Acoustics Research Centre
University of Salford

Acoustics Research Centre
University of Salford, UK

Architectural and building acoustics

Digital signal processing

Outdoor sound propagation

Environmental acoustics

Virtual acoustic prototypes

Acoustics of porous materials

6* 2001 RAE

www.acoustics.salford.ac.uk

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

A time domain Boundary Element Method for compliant surfaces

Jonathan A. Hargreaves
Trevor J. Cox

EPSRC

Talk Structure

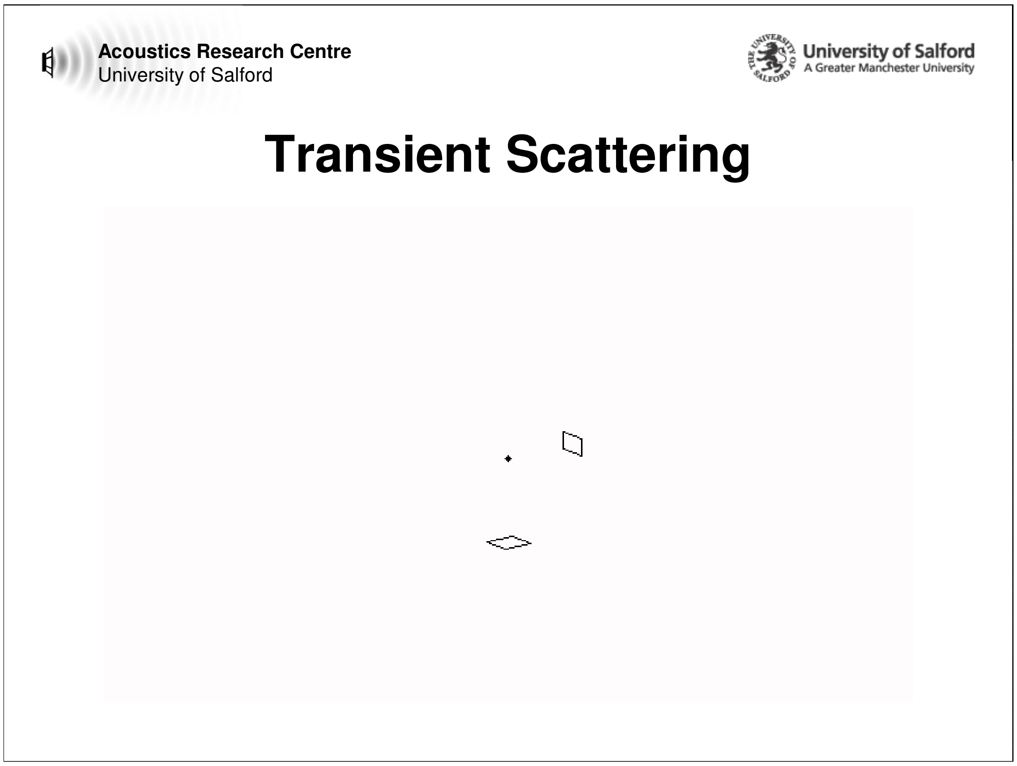
- BEM overview
- Welled obstacle model
- Numerical results
- Scope as a compliant obstacle model
- Conclusions

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

BEM Overview

- Scattering of sound by an obstacle
- Wave method
- Only the surface of the obstacle is modelled
- The obstacle's acoustic properties are stated as surface properties
  - Typically as surface impedance for harmonic excitation
- Time domain BEM permits transient broadband excitation
  - Typically solved by iterating through states in time



Animation to illustrate mutual interaction of scatterers, and the time iterating process by which the BEM solves the scattering problem.

For more implementation info see A. A. Ergin, B. Shanker and E. Michielssen, “Analysis of transient wave scattering from rigid bodies using a Burton-Miller approach”, J. Acoust. Soc. Am. **106** (5): 2396 – 2404 (1999)

Acoustics Research Centre
University of Salford

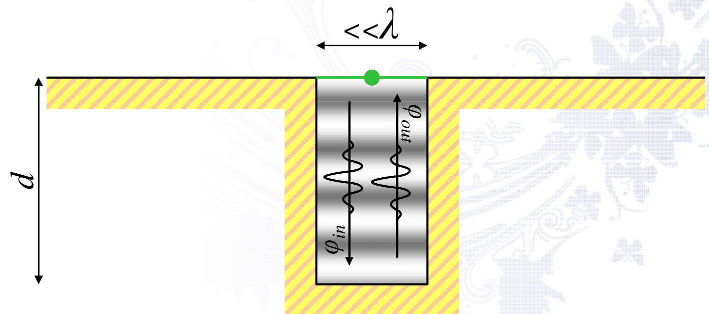
University of Salford
A Greater Manchester University

Welled Obstacle

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Welled region



Consider an obstacle whose surface contains a well with cross-section small w.r.t λ

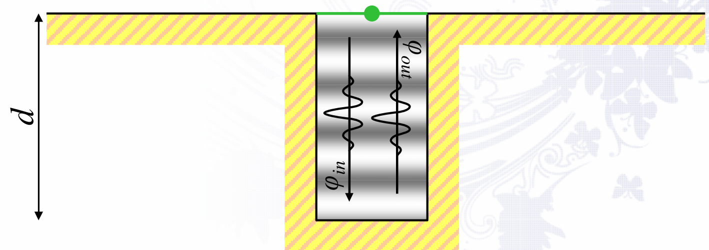
This can support waves travelling in two directions – up & down the well

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c)$$



If we consider a point at the mouth of the well, we can express the outgoing wave in terms of the in incoming wave

Acoustics Research Centre
University of Salford

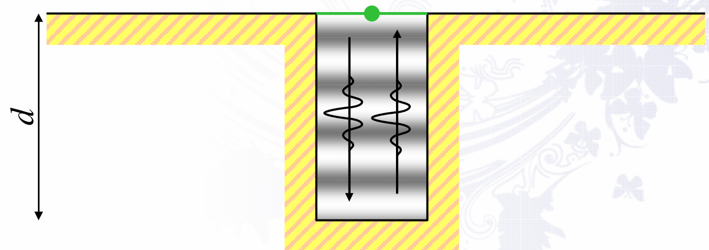
University of Salford
A Greater Manchester University

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c)$$

$$p(\mathbf{x},t) = -\rho_0 \dot{\varphi}(\mathbf{x},t)$$

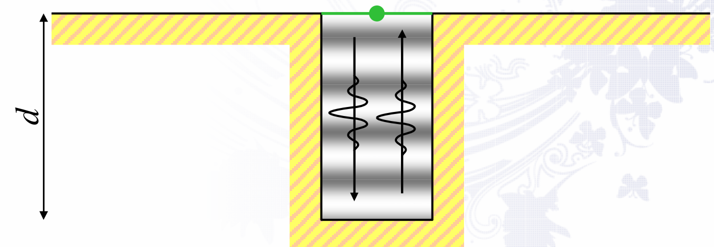
$$\mathbf{v}(\mathbf{x},t) = \nabla \varphi(\mathbf{x},t)$$



Note: the model is in terms of velocity potential which, while not a physical quantity, is useful as both pressure & particle velocity may be found from it

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c)$$

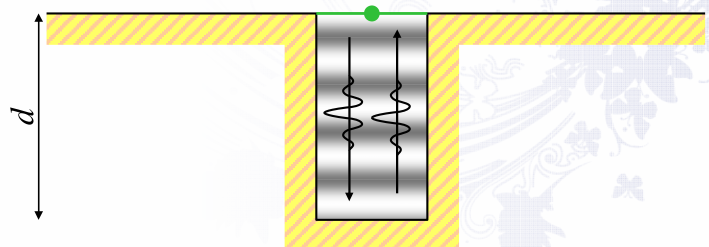


Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c) \qquad \Phi_{out}(\omega) = \Phi_{in}(\omega)e^{i2kd}$$



We can state the same relationship as a phase change in the frequency domain

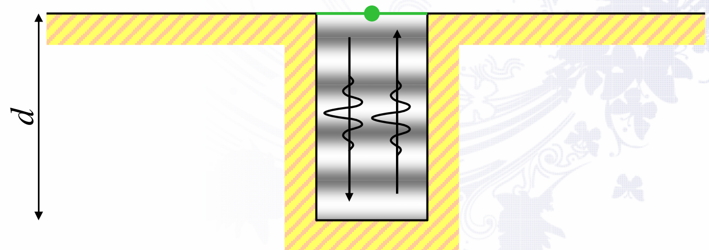
Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c) \qquad \Phi_{out}(\omega) = \Phi_{in}(\omega)e^{i2kd}$$

$$\frac{P_t(\omega)}{-V_n(\omega)} = Z(\omega) = i\rho_0 c \cot(kd)$$



However in the frequency domain it’s more typical to use surface impedance, which relates total pressure with total inward particle velocity

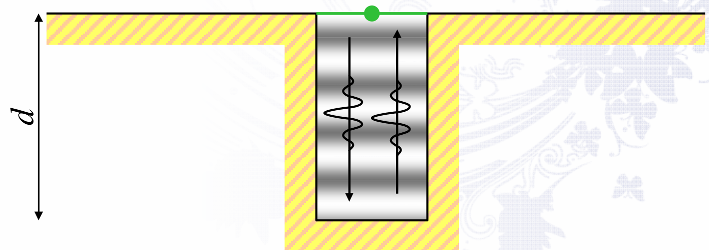
Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c) \qquad \Phi_{out}(\omega) = \Phi_{in}(\omega) e^{i2kd}$$

$$p_t(t) = -v_n(t) * z(t) \qquad \frac{P_t(\omega)}{-V_n(\omega)} = Z(\omega) = i\rho_0 c \cot(kd)$$

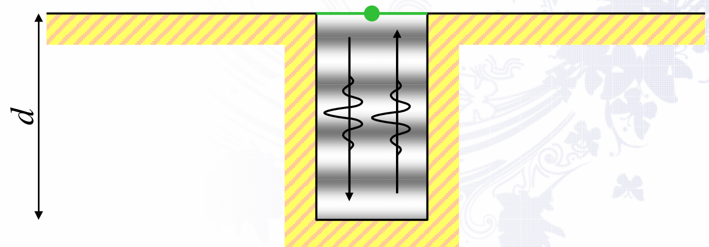


This can be written in the time domain as a convolution

Welled region

$$\varphi_{out}(t) = \varphi_{in}(t - 2d/c) \qquad \Phi_{out}(\omega) = \Phi_{in}(\omega) e^{i2kd}$$

$$\cancel{p_t(t) = -v_n(t) * z(t)} \qquad \frac{P_t(\omega)}{-V_n(\omega)} = Z(\omega) = i\rho_0 c \cot(kd)$$



However, However, a found by inverse discrete Fourier transform of $Z(\omega)$ is typically non-compact in time and requires future values of $v_n(t)$. This is due to the aggregation of cause and effect in the quantities $p_t(t)$ and $v_n(t)$, and means that this form cannot be used with a time-marching solver. Further to this, the inverse Fourier transform of well mouth surface impedance equation appears to be a non-trivial operation

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Compliant Surface

$$\varphi_t(\mathbf{x},t) = \varphi_{in}(\mathbf{x},t) + \varphi_{in}(\mathbf{x},t - 2d(\mathbf{x})/c)$$

So let’s abstract the well to a compliant surface. Total velocity potential for the well can be written as above, where $d(\mathbf{x})$ is the well depth d on the mouth of the well and zero elsewhere.

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Compliant Surface

$$\varphi_t(\mathbf{x},t) = \varphi_{in}(\mathbf{x},t) + \varphi_{in}(\mathbf{x},t - 2d(\mathbf{x})/c)$$

$$p_t(\mathbf{x},t) = -\rho_0\left[\dot{\varphi}_{in}(\mathbf{x},t) + \dot{\varphi}_{in}(\mathbf{x},t - 2d(\mathbf{x})/c)\right]$$

We can also find pressure at the surface…

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Compliant Surface

$$\varphi_t(\mathbf{x},t) = \varphi_{in}(\mathbf{x},t) + \varphi_{in}(\mathbf{x},t-2d(\mathbf{x})/c)$$

$$p_t(\mathbf{x},t) = -\rho_0[\dot{\varphi}_{in}(\mathbf{x},t) + \dot{\varphi}_{in}(\mathbf{x},t-2d(\mathbf{x})/c)]$$

$$v_n(\mathbf{x},t) = \frac{1}{c}[\dot{\varphi}_{in}(\mathbf{x},t) - \dot{\varphi}_{in}(\mathbf{x},t-2d(\mathbf{x})/c)]$$

…and the surface normal component of velocity.

Compliant Surface

$$\varphi_t(\mathbf{x},t)=\varphi_{in}(\mathbf{x},t)+\varphi_{in}(\mathbf{x},t-2d(\mathbf{x})/c)$$

$$p_t(\mathbf{x},t)=-\rho_0\left[\dot{\varphi}_{in}(\mathbf{x},t)+\dot{\varphi}_{in}(\mathbf{x},t-2d(\mathbf{x})/c)\right]$$

$$v_n(\mathbf{x},t)=\frac{1}{c}\left[\dot{\varphi}_{in}(\mathbf{x},t)-\dot{\varphi}_{in}(\mathbf{x},t-2d(\mathbf{x})/c)\right]$$

$$\varphi_s(\mathbf{y},t)=\iint_S\left(\varphi_t(\mathbf{x},t)*\hat{\mathbf{n}}\cdot\nabla g(R,t)-g(R,t)*v_n(\mathbf{x},t)\right)d\mathbf{x}$$

Aarrgggghhhh! – it the Kirchhoff Helmholtz Integral Equation (KIE). Looks scary, but helpfully can tell us what sound is scattered well a given velocity potential and normal velocity distribution exist on the surface. Let's look at it a step at a time:…

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Compliant Surface

$$\varphi_t(\mathbf{x},t) = \varphi_{in}(\mathbf{x},t) + \varphi_{in}(\mathbf{x}, t - 2d(\mathbf{x})/c)$$

$$p_t(\mathbf{x},t) = -\rho_0\left[\dot{\varphi}_{in}(\mathbf{x},t) + \dot{\varphi}_{in}(\mathbf{x}, t - 2d(\mathbf{x})/c)\right]$$

$$v_n(\mathbf{x},t) = \frac{1}{c}\left[\dot{\varphi}_{in}(\mathbf{x},t) - \dot{\varphi}_{in}(\mathbf{x}, t - 2d(\mathbf{x})/c)\right]$$

$$\varphi_s(\mathbf{y},t) = \iint_S \left(\varphi_t(\mathbf{x},t) * \hat{\mathbf{n}} \cdot \nabla g(R,t) - g(R,t) * v_n(\mathbf{x},t)\right) d\mathbf{x}$$

First of all, let’s notice that all the terms inside the integral can be found from the welled surface model.

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Compliant Surface

$$\varphi_t(\mathbf{x},t) = \varphi_{in}(\mathbf{x},t) + \varphi_{in}(\mathbf{x},t-2d(\mathbf{x})/c)$$

$$p_t(\mathbf{x},t) = -\rho_0\left[\dot{\varphi}_{in}(\mathbf{x},t) + \dot{\varphi}_{in}(\mathbf{x},t-2d(\mathbf{x})/c)\right]$$

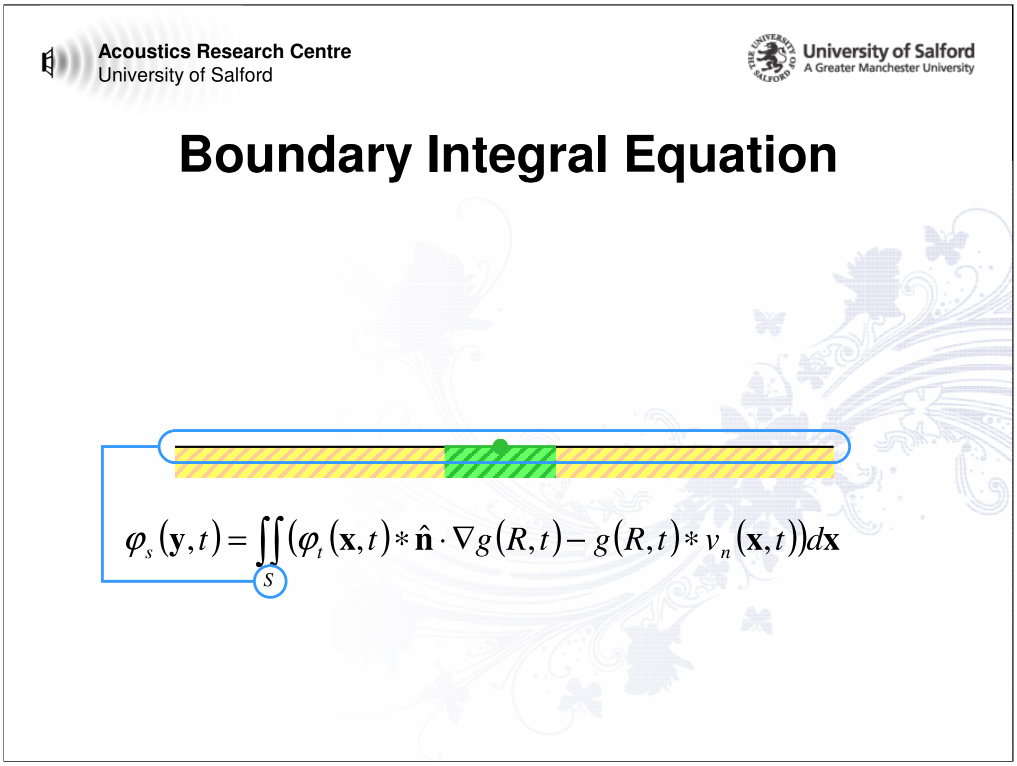
$$v_n(\mathbf{x},t) = \frac{1}{c}\left[\dot{\varphi}_{in}(\mathbf{x},t) - \dot{\varphi}_{in}(\mathbf{x},t-2d(\mathbf{x})/c)\right]$$

$$\varphi_s(\mathbf{y},t) = \iint_S \left(\varphi_t(\mathbf{x},t) * \hat{\mathbf{n}}\cdot\nabla g(R,t) - g(R,t) * v_n(\mathbf{x},t)\right) d\mathbf{x}$$

$\boxed{\varphi_{in} \text{ is the fundamental unknown}}$

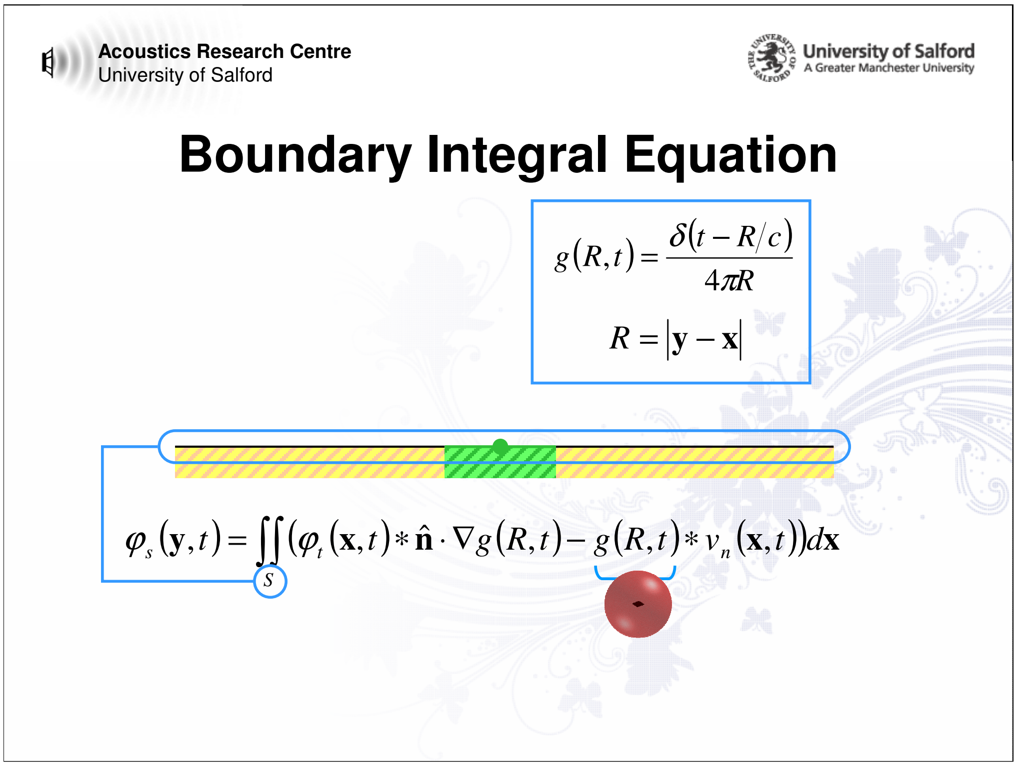
You might notice that this means that φ_{in} is the fundamental surface unknown; all other surface and scattered quantities can be calculated from it.

Hence we will later solve for this numerically.

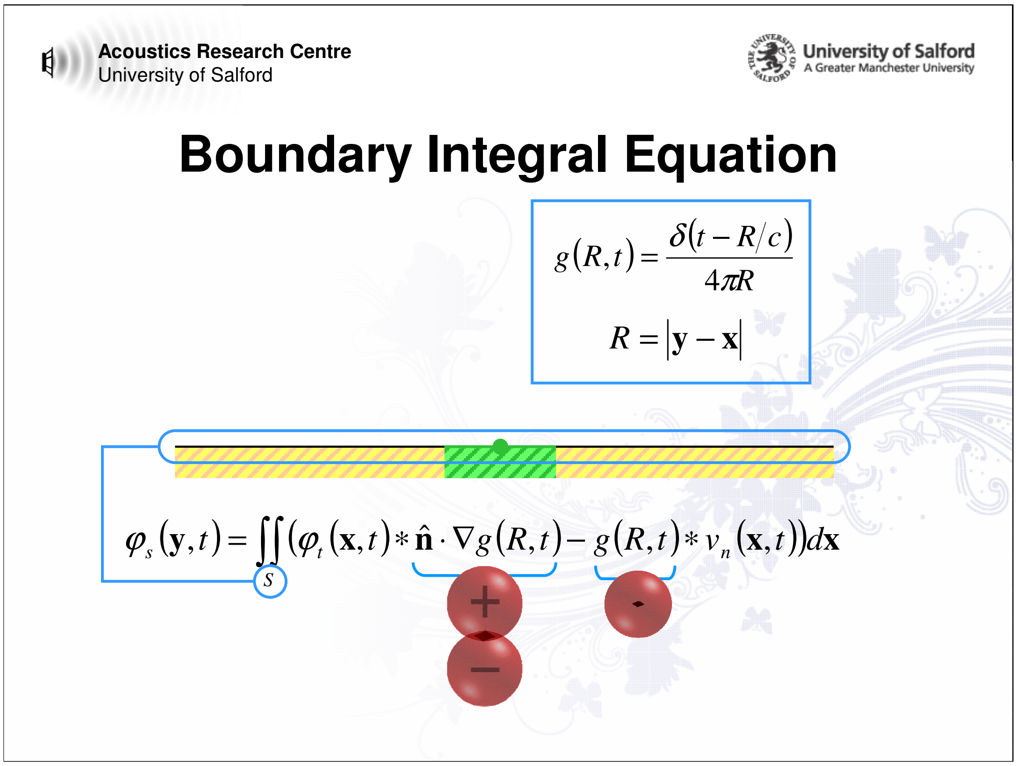


Back to the KIE:

First let’s point out that the integral means that we are adding up the sound scattered by each point on the surface.



g(*R*,*t*) is the free-space time domain greens function, which describes how a instantaneous point source (monopole) at $t = 0$ radiates to an observer at distance R. In the case of the KIE, R is the distance from the integration point **x** to the observation point **y**. This represents particle flow through the surface.

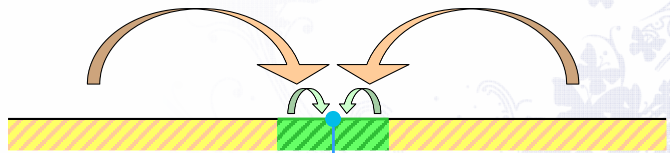


The other term is a dipole. This represents a force with the surface applies to the air.

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Boundary Integral Equation



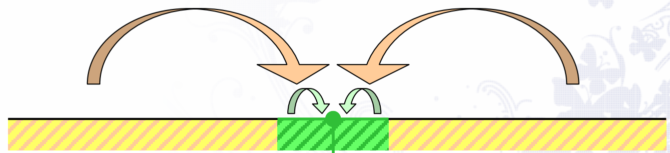
$$\varphi_s(\mathbf{y},t)=\iint_S\left(\varphi_t(\mathbf{x},t)*\hat{\mathbf{n}}\cdot\nabla g(R,t)-g(R,t)*v_n(\mathbf{x},t)\right)d\mathbf{x}$$

Let’s set the observation point **y** to be at the well mouth. Now the KIE tells us what sound is scattered by the whole obstacle, including the well itself, to the mouth of the well. A useful by-product of this is that the KIE incorporates the radiation impedance of the well.

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

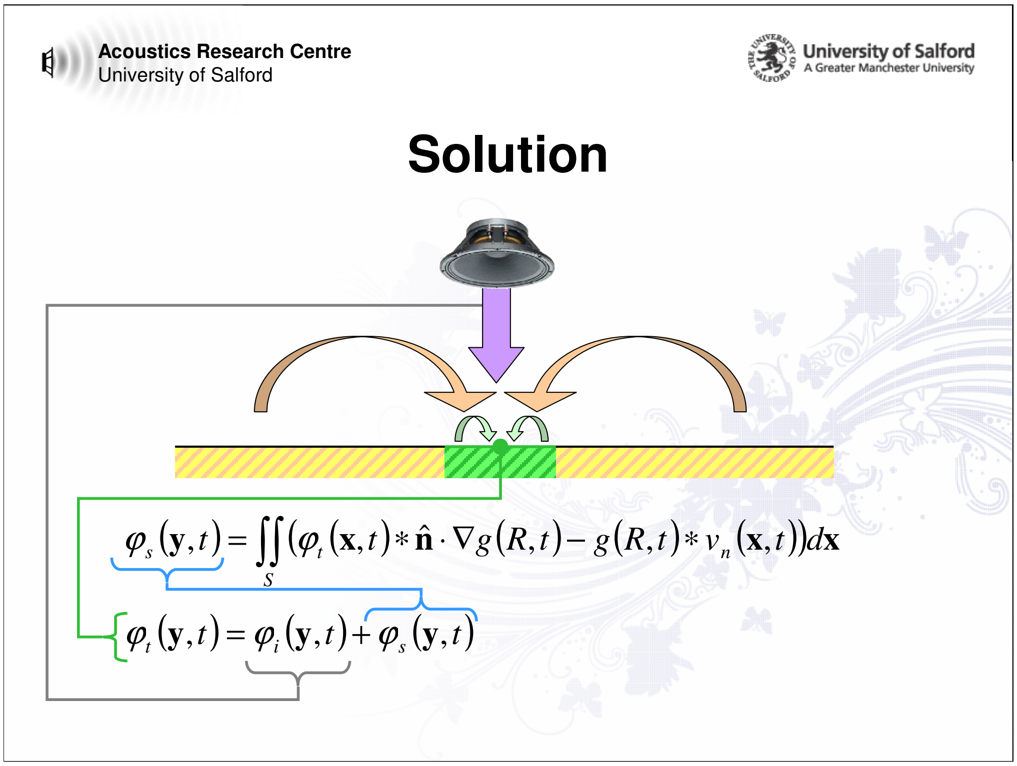
Solution



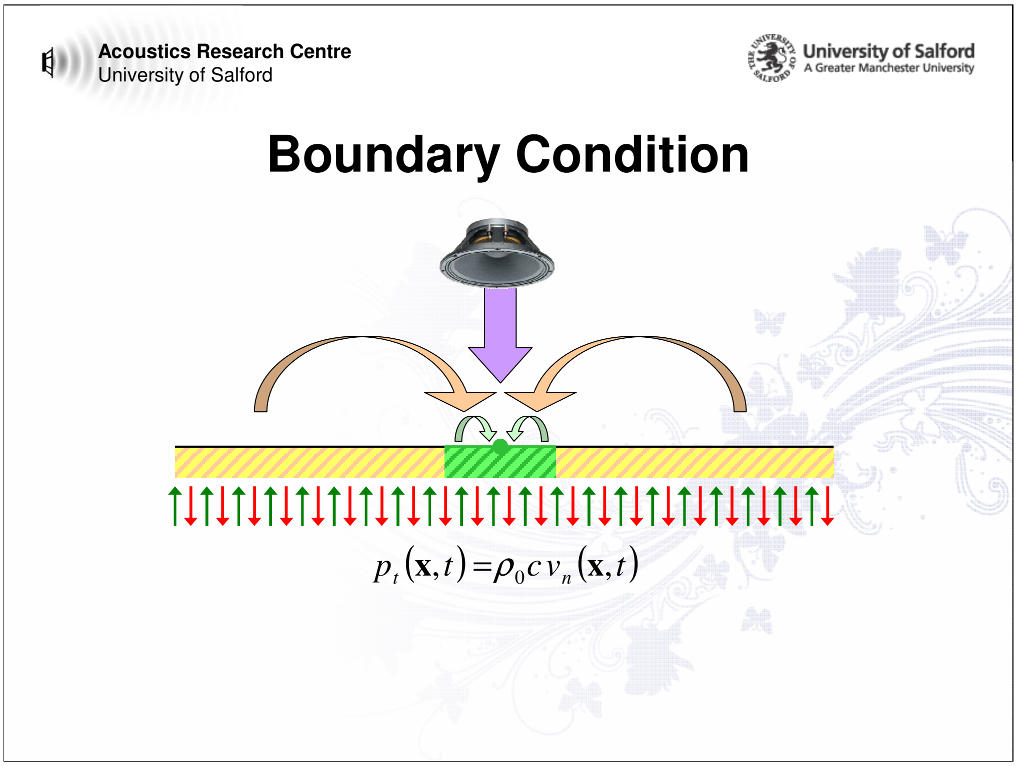
$$\varphi_s(\mathbf{y},t) = \iint_S \left(\varphi_t(\mathbf{x},t) * \hat{\mathbf{n}} \cdot \nabla g(R,t) - g(R,t) * v_n(\mathbf{x},t)\right) d\mathbf{x}$$

$$\varphi_t(\mathbf{y},t) = \varphi_i(\mathbf{y},t) + \varphi_s(\mathbf{y},t)$$

In fact what we really need to know at the mouth of the well is the total velocity potential which is the sum of the scattered sound and…

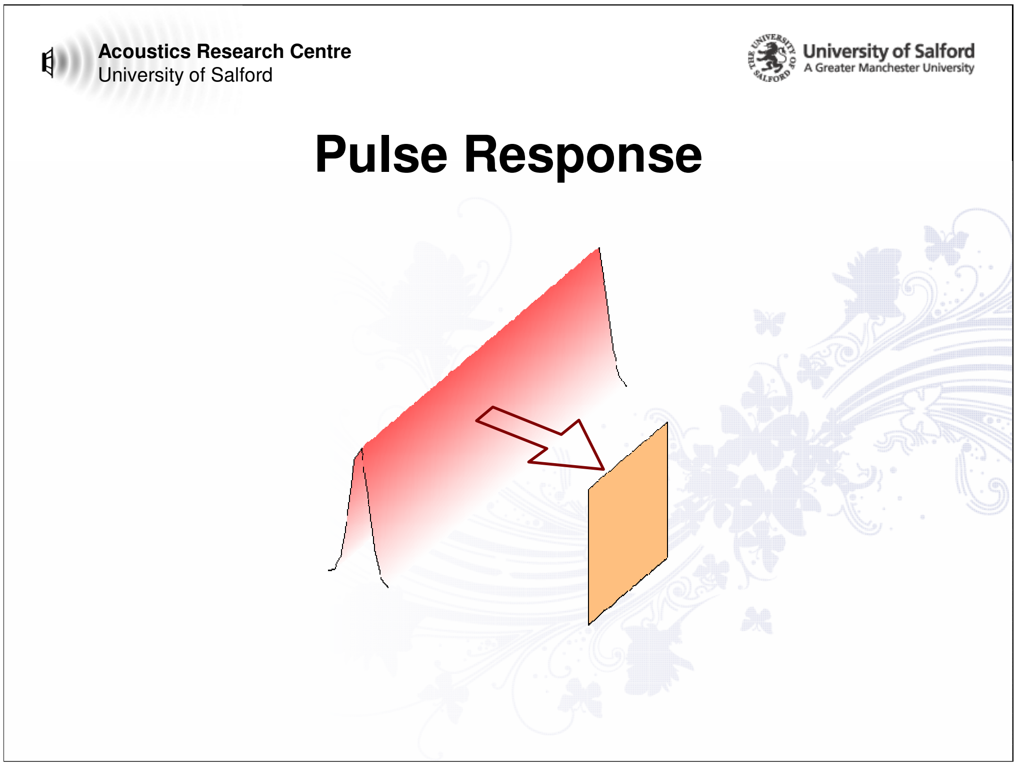


…and the incident sound.

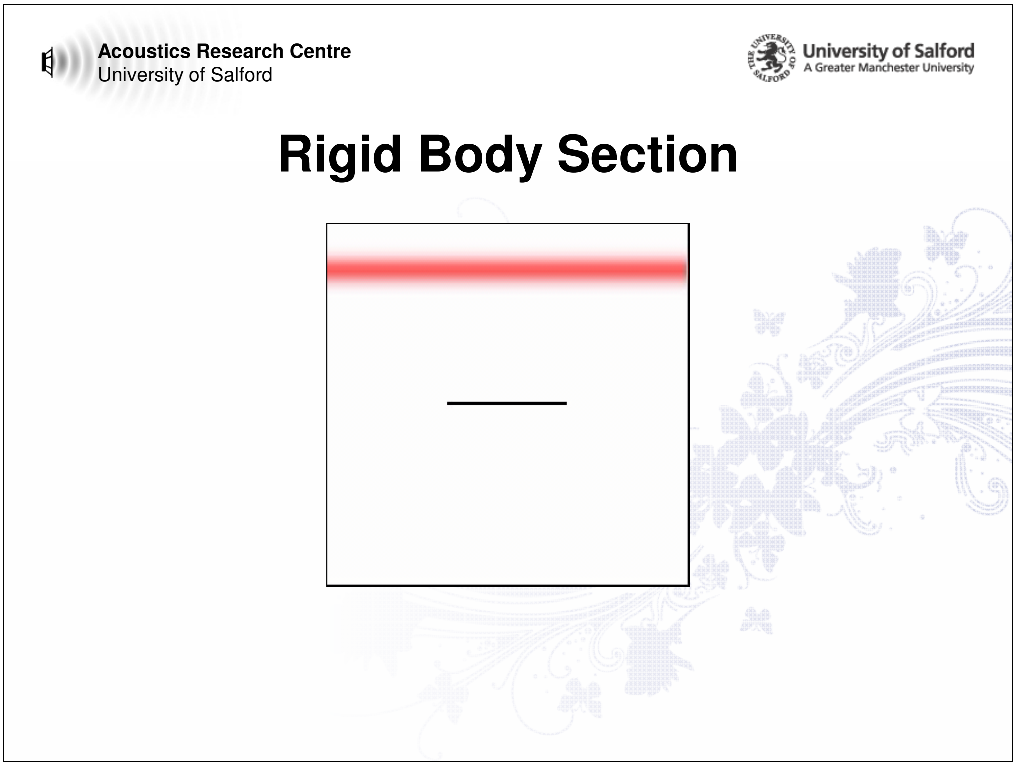


This is because the last piece in the puzzle that lets us solve for surface sound from incident sound, the boundary condition, is written in terms of total sound.

We choose to use the Combined Field Integral Equation as it has been shown to improve stability by suppressing resonances which may occur in the cavity formed by a closed surface. It effectively achieves this by permitting sound from within the cavity (green arrows) to propagate out. It is equivalent to the boundary condition written above.



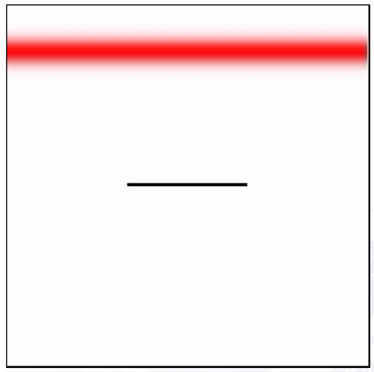
Ok, let's visualise some transient results. We're going to fire a pulse of sound at a plate and watch what happens. First we'll look at a rigid plate, and then a virtual well. Please note: these animations (not the real algorithm!) were generated using the Kirchhoff boundary condition so only show first order diffraction.



Rigid plate:

Notice the reflected wave and the diffraction around the plate of the incident wave. In fact the scattered wave is symmetrical except for a change of sign, so the null behind the plate is caused by cancellation between the incident and scattered waves.

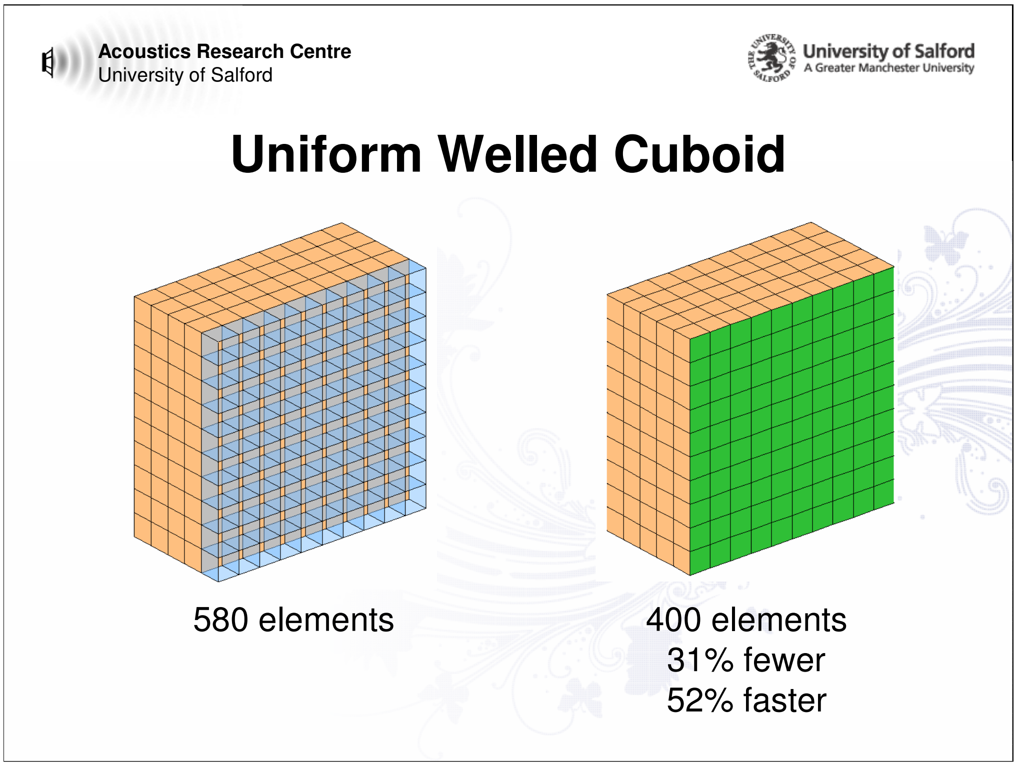
Welled Body Section



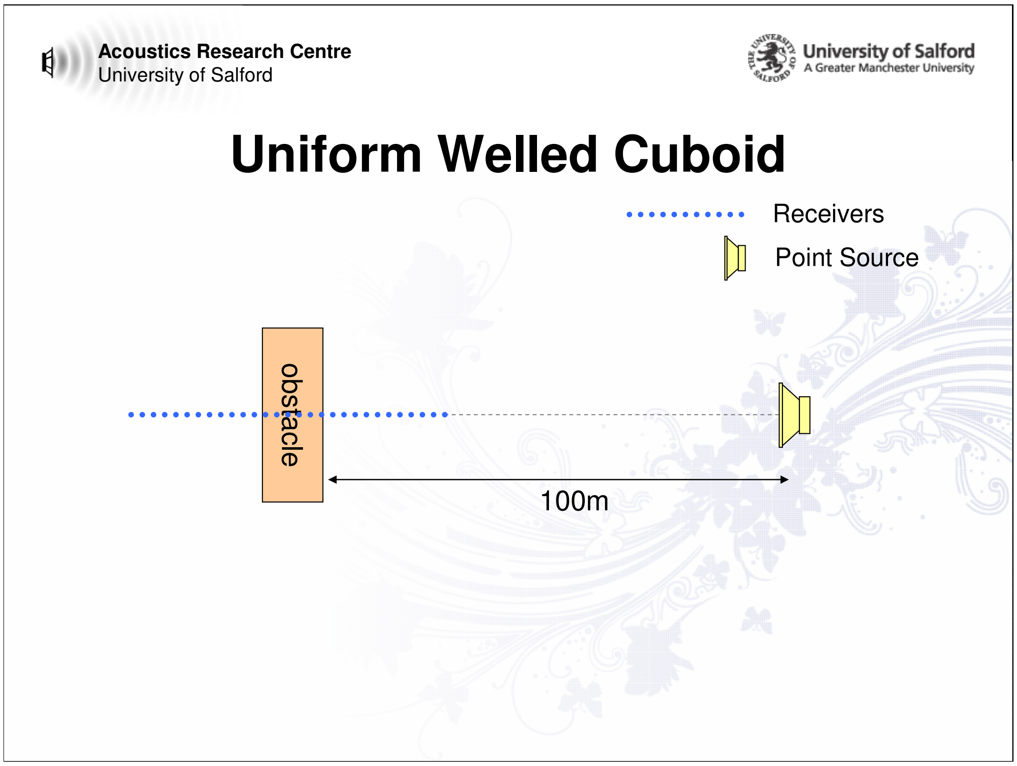
Virtual well:

The cancelling part of the scattered well is still emitted immediately, preventing the total wave from propagating through the obstacle. However the reflected wave is delayed according to the depth of the virtual well.

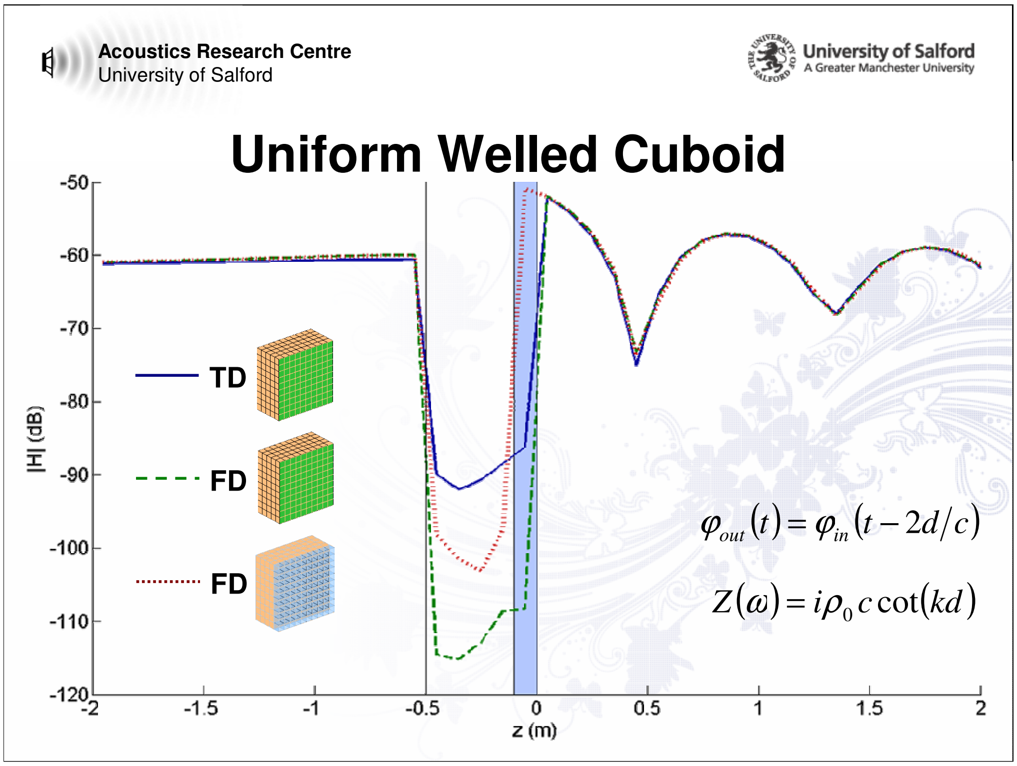
Results



1m sq body, 0.5m deep, 0.1m wells on front face. Equivalent mixed and well models. Because the well model mesh is simpler, it has fewer elements. The BEM must calculate interaction between every element pair, that’s N^2, so that reduction makes the algorithm much faster.



There is a far-field harmonic point source (not transient, but we're going to look for interference patterns) and a line of receivers through the obstacle.



Here we have scattered sound magnitude versus position. We have the time domain model, and two different frequency domain models, all very different implementations of different meshes & boundary conditions. To the right of the figure, the source side of the obstacle, an interference pattern is seen and all models agree very well. Inside the obstacle there is good cancellation.

Error (inc phase) was calculated between the two virtual well models and was generally between 1% and 5% for typical discretisation parameters.

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

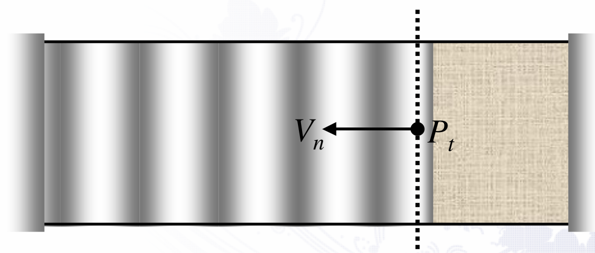
Scope?

What's the future of this compliant surface model?

Acoustics Research Centre
University of Salford

University of Salford
A Greater Manchester University

Impedance Tube

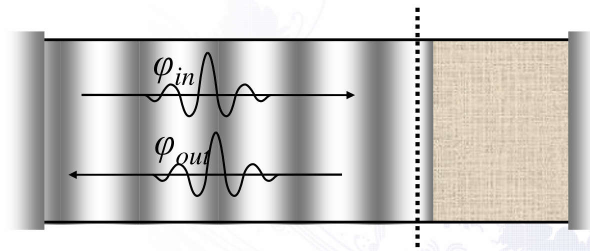


$$\frac{P_t(\mathbf{x},\omega)}{-V_n(\mathbf{x},\omega)} = Z(\omega)$$

$$p_t(\mathbf{x},t) = -v_n(\mathbf{x},t) * z(t)$$

Surface Impedance of materials is usually measured in an impedance tube.

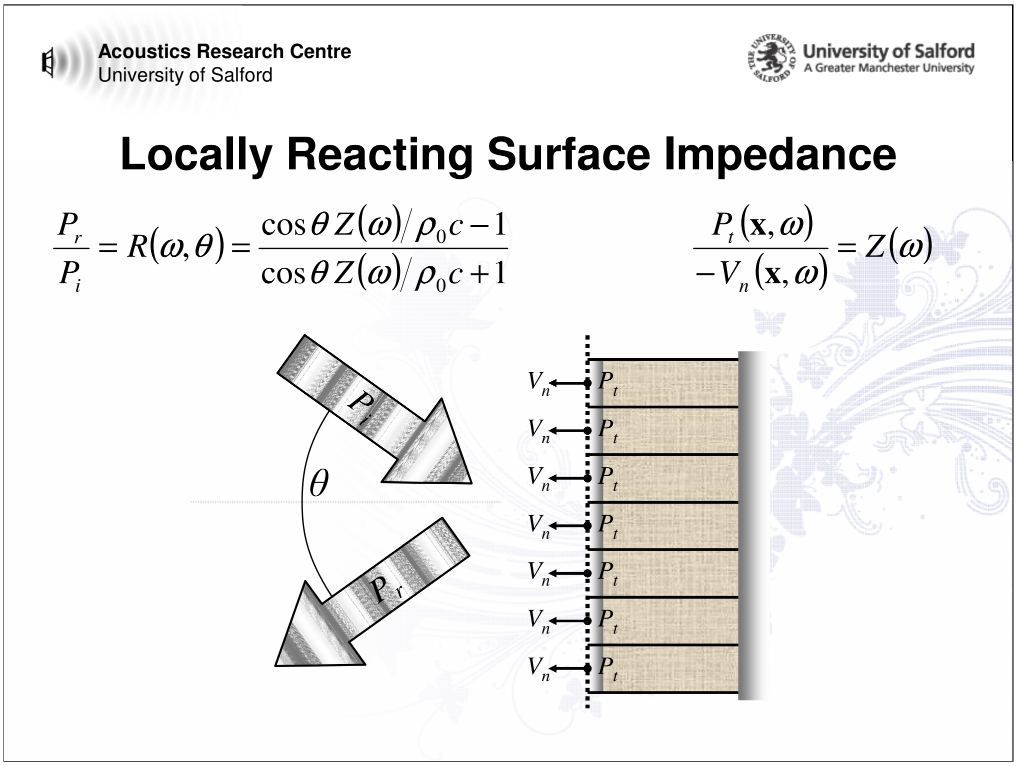
Impedance Tube



$$\varphi_{out}(\mathbf{x},t) = \varphi_{in}(\mathbf{x},t) * w(t)$$

$$\frac{\Phi_{out}(\mathbf{x},\omega)}{\Phi_{in}(\mathbf{x},\omega)} = W(\omega) = \frac{Z(\omega)/\rho_0 c - 1}{Z(\omega)/\rho_0 c + 1}$$

This only supports two directions of wave propagation, so we can use our incoming and outgoing wave model with an unknown surface reflection kernel *w*(*t*), or it's frequency domain counterpart, a surface reflection coefficient *W*(ω)



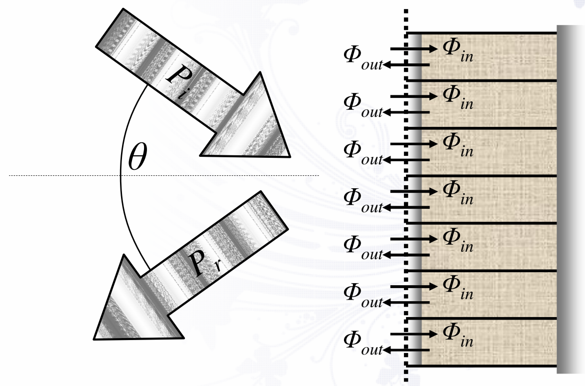
In a frequency domain BEM model this impedance tube data is used, so it's like assuming each element is a little impedance tube.

Acoustics Research Centre
University of Salford

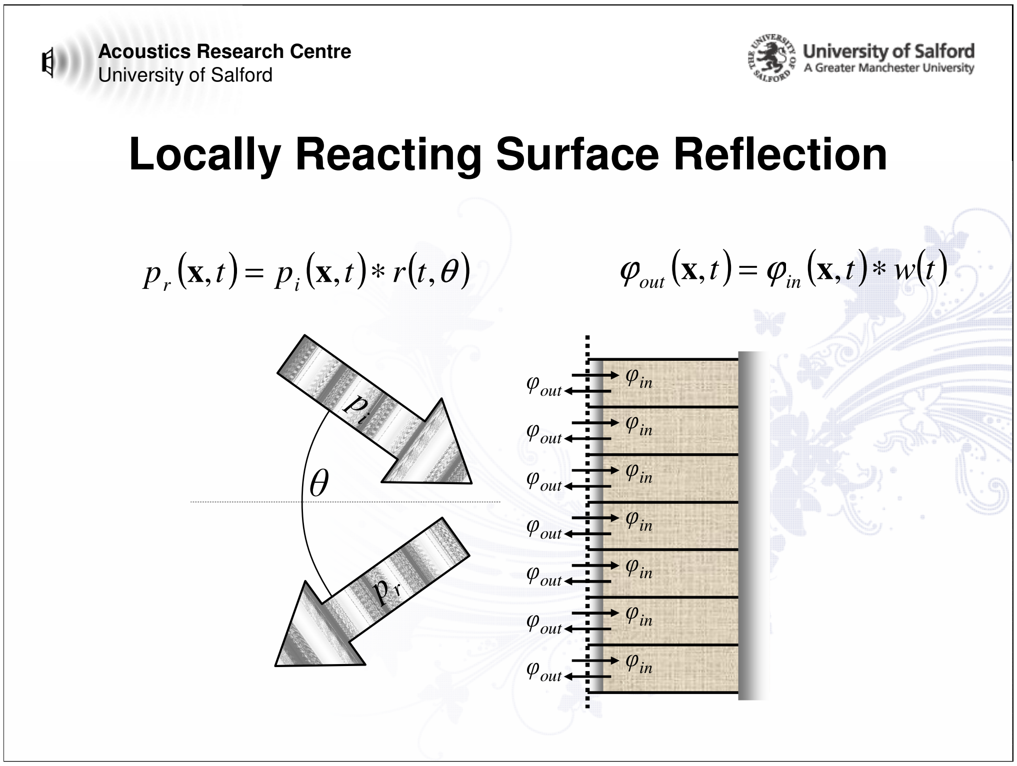
University of Salford
A Greater Manchester University

Locally Reacting Surface Reflection

$$\frac{1+R(\omega,\theta)}{1-R(\omega,\theta)}=\cos\theta\,\frac{1+W(\omega)}{1-W(\omega)} \qquad \frac{\Phi_{out}(\mathbf{x},\omega)}{\Phi_{in}(\mathbf{x},\omega)}=W(\omega)$$



The relationship can be re-written (albeit less concisely) using $W(\omega)$



From which it’s easy to transfer to the time domain.

So in principle we have here a time domain model of locally reaction compliant obstacles which only makes the same assumptions as the generally accepted use of surface impedance.

However, it is clear to see that this is not a realistic model of reality. Further research is required to establish if it is a reasonable approximation for transient scattering, or if a different model need be sought.

Conclusions

- Well mouths can be modelled by a surface impedance
- Equivalent formulation proposed in time domain
  - Based on causal relation between incoming & outgoing waves
- Shown to be accurate and stable
- Could be the basis of a time domain model of locally reacting materials
  - Is this a realistic way of representing the response of a material?

Thank you for your attention

j.a.hargreaves@salford.ac.uk

Acoustics Research Centre
University of Salford

Acoustics Research Centre
University of Salford, UK

Architectural and building acoustics

Digital signal processing

Outdoor sound propagation

Environmental acoustics

Virtual acoustic prototypes

Acoustics of porous materials

6* 2001 RAE

www.acoustics.salford.ac.uk