Helmholtz Solitons in Non-Kerr Media

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1. Introduction

Spatial optical soliton propagation in a planar waveguide is often modelled by a Non-Linear Schrödinger (NLS) type equation. Such systems are paraxial in nature, and cannot describe accurately off-axis propagation at *non-trivial angles*. The NLS equation typically allows for a Kerr non-linearity, but *real optical materials* often possess a response that deviates from this idealization [1]. We report, for the *first* time, exact analytical soliton solutions to a generalized Non-Linear Helmholtz (gNLH) equation. Solutions, and accompanying simulations, thus account for *both* the inherent symmetries of propagation problems *and* more realistic (polynomial-type) non-linearities.

2. Generalized Non-Linear Helmholtz Equation

Models based on the NLH equation are suitable for describing accurately the angular aspects of wave propagation [2]. Since the assumption of beam paraxiality is omitted, such descriptions can support both travelling- and standing-wave solutions. We will present distinct novel families of solutions of the gNLH equation. These will include: *sech-shaped* and *algebraic* (e.g. Lorentzian) *solitons*, and also *spatially-extended* (transverse periodic) *nonlinear waves*.

3. Stability as Robust Attractors

Thorough numerical investigations will demonstrate the role of gNLH solitons as attractors of the system. With few exceptions, we have found that dynamics exhibit limit-cycle qualities (perturbed initial conditions give rise to self-sustained oscillations in the long term). Departure from pure Kerr non-linearity will also be shown to result in much stricter conditions on how closely a launched beam needs to match the corresponding exact soliton solution.

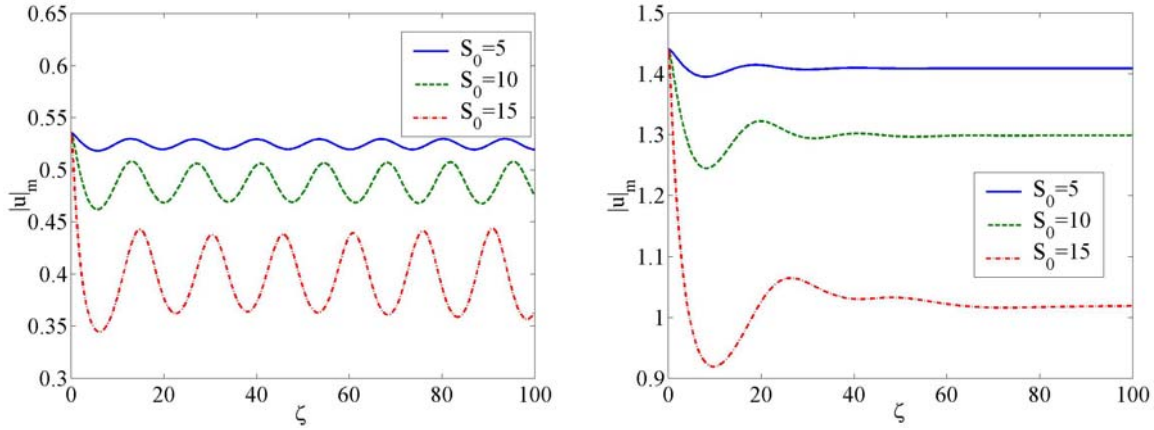

FIG. 1. Reshaping simulations for (left) *sech*-shaped solitons in the pure-focusing regime, and (right) algebraic solitons close to threshold. Initial conditions, corresponding to exact solutions of the corresponding paraxial system [1], do not necessarily give rise to (propagation-invariant) asymptotic Helmholtz solitons.

4. References

[1] R.W. Micallef, V.V. Afanasjev, Y.S. Kivshar and J.D. Love, "Optical Solitons with Power-Law Asymptotics," *Phys. Rev. E* **54**, 2936 (1996).

[2] P. Chamorro-Posada, G.S. McDonald and G.H.C. New, "Exact soliton solutions of the nonlinear Helmholtz equation: communication," *J. Opt. Soc. Am. B* **19**, 1216 (2002).