A STABLE TIME DOMAIN BEM FOR DIFFUSER SCATTERING

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ABSTRACT
Boundary Element Methods (BEMs) may be used to model scattering of sound by surfaces such as diffusers, accelerating prototyping of new Room Acoustics treatments. Unlike the more widely used frequency domain method, the time domain BEM is usually solved in an iterative manner so can exhibit instability, a crucial impediment to its widespread use. These instabilities are primarily associated with resonance of the cavities formed by closed surface sections, but may also be caused by discretisation or integration error corrupting physically relevant damped resonances.

Previous works on time domain BEMs have focused on idealised surfaces, such as spheres and plates, and little is published on their performance for the more complex surfaces treatments typical to Room Acoustics. Consequentially, this paper will apply the method to model the transient behaviour of Schroeder Diffusers and Binary Amplitude Diffusers. Cavity resonances and integration error are addressed through use of the Combined Field Integral Equation and an adaptive contour integration scheme respectively. Thin and absorbing surface sections are implemented as required by the respective diffusers. Accuracy and stability is tested by comparison to verified frequency domain BEMs.

INTRODUCTION
Room acoustic diffusers can be used to treat the acoustics of critical listening environments to improve speech intelligibility and to make music sound better [1]. The development of the modern sound diffuser can be traced back to the pioneering work of Schroeder, who developed the phase grating diffuser [2, 3]. These comprise a series of wells of different depths, determined by a number theoretic sequence and a design frequency, separated by fins. Figure 1 shows a cross section through a Quadratic Residue Diffuser (QRD). Sound waves entering each well emerge following the time taken for them to travel to the bottom of the well, reflect and travel back to the mouth. These delays are optimally decorrelated so the cumulative scattered sound is widely dispersed. Because the wells store sound energy and then reradiate it the scattered sound is diffused in both space and time.
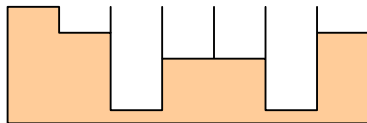

Figure 1: Cross section through a Schroeder diffuser based on the N=7 quadratic residue sequence (0 1 4 2 2 4 1)
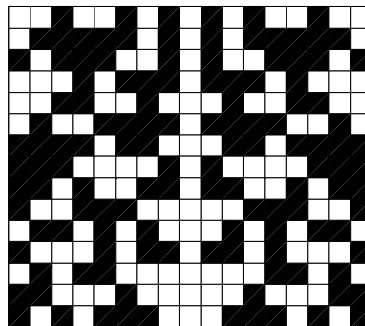

Figure 2: Chinese Remainder Folding of a 255 bit MLS to a 15 by 17 array

An alternate strategy for achieving spatial diffusion is to modulate the reflection amplitude rather than the reflection delay; this is done in the Binary Amplitude Diffuser (BAD) [4]. A unipolar Maximum Length Sequence (MLS) defines a sequence of patches of reflective and absorptive

material, such that the pattern has minimal similarity to a translated version of itself, and this is folded into an array using a process called the Chinese Remainder Theorem [2]. Figure 2 shows a 15 by 17 patch array generated by a 255 bit sequence.

A diffusing surface treatment is characterised by the uniformity of its scattering. This may be measured under anechoic conditions [5], a time consuming and therefore expensive process, particularly for devices that scatter hemispherically (Figure 3). An alternative is to predict this data using a numerical model. The speed and low cost of this approach aids prototyping of new designs, and even allows automated optimisation of treatments to be performed [6]. The Boundary Element Method (BEM) is well suited to this task [7].

In a BEM model only the boundaries between objects and air are modelled as it is known how sound travels unobstructed. This produces smaller, simpler meshes compared to volumetric methods such as Finite Element Modelling. It is ideally suited to free-field scattering situations as, rather than modelling a large expanse of air, one can simply have no outer boundary.

Most BEMs assume time invariant harmonic excitation so the unknowns are time invariant complex numbers. Whilst this frequency domain analyses is a useful tool, the transient behaviour witnessed in the real world may only be recovered by calculation of many frequency domain models and inverse discrete Fourier transform. An alternative is to drop the time invariant assumption and formulate the BEM in the time domain. This algorithm was first published by Friedman and Shaw in 1962 [8] and is presented herein.
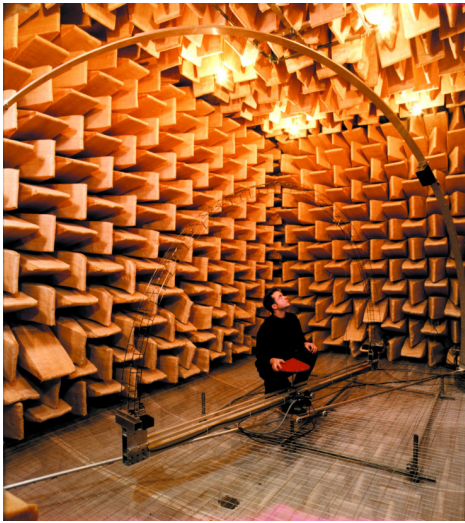

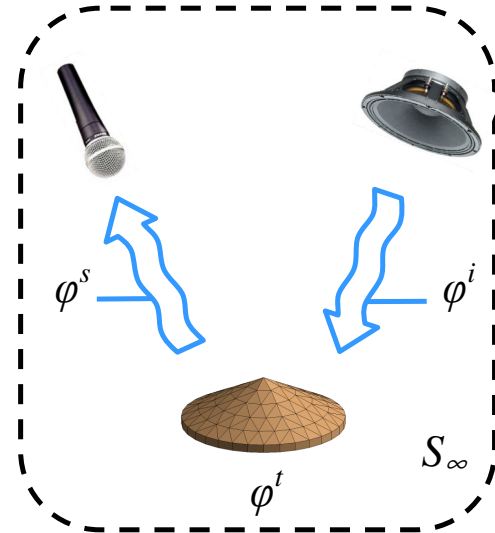Figure 3: Measuring hemispherical scattering from a diffuser

Figure 4: A BEM model equivalent to Figure 3. Incident and scattered sounds are indicated.

THE TIME DOMAIN BEM FOR ACOUSTICS

The Kirchhoff Integral Equation
A BEM to model scattering from an object has three distinct phases: first the sound incident on the object is calculated, then the total sound at the surface of the object is solved for by considering the mutual interactions of parts of the surface S, and finally the scattered sound is calculated from this total surface sound. This is depicted in Figure 4. The scattered sound arising as a consequence of total sound on a surface is described by the Kirchhoff Integral Equation (KIE) (Equation 1); this is the foundation of the time domain BEM:

$$\varphi^s(\mathbf{r},t) = \iint_S \left(\varphi^t(\mathbf{r}',t) * \hat{\mathbf{n}}' \cdot \nabla' g(R,t) - g(R,t) * v_n(\mathbf{r}',t) \right) d\mathbf{r}' \qquad \text{(Eq. 1)}$$

$$p(\mathbf{r},t) = -\rho \dot{\varphi}(\mathbf{r},t) \qquad \text{(Eq. 2)} \qquad\qquad \mathbf{v}(\mathbf{r},t) = \nabla \varphi(\mathbf{r},t) \qquad \text{(Eq. 3)}$$

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r and **r**' are the observation and radiation points respectively and $R = |\mathbf{r} - \mathbf{r}'|$ is the distance between them. φ represents velocity potential, a non-physical quantity from which pressure and velocity may be derived according to Equations 2 and 3, where ρ and c are the density of and speed of sound in air respectively. A dot above a quantity represents temporal differentiation and temporal convolution is represented by $*$. φ^s is the scattered sound and φ^t is total sound. $\hat{\mathbf{n}}'$ is the surface normal vector at **r**' and v_n is the component of velocity in its direction and will be referred to as 'normal velocity'. $g(R,t)$ is the time domain Greens function which describes how sound travels from a point source to an observer. It intuitively comprises a delay term as a numerator and a reduction in magnitude with distance as the denominator:

$$g(R,t) = \frac{\delta\left(t - R/c\right)}{4\pi R} \quad \text{(Eq. 4)}$$

As φ^s is equal to φ^t minus the incident sound φ^i Equation 1 may be rearranged to solve for φ^t from φ^i. The boundary surface S need not be connected (multiple scatterers may exist) but it must be piecewise smooth enough that a unique normal vector $\hat{\mathbf{n}}$ may be defined everywhere on it, perpendicular to S and directed into the enclosed connected volume of air Ω_+. The volume behind S is named Ω_-. For mathematical correctness the air must be fully enclosed by a surface; in free-field models the outer surface is denoted S_∞ and is chosen to be infinitely far away such that sound it scatters never arrives. The scenario is depicted in Figure 5:
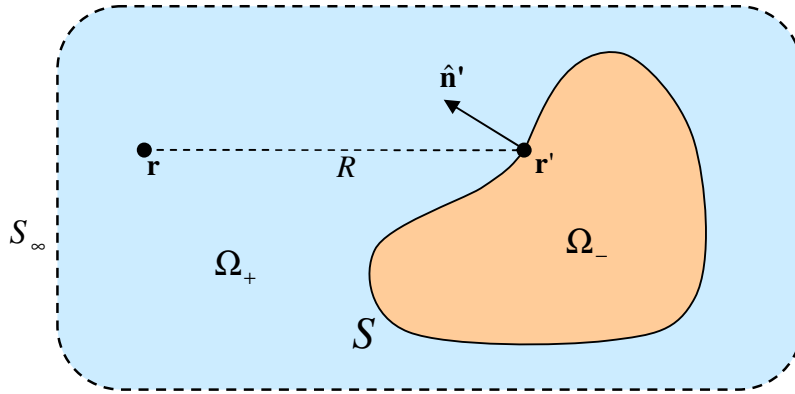

Figure 5: In a free-field model the air must still be enclosed by surfaces; an outer boundary is imagined that so distant that its effects never arrive.

Discretisation and the Marching On in Time solver

In order to solve for the surface quantities numerically a discrete representation is required. The discretisation scheme used herein follows Ergin *et al* [9] as a weighted sum of basis functions. The boundary is partitioned into elements over which sound is considered constant within an instant and interpolated by a piecewise cubic polynomial in time. Spatial resolution is defined by element size and temporal resolution by the time-step duration Δ_t.

To create a BEM the discretisation weights are moved outside the integral of the KIE, creating a weighted sum of integrals that are dependent only on the surface geometry and independent of system excitation. Upon evaluation these integrals become interaction coefficients that express scattered sound from the discretisation weights. To ensure accurate and efficient evaluation the surface integral over each element is replaced by a contour integral around its edge using a coordinate transformation similar to that used by Kawai and Terai [10] and Ha Duong *et al* [11]

The discretisation weights are found by numerical solution of the matrix equation that results from combination of the KIE with boundary conditions that described the nature of the surface. Causality dictates that past surface sound cannot be changed and future sound is irrelevant, hence at each time-step $t_j = j\Delta_t$ the algorithm is only solving for the current unknown weights. This is described by Equation 5 comprising matrices of excitation independent interaction coefficients $\mathbf{Z}_l$, vectors of discretisation weights $\mathbf{w}_i$ and excitation vectors $\mathbf{e}_j$. This is commonly referred to as the Marching On in Time (MOT) solver.

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$$\mathbf{Z}_0 \mathbf{w}_j = \mathbf{e}_j - \sum_{l>0}^{l \le l_{max}} \mathbf{Z}_l \mathbf{w}_{j-l} \qquad \text{(Eq 5)}$$

Stability and Cavity Resonances

Because the MOT solver is iterative there exists the possibility that it will diverge from the true solution and become unstable, a crucial impediment to the widespread application of the algorithm. Rynne [12] observed that similar instabilities affect all models regardless of the application or discretisation, implying that this behaviour is fundamental to the method, and correlated their properties with the resonances of the cavity Ω_-. Such are possible because the restriction that air is present only in Ω_+ is lost in the process of conversion from volume differential equation to boundary integral equation. Cavity resonances are unitary poles of the MOT solver and error introduced in the discretisation process may corrupt a pole so that its response grows exponentially. Ergin *et al's* [9] solution was to use the Combined Field Integral Equation (CFIE) boundary condition which allows sound energy to dissipate from the cavity, damping its resonances and reducing the magnitude of the corresponding poles.

Boundary Conditions

Boundary conditions place restrictions of the values of φ^t and v_n so that Equation 1 may be solved. If the body is rigid (Neumann problem) then the total normal velocity must be zero so the surface normal component of incident and scattered velocity must cancel (Equation 6). An absorbing boundary condition permits an inward propagating plane wave perpendicular to the surface (Equation 7); a similar form has been used by Groenenboom [13] and Ha Duong *et al* [11]. The CFIE is equivalent to a radiating boundary condition, permitting sound energy to leave but not enter Ω_- (Equation 8).

$$\hat{\mathbf{n}} \cdot \mathbf{v}^t = 0 \quad \Rightarrow \quad \hat{\mathbf{n}} \cdot \mathbf{v}^i = -\hat{\mathbf{n}} \cdot \mathbf{v}^s \qquad \text{(Eq. 6)}$$

$$c\,\hat{\mathbf{n}} \cdot \mathbf{v}^t = \dot{\varphi}^t \quad \Rightarrow \quad c\,\hat{\mathbf{n}} \cdot \mathbf{v}^i - \dot{\varphi}^i = \dot{\varphi}^s - c\,\hat{\mathbf{n}} \cdot \mathbf{v}^s \qquad \text{(Eq. 7)}$$

$$c\,\hat{\mathbf{n}} \cdot \mathbf{v}^t = -\dot{\varphi}^t \quad \Rightarrow \quad c\,\hat{\mathbf{n}} \cdot \mathbf{v}^i + \dot{\varphi}^i = -\dot{\varphi}^s - c\,\hat{\mathbf{n}} \cdot \mathbf{v}^s \qquad \text{(Eq. 8)}$$

Thin and Mixed Surfaces

Real thin bodies, such as the fins of a Schroeder diffuser, have some finite thickness. However attempting to model these with two surfaces, each conformal to a body-air interface, results in a phenomena known as Thin Shape Breakdown (TSB) [14]. TSB can be avoided by taking the limit as thickness approaches zero and approximating the two body-air interfaces by a single open surface; an air-air interface. Pressure is unknown on both sides of the surface so only the rigid boundary condition of Equation 6 may be used. φ^t in Equation 1 is replaced by the jump in velocity potential across the surface.

The first time domain BEM to use this formulation was Kawai & Terai's [10] implementation for modelling thin plates. However there remained the question of what is best when it is desired to model a plate near a solid body, or a solid body with a protruding fin. Ergin *et al* [9] found that Equation 6 supports cavity resonances within closed bodies so promoted use of the CFIE (Equation 8). Wu [15] addressed the same concerns in the frequency domain and implemented a BEM with different boundary conditions on open and closed surface sections; the same may be done in the time domain using Equations 6 and 8 respectively.

THE DIFFUSER MODELS

The meshes

Both meshes are of a single period of the corresponding diffuser outlined in the introduction. The QRD mesh comprises 900 elements and has a design frequency of approximately 245Hz, a well width of 0.25m, and a height of 1.0m. The BAD mesh comprises 702 elements is 0.15m by 0.17m by 0.03m; it's elements are much smaller than those of the QRD hence the frequencies modelled are proportionally higher. The meshes are depicted below where thin elements are shown in translucent blue and absorbing elements are white. The CFIE boundary condition is used on thick rigid elements to suppress cavity resonances and aid stability.
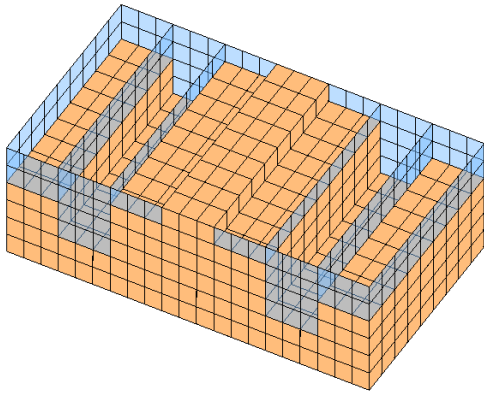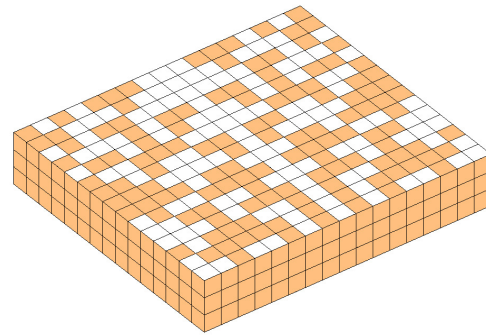
Figure 6: QRD mesh.

Figure 7: BAD mesh.

Accuracy compared to a frequency domain BEM

The time domain BEM models were compared to equivalent frequency domain BEM models using implementations that have previously been verified against experimental results. Excitation was a harmonic point source and the time domain results were discrete Fourier transformed so the source pressure to surface pressure transfer functions could be calculated for all models. The mean magnitude of the complex difference between these results was calculated and normalised to the mean magnitude of the frequency domain result. Agreement is typically better than 3% for most meshes within the frequency range corresponding to suggested temporal and spatial resolution. However, some instability was experienced modelling the QRD mesh resulting in poorer agreement. This is thought to be due to corruption of the lightly damped system poles that correspond to the wells of the QRD.

Figure 8 shows the magnitude of the transfer function from the source to sound scattered from the BAD to an arc of receivers of 5m radius. The wiggle in the frequency domain line is due to truncation error in the process of extracting scattered pressure. Grating lobes are evident and agreement is good between the implementations. The source was located normal to the surface and angles are relative to that.
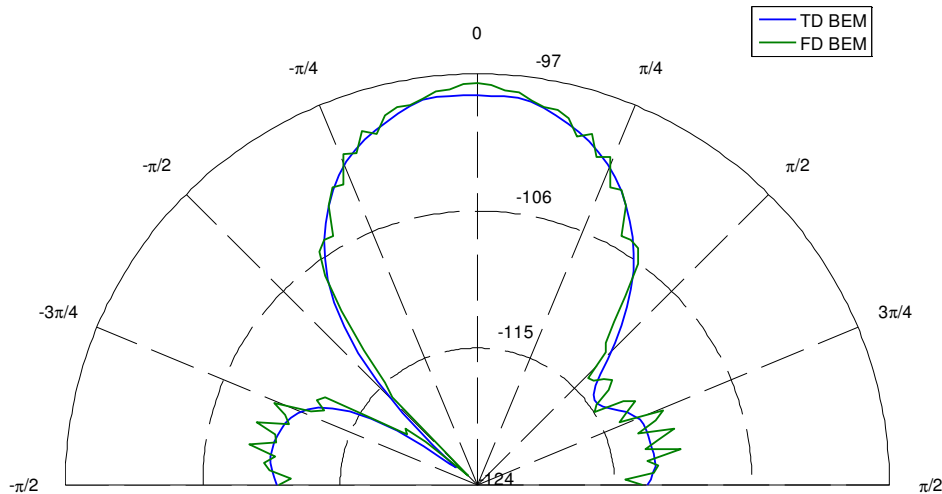

Figure 8: Scattered sound 5m from the BAD at 2695Hz

Transient Scattering

The above harmonic excitation verification examples are clearly inefficient applications of the time domain BEM, its advantages occur when modelling transient behaviour. Figure 9 shows some preliminary results of transient scattering by the above surfaces to the 5m arc of receivers used to generate Figure 8. Magnitude of velocity potential is plotted in dB (normalised to its maximum value) versus time and receiver angle. The results have not yet been verified but it

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can clearly be seen that the scattering from the QRD is much more temporally diffuse than that from the BAD. Farina [16] has produced similar figures from measured data. These results hope to inspire some thoughts on the applications of the time domain BEM as a predictive tool.
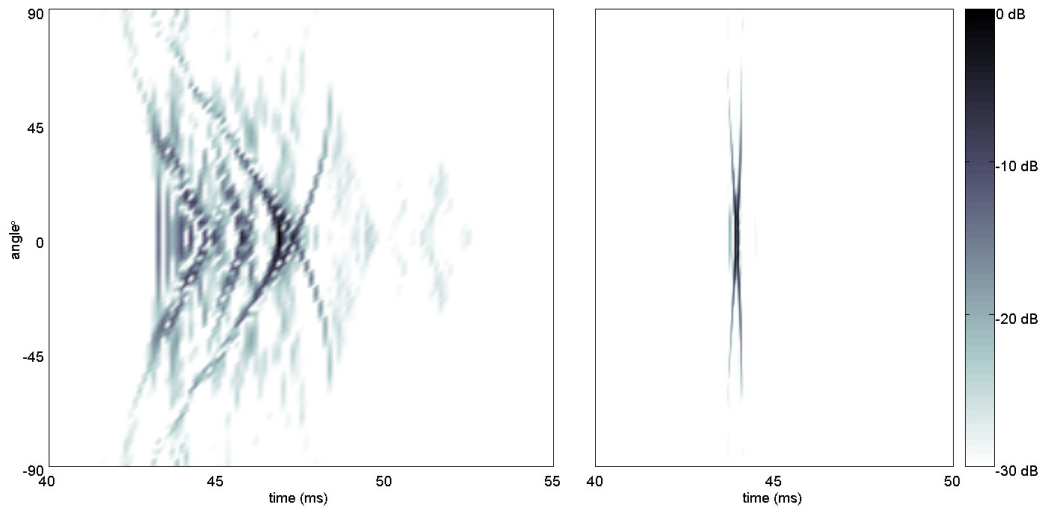

Figure 9: Transient scattering by the QRD (left) and BAD (right)
in response to an impulsive source

CONCLUSIONS

The time domain BEM algorithm has been described. Stability has been highlighted as a critical issue and its causes and treatment discussed. Absorbing and mixed surface boundary conditions have been used to model a Quadratic Residue Diffuser and a Binary Amplitude Diffuser. Agreement versus a verified frequency domain been is good although some instability persists for devices with lightly damped resonances. A preliminary glimpse into the transient results possible with the time domain BEM has been given.

References

[1] P D'Antonio and T J Cox: "Diffusor Application in Rooms". Appl. Acoust. **60** (2): 113 – 142 (2000).

[2] M. R. Schroeder: "Diffuse sound reflection by maximum-length sequences" J. Acoust. Soc. Am. **57** (1): 149 – 150 (1975)

[3] M. R. Schroeder: "Binaural dissimilarity and optimum ceilings for concert halls: more lateral sound diffusion", J. Acoust. Soc. Am. **65** (4): 958-963 (1979).

[4] J. A. S. Angus and P. D'Antonio: "Two dimensional Binary Amplitude Diffusers", Proc. Audio. Eng. Soc. Preprint 5061 (D-5) (1999)

[5] AES information document for room acoustics and sound reinforcement systems – Characterisation and measurement of surface scattering uniformity. Audio Engineering Society (2001)

[6] T. J. Cox and P. D'Antonio: Acoustic Absorbers and Diffusers, Spon Press (2004)

[7] T. J. Cox, Y. W. Lam: "Prediction and Evaluation of the Scattering from Quadratic Residue Diffusers", J. Acoust. Soc. Am. **95** (1) 297-305 (1994)

[8] M. B. Friedman and R. P. Shaw: "Diffraction of Pulses by Cylindrical Obstacles of Arbitrary Cross Section", J. Appl. Mech. **29**: 40 – 46 (1962)

[9] A. A. Ergin, B. Shanker and E. Michielssen: "Analysis of transient wave scattering from rigid bodies using a Burton-Miller approach", J. Acoust. Soc. Am. **106** (5): 2396 – 2404 (1999)

[10] Y. Kawai and T. Terai: "A Numerical Method for the Calculation of Transient Acoustic Scattering from Thin Rigid Plates", J. Sound Vib. **141** (1): 83 – 96 (1990)

[11] T. Ha Duong, B. Ludwig and I.Terrasse: "A Galerkin BEM for transient acoustic scattering by an absorbing obstacle", Int. J. Numer. Meth. Engng. **57**: 1845 – 1882 (2003)

[12] B. P. Rynne: "Instabilities in Time Marching Methods for Scattering Problems", Electromagnetics **6**: 129 – 144 (1986)

[13] P. H. L. Gronenboom: "The applications of boundary elements to steady and unsteady potential fluid flow problems in two and three dimensions", Appl. Math. Modelling **6**: 35 – 40 (1982)

[14] R. Martinez: "The thin-shape breakdown (TSB) of the Helmholtz integral equation", J. Acoust. Soc. Am. **90** (5): 2728 – 2738 (1991)

[15] T. W. Wu: "A direct boundary element method for acoustic radiation and scattering from mixed regular and thin bodies", J. Acoust. Soc. Am. **97** (1): 84 – 91 (1995)

[16] A. Farina: "A new method for measuring the scattering coefficient and the diffusion coefficient of panels" Acustica **86** (6): 928 – 942 (2000)

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