Unstable cavity lasers – from kaleidoscopes to snowflakes

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Summary

The snowflake laser is proposed − a new class of unstable resonator whose feedback mirror has the shape of the von Koch snowflake. Theoretical analysis is undertaken with two-dimensional virtual source theory, and predictions of novel mode patterns, eigenvalue spectra, and convergence phenomena are presented.

Unstable resonators: *kaleidoscope lasers*

Unstable cavity lasers have linear resonators with inherent magnification whose mode patterns possess fractal characteristics (proportional levels of detail across decimal orders of spatial scale) [1]. The origin of fractality in strip resonators has been explained by considering repeated diffraction of the circulating field at the feedback mirror (which subsequently plays a key role in determining mode characteristics) [2]. Kaleidoscope lasers involve fully two-dimensional (2D) transverse geometries where the feedback mirror has the shape of a regular polygon [3].

Previously, we have analyzed cavities with a full range of kaleidoscopic symmetries by deploying a 2D generalization of Southwell's virtual source (VS) theory [4] and exact mathematical descriptions of edge-wave patterns diffracted from a set of (virtual) polygonal apertures [5]. The 2D-VS approach is semi-analytical, and has several distinct advantages over more traditional Fox-Li methods (which are based on paraxial ABCD matrix optics and fast Fourier transforms [6]). For example, one application predicts the entire spectrum of modes while Fox-Li, in contrast, yields only one pattern per application and higher-order modes are notoriously difficult to compute. Moreover, 2D-VS theory allows us to accurately describe systems with *arbitrary cavity parameters* (i.e., the equivalent Fresnel number N_{eq} and round-trip magnification M − see Fig. 1). Previous studies have been restricted to regimes with $N_{eq} = O(1)$ (using purely-numerical methods [6]) or $N_{eq} >> O(1)$ (in which case the mode patterns may be approximated using asymptotic methods [7]).
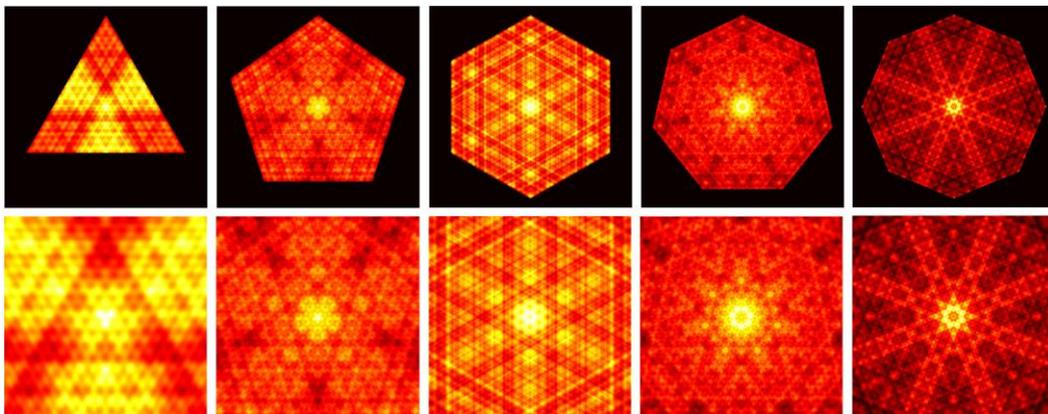
Fig. 1. Top row: 2D-VS computations of the lowest-loss mode patterns for kaleidoscope lasers. Bottom row: magnification of the central portion of the corresponding pattern (the Fox-Li method cannot facilitate such a calculation). Cavity parameters: $N_{eq} = 20$ and $M = 1.5$.
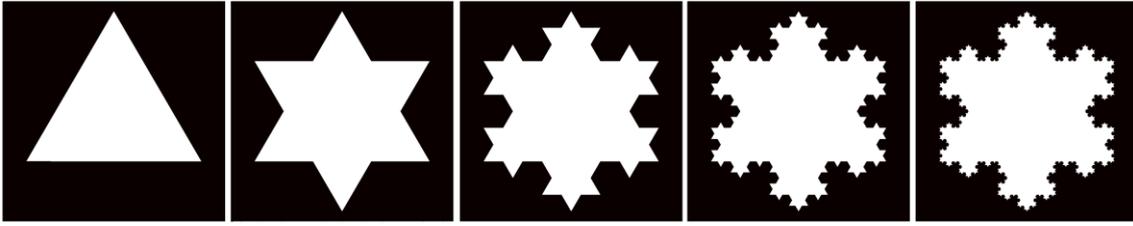
Fig. 2. Construction of the von Koch snowflake from the initiator stage (an equilateral triangle) through four applications of the generator algorithm (adding self-similar sequences triangles).

Unstable resonators: *snowflake lasers*

Here, we propose a new class of unstable resonator and introduce *the snowflake laser*. This novel system has a feedback mirror whose shape corresponds to a classic fractal curve − the von Koch snowflake (an iterated function system involving self-similar sequences of equilateral triangles − see Fig. 2). As such, we consider a cavity whose modes are inherently fractal, and where successive round trips involve the interplay of the re-circulated fractal light beam with a fractal aperturing element.

In this presentation, we will show how the 2D-VS method deployed for modelling kaleidoscope geometries can be applied to the snowflake laser (see Fig. 3). A pivotal development has been a reformulation of the Fresnel diffraction problem for snowflake apertures using a line integral [5]. The eigenvalue spectra of this new class of unstable resonator will also be detailed, along with convergence phenomena concerning pre-fractal snowflake elements. Finally, preliminary results from dimension calculations using specialist fractal analysis software [8] will be reported.
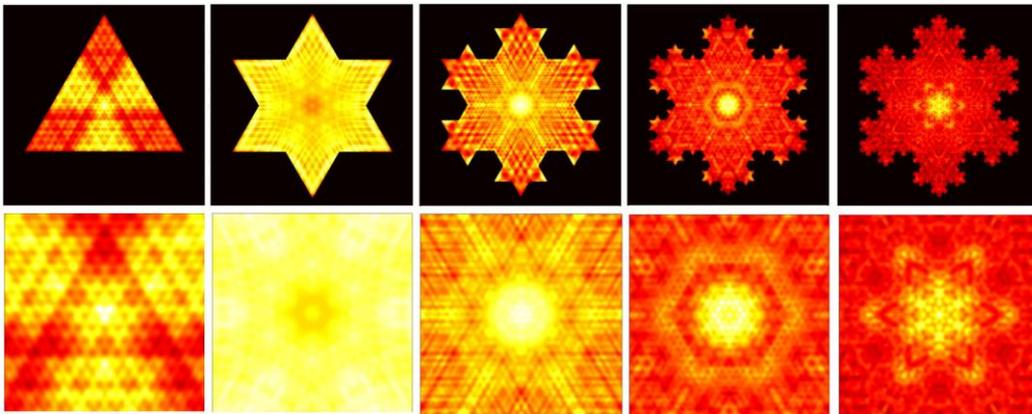
Fig. 3. Top row: 2D-VS computations of the lowest-loss mode patterns for a snowflake laser whose feedback mirror is progressing through the first four applications of the generator algorithm (c.f. Fig. 2). Bottom row: magnification of the central portion of the corresponding pattern. Cavity parameters: $N_{eq} = 20$ and $M = 1.5$.

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