Helmholtz-Manakov Solitons

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1. Introduction

The propagation of a spatial vector soliton beam in a Kerr planar waveguide is typically described by the Manakov equation [1]. However, the assumption of beam paraxiality breaks the rotational symmetry of the wave propagation problem. Manakov-based descriptions are, for example, incapable of describing physical effects associated with off-axis propagation at non-trivial angles. We will report the *first* Helmholtz generalizations of the Manakov equation *and* its soliton solutions, along with a thorough investigation of the dynamical properties of the new solutions.

2. Helmholtz-Manakov Solitons

We introduce the Helmholtz-Manakov (H-M) equation as a vector generalization of the scalar Non-Linear Helmholtz (NLH) equation [2], whereby the guided electric field has two transverse orthogonal components. Exact analytical soliton solutions of the H-M equation will be derived for *both* focusing and defocusing media; the classic Manakov solitons are a subset of these new results. H-M solitons are found to exhibit non-trivial features that are absent from the paraxial-based descriptions (these new features will be shown to influence propagation characteristics).

3. Stability as Robust Attractors

Well-tested numerical perturbative techniques will be employed to demonstrate the role of H-M solitons as robust attractors (in a non-linear dynamical sense). Rich dynamical behaviour will be summarised, including evolution characteristics associated with both fixed-point (see Fig. 1) and limit-cycle attractors.
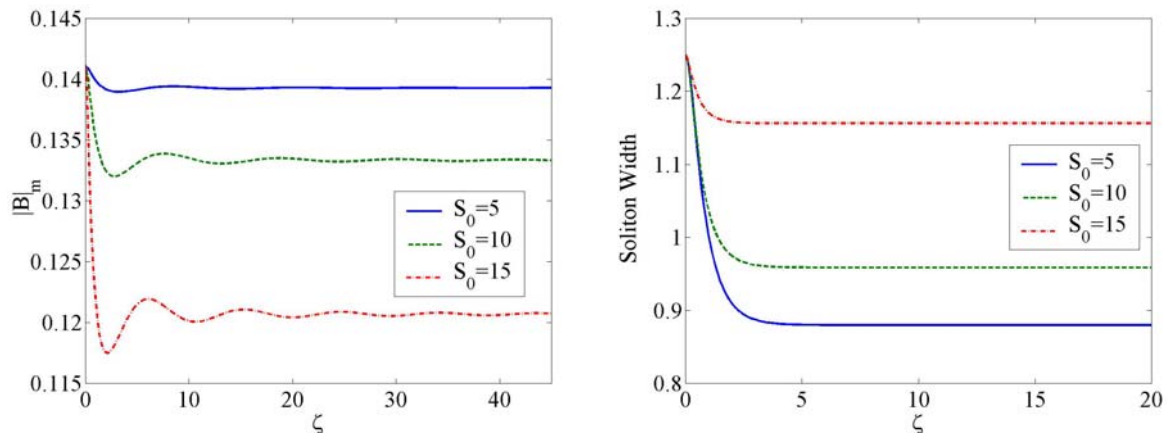

Fig. 1. Reshaping simulations using exact Manakov (paraxial) solitons as initial conditions for the H-M equation. Simulations correspond to the reshaping of (left) dark-bright, and (right) dark-dark solitons in a defocusing medium - both are classed as *stable fixed points*.

4. References

[1] S.V. Manakov, "On the theory of two-dimensional stationary self-focusing of electromagnetic waves," *Sov. Phys. JETP* **38**, 248 (1974).
[2] P. Chamorro-Posada, G.S. McDonald and G.H.C. New, "Exact soliton solutions of the nonlinear Helmholtz equation: communication," *J. Opt. Soc. Am. B* **19**, 1216 (2002).