Fig. 1. Input Weierstrass fractal in Eq. (1) when $\gamma = 3.0$ and the parameter D (often interpreted as the Hausdorff-Besicovich dimension) is set to: (a) $D = 1.37$, (b) $D = 1.5$, and (c) $D = 1.8$. The phases $\phi_n = 0$ for all n, in which case the input field is symmetric in coordinate ξ.

Diffraction of fractal light waves

Exact mathematical descriptions of near-field (Fresnel) patterns have been obtained using Young's edge waves [4] as elemental spatial structures (see Fig. 2). Moreover, analysis using specialist software [5] has suggested that calculated diffraction patterns are bandwidth-limited fractals, as generally occur in Nature (see Fig. 3). Far-field (Fraunhofer) predictions of diffraction patterns emerge asymptotically from our near-field results (as they must) in the limit of vanishing Fresnel number.

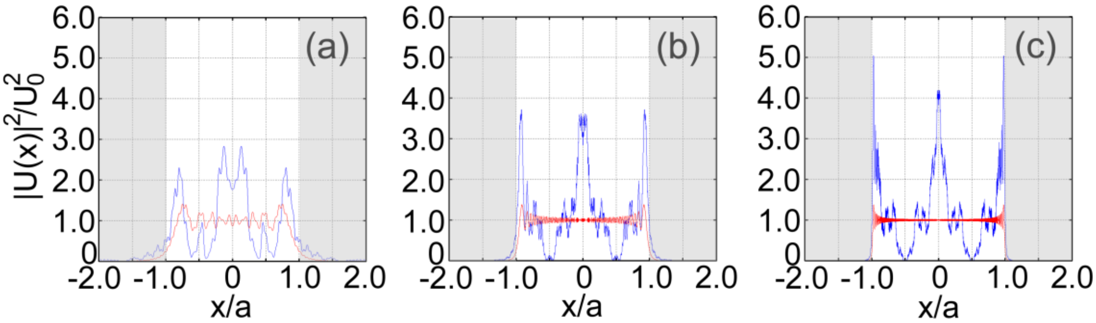

Fig. 2. Diffraction pattern produced by a single slit of width 2*a* when illuminated by a Weierstrass fractal with dimension D = 1.37 (blue lines) [input field corresponds to the wave in Fig. 1(a)]. Comparison is made with a plane wave illuminating field (ε = 0, red lines). Fresnel numbers are: (a) N_F = 10, (b) N_F = 100, (c) N_F = 1000. Shaded areas denote geometrical shadow region.

As the diffracted field propagates away from the aperture, the fractal signature of the illuminating wave is modified until one is left, ultimately, with a far-field pattern characterized by a *single scale-length* proportional to $1/N_F$ (where N_F is the Fresnel number of the aperture). This surprising result is profound, and goes to the heart of many considerations involving fractal wave diffraction. In contrast to analyses of infinitely-wide systems, finite transverse aperturing effects appear to have a strong impact on the complexity of the outgoing wave. This result has major implications for, and applications in, a diverse range of fields [6] such as fractal antenna engineering [7] and surface-roughness measurement techniques [8].

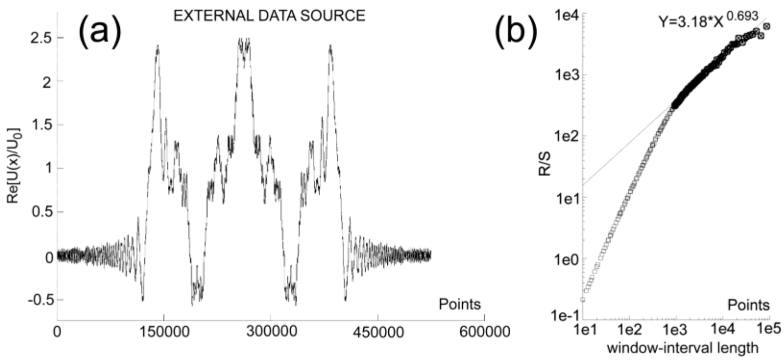

Fig. 3. (a) Diffracted Weierstrass fractal with γ = 3 and D = 1.37 for an aperture Fresnel number of N_F = 1000. (b) The *rescaled-range* (denoted with R/S) dimension of the pattern in (a) is found to be [5] $D_{R/S}$ ≈ 1.307. Analysis suggests that the diffracted field may be classified as a bandwidth-limited fractal whose dimension is generally less than that of the input field.

References

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