Nonparaxial refraction laws in optics: from non-Kerr interfaces to waveguide arrays

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Summary

Angular effects play a central role in essentially all non-trivial optical configurations, and can be well-described within a Helmholtz-type nonparaxial framework. We report recent results modelling spatial solitons interacting with cubic-quintic material interfaces, and extend related considerations to periodically-patterned optical media.

Introduction

A light beam impinging on the boundary between two dissimilar dielectric materials is an elemental optical geometry. The seminal papers of Aceves, Moloney and Newell [1] in the late 1980s considered a simple scenario, where a spatial soliton is incident on the planar boundary between two different Kerr-type materials. Their intuitive approach reduced the full electromagnetic interface problem to the solution of a scalar equation of the inhomogeneous nonlinear Schrödinger type. Over the past two decades, investigations of single and multiple-layer interface geometries have paved the way to deeper understandings of how light behaves inside patterned nonlinear structures such as coupled-waveguide arrays and photonics crystals.

Oblique incidence effects are pivotal in the understanding and manipulation of light wave-interface interactions. For instance, one might envisage the ingoing spatial soliton being arbitrarily inclined with respect to a planar boundary, either through changing the orientation of the light beam (relative to the interface/array) or by rotating the interface/array (relative to the beam). It is thus desirable, essential even, for theoretical descriptions to capture this type of intrinsic angular property. Unfortunately, the ubiquitous assumption of slowly-varying envelopes renders traditional (paraxial) modelling applicable only when angles of incidence, reflection and refraction *in the laboratory frame* are negligibly (or near-negligibly) small.

Soliton refraction at a non-Kerr interface

Earlier modelling of arbitrary-angle refraction at planar interfaces has been largely confined to dissimilar focusing and defocusing Kerr-type media [2]. These considerations permitted us to derive a simple and compact generalization of the familiar Snell's law (describing *plane waves* at *linear dielectric interfaces*) for spatial solitons. Here, we present our latest research in non-Kerr regimes, where the nonlinearity is of the cubic-quintic type [3]. The normalized governing equation is

$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+i\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}+|u|^2u+\sigma|u|^4u=\left[\frac{\Delta}{4\kappa}+(1-\alpha)|u|^2+(1-\nu)\sigma|u|^4\right]h(\xi,\zeta)u, \quad (1)$$

where u is the electric field envelope, ξ / ζ are the transverse / longitudinal coordinates, κ << $O(1)$ measures the inverse beam width, and σ is the strength of the quintic response. Parameters Δ and (α, ν) characterize discontinuities in the linear and nonlinear parts of the refractive index, respectively, while h is a Heaviside unit function specifying the location of the boundary in the (ξ, ζ) plane. A Snell's law for cubic-quintic nonlinearity has been derived; its predictions have been tested, and confirmed, through extensive computer simulations. Excellent theory-numerics agreement has been found in wide ranges of parameter space (see Fig. 1). Research highlights from some of our other key analyses [4] will be reviewed, where qualitatively new phenomena have been uncovered in non-Kerr regimes.

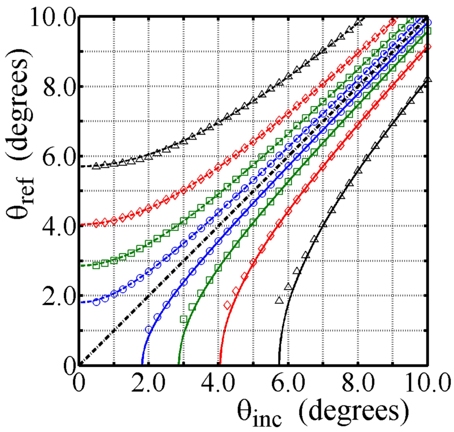

Fig 1. Computational testing of the new Snell's law for cubic-quintic Helmholtz solitons. Incidence and refraction angles are denoted by θ_{inc} and θ_{ref}, respectively.

Oblique injection into waveguide arrays

The coupling of spatial solitons into, and their subsequent propagation inside, periodically-patterned nonlinear optical materials is a problem of wide interest [5]. To date, nearly all theoretical and experimental investigations of both head-on and side-coupling geometries have been within the arena of paraxial wave optics. We will present an overview of our current research into how spatial solitons interact *at arbitrary angles* with such periodic structures. This is, to the best of our knowledge, the first investigation of its type. Particular emphasis will be placed on side-coupling arrangements, where an incident beam travelling in a continuum (e.g., a homogeneous Kerr medium) is injected obliquely into a waveguide array from the side (see Fig. 2). A model based on the scalar nonlinear Helmholtz equation, and similar in spirit to Eq. (1), will be detailed. A selection of new qualitative phenomena predicted by that model will also be reported.

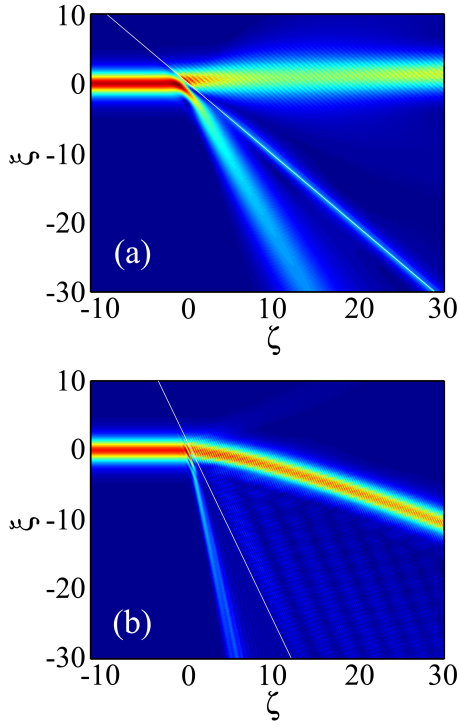

Fig 2. Side-coupling of a spatial soliton into a waveguide array for (a) a quasi-paraxial incidence angle (θ_{inc} = 4.5°) and (b) a nonparaxial incidence angle (θ_{inc} = 10.0°).

References

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