Spontaneous spatial fractal patterns: towards nonparaxial nonlinear ring cavities

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Spontaneous pattern formation in optical ring cavities containing a nonlinear (e.g., Kerr-type) material [see Fig. 1(a)] has been studied extensively for the past three decades. A notable trend in the literature over recent years has been a shift away from the (bulk) *cavity + boundary condition* models of McLaughlin *et al.* [1] toward the (longitudinally-averaged) *mean field* descriptions of Lugiato and Lefever [2]. While this latter approach is analytically more tractable, it does not yield Turing instability spectra with the multiple-minimum characteristic proposed as necessary for spontaneous fractal (i.e., multi-scale) pattern formation [3,4] [see Fig. 1(b)]. Here, we revisit the approach taken by McLaughlin *et al.* [1] but instead allow for the full generality of Helmholtz (broadband) as opposed to paraxial (narrowband) diffraction. Such a restoration of spatial symmetry (whereby diffraction occurs in both transverse and longitudinal dimensions) allows a much more reliable description of small-scale spatial structure in the circulating cavity field. Our analysis also goes some way toward addressing the issue of fractal pattern formation in systems with finite light-medium interaction lengths [3,4]. Linear analysis has predicted the threshold condition for spatial pattern emergence, and simulations have begun to investigate these new instabilities.

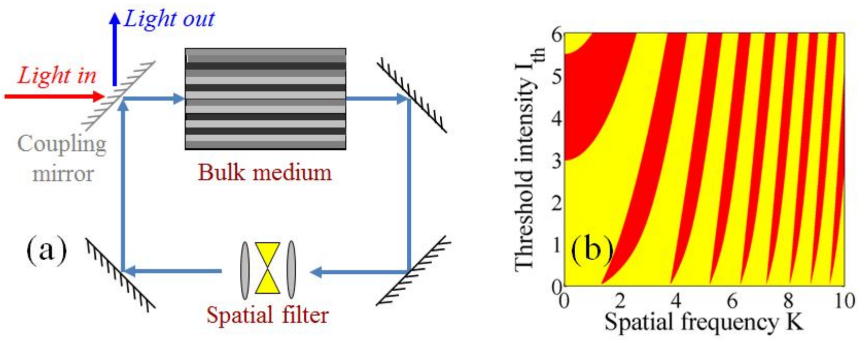

Figure 1. (a) Schematic diagram of an optical ring cavity filled with a bulk nonlinear (e.g., Kerr) material so that light-medium interactions can take place over a finite propagation length. (b) Typical multi-Turing threshold instability curves for the thin-slice cavity system [4]. The introduction of finite medium length changes some key characteristics of the spectrum.

References

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