

# Spatial dispersion and spatiotemporal solitons in cubic-quintic optical waveguides

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## Summary

We report on fresh developments in the field of spatiotemporal optical solitons. A host of new analytical and numerical predictions for light pulses in nonlinear waveguides will be detailed, which have a simple physical interpretation. Tantalizing connections to Einstein's special relativity theory have also been uncovered.

## Introduction

It can be safely said that optical soliton pulses are one of the most thoroughly investigated phenomena in nonlinear photonics. Since the seminal works of Hasegawa and Tappert [1,2], the cornerstone of many investigations has been the slowly-varying envelope approximation (SVEA). In combination with a subsequent Galilean boost to a local time frame, the SVEA provides a mathematical device commonly used to reduce the complexity of the longitudinal (spatial) part of the wave operator. While this approach has some clear-cut advantages [e.g. by replacing the elliptic (or hyperbolic) governing equation with a parabolic one], there are some physical effects that fall outside its remit. One such effect is spatial dispersion, recently discussed by Biancalana and Creatore in the context of pulse envelope equations in some semiconductor (e.g., ZnCdSe / ZnSe superlattice) waveguides [3].

## A new spatiotemporal model

Here, we report on our new approach to nonlinear pulse modelling, where the classic "SVEA + Galilean boost" device is omitted. Mathematical and computational methods may be deployed that are similar to those used over the past 12 years to analyze nonlinear beams [4,5]. Of interest is the normalized scalar pulse equation [6]

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i \left( \frac{\partial u}{\partial \zeta} + \alpha \frac{\partial u}{\partial \tau} \right) + \frac{s}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u + \gamma |u|^4 u = 0, \quad (1)$$

where  $u$  is the envelope,  $(\tau, \zeta)$  denote time and (longitudinal) space coordinates, respectively,  $\alpha$  is a ratio of group speeds,  $s = \pm 1$  flags the sign of the temporal dispersion (+1 for anomalous, -1 for normal),  $\kappa$  is a small parameter [that can assume *either sign* but has magnitude  $\ll O(1)$ ] capturing spatial dispersion [3,6], and  $\gamma$  quantifies the strength of the quintic nonlinearity relative to the Kerr (cubic) effect.

The precedent for using models such as Eq. (1) was set more than 30 years ago [7], but the approach seems to have received little subsequent attention [6]. Full convergence to conventional pulse theory is uncovered in a simultaneous multiple limit. One of the key results, and also one of the most interesting, is the velocity combination rule for spatiotemporal solitons. This law, which is geometric in nature

and *independent of system nonlinearity*, is strongly reminiscent of the way velocities add together in relativistic kinematics. In fact, when  $\text{sgn}(s\kappa) = +1$ , there is a one-to-one mapping with special relativity theory. Deeper insight can be gained by considering the transformation laws for Eq. (1), which show that the velocity combination rule is an intrinsic property of the model itself, rather than a property of particular (e.g., soliton) solutions.

### Solitons and stability

We will give an overview of our investigations into Eq. (1) and related models (where the nonlinearity is generalized to dual power-law form with rationally-related exponents). The fully-second order modulational instability problem for continuous-wave solutions can be solved exactly, and numerical computations have confirmed the validity of predictions made by linear analysis (see Fig. 1).

A range of exact analytical solitons will be reported, including: bright-hyperbolic [8], bright-[8] and dark-[9] algebraic, and boundary [10] solutions. Their space-time geometry will be considered in detail. New parameter regimes (including a type of intrinsic bistability) that have no counterpart in the spatial domain [4,5] (namely, regimes where  $\kappa < 0$ ) will be given, and a wide range of generic features identified. Crucially, the properties of spatiotemporal solitons are found to depend on the sign of the product  $s\kappa$ , rather than  $\text{sgn}(s)$  or  $\text{sgn}(\kappa)$  separately. Extensive simulations also examine their role as robust attractors in the system dynamics (see Fig. 2).

### References

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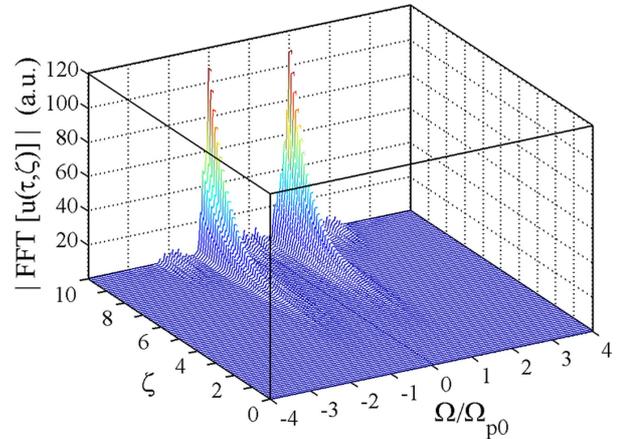


Fig 1. Spontaneous growth and development (in frequency space) of modulational instability in a continuous-wave solution of Eq. (1). The most unstable frequency is denoted by  $\Omega_{p0}$ .

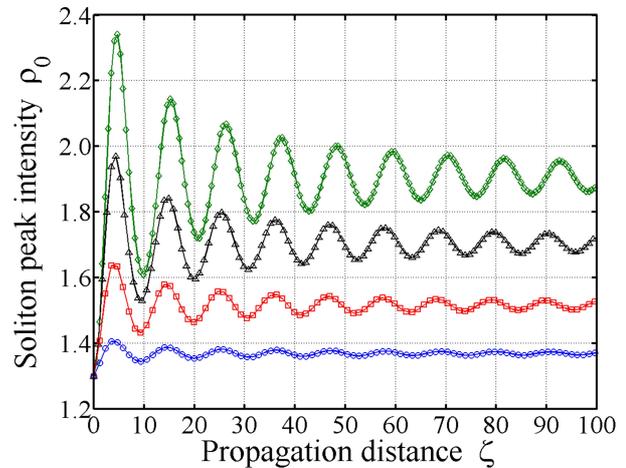


Fig 2. Self-reshaping of a perturbed bright pulse towards a stationary state (i.e., an exact soliton) of Eq. (1).