# Helmholtz dark spatial optical solitons for a defocusing saturable nonlinearity

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## Summary

We report on our latest developments in the field of Helmholtz soliton theory: the derivation of exact dark spatial optical solitons for a defocusing saturable nonlinearity. A raft of new physical predictions is made, and extensive analysis (via both mathematical and computational methods) investigates solution properties.

## Introduction

Off-axis propagation constitutes a truly fundamental class of problem in the theory of nonlinear waves. In photonics, such geometrical problems cannot be solved in general form by conventional methods – classic *Schrödinger*-type models, obtained under the paraxial approximation, restrict propagation angles [measured with respect to the reference (longitudinal) direction in the laboratory frame] to near-negligible magnitudes. Indeed, this elementary context defines an angular type of nonparaxiality that may potentially dominate evolution. Many of the physical phenomena associated with the oblique propagation of continuous-wave scalar optical fields can be well described by *Helmholtz*-type governing equations [1].

In this presentation, our interest lies with dark spatial solitons [2] in materials with a defocusing saturable nonlinearity. Saturation under high-intensity illumination is a property of many optical media. Phenomenological descriptions of a saturable refractive index must go beyond polynomial-type expansions in the (local) light intensity that eventually break down. Such approaches almost always result in a model that does not possess exact soliton solutions. A notable exception, and hence one that is particularly important here, is the form proposed by Wood *et al.* [3].

## Helmholtz soliton theory

We consider a dimensionless nonlinear Helmholtz equation of the form

$$\kappa\frac{\partial^2 u}{\partial\zeta^2}+i\frac{\partial u}{\partial\zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial\xi^2}-\frac{1}{2}\frac{2+|u|^2/\rho_{\mathrm{sat}}}{\left(1+|u|^2/\rho_{\mathrm{sat}}\right)^2}|u|^2 u=0, \tag{1}$$

where $u$ is the electric field envelope, $\xi/\zeta$ are the transverse / longitudinal coordinates, $\kappa << O(1)$ measures the inverse beam width, and $\rho_{\mathrm{sat}}$ is the saturation intensity. This choice of nonlinearity captures the essence of saturation [3] while allowing us to find exact solutions of Eq. (1). By deploying a unified combination of analytical methods (symmetry reduction, coordinate transformations, and direct integration), we have been able to derive exact dark solitons (both black and, more generally, *grey* variants) of Eq. (1).

The new Helmholtz dark solitons, which complement their bright counterparts [4], possess many physical features that are absent from paraxial theory [5]. Such features include: (i) forward- *and backward*-propagating solution families, (ii) potentially non-trivial angular beam broadening corrections (see Fig. 1), and (iii) an intrinsic velocity that is fully consistent with off-axis evolution. These three key properties are tied together, somewhat subtly, by the preservation of spatial symmetry in Eq. (1) (in the laboratory frame, the two spatial directions naturally have equal status). Crucially, we have also proven convergence to known results (i.e., defocusing-Kerr [6] and paraxial saturable dark [5] solutions) in appropriate multi-fold asymptotic limits.

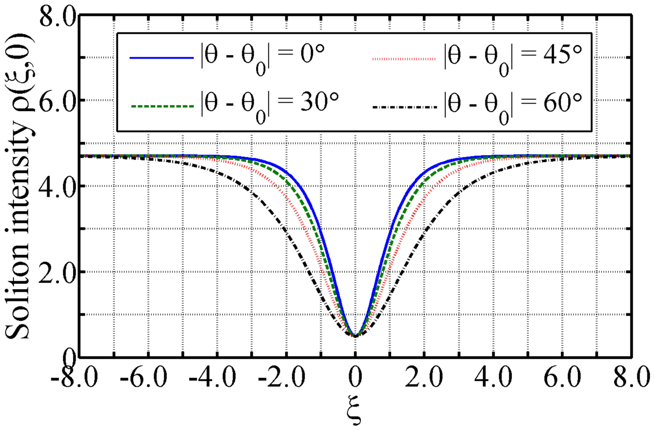

Fig 1. Calculation of Helmholtz grey soliton intensity profiles as a function of the net propagation angle (denoted by $\theta - \theta_0$) in the laboratory frame [6]. Even in moderate angular regimes (for instance, where $|\theta - \theta_0| = 60°$), the Helmholtz solution embodies a 100% correction to the perceived width (i.e., the transverse projection of the beam waist) as compared to that predicted by paraxial theory (blue line) [5].

## Dark soliton stability

Linearization techniques predict, and simulations have subsequently confirmed, the stability of plane waves of Eq. (1) against small modulations. We have also investigated the robustness of the new dark solitons, which exhibit a bistable type of characteristic [4,5]. Perturbations to the local beam shape are found to disappear in the long term, leaving a stationary state (see Fig. 2). This result suggests that Helmholtz dark solitons in saturable media, like their Kerr counterparts [6], are likely to be highly stable entities that may be exploited in a novel optical device designs and architectures.

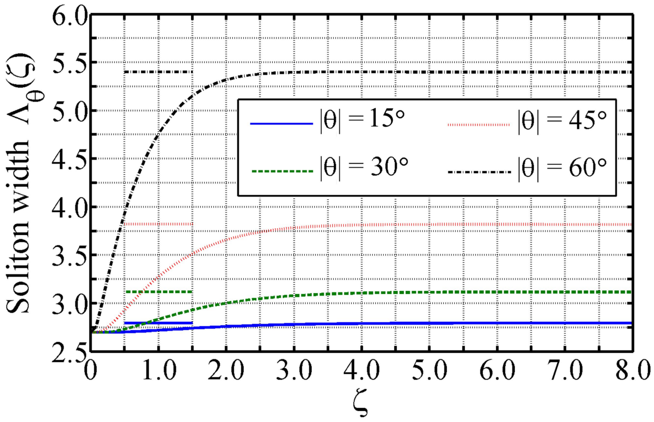

Fig 2. Evolution of the full-width of a perturbed dark Helmholtz soliton (horizontal bars denote asymptotic theoretical predictions). The initial condition used for Eq. (1) is a black beam ($\theta_0 = 0$) with a nonparaxial launching angle (e.g., 15°, 30°, 45° and 60°). In each case, the intensity profile corresponds to an exact paraxial dark soliton [5], so lacks the necessary beam broadening effect shown in Fig. 1.

## References

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