The spatiotemporal wave equation with a dual power-law nonlinearity

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Solitons and soliton-like emergent phenomena appear ubiquitously throughout nonlinear science. These waves are truly fundamental entities, transcending the physical details of a system and resulting whenever linear dispersion (which tends to broaden a pulse in time) is opposed by nonlinearity (which tends to compress it). Such dynamic competition (feedback loops) may then facilitate the spontaneous formation of self-localizing and self-stabilizing wavepackets (solitons) whose profiles are often bell-shaped in time and stationary (invariant) throughout evolution in space. Here, we consider the classic dual power-law nonlinearity model [Micallef *et al.*, Phys. Rev. E vol. 54, 2936 (1994)] in the recently-proposed context of a spatiotemporal wave equation [Christian *et al.*, Phys. Rev. Lett. vol. 108, art. no. 034101 (2012)]. This type of governing equation has wide applicability in the field of photonics.

Our analysis begins with simple continuous waves (solutions that are temporally flat) and a perturbative investigation of their robustness against small disturbances (e.g., fluctuations due to random background noise). Our attention will then shift to hyperbolic solitons [that are quite tightly (i.e., exponentially) localized in time] and algebraic solitons (that have a much weaker Lorentzian-like localization). These two families are intimately related, with the latter appearing at the threshold for linear wave propagation (analytic continuation subsequently yields a class of temporally-delocalized periodic wave). We have also derived so-called boundary solitons, which connect (in a monotonic way) regions of finite-amplitude 'flat' solution to regions of zero amplitude. For each new class of wave, extensive simulations have been used to assess solution stability.